PSET-2.

. Q1   State vector at each step
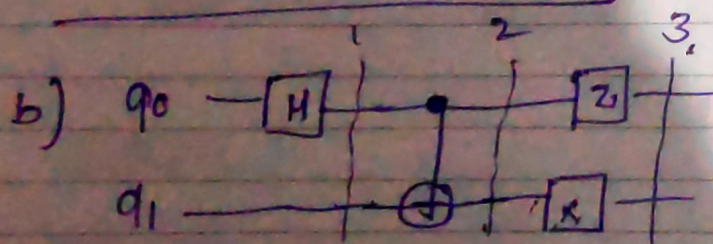
a) q_0 —[H]—

      q_1 —[Z]—
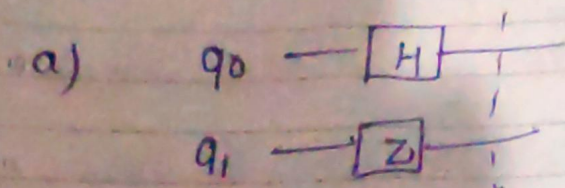
Since both the states start for $|0\rangle$ state

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$Z|0\rangle = |0\rangle$$

So the final state

$$\boxed{\frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)}$$

b) q_0 —[H]——•——[Z]—

      q_1 ————⊕——[x]—

After ①

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right) \quad \boxed{\frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)}$$

After ②   Applying CNOT

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle \right)$$

After ③   Applying a Z and X gate

$$Z|1\rangle = -|1\rangle$$
$$X|0\rangle = |1\rangle$$
$$X|1\rangle = |0\rangle$$

$$\frac{1}{\sqrt{2}}\left(\cancel{|0\rangle} + \cancel{X}-\cancel{X|0\rangle} \right)$$

$$\frac{1}{\sqrt{2}}\left(-|01\rangle + |10\rangle \right)$$

ⓒ

q0 — [H] ——————— q ——

q1 — [X] — [H] — ⊕ —

      1   2   3
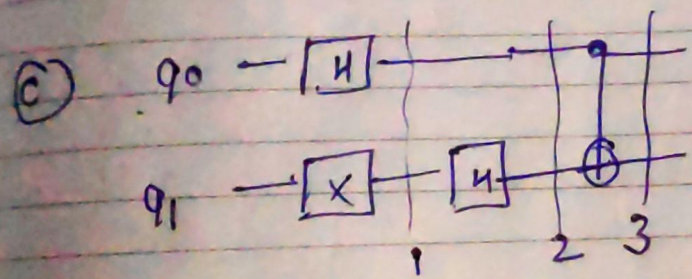
After ①   Hadamart entagls, X bit flip

$$\frac{1}{\sqrt{2}}\left(|10\rangle + |11\rangle \right)$$

After ②   another Hadamaro

$$\frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle \right)$$

After ③ applying CNOT

$$\frac{1}{2}\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle \right)$$