Self-Organized Criticality: Sandpile Model*

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(Term Paper)
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A dynamic system with a spatial degree of freedom will evolve into a self-organized criticality. The **1/f** noise will be identified with the dynamics of the critical state.

I. BASICS

The classic example of SOC is a pile of sand. Sandpile exhibits punctuated equilibrium expression, where intermittent sand slides obstruct periods of stasis. The sand slides, are produced by a domino effect, in which a individual grain of sand shifts one or more other grains and causes them to topple. In turn, those grains of sand may associate with other grains in a chain reaction. Large avalanches, not a gradual change, link quantitative and qualitative behaviour and form the basis for emergent phenomena. If this spectacle is suitable for the physical environment, then we must allow change and catastrophes as inevitable in biology, history, and economics. Because the outcome is contingent upon specific minor events in the past, we must also abandon any idea of detailed long-term determinism or predictability. From a selfish point of view, economics, the best we can do, is to shift disasters to our neighbours. Large catastrophic accidents occur as a result of the same dynamics that create small regular everyday events. This conclusion runs counter to the usual way of thinking about large events, which, as we have seen, looks for specific reasons (for instance, a falling meteorite causing the extinction of dinosaurs) to explain large cataclysmic events. Even though there are many more small events than large ones, most of the system's changes are associated with the large, catastrophic events. Self-organized criticality can be viewed as the theoretical justification for catastrophism.

In 1987 Chao Tang, Kurt Wiesenfeld, and Per Bak created the simplistic, prototypical example of self-organized criticality, the sandpile model. Their discretions on the model showed how a system that performs mild, local rules can arrange itself into a poised state that results in flashing, sporadic bursts rather than resulting in a smooth path. They did not set out with the purpose of inquiring about sandpiles. As with many other discoveries in science, the discovery of sandpile dynamics was accidental.

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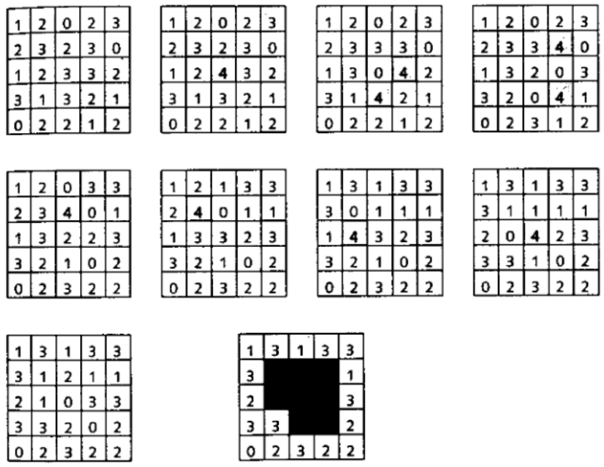
II. HOW IS IT MADE?

Think of a flat table, and there the sand is added a little, one letter at a time. Characters can be added to random positions or can only be added at once, for example, in the middle of the table. The balanced state represents the general state of equality; this the state has low power because we will have to add power to rearrange sand to form piles of any shape. If we had used water, the system would always return to a flat surface as the water would recede to the table's edge. Because letters are often caught regularly, The sand's shape will not automatically return to the ground, a situation where we stop adding sand. Initially, the grains of sand will remain more or less where they land. As we add extra sand, pile solid and small sand slides or avalanches are possible. Letters may continue on top of other characters and up to a low level. This can cause some characters to fall. Adding an individual bit of sand can cause a disturbance of space, but nothing is noticeable it happened in a pile. In particular, events in one part of the pile do not affect sand grains in the farthest parts of the pile. No global communication within the pile at this stage, many individual particles. As the slope increases, one grain is more likely to cause other grains to fall.

Finally, the slope reaches a certain amount and cannot increase continuously because extra sand is estimated to be sand that leaves a lot to fall on the edges. This is called a static condition because the average amount of sand and the general slope is not permanent. It is clear that having this limited range in the middle of the sand added to the pile, say, in the middle of the sand leaving the edges, should be communicated throughout the program. There will be the occasional avalanche covering the whole mass. This is a self-organizing process (SOC) status. The addition of sand grains has changed the system from a state where each grain follows its local strength to a critical state where the power of emergence is present worldwide. There is one complex system in the standing position of the SOC, the sandpile, and its emerging power. The appearance of the sandpile would not be expected from the structures of each character. Sandpile is an open system that is flexible since sand is applied from the outside without. It has many forms of freedom or grains of sand. The descent of the sand the sheaf represents the potential for power, measured as the



grain height above the table.

Figure ?? shows a boat. When the grain is depleted, this energy is converted into kinetic energy. When the cooled grain arrives at rest, the kinetic energy is depleted, converted into heat in a pile. There is a flow of energy through the system. The critical condition can only be maintained because of the new sand structure's strength being given externally. The crisis must be firm about remediation. This is important in considering the importance of good planning to have an opportunity to describe the real world; in fact, this is a complete concept.
After the same system has reached its critical stand, it begins abruptly dispose of wet sand instead of dry sand. Wet sand has more friction than dry sand. So, for a while, the avalanches will be small and local. Below the material will leave the system because small avalanches cannot reach the edge of the table. The pile becomes worse. This will also cause ice floods. The model's formulation is provided in integer height variable z_i at each side of the square lattice. Particles are added at random, and each addition increases the height of the site by one. If the size exceeds the value $z_c = 4$, then the site topples and decrease the height by four and elevation at each location of its neighbours by one. These can become unstable and repeat the same process. It starts a chain reaction, and it goes on until the lattice gets stable again. The change of the sites is done concurrently, and open boundary conditions are assumed.

This system evolves slowly to a critical state with a constant flux of particles with the same properties as the second-order phase transitions. Therefore it obeys the power-law distribution. These critical properties are independent of the initial conditions and the configurations, the flux rate determines them, so there's no fine-tuning or any control parameters.

III. DYNAMIC SYSTEM

A dynamic system with a spatial degree of freedom will evolve into a self-organized criticality. The 1/f noise will be identified with the dynamics of the critical state. The 1/f noise is the most occurring and ubiquitous, detected in the complex system modelling, like in the hourglass, the river flow, and a star's luminosity.

The low-frequency power spectra for the systems mentioned above show a power-law behaviour f^{-1}. Despite all the efforts, there isn't any general theory that explains this widespread existence of the 1/f noise.

The same is with the self-similar fractal structures, like the mountain landscapes and the coastal lines. In Turbulence, self-similarity occurs both in time and space.

Here we'll discuss the dynamic systems with an extended spatial degree of freedom and how they naturally evolve into a self-organized criticality. This self-organized criticality is the primary underlying mechanism behind all the system mentioned above. The noise propagates through the scaling clusters employing a "domino" effect.

This criticality is different from the one in phase transitions of equilibrium statistical mechanics, where the parameter like, Temperature is tuned for getting the criticality. The critical point is an attractor in the dynamic system is reached by starting from an equilibrium. That's why there' no need to fine-tune to get the 1/f noise.

Self Organized Criticality is the most central notion of advanced physics. Bak et al. had put forth the new type of Cellular Automata, known as the "sandpile" model, because of the sand grains' analogy toppling from a pile.

IV. FLUX STATE

We need to derive the mean-field equations, and We'll let the lattice be populated with one of the four species A, B, C or D. These are the only four stable heights of the original model. Some new particles are added to the lattice with some foreign force, initiating an avalanche(ϕ). In this chain reaction, the ϕ species change all the other species and topple the states according to some rules.

$$A + \phi \to B,$$

$$B + \phi \to C,$$

$$C + \phi \to D,$$

$$D + \phi \to p_1 : D + \tilde{\phi}, p_2 : C + 2\tilde{\phi}, p_3 : B + 3\tilde{\phi}, p_4 : A + 4\tilde{\phi}$$

Here ϕ and $\tilde{\phi}$ denote the sand grains or the particles obtained from the neighbouring sites, and the one transferred to the neighbouring sites, respectively. It can be generalized by the branching probabilities

$$p = (p_1, p_2, p_3, p_4), p_1 + p_2 + p_3 + p_4 = 1$$

It generally refers to the total conservation of the sand grains or the particles used in the sandpile model's basic

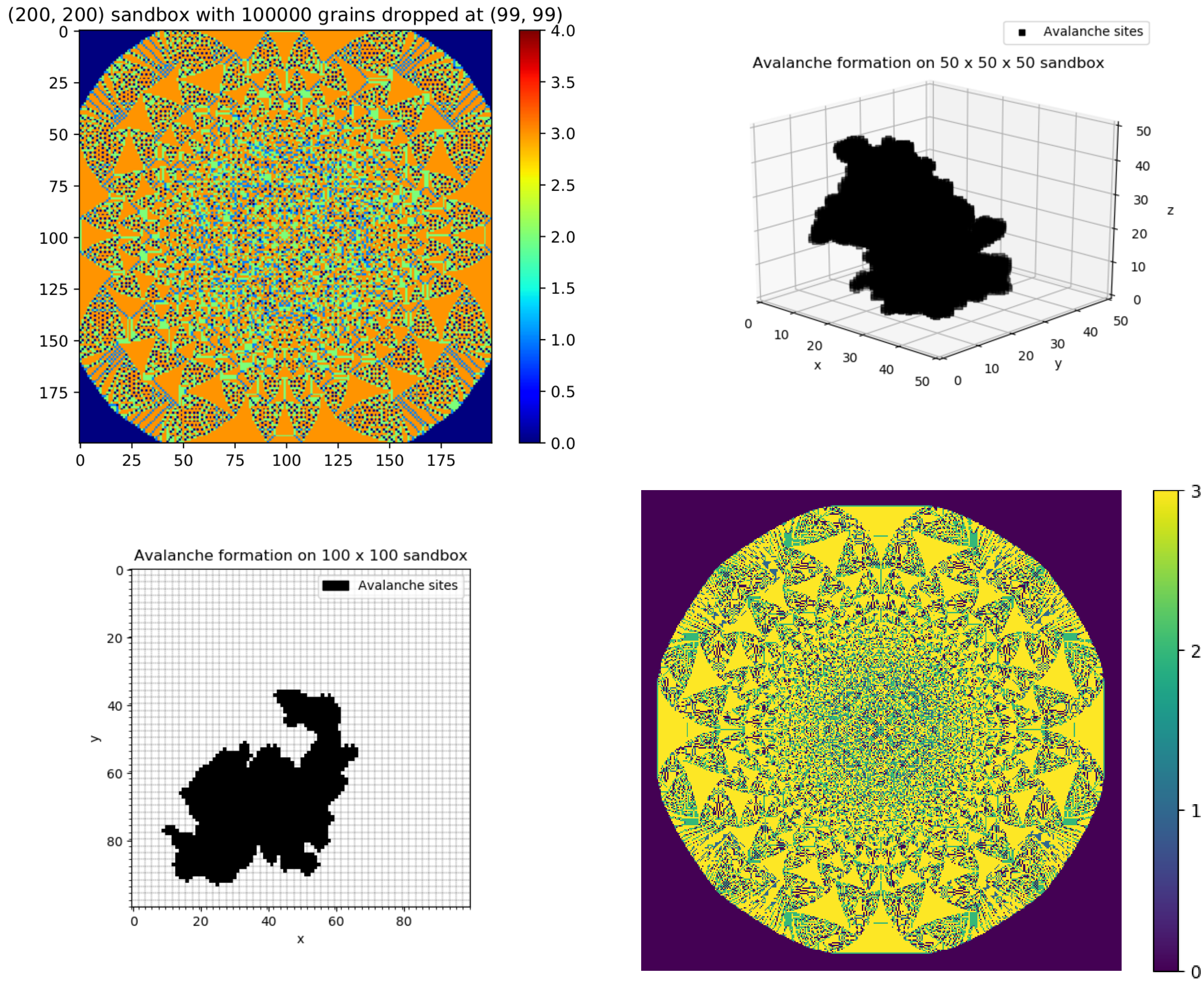


FIG. 1. 2^{20} grains

formulation. The mean-field approximation can make the most detailed description. There are many ways to find the mean-field equation

Self-organized critical systems evolve to multiple decisive states without interference from any external cause. The process of self-organization takes place over a very long momentary period. A long process of development always creates fixation performance, whether in geophysics or biology. Examining the methods within a period frame that is short contrasted with this evolutionary method cannot be explained.

V. THE 1/F NOISE

The origin of the mysterious phenomenon of $\mathbf{1/f}$ noise that is transmitted by copious sources on earth and outside in the universe is a significant matter of debate.

Most attempts to explain $\mathbf{1/f}$ noise were ad hoc theories for a single system, with no general applicability, which appeared unsatisfactory to many. Since the phenomenon occurs everywhere, people believed that there must be a broad, robust explanation. Systems with lesser degrees of freedom, like the angle and velocity of a pendulum and equilibrium systems, cannot generally show $\mathbf{1/f}$ noise or any other complex behaviour since fine-tuning is always necessary. Thus, people concluded that $\mathbf{1/f}$ noise would have to be a cooperative phenomenon where the diverse components of comprehensive systems collaborate in some mutual way. Indeed, all the sources of $\mathbf{1/f}$ noise were such large systems with many parts. For example, the inconstancies of the river level of the Nile must be related to the geographical or climate model of Africa, which can surely not be diminished to a mere dynamical system. One thought was that $\mathbf{1/f}$ noise could be related to matter's spatial structure. The systems had to be "open," Energy had to be supplied from outside since closed systems would approach an ordered or disordered equilibrium state without complex behaviour. However,

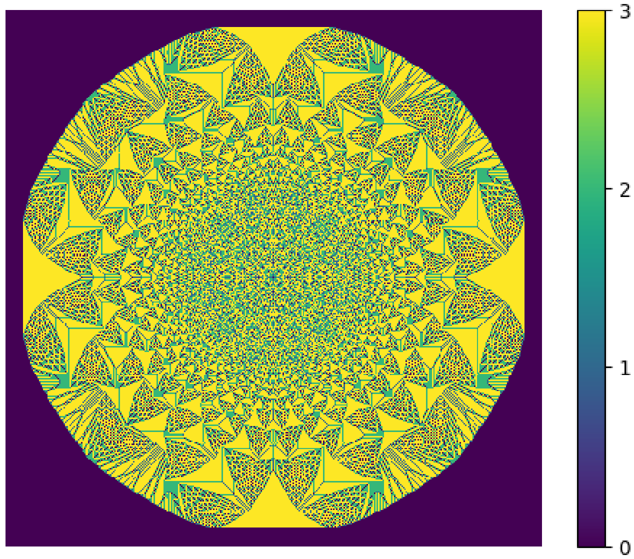


FIG. 2. 2^{18} grains

there were no known general principles for an open system with many degrees of freedom.

VI. LIFE IN A SANDPILE WORLD

The volatility of nonequilibrium cannot be further enhanced different from the quiet power of the flat sea. How could a local viewer do it? During the passing phase, where the sandpile was little by little, his experience can be tedious. There may be a slight disturbance passing when a few grains fall into neighbours. If we throw one grain of sand in one place instead of otherwise, this results in a slight change of location in the configuration. There is no means that disruption can spread the whole system. Feedback on minor disturbances is minor. In a world without conscience, nothing is noticeable. It is easy to become a weather forecaster (sand) in a non-flat critical system. The action elsewhere does not depend on events that took place long before remote areas. Unexpected status does not work. When the masses have reached a critical state of affairs, however, the situation is quite different. A single grain of sand can cause a fall which includes the whole mass. A small configuration change could result. The sand the forecaster can still make short-term predictions by carefully identifying laws and environmental protection. When he sees an avalanche coming, he can guess when it will hit with a certain level of accuracy. However, he can not predict when a significant event will occur because this depends on the smallest details of the entire sand's preparation.

[1] Check the code on GitHub.
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[6] PerBak,ChaoTang,andKurtWiesenfeld *Self-OrganizedCriticality:An Explanation of 1/f Noise.* [*1987*]
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[8] Per Bak, *How Nature works?*.

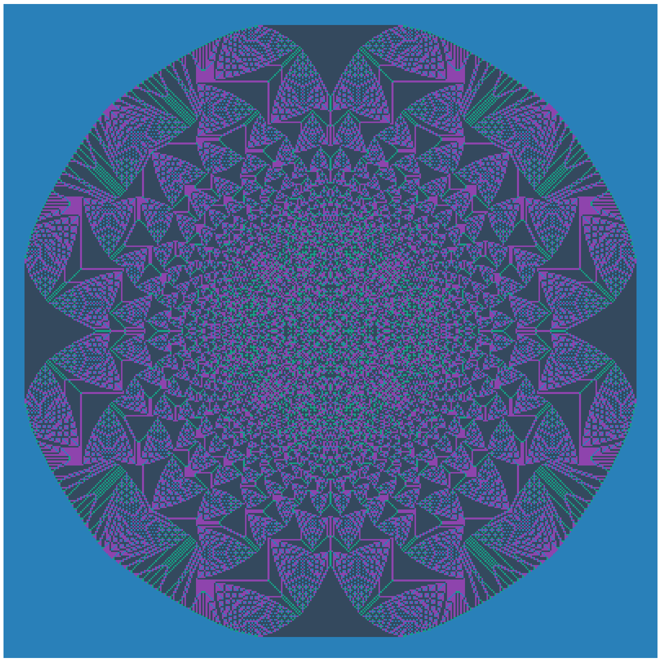

FIG. 3. 2^{18} grains

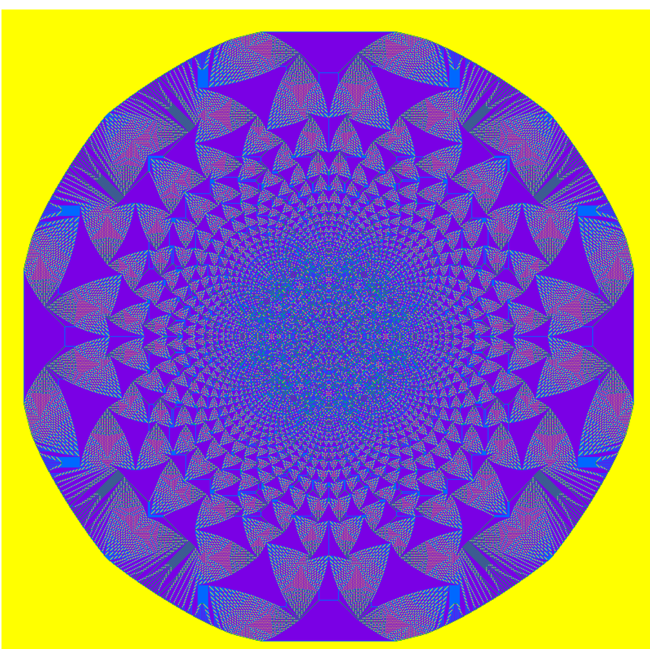

FIG. 4. 2^{20} grains

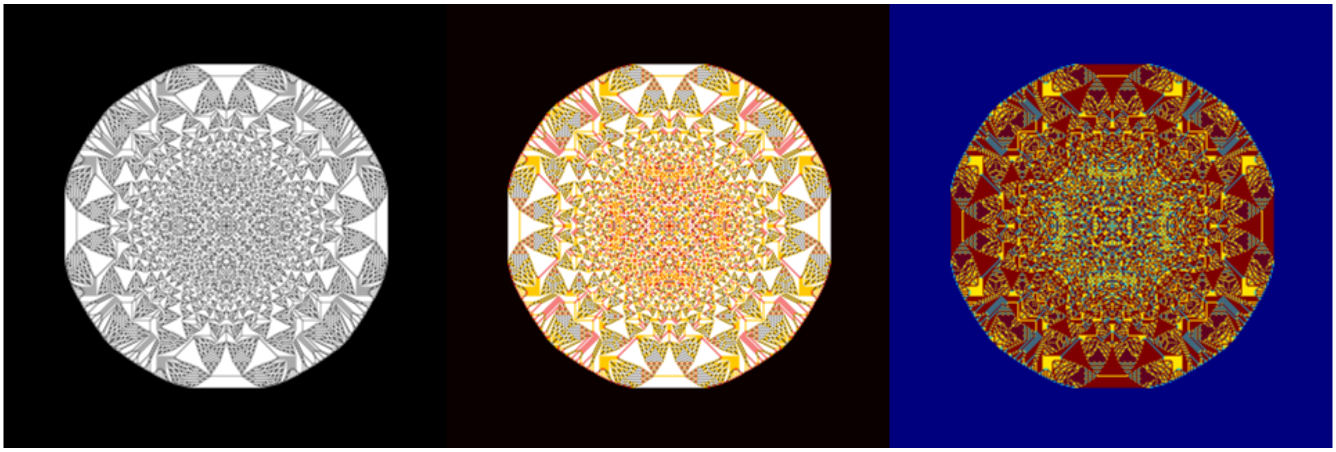

FIG. 5. Abelian Sandpiles

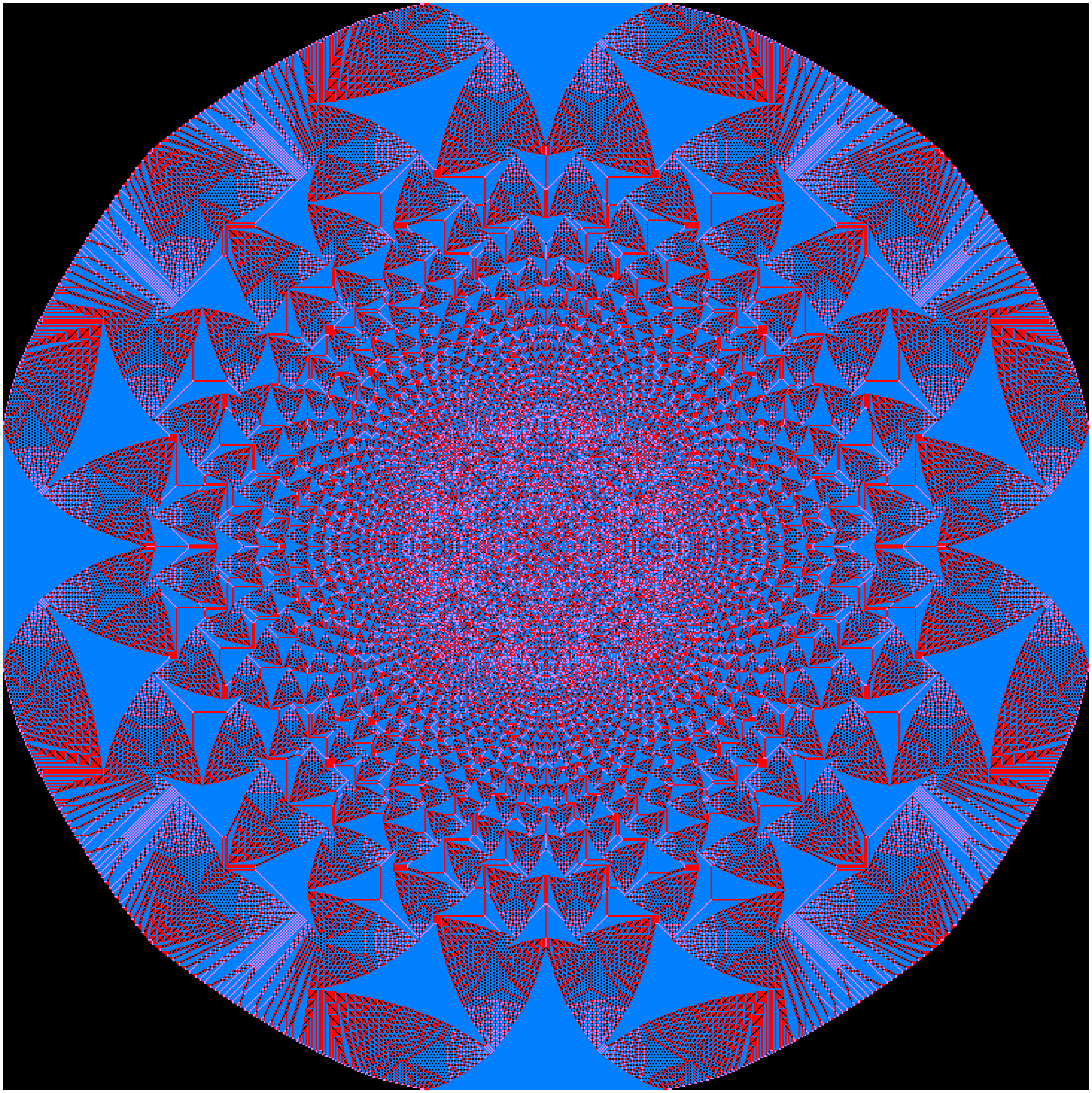

FIG. 6. Abelian Sandpiles