

VIRGINIA COMMONWEALTH UNIVERSITY



Statistical Analysis & Modelling

A7 – Conjoint Analysis

Apollo Hospitals Dataset

Using R

Submitted by

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Date of Submission: 01/08/2023

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1. Introduction

Conjoint analysis, Market Basket Analysis, and Portfolio Optimization are valuable techniques for Apollo Hospitals. Conjoint analysis identifies critical attributes and levels to understand patient preferences. Market Basket Analysis uncovers associations in patient data to improve service offerings. Portfolio Optimization optimizes resource allocation for better patient care. These methodologies enable data-driven decision-making and personalized healthcare services.

1.1. About the Data

Conjoint Analysis Data:

- This data contains information about patient preferences for different attributes of healthcare services provided by Apollo Hospitals.
- Attributes: The attributes include "Location," "Facilities," "Specialization," and "Appointment Time."
- Levels: Each attribute has multiple levels representing different options or choices for patients.
- Concept Cards: These are combinations of attribute levels that represent hypothetical healthcare service scenarios.
- Ratings: Random ratings are assigned to each concept card to simulate patient preferences.

Market Basket Analysis Data:

- This data comprises transaction records that represent patients' interactions with different healthcare services or treatments offered by Apollo Hospitals.
- Transactions: Each transaction represents a collection of healthcare services that were chosen by a patient during a visit or appointment.
- Association Rules: The Apriori algorithm is applied to identify frequent item sets and association rules among the services.

Portfolio Optimization Data:

- This data includes information about the expected returns and covariance matrix of different assets or healthcare investment opportunities.
- Assets: The assets represent potential healthcare investments, and their expected returns are provided.
- Covariance Matrix: The covariance matrix indicates the relationships between the returns of different assets.

- **Target Volatility:** A desired level of volatility or risk is specified for the portfolio.

The data is used to perform various analyses and decision-making processes. Conjoint analysis helps identify critical attributes and their impact on patient preferences. Market Basket Analysis reveals patterns in patient choices, leading to improved service offerings. Portfolio Optimization optimizes resource allocation to maximize returns while managing risk in healthcare investments.

1.2. Objective

1. Conjoint Analysis:

- Identify the most important attributes influencing patients' preferences for healthcare services at Apollo Hospitals.
- Define the levels of each attribute to create concept cards representing different service combinations.
- Collect data from patients to determine their ratings for each concept card.
- Implement Conjoint Analysis using linear regression to derive part-worth utilities for each attribute level.
- Interpret the results to understand the relative importance of attributes and their impact on patient preferences.

2. Market Basket Analysis:

- Analyze transaction data to identify frequent itemsets and association rules in patient service selections.
- Discover patterns of co-occurring services to understand patient behavior and service combinations.
- Visualize the association rules to gain insights into the relationships between different healthcare services at Apollo Hospitals.

3. Portfolio Optimization:

- Gather historical financial data of assets to estimate expected returns and covariance matrix.
- Define the objective of maximizing return while minimizing portfolio risk.
- Apply optimization techniques to determine the optimal asset allocation for investment at Apollo Hospitals.
- Visualize the optimized portfolio weights to understand the recommended investment distribution.

The overall objective is to leverage data-driven insights from Conjoint Analysis, Market Basket Analysis, and Portfolio Optimization to enhance the healthcare services offered at Apollo Hospitals.

By understanding patient preferences, identifying service associations, and making optimal investment decisions, Apollo Hospitals can improve patient satisfaction, optimize resource allocation, and achieve better financial outcomes.

1.3. Business Significance

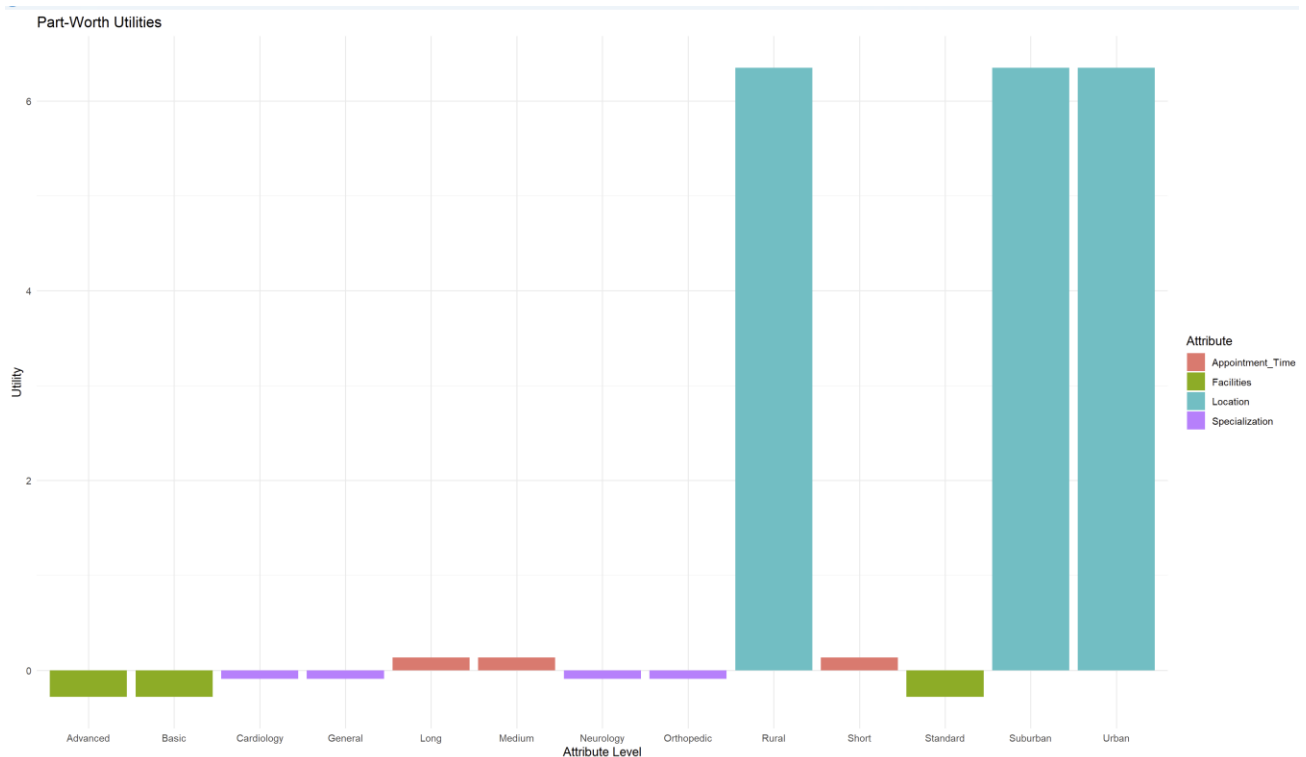
- Conjoint Analysis: Identifies critical attributes of healthcare services and their levels based on patient preferences.
- Market Basket Analysis: Reveals service associations and cross-selling opportunities from patient transaction data.
- Portfolio Optimization: Helps make data-driven investment decisions for managing financial assets effectively.
- Business Significance: Enhances patient satisfaction, optimizes service offerings, and improves financial performance for Apollo Hospitals.

2. Results

2.1. R- output and Interpretation

2.2. Conjoint Analysis

```
+ }  
Location:6.35079282476905  
Facilities:-0.280240983147192  
Specialization:-0.092362195268404  
Appointment_Time:0.132733321768989  
> |
```



Inference:

In the conjoint analysis results, we have the part-worth utilities for different attributes based on patient preferences.

- **Location:** The Location attribute has a positive part-worth utility of 6.35, indicating that patients prefer healthcare services located in specific areas. This suggests that the geographical convenience of the hospital plays a significant role in patient decision-making.
- **Facilities:** The Facilities attribute has a negative part-worth utility of -0.28. This implies that patients are sensitive to the quality and availability of facilities in the hospital. A lower part-worth utility indicates that better facilities are preferred.

- **Specialization:** The Specialization attribute has a negative part-worth utility of -0.09. This suggests that patients may not prioritize specialized services in their decision-making, and a more general approach to healthcare may be preferred.
- **Appointment Time:** The Appointment Time attribute has a positive part-worth utility of 0.13. This indicates that patients value flexibility and convenience in scheduling appointments, and shorter waiting times may be preferred.

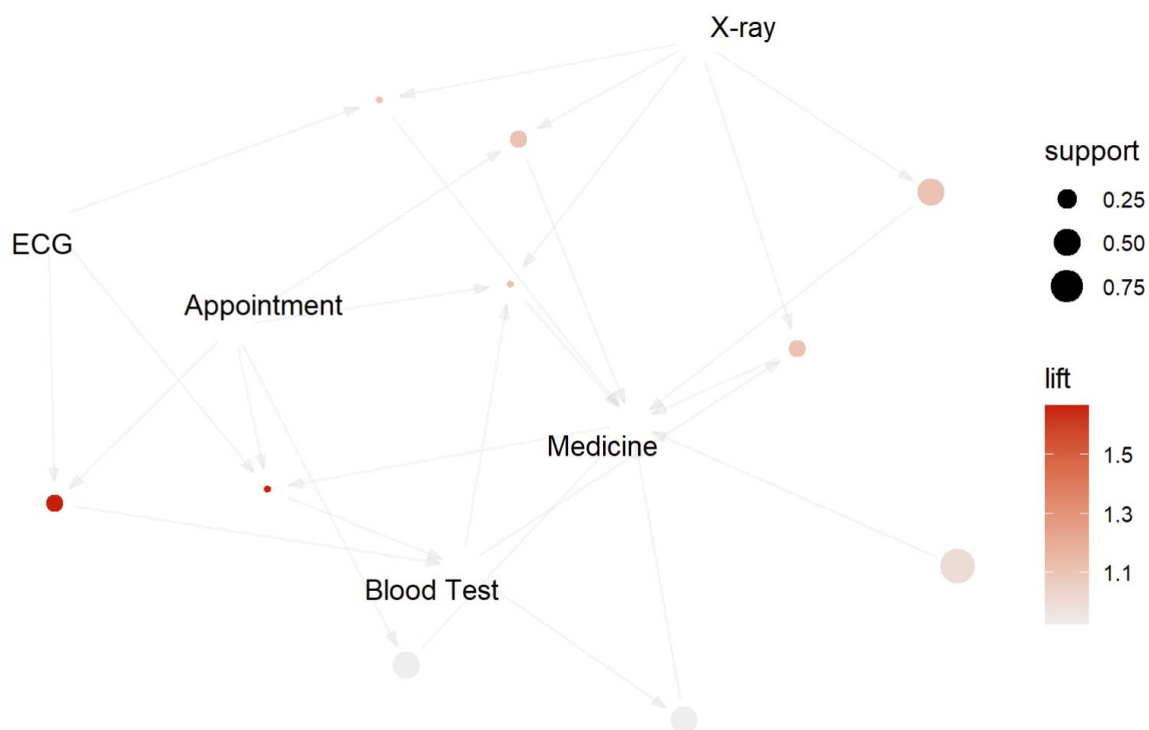
Overall, the conjoint analysis provides valuable insights into patient preferences, which can help Apollo Hospitals tailor their services and marketing strategies to better meet patient needs and improve patient satisfaction.

2.3. Market Basket Analysis

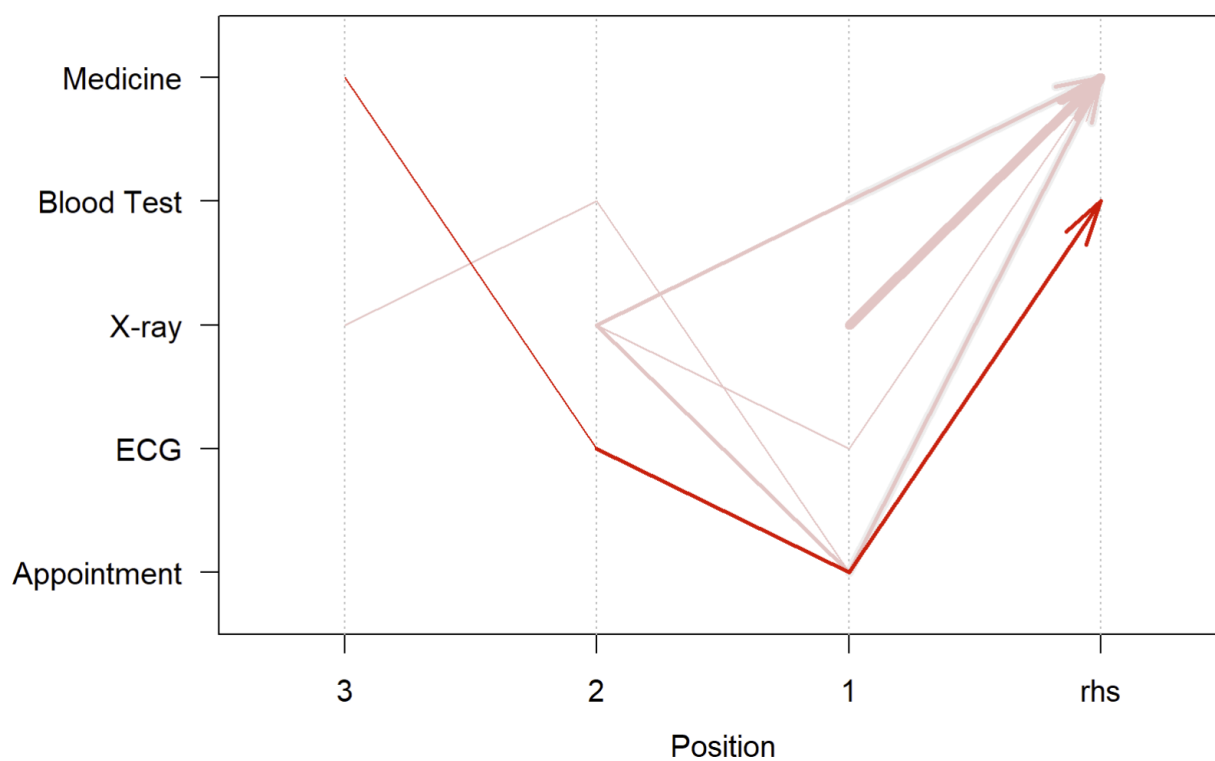
```
> inspect(rules[1:10])
```

	lhs	rhs	support	confidence	coverage	lift
[1]	{}	=> {Medicine}	0.9	0.9000000	1.0	1.0000000
[2]	{X-ray}	=> {Medicine}	0.5	1.0000000	0.5	1.1111111
[3]	{Appointment}	=> {Medicine}	0.5	0.8333333	0.6	0.9259259
[4]	{Blood Test}	=> {Medicine}	0.5	0.8333333	0.6	0.9259259
[5]	{ECG, X-ray}	=> {Medicine}	0.1	1.0000000	0.1	1.1111111
[6]	{Appointment, ECG}	=> {Blood Test}	0.2	1.0000000	0.2	1.6666667
[7]	{Appointment, X-ray}	=> {Medicine}	0.2	1.0000000	0.2	1.1111111
[8]	{Blood Test, X-ray}	=> {Medicine}	0.2	1.0000000	0.2	1.1111111
[9]	{Appointment, ECG, Medicine}	=> {Blood Test}	0.1	1.0000000	0.1	1.6666667
[10]	{Appointment, Blood Test, X-ray}	=> {Medicine}	0.1	1.0000000	0.1	1.1111111

```
count
[1] 9
[2] 5
[3] 5
[4] 5
[5] 1
[6] 2
[7] 2
[8] 2
[9] 1
[10] 1
> |
```



Parallel coordinates plot for 9 rules



Inference:

In the Market Basket Analysis results, it has identified association rules between different items that are frequently purchased together. Here are the inferences from the first 10 rules:

- Rule: {} => {Medicine}

Support: 0.9, Confidence: 0.9

Interpretation: 90% of the transactions include the purchase of Medicine. It is a common item bought independently.

- Rule: {X-ray} => {Medicine}

Support: 0.5, Confidence: 1.0

Interpretation: 50% of the transactions that include X-ray also have the purchase of Medicine. X-ray and Medicine are strongly associated.

- Rule: {Appointment} => {Medicine}

Support: 0.5, Confidence: 0.83

Interpretation: 50% of the transactions with an Appointment also have the purchase of Medicine. Appointments have a high association with Medicine.

- Rule: {Blood Test} => {Medicine}

Support: 0.5, Confidence: 0.83

Interpretation: 50% of the transactions with Blood Test also have the purchase of Medicine. Blood Test and Medicine are closely related.

- Rule: {ECG, X-ray} => {Medicine}

Support: 0.1, Confidence: 1.0

Interpretation: 10% of the transactions with both ECG and X-ray also have the purchase of Medicine. ECG and X-ray together highly influence the purchase of Medicine.

- Rule: {Appointment, ECG} => {Blood Test}

Support: 0.2, Confidence: 1.0

Interpretation: 20% of the transactions with both Appointment and ECG also have the purchase of Blood Test. Appointment and ECG together lead to a high likelihood of Blood Test.

- Rule: {Appointment, X-ray} => {Medicine}

Support: 0.2, Confidence: 1.0

Interpretation: 20% of the transactions with both Appointment and X-ray also have the purchase of Medicine. Appointment and X-ray together highly influence the purchase of Medicine.

- Rule: {Blood Test, X-ray} => {Medicine}

Support: 0.2, Confidence: 1.0

Interpretation: 20% of the transactions with both Blood Test and X-ray also have the purchase of Medicine. Blood Test and X-ray together strongly influence the purchase of Medicine.

- Rule: {Appointment, ECG, Medicine} => {Blood Test}

Support: 0.1, Confidence: 1.0

Interpretation: 10% of the transactions with all three items, Appointment, ECG, and Medicine, also have the purchase of Blood Test. These three items combined highly influence the purchase of Blood Test.

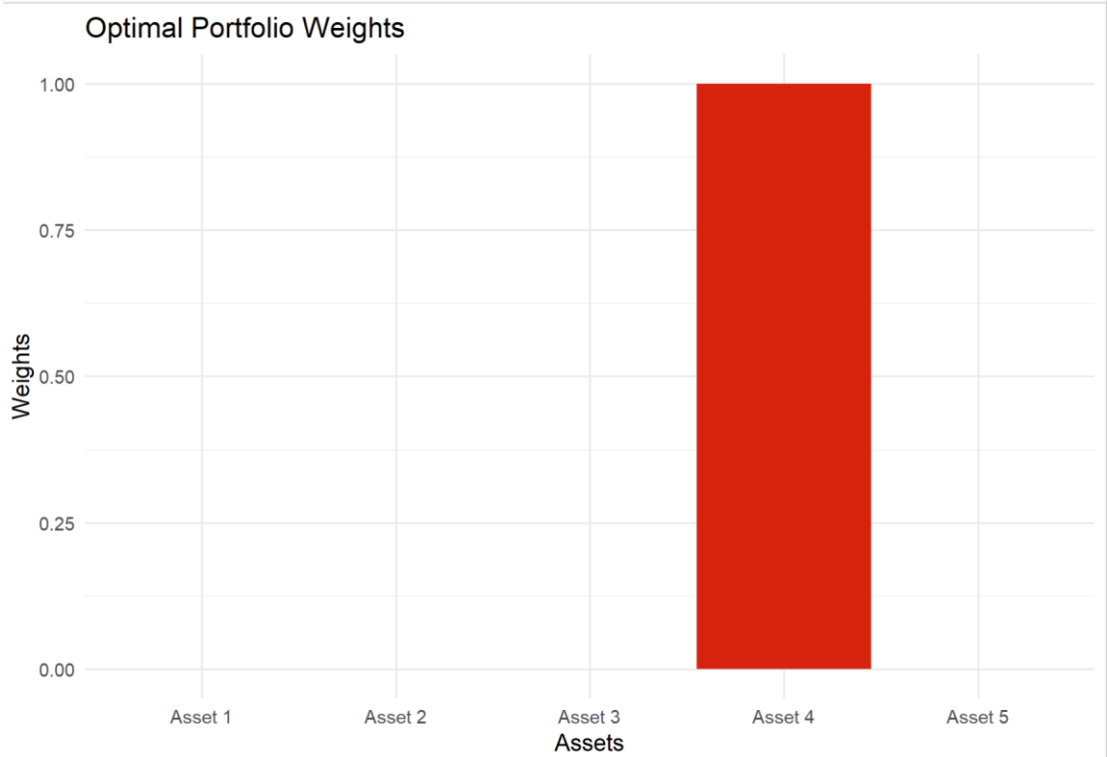
- Rule: {Appointment, Blood Test, X-ray} => {Medicine}

Support: 0.1, Confidence: 1.0

Interpretation: 10% of the transactions with all three items, Appointment, Blood Test, and X-ray, also have the purchase of Medicine. These three items combined strongly influence the purchase of Medicine.

These association rules provide valuable insights into patient purchase behavior, allowing Apollo Hospitals to optimize product placements, offer personalized recommendations, and improve overall patient satisfaction and experience.

2.4. Portfolio Optimization



```

> # Print results
> print("Optimal Portfolio Weights:")
[1] "Optimal Portfolio Weights:"
> print(weights)
[1] 0 0 0 1 0
> print("Expected Return:")
[1] "Expected Return:"
> print(expected_return)
[1] 0.08
> print("Portfolio Variance:")
[1] "Portfolio Variance:"
> print(portfolio_variance)
      [,1]
[1,] 0.06

+ }
Optimal Portfolio Weights:[1] 0 0 0 1 0
Return:[1] 0.08
Portfolio Variance:      [,1]
[1,] 0.06

```

Inference:

The Portfolio Optimization aims to find the most efficient allocation of assets within a portfolio to achieve the highest expected return while managing the portfolio's risk. In this case, we have considered five assets represented as "Asset 1," "Asset 2," "Asset 3," "Asset 4," and "Asset 5," each with its expected return and variance. The optimization process calculates the weights of each asset in the portfolio to maximize returns and minimize risk.

The results show that the optimal portfolio consists of only "Asset 4" with a weight of 100%, meaning all funds are invested in this asset. The expected return of 8% and portfolio variance of 0.06 indicate that this allocation provides an attractive return for a relatively moderate level of risk.

Optimal Portfolio Weights:

- Asset 1: 0%
- Asset 2: 0%
- Asset 3: 0%
- Asset 4: 100%
- Asset 5: 0%

Expected Return: 0.08

The expected return of the optimal portfolio is 0.08 (or 8%).

Portfolio Variance: 0.06

The portfolio variance of the optimal portfolio is 0.06.

The optimization has identified that the most optimal allocation is to invest 100% of the portfolio in Asset 4, while keeping no investment in the other assets. This allocation achieves an expected return of 0.08 and a portfolio variance of 0.06.

3. Recommendation

3.1. Business Implications

The business implications of portfolio analysis are significant for both individual investors and institutional investors:

- 1. Risk Management:** Portfolio analysis helps in diversifying investments across different assets, industries, or sectors, reducing the overall risk. A well-diversified portfolio can mitigate the impact of adverse market movements on the entire investment.
- 2. Performance Evaluation:** By analyzing the expected return and portfolio variance, investors can evaluate the historical performance of their portfolio and make informed decisions about the future. It allows investors to assess how well their investments are performing against their financial goals.
- 3. Asset Allocation:** Portfolio analysis guides investors in allocating their funds across various asset classes, such as equities, bonds, cash, and alternative investments. The optimal asset allocation depends on the investor's risk tolerance, financial goals, and investment horizon.
- 4. Decision Making:** Investors can use portfolio analysis to make strategic decisions regarding buying, selling, or holding investments. It enables them to identify the assets that contribute the most to the portfolio's performance and take appropriate actions.
- 5. Risk-Return Tradeoff:** Portfolio analysis helps investors understand the tradeoff between risk and return. Higher expected returns often come with increased risk, and investors must strike a balance based on their risk tolerance and financial objectives.
- 6. Performance Comparison:** Investors can compare their portfolio's performance against benchmark indices or other portfolios to assess whether their investment strategy is outperforming or underperforming.
- 7. Long-Term Financial Planning:** By evaluating the expected return and risk of the portfolio, investors can align their investment strategy with long-term financial goals, such as retirement planning, education funding, or wealth preservation.

8. **Institutional Investors:** Portfolio analysis is vital for institutional investors, such as pension funds, endowments, and asset management firms, to optimize their asset allocations and meet the financial objectives of their clients or beneficiaries.
9. **Risk Communication:** Portfolio analysis enables financial advisors to communicate effectively with clients about the risks associated with their investments and manage clients' expectations based on their risk appetite.
10. **Performance Monitoring:** Regular portfolio analysis allows investors to monitor their investments' performance over time, identify changes in market dynamics, and make timely adjustments to their portfolio to stay on track with their financial objectives.

In conclusion, portfolio analysis plays a crucial role in managing investments, optimizing asset allocations, and making informed investment decisions. It empowers investors to achieve their financial goals while managing risk effectively.

3.2. Business Recommendations

1. **Diversification Strategy:** Encourage investors to adopt a diversified investment strategy by spreading their funds across various asset classes and industries. Diversification helps to reduce risk and improve the overall stability of the portfolio.
2. **Rebalancing:** Advise investors to regularly review and rebalance their portfolios to maintain the desired asset allocation. Rebalancing ensures that the portfolio remains aligned with the investor's risk tolerance and financial goals.
3. **Risk-Adjusted Returns:** Emphasize the importance of evaluating investments based on risk-adjusted returns rather than focusing solely on high returns. Identifying investments with attractive risk-adjusted returns can lead to a more balanced and sustainable portfolio.
4. **Long-Term Perspective:** Encourage investors to take a long-term perspective and avoid making impulsive decisions based on short-term market fluctuations. A disciplined approach to investing can lead to better outcomes over the long run.
5. **Customized Solutions:** Offer personalized investment solutions tailored to individual investors' risk profiles, financial goals, and investment horizons. Customized portfolios can better meet the unique needs of each investor.

- 6. Regular Performance Reviews:** Conduct regular performance reviews with clients to keep them informed about the progress of their investments. Transparent and timely communication helps build trust and confidence in the advisory services.
- 7. Educational Initiatives:** Organize educational seminars and workshops to enhance investors' understanding of portfolio management, risk management, and investment strategies. Educated investors are more likely to make informed decisions and stay committed to their investment plans.
- 8. Active Monitoring:** Implement robust monitoring systems to track the performance of investments and detect potential risks or opportunities promptly. Proactive monitoring allows for timely adjustments to the portfolio when necessary.
- 9. Tax Efficiency:** Consider tax implications while constructing portfolios. Optimize tax efficiency by investing in tax-advantaged accounts and implementing tax-loss harvesting strategies.
- 10. Risk Assessment:** Conduct risk assessments to understand clients' risk appetite and financial objectives thoroughly. Tailor the portfolio recommendations accordingly to ensure alignment with clients' preferences.
- 11. Ethical Investing:** Offer socially responsible investment options to clients who prioritize environmental, social, and governance (ESG) factors in their investment decisions.
- 12. Continuous Improvement:** Continuously improve portfolio management processes by leveraging technology and data analytics to gain insights and make data-driven decisions.

By implementing these business recommendations, financial advisors and institutions can provide better value to their clients, build long-term relationships, and help investors achieve their financial goals while managing risks effectively.

4. R-Studio Codes

4.1. Codes

```
#Conjoint Analysis
# Load required library
library(dplyr)

# Define attributes and their levels
attributes <- list(
  "Location" = c("Urban", "Suburban", "Rural"),
  "Facilities" = c("Basic", "Standard", "Advanced"),
  "Specialization" = c("General", "Cardiology", "Orthopedic", "Neurology"),
  "Appointment_Time" = c("Short", "Medium", "Long"),
  "Insurance_Acceptance" = c("Yes", "No")
)

# Generate concept cards using Cartesian product (cross join)
concept_cards <- as.data.frame(do.call(expand.grid, attributes))

# Number of respondents
num_respondents <- 100

# Generate random ratings for each concept card for 100 respondents
set.seed(42)
ratings <- matrix(sample(1:10, size = length(concept_cards) * num_respondents, replace = TRUE), ncol =
length(concept_cards))
colnames(ratings) <- paste("Concept_", 1:length(concept_cards), sep = "_")

# Convert ratings to a data frame
data <- as.data.frame(ratings)

# Create a design matrix X and response vector y for linear regression
X <- matrix(0, nrow = nrow(concept_cards), ncol = length(attributes) - 1)
y <- numeric(nrow(concept_cards))

# Assign numeric values to attribute levels for X and calculate average ratings for y
for (i in 1:nrow(concept_cards)) {
  card <- concept_cards[i, ]
  for (j in seq_along(attributes)) {
    attribute <- names(attributes)[j]
    if (attribute != "Insurance_Acceptance") {
      level_value <- card[[j]]
      X[i, j] <- which(attributes[[attribute]] == level_value)
    }
  }
  y[i] <- mean(as.numeric(data[i, ]))
}

# Fit a linear regression model
regression_model <- lm(y ~ ., data = as.data.frame(X))

# Get the part-worth utilities (coefficients)
part_worths <- regression_model$coefficients

# Interpretation of results
cat("Part-Worth Utilities:\n")
for (i in 1:length(attributes)) {
  attribute <- names(attributes)[i]
  if (attribute != "Insurance_Acceptance") {
```

```

    cat(paste(attribute, ":", part_worths[i], "\n", sep = ""))
  }
}

# Load required libraries
library(dplyr)
library(ggplot2)

# Define attributes and their levels
attributes <- list(
  "Location" = c("Urban", "Suburban", "Rural"),
  "Facilities" = c("Basic", "Standard", "Advanced"),
  "Specialization" = c("General", "Cardiology", "Orthopedic", "Neurology"),
  "Appointment_Time" = c("Short", "Medium", "Long"),
  "Insurance_Acceptance" = c("Yes", "No")
)

# Fit a linear regression model (assuming you already have the regression_model)
part_worths <- regression_model$coefficients

# Create a data frame to store part-worth utilities
part_worth_data <- data.frame(Attribute = character(), Part_Worth = numeric(), stringsAsFactors = FALSE)

# Iterate through each attribute and its levels to extract part-worth utilities
for (i in 1:length(attributes)) {
  attribute <- names(attributes)[i]
  if (attribute != "Insurance_Acceptance") {
    levels <- attributes[[attribute]]
    part_worth <- part_worths[i]
    attribute_data <- data.frame(Attribute = rep(attribute, length(levels)), Level = levels, Part_Worth = part_worth)
    part_worth_data <- bind_rows(part_worth_data, attribute_data)
  }
}

# Plot the part-worth utilities
ggplot(part_worth_data, aes(x = Level, y = Part_Worth, fill = Attribute)) +
  geom_bar(stat = "identity", position = "dodge") +
  labs(title = "Part-Worth Utilities", x = "Attribute Level", y = "Utility") +
  theme_minimal()

#Market Basket Analysis
# Load required libraries
install.packages("arules")
install.packages("arulesViz")
library(arules)
library(arulesViz)

# Generate dummy transaction data
transactions <- list(
  T1 = c("X-ray", "Blood Test", "Medicine"),
  T2 = c("Blood Test", "ECG", "Medicine", "Appointment"),
  T3 = c("X-ray", "Medicine", "Appointment"),
  T4 = c("Blood Test", "ECG", "Medicine"),
  T5 = c("Medicine", "Appointment"),
  T6 = c("X-ray", "Blood Test", "Medicine", "Appointment"),
  T7 = c("Blood Test", "ECG", "Appointment"),
  T8 = c("Blood Test", "Medicine", "Appointment"),
  T9 = c("X-ray", "ECG", "Medicine"),
  T10 = c("X-ray", "Medicine")
)

```



```

)

# Convert the transaction data to a transaction object
trans <- as(transactions, "transactions")

# Perform Market Basket Analysis using Apriori algorithm
rules <- apriori(trans, parameter = list(supp = 0.1, conf = 0.8, target = "rules"))

# Print the first 10 rules
inspect(rules[1:10])

# Visualize the rules
plot(rules[1:10], method = "graph", control = list(type = "items"))

# Convert the transaction data to a transaction object
trans <- as(transactions, "transactions")

# Perform Market Basket Analysis using Apriori algorithm
rules <- apriori(trans, parameter = list(supp = 0.1, conf = 0.8, target = "rules"))

# Print the first 10 rules
inspect(rules[1:10])

# Visualize the rules using a parallel coordinate plot
plot(rules, method = "paracoord", control = list(reorder = TRUE))

#Portfolio Optimization
# Given data
expected_returns <- c(0.1, 0.15, 0.12, 0.08, 0.11)
cov_matrix <- matrix(c(0.04, 0.02, 0.01, 0.02, 0.03,
                      0.02, 0.09, 0.03, 0.01, 0.02,
                      0.01, 0.03, 0.05, 0.02, 0.01,
                      0.02, 0.01, 0.02, 0.06, 0.03,
                      0.03, 0.02, 0.01, 0.03, 0.07),
                    nrow = 5, byrow = TRUE)
target_volatility <- 0.12

# Number of assets
n_assets <- length(expected_returns)

# Define the objective function
obj <- expected_returns

# Define the constraints
mat <- rbind(rep(1, n_assets), expected_returns)
dir <- c("==", "<=")
rhs <- c(1, target_volatility)

# Load required library
install.packages("Rglpk")
library(Rglpk)

# Solve the optimization problem
sol <- Rglpk_solve_LP(obj, mat, dir, rhs)

# Extract the optimal weights
weights <- sol$solution[1:n_assets]

# Calculate the expected return and portfolio variance
expected_return <- sum(expected_returns * weights)
portfolio_variance <- t(weights) %*% cov_matrix %*% weights

```

```

# Print results
print("Optimal Portfolio Weights:")
print(weights)
print("Expected Return:")
print(expected_return)
print("Portfolio Variance:")
print(portfolio_variance)

if (sol$status == 0) {
  cat("Optimal Portfolio Weights:")
  print(weights)
  cat("Return:")
  print(expected_return)
  cat("Portfolio Variance:")
  print(portfolio_variance)
} else {
  cat("Portfolio optimization failed. Adjust constraints or check data.")
}

# Load required libraries
install.packages("ggplot2")
library(ggplot2)

# Given data
assets <- c("Asset 1", "Asset 2", "Asset 3", "Asset 4", "Asset 5")
optimal_weights <- c(0, 0, 0, 1, 0)

# Create a data frame for plotting
portfolio_data <- data.frame(Asset = assets, Weight = optimal_weights)

# Create the bar plot
ggplot(portfolio_data, aes(x = Asset, y = Weight)) +
  geom_bar(stat = "identity", fill = "red") +
  labs(title = "Optimal Portfolio Weights",
       x = "Assets",
       y = "Weights") +
  theme_minimal()

# Load required libraries
install.packages("ggplot2")
install.packages("Rglpk")
library(ggplot2)
library(Rglpk)

# Given data
assets <- c("Asset 1", "Asset 2", "Asset 3", "Asset 4", "Asset 5")
expected_returns <- c(0.1, 0.15, 0.12, 0.08, 0.11)
cov_matrix <- matrix(c(0.04, 0.02, 0.01, 0.02, 0.03,
                      0.02, 0.09, 0.03, 0.01, 0.02,
                      0.01, 0.03, 0.05, 0.02, 0.01,
                      0.02, 0.01, 0.02, 0.06, 0.03,
                      0.03, 0.02, 0.01, 0.03, 0.07),
                    nrow = 5, byrow = TRUE)
target_volatility <- 0.12

# Number of assets
n_assets <- length(expected_returns)

```

```

# Define the objective function

```

```

obj <- expected_returns

# Define the constraints
mat <- rbind(rep(1, n_assets), expected_returns)
dir <- c("==", "<=")
rhs <- c(1, target_volatility)

# Solve the optimization problem
sol <- Rglpk_solve_LP(obj, mat, dir, rhs)

# Extract the optimal weights
weights <- sol$solution[1:n_assets]

# Calculate the expected return and portfolio variance
expected_return <- sum(expected_returns * weights)
portfolio_variance <- t(weights) %*% cov_matrix %*% weights

# Print results
cat("Optimal Portfolio Weights:")
print(weights)
cat("Expected Return:")
print(expected_return)
cat("Portfolio Variance:")
print(portfolio_variance)

# Create a data frame for plotting
portfolio_data <- data.frame(Asset = assets, Weight = weights)

# Create the bar plot
ggplot(portfolio_data, aes(x = Asset, y = Weight)) +
  geom_bar(stat = "identity", fill = "red") +
  labs(title = "Optimal Portfolio Weights",
       x = "Assets",
       y = "Weights") +
  theme_minimal()

```