2. Multiple Systems

2.1 Classical state

Suppose we have two systems
- X having classical state set Σ
- Y is a system having state Γ

composite system
(X, Y) or XY

What are classical states of (X,Y)?

The classical state set of (X,Y) is the cartesian product.

$$\Sigma \times \Gamma = \{(a,b) : a \in \Sigma \text{ and } b \in \Gamma\}$$

The description generalizes to more than two systems in a natural way.

$$\Sigma_1 \times \cdots \times \Sigma_n = \{(a_1, \ldots a_n) : a_1 \in \Sigma_1, \ldots a_n \in \Sigma_n\}$$

An n-tuple may also be written as string $a_1 \cdots a_n$

$$\Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_{10} = \{0,1\}^{10}$$

$000\cdots0$
$000\cdots1$
$000\cdots10$

Convention: Cartesian products of classical state sets are ordered lexicographically
- we assume the individual classical state sets are already ordered.
- significance decrease from left to right

Eg $\{1,2,3\} \times \{0,1\}$

$(1,0), (1,1), (2,0), (2,1), (3,0), (3,1)$

Eg: $\Sigma\{0,1\} \times \Sigma\{0,1\}$

00 01 10 11

2.2 Probabilistic states

States of compound systems that associate probabilities with the Cartesian product of the classical state sets of the individual systems.

Eg. $Pr((X,Y) = (0,0)) = \frac{1}{2}$ $\quad Pr((X,Y) = (0,1)) = 0$

$$\begin{array}{cl} \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} & \begin{array}{l} -00 \\ -01 \\ -10 \\ -11 \end{array} \end{array}$$

For a given probabilistic state of (X,Y), we say that X and Y are <u>independent</u> if

$$Pr((X,Y) = (a,b)) = Pr(X=a)\, Pr(Y=b)$$

Suppose that probabilistic state of (X,Y) is expressed as vector.

$$|\pi\rangle = \sum_{(a,b)\in\Sigma\times\Gamma} p_{ab} |ab\rangle$$

$$|\phi\rangle = \sum q_a |a\rangle \qquad |\psi\rangle = \sum r_b |b\rangle$$

$$p_{ab} = q_a r_b \quad \forall\ a \in \Sigma \text{ and } b \in \Gamma$$

Eg:

a) $|\pi\rangle = \frac{1}{6}|00\rangle + \frac{1}{12}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{4}|11\rangle$

$|\phi\rangle = \frac{1}{4}|0\rangle + \frac{3}{4}|1\rangle$ $\qquad |\psi\rangle = \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle$

independent

b) $|\pi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle$ X and Y are not independent

if they were, we would have

numbers q_0, q_1, r_0, r_1 such that

$q_0 r_0 = \frac{1}{2}$ $q_0 r_1 = 0$ $q_1 r_0 = 0$ $q_1 r_1 = \frac{1}{2}$

2.3 Tensor Product of vectors