

PHY425: Computational Methods in Physics II

Lab 2

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1. Solve the following IVP on the interval $[1, 3]$

$$\begin{cases} x' &= t^{-2}(tx - x^2) \\ x(1) &= 2 \end{cases}$$

- (a) Use Adam-Bashforth-Moulton formula of order 4 (ABM4) for various step sizes.
- (b) Use Milne-Simpson Method.

Compare your approximation with the **exact solution**: $x(t) = \left(\frac{1}{2} + \ln t\right)^{-1} t$

2. Solve the following system of ODEs using RK4 method over the interval $[0.0, 0.2]$

$$\begin{cases} x'(t) &= x + 2y \\ y'(t) &= 3x + 2y \\ x(0) &= 6 \\ y(0) &= 4 \end{cases}$$

Exact solution:

$$\begin{aligned} x(t) &= 4e^{4t} + 2e^{-t} \\ y(t) &= 6e^{4t} - 2e^{-t} \end{aligned}$$

3. Solve the boundary value problem

$$x''(t) = \frac{2t}{1+t^2}x'(t) - \frac{2}{1+t^2}x(t) + 1$$

with boundary conditions $x(0) = 1.25$ and $x(4) = -0.95$ over the interval $[0, 4]$ using

- (a) Linear shooting method.
- (b) Finite-differences method.

Compare your results with the **exact solution**

$$x(t) = 1.25 + 0.4860896526t - 2.25t^2 + 2t \arctan(t) - \frac{1}{2} \ln(1 + x^2) + \frac{1}{2}t^2 \ln(1 + t^2)$$