Evaluating gradient, curl and divergence

Aim:

To write Matlab codes to visualize the vector field of 2-Dimensions as well as 3-Dimensions.
To find and visualize the gradient of scalar function, divergence and curl of a vector function.

Gradient vector of a scalar function $f(x, y, z)$

The vector function ∇f is defined as the gradient of the scalar function f and written as *grad f.*

$$grad\, f = \nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}.$$

Divergence of a vector $\vec{F}$

Divergence of a continuously differentiable vector point function $\vec{F}$ is denoted by $div\,\vec{F}$

$$div\,\vec{F} = \nabla \bullet \vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \bullet \left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

Curl of a vector $\vec{F}$

Curl of a continuously differentiable vector point function $\vec{F}$ is denoted by $curl\,\vec{F}$

$$curl\,\vec{F} = \nabla \times \vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}.$$

MATLAB Syntax used:

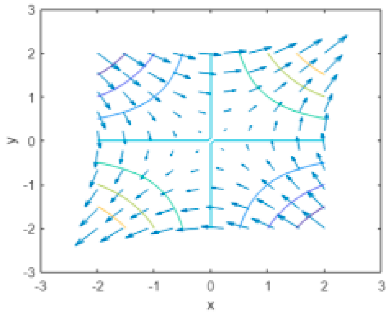
`quiver(x,y,u,v)`	Displays velocity vectors as arrows with components (u,v) at the points (x,y)
`quiver3(x,y,z,u,v,w)`	Plots vectors with components (u,v,w) at the points (x,y,z)
`gradient(f,v)`	Finds the gradient vector of scalar function `f` with respect to vector `v` in Cartesian coordinates.
`divergence(f,v)`	Finds the divergence of vector field `f` with respect to vector v in Cartesian coordinates.
`curl(V,X)`	Finds the curl of vector field `f` with respect to vector v in Cartesian coordinates.
`pcolor(x,y,C)`	When x,y and C are matrices of the same size, pcolor(x,y,C) plots the colored patches of vertices (x(i,j), y(i,j)) and color C(i,j).

Example 1: Find the Gradient of the function $f = 2xy$.

```
clear
clc
syms x y
f=input('Enter the function f(x,y):');
grad=gradient(f,[x,y])
P(x,y)=grad(1);Q(x,y)=grad(2);
x=linspace(-2,2,10);y=x;
[X,Y]=meshgrid(x,y);
U=P(X,Y); V=Q(X,Y);
quiver(X,Y,U,V,1)
axis on
xlabel('x'); ylabel('y')
hold on
fcontour(f,[-2,2])
```

Input:
```
Enter the function f(x,y):2*x*y
```

Output:
```
grad =
 2*y
 2*x
```



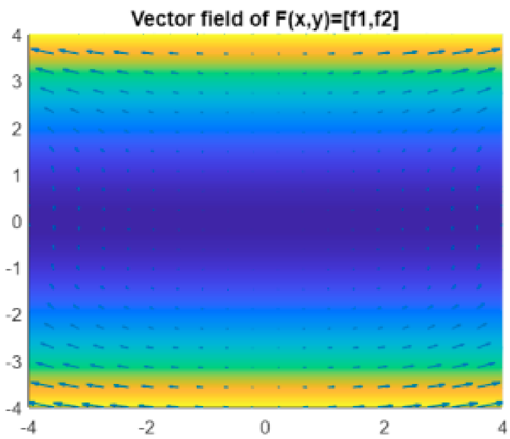
Example 2: Find the divergence of the vector field $\vec{F} = xy^2\hat{i} + x^2\hat{j}$ and visualize it.

```
clear
clc
syms x y
f=input('Enter the 2D vector function in the form [f1,f2]:');
div(x,y)=divergence(f,[x,y])
P(x,y)=f(1);Q(x,y)=f(2);
x=linspace(-4,4,20);y=x;
[X,Y]=meshgrid(x,y);
U=P(X,Y);V=Q(X,Y);
figure
pcolor(X,Y,div(X,Y));
shading interp
hold on;
quiver(X,Y,U,V,1)
axis on
hold off;
title('Vector field of F(x,y)=[f1,f2]');
```

Intput:
```
Enter the 2D vector function in the form [f1,f2]:
[x*y^2, x^2]
```

Output:

```
div(x,y)=
y^2
```



Example 3. Find and visualize the curl of a vector function $\overline{F} = -yi + xj$.

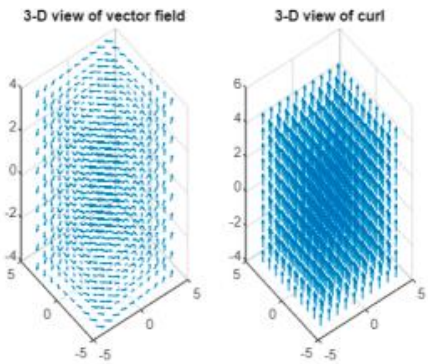
```
clear
clc
syms x y z
f=input('Enter the 3D vector function in the form [f1,f2,f3]:');
P(x,y,z)=f(1);Q(x,y,z)=f(2);R(x,y,z)=f(3); %Components of vector f
crl=curl(f,[x,y,z]) %Calculating curl
C1(x,y,z)=crl(1);C2(x,y,z)=crl(2);C3(x,y,z)=crl(3);%Components of curl(f)
x=linspace(-4,4,10);y=x;z=x;
[X,Y,Z]=meshgrid(x,y,z);
U=P(X,Y,Z);V=Q(X,Y,Z);W=R(X,Y,Z);
CR1=C1(X,Y,Z);CR2=C2(X,Y,Z);CR3=C3(X,Y,Z);
figure;
subplot(1,2,1);
quiver3(X,Y,Z,U,V,W);
title('3-D view of vector field');
subplot(1,2,2);
quiver3(X,Y,Z,CR1,CR2,CR3);
title('3-D view of curl');
```

Input:

```
Enter the 3D vector function in the form [f1,f2,f3]:
[-y,x,0]
```

Output

```
crl =
0
0
2
```



Exercise:

1. Draw the two dimensional vector field for the vector $2xi + 3yj$.
2. Find the Gradient of the function $f = x^2 y^3 - 4y$.
3. Find the divergence of a vector field $f = [xy, x^2]$.
4. Visualize the curl of a vector function $f = [yz, 3zx, z]$.