光本质是电磁波

折射反射问题本质上是电磁波在不同介质间的传播问题

一、先研究电磁波

一般情形下线性介质中的 Maxwell's E.Q

$$\begin{cases} \nabla\cdot\vec{D}=\rho_0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{H}=\vec{J}_0+\dfrac{\partial\vec{D}}{\partial t} \end{cases}$$

性能方程： $\vec{D}=\underbrace{\varepsilon_r\varepsilon_0\vec{E}}_{=\varepsilon\vec{E}}$ $\quad\vec{B}=\underbrace{\mu_r\mu_0\vec{H}}_{=\mu\vec{H}}$ (线性介质中成立)

一般地 $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$ $\quad\vec{B}=\mu_0\vec{H}+\mu_0\vec{M}$ $\quad\vec{M}=\chi\vec{H}$

$\oint\vec{P}\cdot d\vec{S}=-q'$ $\quad\oint\vec{M}\cdot d\vec{l}=I'$ $\quad\vec{B}=\mu_0(1+\chi)\vec{H}=\mu_0\mu_r\vec{H}$

$\oint\vec{E}\cdot d\vec{S}=\dfrac{q_0+q'}{\varepsilon_0}$ $\quad\oint\vec{H}\cdot d\vec{l}=I_0$

在处处 $\rho_0=0$，$\vec{J}_0=0$ 的各向同性线性介质中有

$$\begin{cases} \nabla\cdot\vec{D}=0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{H}=\dfrac{\partial\vec{D}}{\partial t} \end{cases}$$

代入性能方程

$$\begin{cases} \nabla\cdot\vec{E}=0 & (1) \\ \nabla\times\vec{E}=-\mu\dfrac{\partial\vec{H}}{\partial t} & (2) \\ \nabla\cdot\vec{H}=0 & (3) \end{cases}$$

$\nabla\times\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t}$ ④

$\nabla\times$②得

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}$$

代入 $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$ ②, $\nabla\cdot\vec{E} = 0$ ①

$$-\mu\frac{\partial}{\partial t}(\nabla\times\vec{H}) = 0 - \nabla^2\vec{E}$$

$$-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = -\nabla^2\vec{E}$$

$$\frac{1}{\mu\varepsilon}\nabla^2\vec{E} - \frac{\partial^2\vec{E}}{\partial t^2} = 0$$

同理

$$\frac{1}{\mu\varepsilon}\nabla^2\vec{H} - \frac{\partial^2\vec{H}}{\partial t^2} = 0$$

显然这是波动方程

解为 $\vec{E} = \vec{E_0}e^{i(\omega t - \vec{k}\cdot\vec{r} + \varphi_0)}$ ⑤

$\vec{H} = \vec{H_0}e^{i(\omega t - \vec{k}\cdot\vec{r} + \varphi_0)}$ ⑥

其中 $v = \frac{1}{\sqrt{\mu\varepsilon}}$ $k = \frac{2\pi}{\lambda}$ $T = \frac{\lambda}{v} = \lambda\sqrt{\mu\varepsilon}$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{\lambda\sqrt{\mu\varepsilon}}$

解⑤⑥代入②得

$$\vec{k}\times\vec{E} = \mu\omega\vec{H}$$

reason: $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$

$$\nabla\times(-i\vec{k}\cdot\vec{r}\vec{E_0})e^{i(\omega t - \vec{k}\cdot\vec{r} + \varphi_0)} = -\mu\vec{H_0}i\omega e^{i(\omega t - \vec{k}\cdot\vec{r} + \varphi_0)}$$

$$-i\,\nabla(\vec{k}\cdot\vec{r})\times\vec{E_0}\,e^{i(\omega t-\vec{k}\cdot\vec{r})+\varphi_0)} = -\mu\vec{H_0}\,i\omega\,e^{i(\omega t-\vec{k}\cdot\vec{r}+\varphi_0)}$$

reason: $\nabla\times\left(\varphi(r)\,\vec{E_0}\right) = \left(\nabla\varphi(r)\right)\times\vec{E_0}$

$$\nabla(\vec{A}\cdot\vec{B}) = \nabla_A(\vec{A}\cdot\vec{B}) + \nabla_B(\vec{A}\cdot\vec{B}) = (\vec{A}\cdot\vec{B})\nabla_A + (\vec{A}\cdot\vec{B})\nabla_B$$

$$\vec{A}\times(\nabla_B\times\vec{B}) = \nabla_B(\vec{A}\cdot\vec{B}) - (\nabla_B\cdot\vec{A})\vec{B}$$

$$\vec{B}\times(\nabla_A\times\vec{A}) = \nabla_A(\vec{A}\cdot\vec{B}) - (\nabla_A\cdot\vec{B})\vec{A}$$

so

$$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + (A\cdot\nabla)\vec{B} + \vec{B}\times(\nabla\times\vec{A}) + (\vec{B}\cdot\nabla)\vec{A}$$

$$\nabla(\vec{k}\cdot\vec{r}) = \vec{k}\times(\nabla\times\vec{r}) + (\vec{k}\cdot\nabla)\vec{r} + \vec{r}\times(\nabla\times\vec{k}) + (\vec{r}\cdot\nabla)\vec{k}$$

$$= 0 + k_i\partial_i x_i\vec{e_i} + 0 + 0$$

$$= k_i\vec{e_i} = \vec{k}$$

so $\nabla(\vec{k}\cdot\vec{r})\times\vec{E_0} = \vec{k}\times\vec{E_0}$

化简得

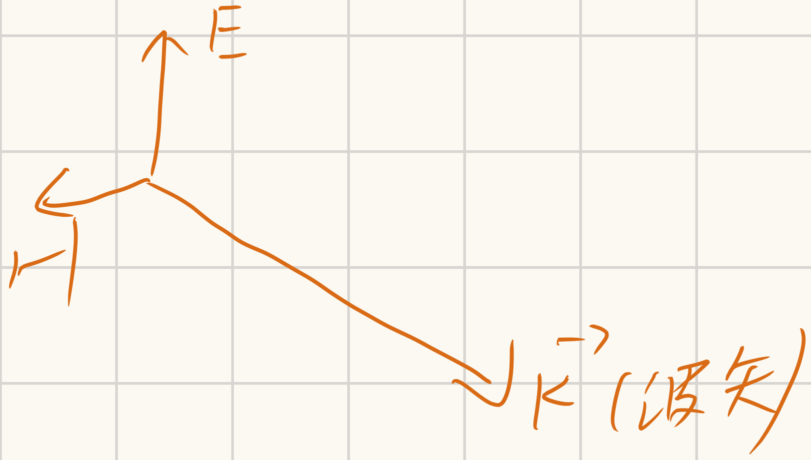
$$\vec{k}\times\vec{E} = \mu\omega\vec{H}$$

$v = \frac{1}{\sqrt{\mu\varepsilon}}$ $\quad n = \sqrt{\mu_r\varepsilon_r} \approx$

(其中 $k = \frac{2\pi}{\lambda}$ $\quad T = \frac{\lambda}{v} = \lambda\sqrt{\mu\varepsilon}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{\lambda\sqrt{\mu\varepsilon}}$)

可得图像



$$|E| = \frac{\mu\omega}{k}|H| \qquad E = \mu\cdot vH = \frac{\mu}{\sqrt{\mu\varepsilon}}H = \sqrt{\frac{\mu}{\varepsilon}}H = \sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}}H$$

$\mu_r \approx 1$ $\quad n = \sqrt{\varepsilon_r\mu_r} \approx \sqrt{\varepsilon_r}$

$$E_0 \approx \frac{1}{n}\sqrt{\frac{\mu_0}{\varepsilon_0}}\, H_0$$

$$H_0 \approx n\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0 \qquad H_0 \propto n E_0$$

二、边界条件

1. 由 Maxwell 之积分形式

$$\oint_S \vec{D}\cdot d\vec{S} = q_0$$

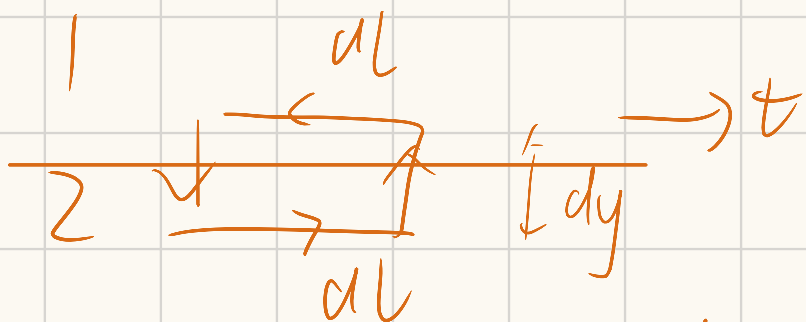
$$\oint_L \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\oint_{S_1}\vec{B}\cdot d\vec{s}$$

$$\oint_{S闭} \vec{B}\cdot d\vec{S} = 0$$

$$\oint \vec{H}\cdot d\vec{l} = I + I' = I_0 + \oint \frac{\partial \vec{D}}{\partial t}\cdot d\vec{S}$$

不难得边界条件（n代表法向，t代表切向）

$$(\vec{D}_2-\vec{D}_1)\cdot\vec{e}_{21} = \sigma_0$$



$+E_{2t}\,dl - E_{1t}\,dl = -$

$dy \to 0 \quad E_{1t} = E_{2}$

$E_{2t} = E_{1t}$

$$(\vec{B}_2-\vec{B}_1)\cdot\vec{e}_{21} = \vec{0}$$

$$(\vec{H}_2-\vec{H}_1)\times\vec{e}_n = \vec{\alpha}_0$$ （面电流密度，$\vec{e}_n$ 面矢量）

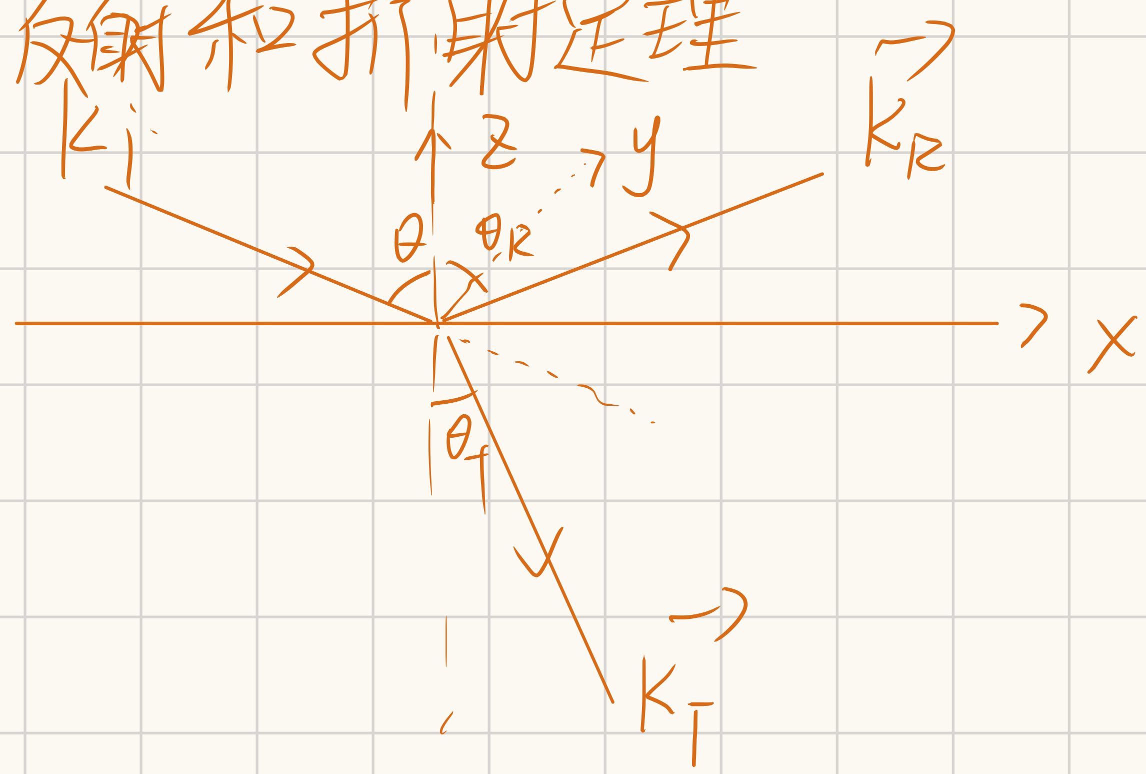
在 $\vec{J_s}=0$ $\vec{\rho_0}=0$ 时

$$\begin{cases} D_{2n}=D_{1n} \\ E_{2t}=E_{1t} \\ B_{2n}=B_{1n} \\ H_{2t}=H_{1t} \end{cases}$$

结合性质能 $\vec{D}=\varepsilon\vec{E}$ $\vec{B}=\mu\vec{H}$

$$\begin{cases} \varepsilon_2 E_{2n}=\varepsilon_1 E_{1n} \\ E_{2t}=E_{1t} \\ \mu_2 H_{2n}=\mu_1 H_{1n} \\ H_{2t}=H_{1t} \end{cases}$$

II 反射和折射定理



x, y, z 方向如图所示

I角标入射光，R角标反射光，T角标透射光

在 $z=0$ 处有 $\tilde{u}_I + \tilde{u}_R = \tilde{u}_T$

(从逻辑上讲此处反射光和透射光的频率都与入射光的相同是需要证明的，我们暂时略去) $v=\lambda f$ $v=\frac{w}{k}$

w 同，$v_1 k_1 = v_1 k_R = v_2 k_T \Rightarrow \frac{k_I}{n_1} = \frac{k_R}{n_1} = \frac{k_T}{n_2}$

$$A_I e^{i(wt-\vec{k_I}\cdot\vec{r}+\varphi_I)} + A_R e^{i(wt-\vec{k_R}\cdot\vec{r}+\varphi_R)} = A_T e^{i(wt-\vec{k_T}\cdot\vec{r}+\varphi_R)}$$

约掉 wt，$\forall \vec{r}$，欲使上式成立

需有 $\vec{k_I}\cdot\vec{r} = \vec{k_R}\cdot\vec{r} = \vec{k_T}\cdot\vec{r}$

可立刻得 $k_{1x} = k_{2x} \Rightarrow \theta_1 = \theta_R$

$k_{1x} = k_{Tx} \Rightarrow k_1 \sin\theta_1 = k_T \sin\theta_2 \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$

三、光的折射反射

结合一、二章

假设在

无自由电荷 ($\rho_0=0$)，无自由电流 ($\vec{J}_0=0$)

的各向同性的线性介质中有

① 电磁波的形式为

$$\vec{E}=\vec{E}_0 e^{i(\omega t-\vec{k}\cdot\vec{r}+\varphi_0)} \quad (5)$$

$$\vec{H}=\vec{H}_0 e^{i(\omega t-\vec{k}\cdot\vec{r}+\varphi_0)} \quad (6)$$

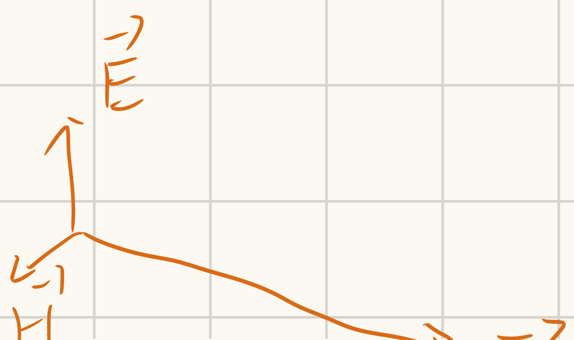
$$\vec{k}\times\vec{E}=\mu\omega\vec{H}$$

$$v=\frac{1}{\sqrt{\mu\varepsilon}} \quad n=\sqrt{\mu_r\varepsilon_r}\approx\sqrt{\varepsilon_r}$$

(其中 $k=\frac{2\pi}{\lambda}$ $T=\frac{\lambda}{v}=\lambda\sqrt{\mu\varepsilon}$

$\omega=\frac{2\pi}{T}=\frac{2\pi}{\lambda\sqrt{\mu\varepsilon}}$)

$$E_0\approx\frac{1}{n}\sqrt{\frac{\mu_0}{\varepsilon_0}}H_0$$



$H_0 = n\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0$ $\vec{k}$

② 边界条件

$$\begin{cases} \varepsilon_2 E_{2n} = \varepsilon_1 E_{1n} \\ E_{2t} = E_{1t} \\ \mu_2 H_{2n} = \mu_1 H_{1n} \\ H_{2t} = H_{1t} \end{cases} \Rightarrow \begin{cases} \\ \end{cases}$$

③ 折射反射

$$\begin{cases} \theta_1 = \theta_R \\ n_1 \sin\theta_1 = n_2 \sin\theta_T \end{cases}$$

all cards are ready!

由于 E, H 都垂直于 $\vec{k}$，取正交确定关系有一个方向

称平行于研究平面的方向为 P 向 parallel

垂直于研究平面为 S 向

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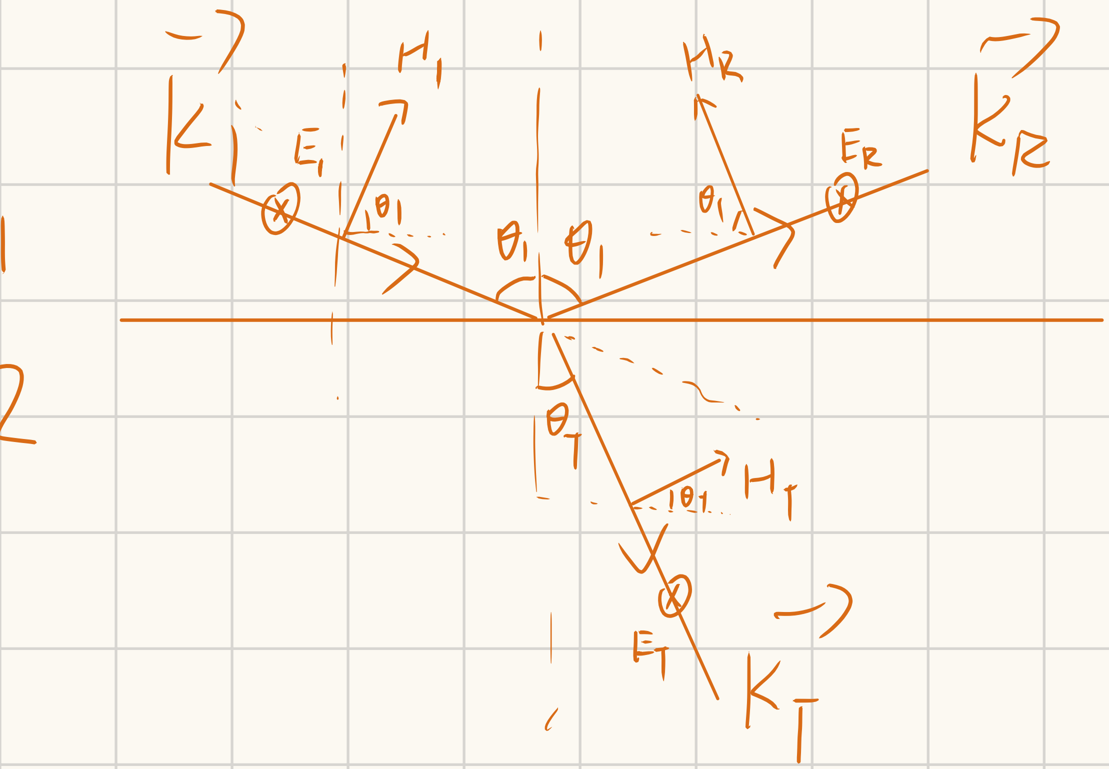
I E为S面



E_n 面: $0=0$

E_t : $E_1+E_R=E_T$

H_n : $\mu_1 H_1 \sin\theta_1 + \mu_1 H_R \sin\theta_1 = \mu_2 H_T \sin\theta_T$

H_t : $H_1\cos\theta_1 - H_R\cos\theta_1 = H_T\cos\theta_T$

$\mu v H = E \Rightarrow H = \frac{n}{\mu}E = \frac{E}{\mu v}$

$E_1+E_R=E_T$

$$\frac{\sin\theta_1}{v_1}E_1 + \frac{\sin\theta_1}{v_1}E_R = \frac{\sin\theta_T}{v_2}E_T$$

$$\frac{\cos\theta_1}{\mu_1 v_1}(E_1 - E_R) = \frac{\cos\theta_T}{\mu_2 v_2}E_T$$

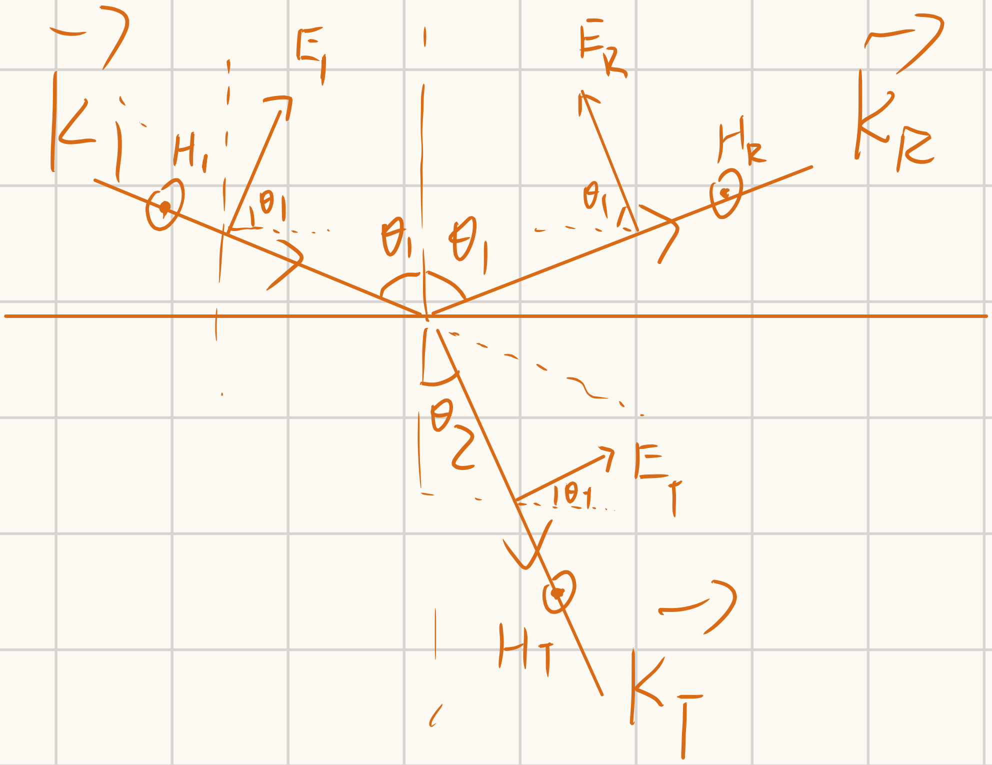
$$E_1 + E_R = \frac{v_1\sin\theta_T}{v_2\sin\theta_1}E_T = p E_T$$

$$E_1 - E_R = \frac{\mu_1 v_1\cos\theta_T}{\mu_2 v_2\cos\theta_1}E_T = q E_T$$

令 $\alpha = \dfrac{\cos\theta_T}{\cos\theta_1}$ $\beta = \dfrac{\mu_1 v_1}{\mu_2 v_2} \approx \dfrac{n_2}{n_1}$ $\beta = \dfrac{n_2}{n_1}$ $\mu \approx 1$

$$\left.\begin{aligned} E_1 - E_R &= \alpha\beta E_T \\ E_1 + E_R &= E_T \end{aligned}\right\} \Rightarrow \begin{cases} E_T = \dfrac{2}{1+\alpha\beta} E_1 \approx \dfrac{2}{1+\frac{\cos\theta_2}{\cos\theta_1}\frac{n_2}{n_1}} = \dfrac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2 \ldots} \\ E_R = \dfrac{1-\alpha\beta}{1+\alpha\beta} E_1 \approx \dfrac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} E_1 \end{cases}$$

II E为P方向



E_n: $\varepsilon_1 \sin\theta_1 (E_1 + E_R) = \varepsilon_2 \sin\theta_2 E_T$

E_t: $E_1\cos\theta_1 - E_R\cos\theta_1 = E_T\cos\theta_2$

H_n: $0 = 0$

H_t: $H_1 + H_R = H_T \Rightarrow \dfrac{E_1}{\mu_1 v_1} + \dfrac{E_R}{\mu_1 v_1} = \dfrac{E_T}{\mu_2 v_2}$

$H = \dfrac{1}{\mu v} E$

$$E_1 + E_R = \frac{\varepsilon_2 \sin\theta_2}{\varepsilon_1 \sin\theta_1} E_T$$

$$E_1 - E_R = \frac{\cos\theta_2}{\cos\theta_1} E_T = \alpha E_T$$

$$E_1 + E_R = \frac{u_1 v_1}{u_2 v_2} E_T = \beta E_T$$

$$E_T = \frac{2}{\beta+\alpha} E_I = \frac{2}{\frac{n_2}{n_1} + \frac{\cos\theta_2}{\cos\theta_1}} = \frac{2 n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$E_R = \frac{\beta-\alpha}{\beta+\alpha} E_I = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 - n_1 \cos\theta_2}$$

（注意 格里菲斯书上与此不同

因为 E_R 的正方向选取不同）

综上 $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$，$\beta = \frac{u_1 v_1}{u_2 v_2} \approx \frac{n_2}{n_1}$

p: $r_p = \frac{\beta-\alpha}{\beta+\alpha}$

$t_p = \frac{2}{\beta+\alpha}$

t: $r_p = \frac{1-\alpha\beta}{1+\alpha\beta}$

$t_p = \frac{2}{1+\alpha\beta}$