Aplicación del MEF para la 2da ecuación

Contexto

La empresa de videojuegos desea lanzar el juego de supervivencia más realista que se haya visto jamás y ha prometido tanto los gráficos más reales nunca vistos, también ha prometido las mecánicas más parecidas a la realidad posibles, de modo que se han puesto una gran presión encima y saben que si no logran lo que prometieron los usuarios los cancelaran en cualquier red social existente y por existir, para ello los desarrolladores deben evitar las clásicas cajas de hit box y diseñaron una manera de poder calcular la cantidad de daño que el jugador reciba o ejerza sobre otro jugador u objeto y como no solo se verá afectada le zona donde reciba el impacto sino el resto de partes del cuerpo o del objeto recibirán un porcentaje de daño.

A esta ecuación le llamaron *"índice de daño recibido o IDR"* la cual es: $-\epsilon^2\nabla\cdot(\eta^2\nabla X)=\eta^2+\eta$

Modelo

$$-\epsilon^2\nabla\cdot(\eta^2\nabla X)=\eta^2+\eta$$

Paso 1: Localización

$$N_1 = 1-\epsilon-\eta-\phi$$
$$N_2 = \epsilon$$
$$N_3 = \eta$$
$$N_4 = \phi$$

Paso 2: Interpolación

$$X \approx N_1X_1 + N_2X_2 + N_3X_3 + N_4X_4$$

$$= \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \mathbf{N}\mathbf{X} \qquad X \approx \mathbf{N}\mathbf{X}$$

Paso 3: Aproximación del modelo

$$-\epsilon^2\nabla\cdot(\eta^2\nabla X)=\eta^2+\eta$$
$$-\epsilon^2\nabla\cdot(\eta^2\nabla(\mathbf{N}\mathbf{X}))=\eta^2+\eta$$
$$R=\eta^2+\eta+\epsilon^2\nabla\cdot(\eta^2\nabla(\mathbf{N}\mathbf{X}))$$

Paso 4: Método de los Residuos Ponderados

$$R = \eta^2 + \eta + \epsilon^2 \nabla \cdot (\eta^2 \nabla (N x))$$

$$\int_V W R \, \partial V = 0$$

$$\int_V W (\eta^2 + \eta + \epsilon^2 \nabla (\eta^2 \nabla (N x))) \partial V = 0$$

Paso 5: Método de Galerkin

$$W = N^T$$

$$\int_V N^T (\eta^2 + \eta + \epsilon^2 \nabla \cdot (\eta^2 \nabla (N X))) \partial V = 0$$

Interludio y "Formateo" de los términos

$$\int_V (N^T \eta^2 + N^T \eta + N^T \epsilon^2 \nabla \cdot (\eta^2 \nabla \cdot (N X))) \, \partial V = 0$$

$$\int_V N^T \eta^2 \partial V + \int_V N^T \eta \, \partial V + \int_V N^T \epsilon^2 \nabla (\eta^2 \nabla (N X)) \, \partial V = 0$$

$$\int_V N^T \eta^2 \partial V + \int_V N^T \eta \, \partial V + \left(\int_V N^T \epsilon^2 \nabla \cdot (\eta^2 \nabla N) \, \partial V \right) X = 0$$

$$-\left(\int_V N^T \epsilon^2 \nabla \cdot (\eta^2 \nabla N) \partial V \right) X = \int_V N^T \eta^2 \partial V + \int_V N^T \eta \, \partial V$$

Paso 6: Resolución de integrales

Lado derecho

$$\int_V N^T \eta^2 \, dV + \int_V N^T \eta \, dV$$

$$\int_V \begin{bmatrix} 1-\epsilon-\eta-\phi \\ \epsilon \\ \eta \\ \phi \end{bmatrix} \eta^2 \, dV + \int_V \begin{bmatrix} 1-\epsilon-\eta-\phi \\ \epsilon \\ \eta \\ \phi \end{bmatrix} \eta \, dV$$

$$\int_V \begin{bmatrix} \eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2 \\ \epsilon\eta^2 \\ \eta^3 \\ \phi\eta^2 \end{bmatrix} dV + \int_V \begin{bmatrix} \eta-\epsilon\eta-\eta^2-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} dV \qquad dV = dx\,dy\,dz$$

$$\int_V \begin{bmatrix} \eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2 \\ \epsilon\eta^2 \\ \eta^3 \\ \phi\eta^2 \end{bmatrix} dx\,dy\,dz + \int_V \begin{bmatrix} \eta-\epsilon\eta-\eta^2-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} dx\,dy\,dz$$

$$dx\,dy\,dz = J\,d\epsilon\,d\eta\,d\phi$$

$$\int_V \begin{bmatrix} \eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2 \\ \epsilon\eta^2 \\ \eta^2 \\ \phi\eta^2 \end{bmatrix} J\,d\epsilon\,d\eta\,d\phi + \int_V \begin{bmatrix} \eta-\epsilon\eta-\eta^2-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} J\,d\epsilon\,d\eta\,d\phi$$

$$J\left(\int_V \begin{bmatrix} \eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2 \\ \epsilon\eta^2 \\ \eta^2 \\ \phi\eta^2 \end{bmatrix} d\epsilon\,d\eta\,d\phi + \int_V \begin{bmatrix} \eta-\epsilon\eta-\eta^2-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon\,d\eta\,d\phi \right)$$

Resolviendo la primer integral

$$\int_V \begin{bmatrix} \eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2 \\ \epsilon\eta^2 \\ \eta^3 \\ \phi\eta^2 \end{bmatrix} d\epsilon\, d\eta\, d\phi$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi} \begin{bmatrix} \eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2 \\ \epsilon\eta^2 \\ \eta^3 \\ \phi\eta^2 \end{bmatrix} d\epsilon\, d\eta\, d\phi$$

$$\begin{bmatrix} \int_0^1\int_0^{1-\phi}\int^{1-\eta-\phi}(\eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2)\, d\epsilon\, d\eta\, d\phi \\ \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi} \epsilon\eta^2\, d\epsilon\, d\eta\, d\phi \\ \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi} \eta^3\, d\epsilon\, d\eta\, d\phi \\ \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi} \phi\eta^2\, d\epsilon\, d\eta\, d\phi \end{bmatrix}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi} \epsilon\eta^2\, d\epsilon\, d\eta\, d\phi$$

$$\int_0^{1-\eta-\phi} \epsilon\eta^2\, d\epsilon \Rightarrow \eta^2\int_0^{1-\eta-\phi} \epsilon\, d\epsilon$$

$$= \eta^2\left(\frac{1-2\eta-2\phi+\eta^2+2\eta\phi+\phi^2}{2}\right) = \frac{\eta^2-2\eta^3-2\eta^2\phi+\eta^4+2\eta^3\phi+\eta^2\phi^2}{2}$$

$$\int^{1-\phi} \frac{\eta^2-2\eta^3-2\eta^2\phi+\eta^4+2\eta^3\phi+\eta^2\phi^2}{2}\, d\eta$$

$$= \frac{2-7\phi+8\phi^2-2\phi^3-2\phi^4+\phi^5-3(1-\phi)^4}{12} + \frac{(1-\phi)^5}{10}$$

$$\int_0^1 \frac{2-7\phi+8\phi^2-2\phi^3-2\phi^4+\phi^5-3(1-\phi)^4}{12}+\frac{(1-\phi)^5}{10}\,d\phi$$

$$=\frac{1}{360}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^3\,d\epsilon\,d\eta\,d\phi$$

$$\int_0^{1-\eta-\phi}\eta^3\,d\epsilon \Rightarrow \eta^3\int_0^{1-\eta-\phi}d\epsilon=\eta^3(1-\eta-\phi)=\eta^3-\eta^4-\eta^3\phi$$

$$\int_0^{1-\phi}\eta^3-\eta^4-\eta^3\phi\,d\eta$$

$$=\frac{(1-\phi)^4-(1-\phi)^4\phi}{4}-\frac{(1-\phi)^5}{5}$$

$$\int_0^1\frac{(1-\phi)^4-(1-\phi)^4\phi}{4}-\frac{(1-\phi)^5}{5}\,d\phi$$

$$=\frac{1}{120}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\phi\eta^2\,d\epsilon\,d\eta\,d\phi \xrightarrow[\text{la integral}]{\text{Al reordenar}} \underbrace{\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\phi\eta^2\,d\phi\,d\eta\,d\epsilon}_{\text{Tiene la misma forma que la primera integral}}$$

$$=\frac{1}{360}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}(\eta^2-\epsilon\eta^2-\eta^3-\phi\eta^2)\,d\epsilon\,d\eta\,d\phi$$

$$=\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^2\,d\epsilon\,d\eta\,d\phi-\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\epsilon\eta^2\,d\epsilon\,d\eta\,d\phi$$

$$-\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^3\,d\epsilon\,d\eta\,d\phi-\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^2\phi\,d\epsilon\,d\eta\,d\phi$$

$$=\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^2\,d\epsilon\,d\eta\,d\phi-\frac{1}{360}-\frac{1}{120}-\frac{1}{360}$$

$$\int_0^{1-\eta-\phi} \eta^2 \, d\theta \Rightarrow \eta^2 \int_0^{1-\eta-\phi} d\theta = \eta^2(1-\eta-\phi)$$

$$= \eta^2 - \eta^3 - \eta^2\phi$$

$$\int_0^{1-\phi} \eta^2 - \eta^3 - \eta^2\phi \, d\eta$$

$$= \frac{\phi^4 - 4\phi^3 + 6\phi^2 - 4\phi + 1}{3} - \frac{(1-\phi)^4}{4}$$

$$\int_0^1 \frac{\phi^4 - 4\phi^3 + 6\phi^2 - 4\phi + 1}{3} - \frac{(1-\phi)^4}{4} \, d\phi$$

$$= \frac{1}{60}$$

$$= \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^2 \, d\theta \, d\eta \, d\phi - \frac{1}{360} - \frac{1}{120} - \frac{1}{360}$$

$$= \frac{1}{60} - \frac{1}{360} - \frac{1}{120} - \frac{1}{360}$$

$$= \frac{1}{360}$$

$$\begin{bmatrix} \frac{1}{360} \\ \frac{1}{360} \\ \frac{1}{120} \\ \frac{1}{360} \end{bmatrix}$$

Resolviendo la segunda integral

$$\int_V \begin{bmatrix} \eta-\epsilon\eta-\eta^2-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon\, d\eta\, d\phi$$

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \begin{bmatrix} \eta-\epsilon\eta-\eta^2-\phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta \end{bmatrix} d\epsilon\, d\eta\, d\phi$$

$$\begin{bmatrix} \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} (\eta-\epsilon\eta-\eta^2-\phi\eta)\, d\epsilon\, d\eta\, d\phi \\ \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta\, d\epsilon\, d\eta\, d\phi \\ \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta^2\, d\epsilon\, d\eta\, d\phi \\ \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \phi\eta\, d\epsilon\, d\eta\, d\phi \end{bmatrix}$$

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon\eta\, d\epsilon\, d\eta\, d\phi$$

$$\int_0^{1-\eta-\phi} \epsilon\eta\, d\epsilon \Rightarrow \eta \int_0^{1-\eta-\phi} \epsilon\, d\epsilon$$

$$= \eta\left(\frac{1-2\eta-2\phi+\eta^2+2\eta\phi+\phi^2}{2}\right) = \frac{\eta-2\eta^2-2\eta\phi+\eta^3+2\eta^2\phi+\eta\phi^2}{2}$$

$$\int_0^{1-\phi} \frac{\eta-2\eta^2-2\eta\phi+\eta^3+2\eta^2\phi+\eta\phi^2}{2}\, d\eta$$

$$= \frac{-\phi^4+4\phi^3-6\phi^2+4\phi-1}{12} + \frac{(1-\phi)^4}{8}$$

$$\int_0^1 \frac{-\phi^4+4\phi^3-6\phi^2+4\phi-1}{12} + \frac{(1-\phi)^4}{8}\, d\phi$$

$$= \frac{1}{120}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^2\,d\epsilon\,d\eta\,d\phi$$ Esta integral se resolvio anteriormente

$$= \frac{1}{60}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\phi\eta\,d\epsilon\,d\eta\,d\phi \underset{\text{al reordenar la integral}}{\Longrightarrow} \underbrace{\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\phi\eta\,d\phi\,d\eta\,d\epsilon}_{\text{Tiene la misma forma de la primera integral}}$$

$$= \frac{1}{120}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}(\eta-\epsilon\eta-\eta^2-\phi\eta)\,d\epsilon\,d\eta\,d\phi$$

$$= \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta\,d\epsilon\,d\eta\,d\phi - \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\epsilon\eta\,d\epsilon\,d\eta\,d\phi$$

$$- \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta^2\,d\epsilon\,d\eta\,d\phi - \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\phi\eta\,d\epsilon\,d\eta\,d\phi$$

$$= \int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta\,d\epsilon\,d\eta\,d\phi - \frac{1}{120} - \frac{1}{60} - \frac{1}{120}$$

$$\int_0^1\int_0^{1-\phi}\int_0^{1-\eta-\phi}\eta\,d\epsilon\,d\eta\,d\phi$$

$$\int_0^{1-\eta-\phi}\eta\,d\epsilon \Rightarrow \eta\int_0^{1-\eta-\phi}d\epsilon = \eta(1-\eta-\phi)$$

$$= \eta-\eta^2-\phi\eta$$

$$\int_0^{1-\phi}\eta-\eta^2-\phi\eta\,d\eta$$

$$= \frac{-\phi^3+3\phi^2-3\phi+1}{6}$$

$$\int_0^1\frac{-\phi^3+3\phi^2-3\phi+1}{6}\,d\phi$$

$$= \frac{1}{24}$$

$$= \int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \eta \, d\epsilon \, d\eta \, d\phi - \frac{1}{120} - \frac{1}{60} - \frac{1}{120}$$

$$= \frac{1}{24} - \frac{1}{120} - \frac{1}{60} - \frac{1}{120}$$

$$= \frac{1}{120}$$

$$\begin{bmatrix} \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{60} \\ \frac{1}{120} \end{bmatrix}$$

$$J \left(\int_V \begin{bmatrix} \eta^2 - \epsilon\eta^2 - \eta^3 - \phi\eta^2 \\ \epsilon\eta^2 \\ \eta^2 \\ \phi\eta^2 \end{bmatrix} d\epsilon \, d\eta \, d\phi + \int_V \begin{bmatrix} \eta - \epsilon\eta - \eta^2 - \phi\eta \\ \epsilon\eta \\ \eta^2 \\ \phi\eta^2 \end{bmatrix} d\epsilon \, d\eta \, d\phi \right)$$

$$J \left(\begin{bmatrix} \frac{1}{360} \\ \frac{1}{360} \\ \frac{1}{120} \\ \frac{1}{360} \end{bmatrix} + \begin{bmatrix} \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{60} \\ \frac{1}{120} \end{bmatrix} \right) = J \left(\begin{bmatrix} \frac{1}{90} \\ \frac{1}{90} \\ \frac{1}{40} \\ \frac{1}{90} \end{bmatrix} \right) = b$$

Lado izquierdo

$$-\int_V N^T \epsilon^2 \nabla\cdot(\eta^2 \nabla N)\,dV$$

$$\int_V u\,dv = [uv]|_V - \int_V du\,v$$

$$u = N^T \epsilon^2 \qquad dv = \nabla(\eta^2 \nabla N)$$

$$du = \nabla N^T \epsilon^2 \qquad v = \eta^2 \nabla N$$

$$-\left([N^T \epsilon^2 \eta^2 \nabla N]|_V - \int_V \nabla N^T \epsilon^2 \eta^2 \nabla N\,dV\right)$$

$$-[N^T \epsilon^2 \eta^2 \nabla N]|_V + \underline{\int_V \nabla N^T \epsilon^2 \eta^2 \nabla N\,dV}$$

$$\int_V \nabla N^T \epsilon^2 \eta^2 \nabla N\,dV$$

$$\nabla N = \begin{bmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{bmatrix} N = \begin{bmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{bmatrix} \begin{bmatrix} 1-\epsilon-\eta-\phi & \epsilon & \eta & \phi \end{bmatrix}$$

$$\begin{bmatrix} \frac{\delta}{\delta \epsilon} \\ \frac{\delta}{\delta \eta} \\ \frac{\delta}{\delta \phi} \end{bmatrix} = J \begin{bmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\delta}{\delta \epsilon} \\ \frac{\delta}{\delta \eta} \\ \frac{\delta}{\delta \phi} \end{bmatrix}$$

$$\nabla N = J^{-1} \begin{bmatrix} \frac{\delta}{\delta \epsilon} \\ \frac{\delta}{\delta \eta} \\ \frac{\delta}{\delta \phi} \end{bmatrix} \begin{bmatrix} 1-\epsilon-\eta-\phi & \epsilon & \eta & \phi \end{bmatrix}$$

$$= J^{-1} \begin{bmatrix} \frac{\delta}{\delta \epsilon}(1-\epsilon-\eta-\phi) & \frac{\delta}{\delta \epsilon}\epsilon & \frac{\delta}{\delta \epsilon}\eta & \frac{\delta}{\delta \epsilon}\phi \\ \frac{\delta}{\delta \eta}(1-\epsilon-\eta-\phi) & \frac{\delta}{\delta \eta}\epsilon & \frac{\delta}{\delta \eta}\eta & \frac{\delta}{\delta \eta}\phi \\ \frac{\delta}{\delta \phi}(1-\epsilon-\eta-\phi) & \frac{\delta}{\delta \phi}\epsilon & \frac{\delta}{\delta \phi}\eta & \frac{\delta}{\delta \phi}\phi \end{bmatrix}$$

$$= J^{-1} \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = J^{-1} B$$

$$J^{-1} = \frac{1}{J} A$$

$$\nabla N = \frac{1}{J} A B$$

$$\nabla N^T = \left(\frac{1}{J}AB\right)^T \Rightarrow \frac{1}{J}(AB)^T$$

$$\nabla N^T = \frac{1}{J}B^T A^T$$

$$\int_V \nabla N^T \epsilon^2 \eta^2 \nabla N \, dV$$

$$= \int_V \frac{1}{J} B^T A^T \epsilon^2 \eta^2 \frac{1}{J} AB \, dV$$

$$= \frac{1}{J^2} B^T A^T AB \int_V \epsilon^2 \eta^2 \, dV$$

$$= \frac{1}{J^2} B^T A^T AB \int_V \epsilon^2 \eta^2 \, dx\, dy\, dz = \frac{1}{J^2} B^T A^T AB \int_V \epsilon^2 \eta^2 J \, d\epsilon\, d\eta\, d\phi$$

$$= \frac{1}{J^2} B^T A^T AB J \int_V \epsilon^2 \eta^2 \, d\epsilon\, d\eta\, d\phi = \frac{1}{J} B^T A^T AB \int_V \epsilon^2 \eta^2 \, d\epsilon\, d\eta\, d\phi$$

$$\int_V \epsilon^2 \eta^2 \, d\epsilon\, d\eta\, d\phi$$

$$\int_0^1 \int_0^{1-\phi} \int_0^{1-\eta-\phi} \epsilon^2 \eta^2 \, d\epsilon\, d\eta\, d\phi$$

$$\int_0^{1-\eta-\phi} \epsilon^2 \eta^2 \, d\epsilon$$

$$= \frac{\eta^2 - \eta^5 - \eta^2\phi^3 - 3\eta^3 - 3\eta^2\phi + 3\eta^4 - 3\eta^4\phi + 3\eta^2\phi^2 - 3\eta^3\phi^2 + 6\eta^3\phi}{3}$$

$$\int_0^{1-\phi} \frac{\eta^2 - \eta^5 - \eta^2\phi^3 - 3\eta^3 - 3\eta^2\phi + 3\eta^4 - 3\eta^4\phi + 3\eta^2\phi^2 - 3\eta^3\phi^2 + 6\eta^3\phi}{3} \, d\eta$$

$$= \frac{(\phi - 1)^6}{180}$$

$$\int_0^1 \frac{(\phi - 1)^6}{180} \, d\phi$$

$$= \frac{1}{1260}$$

$$= \frac{1}{J} B^T A^T A B \int_V \epsilon^2 \eta^2 \, d\epsilon \, d\eta \, d\sigma$$

$$= \frac{1}{J} B^T A^T A B \left(\frac{1}{1260}\right)$$

$$= \frac{1}{1260 J} B^T A^T A B$$

$$-\left[N^T \epsilon^2 \eta^2 \nabla N\right]\Big|_V + \left(\frac{1}{1260 J} B^T A^T A B\right) X = J \begin{bmatrix} \frac{1}{90} \\ \frac{1}{90} \\ \frac{1}{40} \\ \frac{1}{90} \end{bmatrix}$$

$$\left(\frac{1}{1260 J} B^T A^T A B\right) X = J \begin{bmatrix} \frac{1}{90} \\ \frac{1}{90} \\ \frac{1}{40} \\ \frac{1}{90} \end{bmatrix}$$

$$KX = b$$