New Idea: No computing d'f x'=f(t,x)Use only = fland Runge-Kutla Method

1st order $x_{k+1} = x_k + h \cdot f(t_k, x_k)$ 2nd oeider $x_{k+1} = x_k + \frac{1}{2}x_1 + \frac{1}{2}x_2$ where f ky = h.f(tk, xk) L ky = h.f(tk+h, xk+k) x'= f the think t Heuris method. Theorem: Heun's Method is 2nd order.

It suffices to show local evolute is $O(h^3)$ Taylor series for $f(t_K+h, \chi_K+K_1)$ expanded at (t_K, χ_K) f(+k+h, 2x+K1) = f(tx,2x)+ h.f. (tx,2x)+ K,fx (tx,2x)+ O(1,2x) O(h2) =) Kg = h. (++hf+hf.fx+0(h2)) Multi-variable Taylor series $f(t_{K}+h_{1},x_{K}+K_{1}) = \text{series}$ $f(t_{K}+h_{1},x_{K}+K_{1}) = f(t_{K},n_{K}) + h_{1}f(t_{K},n_{K}) + k_{1}f_{1}(t_{K},n_{K}) + 0(h_{1}^{2},h_{1}^{2})$ $f(t_{K}+h_{1},x_{K}+k_{1}) = f(t_{K},n_{K}) + h_{1}f_{1}(t_{K},n_{K}) + k_{1}f_{1}(t_{K},n_{K}) + 0(h_{1}^{2},h_{1}^{2})$ => ky= h(++ hf++ ++ + (h2)) 2x+= 2x+ 1h++ 1h(f+ hf+ hf.fx+0(2)) =) xx+ 1 hof+ 1 hof + 12f+ + 12f+ + 0(12) = 7k + h.f + 12 (fet f.fx) + O(h3) Exact solv. x(t) = x'(t) = f(x,t) = x(tu) = x

Enact salmost a(txth)
using Touglou serus => 2 (th) + h- x'(tx) + h2x"(tx) + 0(18) a x(ti) + b-f + h^2 [ft + fx o n'] + O(t) a ax + hf + h [ft + fx of] + o(h3) hocal Event is their order. general form of mit wooder Runge kuller 9/K+1 = 26 K + W, K, + W2 K2 + ... + Wm Km whole k, = hf(tx,nx) $K_2 = h \cdot f \left(\pm_{K} + Q_2 \cdot h , \chi_{K} + b_2 k_1 \right)$ $K_3 = h \cdot f \left(\pm_{K} + Q_3 h , \chi_{K} + b_3 k_1 + C_3 k_2 \right)$ Classical Runge-Kutta 4 $\alpha_{K+1} = \alpha_{K} + \frac{1}{6} \left[k_1 + 2k_2 + 2k_3 + k_4 \right]$ $K_1 = h \cdot f(t_k, \chi_k)$ $K_2 = h \cdot f(t_k + \frac{1}{2}h, \chi_k + \frac{1}{2}k_1)$ $K_3 = h \cdot f(t_k + \frac{1}{2}h, \chi_k + \frac{1}{2}k_2)$ $K_4 = h \cdot f(t_k + h, \chi_k + k_3)$ elmpson's method > Adaptive Methods Observation 1 Small to . > Smaller evels Higher ander - smaller every 1 Uniform guid, give varying local woods

New Idea: No computing diff x'=f(t,x)Runge-Kutla Method

1st order $x_{K+1} = x_K + h \cdot f(t_K x_K)$ 2nd order $x_{K+1} = x_K + \frac{1}{2}x_1 + \frac{1}{2}x_2$ where f Kol = h.fltx, Xx)

[Kol = h.fltx, Xx+ke) x'= f the test t Heuris method. Theorem: Heun's Method is 2rd order.
Pf: It suffices to show local evolve is 0/h3) Taylor series for f(tk+h, xk+k,) expanded at ltk, h f(+k+h, 2x+K,) = f(tx,2x)+h-f(tx,2x)+K,fx(tx,2x)+O(h,k,2) O(h2) =) Kg = h. (+ h f + h f fx + 0(102)) Multi-variable Taylor Series $f(t_{k}+h, x_{k}+k_{1}) = \exp \operatorname{anded} \operatorname{at} (t_{k}, x_{k})$ $f(t_{k}+h, x_{k}+k_{1}) = f(t_{k}, x_{k}) + h \cdot f(t_{k}, x_{k}) + k_{1} \cdot f_{1}(t_{k}, x_{k}) + 0(h^{2}, x_{1}^{2})$ => ky= h(f+ hf+ hf+x+0(h2)) 2x+1= 2x+ 1hof + 1h(f+ hft + hf.fx +0(h2)) =) 2x+ 1 hof+ 1 hof+ 12f+ + 12f+ + 0(h3) A 7 x + hif + 12 (ft+ffx) + O(103) Exact solu. x(t) = x'(t) = f(x,t) $x(t_0) = x$

Enact salmout altach) using Touthor sous => x(th) + h- x'(th) + h2x"(th) +0(h3) = x(ti) + b-f + h^2 [f+ fx n'] + O(t) a ax + hf + h2 Tft + fx of] + o(h3) Local Event $\ni |\chi_{K+1} - \chi(tn+h)| = O(h^3)$ hocal Event is their order. general form of mit wider Runge kuller 9(K+1 = 20K + W, K, + W2 K2 + ... + Wm Km whole k, = hf(tx, nx) K2 = h.f (+x+a2.h, 2x+b2k1) k3 = h-f(tx+ 93h, xx + b3k1+ C3k2) Classical Runge-Kutta 4 2KH = 2K+ 1 [K1+2K2+2K3+ K4] $K_1 = h \cdot f(t_k, \chi_k)$ $K_2 = h \cdot f(t_k + \frac{1}{2}h, \chi_k + \frac{1}{2}k_1)$ $K_3 = h \cdot f(t_k + \frac{1}{2}h, \chi_k + \frac{1}{2}k_2)$ $K_4 = h \cdot f(t_k + h, \chi_k + k_3)$ simpson's method - Adaptive Methods Observation 1 Small to . > Smaller evels D Higher ander → Smaller everous

@ Uniform glad, gives varying local voors

rala: - xk+1
> 38 K+1 (better method)
7k+1- rik+1]: evolue measurement
· if Eurn>> tolerance, reject, half the slep size (h)
· if ever << tolerance, accept, double the slep size (R)
"if ever & tolerans, a ccept., continue
1 method 1 of take RK4, one Step w/ L
Method 2 & take RK4, one Steps w/ h/2
1001012 6 me KK4, 400 sup 10/2
Use KK4, RK5 -> Rumar - Kuttar - Fallonka
llse KK4, RK5 → Rung - Kutta - Ferthery