PROBLEM OF THE WEEK
S25 W20 (Nov 15 - Nov 19)

i) Let 'n' be the answer to the question. Find the number of positive divisors of the integer

$$\left(\frac{n-25}{75}\right)^{\frac{n-25}{75}}$$

Sol) Let $x = \frac{n-25}{75}$

Since n is the answer, hence the divisor of the integer, hence we can say

x^x has $75x+25$ divisor

Now, the prime factorization of x is

$$\prod_{i=1}^{n} p_i^{e_i}$$

So, according to above

$$\prod_{i=1}^{n} (xe_i + 1) = 75x + 25$$

Taking modulo x on both side we have

$$1 \equiv 25 \pmod{x}$$

that $\Rightarrow$ $24 \equiv 0 \pmod{x}$

hence x divides 24.

Take $p_1 = 2$, $p_2 = 3$ and $n = 2$, so our condition get

$$(xe_1 + 1)(xe_2 + 1) = 75x + 25$$

$$\Rightarrow (xe_1e_2 + e_1 + e_2 - 75)x = 24$$

Hence. $xe_1e_2 + e_1 + e_2 > 75$, now since

$e_1 \leq 3$ and $e_2 \leq 1$

$\Rightarrow 3x + 4 > 75$

this rules out everything except $x = 24$,

So we end up getting

$$xe_1e_2 + e_1 + e_2 - 75$$

$$\Rightarrow 72 + 3 + 1 - 75 = 1$$

Hence, the answer is

$$75 \cdot 24 + 25$$

$$\Rightarrow 1825$$