

**Exercise 1:**

$x_1$	$x_2$	$y$	Lagrange Multiplier
0.3858	0.4687	1	65.5261
0.4871	0.611	-1	65.5261
0.9218	0.4103	-1	0
0.7382	0.8936	-1	0
0.1763	0.0579	1	0
0.4057	0.3529	1	0
0.9355	0.8132	-1	0
0.2146	0.0099	1	0

$$\mathbf{w} = \sum_{i=1}^N \lambda_i y_i \mathbf{x}_i,$$

Given:  $N=2$ ;  $x_{i=1,2} = x_1$  and  $x_2$  which is a vector

$$y_1 = 1$$

$$y_2 = -1$$

$$\lambda_1 = 65.5261$$

$$\lambda_2 = 65.5261$$

**Vector W computation**

Vector  $\mathbf{W} = \mathbf{w}_1$  &  $\mathbf{w}_2$

$$w_1 = \lambda_1 * y_1 * x_1 + \lambda_2 * y_2 * x_2$$

$$w_2 = \lambda_1 * y_1 * x_1 + \lambda_2 * y_2 * x_2$$

$$w_1 = 65.5261 \times 1 \times 0.3858 + 65.5261 \times -1 \times 0.4871 = -6.64.$$

$$w_2 = 65.5261 \times 1 \times 0.4687 + 65.5261 \times -1 \times 0.611 = -9.32.$$

**Bias Computation**

$$y_i(\mathbf{w} \cdot \mathbf{x}_i + b) = 1$$

substituting  $y_1 = 1$  to get  $b_1$  that is  $i = 1$

$$b_1 = 1 - \mathbf{w} \cdot \mathbf{x}_i$$

substituting  $y_2 = -1$  to get  $b_2$  that is  $i = 2$

$$-\mathbf{w} \cdot \mathbf{x}_i - b_2 = 1$$

$$b_2 = -1 - \mathbf{w} \cdot \mathbf{x}_i$$

$b_1 = 1 - \mathbf{w} \cdot \mathbf{x}_1 = 1 - (-6.64)(0.3858) - (-9.32)(0.4687) = 7.9300.$   
 $b_2 = -1 - \mathbf{w} \cdot \mathbf{x}_2 = -1 - (-6.64)(0.4871) - (-9.32)(0.611) = 7.9289$   
 Averaging these values to obtain  $b = 7.93.$

**Decision boundary**  $w_1x_1 + w_2x_2 + b = 0$   
 $(-6.64)*x_1 + (-9.32)*x_2 + 7.93 = 0$

### Exercise 2:

Note: Refer Slide No.99 – 104 from Lecture 20 to 29\_Rule Extraction\_SVM\_Univariate\_Multivariate\_Regression Trees

The decision boundary is  $f(x_1, x_2) = x_1x_2.$

### Exercise 3:

1. Consider the 1-dimensional data set with 10 data points  $\{1, 2, 3, \dots, 10\}$ . Show three iterations of the k-means algorithms when  $k = 2$ , and the random seeds are initialized to  $\{1,2\}$ . Repeat the problem with random seeds  $\{2,9\}$ . How did the different choice of the seed set affect the quality of the results?

**Use Manhattan**

Solution:

dp	m1(1)	m2(2)	MANHATTAN		
1	0	1			
2	1	0			
3	2	1			
4	3	2			
5	4	3			
6	5	4			
7	6	5			
8	7	6			
9	8	7			
10	9	8			
INITIAL:	C1={1}	C2={2,3,4,5,6,7,8,9,10}	6		
dp	m1(1)	m2(6)			
1	0	5			
2	1	4			
3	2	3			

4	3	2			
5	4	1			
6	5	0			
7	6	1			
8	7	2			
9	8	3			
10	9	4			
ITERATION 1	C1={1,2,3}	C2={4,5,6,7,8,9,10}		7	
dp	m1(2)	m2(7)			
1	1	6			
2	0	5			
3	1	4			
4	2	3			
5	3	2			
6	4	1			
7	5	0			
8	6	1			
9	7	2			
10	8	3			
ITERATION 2:	C1={1,2,3,4}	2.5	C2={5,6,7,8,9,10}	7.5	
dp	m1(2.5)	m2(7.5)			
1	1.5	6.5			
2	0.5	5.5			
3	0.5	4.5			
4	1.5	3.5			
5	2.5	2.5			
6	3.5	1.5			
7	4.5	0.5			
8	5.5	0.5			
9	6.5	1.5			
10	7.5	2.5			
ITERATION 3: c1={1,2,3,4,5}					
	c2={6,7,8,9,10}				

- √ With the initial seed as  $\{2,9\}$ ; the quality of the algorithm is improvised, and the algorithm converges in the first iteration itself.
- √ Initial:  $C1(m1=2) = \{1,2,3,4,5\}$   
 $C2(m2=9) = \{6,7,8,9,10\}$
- √ Iteration 1:  $C1(m1=3) = \{1,2,3,4,5\}$   
 $C2(m2=8) = \{6,7,8,9,10\}$