PHY425: Computational Methods in Physics II
Lab 1

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1. Solve the following initial-value problem (IVP) using Euler's method:

$$\begin{cases} x'(t) &= -\sin(t+x) \qquad (0 \le t \le 2) \\ x(0) &= -\pi/2 \end{cases}$$

Exact solution:

$$x = -\sin^{-1}\left(\frac{1-t^2}{1+t^2}\right) - t$$

2. Solve the following IVP in the interval $[-1, 1]$ using Taylor-series method of order 4:

$$\begin{cases} x' &= \cos t - \sin x + t^2 \\ x(-1) &= 3 \end{cases}$$

3. Solve the following delay differential equation using Taylor-series method of order 3:

$$\begin{cases} x'(t) &= x(t-1) \qquad (t \ge 0) \\ x(t) &= t^2 \qquad (-1 \le t \le 0) \end{cases}$$

Exact solution:

$$x(t) = \begin{cases} \frac{1}{3}(t-1)^3 + \frac{1}{3} & (0 \le t \le 1) \\ \frac{1}{12}(t-2)^4 + \frac{1}{3}t - \frac{1}{12} & (1 \le t \le 2) \end{cases}$$

4. Use Taylor-series method of order 4 to compute $\int_0^2 e^{-s^2} ds$ by solving the IVP on the interval $t \in [0, 2]$

$$\begin{cases} x' &= e^{-t^2} \\ x(0) &= 0 \end{cases}$$

Check: from the table of error function $\text{erf}(t) = \frac{2}{\pi}\int_0^t e^{-s^2} ds$, we obtain $x(2) = (\pi/2) \cdot \text{erf}(2) \approx 0.8820813907$

5. Solve the following IVP on the interval $[1, 3]$

$$\begin{cases} x' &= t^{-2}(tx - x^2) \\ x(1) &= 2 \end{cases}$$

(a) Use Runge-Kutta method of order 4 (RK4) with steps of sizes up to $h = 1/128$.

(b) Use Runge-Kutta-Fehlberg (RKF45) method.

Compare your approximation with the exact solution: $x(t) = \left(\frac{1}{2} + \ln t\right)^{-1} t$