Inventory Management- Decision Models

Introduction:

Inventories play a pivotal role in the operational and financial health of any organization. Efficient inventory management is crucial as excess inventory can signal poor management practices, while inadequate inventory levels can lead to business disruptions and potential failure. The fundamental decisions in inventory management revolve around determining the optimal quantity to order and the timing of these orders to minimize overall inventory costs.

Holding costs, represent expenses associated with maintaining inventory, encompass various factors such as interest on tied-up capital, storage expenses, warehousing costs, and potential losses due to deterioration or obsolescence. On the other hand, ordering costs comprise expenses related to replenishing inventory, including overheads, clerical work, and transportation costs.

Our objective is to develop and implement a decision model to assist in making informed inventory management decisions regarding a key engine component. This decision model will enable the company to optimize its inventory levels, balancing the costs of holding inventory against the expenses incurred in replenishing it.

We will leverage mathematical modeling techniques to quantify the annual ordering and holding costs based on average inventory levels throughout the year. This mathematical model will be implemented using both Excel and R, facilitating a comprehensive analysis and comparison of results.

Furthermore, we will employ data tables in Excel to identify the order quantity that minimizes total inventory costs and visualize the relationship between total cost and order quantity through plotting. Excel Solver will be utilized to validate the optimal order quantity obtained from the model.

Moreover, we will conduct what-if analyses using two-way tables in Excel to explore the sensitivity of total costs to changes in various model parameters, providing insights for decision-making.

In the subsequent phase of the project, conducted exclusively in R, we will simulate 1000 occurrences considering a triangular probability distribution for annual demand. Through statistical analysis, including estimation of confidence intervals and determination of probability distributions, we aim to provide a robust understanding of expected outcomes and associated uncertainties.

Part I:

In this part, we developed a decision model using both Excel and R to minimize total inventory cost. Our objective was to define data, model parameters, and decision variables influencing total inventory cost. We calculated annual ordering and holding costs, implemented the model in Excel and R, and analyzed the results. Using data tables, we identified the order quantity minimizing total cost and validated it using Excel Solver. Sensitivity analysis was conducted to assess the impact of parameter changes, and findings were communicated to the vice president of operations. Let's explore every step in detail.

Decision Model Using Excel:

Our objective is to build a decision model that will minimize the total inventory cost. Following information is already provided and we need to optimize the inventory cost:

Given Information		
Number of units	15000	Uncontrollable
Unit Price	78	Parameter
Opportunity cost per unit per year in percentage	18%	Parameter
Cost per order	1180	Uncontrollable
Total inventory cost		Desirable
Opportunity cost per unit per month	0.015	Parameter

Here, uncontrollable factors are beyond the direct control of the decision-maker, such as the number of units (e.g., annual demand) and the cost per order (e.g., supplier charges). These variables are given and cannot be changed by the decision-maker. A Desirable factor is the outcome or result that the decision-maker aims to achieve. Decision variables the decision-maker can adjust or control to achieve the desired outcome. In our example, decision variable is number of orders per year. Parameters are the decision-maker can adjust or control to achieve the desired outcome.

To build a decision model, used the following formulae in excel:

Formulae
Total Inventory Cost= Ordering Cost + Holding Cost
Ordering Cost= No. of Orders per year + Cost per order
Unit Value= No. of units X Unit Price
Holding Cost per month= (opportunity cost per month X unit value per month)
Holding cost per year= ∑ Holding cost in each month

Approach: To find the hosting cost for a given number of order per year, I calculated the holding cost per month and then did the summation of the holding cost per year. I did this because, since the inventory will be in use, the number of items in use will keep on reducing over time and hence the hosting cost will also keep on reducing. For instance, when an order is placed only once, there will be 15000 units in the inventory at the beginning of the year but in the next month there will be only 15000-(15000/12) units in the inventory given the rate of use is constant throughout the year. Hence the holding cost in the first month will be the highest and then it will keep on reducing until the next order is placed. This was solved mathematically to calculate the holding cost per year formula:

Holding cost= Opportunity cost per month*unit price*number of units ordered per order*number of orders per year*(reordering time+1)/2

Where, reordering time is given in months and calculated as: 12/no or orders per year

The above formulae were used in the excel to obtain total inventory cost for different number of orders per year. The number of orders per year were taken from 1 meaning all 15000 units were ordered in one go to 36 meaning every month 3 orders were given to complete the order of 15000 units throughout the year. Ordering cost and hosting cost was calculated by using the above formulae in the table given below:

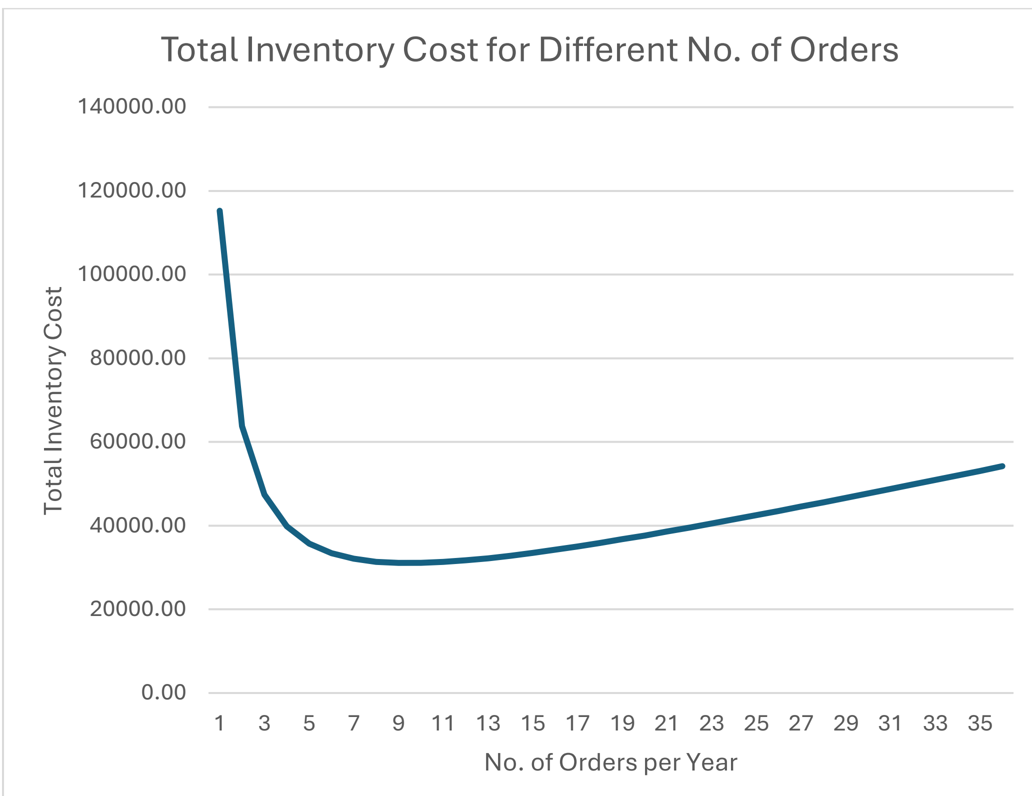
Part I: One Way Model for Total Inventory Cost Minimization

Table for Optimization

Number of Orders Per Year	Number of units in each order	Reordering time	Ordering Cost	Holding Cost	Total Inventory Cost
1	15000.00	12.00	1180	114075.00	115255.00
2	7500.00	6.00	2360	61425.00	63785.00
3	5000.00	4.00	3540	43875.00	47415.00
4	3750.00	3.00	4720	35100.00	39820.00
5	3000.00	2.40	5900	29835.00	35735.00
6	2500.00	2.00	7080	26325.00	33405.00
7	2142.86	1.71	8260	23817.86	32077.86
8	1875.00	1.50	9440	21937.50	31377.50
9	1666.67	1.33	10620	20475.00	31095.00
10	1500.00	1.20	11800	19305.00	31105.00
11	1363.64	1.09	12980	18347.73	31327.73
12	1250.00	1.00	14160	17550.00	31710.00
13	1153.85	0.92	15340	16875.00	32215.00
14	1071.43	0.86	16520	16296.43	32816.43
15	1000.00	0.80	17700	15795.00	33495.00
16	937.50	0.75	18880	15356.25	34236.25
17	882.35	0.71	20060	14969.12	35029.12
18	833.33	0.67	21240	14625.00	35865.00
19	789.47	0.63	22420	14317.11	36737.11
20	750.00	0.60	23600	14040.00	37640.00

21	714.29	0.57	24780	13789.29	38569.29
22	681.82	0.55	25960	13561.36	39521.36
23	652.17	0.52	27140	13353.26	40493.26
24	625.00	0.50	28320	13162.50	41482.50
25	600.00	0.48	29500	12987.00	42487.00
26	576.92	0.46	30680	12825.00	43505.00
27	555.56	0.44	31860	12675.00	44535.00
28	535.71	0.43	33040	12535.71	45575.71
29	517.24	0.41	34220	12406.03	46626.03
30	500.00	0.40	35400	12285.00	47685.00
31	483.87	0.39	36580	12171.77	48751.77
32	468.75	0.38	37760	12065.63	49825.63
33	454.55	0.36	38940	11965.91	50905.91
34	441.18	0.35	40120	11872.06	51992.06
35	428.57	0.34	41300	11783.57	53083.57
36	416.67	0.33	42480	11700.00	54180.00

Total inventory costs were plotted in a line chart to see if there exists a global minimum.



From the above graph, it can be observed that when the orders are between 7 to 10, the total inventory cost is minimum.

In order to find the exact number of orders for minimum total inventory cost, I used 'Solver' add-in in excel. For solver, total inventory cost was minimized and for which the number of orders were calculated.

Number of Orders Per Year	Number of units in each order	Reordering time	Ordering Cost	Holding Cost	Total Inventory Cost
1	15000.00	12.00	1180	114075.00	115255.00
2	7500.00	6.00	2360	61425.00	63785.00
9.446549007	**1587.88**	**1.27**	11146.93	19921.93	**31068.86**

As can be seen in the above image, the minimum total inventory cost was 31068 and the number of orders per year was 9.44. That means, if an order is given 9 times, then the total inventory cost will be minimum.

Decision Model using R:

A similar model was obtained using R. Total inventory cost was calculated by considering both ordering and holding costs. The code varies the number of orders per year, calculates the corresponding total inventory cost, and plots it. It then identifies the number of orders per year that minimizes the total cost, thereby determining the most cost-effective ordering strategy.

```
# Part 1: Decision model to minimize the total inventory cost
unt_prc<- 78
opp_cst_pr_yr<- 0.18
cst_pr_ordr<- 1180
opp_cst_pr_mnth <- 0.015

no_ordrs_pr_yr<-seq(1,36)
reordr_time<- 12/no_ordrs_pr_yr
unts_pr_ordr <- 15000/no_ordrs_pr_yr
ordrng_cst <- cst_pr_ordr*no_ordrs_pr_yr

hldng_cst<- opp_cst_pr_mnth*unt_prc*unts_pr_ordr*no_ordrs_pr_yr*(reordr_time+1)/2

#Taking log scale as the numbers are large
ttl_invntry_cst<- ordrng_cst+hldng_cst
#Plot
plot(ttl_invntry_cst)

#Finding the index(number of orders per year) of the observation with minimum total inventory cost:
which.min(ttl_invntry_cst)

#Minimum total inventory cost:
ttl_invntry_cst[which.min(ttl_invntry_cst)]
```

```
> which.min(ttl_invntry_cst)
[1] 9
> print(ttl_invntry_cst[which.min(ttl_invntry_cst)])
[1] 31095
```

It can be seen that even with R, minimum total inventory cost is around 31000 dollars and the number of orders for which the total inventory cost is minimum is 9.

Hence, we can confirm that the decision model obtained using excel and r are similar to each other.

Two-way table for inventory cost minimization:

In the above part, only one parameter was changed to find the minimum inventory cost but in the two way table, two parameters are changed to find the minimum inventory cost. In this part, two parameters were changed, one is the number of orders per year and the other is the opportunity cost per year.

In order to calculate the total inventory cost in terms of number of orders per year and opportunity cost, the following formula was obtained mathematically:

Total inventory cost: (13*75*Q/2)*((12/P)+1) + (1180*P)

Where, P represents the number of orders per year and Q represents the opportunity cost.

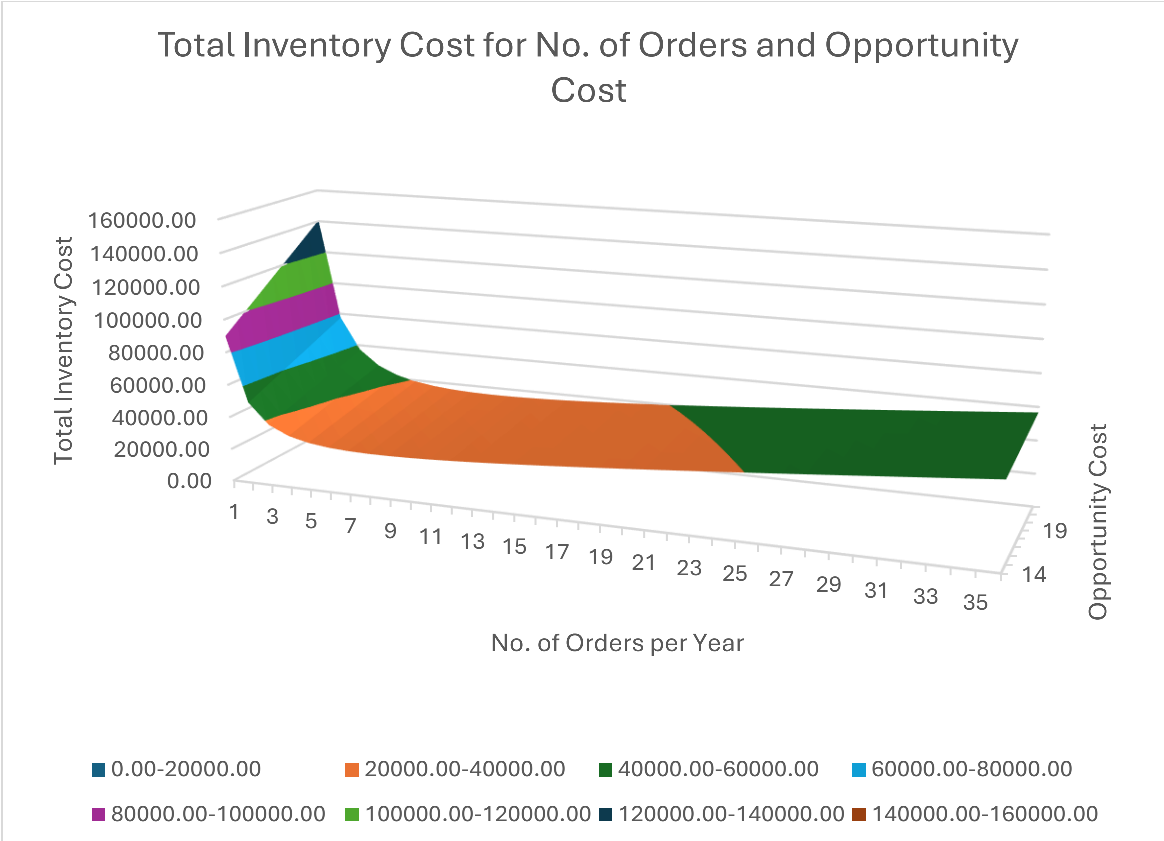
The number of orders per year was taken from 1 to 36 and opportunity costs were taken from 14 to 22%.

The following table was obtained by using the formula till 36 orders per year.

Part I: Two Way Table for Inventory Cost Minimization

orders per year)	Q(Opportunity Cost)								
	14	15	16	17	18	19	20	21	2
1	89905.00	96242.50	102580.00	108917.50	115255.00	121592.50	127930.00	134267.50	140605.0
2	50135.00	53547.50	56960.00	60372.50	63785.00	67197.50	70610.00	74022.50	77435.0
3	37665.00	40102.50	42540.00	44977.50	47415.00	49852.50	52290.00	54727.50	57165.0
4	32020.00	33970.00	35920.00	37870.00	39820.00	41770.00	43720.00	45670.00	47620.0
5	29105.00	30762.50	32420.00	34077.50	35735.00	37392.50	39050.00	40707.50	42365.0
6	27555.00	29017.50	30480.00	31942.50	33405.00	34867.50	36330.00	37792.50	39255.0
7	26785.00	28108.21	29431.43	30754.64	32077.86	33401.07	34724.29	36047.50	37370.7
8	26502.50	27721.25	28940.00	30158.75	31377.50	32596.25	33815.00	35033.75	36252.5
9	26545.00	27682.50	28820.00	29957.50	31095.00	32232.50	33370.00	34507.50	35645.0
10	26815.00	27887.50	28960.00	30032.50	31105.00	32177.50	33250.00	34322.50	35395.0
11	27250.45	28269.77	29289.09	30308.41	31327.73	32347.05	33366.36	34385.68	35405.0
12	27810.00	28785.00	29760.00	30735.00	31710.00	32685.00	33660.00	34635.00	35610.0
13	28465.00	29402.50	30340.00	31277.50	32215.00	33152.50	34090.00	35027.50	35965.0
14	29195.00	30100.36	31005.71	31911.07	32816.43	33721.79	34627.14	35532.50	36437.8
15	29985.00	30862.50	31740.00	32617.50	33495.00	34372.50	35250.00	36127.50	37005.0
16	30823.75	31676.88	32530.00	33383.13	34236.25	35089.38	35942.50	36795.63	37648.7
17	31702.65	32534.26	33365.88	34197.50	35029.12	35860.74	36692.35	37523.97	38355.5
18	32615.00	33427.50	34240.00	35052.50	35865.00	36677.50	37490.00	38302.50	39115.0

The highlighted cell represents the minimum total inventory cost. The total inventory cost is minimum when the opportunity cost is 14% and the number of orders placed per year is 8.



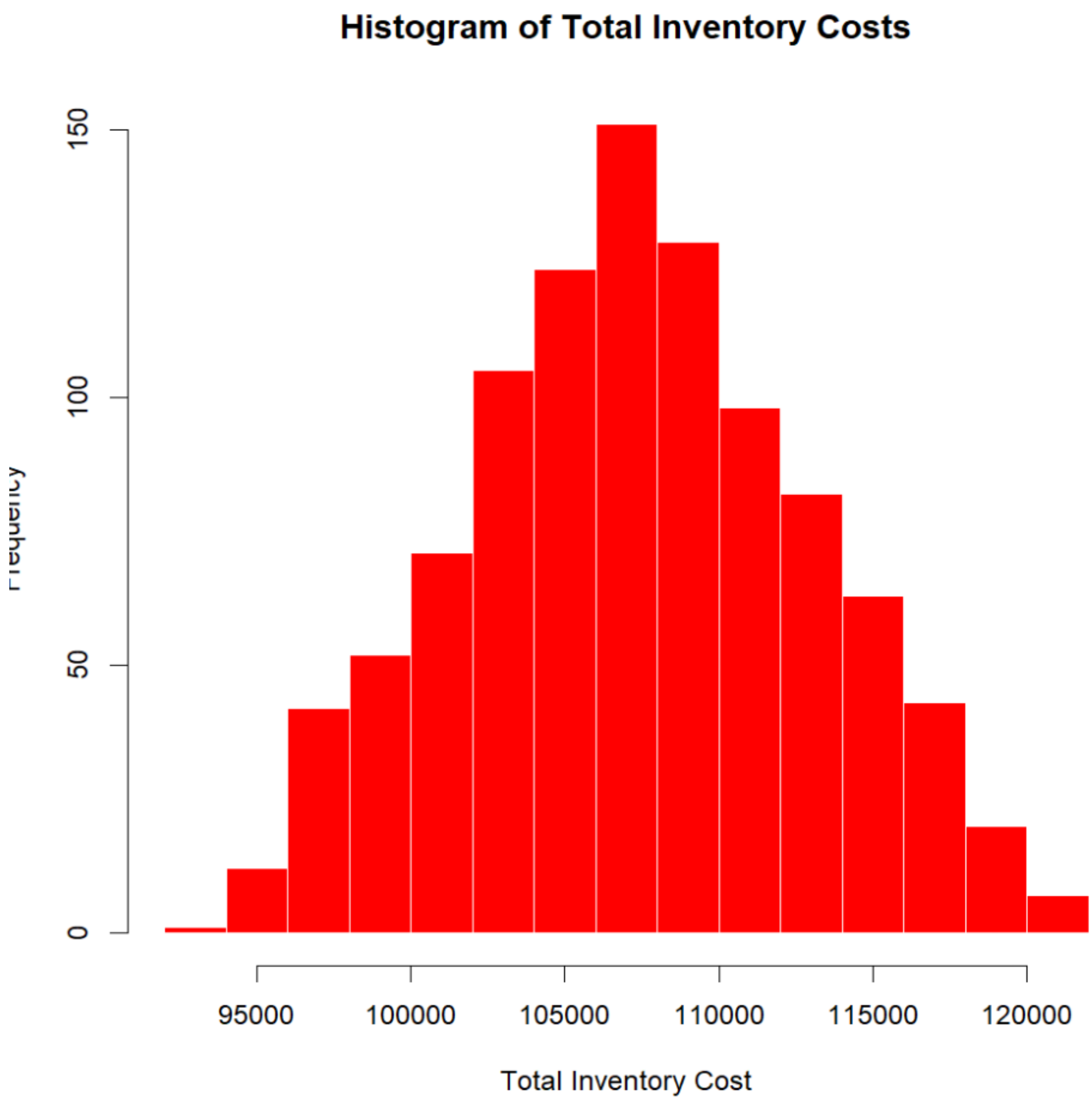
The above can be concluded from the graph as well. The surface is minimized for lower opportunities cost between 7 to 9 number of orders per year. Let's now dive into the second part.

Part II:

In this part, we are supposed to extend the analysis using R, incorporating a triangular probability distribution for annual demand. With parameters consistent with Part I, 1000 simulations are conducted to compute minimum total costs. Subsequently, we will estimate the expected minimum total cost, the order quantity and annual number of orders with a 95% confidence interval and select respective appropriate probability distributions.

- **Estimating total minimum inventory cost for 1000 different values of number of units required:**

In this task, a simulation with 1000 occurrences was conducted in R to calculate the minimum total cost under a triangular probability distribution for annual demand. The function ttl_cst_fun() was defined to compute the total inventory cost for each occurrence. A histogram was plotted to visualize the distribution of total inventory costs. Finally, the average total inventory cost, standard deviation, and a 95% confidence interval were calculated and presented, offering insights into the expected costs and their variability in the decision-making process. To calculate the values for different annual demands, the number of orders placed per year was kept constant and was taken as 2 orders per year.



Observations: It can be seen that the total inventory cost is higher than the one obtained in part I. This could be because the orders were placed twice instead of the optimal value of 9. It can also be seen that the total inventory cost also follows triangular distribution like annual demand.

```
> cat("Average Total Inventory Cost:", avg_ttl_invntry_csts, "\n")
Average Total Inventory Cost: 107306.9
> cat("Standard Deviation of Total Inventory Cost:", sd_ttl_invntry_csts, "\n")
Standard Deviation of Total Inventory Cost: 5593.494
> cat("Confidence Interval (95%) for the  Total Inventory Cost:", cnf_intrvl_ttl_cst[1], "-", cnf_intrvl_t
tl_cst[2], "\n")
Confidence Interval (95%) for the  Total Inventory Cost: 96968.28 - 118063.6
```

Additionally, the average total inventory cost amounted to $107,306.90, with a standard deviation of $5,593.49. Constructing a 95% confidence interval revealed a range of $96,968.28 to $118,063.60 for the total inventory cost.

- **Estimating average order quantity:**

n this task, we aimed to estimate the expected order quantity by constructing a 95% confidence interval and determining the probability distribution that best fit its distribution. Through analysis using R, we calculated the average order quantity alongside its standard deviation, providing valuable insights into inventory management. Additionally, we identified the probability distribution that best fitted the data, allowing for further validation of our findings.

```
> #Output
> cat("Average of Order Quantity:",avg_ordr_qty, "\n")
Average of Order Quantity: 7474.849
> cat("Standard Deviation of Order Quantity:",sd_ordr_qty , "\n")
Standard Deviation of Order Quantity: 398.397
> cat("Confidence Interval (95%) for the  Order Quantity:", cnf_intrvl_ordr_qty[1], "-", cnf_intrvl_ordr_
ty[2], "\n")
Confidence Interval (95%) for the  Order Quantity: 6738.481 - 8241
>
> #Probability Distribution
> ordr_qntty_ft <- fitdistr(ordrs_pr_yr / 2, "gamma")
> print("Best-Fit Distribution for Order Quantity:")
[1] "Best-Fit Distribution for Order Quantity:"
> print(ordr_qntty_ft)
      shape           rate
  3.520245e+02   4.709452e-02
 (1.429422e+01) (1.913235e-03)
```

Observations: It can be observed that the average order quantity is approximately 7474.849 units, with a standard deviation of 398.397 units. This is because the orders were considered to be placed twice a year. The 95% confidence interval for the order quantity ranged from 6738.481 to 8241 units, providing a range within which the true average order quantity is likely to lie. Furthermore, fitting a gamma distribution to the order quantity data identified a best-fit distribution with shape = 352.025 and rate = 0.0471, indicating a skewed distribution with a right tail. These findings offer valuable insights into the variability and distribution of order quantities, aiding in the optimization of inventory management strategies.

- **Estimating average number of orders placed per year:**

In this task, we aimed to estimate the expected annual number of orders by constructing a 95% confidence interval. Through R analysis, we computed the average number of orders per year and its standard deviation, providing insights into the frequency of orders. Additionally, we determined the best-fitting probability distribution, facilitating a deeper understanding of order dynamics in inventory management.

```
> #Output
> cat("Average Number of Orders per Year:", avg_no_ordrs, "\n")
Average Number of Orders per Year: 2
> cat("Standard Deviation of Number of Orders per Year:", sd_no_ordrs, "\n")
Standard Deviation of Number of Orders per Year: 0.1065967
> cat("95% Confidence Interval for Average Number of Orders per Year:", cnf_intrvl_no_ordrs[1], "-", cnf_i
ntrvl_no_ordrs[2], "\n")
95% Confidence Interval for Average Number of Orders per Year: 0.5 - 0.5
>
> #Probability Distribution
> no_ordrs_ft <- fitdistr(ordrs_pr_yr / (avg_ordr_qty), "gamma")
>
> # Output results for number of orders distribution
> print("Parameters of Best-Fit Distribution for Number of Orders per Year:")
[1] "Parameters of Best-Fit Distribution for Number of Orders per Year:"
> print(no_ordrs_ft)
     shape         rate    
  352.024459   176.012230 
 ( 15.725591) (  7.868383)
```

Observations: As expected, the average number of orders per year is approximately 2, with a standard deviation of 0.107. The 95% confidence interval for the average number of orders per year ranged from 0.5 to 0.5, indicating a narrow range of uncertainty. Fitting a gamma distribution to the data identified parameters shape = 352.024 and rate = 176.012, suggesting a skewed distribution with right tail behavior. These findings provide valuable insights into the frequency of orders and offer guidance for inventory management strategies.

Conclusion:

In conclusion, this project delved into the realm of decision models for inventory management, exploring both Excel and R methodologies to optimize inventory costs. Through meticulous analysis and modeling, we uncovered insights into the dynamics of inventory systems. Part I focused on developing and implementing decision models in Excel and R, culminating in the identification of optimal order quantities to minimize total inventory costs. Sensitivity analyses provided further clarity on the impact of parameter changes. Transitioning to Part II, conducted exclusively in R, we simulated 1000 occurrences to compute minimum total costs under a triangular probability distribution for annual demand. The subsequent estimation of expected minimum total cost, order quantity, and annual number of orders, alongside probability distribution analysis, enriched our understanding of inventory management dynamics. These findings underscore the significance of robust decision models in navigating the complexities of inventory management, offering valuable insights for operational efficiency and cost optimization.

References:

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