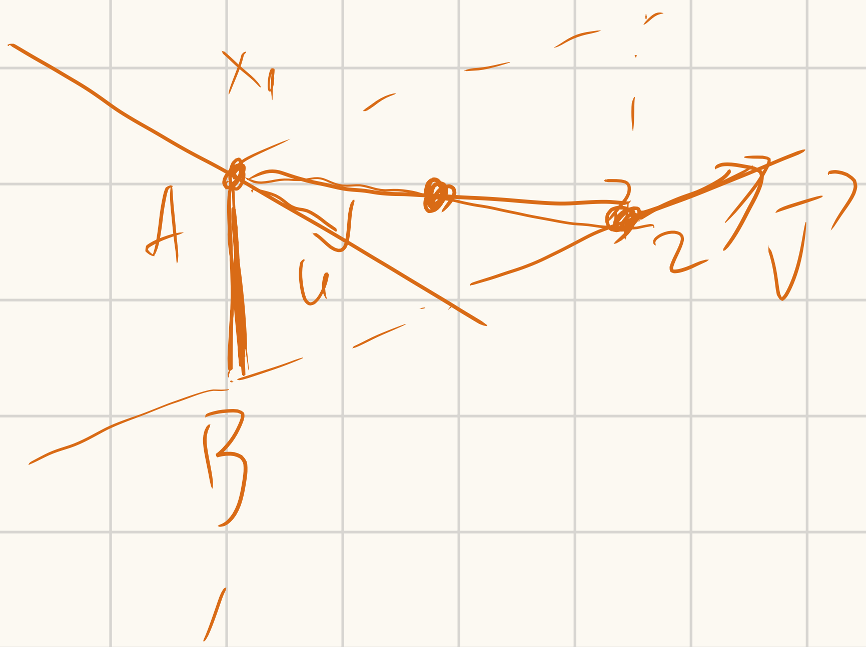
8.1A



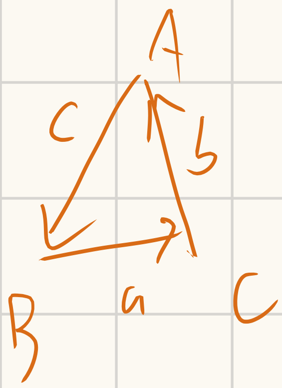
2.
$$\vec{m} = (2\vec{a}+\vec{b})\times(\vec{c}-\vec{a}) + (\vec{b}+\vec{c})\times(\vec{a}+\vec{b})$$
$$= 2\vec{a}\times\vec{c} + \vec{b}\times\vec{c} - 0 - \vec{b}\times\vec{a} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a}$$
$$+ 0 + \vec{c}\times\vec{b}$$
$$= \vec{a}\times\vec{c} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} = (1-2, -(1+1), 2+1)$$
$$= (-1, -2, 3)$$

3. $|a| = \sqrt{11^2+10^2+12^2} = \sqrt{121+100+144} = \sqrt{365}$

$|b| = 5$

$$\cos\langle\vec{a},\vec{b}\rangle = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{44+36}{|\vec{a}||\vec{b}|} = \frac{80}{\sqrt{365}\times 5} = \frac{16}{\sqrt{365}}$$

$a\cos\langle\vec{a},\vec{b}\rangle = 16$

4. 

$\vec{a} = \vec{BC} = (2, 2, 9)$ $\quad a = \sqrt{4+4+81} = \sqrt{89}$

$\vec{b} = \vec{CA} = (-4, 0, -6)$ $\quad b = \sqrt{16+36} = \sqrt{52}$

$\vec{c} = \vec{AB} = (2, -2, -3)$ $\quad c = \sqrt{4+4+9} = \sqrt{17}$

$$\begin{array}{r} 89 \\ 69 \\ 8 \\ \hline 20 \end{array}$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{52+17-89}{2\sqrt{52\times17}} = \frac{-10}{\sqrt{52\times17}}$$

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{100}{52\times17}} = \sqrt{\frac{784}{52\times17}}$$

$$h = b\sin A = \sqrt{\frac{784}{17}}$$

$$S = \frac{1}{2}bc\sin A = \frac{1}{2}\sqrt{784}$$

5. $\lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\vec{c} = 0$, $\lambda_1\lambda_2\lambda_3$

写作矩阵

$$A\vec{x}=\vec{0}$$

$$\begin{bmatrix} -1 & 3 & 2 \\ 2 & -3 & -4 \\ -3 & 12 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\vec{0}$$

扩增矩阵

$$\begin{bmatrix} -1 & 3 & 2 & 0 \\ 2 & -3 & -4 & 0 \\ -3 & 12 & 6 & 0 \end{bmatrix} \sim \begin{array}{c} \begin{matrix} 2 & -6 & -4 & 0 \end{matrix} \\ \begin{bmatrix} 1 & -3 & -2 & 0 \\ 2 & -3 & -4 & 0 \\ -1 & 4 & 2 & 0 \end{bmatrix} \end{array}$$

$$\sim \begin{bmatrix} 1 & -3 & -2 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

解为 $\lambda_1=3\lambda_2+2\lambda_3=2\lambda_3$

$\lambda_2=0$

λ_3 自由变量

故 $2\lambda_3\vec{a}+0\vec{b}+\lambda_3\vec{c}=\vec{0}$

$\vec{c}=-2\vec{a}+0\vec{b}$

这同时证明了 $\vec{a},\vec{b},\vec{c}$ 共面

6. $\begin{cases} (\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b})=0 & ① \\ (\vec{a}-4\vec{b})\cdot(7\vec{a}-2\vec{b})=0 & ② \end{cases}$

由①得 $7a^2+(21-5)\vec{a}\cdot\vec{b}-15b^2=0$

$7a^2+16\vec{a}\cdot\vec{b}-15\vec{b}^2=0$ ③

由②得 $7a^2+(-28-2)\vec{a}\cdot\vec{b}+8b^2=0$

$7a^2-30\vec{a}\cdot\vec{b}+8b^2=0$ ④

④-③得 $-46\vec{a}\cdot\vec{b}+23b^2=0$

$2\vec{a}\cdot\vec{b}=b^2$ ⑤

代入④得 $7a^2-30\vec{a}\cdot\vec{b}+8\cdot2\vec{a}\cdot\vec{b}=0$

$7a^2-14\vec{a}\cdot\vec{b}=0$

$2\vec{a}\cdot\vec{b}=a^2$

可见 $|\vec{a}|=|\vec{b}|$

$2ab\cos\theta=a^2$

$\cos\theta=\frac{1}{2}$

$\theta=30^\circ$

7. $\vec{x}=\lambda\vec{a}=\lambda\begin{bmatrix}2\\-1\\2\end{bmatrix}$

$\vec{a}\cdot\vec{x}=\lambda(4+1+4)=9\lambda=-18$

$\lambda=-2$

$\vec{x}=-2\begin{bmatrix}2\\-1\\2\end{bmatrix}=\begin{bmatrix}-4\\2\\-4\end{bmatrix}$

8.2 A

2、$\vec{n_1}=\begin{bmatrix}2\\1\\3\end{bmatrix}$ $\vec{n_2}=\begin{bmatrix}3\\2\\1\end{bmatrix}$

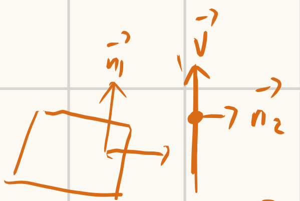
$$\vec{n}=\vec{n_1}\times\vec{n_2}=\begin{vmatrix}i & j & k\\2 & 1 & 3\\3 & 2 & 1\end{vmatrix}=\begin{bmatrix}1-6\\-(2-9)\\4-3\end{bmatrix}=\begin{bmatrix}-5\\7\\1\end{bmatrix}$$

取 $x=1$ 时 $\begin{cases}y+3z=-2\\2y+z=1\end{cases}$ 解出 $\begin{cases}y=1\\z=-1\end{cases}$

提点向式 $\dfrac{x-1}{-5}=\dfrac{y-1}{7}=\dfrac{z+1}{1}$

$$\begin{cases}x=-5t+1\\y=7t+1\\z=t-1\end{cases}$$

3、



设 $\vec{n_1}$ 为已知平面法向量，$\vec{n_2}$ 为所求平面法向量，$\vec{v}$ 为直线方向向量

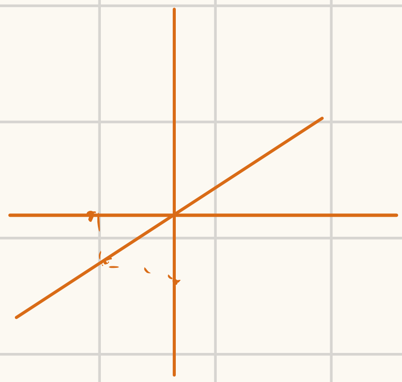
$$\vec{n_2}=\vec{n_1}\times\vec{v}=\begin{bmatrix}i & j & k\\3 & 2 & -1\\2 & -3 & 2\end{bmatrix}=\begin{bmatrix}4-3\\-(6+2)\\-9-4\end{bmatrix}=\begin{bmatrix}1\\-8\\-13\end{bmatrix}$$

平面：$x-1-8(y+2)-13(z-2)=0$

4、

直线方向向量 $\vec{v}=\vec{n_1}\times\vec{n_2}=\begin{bmatrix}i & j & k\\3 & 1 & -1\\2 & -1 & 4\end{bmatrix}=\begin{bmatrix}4-1\\-(12+2)\\-3-2\end{bmatrix}=\begin{bmatrix}3\\-14\\-5\end{bmatrix}$

夹角正弦 $\sin\theta=\cos\langle\vec{v},\vec{n_3}\rangle=\dfrac{\vec{v}\cdot\vec{n_3}}{|\vec{v}||\vec{n_3}|}=\dfrac{3+14\times8-15}{\sqrt{9+14^2+25}\sqrt{1+64+9}}=\dfrac{100}{\sqrt{230}\sqrt{74}}$

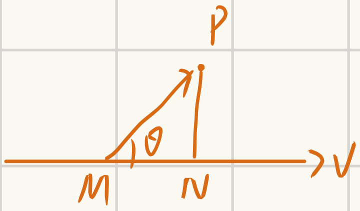


5、$\dfrac{x}{-\frac{5}{6}}+\dfrac{y}{-5}+\dfrac{z}{-\frac{5}{6}}=1$

设平面为 $\dfrac{x}{\frac{D}{6}}+\dfrac{y}{D}+\dfrac{z}{\frac{D}{6}}=1$

$V=\frac{1}{3}\times\frac{1}{2}\times\frac{D}{6}\times D\times\frac{D}{6}=\frac{D^3}{6^3}=1$

$D=\pm 6$

6. 

$\frac{\vec{N}\times\vec{MP}}{|\vec{V}|\cancel{|MP|}}\cdot\cancel{MP}=NP$

$$\vec{v}=\begin{bmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}=\begin{bmatrix} 1-1 \\ -(-2-1) \\ 2+1 \end{bmatrix}=\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$$

$x=-2,\ y=8$

直线上一点 $M(-2,1,1)$

$\vec{MP}=(5,-2,2)$

$$d=\frac{|\vec{v}\times\vec{MP}|}{|\vec{v}|}=\frac{\begin{vmatrix} i & j & k \\ 0 & 3 & 3 \\ 5 & -2 & 2 \end{vmatrix}}{3\sqrt{2}}=\frac{|(6+6,\ -(0-15),\ 0-15)|}{3\sqrt{2}}$$

$$=\frac{|(12,15,-15)|}{3\sqrt{2}}=\frac{3\sqrt{66}}{3\sqrt{2}}=\sqrt{33}$$

7、 $M(1,2,-2)$

$\vec{MP}=(0,0,5)$

$\vec{v}=(2,-3,1)$

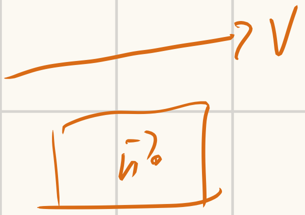
$$\vec{MP}\times\vec{v}=\begin{vmatrix} i & j & k \\ 0 & 0 & 5 \\ 2 & -3 & 1 \end{vmatrix}=(-15,\ -(0-10),\ 0)=(-15,10,0)$$

$|\vec{MP}\times\vec{v}|=5\sqrt{13}$

$|\vec{v}|=\sqrt{4+9+1}=\sqrt{14}$

$d=\frac{|\vec{MP}\times\vec{v}|}{|\vec{v}|}=\frac{5\sqrt{13}}{\sqrt{14}}=\frac{5}{14}\sqrt{182}$

8.



直线向 $\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 3 & -2 & -2 \end{vmatrix} = (-2-2, -(-4+3), -4-3)$

$= (-4, 1, -7)$

$\vec{n_4} = \vec{v} \times \vec{n_3} = \begin{vmatrix} i & j & k \\ -4 & 1 & -7 \\ 3 & 2 & 3 \end{vmatrix} = (3+14, -(-12+21), -8-3)$

$= (17, -9, -11)$

令 $x=1$ $\begin{cases} 2+y-z-2=0 \\ 3-2y-2z+1=0 \end{cases}$ 得 $y=z=1$

$17(x-1) - 9(y-1) - 11(z-1) = 0$

9.

$\vec{v_1} = \begin{vmatrix} i & j & k \\ 2 & -4 & 1 \\ 1 & 3 & 0 \end{vmatrix} = (0-3, -(0-1), 6+4) = (-3, 1, 10)$

$\vec{v_2} = (4, -1, 2)$

$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} i & j & k \\ -3 & 1 & 10 \\ 4 & -1 & 2 \end{vmatrix} = (2+10, -(-6-40), 3-4)$

$= (12, 46, -1)$

$12(x+1) + 46(y+4) - (z-3) = 0$

10、$\vec{v_1} = (2, 9, 6)$ $\vec{v_2} = (3, 6, 2)$

$\cos\langle\vec{v_1}, \vec{v_2}\rangle = \frac{\vec{v_1}\cdot\vec{v_2}}{|\vec{v_1}||\vec{v_2}|} = \frac{6+54+12}{\sqrt{4+81+36}\sqrt{9+36+4}} = \frac{72}{\sqrt{121}\sqrt{49}}$

$= \frac{72}{11\times7} = \frac{72}{77}$

11、

设直线的方向向量为 $\vec{v}$

$$\vec{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = (2-1, -(1+2), -1-4) = (1, -3, -5)$$

$3x+y-5=0$

令 $y=-1$ 则 $x=2$　$z=-6$ 过点 $(2,-1,-6)$

直线的参数方程

$$\begin{cases} x = 2+t \\ y = -1-3t \\ z = -6-5t \end{cases}$$

代入平面方程

$2+t-1-3t-6-5t=9$

$-7t-5=9$

$7t=-14$

$t=-2$

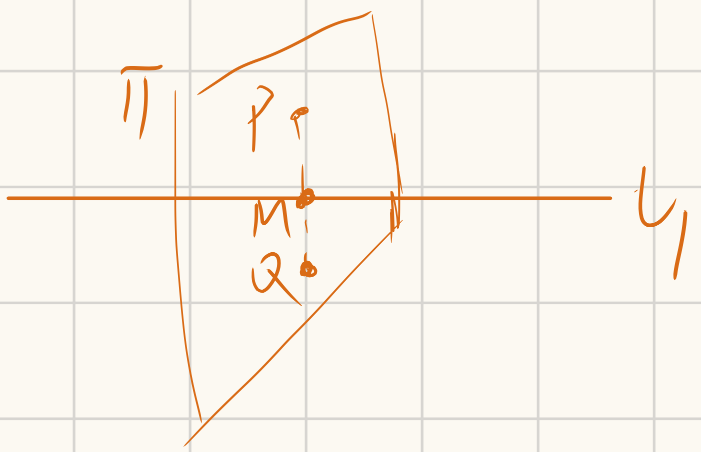
交点：$(0,5,4)$

夹角的θ　$\vec{n}=(1,1,1)$

$\sin\theta=|\cos\langle\vec{n},\vec{v}\rangle|=\left|\frac{\vec{n}\cdot\vec{v}}{|\vec{n}||\vec{v}|}\right|=\left|\frac{1-3-5}{\sqrt{3}\sqrt{1+9+25}}\right|=\frac{7}{\sqrt{3}\sqrt{35}}$

$=\frac{7}{105}\sqrt{105}$

12.



平面π为 $2(x-4)+4(y-3)+5(z-10)=0$

下求π与直线 L_1 交点

L_1 参数方程：

$$\begin{cases} x=1+2t, \\ y=2+4t, \\ z=3+5t, \end{cases}$$

代入π方程得

$$2(1+2t-4)+4(2+4t-3)+5(3+5t-10)=0$$

$$2(2t-3)+4(4t-1)+5(5t-7)=0$$

$$4t-6+16t-4+25t-35=0$$

$$45t=45$$

$$t=1$$

交点M(3, 6, 8)

交点为 $\vec{OM}+\vec{PM}=2\vec{OM}-\vec{OP}=(6,12,16)-(4,3,10)$

$$=(2,9,6)$$

8.3A

1.

$$\begin{cases} x^2+y^2+z=2 \\ x^2-2x+y^2-2y-z=-2 \end{cases}$$

① 在yz平面上

$$x=\pm\sqrt{2-y^2-z}$$

$$2-y^2-z\pm2\sqrt{2-y^2-z}+y^2-2y-z+2=0$$

$$\begin{cases} 4-2z\pm2\sqrt{2-y^2-z}-2y=0 \\ x=0 \end{cases}$$

② xz 平面上 消 y

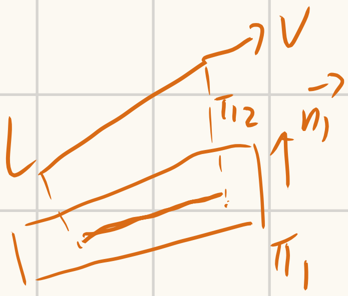
由于 x y 对称, ①中 x 换为 y 即可

$$\begin{cases} 4-2z\pm 2\sqrt{2-x^2-z}-2x=0 \\ y=0 \end{cases}$$

③ xy 平面上 消 z

$$\begin{cases} 2-x^2-y^2=(x-1)^2+(y-1)^2 \\ z=0 \end{cases}$$

2、



$$\vec{v}=\begin{vmatrix} i & j & k \\ 2 & -4 & 1 \\ 3 & -1 & -2 \end{vmatrix}=(8+1,\ -(4-3),\ -2+12)=(9,7,10)$$

$$\vec{n_2}=\vec{n_1}\times\vec{v}=\begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 9 & 7 & 10 \end{vmatrix}=(-10-7,\ -(40-9),\ 28+9)$$

$$=(-17,-31,37)$$

直线l上过点

$$\left(1,-\frac{2}{9},-\frac{26}{9}\right)$$

$$2-4y+z=0$$

$$-6-y-2z=0$$

$$4-8y+2z=0$$

$$-2-9y=0 \quad y=-\frac{2}{9}$$

$$2z=-6-y=-6+\frac{2}{9} \qquad z=-3+\frac{1}{9}=\frac{-27+1}{9}=\frac{-26}{9}$$

$$\begin{cases} -17(x-1)-31\left(y+\frac{2}{9}\right)+37\left(z+\frac{26}{9}\right) \\ 4x-y+z=1 \end{cases}$$

4、

① yz平面，消x

$$\begin{cases} 6x+2y-2z+2=0 \\ 6x-3y+12z-6=0 \end{cases} \Rightarrow 5y-14z+8=0$$

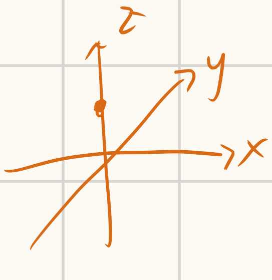
② xz平面 消y

$$5x+3z-1=0$$

③ xy平面，消z

$$\begin{cases} 12x+4y-4z+4=0 \\ 2x-y+4z-2=0 \end{cases} \Rightarrow 14x+3y+2=0$$

5、



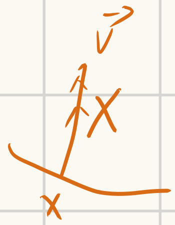
$$\sqrt{x^2+y^2+(z-1)^2}=\frac{1}{2}|z-4|$$

$$4x^2+4y^2+4(z-1)^2=(z-4)^2$$

$$4x^2+4y^2+4z^2-8z+4=z^2-8z+16$$

$$4x^2+4y^2+3z^2=12$$

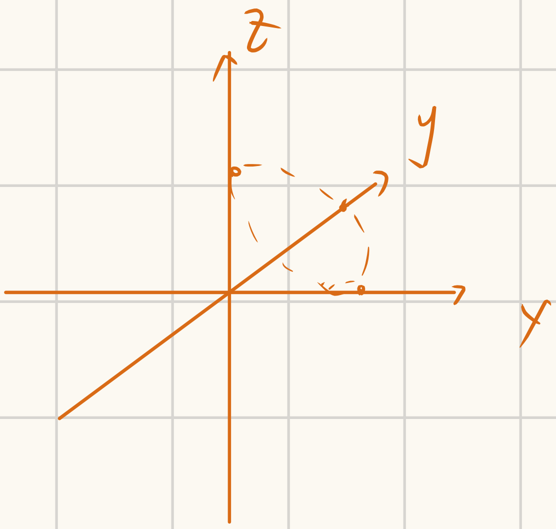
6.



$$\begin{cases} f(x,z)=0 \\ y=0 \\ \dfrac{X-x}{l}=\dfrac{Y-y}{m}=\dfrac{Z-z}{n} \end{cases}$$

7. $$\begin{cases} x^2+y^2=R^2 \\ z=0 \\ \dfrac{X-x}{1}=\dfrac{Y-y}{2}=\dfrac{Z-z}{3} \end{cases}$$

9.



准线 $$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1 \\ \dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1 \end{cases}$$

锥面 $$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1 \\ \dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1 \\ \dfrac{X-0}{x-0}=\dfrac{Y-0}{y-0}=\dfrac{Z-0}{z-0}=t \end{cases}$$

$$x=\frac{X}{t} \quad y=\frac{Y}{t} \quad z=\frac{Z}{t}$$

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}+\frac{Z^2}{c^2}=t^2=\left(\frac{X}{a}+\frac{Y}{b}+\frac{Z}{c}\right)^2$$

即 $$\frac{1}{ab}XY+\frac{1}{bc}YZ+\frac{1}{ac}XZ=0$$