24 Density matrix of a single qubit

$$\rho = \frac{1}{2}\left(I + \vec{r}\cdot\vec{\sigma}\right)$$

position
vector

qubit initializes $|0\rangle$ with a Hadamard gate

our final state

$$|q\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$\rho = \frac{1}{2}\left(I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z\right)$$

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \quad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \quad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

and Bloch sphere

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\varphi}|1\rangle$$

$$\vec{r} = \begin{pmatrix}\cos\phi\ \sin\theta\\ \sin\phi\ \sin\theta\\ \cos\theta\end{pmatrix}$$

$$\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}$$

$$\sin\frac{\theta}{2} e^{i\phi} = \frac{1}{\sqrt{2}} \qquad \sin\frac{\theta}{2}(\cos\phi + i\sin\phi) = \frac{1}{\sqrt{2}}$$

$$\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}. \qquad \theta = 90^\circ \text{ or } \pi/2$$

$$\sin\frac{\theta}{2} = \frac{1}{\sqrt{2}}.$$

$$\cos\phi + i\sin\phi = 1$$

$$\phi = 0^\circ$$

Now, $\vec{r} = \begin{pmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$P = \frac{1}{2}(I + r_x \sigma_x)$$

$$\cancel{\frac{1}{2}\left(I + (1)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)}$$

$$P = \frac{1}{2}\left(I_{2\times2} + 1\cdot\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$$

$$\rho = \frac{1}{2}\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$$

$$\boxed{\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}$$

the density Matrix