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A joint test for serial correlation and random individual effects

Badi H. Baltagi and Qi Li

Texas A & M University, Department of Economics, College Station, TX 77843-4228, USA

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Abstract: This paper derives a simple lagrange multiplier (LM) test which jointly tests the presence of random individual effects and serial correlation. This test is an extension of the Breusch and Pagan (1980) LM test. It is computationally simple and requires only the OLS residuals. It should prove useful for panel data applications where both serial correlation and random individual effects are suspect.

Keywords: Lagrange multiplier test, error components, Breusch–Pagan test, serial correlation.

1. Introduction

In panel data applications, the error component model, popularized by Balestra and Nerlove (1966), is by far the most widely used specification in economics. This model has been recently extended to take into account serial correlation in the remainder disturbance term by Lillard and Willis (1978). One can test for the existence of the random effects, assuming there is no serial correlation, using the lagrange multiplier (LM) test developed by Breusch and Pagan (1979, 1980). Also, one can test for serial correlation, assuming there are no random effects, using the LM test derived in Godfrey (1978), Breusch (1978) and Breusch and Pagan (1980). This paper considers the question of jointly testing for serial correlation and the random individual effects. In this context, it is important to note that Bhargava, Franzini and Narendranathan (1982) modified the Durbin-Watson statistic to test for serial correlation when the individual effects are assumed fixed. When these individual effects are assumed random, this paper shows that a simple lagrange multiplier test can be derived which jointly tests for serial correlation and the random effects. As expected, this LM test is much simpler to compute than the likelihood ratio test which requires the estimation of the unrestricted model, see Breusch and Pagan (1980), Engle (1984), and the recent monograph by Godfrey (1989) for other econometric examples that show the easy applicability of the LM test. In this case, the LM computation requires OLS residuals only, and the resulting test statistic should prove useful for panel data applications.

2. The model and the LM test

Consider the following panel data regression:

$$y_{it} = x'_{it}\beta = u_{it}, \quad i = 1, 2, \ldots, N, \ t = 1, 2, \ldots, T, \tag{1}$$

where β is a $k \times 1$ vector of regression coefficients including the intercept, i denoting individuals and t denoting time-periods. Following Lillard and

Willis (1978), we model the disturbance terms as follows:

$$u_{it} = \mu_i + \nu_{it} \tag{2}$$

with $\mu_i \sim \text{IIN}(0, \sigma_\mu^2)$ and $\nu_{it} = \rho\nu_{i,t-1} + \varepsilon_{i,t}$, $|\rho| < 1$ and $\varepsilon_{it} \sim \text{IIN}(0, \sigma_\varepsilon^2)$. The μ_i's are independent of the ν_{it}'s, and $\nu_{i,0} \sim \text{N}(0, \sigma_\varepsilon^2/(1-\rho^2))$. This is a one-way error component model with individual effects and a serially correlated remainder term.

It is well established, see for e.g. Kadiyala (1968), that

$$C = \begin{bmatrix} (1-\rho^2)^{1/2} & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\rho & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\rho & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -\rho & 1 \end{bmatrix}$$

will transform the usual AR(1) model into a serially uncorrelated regression with independent observations. The initial observation for each individual i, will be transformed as follows:

$$\begin{aligned} u_{i1}^* &= \sqrt{1-\rho^2}\, u_{i1} \\ &= \sqrt{1-\rho^2}\,(\mu_i + \nu_{i1}) \\ &= \alpha(1-\rho)\mu_i + \sqrt{1-\rho^2}\,\nu_{i1} \end{aligned} \tag{3}$$

for $i = 1, \ldots, N$, where $\alpha = \sqrt{(1+\rho)/(1-\rho)}$. The remaining observations are transformed, as expected,

$$\begin{aligned} u_{it}^* &= (u_{it} - \rho u_{i,t-1}) \\ &= \mu_i(1-\rho) + \varepsilon_{it}, \quad t = 2, \ldots, T,\ i = 1, \ldots, N. \end{aligned} \tag{4}$$

In vector form:

$$u = (I_N \otimes e_T)\mu + \nu$$

where e_T is a vector of ones of dimension T, I_N is an identity matrix of dimension N, $\mu' = (\mu_1, \ldots, \mu_N)$ and $\nu' = (\nu_{11}, \ldots, \nu_{1T}, \ldots, \nu_{N1}, \ldots, \nu_{NT})$.

$$\begin{aligned} u^* = (I_N \otimes C)u &= (I_N \otimes Ce_T)\mu + (I_N \otimes C)\nu \\ &= (1-\rho)(I_N \otimes e_T^\alpha)\mu + (I_N \otimes C)\nu \end{aligned} \tag{5}$$

where $e_T^{\alpha\prime} = (\alpha, e_{T-1}')$ and $Ce_T = (1-\rho)e_T^\alpha$. Therefore,

$$\begin{aligned} \Omega^* &= E(u^* u^{*\prime}) \\ &= \sigma_\mu^2(1-\rho)^2\left[I_N \otimes e_T^\alpha e_T^{\alpha\prime}\right] + \sigma_\varepsilon^2(I_N \otimes I_T) \end{aligned} \tag{6}$$

since $(I_N \otimes C)E(\nu\nu')(I_N \otimes C') = \sigma_\varepsilon^2(I_N \otimes I_T)$. This can be re-written as

$$\Omega^* = d^2\sigma_\mu^2(1-\rho)^2\left[I_N \otimes e_T^\alpha e_T^{\alpha\prime}/d^2\right] + \sigma_\varepsilon^2(I_N \otimes I_T) \tag{7}$$

where $d^2 = e_T^{\alpha\prime} e_T^\alpha = \alpha^2 + (T-1)$. This replaces $e_T^\alpha e_T^{\alpha\prime}$ by its idempotent counterpart $\bar{J}_T^\alpha = e_T^\alpha e_T^{\alpha\prime}/d^2$. Replacing I_T by $E_T^\alpha + \bar{J}_T^\alpha$, where $E_T^\alpha = I_T - \bar{J}_T^\alpha$ and collecting terms with the same matrices, see Wansbeek and Kapteyn (1982, 1983), one gets the spectral decomposition of Ω^*:

$$\begin{aligned} \Omega^* &= I_N \otimes \left[\sigma_\alpha^2 \bar{J}_T^\alpha + \sigma_\varepsilon^2 E_T^\alpha\right] \\ &= \sigma_\alpha^2(I_N \otimes \bar{J}_T^\alpha) + \sigma_\varepsilon^2(I_N \otimes E_T^\alpha) \end{aligned} \tag{8}$$

where $\sigma_\alpha^2 = d^2\sigma_\mu^2(1-\rho)^2 + \sigma_\varepsilon^2$ is the first characteristic root of Ω^* of multiplicity N and σ_ε^2 is the second characteristic root of Ω^* of multiplicity $N(T-1)$, see Baltagi and Li (1990). Therefore,

$$\Omega^{*p} = (\sigma_\alpha^2)^p(I_N \otimes \bar{J}_T^\alpha) + (\sigma_\varepsilon^2)^p(I_N \otimes E_T^\alpha) \tag{9}$$

where p is any arbitrary scalar. $p = -1$ obtains the inverse, while $p = -\frac{1}{2}$ obtains $\Omega^{*-1/2}$. $\Omega = E(uu')$ is related to Ω^* by $\Omega^* = (I_N \otimes C)\Omega(I_N \otimes C')$ and $|C| = \sqrt{1-\rho^2}$, $|I_N \otimes C| = |C|^N$ and $|\Omega^*| = (\sigma_\varepsilon^2)^{N(T-1)}(\sigma_\alpha^2)^N$. Therefore, the log likelihood function can be written as:

$$\begin{aligned} L(\beta, \sigma_\mu^2, \rho, \sigma_\varepsilon^2) = \text{const.} &- \tfrac{1}{2}N(T-1)\log\sigma_\varepsilon^2 \\ &+ \tfrac{1}{2}N\log(1-\rho^2) \\ &- \tfrac{1}{2}N\log\left[\sigma_\varepsilon^2 + \sigma_\mu^2 d^2(1-\rho)^2\right] \\ &- \tfrac{1}{2}u^{*\prime}\Omega^{*-1}u^* \end{aligned} \tag{10}$$

where Ω^{*-1} is given by (9).

Following Breusch and Pagan (1980), we let $\theta = (\sigma_\mu^2, \rho, \sigma_\varepsilon^2)'$. The information matrix will be block diagonal between the θ and β parameters, and since the hypothesis tested H_0; $\rho = 0$ and $\sigma_\mu^2 = 0$, involves only the θ parameters, the part of the information matrix corresponding to β will be ignored in computing the LM statistic, see equation (7) of Breusch and Pagan (1980, p. 241).

$$\mathrm{LM} = \tilde{D}_1' \tilde{J}_{11}^{-1} \tilde{D}_1 \tag{11}$$

where $\tilde{D}_1 = (\partial L/\partial\theta)(\tilde{\theta})$ is a 3×1 vector of partial derivatives of the likelihood function with respect to each element of θ, evaluated at the restricted mle $\tilde{\theta}$. $J_{11} = E[-\partial^2 L/\partial\theta\,\partial\theta']$ is the part of the information matrix corresponding to θ, and $\tilde{J}_{11}$ is J_{11} when the null hypothesis is true, evaluated at the restricted mle $\tilde{\theta}$. Under the null hypothesis, the variance–covariance matrix reduces to $\Omega^* = \Omega = \sigma_\varepsilon^2 I_{NT}$ and the restricted mle of β is $\tilde{\beta}_{\mathrm{OLS}}$, so that $\tilde{u} = y - x'\tilde{\beta}_{\mathrm{OLS}}$ are the OLS residuals and $\tilde{\sigma}_\varepsilon^2 = \tilde{u}'\tilde{u}/NT$. In fact,

$$\tilde{D}_1 = \begin{bmatrix} \dfrac{-NT}{2\tilde{\sigma}_\varepsilon^2}\left(1 - \tilde{u}'(I_N \otimes J_T)\tilde{u}/\tilde{u}'\tilde{u}\right) \\ NT(\tilde{u}'\tilde{u}_{-1}/\tilde{u}'\tilde{u}) \\ 0 \end{bmatrix},$$

$$\tilde{J}_{11} = \frac{NT}{2\tilde{\sigma}_\varepsilon^4}\begin{bmatrix} T & \dfrac{2(T-1)\tilde{\sigma}_\varepsilon^2}{T} & 1 \\ \dfrac{2(T-1)\tilde{\sigma}_\varepsilon^2}{T} & \left(\dfrac{T-1}{T}\right)2\tilde{\sigma}_\varepsilon^4 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

and the LM statistic is given by

$$\mathrm{LM} = \frac{NT^2}{2(T-1)(T-2)}\left[A^2 + 4AB + 2TB^2\right] \tag{12}$$

and is asymptotically distributed as χ_2^2 under the null hypothesis.

$$A = 1 - \left[\tilde{u}'(I_N \otimes J_T)\tilde{u}/\tilde{u}'\tilde{u}\right] \quad \text{and}$$

$$B = \tilde{u}'\tilde{u}_{-1}/\tilde{u}'\tilde{u}.$$

A is the familiar term used in testing $\sigma_\mu^2 = 0$, see Breusch and Pagan (1980), while $B = \tilde{\rho}$ is the familiar estimate of ρ used in testing $\rho = 0$. These terms can be easily computed from the OLS residuals.

If $\rho = 0$, and one is testing H_0: $\sigma_\mu^2 = 0$, the same derivation applies except $\theta = (\sigma_\mu^2, \sigma_\varepsilon^2)'$ and we ignore the second element of $\tilde{D}_1$ and the second row and column of $\tilde{J}_{11}$. In this case, the LM statistic reverts back to $(NT)A^2/2(T-1)$ reported in Breusch and Pagan (1980). Similarly, if $\sigma_\mu^2 = 0$ and one is testing H_0: $\rho = 0$, then $\theta = (\rho, \sigma_\varepsilon^2)'$ and we ignore the first element of $\tilde{D}_1$ and the first row and column of $\tilde{J}_{11}$. In this case, the LM statistic becomes $[NT^2/(T-1)] \times (\tilde{u}'\tilde{u}_{-1}/\tilde{u}'\tilde{u})^2 = NT^2\tilde{\rho}^2/(T-1)$.

In conclusion, the LM statistic for the joint test H_0: $\rho = 0$ and $\sigma_\mu^2 = 0$, given in (12), involves an interaction term $(4AB)$ in addition to the familiar A^2 and B^2 terms. This clearly demonstrates the importance of carrying out this joint test whenever both serial correlation and random individual effects are suspect.

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