

# Grav Sim

Simulation Date:

$$1d = 86,400 s$$

•  $\Delta t$

Real time is continuous, but computers can only take step in finite chunks

positions and velocity are updated by:

$$\begin{aligned} v(t + \Delta t) &= v(t) + a \cdot \Delta t \\ x(t + \Delta t) &\approx x(t) + v \cdot \Delta t \end{aligned}$$

Date:

• Vector position of object

$$\text{a 3D vector} = \vec{r} = (x, y, z)$$

describes where the object is in space

• Velocity vector for objects in space

$$v = \frac{dr}{dt}$$

To find velocity of objects in space

rate of change of position, both direction and speed.

• Acceleration Vector for objects in space

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

To find acceleration of objects



# Important

Date:

● MASS  $\rightarrow$  Source of gravity

Newton's law:

$$F_{ij} = G \frac{m_i m_j}{r_{ij}^2}$$

[ we will use both gravitational pull and visual depth of space and time ]

Date:

● gravitational force and acceleration

$$G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ Kg}^{-1} \text{ s}^{-2}$$

[ rescaling of  $G$  for proper simulation ]

● Separation of two bodies

$$Y_{ij} = r_j - r_i$$

[ where  $i, j$  will be co-ordinates of bodies ]

● Distance between bodies and scalar vector position

$$r = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$



Date:

- ① Newton Law of Gravitation → acceleration

$$a_j = G \sum_{j \neq i} \frac{m_j}{|r_{ij}|^3} r_{ij}$$

• Note

1. Each body accelerates towards all others.

2. The denominator  $r^3$  arises because force  $\propto 1/r^2$  and acceleration is force/mass

$$\text{force} \propto \frac{1}{r^2}$$

$$\text{acceleration} = \frac{\text{force}}{\text{mass}}$$

For Simulation

Date:

- ② Orbit Initialization (Circular Speed)

Velocity of Circular orbit,  $v$  of radius  $r$  around Mass  $M$

$$v = \sqrt{\frac{GM}{r}}$$

Centripetal force = gravitational force

$$\boxed{\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}}$$

• Numerical Physics for updation of Velocity and position of bodies.



Date:

$$\begin{aligned} V(t + \Delta t) &= V(t) + a(t) \Delta t \\ r(t + \Delta t) &= r(t) + v(t + \Delta t) \Delta t \end{aligned}$$

Motion of bodies in simulation is approximated continuously step by step.

## 7. Fabric of space time

$$\phi(x, z) = \sum_{\text{bodies}} (-0.1 \cdot m) e^{-\frac{(x - x_{ij})^2 + (z - z_{ij})^2}{10m}}$$

$$\psi(x, z) = \sum_{\text{bodies}} (-0.1 \cdot m) e^{-\frac{(x - x_{ij})^2 + (z - z_{ij})^2}{10m}}$$

Date:

Gaussian function is used to or each body.

mimics "gravitational well"

but it is not real general relativity

◦ Not real general relativity.

Using gaussian for "gravitational well" to look smooth



Date:

~~File layout~~

[My approach]

1) Using actual physics and general relativity for simulation of solar system not just random values

2) Making it optimize for real time computational

3) each planet with it's info for astronomical lover's.

4) Most important [approximating physical

Date:

quantities for my computer to calculate] :)

5) Future updates like

•) brief info or video of each planet

•) move galaxies galaxy and celestial bodies.

•) A python based space simulation system with actual physics.



Date:

## 1) Units and Constants (keeping no.s same)

distance = AU

time = Year

mass = solar masses.

$$G = 4\pi^2 (AU^3 \cdot M_{\odot}^{-1} \cdot yr^{-2})$$

Speed of light  $c \approx 63241 AU/yr$

Date:

## 2) Body state & derived quantities.

each planet and sun has

$m$ , radius  $R$  (approximate  
note realistic  
but actual no.  
shorten down)

$$V = \frac{4}{3} \pi R^3 \quad - \text{Volume}$$

$$\rho = m/v \quad - \text{density}$$

$$\text{Orbital period } T = 2\pi \sqrt{\frac{a^3}{G(M_{\odot} + m)}}$$

$$\text{angular momentum } h = r \times v, |h| = \sqrt{G(M_{\odot} + m) a (1 - e^2)}$$



## Important

Date:

### 3) Newton's N-body Core

$$\boxed{a_i^{(n)} = G \sum_{j \neq i} m_j \frac{r_{ij}}{|r_{ij}|^3}}$$

where,  $r_{ij} = r_j - r_i$

N massive bodies (Sun + 8 planets)  
at each step.

Update rule ( $\Delta t = dt$ ):

$$\boxed{v_i^{n+1/2} = v_i^n + \frac{1}{2} a_i^n dt}$$

$$\boxed{r_i^{n+1} = r_i^n + v_i^{n+1/2} dt}$$

Recomputing Values

$$v_i^{n+1} = v_i^{n+1/2} + \frac{1}{2} a_i^{n+1} dt$$

## My approach important part

Date:

### 4) Relativity without a Super computer : 1-Post-Newtonian (1PN)

$$\boxed{a_{PN} = \frac{GM}{c^2 r^3} \left[ \left( 4 \frac{GM}{r} - v^2 \right) r + 4(r \cdot v)v \right]}$$

$$\boxed{\Delta \omega \approx \frac{6\pi GM}{a(1-e^2)c^2}}$$



Date:

5) Gravitational Well  
 (actual one with shorten  
 down values of bodies)

o Newton's potential surface

$$\Phi(n, z) = G \sum_k \frac{m_{ik}}{\sqrt{(n-n_k)^2 + (z-z_k)^2 + e^2}}$$

Render height

$$y(n, z) = a\Phi(n, z)$$

o Schwarzschild embedding for  
 sun (LRR looks near the sun)

$$y(r) = \pm 2\sqrt{r_s(r-r_s)}$$

Date:

file layout of project  
 (By my logic)

Solara-Sims /

constant.py → all the constants  
 physics /

element.py

nbody.py

int.py

osculating.py

diagnostics.py

!

data /

Solar - data.json

model /

body and system

Planets /

planets with their physical quantities  
 in.py.