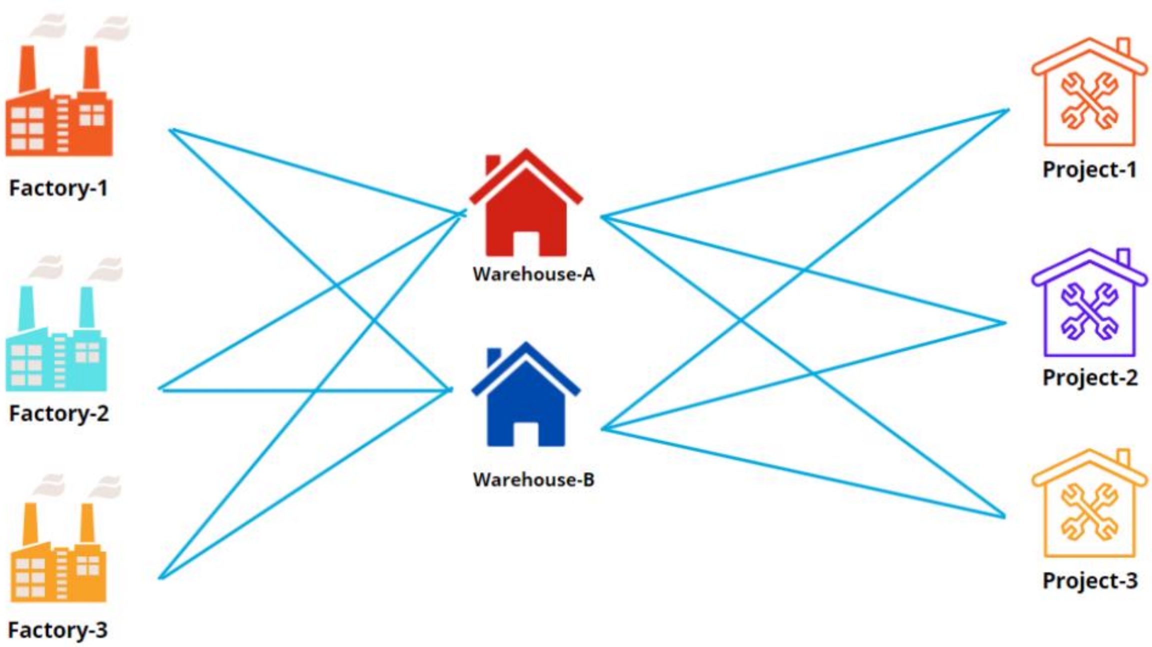
Northeastern University

ALY6050.20469.202425: Intro to Enterprise Analytics

Project 6 - Module 6

Transshipment and Risk Minimizing



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30th March, 2024

Introduction:

In this project, we are presented with two distinct challenges, each demanding careful analysis and strategic decision-making. Part 1 involves assisting Allen, a manager at Rockhill Shipping & Transport Company, in negotiating a shipping contract with Chimotoxic, a chemical manufacturer. The task at hand is to efficiently transport hazardous waste from six plants to three disposal sites, considering safety, cost-effectiveness, and logistical constraints.

Optimization, in simple terms, refers to the process of finding the best solution among a set of feasible options. In Part 1, we will utilize optimization techniques to minimize Rockhill's total transportation costs while meeting Chimotoxic's waste disposal needs. This entails determining the most economical shipping routes, considering factors such as distance, shipping costs, waste volumes, and potential regulatory constraints.

Meanwhile, Part 2 shifts our focus to investment allocations. Here, we will assist an investor in optimizing their portfolio to achieve a minimum baseline expected return of 11% while minimizing risk. This involves allocating funds across various asset types such as bonds, stocks, options, and gold. To achieve this optimization, we will employ non-linear optimization techniques, aiming to strike a balance between maximizing returns and minimizing portfolio volatility.

Non-linear optimization is a mathematical approach used to optimize functions that do not follow a linear relationship. In the context of Part 2, we will utilize non-linear optimization to determine the optimal allocation of funds across different asset classes, considering the expected returns and covariance matrix of each asset type. By exploring different combinations of asset allocations, we aim to identify the portfolio composition that achieves the desired balance of risk and return.

Through the application of optimization techniques in both parts of the project, we aim to provide actionable insights and solutions that enable informed decision-making. Let’s now delve deeper in the project.

Part 1:

In part 1, we need to come up with an optimization model that will minimize the cost of shipment from Rockhill to transport waste products from its six plants to three designated waste disposal sites. There are a total of 6 plants Denver, Morganton, Morrisville, Pineville, Rockhill, and Statesville and three designated waste proposal sites, Orangeburg, Florence, and Macon.

The following table shows the transportation cost per barrel of waste from the given plants to waste disposal sites:

Cost: Waste Proposal Site			
Plant:	***Orangeburg***	***Florence***	***Macon***
Denver	12	15	17
Morganton	14	9	10
Morrisville	13	20	11
Pineville	17	16	19
Rockhill	7	14	12
Statesville	22	16	18

Each plant produces following waste every week:

Plant:	***Waste per Week (bbl)***
Denver	45
Morganton	26
Morrisville	42
Pineville	53
Rockhill	29
Statesville	38

Now the objective is to reduce the total transportation cost, we can write the objective function as:

Objective function = Minimize total cost= $\sum X_{ij}C_{ij}$

Where X_{ij} denote the number of waste barrels transported from a plant to a disposal site and C_{ij} denotes the respective transportation cost.

In order to solve the problem, we have following constraints:

Plants produce a constant amount of waste per week as given in the above table and Orangeburg, Florence, and Macon can accommodate maximum of 65, 80, and 105 barrels per week.

We also have non-negativity constraint according to which all the decision variables should be positive.

Decision Variables: Waste Proposal Site				**Shipped from**		**Waste Production**
Plant:	***Orangeburg***	***Florence***	***Macon***			
Denver	36	9	0	45	=	45
Morganton	0	0	26	26	=	26
Morrisville	0	0	42	42	=	42
Pineville	0	53	0	53	=	53
Rockhill	29	0	0	29	=	29
Statesville	0	18	20	38	=	38
Shipped to:	65	80	88	233		233
	<=	<=	<=			
Max Accommodation:	65	80	105	250		

Total Cost	**2988**

To minimize the total cost, I created the above table and applied Solver to the total cost cell. The decision variables are the number of barrels shipped from one plant to the disposal site per week. The constraints are highlighted in the yellow cell. Here, the total number of shipped disposals from the given plant is constant, hence the equal sign is used in the constraint. Whereas the maximum waste that a plant can accommodate is given, so in the problem, the constraint should have less than or equal to sign.

Observations:
It can be observed that the minimum total cost applied by the solver is $2988 per week. The solution suggests that, from Denver, out of 45 total barrels, 36 should be sent to Orangeburg and 9 should be sent to Florence. From Morganton and Morrisville, all the barrels should be sent to Macon. From Pineville, all the barrels should be sent to Florence, from Rockhill, all the barrels should be sent to Orangeburg, whereas from Statesville, 18 barrels should be sent to Florence and 20 barrels should be sent to Macon. The solver is showing only 233 barrels to accommodate the first constraint. The first constraint can also be changed to greater than or equal to, as sometimes more waste product can be generated.

Now, in the subsequent part, Allen is exploring the possibility of utilizing intermediate shipping points. These intermediate points would involve dropping off loads at plants or disposal sites to be picked up and transported to the final destination by another truck. The intermediate points are referred to as nodes, and Allen is considering that either plants or destinations can act as nodes. Thus, there are a total of 9 nodes, consisting of 6 plants and 3 destination sites.

A similar solver method is applied to solve this problem as well, but the number of constraints is higher. We need to ensure that the number of barrels shipped to a node equals the number of barrels shipped from the node. The other two constraints will remain the same.

The following solution is obtained through the solver:

Plant:	Node1: Denver	Node2: Morganton	Node3:Morrisville	Node 4: Pineville	Node 5: Rockhill	Node 6: Satesville	Node 7: Orangeburg	Node 8: Florence	Node 9: Macon	Orangeburg	Florence	Macon	Shipped from		
Denver	0	45	0	0	0	0	0	0	0	0	0	0	45	=	45
Morganton	0	0	26	0	0	0	0	0	0	0	0	0	26	=	26
Morrisville	0	0	0	0	42	0	0	0	0	0	0	0	42	=	42
Pineville	0	30	0	0	23	0	0	0	0	0	0	0	53	=	53
Rockhill	0	0	29	0	0	0	0	0	0	0	0	0	29	=	29
Statesville	0	38	0	0	0	0	0	0	0	0	0	0	38	=	38
Node1: Denver	0	0	0	0	0	0	0	0	0	0	0	0	0		
Node2: Morganton	0	0	0	0	0	0	0	0	0	0	80	33	113		
Node3:Morrisville	0	0	0	0	0	0	0	0	0	0	0	55	55		
Node 4: Pineville	0	0	0	0	0	0	0	0	0	0	0	0	0		
Node 5: Rockhill	0	0	0	0	0	0	0	0	0	65	0	0	65		
Node 6: Satesville	0	0	0	0	0	0	0	0	0	0	0	0	0		
Node 7: Orangeburg	0	0	0	0	0	0	0	0	0	0	$0	$0	0		
Node 8: Florence	0	0	0	0	0	0	0	0	0	0	0	0	0		
Node 9: Macon	0	0	0	0	0	0	0	0	0	0	0	0	0		
Shipped	0	113	55	0	65	0	0	0	0	65	80	88			
Sign										<=	<=	<=			
Waste Production										65	80	105			

Total Cost:	3195

Observations: It can be observed that the total minimum cost is $3195 per week when transshipment is considered. This value is higher than the value obtained in direct transportation. Additionally, it can be observed that the nodes selected in the table are other plants instead of disposal sites. The model suggests transporting all the barrels from Denver to the plant Morganton, barrels from Morganton to Morrisville, 30 barrels from Pineville, and 23 barrels from Pineville to Rockhill, all barrels from Rockhill to Morrisville, and all barrels from Statesville to Morganton. From these nodes, the barrels can be transported to the final disposal sites. However, the solver is still showing only 233 barrels to accommodate the first constraint, which provides information about the amount of waste generated at every plant.

Recommendation: While the initial analysis focused on direct transportation routes from plants to disposal sites, it is evident that considering intermediate nodes can impact the total transportation cost. The solution obtained through transshipment suggests that utilizing other plants as intermediate nodes can lead to higher transportation costs compared to direct transportation to disposal sites therefore, direct transportation will be a better option in this scenario.

Part 2:

In Part 2 of our project, we shift our attention to optimizing an investor's portfolio composition. The investor has chosen different types of assets to include in their portfolio, each with its own expected return determined through historical data analysis. These assets encompass bonds, high-tech stocks, foreign stocks, call options, put options, and gold, as detailed in Table 5, providing valuable insights into their potential profitability. Additionally, we have been furnished with a covariance matrix illustrating the relationships between different pairs of assets in terms of their variance and covariance. This matrix aids in evaluating the diversification benefits and strategies for mitigating risk within the portfolio. Our main goals in this segment of the project are twofold: first, to ascertain the optimal allocation of a $10,000 investment across the diverse asset types to attain a minimum baseline expected return of 11%; and second, to delve into the relationship between risk and expected portfolio return by analyzing various solution pairs denoted by (r, e), where "r" signifies minimized risk and "e" denotes expected portfolio return. By employing optimization techniques, our aim is to strike a harmonious balance between maximizing portfolio returns and minimizing exposure to risk. Let's now delve deeper into it.

We have been given different types of investments and their expected individual returns as follows:

	Expected Returns
Bonds	**6%**
High tech stocks	**26%**
Foreign stocks	**27%**
Call options	**18%**
Put options	**19%**
Gold	**7%**

We have also been given the covariance matrix of asset's return. It gives information about how the returns of the different asset classes move together or covary with each other. A positive covariance means the returns tend to move in the same direction, while a negative covariance indicates they move in opposite directions.

	Bonds	**High tech stocks**	**Foreign stocks**	**Call options**	**Put options**	**Gold**
Bonds	**0.001**	**0.0003**	**-0.0003**	**0.00035**	**-0.00035**	**0.0004**

High tech stocks		**0.009**	**0.0004**	**0.0016**	**-0.0016**	**0.0006**
Foreign stocks			**0.008**	**0.0015**	**-0.0055**	**-0.0007**
Call options				**0.012**	**-0.0005**	**0.0008**
Put options					**0.012**	**-0.0008**
Gold						**0.005**

Table 1: The Covariance matrix of assets' returns

The covariance matrix is a crucial input because it captures not just the individual risk of each asset, but also how the risks of the assets interact and offset (or amplify) each other when combined in a portfolio.

Here, the diagonal entries (e.g. 0.001 for bonds) represent the variance of each individual asset's returns. This shows how volatile or risky that asset is on its own.The off-diagonal entries (e.g. 0.0003 for bonds vs high-tech stocks) represent the covariance between the returns of the corresponding pair of assets.

Observations: It can be observed that bonds and high-tech stocks have a positive covariance (0.0003), implying their returns move together somewhat. Bonds and call options have a slightly negative covariance (-0.00035), suggesting their returns tend to offset each other. Put options and gold have a relatively high negative covariance (-0.0008), indicating strong negative co-movement.

Now the products of deviations of each asset's return from the expected portfolio return are calculated in the following table. These products are needed to calculate the covariance between each pair of assets.

Auto Correlation		
x1^2	0.036026808	0.001
x2^2	0.011800532	0.009
x3^2	0.073347664	0.008
x4^2	0.002298489	0.012
x5^2	0.064754966	0.012
x6^2	0.016466744	0.005
x1x2	0.041237629	0.0003
x1x3	0.102810159	-0.0003
x1x4	0.018199696	0.00035
x1x5	0.096600512	-0.00035
x1x6	0.04871321	0.0004
x2x3	0.058840173	0.0004
x2x4	0.010416026	0.0016
x2x5	0.055286275	-0.0016
x2x6	0.027879479	0.0006
x3x4	0.025968352	0.0015
x3x5	0.137835053	-0.0055
x3x6	0.069506753	-0.0007
x4x5	0.024399886	-0.0005
x4x6	0.012304249	0.0008

x5x6	0.065308604	-0.0008

The variance of the portfolio is calculated as a weighted sum of the individual asset variances and the covariance terms between each pair of assets. Our objective function is the variance and our goal is to minimize variance using solver function in excel.

The decision variables for this problem is asset allocation percentages. They are obtained using solver in the following table:

	Percentage	Investment allocated
Bonds	0.190	1898.07292
High tech stocks	0.109	1086.30255
Foreign stocks	0.271	2708.27739
Call options	0.048	479.425638
Put options	0.254	2544.69971
Gold	0.128	1283.22811
Total	1.000	10000.0063

The constraints for the objective function and decision variables are: The total percentage of all the types of investments should be equal to 100% or 1. And the returns obtained is more than or equal to the targeted returns.

The following variance was obtained for 11% targeted returns.

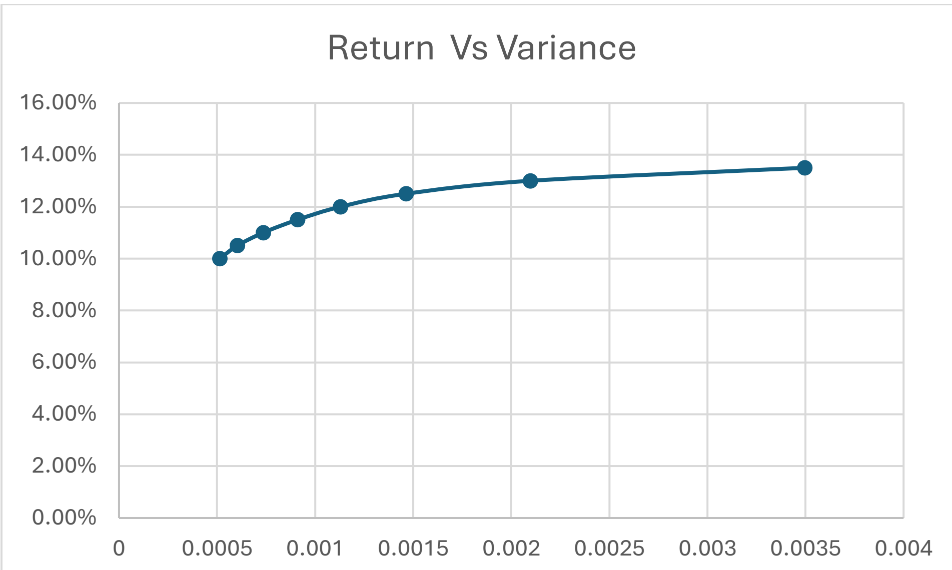
Variance	0.000735634

Observation: The optimization model has determined the optimal asset allocation strategy that minimizes the portfolio's variance (risk) while achieving the targeted expected return of 11%. The minimized variance of 0.000735634 represents the smallest level of risk the investor can expect for it. It’s important to note that this solution is based on the input parameters (expected returns, covariance matrix) and the targeted return level. If any of these inputs change or if the investor's return objective changes, the optimal allocations and minimized variance would need to be recalculated accordingly.

In the next part, we solved the optimization problem using successive target expected return levels from 10% to 13.5% to obtain pairs of minimized portfolio risk (r) and achieved expected return (e). We then plotted these (r, e) pairs to visualize and analyze the relationship between portfolio risk and expected return as the return target increases. By examining the pattern in this plot, we can gain insights into the mathematical nature of the risk-return tradeoff and make more informed investment decisions based on the investor's risk tolerance.

Expexted return	Variance	observed Return
10.00%	0.000513907	0.100

10.50%	0.000603273	0.105
11.00%	0.000735634	0.110
11.50%	0.000910995	0.115
12.00%	0.001129351	0.120
12.50%	0.001463498	12.500
13.00%	0.00209792	0.130
13.50%	0.003496248	0.135



Observations: It can be observed that as the target expected return (e) increases from 10% to 13.5%, the minimized portfolio variance (r) also increases monotonically from 0.000513907 to 0.003496248. This confirms the fundamental risk-return tradeoff in portfolio theory, where higher expected returns can only be achieved by taking on higher levels of risk (variance). The relationship between the minimized variance (r) and the target expected return (e) appears to be non-linear and convex. The variance increases at a faster rate as the expected return target gets higher. The achieved observed returns closely match the target expected returns, indicating that the optimization model is effectively allocating the assets to meet the desired return levels while minimizing risk.

Conclusion:

This project tackled two distinct optimization problems - minimizing transportation costs for hazardous waste shipment, and optimizing portfolio allocation to achieve target returns while minimizing risk.

For the transportation problem, the analysis showed that direct shipment from plants to disposal sites resulted in lower costs compared to using intermediate transshipment nodes. Therefore, it is recommended to pursue direct transportation routes for cost-effectiveness.

In the portfolio optimization problem, the analysis demonstrated the fundamental risk-return trade-off, where higher expected returns can only be achieved by taking on higher portfolio risk (variance). The optimization model successfully determined the asset allocations that minimized portfolio variance while meeting target expected returns ranging from 10% to 13.5%.

Overall, the application of optimization techniques in this project provided valuable insights and actionable solutions for both the hazardous waste transportation problem and the portfolio allocation problem.

References:

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