18CSE751 – Introduction to Machine Learning Lecture 20,21: RULE EXTRACTION FROM TREES, LEARNING RULES FROM DATA

Dr.Vani Vasudevan

Professor –CSE, NMIT

LECTURE 20

Dr.Vani V

Rule-Based Classifier

■ Classify records by using a collection of "if…then…" rules

■ Rule: (*Condition*) → *y*
  – *where*
    ■ *Condition* is a conjunctions of attributes
    ■ *y* is the class label
  – *LHS: rule antecedent or condition*
  – *RHS: rule consequent*
  – *Examples of classification rules:*
    ■ (Blood Type=Warm) ∧ (Lay Eggs=Yes) → Birds
    ■ (Taxable Income < 50K) ∧ (Refund=Yes) → Evade=No

Rule-based Classifier (Example)

Name	Blood Type	Give Birth	Can Fly	Live in Water	Class
human	warm	yes	no	no	mammals
python	cold	no	no	no	reptiles
salmon	cold	no	no	yes	fishes
whale	warm	yes	no	yes	mammals
frog	cold	no	no	sometimes	amphibians
komodo	cold	no	no	no	reptiles
bat	warm	yes	yes	no	mammals
pigeon	warm	no	yes	no	birds
cat	warm	yes	no	no	mammals
leopard shark	cold	yes	no	yes	fishes
turtle	cold	no	no	sometimes	reptiles
penguin	warm	no	no	sometimes	birds
porcupine	warm	yes	no	no	mammals
eel	cold	no	no	yes	fishes
salamander	cold	no	no	sometimes	amphibians
gila monster	cold	no	no	no	reptiles
platypus	warm	no	no	no	mammals
owl	warm	no	yes	no	birds
dolphin	warm	yes	no	yes	mammals
eagle	warm	no	yes	no	birds

R1: (Give Birth = no) $\wedge$ (Can Fly = yes) $\rightarrow$ Birds

R2: (Give Birth = no) $\wedge$ (Live in Water = yes) $\rightarrow$ Fishes

R3: (Give Birth = yes) $\wedge$ (Blood Type = warm) $\rightarrow$ Mammals

R4: (Give Birth = no) $\wedge$ (Can Fly = no) $\rightarrow$ Reptiles

R5: (Live in Water = sometimes) $\rightarrow$ Amphibians

Application of Rule-Based Classifier

■ A rule *r* covers an instance x if the attributes of the instance satisfy the condition of the rule

R1: (Give Birth = no) ∧ (Can Fly = yes) → Birds

R2: (Give Birth = no) ∧ (Live in Water = yes) → Fishes

R3: (Give Birth = yes) ∧ (Blood Type = warm) → Mammals

R4: (Give Birth = no) ∧ (Can Fly = no) → Reptiles

R5: (Live in Water = sometimes) → Amphibians

Name	Blood Type	Give Birth	Can Fly	Live in Water	Class
hawk	warm	no	yes	no	?
grizzly bear	warm	yes	no	no	?

The rule R1 covers a hawk => Bird

The rule R3 covers the grizzly bear => Mammal

Rule Coverage and Accuracy

- Coverage of a rule:
  - *Fraction of records that satisfy the antecedent of a rule*

- Accuracy of a rule:
  - *Fraction of records that satisfy both the antecedent and consequent of a rule*

Tid	Refund	Marital Status	Taxable Income	Class
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

(Status=Single) $\rightarrow$ No

Coverage = 40%, Accuracy = 50%

How does Rule-based Classifier Work?

R1: (Give Birth = no) $\wedge$ (Can Fly = yes) $\rightarrow$ Birds

R2: (Give Birth = no) $\wedge$ (Live in Water = yes) $\rightarrow$ Fishes

R3: (Give Birth = yes) $\wedge$ (Blood Type = warm) $\rightarrow$ Mammals

R4: (Give Birth = no) $\wedge$ (Can Fly = no) $\rightarrow$ Reptiles

R5: (Live in Water = sometimes) $\rightarrow$ Amphibians

Name	Blood Type	Give Birth	Can Fly	Live in Water	Class
lemur	warm	yes	no	no	?
turtle	cold	no	no	sometimes	?
dogfish shark	cold	yes	no	yes	?

A lemur triggers rule R3, so it is classified as a mammal

A turtle triggers both R4 and R5

A dogfish shark triggers none of the rules

Characteristics of Rule-Based Classifier

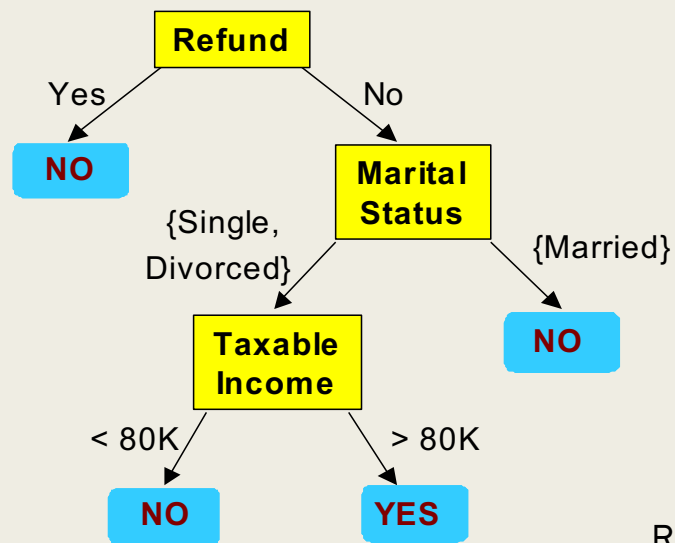- Mutually exclusive rules
  - *Classifier contains mutually exclusive rules if the rules are independent of each other*
  - *Every record is covered by at most one rule*

- Exhaustive rules
  - *Classifier has exhaustive coverage if it accounts for every possible combination of attribute values*
  - *Each record is covered by at least one rule*

From Decision Trees To Rules

Refund

Yes | No

NO

Marital Status

{Single, Divorced} | {Married}

Taxable Income

NO

< 80K | > 80K

NO | **YES**

Classification Rules

(Refund=Yes) ==> No

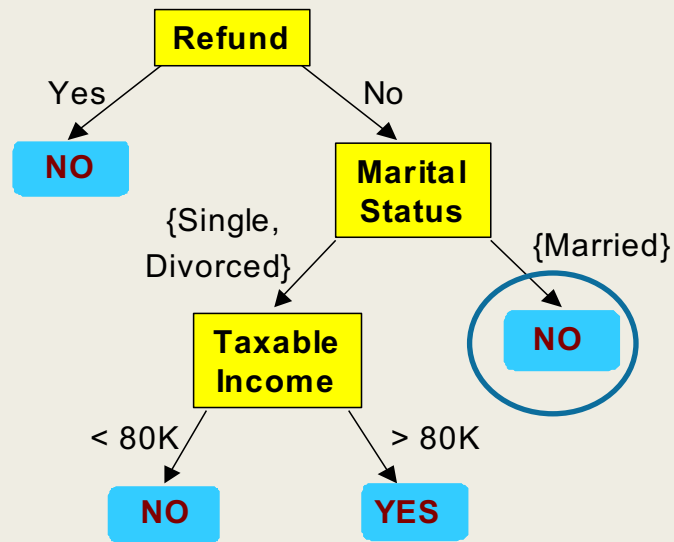(Refund=No, Marital Status={Single,Divorced}, Taxable Income<80K) ==> No

(Refund=No, Marital Status={Single,Divorced}, Taxable Income>80K) ==> Yes

(Refund=No, Marital Status={Married}) ==> No

Rules are mutually exclusive and exhaustive

Rule set contains as much information as the tree

Rules Can Be Simplified

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Initial Rule: (Refund=No) $\wedge$ (Status=Married) $\rightarrow$ No

Simplified Rule: (Status=Married) $\rightarrow$ No

Effect of Rule Simplification

- Rules are no longer mutually exclusive
  - *A record may trigger more than one rule*
  - *Solution?*
    - Ordered rule set
    - Unordered rule set – use voting schemes

- Rules are no longer exhaustive
  - *A record may not trigger any rules*
  - *Solution?*
    - Use a default class

Rule Ordering Schemes

- ■ Rule-based ordering
  - – *Individual rules are ranked based on their quality*

- ■ Class-based ordering
  - – *Rules that belong to the same class appear together*

Rule-based Ordering
(Refund=Yes) ==> No
(Refund=No, Marital Status={Single,Divorced}, Taxable Income<80K) ==> No
(Refund=No, Marital Status={Single,Divorced}, Taxable Income>80K) ==> Yes
(Refund=No, Marital Status={Married}) ==> No

Class-based Ordering
(Refund=Yes) ==> No
(Refund=No, Marital Status={Single,Divorced}, Taxable Income<80K) ==> No
(Refund=No, Marital Status={Married}) ==> No
(Refund=No, Marital Status={Single,Divorced}, Taxable Income>80K) ==> Yes

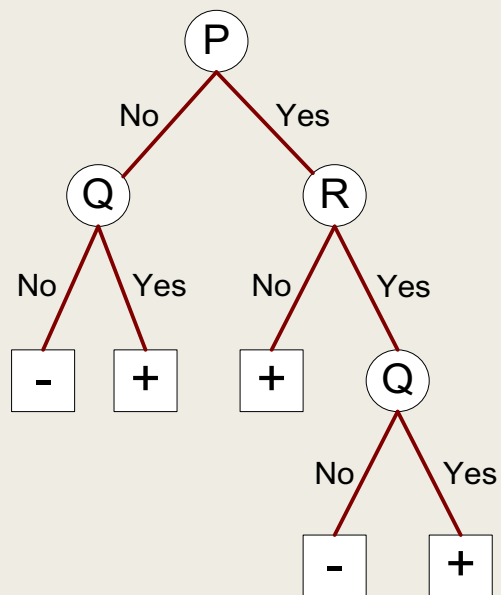# Building Classification Rules

- Direct Method:
    - Extract rules directly from data
    - e.g.: RIPPER, CN2, Holte's 1R

- Indirect Method:
    - Extract rules from other classification models (e.g. decision trees, neural networks, etc).
    - e.g: C4.5 rules

Indirect Methods

Rule Set

r1: (P=No,Q=No) ==> -
r2: (P=No,Q=Yes) ==> +
r3: (P=Yes,R=No) ==> +
r4: (P=Yes,R=Yes,Q=No) ==> -
r5: (P=Yes,R=Yes,Q=Yes) ==> +

Indirect Method: C4.5rules

- Extract rules from an unpruned decision tree

- For each rule, r: A $\rightarrow$ y,
    - *consider an alternative rule r': A' $\rightarrow$ y where A' is obtained by removing one of the conjuncts in A*
    - *Compare the pessimistic error rate for r against all r's*
    - *Prune if one of the r's has lower pessimistic error rate*
    - *Repeat until we can no longer improve generalization error*

Indirect Method: C4.5rules

- Instead of ordering the rules, order subsets of rules (class ordering)
  - *Each subset is a collection of rules with the same rule consequent (class)*
  - *Compute description length of each subset*
    - Description length = L(error) + g L(model)
    - g is a parameter that considers the presence of redundant attributes in a rule set (default value = 0.5)

Sequential Covering Algorithm...

1. Used to extract rules directly from data.

2. Rules are grown in a greedy fashion based on a certain evaluation measure.

3. The algorithm extracts the rules one class at a time for data sets that contain more than two classes.

   - *For the vertebrate classification problem,*
   - *the sequential covering algorithm may generate rules for classifying birds first, followed by rules for classifying mammals, amphibians, reptiles, and finally, fishes*

Class-Based Ordering

(Skin Cover=feathers, Aerial Creature=yes)
  ==> Birds

(Body temperature=warm-blooded,
Gives Birth=no) ==> Birds

(Body temperature=warm-blooded,
Gives Birth=yes) ==> Mammals

(Aquatic Creature=semi)) ==> Amphibians

(Skin Cover=none) ==> Amphibians

(Skin Cover=scales, Aquatic Creature=no)
  ==> Reptiles

(Skin Cover=scales, Aquatic Creature=yes)
  ==> Fishes

Direct Method: Sequential Covering...

1. Start from an empty rule

2. Grow a rule using the Learn-One-Rule function

3. Remove training records covered by the rule

4. Repeat Step (2) and (3) until stopping criterion is met

Sequential Covering Algorithm...

The criterion for deciding which class should be generated

1. Depends on a number of factors, such as the **class prevalence** (i.e., fraction of training records that belong to a particular class) or

2. The **cost of misclassifying records** from a given class.

Algorithm 5.1 Sequential covering algorithm.

1: Let E be the training records and A be the set of attribute-value pairs, $\{(A_j, v_j)\}$.
2: Let Y_o be an ordered set of classes $\{y_1, y_2, \ldots, y_k\}$.
3: Let $R = \{\ \}$ be the initial rule list.
4: **for** each class $y \in Y_o - \{y_k\}$ **do**
5:     **while** stopping condition is not met **do**
6:        $r \leftarrow$ Learn-One-Rule (E, A, y).
7:        Remove training records from E that are covered by r.
8:        Add r to the bottom of the rule list: $R \longrightarrow R \vee r$.
9:     **end while**
10: **end for**
11: Insert the default rule, $\{\} \longrightarrow y_k$, to the bottom of the rule list R.

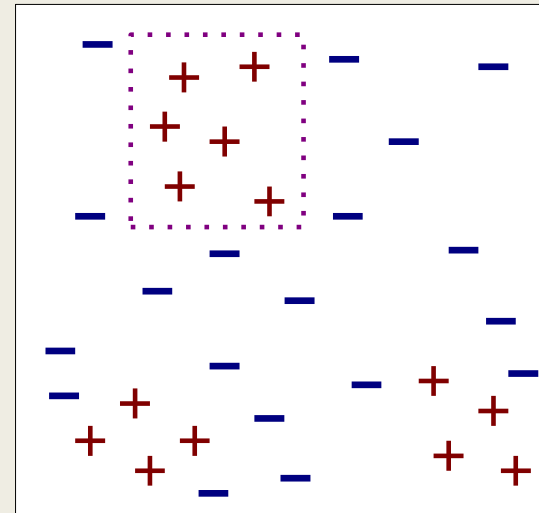# Sequential Covering Algorithm...

1. The algorithm **starts with an empty decision list, R.**

2. The **Learn-One- Rule function** is then used to **extract the best rule for class y that covers the current set of training records.**

3. During rule extraction, **all training records for class y are considered to be positive examples,** while **those that belong to other classes are considered to be negative examples.**

4. A **rule is desirable if it covers most of the positive examples and none (or very few) of the negative examples.** Once such a rule is found, **the training records covered by the rule are eliminated.**

5. **The new rule is added to the bottom of the decision list R.**

6. This procedure is **repeated until the stopping criterion is met.**

7. The algorithm then proceeds to generate rules for the next class.

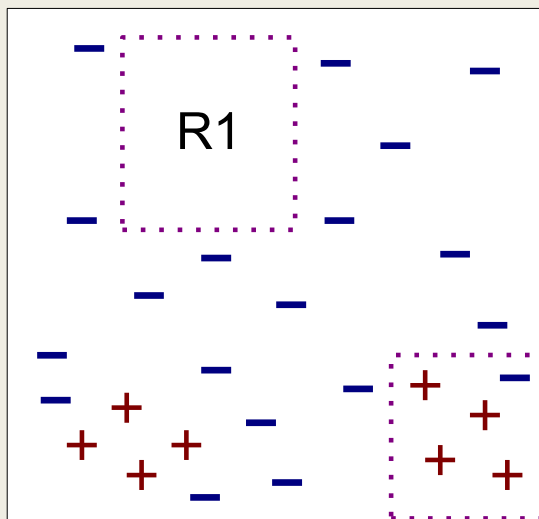# Example of Sequential Covering...
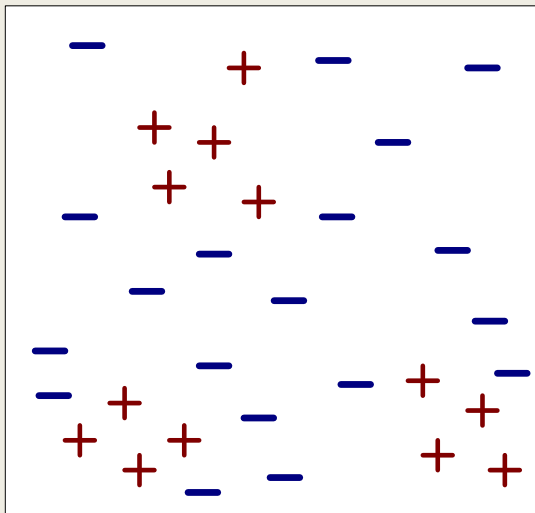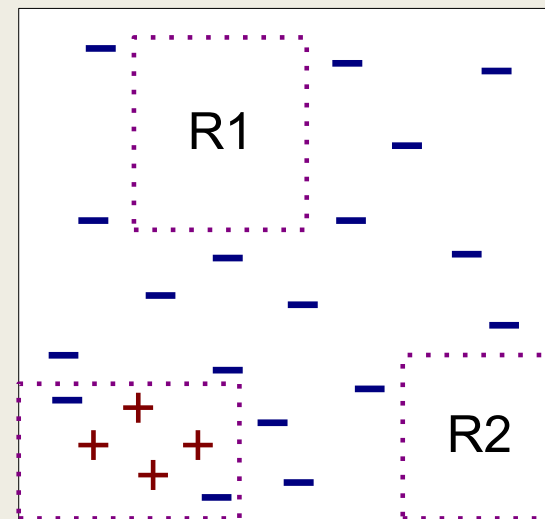

(i) Original Data

(ii) Step 1

Example of Sequential Covering

(iii) Step 2

(iv) Step 3

Learn-One Rule Function

- The **objective of the Learn-One-Rule function is to extract a classification rule that covers many of the positive examples** and none (or very few) of the negative examples in the training set.
- Finding an optimal rule is computationally expensive given the exponential size of the search space.
- The **Learn-One-Rule function addresses the exponential search problem by growing the rules in a greedy fashion.**
- It generates an **initial rule r and keeps refining the rule until a certain stopping criterion is met.**
- The rule is then pruned to improve its generalization error.
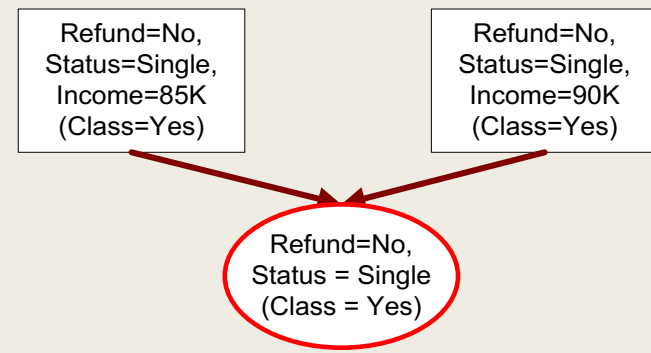
Aspects of Sequential Covering

- Rule Growing

- Instance Elimination

- Rule Evaluation

- Stopping Criterion

- Rule Pruning

Rule Growing

- Two common strategies
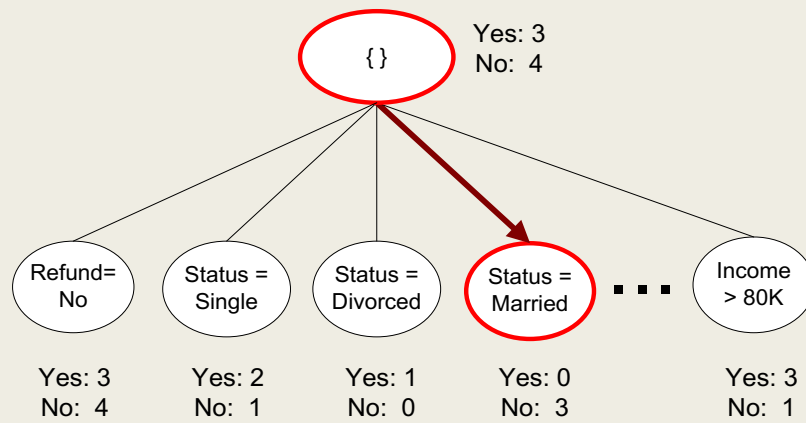
(a) General-to-specific

(b) Specific-to-general

Rule Evaluation…

- Metrics:
  - $Accuracy = \dfrac{n_c}{n}$

  - $Laplace = \dfrac{n_c + 1}{n + k}$

  - $M\text{-}estimate = \dfrac{n_c + kp}{n + k}$

n : Number of instances covered by rule

n_c : Number of c instances covered by rule

k : Number of classes

p : Prior probability

Rule Evaluation...

- An evaluation metric is needed to determine which conjunct should be added (or removed) during the rule-growing process.

- Accuracy is an obvious choice because it explicitly measures the fraction of training examples classified correctly by the rule.

- However, **a potential limitation of accuracy is that it does not consider the rule's coverage.**

- For example, consider a training set that contains 60 positive examples and 100 negative examples. Suppose we are given the following two candidate rules:
  - *Rule r1: covers 50 positive examples and 5 negative examples,*
  - *Rule r2: covers 2 positive examples and no negative examples.*

- The accuracies for r1 and r2 are 90.9% and 100%, respectively.
  - *r1 is the better rule despite its lower accuracy. The high accuracy for r2 is potentially spurious because the coverage of the rule is too low.*

Rule Evaluation...

■ The following approaches can be used to handle this problem.

1. A statistical test can be used to prune rules that have poor coverage.

■ Likelihood ratio statistic:

$$R = 2\sum_{i=1}^{k} f_i \log(f_i/e_i),$$

where k is the number of classes,

f_i is the observed frequency of class i examples that are covered by the rule, and

e_i is the expected frequency of a rule that makes random predictions.

Rule Evaluation...

Rule r1: covers 50 positive examples and 5 negative examples,

Rule r2: covers 2 positive examples and no negative examples.

- For example, since *r1* covers 55 examples,
  - $e_+ = 55x\ 60/160 = 20.625,$

    $e_- = 55\ x100/160 = 34.375.$

- The likelihood ratio for r1 is $R(r_1) = 2 \times [50 \times \log_2(50/20.625) + 5 \times \log_2(5/34.375)] = 99.9.$

- Similarly, $e_+ = 2x\ 60/160 = 120/160 = \frac{3}{4} = 0.75,$ $e_- = 0$, the likelihood ratio for r2 is

  $$R(r_2) = 2 \times [2 \times \log_2(2/0.75) + 0 \times \log_2(0/1.25)] = 5.66.$$

- This statistic therefore suggests that **r1 is a better rule than r2.**

Rule Evaluation...

Rule r1: covers 50 positive examples and 5 negative examples,

Rule r2: covers 2 positive examples and no negative examples.

2. An **evaluation metric that considers the rule coverage can be used.**

$$\text{Laplace} = \frac{f_+ + 1}{n + k},$$

$$\text{m-estimate} = \frac{f_+ + kp_+}{n + k},$$

where , n is the number of examples covered by the rule,

f_+ is the number of positive examples covered by the rule,

k is the total number of classes, and

p_+ is the prior probability for the positive class.

Note : m-estimate is equivalent to the Laplace measure by choosing $p_+ = 1/k$.

The Laplace measure for r1 is 51/57 = 89.47%, r2 is 3/4=75%

LECTURE 21

Dr.Vani V

Rule Evaluation...

3. An evaluation metric that considers the **support count** of the rule can be used. One such metric is the **FOIL's information gain.**

- The **support count of a rule corresponds to the number of positive examples covered by the rule.**

- Suppose the rule r : A → +
  - *covers p_0 positive examples and n_0 negative examples.*
- After adding a new conjunct B, the extended rule r : A ∧ B → +
  - *covers p1 positive examples and n1 negative examples.*
- Given this information, the FOIL's information gain of the extended rule is defined as follows:

$$\text{FOIL's information gain} = p_1 \times \left(\log_2 \frac{p_1}{p_1 + n_1} - \log_2 \frac{p_0}{p_0 + n_0} \right).$$

Rule Growing (Examples)

- CN2 Algorithm:
  - *Start from an empty conjunct: {}*
  - *Add conjuncts that minimizes the entropy measure: {A}, {A,B}, ...*
  - *Determine the rule consequent by taking majority class of instances covered by the rule*

- RIPPER Algorithm:
  - *Start from an empty rule: {} => class*
  - *Add conjuncts that **maximizes FOIL's information gain measure:***
    - R0: {} => class   (initial rule)
    - R1: {A} => class (rule after adding conjunct)
    - Gain(R0, R1) = t [log (p1/(p1+n1)) – log (p0/(p0 + n0))]
    - where   t: number of positive instances covered by both R0 and R1

      p0: number of positive instances covered by R0

      n0: number of negative instances covered by R0

      p1: number of positive instances covered by R1

      n1: number of negative instances covered by R1

Instance Elimination

- Why do we need to eliminate instances?
    - *Otherwise, the next rule is identical to previous rule*

- Why do we remove positive instances?
    - *Ensure that the next rule is different*

- Why do we remove negative instances?
    - *Prevent underestimating accuracy of rule*
    - *Compare rules R2 and R3 in the diagram*
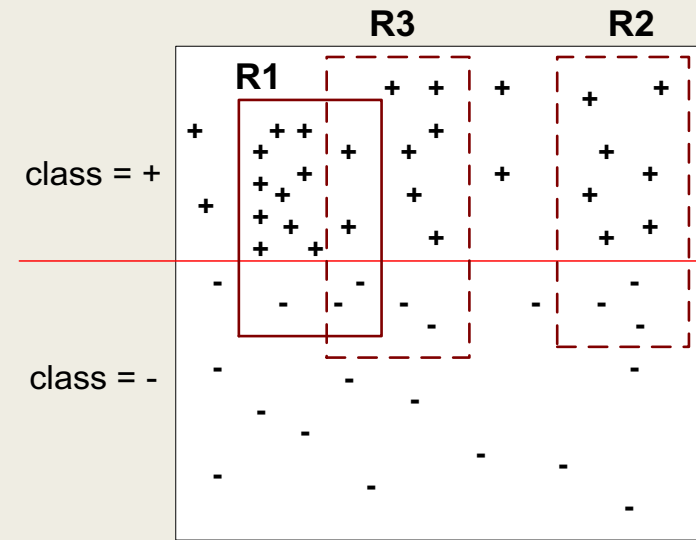
Stopping Criterion and Rule Pruning

- Stopping criterion
  - *Compute the gain*
  - *If gain is not significant, discard the new rule*

- Rule Pruning
  - *Reduced Error Pruning:*
    - Remove one of the conjuncts in the rule
    - Compare error rate on validation set before and after pruning
    - If error improves, prune the conjunct

Summary of Direct Method

- Grow a single rule

- Remove Instances from rule

- Prune the rule (if necessary)

- Add rule to Current Rule Set

- Repeat

Direct Method: RIPPER

- For 2-class problem, choose one of the classes as positive class, and the other as negative class
  - *Learn rules for positive class*
  - *Negative class will be default class*
- For multi-class problem
  - *Order the classes according to increasing class prevalence (fraction of instances that belong to a particular class)*
  - *Learn the rule set for smallest class first, treat the rest as negative class*
  - *Repeat with next smallest class as positive class*

Direct Method: RIPPER

- Growing a rule:
  - *Start from empty rule*
  - *Add conjuncts if they improve FOIL's information gain*
  - *Stop when rule no longer covers negative examples*
  - *Prune the rule immediately using incremental reduced error pruning*
  - *Measure for pruning: v = (p-n)/(p+n)*
    - p: number of positive examples covered by the rule in the validation set
    - n: number of negative examples covered by the rule in the validation set
  - *Pruning method: delete any final sequence of conditions that maximizes v*

Direct Method: RIPPER

- Building a Rule Set:
  - *Use sequential covering algorithm*
    - Finds the best rule that covers the current set of positive examples
    - Eliminate both positive and negative examples covered by the rule
  - *Each time a rule is added to the rule set, compute the new description length*
    - stop adding new rules when the new description length is d bits longer than the smallest description length obtained so far

Direct Method: RIPPER

- Optimize the rule set:
  - *For each rule r in the rule set **R***
    - Consider 2 alternative rules:
      - *Replacement rule (r*): grow new rule from scratch*
      - *Revised rule(r'): add conjuncts to extend the rule r*
    - Compare the rule set for *r* against the rule set for r* and r'
    - Choose rule set that minimizes MDL (Minimum Description Length) principle
  - *Repeat rule generation and rule optimization for the remaining positive examples*

Example

Name	Give Birth	Lay Eggs	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	no	yes	mammals
python	no	yes	no	no	no	reptiles
salmon	no	yes	no	yes	no	fishes
whale	yes	no	no	yes	no	mammals
frog	no	yes	no	sometimes	yes	amphibians
komodo	no	yes	no	no	yes	reptiles
bat	yes	no	yes	no	yes	mammals
pigeon	no	yes	yes	no	yes	birds
cat	yes	no	no	no	yes	mammals
leopard shark	yes	no	no	yes	no	fishes
turtle	no	yes	no	sometimes	yes	reptiles
penguin	no	yes	no	sometimes	yes	birds
porcupine	yes	no	no	no	yes	mammals
eel	no	yes	no	yes	no	fishes
salamander	no	yes	no	sometimes	yes	amphibians
gila monster	no	yes	no	no	yes	reptiles
platypus	no	yes	no	no	yes	mammals
owl	no	yes	yes	no	yes	birds
dolphin	yes	no	no	yes	no	mammals
eagle	no	yes	yes	no	yes	birds

C4.5 versus C4.5rules versus RIPPER
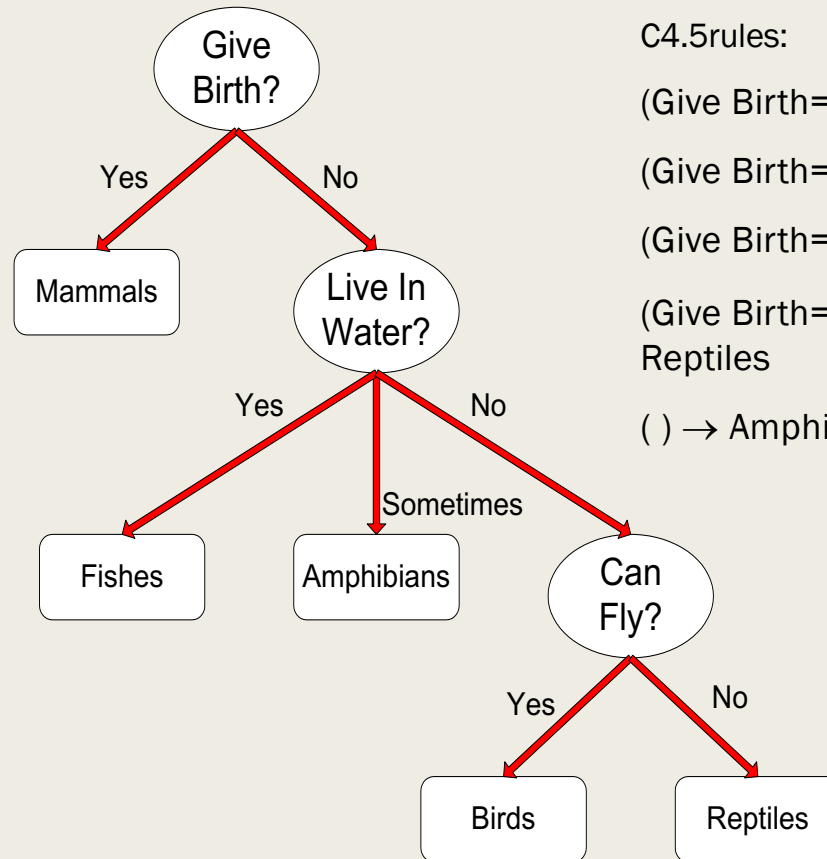
C4.5rules:

(Give Birth=No, Can Fly=Yes) → Birds

(Give Birth=No, Live in Water=Yes) → Fishes

(Give Birth=Yes) → Mammals

(Give Birth=No, Can Fly=No, Live in Water=No) → Reptiles

RIPPER:

( ) → Amphibians

(Live in Water=Yes) → Fishes

(Have Legs=No) → Reptiles

(Give Birth=No, Can Fly=No, Live In Water=No)
    → Reptiles

(Can Fly=Yes,Give Birth=No) → Birds

() → Mammals

Advantages of Rule-Based Classifiers

- As highly expressive as decision trees

- Easy to interpret

- Easy to generate

- Can classify new instances rapidly

- Performance comparable to decision trees

Characteristics of Rule-Based Classifiers

A rule-based classifier has the following characteristics:

- The expressiveness of a rule set is almost equivalent to that of a decision tree

- **If the rule-based classifier allows multiple rules to be triggered for a given record, then a more complex decision boundary can be constructed.**

- **Rule-based classifiers are generally used to produce descriptive models that are easier to interpret but gives comparable performance to the decision tree classifier.**

- **The class-based ordering approach adopted by many rule-based classifiers (such as RIPPER) is well suited for handling data sets with imbalanced class distributions.**

Selected Exercises: 1...

- Consider a binary classification problem with the following set of attributes and attribute values:

  - Air Conditioner = {Working, Broken}

  - Engine = {Good, Bad}

  - Mileage = {High, Medium, Low}

  - Rust = {Yes, No}

- Suppose a rule-based classifier produces the following rule set:

  r1: Mileage = High $-\longrightarrow$ Value = Low

  r2: Mileage = Low $-\longrightarrow$ Value = High

  r3: Air Conditioner = Working, Engine = Good $-\longrightarrow$ Value = High

  r4: Air Conditioner = Working, Engine = Bad $-\longrightarrow$ Value = Low

  r5: Air Conditioner = Broken $-\longrightarrow$ Value = Low

Selected Exercises:1

(a) Are the rules mutually exclusive?

No

(b) Is the rule set exhaustive?

Yes

(c) Is ordering needed for this set of rules?

Yes, because a test instance may trigger more than one rule.

(d) Do you need a default class for the rule set?

No, because every instance is guaranteed to trigger at least one rule.

Selected Exercises:2...

<u>Given:</u>

Suppose R1 is covered by 350 positive examples and 150 negative examples, while R2 is covered by 300 positive examples and 50 negative examples. Compute the FOIL's information gain for the rule R2 with respect to R1.

<u>Solution:</u>

$$\text{FOIL's information gain} = p_1 \times \left(\log_2 \frac{p_1}{p_1 + n_1} - \log_2 \frac{p_0}{p_0 + n_0} \right).$$

For this problem, p0 = 350, n0 = 150, p1 = 300, and n1 = 50. Therefore, the FOIL's information gain for R2 with respect to R1 is:

$$Gain = 300 \; x \; [\log_2 \frac{300}{350} - \log_2 \frac{350}{500}] \; = 87.65$$

2. The RIPPER algorithm (by Cohen [170]) is an extension of an earlier algorithm called IREP (by Fürnkranz and Widmer [184]). Both algorithms apply the **reduced-error pruning** method to determine whether a rule needs to be pruned. The reduced error pruning method uses a validation set to estimate the generalization error of a classifier. Consider the following pair of rules:

$$R_1: \quad A \longrightarrow C$$
$$R_2: \quad A \wedge B \longrightarrow C$$

R_2 is obtained by adding a new conjunct, B, to the left-hand side of R_1. For this question, you will be asked to determine whether R_2 is preferred over R_1 from the perspectives of rule-growing and rule-pruning. To determine whether a rule should be pruned, IREP computes the following measure:

$$v_{IREP} = \frac{p + (N - n)}{P + N},$$

where P is the total number of positive examples in the validation set, N is the total number of negative examples in the validation set, p is the number of positive examples in the validation set covered by the rule, and n is the number of negative examples in the validation set covered by the rule. v_{IREP} is actually similar to classification accuracy for the validation set. IREP favors rules that have higher values of v_{IREP}. On the other hand, RIPPER applies the following measure to determine whether a rule should be pruned:

$$v_{RIPPER} = \frac{p - n}{p + n}.$$

Selected Exercises:2...

Given : Consider a validation set that contains 500 positive examples and 500 negative examples. For R1, suppose the number of positive examples covered by the rule is 200, and the number of negative examples covered by the rule is 50. For R2, suppose the number of positive examples covered by the rule is 100 and the number of negative examples is 5.

(1) Compute $vIREP$ for both rules. Which rule does $IREP$ prefer?

Selected Exercises:2...

(1) For the given problem, P = 500, and N = 500.

For rule R1, p = 200 and n = 50. Therefore,

$$V_{IREP}(R1) = \frac{p + (N - n)}{P + N}$$

$$= \frac{200 + (500 - 50)}{1000} = 0.65$$

For rule R2, p = 100 and n = 5.

$$V_{IREP}(R2) = \frac{p + (N - n)}{P + N}$$

$$= \frac{100 + (500 - 5)}{1000} = 0.595$$

Thus, IREP prefers rule R1.

Selected Exercises:2

(2) Compute v_{RIPPER} for the given problem. Which rule does RIPPER prefer?

For rule R1, p = 200 and n = 50.

$$V_{RIPPER}(R1) = \frac{p-n}{p+n}$$

$$= \frac{200-50}{200+50} = 0.6$$

For rule R2, p = 100 and n = 5.

$$V_{RIPPER}(R2) = \frac{p+n}{p+n}$$

$$= \frac{100-5}{100+5} = 0.9$$

Thus, RIPPER prefers rule R2.

Selected Exercises:3...

C4.5rules is an implementation of an indirect method for generating rules from a decision tree. RIPPER is an implementation of a direct method for generating rules directly from data.

(a) Discuss the strengths and weaknesses of both methods.

The C4.5 rules algorithm generates classification rules from a global perspective. This is because the rules are derived from decision trees, which are induced with the objective of partitioning the feature space into homogeneous regions, without focusing on any classes. In contrast, RIPPER generates rules one-class-at-a-time. Thus, it is more biased towards the classes that are generated first.

Selected Exercises:3

C4.5 rules is an implementation of an indirect method for generating rules from a decision tree. RIPPER is an implementation of a direct method for generating rules directly from data.

(b) Consider a data set that has a large difference in the class size (i.e., some classes are much bigger than others). Which method (between C4.5 rules and RIPPER) is better in terms of finding high accuracy rules for the small classes?

The class-ordering scheme used by C4.5 rules has an easier interpretation than the scheme used by RIPPER.

Selected Exercises:4...

Consider a training set that contains 100 positive examples and 400 negative examples. For each of the following candidate rules,

R1: A → + (covers 4 positive and 1 negative examples),

R2: B → + (covers 30 positive and 10 negative examples),

R3: C → + (covers 100 positive and 90 negative examples),

Determine which is the best and worst candidate rule according to:

(a) Rule accuracy.

(b) FOIL's information gain.

(c) The likelihood ratio statistic.

(d) The Laplace measure.

(e) The m-estimate measure (with k = 2 and p+ = 0.2).

Selected Exercises:4...

(a) Rule accuracy

Solution: R1 : 4/5

R2: 30/40

R3:100/190

The accuracies of the rules are 80% (for R1), 75% (for R2), and 52.6% (for R3), respectively. Therefore, R1 is the best candidate and R3 is the worst candidate according to rule accuracy

Selected Exercises:4...

(b) Foil's information gain

Solution: Assume the initial rule is $\emptyset \rightarrow +$. This rule covers $p0 = 100$ positive examples and $n0 = 400$ negative examples. The rule R1 covers $p1 = 4$ positive examples and $n1 = 1$ negative example. Therefore, the FOIL's information gain for this rule is

$$\text{FOIL's information gain} = p_1 \times \left(\log_2 \frac{p_1}{p_1 + n_1} - \log_2 \frac{p_0}{p_0 + n_0} \right).$$

$$Gain(R1) = 4 \ x \ [\log_2 \frac{4}{5} - \log_2 \frac{100}{500}] \ = 8$$

$$Gain(R2) = 57.2$$

$$Gain(R3) = 139.6$$

Therefore, R3 is the best candidate and R1 is the worst candidate according to FOIL's information gain.

Selected Exercises:4...

(c)The likelihood ratio statistic:

$$R = 2 \sum_{i=1}^{k} f_i \log(f_i/e_i),$$

Solution:

The given training set that contains 100 positive examples and 400 negative examples. For each of the following candidate rules,

R1: A → + (covers 4 positive and 1 negative examples),

For R1, the expected frequency for the positive class is 5 x 100/500 = 1 and the expected frequency for the negative class is 5 x 400/500 = 4. Therefore, the likelihood ratio for R1 is

$R(R1) = 2 \; x \; [4x \; log_2 \frac{4}{1} + 1x \; log_2 \frac{1}{4}] \; = 12$

Selected Exercises:4...

(c)The likelihood ratio statistic:

$$R = 2\sum_{i=1}^{k} f_i \log(f_i/e_i),$$

<u>Solution:</u>

The given training set that contains 100 positive examples and 400 negative examples. For each of the following candidate rules,

R2: B → + (covers 30 positive and 10 negative examples),

For R2, the expected frequency for the positive class is 40x 100/500 = 8 and the expected frequency for the negative class is 40 x400/500 = 32. Therefore, the likelihood ratio for R2 is

R(R2) = $2\,x\,[30x\,log_2\frac{30}{8} + 10x\,log_2\frac{10}{32}]$ = 80.85

Selected Exercises:4...

(c)The likelihood ratio statistic:

$$R = 2\sum_{i=1}^{k} f_i \log(f_i/e_i),$$

Solution:

The given training set that contains 100 positive examples and 400 negative examples. For each of the following candidate rules,

R3: C → + (covers 100 positive and 90 negative examples),

For R3, the expected frequency for the positive class is 190 x 100/500 = 38 and the expected frequency for the negative class is 190x 400/500 = 152. Therefore, the likelihood ratio for R3 is

R(R3) = $2 x [100 x log_2 \frac{100}{38} + 90x log_2 \frac{90}{152}]$ = 143.09

Therefore, R3 is the best candidate and R1 is the worst candidate according to the likelihood ratio statistic.

Selected Exercises:4...

(d) The Laplace measure.

$$\text{Laplace} = \frac{f_+ + 1}{n + k},$$

where , n is the number of examples covered by the rule,

f_+ is the number of positive examples covered by the rule,

k is the total number of classes, and

p_+ is the prior probability for the positive class.

Given: Consider a training set that contains 100 positive examples and 400 negative examples. For each of the following candidate rules,

R1: A → + (covers 4 positive and 1 negative examples),

R2: B → + (covers 30 positive and 10 negative examples),

R3: C → + (covers 100 positive and 90 negative examples),

<u>Solution:</u>

- The Laplace measure of the rules are 71.43% (for R1), 73.81% (for R2), and 52.6% (for R3), respectively. **Therefore, R2 is the best candidate and R3 is the worst candidate according to the Laplace measure.**

Selected Exercises:4

e) The m-estimate measure (with k = 2 and p+ = 0.2).

$$\text{m-estimate} = \frac{f_+ + kp_+}{n + k},$$

Given: Consider a training set that contains 100 positive examples and 400 negative examples. For each of the following candidate rules,

R1: A → + (covers 4 positive and 1 negative examples),

R2: B → + (covers 30 positive and 10 negative examples),

R3: C → + (covers 100 positive and 90 negative examples),

<u>Solution:</u>

The m-estimate measure of the rules are 62.86% (for R1), 73.38% (for R2), and 52.3% (for R3), respectively. Therefore, R2 is the best candidate and R3 is the worst candidate according to the m-estimate measure.

Selected Exercises:5

Consider the one-dimensional data set shown in below table.

x	0.5	3.0	4.5	4.6	4.9	5.2	5.3	5.5	7.0	9.5
y	−	−	+	+	+	−	−	+	−	−

(a) Classify the data point x = 5.0 according to its 1-, 3-, 5-, and 9-nearest neighbors (using majority vote).

(b) Repeat the previous analysis using the distance-weighted voting approach

Solution:

Majority Voting: $y' = \underset{v}{\mathrm{argmax}} \sum_{(x_i,y_i) \in D_z} I(v = y_i),$

Distance-Weighted Voting: $y' = \underset{v}{\mathrm{argmax}} \sum_{(x_i,y_i) \in D_z} w_i \times I(v = y_i).$

(a)
1-nearest neighbor: +,
3-nearest neighbor: −,
5-nearest neighbor: +,
9-nearest neighbor: −.

(b)
1-nearest neighbor: +,
3-nearest neighbor: +,
5-nearest neighbor: +,
9-nearest neighbor: +.

Selected Exercises:6...

Consider the training set for the loan classification problem shown in Figure
Use the MVDM measure to compute the distance between every pair of attribute values for the **Home Owner and Marital Status attributes.**

	binary	categorical	continuous	class
Tid	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Selected Exercises:6

Tid	Home Owner (binary)	Marital Status (categorical)	Annual Income (continuous)	Defaulted Borrower (class)
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

	Marital Status		
Class	Single	Married	Divorced
Yes	2	0	1
No	2	4	1

	Home Owner	
Class	Yes	No
Yes	0	3
No	3	4

$d(\text{Single, Married}) = 1$

$d(\text{Single, Divorced}) = 0$

$d(\text{Married, Divorced}) = 1$

$d(\text{Refund=Yes, Refund=No}) = 6/7$

References

TEXTBOOKS :
1. Pang-Ning Tan, Vipin Kumar, Michael Steinbach: **Introduction to Data Mining**, Pearson, 2012.
2. Jiawei Han and Micheline Kamber: **Data Mining - Concepts and Techniques**, 3rd Edition, MorganKaufmann Publisher, 2014