COMP 3761

Math Review

Math Review

- Logarithm
- Floor and Celling
- Counting
  - Permutations
  - Subsets
- Summation

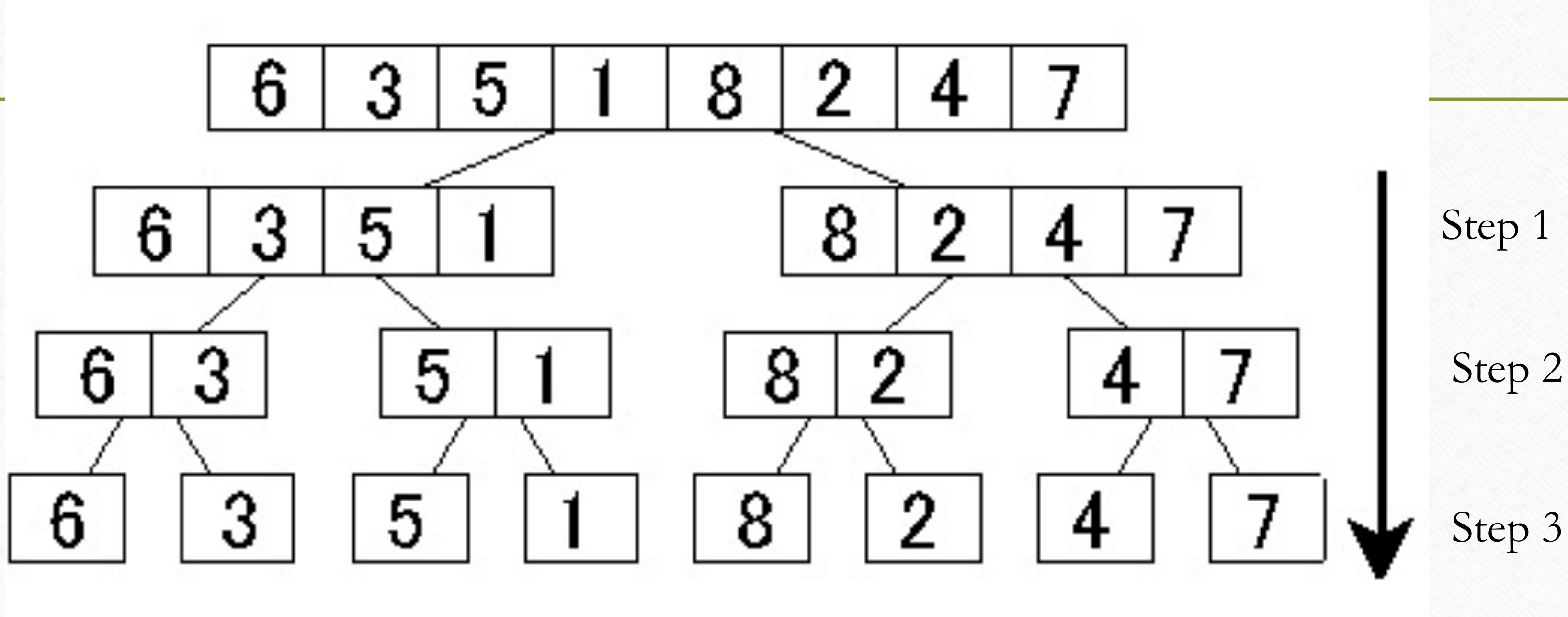
Logarithms

- Mostly what you need to know:
  - $log_b n = e$
  - just means: $b^e = n$
- So these are the same question:
  - $\log_2 16 = ?$
  - $16 = 2^?$
- In words:
  - "What is log base 2 of 16?"
  - "What power of 2 gives 16?"

When We Use Them in This Course

- The most common time to use:
  - Start with n items
  - Divide the group in half at each step
  - How many steps does it take to get down to one?

Example



$\log_2 8 = 3$

Floor and Ceiling

- If *x* is not a whole number, these are useful:
  - $\lceil x \rceil$ = The closest whole number *above* x
    (the *ceiling* of x)
  - $\lfloor x \rfloor$ = The closest whole number *below* x
    (the *floor* of x)

So: $\lceil \log 38 \rceil$ = 6

$\lfloor \log 38 \rfloor$ = 5

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Counting

- Sometimes, we need to count things
- Example

In how many different ways could student sit on the chairs in a class?

Counting

- The trick when counting is this:
  - Divide the problem into a sequence of independent choices
  - See how many options there are for each choice
  - Multiply those number together

Counting Permutations

- A permutation is an arrangement in which order matters. ABC differs from BCA
- How many permutations are there on a collection of *3* items, A, B,C?
- ABC, ACB, BAC, BCA, CAB, CBA
- Permutations for a set of n elements: $n! = n*(n-1)*\ldots*1$

Permutations

- For ABC:
  - n! = n*(n-1)*…*1
  - 3! = 3*2*1
  - In words – 3 * 2 permutations of two letters
    - A combines with BC, CB
    - B combines with AC, CA
    - C combines with AB, BA
    - Total = ABC, ACB, BAC, BCA, CAB, CBA

Permutations

- Suppose you have n items: $A_1,\ldots,A_n$
- Then you have n independent choices: ……………
- Count the # of options for each choice n n-1 …………… 1
- Multiply together:
  - $n*(n-1)*\ldots*1 = n!$ permutations

Permutations

n	*n*!
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200
15	1307674368000
16	20922789888000
17	355687428096000
18	6402373705728000

*Wikipedia

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Subsets

- Given a set of 3 items {a, b, c}, how many different subsets can we make?

- Subsets are:

  {a, b ,c}, {a, b}, {b, c}, {a, c}, {a}, {b}, {c}, {}

Subsets

- Suppose you have n things: $A_1,\ldots,A_n$
- Then you have n items(choices) to consider:

  A_1 .... A_n

- You have 2 options for each item (in/out)

  i/o .... i/o

- Multiply together:
  - $2*2*\ldots*2$ (n times) $= 2^n$ subsets

Subsets

- Example
- Subsets for a, b, c are:

  {a, b ,c}, {a, b}, {b, c}, {a, c}, {a}, {b}, {c}, {}

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Summations

- We use compact notation for summations

Max summation iteration

$$\sum_{i=1}^{n} f(i)$$

f(i) is just a function… Think of it as a formula that has an i in it.

Start of summation iteration.

Substitute 'i' in formula for everything between start & end then add all results

- So this is really just a shorthand for: $\sum_{i=1}^{n} f(i) = f(1) + f(2) + f(3) + \dots + f(n)$

Example

- Evaluate this expression: $\sum_{i=1}^{4}(2+i^2)$

- Start with i=1, end with i=4…

$$(2+1^2)+(2+2^2)+(2+3^2)+(2+4^2)$$

- Now you just have numbers… so you can add

$$= 3 + 6 + 11 + 18$$
$$= 38\ .$$

Sum of a Constant

$$\sum_{i=1}^{n} C$$

- What it means:
  - C+ C +…+ C (n times)
  - So: $\sum_{i=1}^{n} C = n\,C$

Whiteboard

Sum of a Constant

$$\underbrace{\sum_{i=1}^{n} n}$$

- What it means:
  - n+ n +…+ n (n times)
  - So: $\sum_{i=1}^{n} n = n^2$

Whiteboard

Changing the Start and End

- We don't always start from 1 and end at n
- What is this sum: $$\sum_{i=m}^{n} c = \underbrace{c + c + \cdots + c}_{(n-m+1) \text{ times}}$$

$$\sum_{i=m}^{n} c = (n - m + 1) * c$$

Add to cheat sheet

Way to remember

$$\sum_{i=bottom}^{top} c = (\boldsymbol{top} - \boldsymbol{bottom} + 1) * c$$

Simple Question

- What is this sum?

$$\sum_{i=0}^{n} 1$$

$$\sum_{i=bottom}^{top} c = (\boldsymbol{top} - \boldsymbol{bottom} + 1) * c$$

- Careful… before we had i=1

$$\sum_{i=0}^{n} 1 = \underbrace{1 + 1 + \cdots + 1}_{(n-0+1)\text{ times}} = (n+1) * 1 = n + 1$$

Sums of Sums

- Sometimes you have a sum with two parts added together:

$$\sum_{n=s}^{t}[f(n)+g(n)]$$

- You can just break it into two parts:

$$\sum_{n=s}^{t}f(n)+\sum_{n=s}^{t}g(n)$$

Constant Rule

- You can actually move the constant in front for any sum
- RULE:

$$\sum_{n=s}^{t} C \cdot f(n) = C \cdot \sum_{n=s}^{t} f(n), \text{ where } C \text{ is a constant}$$

Summation Rules

- There are many more summation rules in the appendix of your text.
- Important examples:

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} .$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} .$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} .$$

Add to cheat sheet

Practice Problems

- Try to evaluate these:

$$\sum_{i=0}^{3}(5+\sqrt{4^i})$$

$$\sum_{i=1}^{100}(4+3i)$$

Try at home

Solution 1

$$\sum_{i=0}^{3}(5+\sqrt{4^i}) = (5+\sqrt{4^0}) + (5+\sqrt{4^1}) + (5+\sqrt{4^2}) + (5+\sqrt{4^3})$$

$$= (5+\sqrt{1}) + (5+\sqrt{4}) + (5+\sqrt{16}) + (5+\sqrt{64})$$

$$= (5+1) + (5+2) + (5+4) + (5+8)$$

$$= 6 + 7 + 9 + 13$$

$$= 35\ .$$

Solution 2

$$\sum_{i=1}^{100}(4+3i) = \sum_{i=1}^{100}4 + \sum_{i=1}^{100}3i$$

$$= \sum_{i=1}^{100}4 + 3\left(\sum_{i=1}^{100}i\right)$$

$$= 4(100) + 3\left\{\frac{100(100+1)}{2}\right\}$$

$$= 400 + 15{,}150$$

$$= 15{,}550\ .$$

Sums of Sums

- We will often see things like this:

$$\sum_{j=1}^{i}\sum_{k=j}^{n} 1$$

- What does this mean?
  - It means you have a sum of sums
    - (NOT two sums multiplied)
  - In order to solve it… you work from the inside out.

Sum of Sums

- In this example: $$\sum_{j=1}^{i}\sum_{k=j}^{n} 1 = \sum_{j=1}^{i}(n-j+1)$$

- Now you can divide into three sums and solve:

$$\sum_{j=1}^{i} n - \sum_{j=1}^{i} j + \sum_{j=1}^{i} 1 = \mathrm{n} * \mathrm{i} - \frac{\mathrm{i} * (\mathrm{i}+1)}{2} + \mathrm{i}$$

Use cheat sheet

We will solve this kind of sum often in the first part of the course… so make sure you understand how to do it.

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