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MATLAB LABORATORY WORKBOOK

Subject Code	:	**U18MAI1201L - Linear Algebra and Calculus**
Regulations	:	**R18**
Class	:	**I B.E/B.Tech Branches**

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WORKSHEET-1

INTRODUCTION TO MATLAB

1.1 OBJECTIVES

a. To know the history and features of MATLAB

b. To know the local environment of MATLAB

1.2 Introduction

MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming. Using MATLAB, you can analyze data, develop algorithms, and create models and applications. The language, tools, and built-in math functions enable you to explore multiple approaches and reach a solution faster than with spread sheets or traditional programming languages, such as C/C++ or Java. You can use MATLAB for a range of applications, including signal processing and communications, image and video processing, control systems, test and measurement, computational finance, and computational biology. More than a million engineers and scientists in industry and academia use MATLAB, the language of technical computing.

A matrix is a two-dimensional array of numbers.

In MATLAB, you create a matrix by entering elements in each row as comma or space delimited numbers and using semicolons to mark the end of each row.

For example, let us create a 4-by-5 matrix *a* −

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8]
```

MATLAB will execute the above statement and return the following result −

```
a =
      1      2      3      4      5
      2      3      4      5      6
      3      4      5      6      7
      4      5      6      7      8
```

Referencing the Elements of a Matrix

To reference an element in the m^{th} row and n^{th} column, of a matrix *mx*, we write −

```
mx(m, n);
```

For example, to refer to the element in the 2nd row and 5th column, of the matrix *a*, as created in the last section, we type –

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

a(2,5)
```

MATLAB will execute the above statement and return the following result –

```
ans =  6
```

To reference all the elements in the mth column we type A(:,m).

Let us create a column vector v, from the elements of the 4th row of the matrix a –

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

v = a(:,4)
```

MATLAB will execute the above statement and return the following result –

```
v =
      4
      5
      6
      7
```

You can also select the elements in the mth through nth columns, for this we write –

```
a(:,m:n)
```

Let us create a smaller matrix taking the elements from the second and third columns –

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

a(:, 2:3)
```

MATLAB will execute the above statement and return the following result –

```
ans =
      2     3
      3     4
      4     5
      5     6
```

In the same way, you can create a sub-matrix taking a sub-part of a matrix.

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

a(:, 2:3)
```

MATLAB will execute the above statement and return the following result –

```
ans =
      2     3
```

```
       3       4
       4       5
       5       6
```

In the same way, you can create a sub-matrix taking a sub-part of a matrix.

For example, let us create a sub-matrix *sa* taking the inner subpart of a –

```
3      4       5
4      5       6

a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

sa = a(2:3,2:4)
```

MATLAB will execute the above statement and return the following result –

```
sa =
       3       4       5
       4       5       6
```

Deleting a Row or a Column in a Matrix

You can delete an entire row or column of a matrix by assigning an empty set of square braces [] to that row or column. Basically, [] denotes an empty array.

For example, let us delete the fourth row of a –

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

a( 4 , : ) = []
```

MATLAB will execute the above statement and return the following result –

```
a =
       1       2       3       4       5
       2       3       4       5       6
       3       4       5       6       7
```

Next, let us delete the fifth column of a –

```
a = [ 1 2 3 4 5; 2 3 4 5 6; 3 4 5 6 7; 4 5 6 7 8];

a(: , 5)=[]
```

MATLAB will execute the above statement and return the following result –

```
a =
       1      2      3      4
       2      3      4      5
       3      4      5      6
       4      5      6      7
```

Example

In this example, let us create a 3-by-3 matrix m, then we will copy the second and third rows of this matrix twice to create a 4-by-3 matrix.

Create a script file with the following code –

```
a = [ 1 2 3 ; 4 5 6; 7 8 9];

new_mat = a([2,3,2,3],:)
```

When you run the file, it displays the following result –

```
new_mat =
       4      5      6
       7      8      9
       4      5      6
       7      8      9
```

Task 1

1. Try these commands:

```
>> 3*5*6
```

```
>> z1 = 34; z2 = 17; z3 = -8;
```

```
>> z1/z2
```

```
>> z1-z3
```

```
>> z2+z3-z1
```

2. Determine the value of the expression a(b + c(c + d))a, where $a = 2$, $b = 3$, $c = -4$, $d = -3$.

```
>> a = 2; b = 3; c = -4; d = -3;
>> a*(b+c*(c+d))*a
```

3. Evaluate the MATLAB expression

```
>>1+2/3*4-5
```

```
>>1/2/3/4
```

```
>>1/2+3/4*5
```

```
>>5-2*3*(2+7)
```

>>(1+3)*(2-3)/3*4

>>(2-3*(4-3))*4/5

4. Calculate the expressions:

(1) sin 60^0 in radians

(2) sin 60^0 in degree

(3) e^{log4}

(4) cos 45^0 - sin 45^0

(5) log e^2

5. Create two vectors running from one to six and from six to one and then demonstrate the use of the dot arithmetical operations: s+t, s-t, st, s/t, s^2, 1/s, s/2, s+1

s=1:6

t=6:-1:1

(i) s+t

(ii) s-t

(iii) s*t

(iv) s/t

(v) s^2

(vi) 1/s

(vii) s/2

(ix) s+1

6. Construct the polynomial $y = (x + 2)^2(x^3 + 1)$ for values of x from minus one to one in steps of 0.1.

7. Find the roots of the polynomial $y = x^3 - 3x^2 + 2x$

WORKSHEET-2

MATRIX OPERATIONS -

ADDITION, MULTIPLICATION, TRANSPOSE, INVERSE

2.1 OBJECTIVES:

- To evaluate the addition, subtraction and multiplication of two Matrices.
- To evaluate the transpose and inverse of a Matrix.

2.2 MATRIX ADDITION

Example 1: Find the addition of the matrices $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 6 \\ 7 & 8 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 & 6 \\ 7 & 1 & -1 \\ 5 & 7 & -9 \end{pmatrix}$

```
A=[1 2 4 ; 0 5 6; 7 8 4];
B=[0 3 6; 7 1 -1; 5 7 9];
disp('The matrix A= ');A
disp('The matrix B= ');B
 %   to find sum of a and b, c=a+b;
disp('The sum of A and B is ');
C=A+B
```

Output:

```
    The matrix A=

A =

   1   2   4
   0   5   6
   7   8   4

The matrix B=
B =

   0   3   6
   7   1  -1
   5   7   9
```

The sum of A and B is

C =

```
     1     5    10
     7     6     5
    12    15    13
```

MATRIX MULTIPLICATION:

Example 2: Find the multiplication of the matrices $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 6 \\ 7 & 8 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 & 6 \\ 7 & 1 & -1 \\ 5 & 7 & -9 \end{pmatrix}$

```
A=[1 2 4 ; 0 5 6; 7 8 4];
B=[0 3 6; 7 1 -1; 5 7 9];
disp('The matrix A= ');A
disp('The matrix B= ');B
disp('The multiplication of A and B is ');
C=A*B
```

Output:

The matrix A=

A =

```
     1     2     4
     0     5     6
     7     8     4
```

The matrix B=

B =

```
     0     3     6
     7     1    -1
     5     7     9
```

The multiplication of A and B is

```
C =

  34   33   40
  65   47   49
  76   57   70
```

MATRIX TRANSPOSE AND INVERSE.

Example 3: Find the transpose and inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 6 \\ 7 & 8 & 4 \end{pmatrix}$

```
A=[1 2 4 ; 0 5 6; 7 8 4];
disp('The matrix A= ');A
%   to find transpose of A;
disp('The transpose of A is ');A'
%   to find inverse of A;
disp('The inverse of A  is ');
inv(A)
```

Output:

```
 The matrix A=
  A =
   1    2    4
   0    5    6
   7    8    4

The transpose of A is
ans =

   1    0    7
   2    5    8
   4    6    4

The inverse of A  is
ans =

   0.3333  -0.2857   0.0952
  -0.5000   0.2857   0.0714
   0.4167  -0.0714  -0.0595
```

Task 2

1. Find the addition and subtraction of the matrices $A = \begin{pmatrix} 5 & 2 & 12 \\ 0 & 19 & 16 \\ 17 & 8 & 6 \end{pmatrix}$ and

$$B = \begin{pmatrix} 8 & 13 & 8 \\ 15 & 10 & -1 \\ 5 & 9 & -9 \end{pmatrix}$$

2.Find the multiplication of the matrices $A = \begin{pmatrix} 11 & 2 & 14 \\ 10 & 5 & 60 \\ 17 & 8 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 10 & 8 & 6 \\ 7 & 1 & -11 \\ 9 & 7 & -9 \end{pmatrix}$

3. Find the transpose and inverse of the matrix $A = \begin{pmatrix} 1 & 22 & 4 \\ 18 & 5 & 16 \\ 7 & 28 & 4 \end{pmatrix}$

4. Find the matrix multiplication of the two matrices $A = \begin{pmatrix} 6 & 2 & 4 \\ 8 & 5 & 6 \\ 7 & 8 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 12 & 21 & 14 \\ 11 & 15 & 36 \\ 17 & 68 & 24 \end{pmatrix}$.

5. Find the transpose and inverse of the matrix $A = \begin{pmatrix} 91 & 24 & 42 \\ 18 & 52 & 48 \\ 72 & 45 & 64 \end{pmatrix}$

WORKSHEET-3

RANK OF A MATRIX AND SOLUTION OF A SYSTEM OF LINEAR EQUATIONS

3.1 OBJECTIVES:

- To find the rank of a matrix
- To solve the system of linear equations
- To find the echelon form of the matrix.

3.2 PROGRAM

RANK OF THE MATRIX

Example 1: Find the rank of the matrices $A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$

```
A=[1 2 3 2; 2 3 5 1; 1 3 4 5]
disp('The matrix A= ');A
%  To find rank of A
disp('The rank  of A  is ');
C=rank(A)
```

Output:

```
The matrix A=

A =

   1   2   3   2
   2   3   5   1
   1   3   4   5

The rank  of A  is

C =

   2
```

ECHELON FORM OF THE MATRIX

Example 2. Find the echelon form of the matrix $A=\begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$

```
A=[1 2 3 2; 2 3 5 1; 1 3 4 5]
disp('The matrix A= ');A
%   to find rank of A ;
disp('The echelon form  of A  is ');
D=rref(A)
```

Output:

The matrix A=

A =

```
   1   2   3   2
   2   3   5   1
   1   3   4   5
```

The rank of A is
C =

   2

SOLUTION OF SYSTEM OF LINEAR EQUATIONS.

Example 3: Solving system of linear equations 2x+y+z=2; -x+y-z=3; x+2y+3z=-10.

```
syms x y z
eqn1 = 2*x + y + z == 2;
eqn2 = -x + y - z == 3;
eqn3 = x + 2*y + 3*z == -10;
[A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [x, y, z])
X = linsolve(A,B)
```

Output:

```
A =
     [  2, 1,  1]
     [ -1, 1, -1]
      [  1, 2,  3]
B =
     2
     3
   -10

X =
      3
      1
     -5
```

Task 3

1. Find the rank of the matrices $A = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & -1 & 1 \\ 3 & 3 & -5 & 1 \end{pmatrix}$

2. Find the echelon form of the matrix $A = \begin{pmatrix} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 \end{pmatrix}$

3. Solving system of linear equations

$$x + 2y - z - c = 4$$
$$x + 3y - 2z - 7c = 5$$
$$2x - y + 3z = 3$$

4. Find the row echelon form of the matrix $A = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{bmatrix}$

5. Find the rank of the matrix $A = \begin{pmatrix} 4 & -5 & 1 & 2 \\ 3 & 1 & -2 & 9 \\ 1 & 4 & 1 & 5 \end{pmatrix}$

6. Solving system of linear equations

$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$
$$3x - 5y + 5z = 2$$
$$3x + 9y - z = 4$$

WORKSHEET-4

CHARACTERISTIC EQUATION OF A MATRIX AND CAYLEY-HAMILTON THEOREM

4.1 OBJECTIVES

- To verify Cayley Hamilton Theorem and find the inverse of the matrix

4.2 **Example 1**:Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

Program:

```
 A=[1 3;2 4]
disp('The matrix A= ');A
%   to find the polynomial of A ;
X=poly(A)
disp('The polynomial of  A is ');
round(X)
y=polyvalm(X,A)
 disp(' Cayley Hamilton theorem  ');
 round(y)
 disp('The identity matrix  is ');
  i=eye(2,2)
 disp('The inverse of  A is ');
 InvA=(A-5*i)/2
 s=inv(A)
```

Output:

The matrix A=

A =

```
    1    3
    2    4
```

X =

```
   1.0000   -5.0000   -2.0000
```

The polynomial of A is

ans = 1 -5 -2

Cayley Hamilton theorem

ans =

0 0

0 0

The identity matrix is

i =

1 0

0 1

The inverse of A is

InvA =

-2.0000 1.5000

1.0000 -0.5000

s =

-2.0000 1.5000

1.0000 -0.5000

Example 2:Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & 1 \\ 2 & 1 & -1 \end{pmatrix}$

Program:

```
A=[1 -1 4;3 2 1;2 1 -1]
disp('The matrix A= ');A
%   to find the polynomial of A ;
X=poly(A)
disp('The polynomial of  A is ');
round(X)
y=polyvalm(X,A)
disp(' Cayley Hamilton theorem  ');
round(y)
disp('The identity matrix  is ');
i=eye(3,3)
disp('The inverse of  A is ');
```

```
InvA=(-A^2+2*A+7*i)/12
s=inv(A)
```

Output:

The matrix A=

```
A =
    1   -1    4
    3    2    1
    2    1   -1
X =
   1.0000   -2.0000   -7.0000   12.0000
The polynomial of  A is
ans =
    1   -2   -7   12
Cayley Hamilton theorem
ans =
    0    0    0
    0    0    0
    0    0    0
The identity matrix is
i =
    1    0    0
    0    1    0
    0    0    1
The inverse of A is
InvA =
   0.2500   -0.2500    0.7500
  -0.4167    0.7500   -0.9167
   0.0833    0.2500   -0.4167
s =    0.2500   -0.2500    0.7500
      -0.4167    0.7500   -0.9167
       0.0833    0.2500   -0.4167
```

Task 4:

1. Verify Cayley Hamilton theorem for the matrix and find the value of A^{-1}

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

2. Verify Cayley Hamilton theorem for the matrix and find A^{-1}.

$$A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

3. Verify Cayley Hamilton theorem for the matrix and find A^{-1}.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$$

4 . Verify Cayley Hamilton theorem for the matrix and find A^{-1}.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

5. Verify Cayley Hamilton theorem for the matrix and find A^{-1}.

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$$

WORKSHEET-5

EIGENVALUES AND EIGENVECTORS OF HIGHER ORDER MATRICES

5.1 OBJECTIVES

- To find the Eigenvalues and Eigenvectors of the matrix

5.2 Example 1: Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Program:

```
A=[1 -1 0 ; -1 2 1 ; 0 1 1]
disp(' The Eigenvalues of A are  ');
eig(A)
disp(' The Eigenvector of A are  ');
[v,d]=eig(A)
round([v,d])
```

Output:

```
A =
  1  -1   0
 -1   2   1
  0   1   1
The eigenvalues of A are
ans =
   0.0000
   1.0000
   3.0000
The Eigenvector of A are
v =
   0.5774   0.7071   -0.4082
```

```
  0.5774  -0.0000   0.8165
 -0.5774   0.7071   0.4082
d =
  0.0000        0        0
       0   1.0000        0
       0        0   3.0000
ans =
   1   1   0   0   0   0
   1   0   1   0   1   0
  -1   1   0   0   0   3
```

Example 2: Find the eigenvalues and eigenvectors of the matrix

$$B = \begin{pmatrix} 1 & -3 & 2 & -1 \\ -3 & 9 & -6 & 3 \\ 2 & -6 & 4 & -2 \\ -1 & 3 & -2 & 1 \end{pmatrix}$$

Program:

```
B=[1 -3  2 -1 ; -3  9  -6  3 ; 2 -6  4 -2 ; -1  3  -2  1]
  disp(' The Eigenvalues of B are  ');
  eig(B)
  disp(' The Eigenvector of B are  ');
  [v,d]=eig(B)
  round([v,d])
```

Output:

```
B =
   1  -3   2  -1
  -3   9  -6   3
   2  -6   4  -2
  -1   3  -2   1
 The Eigenvalues of B are
```

```
ans =
   -0.0000
   -0.0000
    0.0000
   15.0000
```

The Eigenvector of B are

```
v =
    0.0481    0.9649    0.0069   -0.2582
    0.1516    0.1956    0.5820    0.7746
   -0.2712   -0.1304    0.8017   -0.5164
   -0.9493    0.1173   -0.1357    0.2582
d =
   -0.0000         0         0         0
         0   -0.0000         0         0
         0         0    0.0000         0
         0         0         0   15.0000
ans =
     0     1     0     0     0     0     0     0
     0     0     1     1     0     0     0     0
     0     0     1    -1     0     0     0     0
    -1     0     0     0     0     0     0    15
```

Task 5:

1.Find the eigenvalues and eigenvectors of $A=\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$

2.Find the eigenvalues of A^{-1}, given that $A=\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$

3. Diagonalize the following matrices $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

4. Find the eigenvalues and eigenvectors of $A=\begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$

5. Find the eigenvalues and eigenvectors of $A=\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

WORKSHEET-6

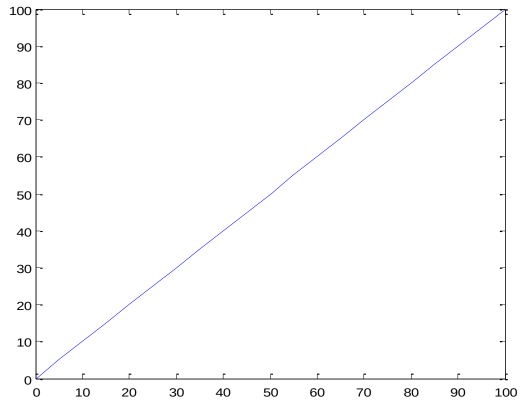
CURVE TRACING

6.1 OBJECTIVES

- To plot a two dimensional plot.
- To plot a three dimensional plot.

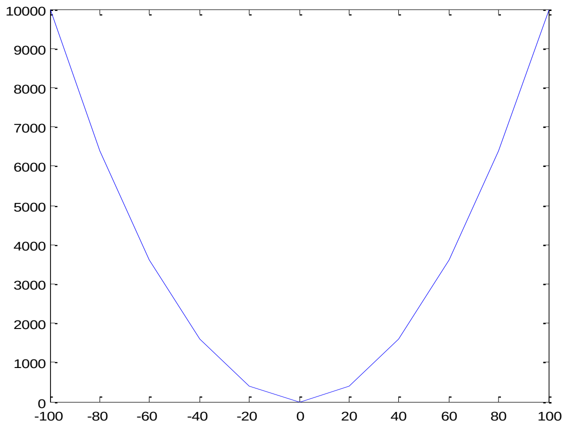
6.2 Example 1: Plot the simple function y = x for the range of values for x from 0 to 100, with an increment of 5.

```
x = [0:5:100];
  y = x;
  plot(x, y)
```



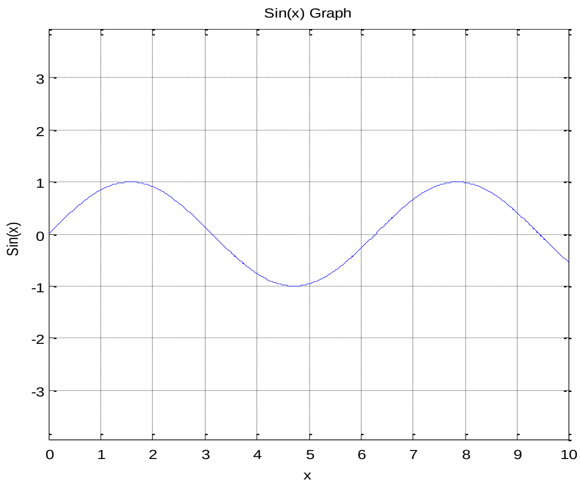
Example 2: Plot the function $y = x^2$

```
x = [-100:20:100];
    y = x.^2;
    plot(x, y)
```



Example 3: Plot the function y=sin x

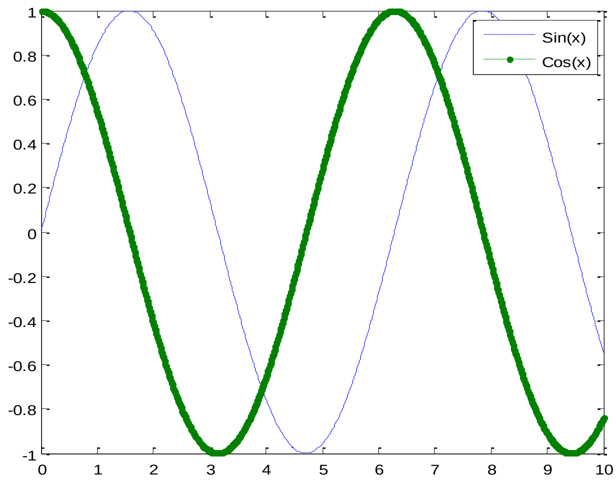
```
x = [0:0.01:10];
y = sin(x);
plot(x, y), xlabel('x'), ylabel('Sin(x)'), title('Sin(x)
Graph'), grid on, axis equal
```



Example 4: Plot multiple functions on the same graph for the curve y=sin *x* and g = cos *x*

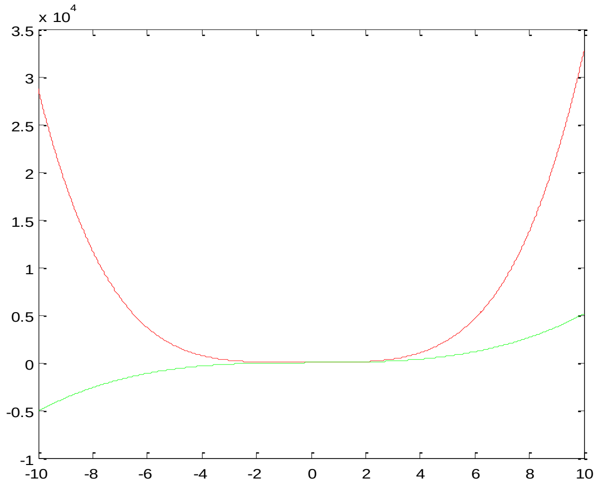
```
x = [0 : 0.01: 10];
y = sin(x);
```

```
g = cos(x);
plot(x, y, x, g, '.-'), legend('sin(x)', 'cos(x)')
```



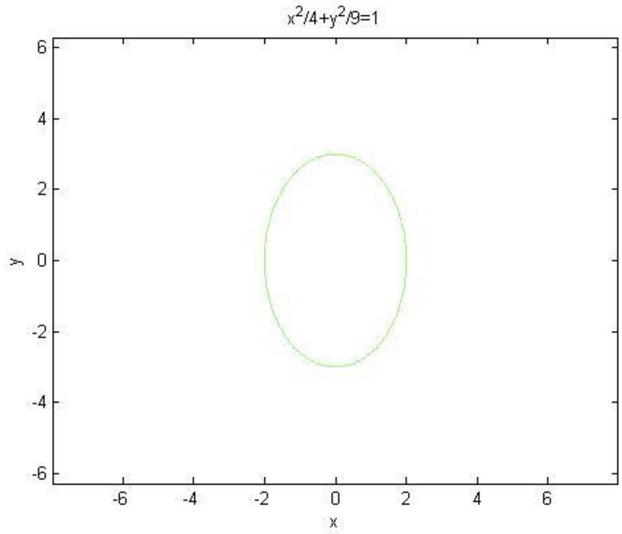
Example 5. Draw the graph of two polynomials $f(x) = 3x^4 + 2x^3 + 7x^2 + 2x + 9$ and $g(x) = 5x^3 + 9x + 2$.

```
x = [-10 : 0.01: 10];
y = 3*x.^4 + 2 * x.^3 + 7 * x.^2 + 2 * x + 9;
g = 5 * x.^3 + 9 * x + 2;
plot(x, y, 'r', x, g, 'g')
```



Example 6. Plot the curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$

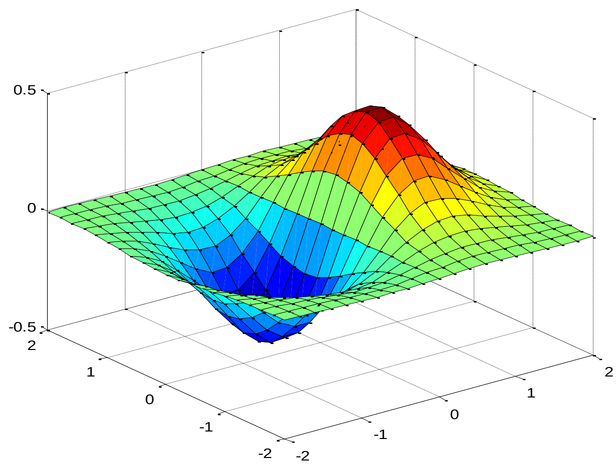
```
ezplot('x^2/4+y^2/9=1');
axis equal;
```



3-D Plot

Example 7. Create a 3D surface map for the function $g = xe^{-(x^2+y^2)}$.

```
[x,y] = meshgrid(-2:.2:2);
g = x .* exp(-x.^2 - y.^2);
surf(x, y, g)
```



Task 6:

1. Plot the sine-wave curve y=sin x such that x varies from 0 to 2π with h=π/100.

2. For $f(x) = 8x^8 - 7x^7 + 12x^6 - 5x^5 + 8x^4 + 13x^3 - 12x + 9$, compute f(2),roots of f(x) and plot for x= 0 to 20 in step of 0.1.

3. Plot the curve $y = e^{-x} \sin(2x + 3)$;

4.Create a 3D surface map for the function $g = xe^{+(x^2+y^2)}$.

5. Plot the curve $y = x^3$

WORKSHEET- 7

SOLVING FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS

7.1 OBJECTIVES

- To solve the first order ordinary differential equations

7.2 **Example 1:** Solve $\frac{dy}{dx} = 5y$

```
%Solving first order ODE
    s = dsolve('Dy = 5*y')
```

Output:

s =
C2*exp(5*t)

Example 2. Solve $\frac{df}{dt} = -2f + \cos t$

```
%Solving First order ODE
   f= dsolve('Df = -2*f + cos(t)','t')
```

Output:

f =
(2*cos(t))/5 + sin(t)/5 + C4*exp(-2*t)

Example 3. Solve $\frac{dx}{dy} + \frac{x}{1+y^2} = \tan^{-1}\left(\frac{y}{1+y^2}\right)$

```
% LEIBNITZ'S FORM - EX2
    x=dsolve('Dx+(1/(1+y^2))*x=atany/(1+y^2)','y')
```

Output:

x = atany - C10*exp(-atan(y))

TASK 7:

1. Solve $\left(\frac{dy}{dt}\right)+4y(t)=e^{-t}, \;\; y(0)=1.$

2. Solve $\frac{dy}{dx}-y\cot x=2x\sin x$

3. Solve $\frac{dy}{dt}=ty$

WORKSHEET- 8

SOLVING SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

8.1 OBJECTIVES

- To solve second order ordinary differential equations

8.2 **Example 1:** Solve $(D^2 - 1)y = 0; y(0) = -1 \ \& \ y'(0) = 2$

```
y=dsolve('D2y - y = 0','y(0) = -1','Dy(0) = 2')
```

Output:

y =exp(t)/2 - (3*exp(-t))/2

Example 2: Solve $(D^2 + 5D + 6)y = e^x$

```
y=dsolve('D2y+5*Dy+6*y=exp(x)','x')
```

Output:

y =

exp(x)/12 + C25*exp(-2*x) + C26*exp(-3*x)

Example 3. Solve $(D^2 + 5D + 6)y = 4e^{-x} \log x$

```
y=dsolve('D2y+5*Dy+6*y=4*exp(-x)*log(x)','x')
```

Output:

y =

C28*exp(-2*x) - exp(-2*x)*(4*ei(x) - 4*exp(x)*log(x)) + C29*exp(-3*x) +

exp(-3*x)*(2*ei(2*x) - 2*exp(2*x)*log(x))

Example 4. Solve $(D^3 - D^2 + D - 1)y = e^x + \cos x, y(0) = y'(0) = y''(0) = 0$

```
y=dsolve('D3y-D2y+Dy-y=exp(x)+cos(x)','y(0)=0','Dy(0)=0','D2y(0)=0','x')
```

Output:

y =

(3*cos(x))/4 + exp(x)/4 - sin(x)/4 + exp(x)*(x/2 - (exp(-x)*cos(x))/4 + (exp(-x)*sin(x))/4) - cos(x)*(x/4 + cos(2*x)/8 + sin(2*x)/8 + (exp(x)*cos(x))/2 + 1/8) - sin(x)*(x/4 - cos(2*x)/8 + sin(2*x)/8 + (exp(x)*sin(x))/2 - 1/8)

Task:8

1. Solve the differential equation $(D^2 - D + 1)y = 0$

2. Solve the differential equation $(D^3 - 3D^2 + 3D - 1)y = 0$

3. Solve the differential equation $(D^4 - 2D^2 + 1)y = 0$

4. Solve the differential equation $(D^4 - 2D^3 + D^2)y = x^2 + e^x$

5. Solve the differential equation $(D+1)^2 y = e^{-x} \cos x$

6. Solve the differential equation $(D^2 + 4)y = \cos 3x$

WORKSHEET- 9

DETERMINING MAXIMA AND MINIMA OF A FUNCTION OF ONE VARIABLE

9.1 OBJECTIVE

- To determine the maxima and minima of a function of a single variable

9.2 **Example 1:** Find the extreme value of the function $f(x)= x^3 – 12x^2 – 20$.

Program:

```
syms x
f=x^3-12*x^2-20
fx=diff(f,x)
a=solve(fx)
double(a)
fxx=diff(fx,x)
D=subs(fxx,x,8)
if D >0
disp('Attains Minima')
else
disp('Attains Maxima')
end
D=subs(fxx,x,0)
if D >0
disp('Attains Minima')
else
disp('Attains Maxima')
end
```

OUTPUT

f =x^3 - 12*x^2 - 20

fx = 3*x^2 - 24*x

a = 0   8

fxx = 6*x - 24

D = 24   **D=-24**

Attains Minima   **Attains Maxima**

Task:9

1. Find the maximum value of the function $f(x) = x^4 - 5x^3 + 3x - 18$

2. Find the minimum value of the function $f(x) = 7x^3 - 2x^2 - 5$

3. Find the extrema of the function $f(x) = x^3 - 3x^2 - 24x + 5$

4. Find the extrema of $f(x) = x(12 - 2x)^2$

WORKSHEET- 10

DETERMINING MAXIMA AND MINIMA OF A FUNCTION OF TWO VARIABLES

10.1 OBJECTIVES

- To determine maxima and minima of a function of two variables.

10.2 Example 1: Find the extreme value of the function

$f(x,y) = x^4 + 2y^4 - 12xy^2 - 20y^2.$

PROGRAM:

```
syms x y
f=x.^4+2*y.^4-12*x*y.^2-20*y.^2
fx=diff(f,x)
fy=diff(f,y)
[a,b]=solve(fx,fy);
double([a,b])
fxx=diff(fx,x)
fxy=diff(fx,y)
fyy=diff(fy,y)
D=fxx*fyy-(fxy)^2
D1=subs(D,[x,y],[3.6247,3.9842])
A1=subs(fxx,[x,y],[3.6247,3.9842])
D2=subs(D,[x,y],[3.6247,-3.9842])
A2=subs(fxx,[x,y],[3.6247,-3.9842])
if D1>0
if A1>0
disp('Attains Minima')
else
disp('Attains Maxima')
    end
else
if D1==0
disp('No conclusion')
```

```
else
disp('Neither Minima nor Maxima')
end
end
if D2>0
if A2>0
disp('Attains Minima')
     else
    disp('Attains Maxima')
    end
  else
    if D2==0
  disp('no conclusion')
  else
  disp('Neither Minima nor maxima')
  end
  end
  ezsurf(f,[0,5,0,5])
```

Output:

f =x^4 - 12*x*y^2 + 2*y^4 - 20*y^2

fx = 4*x^3 - 12*y^2

fy = 8*y^3 - 24*x*y - 40*y

ans =

```
   0.0000 + 0.0000i   0.0000 + 0.0000i
   3.6247 + 0.0000i   3.9842 + 0.0000i
   3.6247 + 0.0000i  -3.9842 + 0.0000i
  -1.8123 - 0.9240i   1.0884 - 1.2734i
  -1.8123 + 0.9240i   1.0884 + 1.2734i
  -1.8123 - 0.9240i  -1.0884 + 1.2734i
  -1.8123 + 0.9240i  -1.0884 - 1.2734i
```

fxx =12*x^2

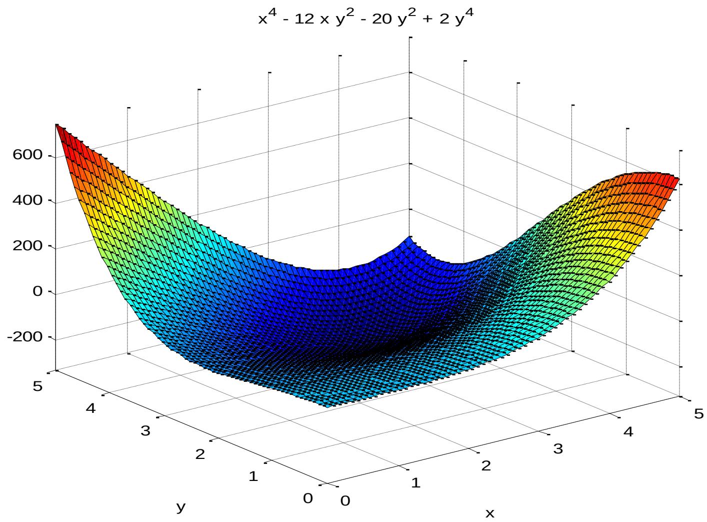
fxy = -24*y

fyy = 24*y^2 - 24*x - 40

D = - 12*x^2*(- 24*y^2 + 24*x + 40) - 576*y^2

ans =

3941535027/25000000



Example 2 :. Find the extreme value of the function

$f(x,y) = x^3 + 2y^3 - 12xy^2 - 20y^2$

Program:

```
syms x y
f=x.^3+2*y.^3-12*x*y.^2-20*y.^2
fx=diff(f,x)
fy=diff(f,y)
[a,b]=solve(fx,fy);
double([a,b])
fxx=diff(fx,x)
fxy=diff(fx,y)
fyy=diff(fy,y)
D=fxx*fyy-(fxy)^2
D1=subs(D,[x,y],[ -1.4815,0.7807])
```

```
A1=subs(fxx,[x,y],[ -1.4815,0.7807])
D2=subs(D,[x,y],[ -1.9048,-0.9524])
A2=subs(fxx,[x,y],[ -1.9048,-0.9524])
if D1>0
if A1>0
disp('Attains Minima')
else
disp('Attains Maxima')
    end
else
if D1==0
disp('No conclusion')
else
disp('Neither Minima nor Maxima')
end
end
if D2>0
if A2>0
disp('Attains Minima')
    else
   disp('Attains Maxima')
   end
  else
   if D2==0
  disp('no conclusion')
  else
  disp('Neither Minima nor maxima')
  end
  end
  ezsurf(f,[0,5,0,5])
```

Output:

f =x^3 - 12*x*y^2 + 2*y^3 - 20*y^2

fx =3*x^2 - 12*y^2

fy =6*y^2 - 24*x*y - 40*y

ans =

```
         0         0
   -1.9048   -0.9524
   -1.4815    0.7407
```

fxx =6*x

fxy =-24*y

fyy =12*y - 24*x - 40

D =- 576*y^2 - 6*x*(24*x - 12*y + 40)

D1=-4935508323/12500000

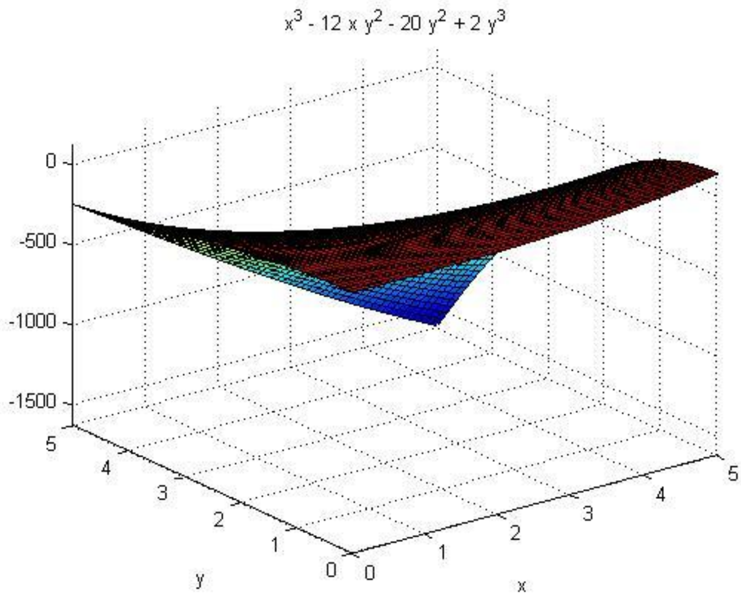
A1 =-8889/1000

D2 =-178582143/390625

A2 =7143/625

Neither Minima nor Maxima

Neither Minima nor maxima



Task :10

1. Find the Maximum value of the function $f(x,y) = x^3 - 3xy^2 - 12xy + 2$

2. Find the Minimum value of the function $f(x,y) = x^4 + 6y^3 - 36xy - 20y$

3. Examine for extreme values of the function $u(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

4. Find the maximum and minimum values of the function

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$$

5. Find the maximum or minimum value of the function

$f(x, y) = 2(x^2 - y^2) - x^4 + y^4$