

2017

PSET - 1

JULY
SATURDAY

22

1. Normalize the vectors if they are not plausible state-vectors.

To be a state vector for a state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

the sum of amplitude squares should be 1.

$$\therefore |\alpha|^2 + |\beta|^2 = 1$$

$$\text{or } ||\psi\rangle| = 1$$

$$a) |\psi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\Rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

Hence it's a state vector.

$$b) |\omega\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{coeff. are } (1), \text{ so}$$

$$\sqrt{1^2 + 1^2} \Rightarrow \sqrt{2}$$

Not a state vector,

Normalize

$$|\hat{\omega}\rangle = \frac{|\omega\rangle}{||\omega\rangle|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Now it's a state vector.

SUNDAY

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2017 AUGUST						
Mon	7	14	21	28		
Tue	1	8	15	22	29	
Wed	2	9	16	23	30	
Thu	3	10	17	24	31	
Fri	4	11	18	25		
Sat	5	12	19	26		
Sun	6	13	20	27		

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JULY
MONDAY

2017

$$c) |\phi\rangle = \begin{pmatrix} 3+2i \\ 0 \\ 3i \end{pmatrix}$$

$$3+2i \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 3i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$||\phi\rangle| = \sqrt{(3+2i)^2 + (3i)^2}$$

$$\Rightarrow 9 + (-4) + 12i + (-9)$$

$$\Rightarrow -4 + 12i$$

Not a state vector

$$|\hat{\phi}\rangle = \frac{|\phi\rangle}{||\phi\rangle|} \Rightarrow \frac{1}{-4(1-3i)} \begin{pmatrix} 3+2i \\ 0 \\ 3i \end{pmatrix}$$

Now it's a state vector

$$d) |\chi\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix}$$

$$0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$||\chi\rangle| = i$$

So, $\frac{1}{i} \begin{pmatrix} 0 \\ i \end{pmatrix}$ normalized and a state vector

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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Notes

2017

JULY
THURSDAY

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c) $|E\rangle = \begin{bmatrix} 1 \\ 1/2 \\ 1/4 \\ 1/8 \\ \vdots \end{bmatrix}$

the infinite series

$$|E\rangle = \begin{bmatrix} 1 \\ 0 \\ \vdots \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} + \dots = \sum_{n=0}^{\infty} \frac{1}{2^n} \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$= \frac{1}{3} \times 2 \left(2^{k+2} - 1 \right)$$

$$||E\rangle| = \sqrt{\sum_{n=0}^{\infty} \frac{1}{2^{2n}}} \Rightarrow \sqrt{1.333 \dots}$$

Not a state vector

after normalizing

$$\frac{1}{\sqrt{1.333}} \begin{bmatrix} 1 \\ 1/2 \\ 1/4 \\ \vdots \end{bmatrix}$$

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