Monika Molnar

Math task

1.1

$$\frac{x^{n+2}}{x^{n-2}} = x^{n+2-(n-2)} = x^{n+2-n+2} = x^4$$

1.2 $x^{-1} \cdot 8 = 2$

$$x^{-1} = \frac{2}{8}$$

$$\frac{1}{x} = \frac{1}{4}$$

$$x = 4$$

1.3 $a = 5$, $b = 10$

$$(a^b)^0 = (5^{10})^0 = 5^0 = 1$$

1.4

$$\frac{\sqrt{4x}}{x} = \frac{\sqrt{4}\sqrt{x}}{\sqrt{x}} = 2\frac{\sqrt{x}}{\sqrt{x}} = 2 \cdot 1 = 2$$

1.5

$$x^2 + (x+1)^2 = (x+2)^2$$

$$2x^2 + 2x + 1 = x^2 + 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$x = -1$ $\quad x = 3$

1.6

$$2^x > 1024$$

$$2^x > 2^{10}$$

$$x > 10$$

2.1 $F(c^\circ)$ = farenheit in terms of c° $\qquad (\underbrace{m}_{\text{slope}} x + \underbrace{b}_{y\text{-intercept}})$

$F(0) = 32$ — y intercept

$F(100) = 212$ $\qquad$ slope $= \frac{212 - 32}{100 - 0} = \frac{180}{100} = 1.8$

$F(c^\circ) = 1.8 \times c^\circ + 32$

$F = C$

$F = 1.8F + 32$ $\qquad \boxed{F = -40}$

2.2 $f(x) = 5x + 4$

$f(3) = 5 \cdot 3 + 4 = 15 + 4 = \underline{19}$

2.3 $x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\underline{x=1} \quad \underline{x=3}$

2.4 $10 \cdot (1+0.02)^{90} = \underline{59.4313}$

2.5 $e^{\ln 5} = \underline{5}$

3.1 $\sum_{i=1}^{\infty} \frac{12}{6^i} = \sum_{i=1}^{\infty} 12 \cdot \left(\frac{1}{6}\right)^i = \frac{12 \cdot \frac{1}{6}}{1 - \frac{1}{6}} = \frac{\frac{2}{6}}{\frac{5}{6}} = \underline{\frac{12}{5}}$

3.2 $\lim_{x \to 1} \frac{6^{1-x}}{x} = \frac{6^{1-1}}{1} = \frac{6^0}{1} = \frac{1}{1} = \underline{1}$

3.3 $f(x) = x^5 - 8$

$f'(x) = 5x^4$

$f'(-3) = 5(-3)^4 = 5 \cdot 81 = \underline{405}$

3.4

$$f'(x) = \frac{(3x^2+2)(x-2) - (x^3+2x-1)(1)}{(x-2)^2} = \underline{\frac{2x^3 - 6x^2 - 3}{(x-2)^2}}$$

3.5 $f(x) = 4x^4 + 4x^2$

$f'(x) = 16x^3 + 8x$

$\underline{f''(x) = 48x^2 + 8}$

3.6

$$f'(x) = \frac{\frac{1}{x} \cdot e^x - \ln x \cdot e^x}{(e^x)^2} = \underline{\frac{\frac{1}{x} - \ln x}{e^x}}$$

3.7 $f(x) = 3x^2 - 5x + 2 = (3x-2)(x-1) = 0$

x-intercepts at $x = \frac{2}{3}$ and $x = 1$

$f'(x) = 6x - 5 \qquad 6x - 5 = 0$

$6x = 5$

$x = \frac{5}{6}$ — min or max

$f''(x) = 6 > 0$ → MIN

also means function always convex

x	$< \frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3} < x < \frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6} < x < 1$	1	$1 <$
$f(x)$	+	0	−	−	−	0	+
$f'(x)$	−	−	−	0	+	+	+
slope	↘	↘	↘	MIN	↗	↗	↗
$f'(x)$	+	+	+	+	+	+	+
convexity	∪	∪	∪	∪	∪	∪	∪

3.8 $f(2,3) = 2^2 + 3^3 = 4 + 27 = \underline{31}$

3.9 $f(x,y) = \ln(x-y)$

$x - y > 0$

$\underline{x > y}$

3.10 $\frac{d}{dx} x^5 + xy^3$

$\underline{5x^4 + y^3}$

3.11 $f(x,y) = x^2 y^2 + 10$

$f'_x(x,y) = 2xy^2 = 0$

$f'_y(x,y) = 2x^2 y = 0 \quad \Rightarrow 2xy^2 = 2x^2 y$

$\underline{x = y}$

$2x^2 x = 2x^3 = 0$

min for all points where $x=0$ or $y=0$

3.12 $\max x^2y^2$ $x+y=10$ $\Rightarrow x+y-10=0$

$$\mathcal{L} = x^2y^2 - \lambda(x+y-10)$$

$$\frac{\partial \mathcal{L}}{x} = 2xy^2 - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{y} = 2x^2y - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\lambda} = x+y-10$$

$2xy^2 - \lambda = 2x^2y - \lambda$

$2xy^2 = 2x^2y$

$x = y$

$\Downarrow$

$x+x-10=0$

$2x=10$

$x=5 \quad y=5$

4.1

$$\begin{bmatrix} 1 & 17 \\ 2 & 82 \end{bmatrix}$$

$$\begin{bmatrix} 26 & 2+12 & 2+68 & 14+12 \\ 51 & 5+2 & 5+8 & 57+12 \\ 19 & 1+18 & 1+9.8 & 1.7+9.2 \end{bmatrix} = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

4.2

$$\begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 191 & 2+4.9+1.1 & 2+9.6+3 \\ 2.12 & 4+4+2 & 4+6+6 \end{bmatrix} = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

4.3

$$\begin{bmatrix} 71 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

4.4

$$\begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix} = 1.8 - 9.2 = -10$$

5.1 order matters, with replacement

$\#\Omega$ = sample size $= n^k = 6^2 = \underline{36}$

$n = 6$

$k = 2$

5.2 Bayes rule

$\mathbb{P}(A)$ ↑ positive drug test on random

$\mathbb{P}(A) =$ $\mathbb{P}(A|B) =$

$\mathbb{P}(B) =$ $\mathbb{P}(B|A) =$

drug user 1% B_1

non-user 99% B_2

$\mathbb{P}(A|B_1) = 99\%$

$\mathbb{P}(A|B_2) = .5\%$ $(100\% - 99.5\%)$

$$\mathbb{P}(A) = \sum_{i=1}^{2} \mathbb{P}(B_i) \cdot \mathbb{P}(A|B_i)$$

$$= \mathbb{P}(B_1) \cdot \mathbb{P}(A|B_1) + \mathbb{P}(B_2) \cdot \mathbb{P}(A|B_2)$$

$$= 1\% \cdot 99\% + 99\% \cdot 0.5\% =$$

$$= 99\%(1\% + 0.5\%) = \underline{1.485\%}$$

5.3 $\mathbb{P}(B_j|A) = \dfrac{\mathbb{P}(A|B_j)\mathbb{P}(B_j)}{\sum_{i=1} \mathbb{P}(B_i)\mathbb{P}(A|B_i)}$ — already known

+test, drug user

$$\mathbb{P}(B_1|A) = \frac{99\% \cdot 1\%}{1.485\%} = \underline{66.44\%}$$