Module 04: Graph Algorithms

Analysis and Design of Algorithms

Ammar Sherif

Nile University

What are Graphs?

Graphs

From a mathematical perspective, consist of a nodes/vertices and edges.

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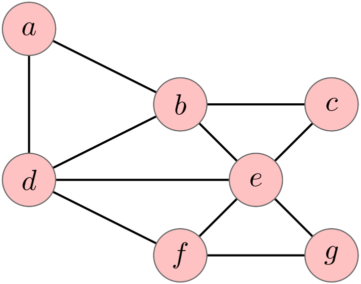
- Nodes/Vertices



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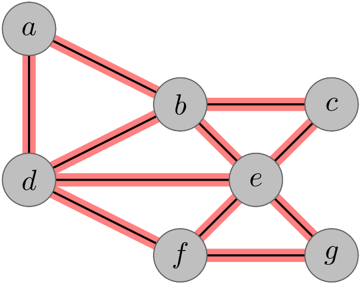
- Edges



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- Nodes/Vertices
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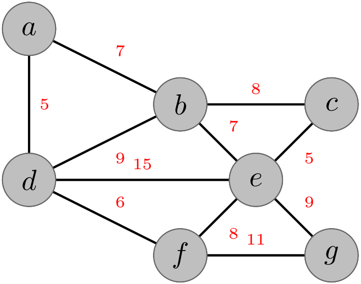

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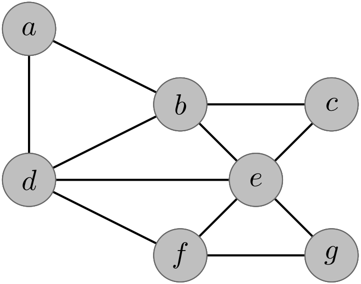

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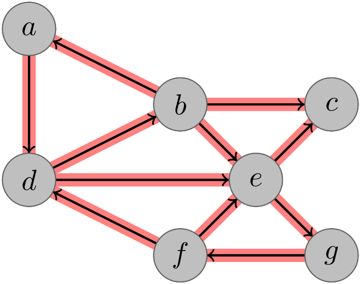

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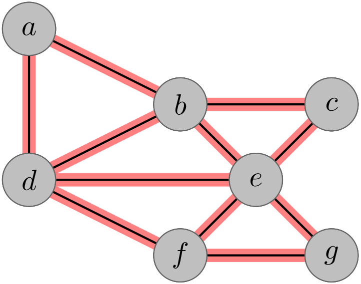

Why Graphs & Graph Algorithms?

- Shortest Path and Route planning

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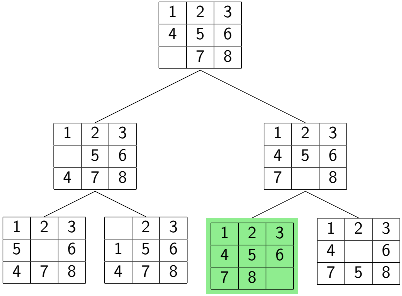

Why Graphs & Graph Algorithms?

- Shortest Path and Route planning
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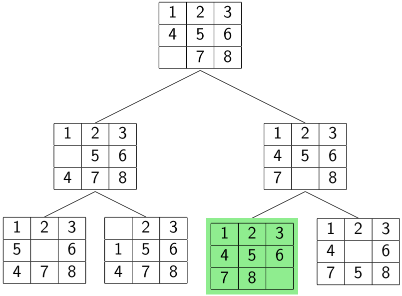
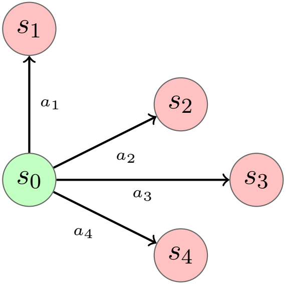
- Games
- Optimization Problems



Why Graphs & Graph Algorithms?

- Shortest Path and Route planning
- Robotics
  - Warehouses
  - Space Robots
  - Rescue Robots
- Games
- Optimization Problems
- Any decision-based problem



Graph Representation

Algorithms are implemented via programming, so how to represent graphs?

What our representation should provide?

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- Weights of links

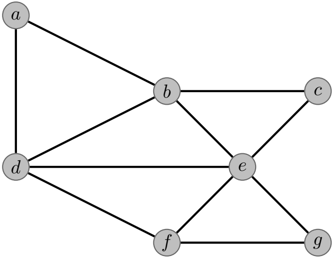
Graph Representation

Algorithms are implemented via programming, so how to represent graphs?

What our representation should provide?

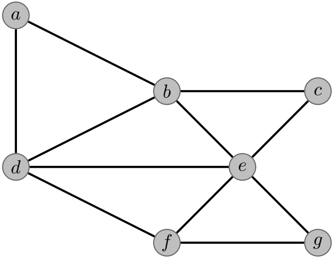
- Know the neighbors
- Weights of links
- list of nodes/edges

Graph Representation



Graph Representation

- Adjacency Matrix



	a	b	c	d	e	f	g
a	–	1		1			
b	1	–	1	1	1		
c		1	–		1		
d	1	1		–	1	1	
e		1	1	1	–	1	1
f				1	1	–	1
g					1	1	–

Graph Representation

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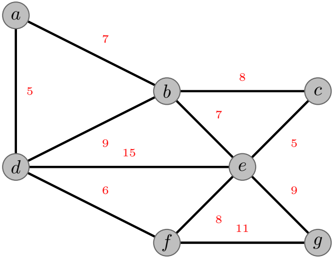
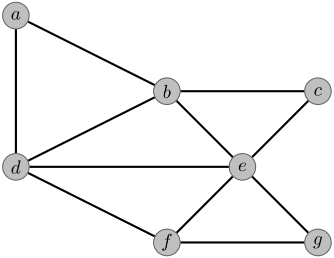


	a	b	c	d	e	f	g
a	–	7		5			
b	7	–	8	9	7		
c		8	–		5		
d	5	9		–	15	8	
e		7	5	15	–	8	9
f				8	8	–	11
g					9	11	–

Graph Representation

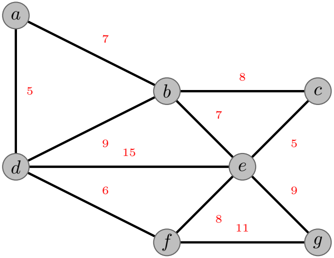


- Adjacency Matrix
- Adjacency List

a	b	c	d	e	f	g
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
b	a	b	a	b	d	e
d	c	e	b	c	e	f
	d		e	d	g	
	e		f	f		
				g		

Graph Representation

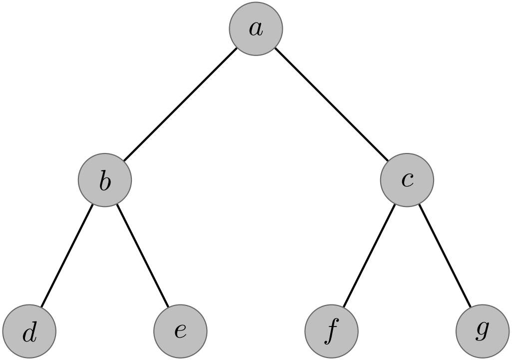
- Adjacency Matrix
- Adjacency List



a	b	c	d	e	f	g
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$b:7$	$a:7$	$b:8$	$a:5$	$b:7$	$d:8$	$e:9$
$d:5$	$c:8$	$e:5$	$b:9$	$c:5$	$e:8$	$f:11$
	$d:9$		$e:15$	$d:15$	$g:11$	
	$e:7$		$f:8$	$f:8$		
				$g:9$		

Depth First Search (DFS)

Depth has the max priority



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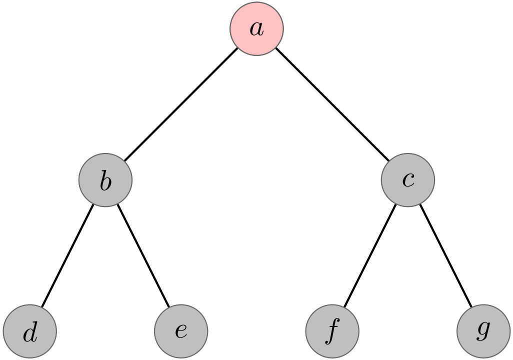

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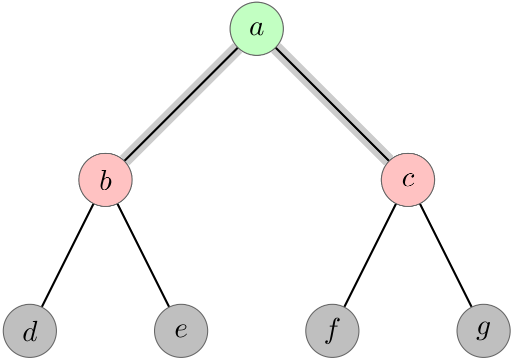
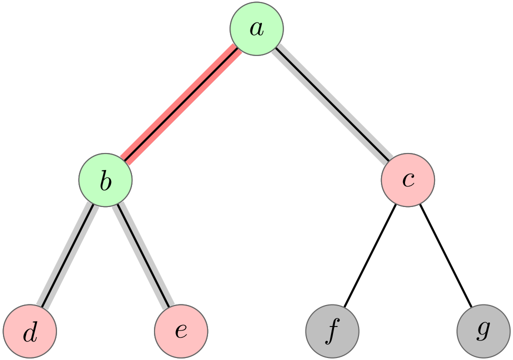


Depth First Search (DFS)

Depth has the max priority

Child $\longleftarrow$ Parent

$b \longleftarrow a$

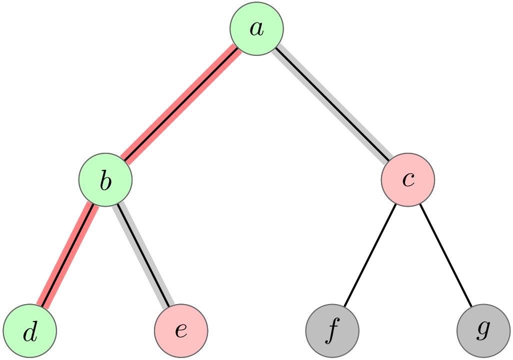


Depth First Search (DFS)

Depth has the max priority

Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$



Depth First Search (DFS)

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Child $\longleftarrow$ Parent

$b \longleftarrow a$
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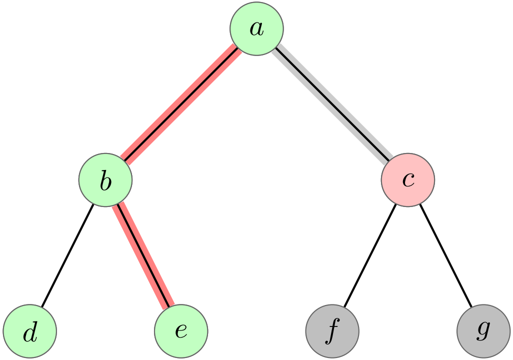

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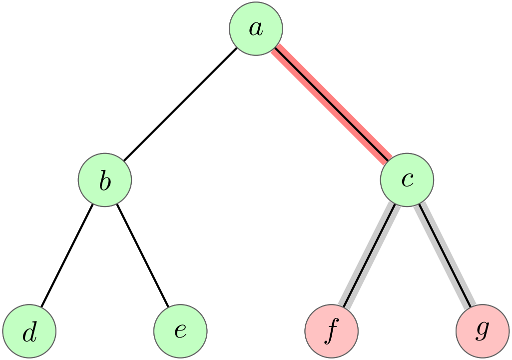

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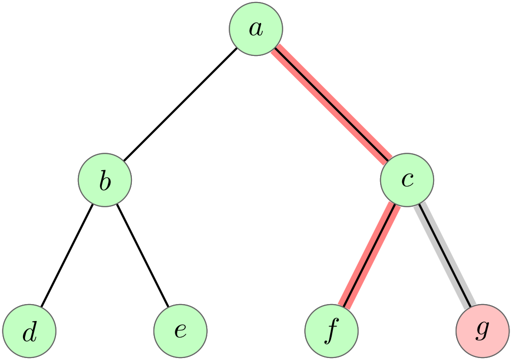
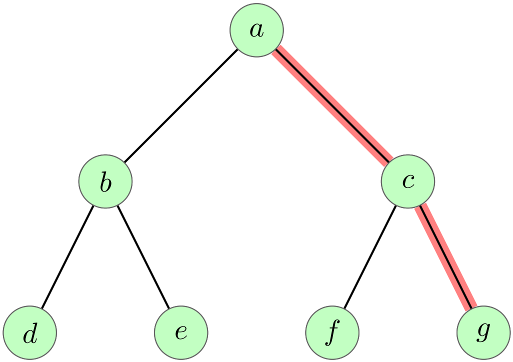


Depth First Search (DFS)

Therefore, the below table summarizes how did we get to any node through our traversal

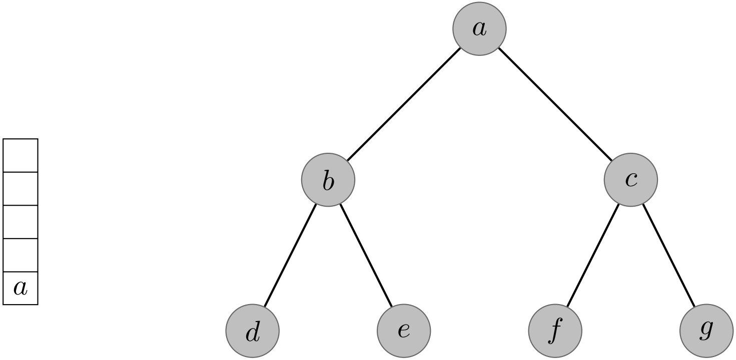
Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$
$e \longleftarrow b$
$c \longleftarrow a$
$f \longleftarrow c$
$g \longleftarrow c$



Depth First Search (DFS)

Let's do it again, to notice the **pattern** of nodes to be visited



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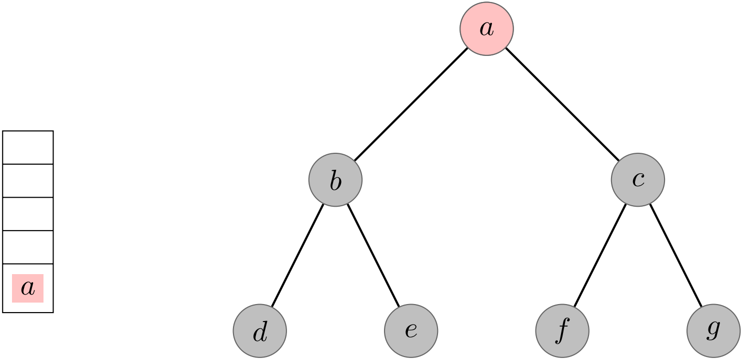

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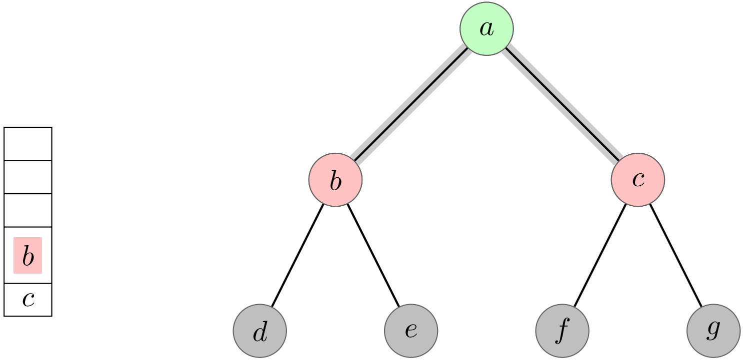

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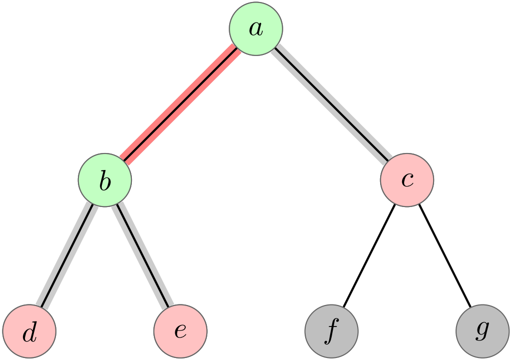

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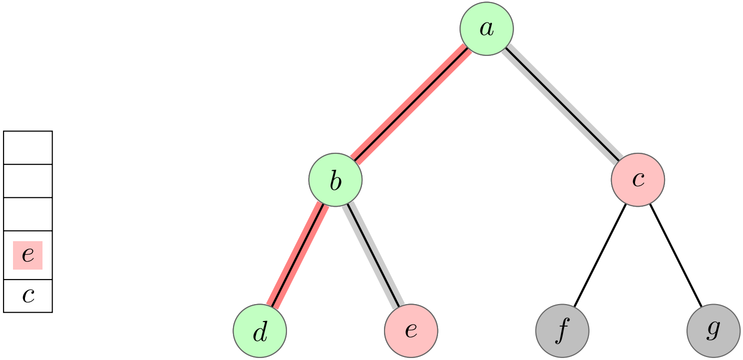
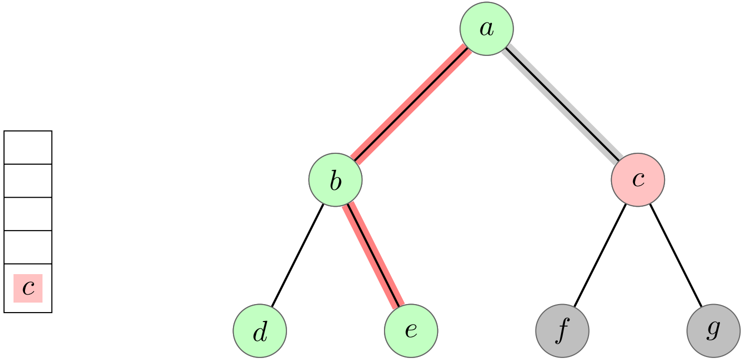


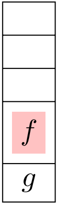
Depth First Search (DFS)

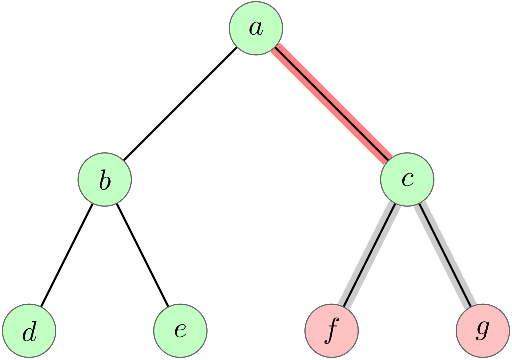
Did you get the pattern of nodes to be visited?



Depth First Search (DFS)

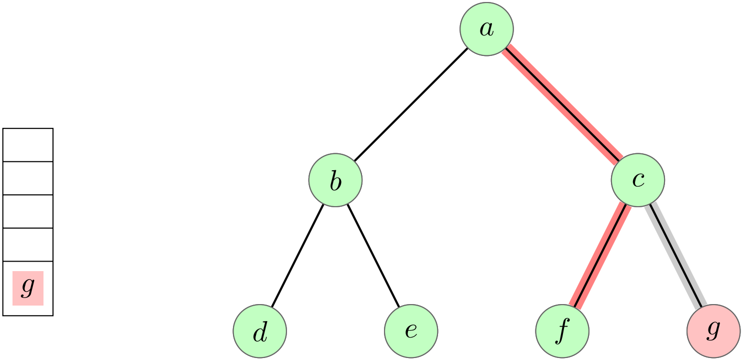
Did you get the pattern of nodes to be visited? **Last** inserted element is **first** to explore.





Depth First Search (DFS)

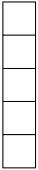
Did you notice what might be this structure? Hint: Last-in First-out

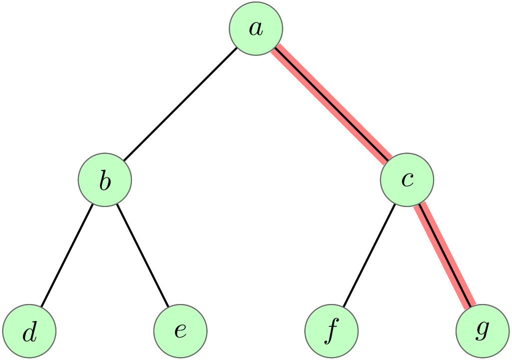


Depth First Search (DFS)

Did you notice what might be this structure? Hint: Last-in First-out

Yes, it is *Stack*





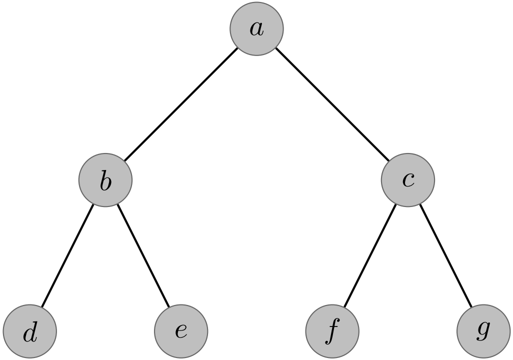
Depth First Search (DFS)

Algorithm 1: Depth-First($root$)

```
def S to be Stack;
visited ← {};
S.push(root);
while S ≠ ϕ do
    node ← S.pop();
    if node ∉ visited then
        visited ← visited ∪ {node};
        for n ∈ adjacent(node) do
            S.push(n);
        end
    end
end
```

Breadth First Search (BFS)

Breadth has the max priority



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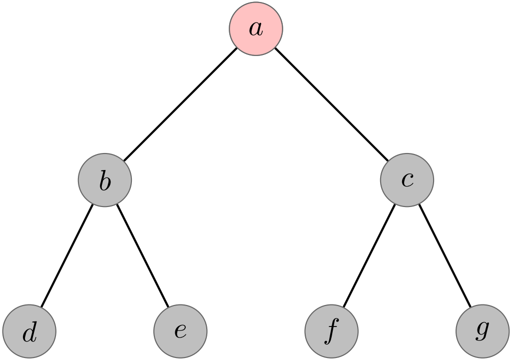

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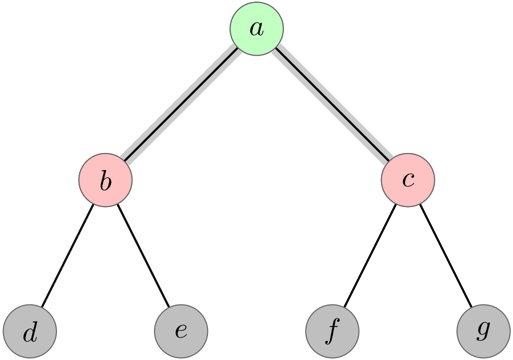

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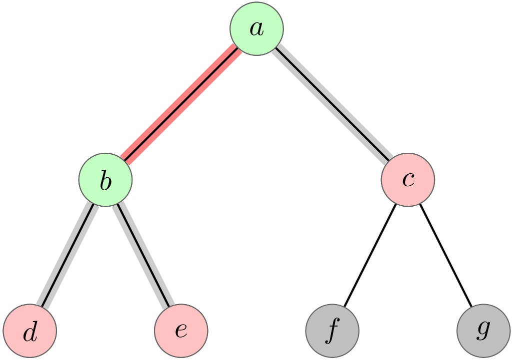

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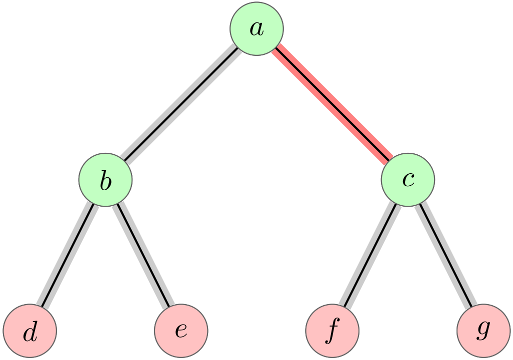

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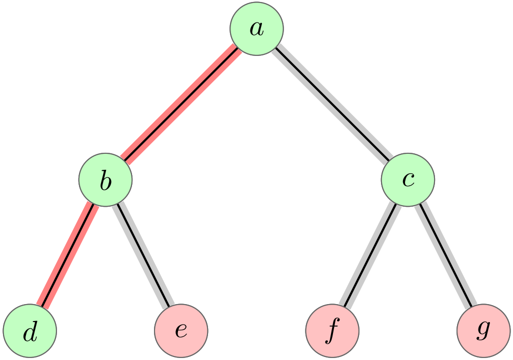

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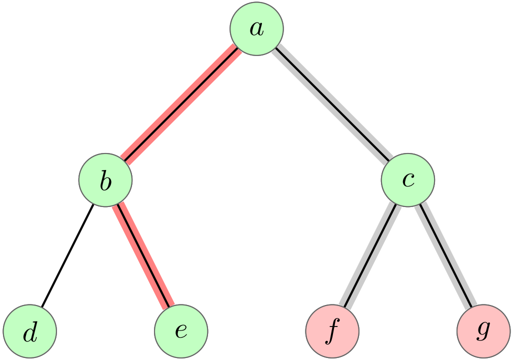

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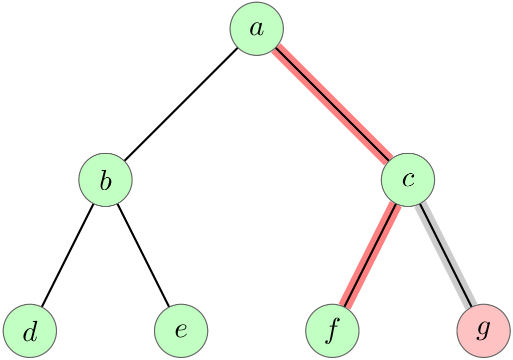

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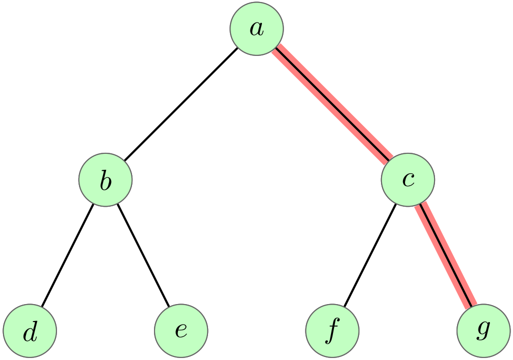
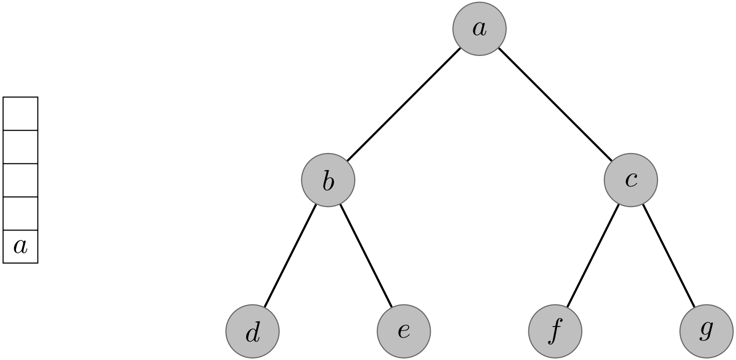


Breadth First Search (BFS)

Let's do it again, to notice the **pattern** of nodes to be visited



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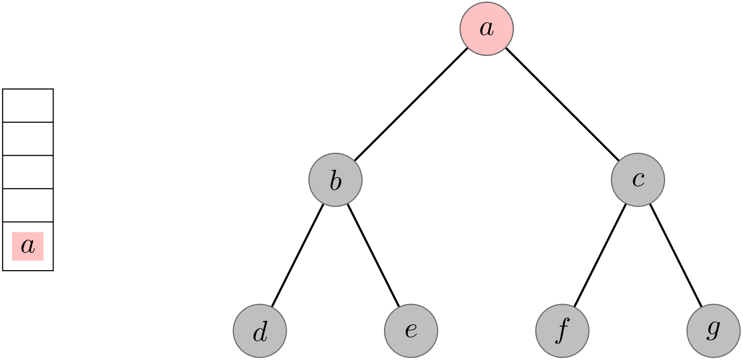

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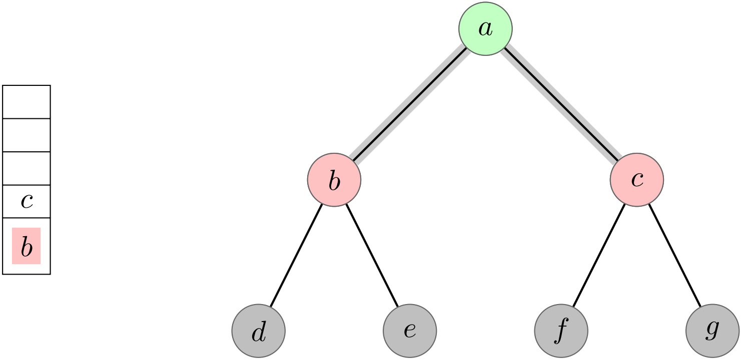

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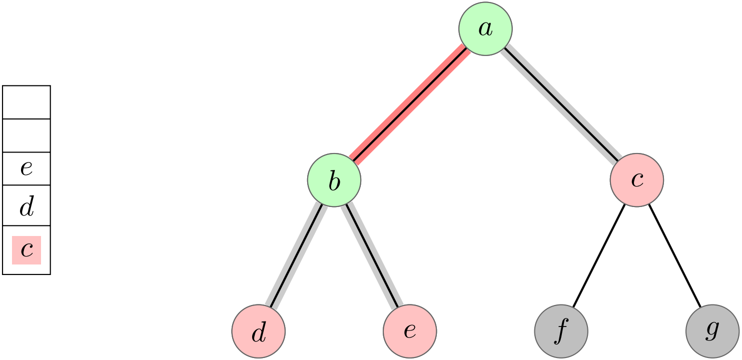
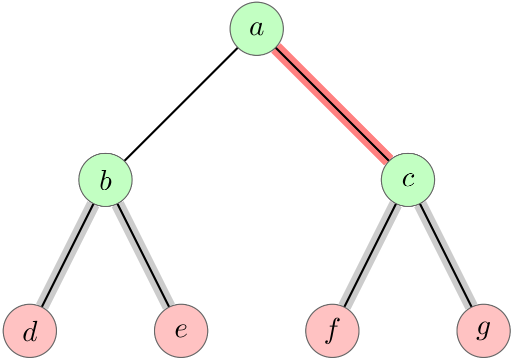


Breadth First Search (BFS)

Let's do it again, to notice the **pattern** of nodes to be visited

g
f
e
d



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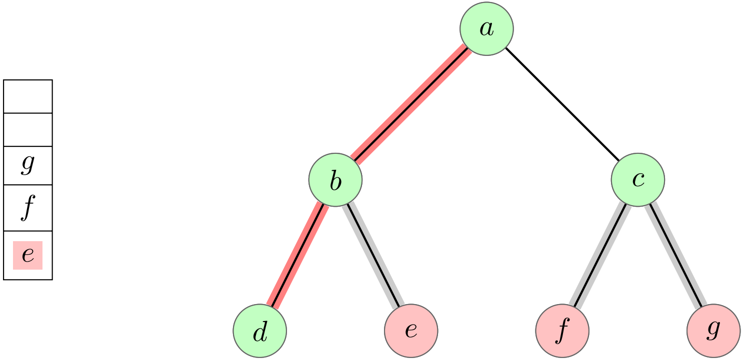
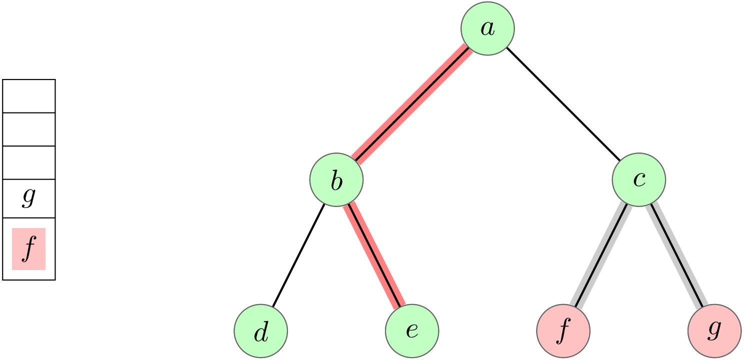
Did you get the pattern of nodes to be visited?



Breadth First Search (BFS)

Did you get the pattern of nodes to be visited? **First** inserted element is **first** to explore.



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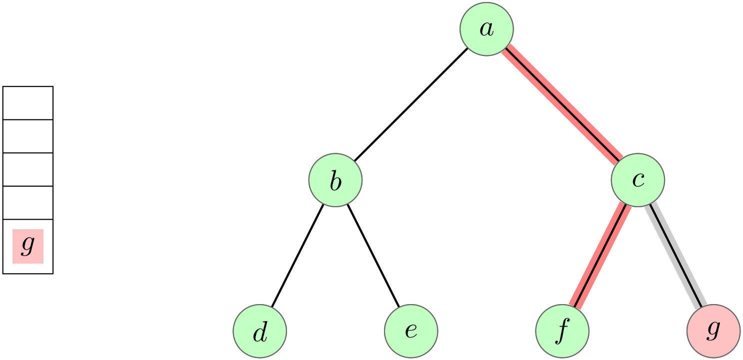

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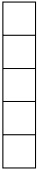
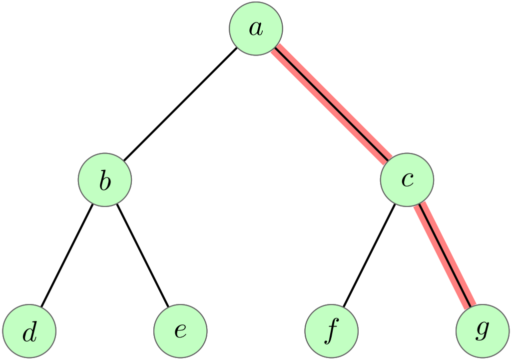
Did you notice what might be this structure? Hint: First-in First-out

Yes, it is *Queue*





Breadth First Search (BFS)

Algorithm 2: Breadth-First($root$)

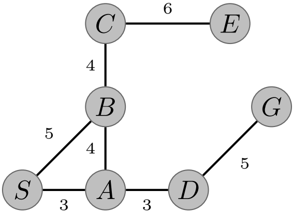
```
def S to be Queue;
visited ← {};
S.enqueue(root);
while S ≠ ϕ do
    node ← S.dequeue();
    if node ∉ visited then
        visited ← visited ∪ {node};
        for n ∈ adjacent(node) do
            S.enqueue(n);
        end
    end
end
```

Branch and Bound

Right now we have weights, now what should we prioritize?

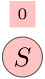
Branch and Bound

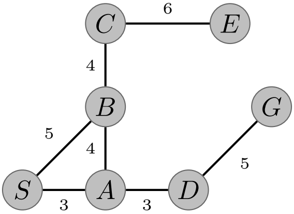
Right now we have weights, now what should we prioritize? Min/Max **weights**



Branch and Bound

Right now we have weights, now what should we prioritize? Min/Max **weights**

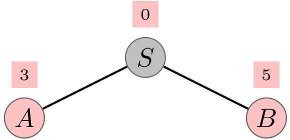


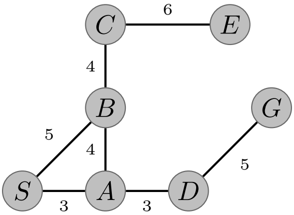


$\boxed{S : 0}$

Branch and Bound

Right now we have weights, now what should we prioritize? Min/Max **weights**



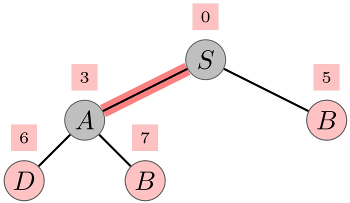


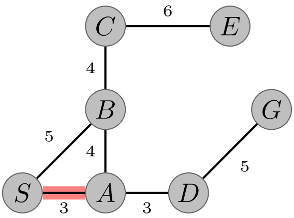
iterations/visits: 1

$A:3$	$B:5$

Branch and Bound

Right now we have weights, now what should we prioritize? Min/Max **weights**



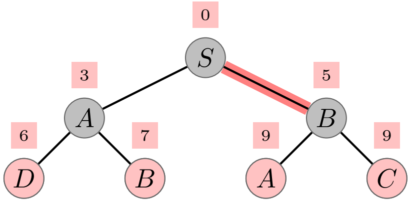


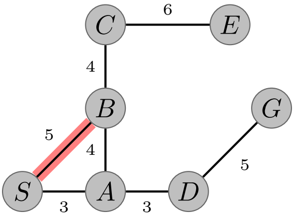
iterations/visits: 2

$B:5$	$D:6$	$B:7$

Branch and Bound

Can you notice the pattern of the structure?



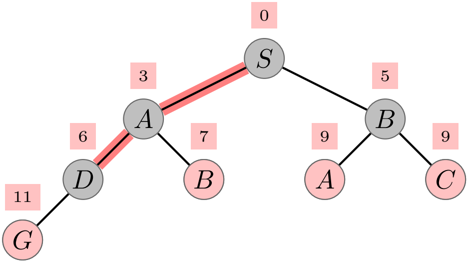


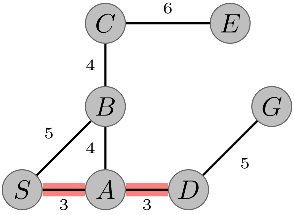
iterations/visits: 3

$D:6$	$B:7$	$A:9$	$C:9$

Branch and Bound

Can you notice the pattern of the structure? Also, we found G, so shall we stop?



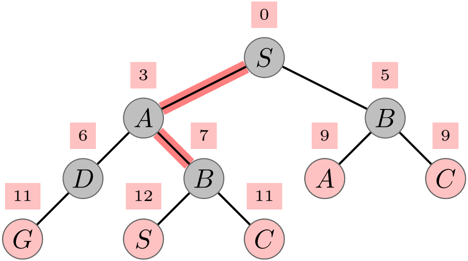


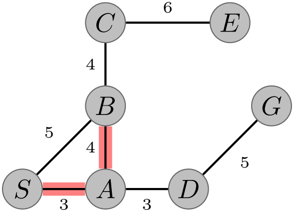
iterations/visits: 4

$B:7$	$A:9$	$C:9$	$G:11$

Branch and Bound

Have you noticed what happened?



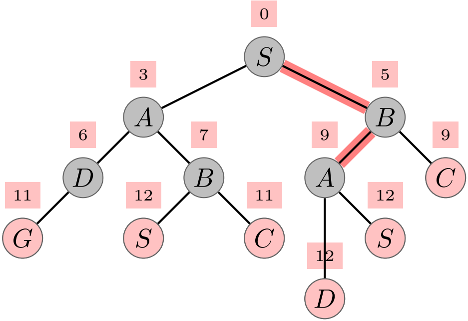


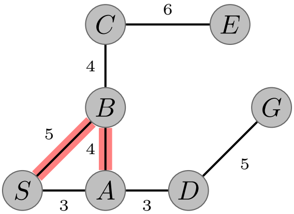
iterations/visits: 5

$A:9$	$C:9$	$C:11$	$G:11$	$S:12$

Branch and Bound

Now, what is such structure?



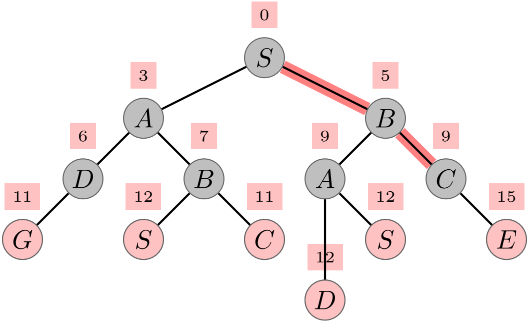


iterations/visits: 6

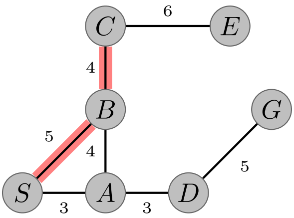
$C:9$	$C:11$	$G:11$	$S:12$	$D:12$	$S:12$

Branch and Bound

Now, what is such structure?



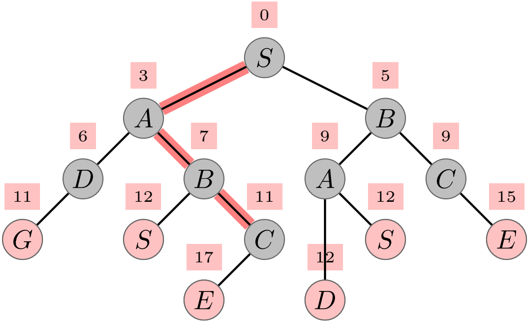
$C:11$	$G:11$	$S:12$	$D:12$	$S:12$	$E:15$

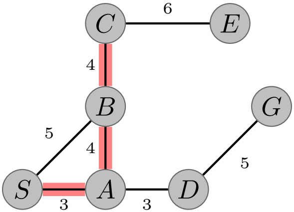


iterations/visits: 7

Branch and Bound

Now, what is such structure?



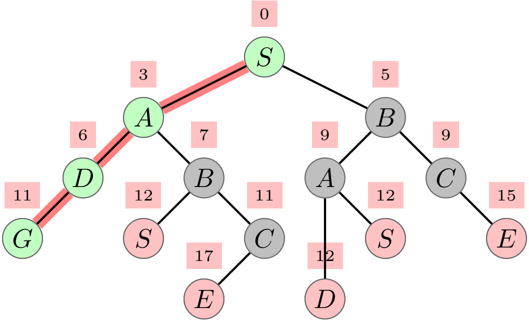


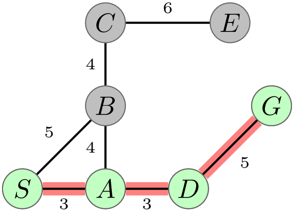
iterations/visits: 8

$G:11$	$S:12$	$D:12$	$S:12$	$E:15$	$E:17$

Branch and Bound

Yes, it is a **Priority Queue**, where the priority is the overall path cost.





iterations/visits: 9

$S:12$	$D:12$	$S:12$	$E:15$	$E:17$

Branch and Bound

Algorithm 3: Branch-Bound$(root, goal)$

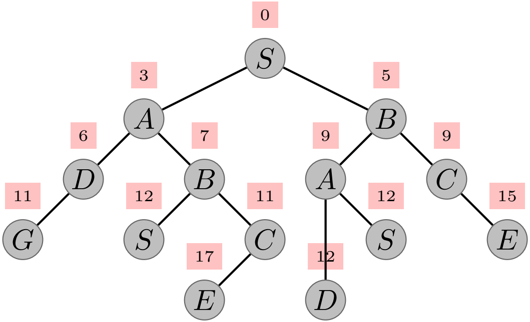
```
def PQ to be Priority Queue
PQ.enqueue(root, 0)
node ← root
while PQ ≠ ϕ ∧ node ≠ goal do
    node, path_cost ← PQ.dequeue()
    for n ∈ adjacent(node) do
        // loop over the links and their costs
        PQ.enqueue(n, path_cost + cost(n))
    end
end
```

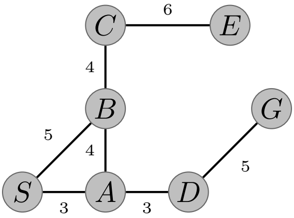
Search Space Pruning

One question over the previous algorithm is: **cannot we do better?**

Search Space Pruning

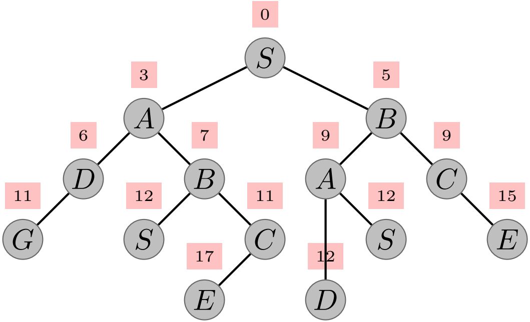
To answer this, we need to check the produced **search space**, on the left.

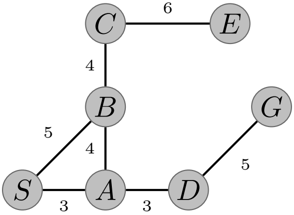




Search Space Pruning

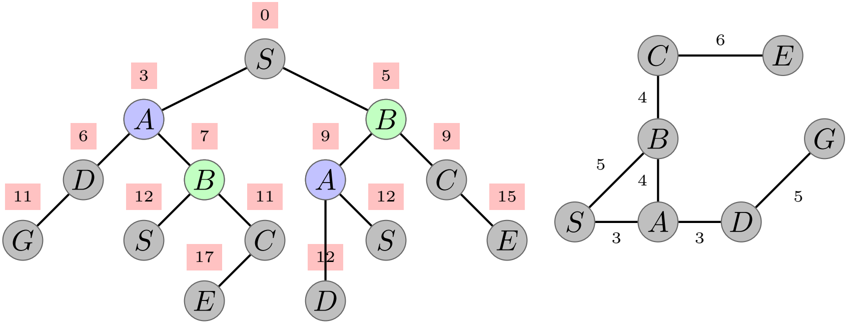
We know the **denser** the search space, more nodes to visit, the more time our algorithms takes. Hence, ***pruning*** it, eliminating some of the nodes, should improve it, how can we do this?





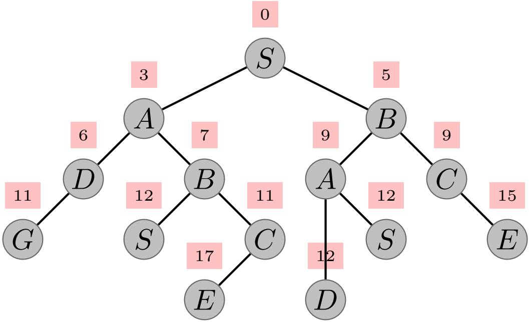
Search Space Pruning

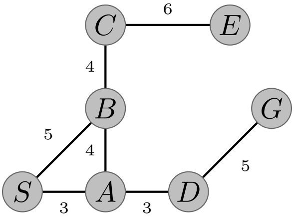
One way is to remove duplicates; do you think we need to check A or B twice? We can do this using a simple **visited list**



Search Space Pruning

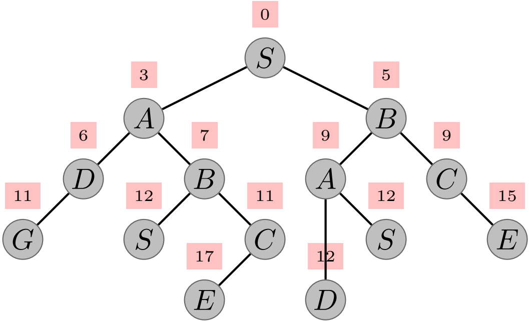
Could you think of other techniques to prune the tree?

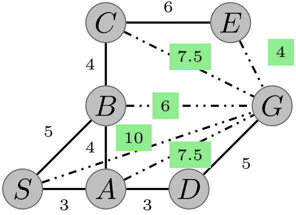




Search Space Pruning

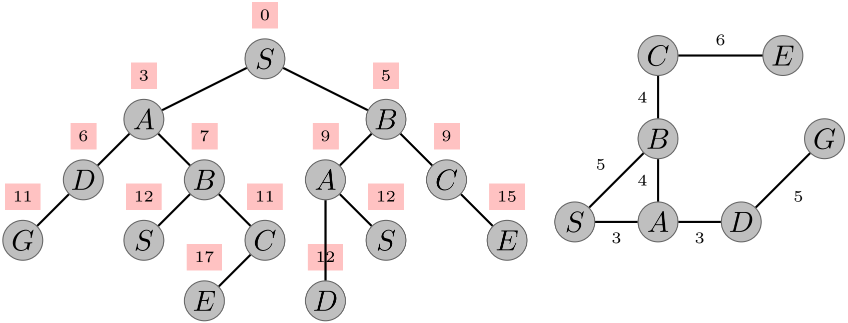
One solution is if we could have an estimate of where the goal is; this may make us more oriented towards searching in particular regions than others; this is briefly, the **heuristics**.





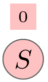
Search Space Pruning

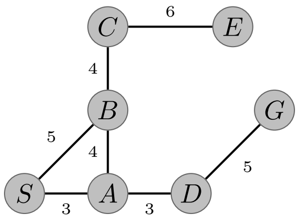
We are covering both the **visited list** and **heuristic** techniques in the next subsections, applying them to branch and bound.



Adding visited list to Branch and Bound

Let's follow on the same graph, but while using a **visited list**.



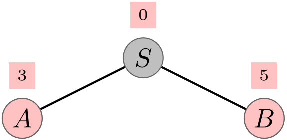


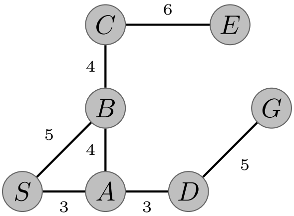
PQ: $\boxed{S:0}$

$visited$:

Adding visited list to Branch and Bound

Let's follow on the same graph, but while using a **visited list**.





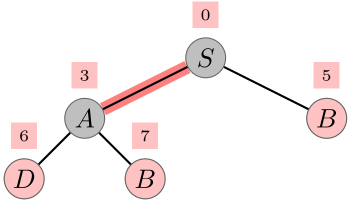
iterations/visits: 1

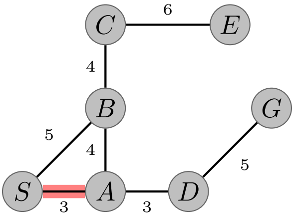
PQ: $\boxed{A:3}\boxed{B:5}$

$\boldsymbol{visited}$: $\boxed{S}$

Adding visited list to Branch and Bound

Let's follow on the same graph, but while using a **visited list**.





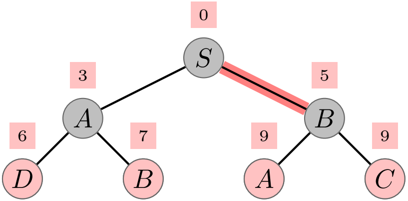
iterations/visits: 2

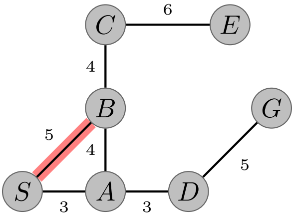
PQ: $\boxed{B:5}\boxed{D:6}\boxed{B:7}$

$visited$: $\boxed{S}\boxed{A}$

Adding visited list to Branch and Bound

Let's follow on the same graph, but while using a **visited list**.





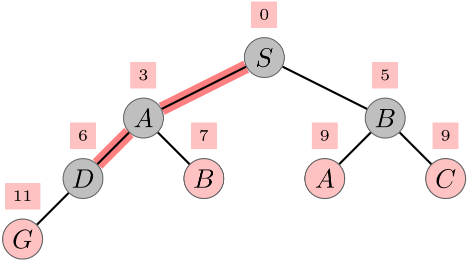
iterations/visits: 3

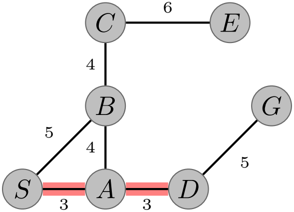
PQ:	$D:6$	$B:7$	$A:9$	$C:9$

visited:	S	A	B

Adding visited list to Branch and Bound

Now, the queue has B, should we visit it next?



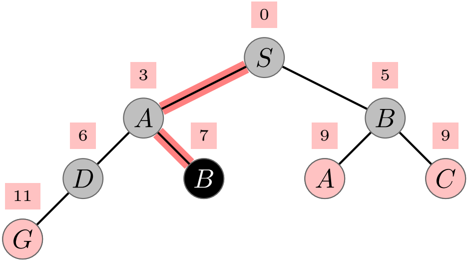


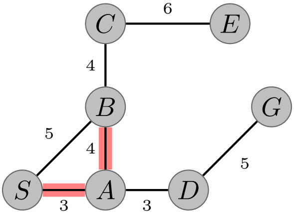
iterations/visits: 4

PQ:		$B:7$	$A:9$	$C:9$	$G:11$
$visited$:	S	A	B	D	

Adding visited list to Branch and Bound

Well, no: B is already visited, so there is already a shorter path to it.





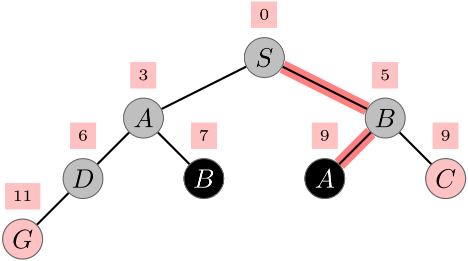
iterations/visits: 5

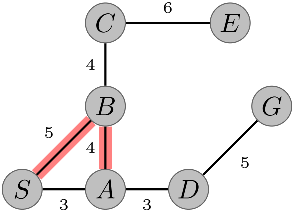
PQ:	$A:9$	$C:9$	$G:11$

$visited$:	S	A	B	D

Adding visited list to Branch and Bound

Again, the next was A, so it is blocked as well.





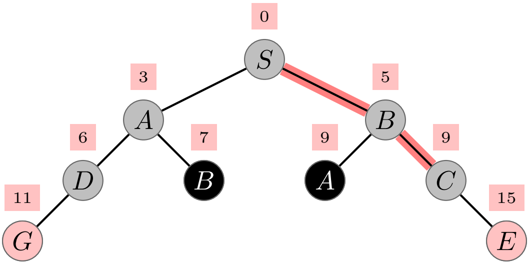
iterations/visits: 6

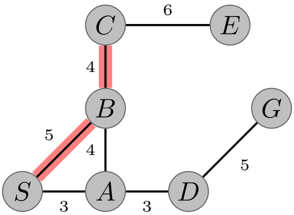
PQ:	$C:9$	$G:11$

$visited$: S | A | B | D

Adding visited list to Branch and Bound

We, then, proceed





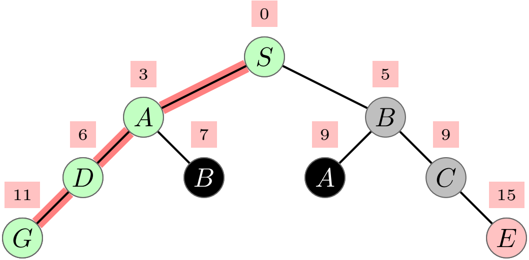
iterations/visits: 7

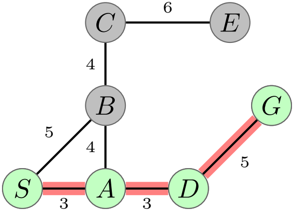
PQ: $\boxed{G:11}\boxed{E:15}$

$visited$: $\boxed{S}\boxed{A}\boxed{B}\boxed{D}\boxed{C}$

Adding visited list to Branch and Bound

Finally, we got our solution. Could you see the pruning effect?





iterations/visits: 8

PQ: $\boxed{E : 15}$

$\texttt{visited}$: $\boxed{S}\boxed{A}\boxed{B}\boxed{D}\boxed{C}$

Adding visited list to Branch and Bound

Algorithm 4: Branch-Bound-Visited$(root, goal)$

$\mathtt{visited} \leftarrow \{\}$
PQ.enqueue$(root, 0)$
$\mathtt{node} \leftarrow root$
while $PQ \neq \phi \wedge \mathtt{node} \neq goal$ **do**
  $\mathtt{node},\ \mathtt{path\_cost} \leftarrow PQ$.dequeue()
  if $\mathtt{node} \notin \mathtt{visited}$ **then**
    $\mathtt{visited} \leftarrow \mathtt{visited} \cup \{\mathtt{node}\}$
    for $n \in adjacent(\mathtt{node})$ **do**
      // loop over the links and their costs
      PQ.enqueue$(n, \mathtt{path_cost} + \text{cost}(n))$
    end
  end
end

Branch and Bound + Heuristics

Now, we start with showing what heuristics are, first. Heuristics are **optimistic estimation** to the future cost.

Branch and Bound + Heuristics

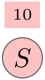
Now, we start with showing what heuristics are, first. Heuristics are **optimistic estimation** to the future cost. Although they are not accurate, but because they capture some of the information, we use them within our algorithms to **guide** our search to **more promising areas** than others, leading to faster findings.

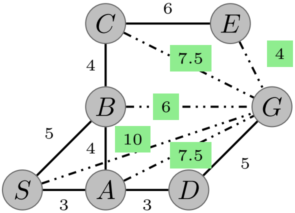
Branch and Bound + Heuristics

We will be using the same map, where our heuristic values, in dashed lines, represent the **direct Euclidean distance** between the nodes. They are not accurate because shorter distances does not necessarily mean we are close to our goal.

Branch and Bound + Heuristics

Our cost, priority, = actual cost to current node + the estimate to the goal. Let's do it without a visited list.

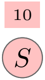


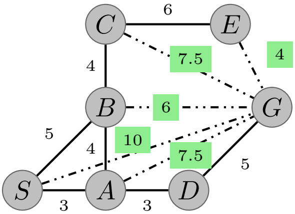


PQ: $\boxed{S:10}$

Branch and Bound + Heuristics

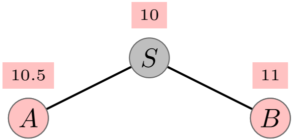
Note our initial cost $= 10$, which is 0 actual cost $+10$ as estimate

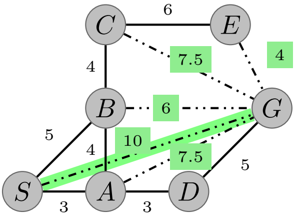




Branch and Bound + Heuristics

Again, the cost = true cost + heuristic



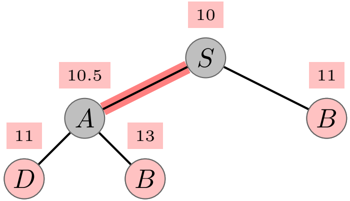


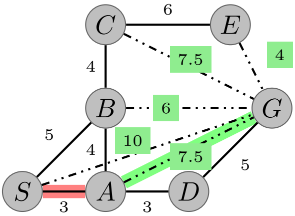
iterations/visits: 1

PQ: $\boxed{A : 10.5}\boxed{B : 11}$

Branch and Bound + Heuristics

Again, the cost = true cost + heuristic





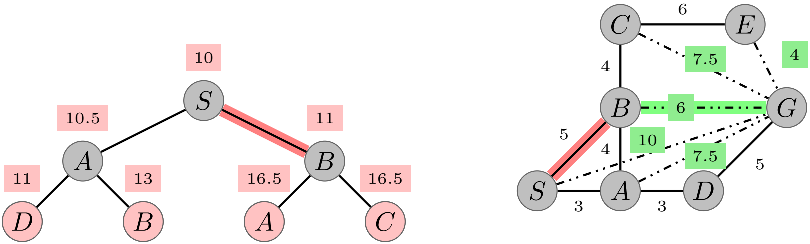
iterations/visits: 2

PQ:

$B:11$	$D:11$	$B:13$

Branch and Bound + Heuristics

Again, the cost = true cost + heuristic



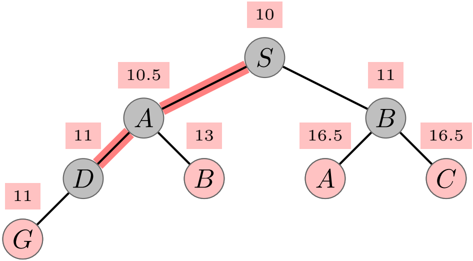
iterations/visits: 3

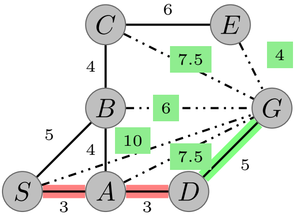
PQ:

$D:11$	$B:13$	$A:16.5$	$C:16.5$

Branch and Bound + Heuristics

Again, the cost = true cost + heuristic





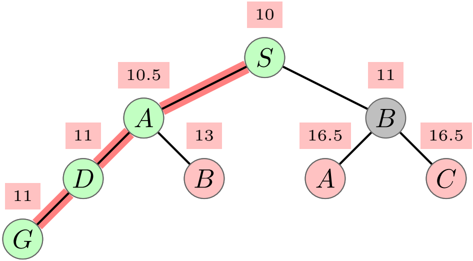
iterations/visits: 4

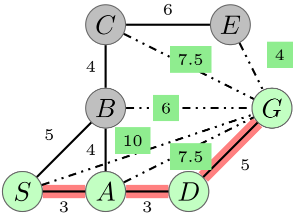
PQ:

$G:11$	$B:13$	$A:16.5$	$C:16.5$

Branch and Bound + Heuristics

and we got it in just 5 iterations.





iterations/visits: 5

PQ:

$B : 13$	$A : 16.5$	$C : 16.5$

Branch and Bound + Heuristics

Algorithm 5: Branch-Bound-Heuristics($root, goal$)

```
def PQ to be Priority Queue
PQ.enqueue(root, h(root))
node ← root
while PQ ≠ φ ∧ node ≠ goal do
    node, node_cost ← PQ.dequeue()
    path_cost ← node_cost − h(node)
    for n ∈ adjacent(node) do
        // h(n) is the heuristic estimate from n to goal
        total_cost ← path_cost + cost(n) + h(n)
        PQ.enqueue(n, total_cost)
    end
end
```

1. Graphs & Graph Algorithms
2. Unweighted Graphs
3. Weighted Graphs
   - Branch and Bound
   - Search Space Pruning
   - Branch and Bound + Visited List
   - Branch and Bound + Heuristics
   - A* Algorithm
   - Heuristic Design
4. Path Construction
5. References

A* Algorithm

We studied two cool search pruning techniques:

- visited list
- heuristics

Now, you might ask what happens if we used both of these cool techniques.

A* Algorithm

We studied two cool search pruning techniques:

- visited list
- heuristics

Now, you might ask what happens if we used both of these cool techniques. We get an even more **awesome** algorithm, which is the **A***. It is typically using both techniques with the Branch-and-Bound algorithm. Let's check the pseudo-code

A* Algorithm

Algorithm 6: A-Star$(root, goal)$

$\mathtt{visited} \leftarrow \{\}$, $\mathtt{node} \leftarrow root$
PQ.enqueue$(root, \mathrm{h}(root))$
while $PQ \neq \phi \wedge \mathtt{node} \neq goal$ **do**
  $\mathtt{node},\ \mathtt{node\_cost} \leftarrow PQ$.dequeue()
  if $\mathtt{node} \notin \mathtt{visited}$ **then**
    $\mathtt{visited} \leftarrow \mathtt{visited} \cup \{\mathtt{node}\}$
    $\mathtt{path\_cost} \leftarrow \mathtt{node\_cost} - \mathrm{h}(\mathtt{node})$
    for $n \in adjacent(\mathtt{node})$ **do**
      PQ.enqueue$(n, \mathtt{path_cost} + \mathrm{cost}(n) + \mathrm{h}(n))$
    end
  end
end

1. Graphs & Graph Algorithms
2. Unweighted Graphs
3. Weighted Graphs
   - Branch and Bound
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How to design a heuristic?

I know you might be thinking: heuristics might be useful, but how could we get, design, them? This is the exact aim of this part. Let's review our information about heuristics first:

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- They make our algorithm able to search in the most promising settings

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Let's review our information about heuristics first:

- Heuristics: are **optimistic estimates** to the goal.
- They make our algorithm able to search in the most promising settings
- We terminate our algorithm when we reach the goal because if the optimistic estimate is already worse than what we have in our hands, why should we explore it?

How to design a heuristic?

Back to our question, we simply could design our have optimistic estimates through ***problem relaxation***.

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Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

Example

Think of the map. The rules to move from city to another is that there must be a route. If we relaxed this constraint, and let anyone move to any other city with the cost of **euclidean distance**, the *shortest possible* distance, this is exactly what we did in the previous map.

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

Let's think about another example, eight-tile puzzle: first, we need to know the puzzle rules

Start

2	8	1
4	6	3
	7	5

Goal

1	2	3
8		4
7	6	5

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

Let's think about another example, eight-tile puzzle: first, we need to know the puzzle rules

- Movement are either vertical or horizontal, no diagonal movement.

Start

2	8	1
4	6	3
	7	5

Goal

1	2	3
8		4
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Let's think about another example, eight-tile puzzle: first, we need to know the puzzle rules

- Movement are either vertical or horizontal, no diagonal movement.
- One move each time.

Start

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Let's think about another example, eight-tile puzzle: first, we need to know the puzzle rules

- Movement are either vertical or horizontal, no diagonal movement.
- One move each time.
- No cell could move unless the it moves to empty cell.

Start

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4	6	3
	7	5

Goal

1	2	3
8		4
7	6	5

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

- Movement are either vertical or horizontal, no diagonal movement.
- One move each time.
- No cell could move unless the it moves to empty cell.

Think how could you relax these rules to have an estimate to the number of moves needed to get the solution?

Start

2	8	1
4	6	3
	7	5

Goal

1	2	3
8		4
7	6	5

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

What if we can move any cell to any other cell without constraints, but one move per time. The **heuristic** for this example would be $h_1 = 1 * 7 = \boxed{7}$ moves. This is just the number of misplacements.

Start

2	8	1
4	6	3
	7	5

Goal

1	2	3
8		4
7	6	5

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

Alternatively, What if we stick to vertical/horizontal movement but we can move over non-empty cells. The **heuristic** for this example would be $h_2 = 1+2+2+2+1+1+1+0 = \boxed{10}$ moves. Could you think of another heuristic?

Start

2	8	1
4	6	3
	7	5

Goal

1	2	3
8		4
7	6	5

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

In brief, to design a heuristic

1. list the problem rules
2. start removing subset of these rules, and consider the solution as an estimate to the actual solution

How to design a heuristic?

Problem Relaxation

Getting estimates through relaxing, removing, some of the problem constraints.

In brief, to design a heuristic

1. list the problem rules
2. start removing subset of these rules, and consider the solution as an estimate to the actual solution

Nevertheless, some heuristics might be good while other might be bad. For that reason, we study the criteria for good heuristics in the following.

Heuristic Criteria

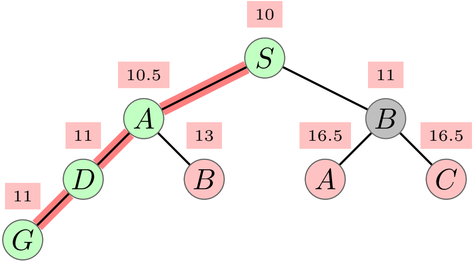
Admissibility

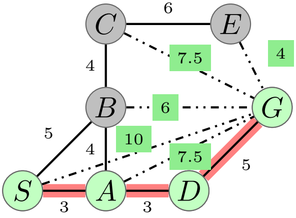
Heuristic values are **optimistic estimates**. If that is not true, it causes troubles.

Heuristic Criteria

Admissibility

Heuristic values are **optimistic estimates**. Let's revisit the search space from the Heuristic implementation



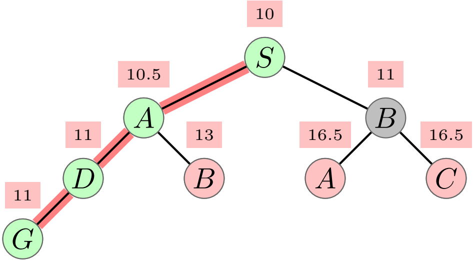


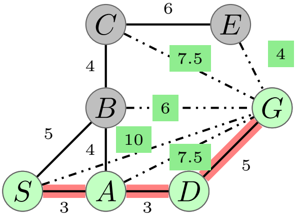
PQ:	$B : 13$	$A : 16.5$	$C : 16.5$

Heuristic Criteria

Admissibility

Heuristic values are **optimistic estimates**. Why did we stop here? Why did not we explore B because 13 is just an estimate?



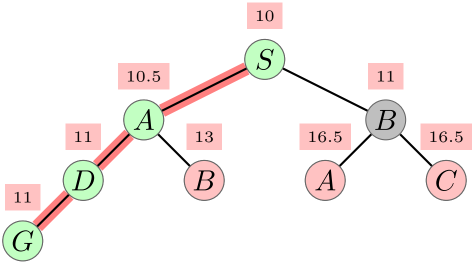


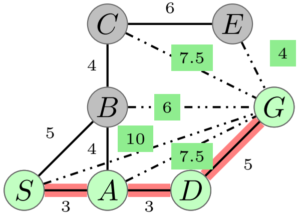
PQ: $\boxed{B:13}\boxed{A:16.5}\boxed{C:16.5}$

Heuristic Criteria

Admissibility

Yes; because 13 is an **optimistic estimate**, we are sure we will never get a lower cost than 13, and we already have 11.



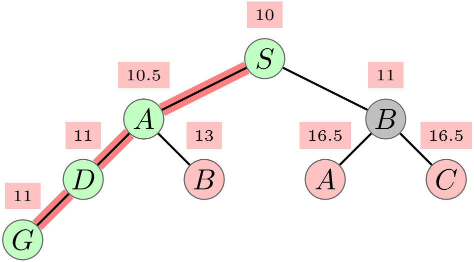


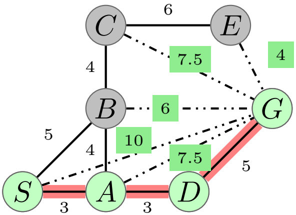
PQ: | $B : 13$ | $A : 16.5$ | $C : 16.5$ |

Heuristic Criteria

Admissibility

Now, what if we cannot guarantee that our estimate is optimistic;



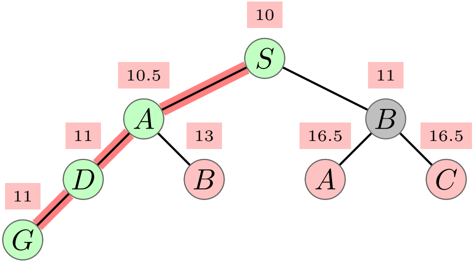


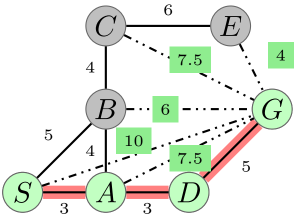
PQ: | $B:13$ | $A:16.5$ | $C:16.5$ |

Heuristic Criteria

Admissibility

Now, what if we cannot guarantee that our estimate is optimistic; in that case, we will need to explore B because we might get a lower cost.





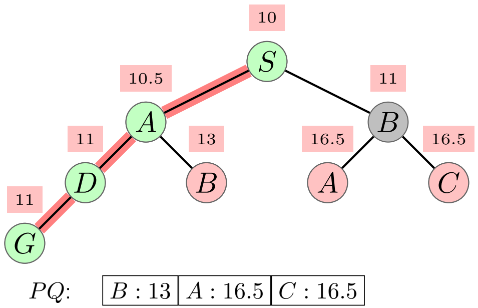
PQ: $\boxed{B:13}\boxed{A:16.5}\boxed{C:16.5}$

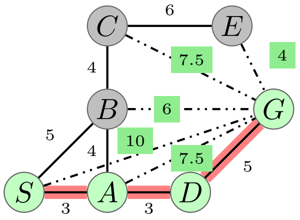
Heuristic Criteria

Admissibility

Admissibility

That is exactly what admissibility criterion is; that is to guarantee that our heuristic estimates are ***optimistic*** $\boxed{h(s) \leq \texttt{cost}(s, goal)}$





Heuristic Criteria

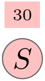
Consistency

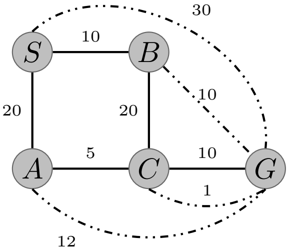
The question now is: is that enough? What if we used a visited list?

Heuristic Criteria

Consistency

Let's change the problem a little bit to see it. like in the below graph.



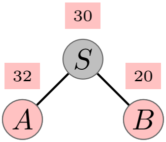


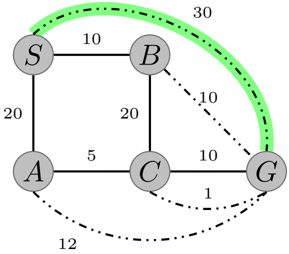
PQ: $\boxed{S : 30}$

$\textit{visited}$:

Heuristic Criteria

Consistency



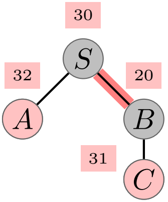


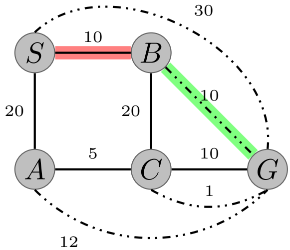
PQ: $\boxed{B : 20}\boxed{A : 32}$

$visited$: $\boxed{S}$

Heuristic Criteria

Consistency



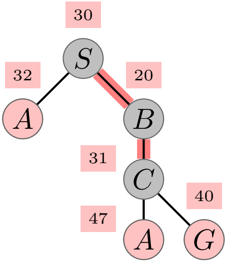


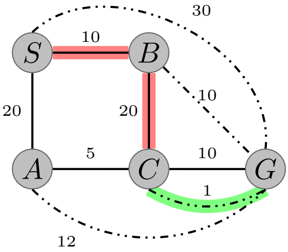
PQ: $\boxed{C : 31}\boxed{A : 32}$

$visited$: $\boxed{S}\boxed{B}$

Heuristic Criteria

Consistency



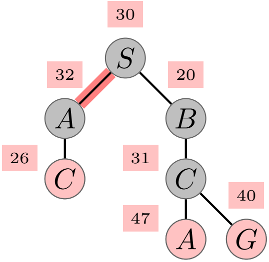


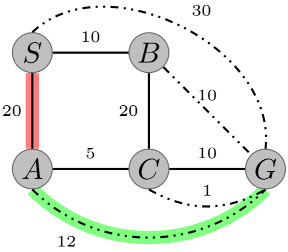
PQ: $\boxed{A:32}\boxed{G:40}\boxed{A:47}$

$\boldsymbol{visited}$: $\boxed{S}\boxed{B}\boxed{C}$

Heuristic Criteria

Consistency





$C:26$	$G:40$	$A:47$

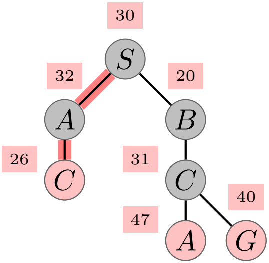
PQ:

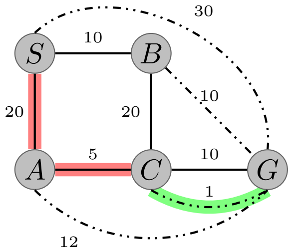
$visited$: S | B | C | A

Heuristic Criteria

Consistency

Now, we are exploring C, producing B, G right?





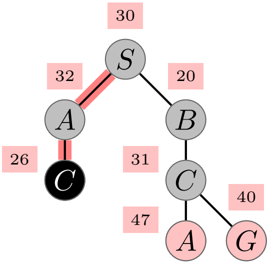
PQ:	$C:26$	$G:40$	$A:47$

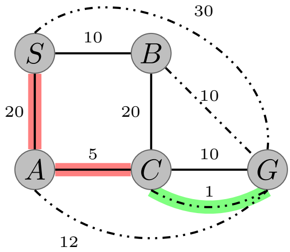
$\boldsymbol{visited}$:	S	B	C	A

Heuristic Criteria

Consistency

Not really, C is already visited



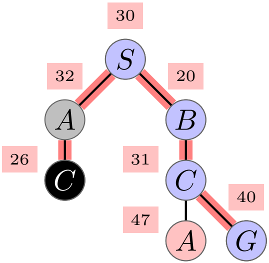


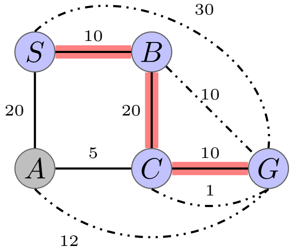
PQ: $\boxed{G:40}\boxed{A:47}$

visited: $\boxed{S}\boxed{B}\boxed{C}\boxed{A}$

Heuristic Criteria
Consistency

Hence, the best solution is S, B, C, G with a cost of 40





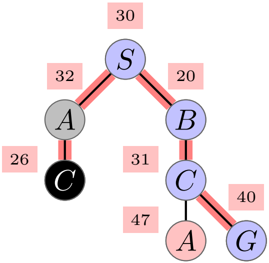
PQ: $\boxed{G:40}\boxed{A:47}$

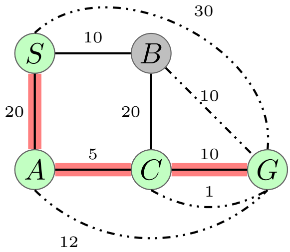
$\boldsymbol{visited}$: $\boxed{S}\boxed{B}\boxed{C}\boxed{A}$

Heuristic Criteria

Consistency

Nevertheless, S, A, C, G with a cost of 35 is better.





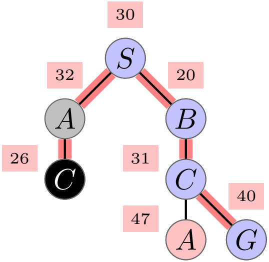
PQ: $\boxed{G : 40}\boxed{A : 47}$

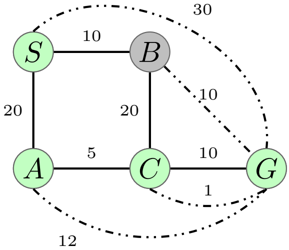
$\mathit{visited}$: $\boxed{S}\boxed{B}\boxed{C}\boxed{A}$

Heuristic Criteria

Consistency

That is why we need another criteria that is **Consistency**.





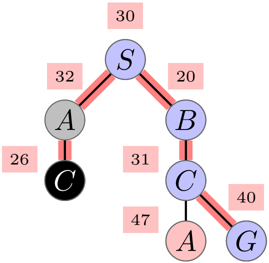
PQ:	$G:40$	$A:47$		
$visited$:	S	B	C	A

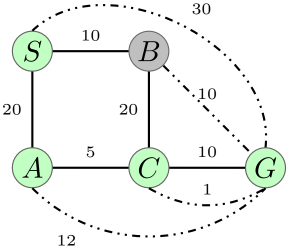
Heuristic Criteria

Consistency

Consistency

enforces that once we visit a node, 1^{st} time, it is the shortest path



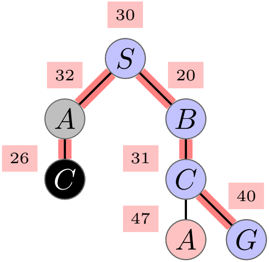


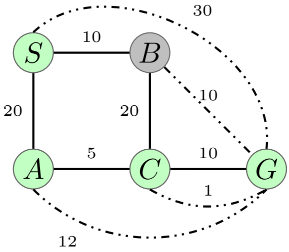
Heuristic Criteria

Consistency

Consistency

$$\forall v \in V, \forall n \in adjacent(v) \quad h(v) \leq cost(v,n) + h(n)$$





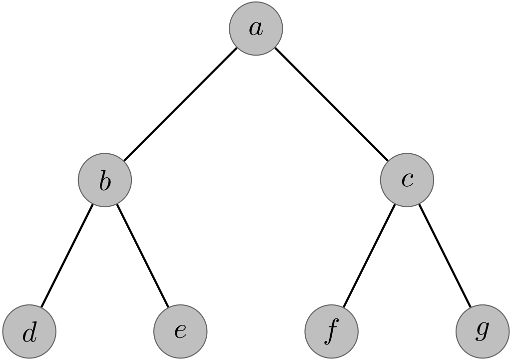
Path Construction from Traversing Algorithm

Cool; now, we have shown how many traversing algorithms, *which visits the nodes*, and checked the pseudo-code, but and important question is

how to construct a path from a traversing algorithm? (DFS for example)

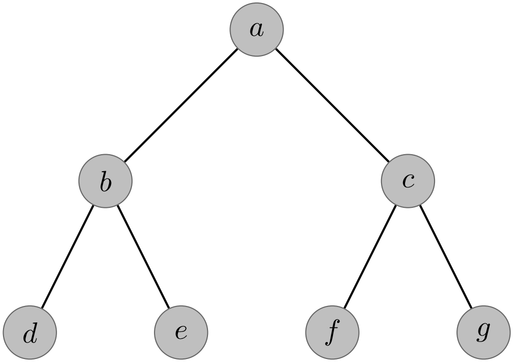
Path Construction from Traversing Algorithm

First, to differentiate between traversing and path, we revisit the same DFS tree as before; the ordering of the visited nodes is a, b, d, e, c, f, g.



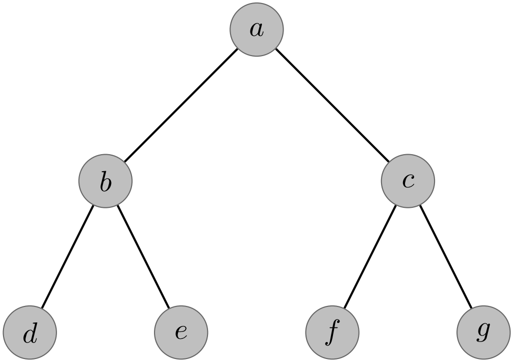
Path Construction from Traversing Algorithm

the ordering of the visited nodes is a, b, d, e, c, f, g. Does this mean that the **path** according to our algorithm from $a \to f$ is a, b, d, e, c, f?



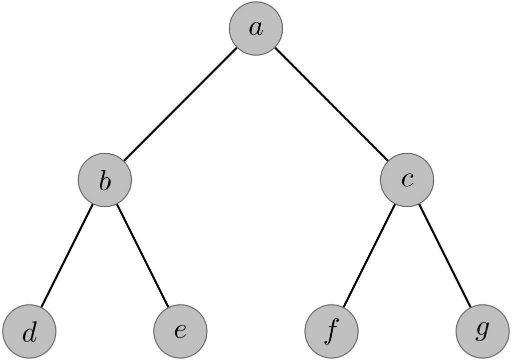
Path Construction from Traversing Algorithm

the ordering of the visited nodes is a, b, d, e, c, f, g. Does this mean that the **path** according to our algorithm from $a \to f$ is a, b, d, e, c, f? well, **No**. This is just a traversing order.



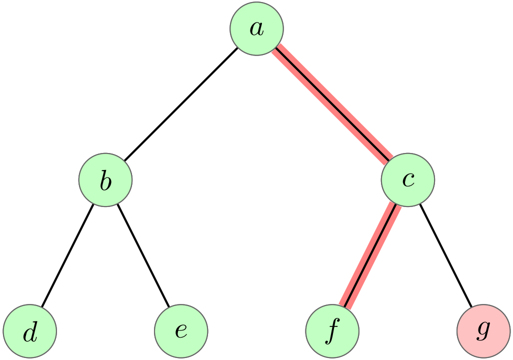
Path Construction from Traversing Algorithm

In that case, what is the path constructed from $a \to f$ according to our algorithm?



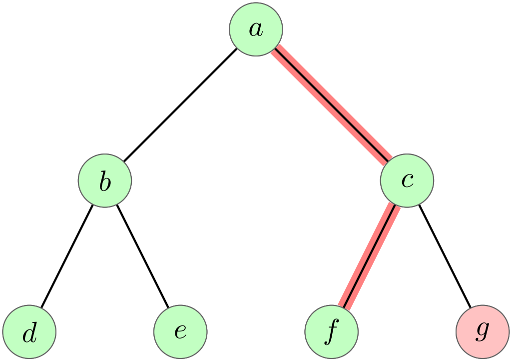
Path Construction from Traversing Algorithm

In that case, what is the path constructed from $a \rightarrow f$ according to our algorithm? It is a, c, f as we see from our previous execution.



Path Construction from Traversing Algorithm

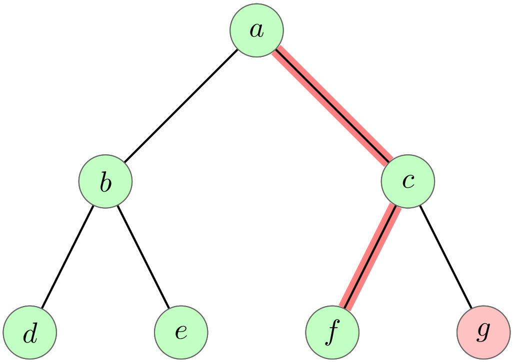
so, how could we construct such path from our algorithm?



Path Construction from Traversing Algorithm

so, how could we construct such path from our algorithm?

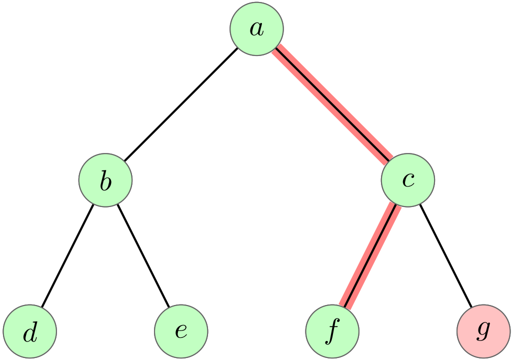
- store the whole paths, instead of just nodes



Path Construction from Traversing Algorithm

so, how could we construct such path from our algorithm?

- store the whole paths, instead of just nodes; Storage hungry



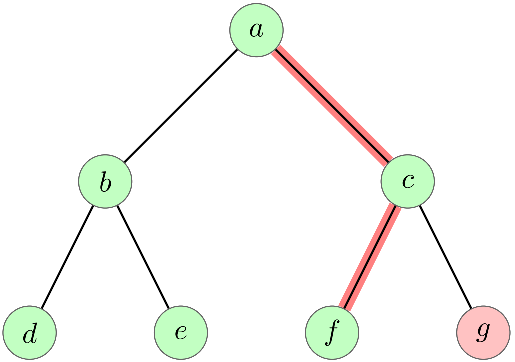
Path Construction from Traversing Algorithm

so, how could we construct such path from our algorithm?

- using **parent-child** structure.

Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$
$e \longleftarrow b$
$c \longleftarrow a$
$f \longleftarrow c$
$g \longleftarrow c$

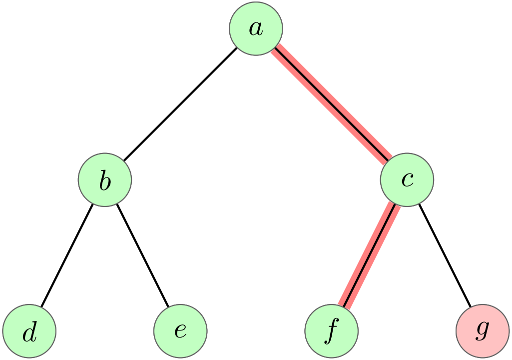


Path Construction from Traversing Algorithm

start from your goal, f, and move backward until getting your start.

Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$
$e \longleftarrow b$
$c \longleftarrow a$
$f \longleftarrow c$
$g \longleftarrow c$

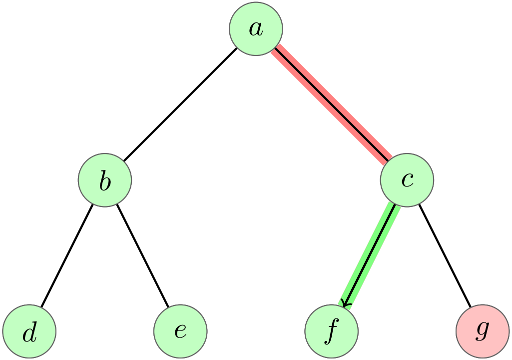


Path Construction from Traversing Algorithm

Path: $c \to f$

Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$
$e \longleftarrow b$
$c \longleftarrow a$
$f \longleftarrow c$
$g \longleftarrow c$

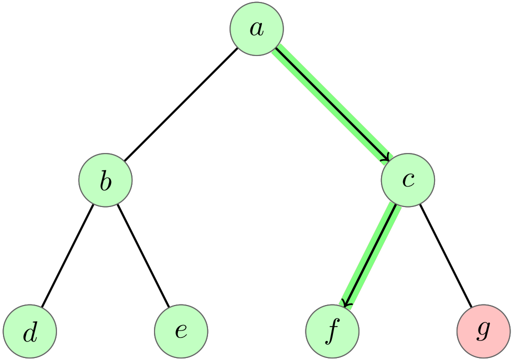


Path Construction from Traversing Algorithm

Path: $\boxed{a \to c \to f}$

Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$
$e \longleftarrow b$
$c \longleftarrow a$
$f \longleftarrow c$
$g \longleftarrow c$

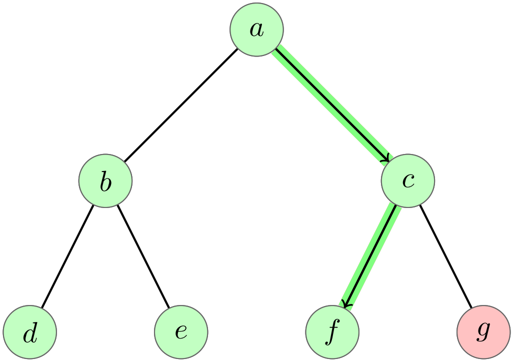


Path Construction from Traversing Algorithm

We can this technique for all the traversing algorithms, mentioned within this module.

Child $\longleftarrow$ Parent

$b \longleftarrow a$
$d \longleftarrow b$
$e \longleftarrow b$
$c \longleftarrow a$
$f \longleftarrow c$
$g \longleftarrow c$



References

- Part VI, Ch 22, in *Introduction to Algorithms*
  Cormen, Leiserson, Rivest, and Stein
- Part II, Ch $3-6$ in AI Modern Approach
  Russell and Norvig

  As well, you might find these links useful
- Red Blob Games including
  - Graph Theory
  - A* and Other search algorithms
- Some visualization tools:
  - VisualAlgo
  - GraphAV
    visualization tool developed by students from previous intakes

Contacts

- Email: ammarsherif90 [at] gmail [dot] com
- Github-ID: ammarSherif

- More Teaching Material: Github Repository

Thanks!