Appendices

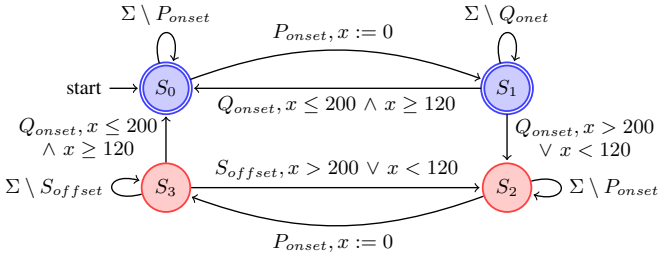

Fig. 1: Policy φ_{ECG1} specified as TA.

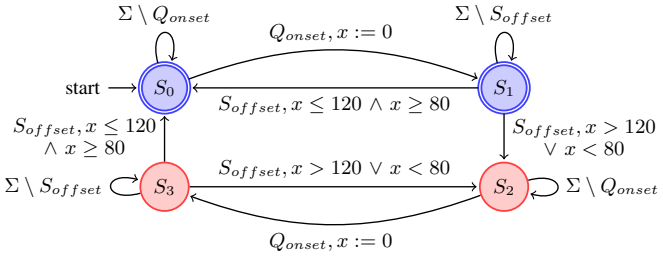

Fig. 2: Policy φ_{ECG2} specified as TA.

Appendix A
Capturing Heart Abnormalities as Timed Automata

We will discuss the cardiac problems caused by temporal feature deviations and the timed policies considered to monitor and classify ECG. A typical ECG signal has various temporal aspects, including PR, QT, RR, and P-wave intervals, as well as conventional ranges. Any variations from these normal electrical patterns can suggest a variety of heart conditions.

The table I presents different ECG waves and intervals, their usual ranges, and abnormalities in the heart when these properties deviate from their safe ranges.

A. Capturing cardiac complications due to extended PR interval as TA

Regular PR intervals average between 0.12 and 0.20 seconds. A prolonged PR interval showing delayed transmission of the SA node impulse to the ventricles is diagnostic of first-degree AV block. Firstdegree heart block is a clinically insignificant finding on its own; however, heart diseases such as acute rheumatic carditis, an overdose of digoxin, or an electrolyte imbalance may arise because of it. A Harvard study found that people with PR intervals longer than 200 ms had an increased risk of atrial fibrillation, pacemaker insertion, and premature death by roughly a factor of 1.5. Wolff-ParkinsonWhite and Lown-Ganong-Levine syndromes are two conditions in which a short PR interval is seen because the heart is able to bypass the AV node delay. The TA in Fig. 1 captures the extended PR interval. The RV monitor raises the alarm whenever the standard PR interval is violated, indicating the possibility of cardiac abnormalities due to the prolonged PR interval.

B. Capturing wide QRS complex as TA

In general, the QRS-complex ranges between 0.08 and 0.10 seconds. When the time span is between 0.10 and 0.12 seconds, we classify it as intermediate or prolonged QRS-complex. This may indicate a left anterior or posterior fascicular block, or an incomplete right or left bundle branch block. When the QRS-complex lasts longer than 0.12 seconds,

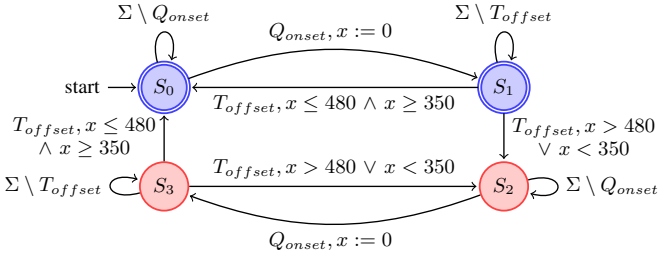

Fig. 3: Policy φ_{ECG3} specified as TA.

medical attention is warranted. An extended QRS duration is a symptom of a variety of heart rhythm disorders, such as right bundle branch block, left bundle branch block, non-specific intraventricular conduction delay, and ventricular arrhythmias such as ventricular tachycardia. Right-sided heart problems are often indicated by a phenomenon known as right bundle branch block (RBBB). However, the left bundle branch block (LBBB) is always associated with heart disease, most frequently in the left ventricle. The prolonged QRS duration is captured by the TA in Fig. 2. Every time the expected duration is exceeded, the RV monitor triggers warning of the potential for cardiac problems caused by the prolonged QRS interval.

C. Capturing prolonged QT interval as TA

The QTc interval should be between 0.35 and 0.48 seconds. It is prolonged in people with certain electrolyte disorders, and it is also prolonged by some drugs. Ventricular tachycardia can be caused by a prolonged QT interval (more than 480 ms). Long QT Interval Syndromes occur when the QT is larger than 480 ms (LQTS). This condition has significant clinical implications because it typically suggests a higher risk of malignant ventricular arrhythmia, syncope, and sudden death. Short QT syndrome occurs when QT is less than 0.35 s and can result in hypercalcemia and malignant arrhythmia. The elongated QT interval is depicted by the TA in Fig. 3. When the typical QT interval is exceeded, the RV monitor triggers a warning because this may be a sign of cardiac problems.

TABLE I: ECG features and heart abnormalities

ECG waves and intervals	Normal Range	Abnormal duration	Heart Abnormality	References
PR interval	0.12-0.20s	$\leq$ 0.12 s	Pre-excitation syndromes	[1]
		$\geq$ 0.2 s	First-degree AV block	[1]
QRS complex	0.08-0.12 s	0.1 - 0.12 s	Incomplete bundle branch block	[1]
		$\geq$ 0.12 s	Complete bundle branch block	[1]
QT interval	350-480 ms	$\geq$ 480 ms	Abnormal ventricular repolarization	[1]
		$\leq$ 390 ms	Abnormal ventricular depolarization or repolarization	[1]
RR interval	0.6-1.2 s	variable	Irregular heart rhythm	[1]
P-wave	$\leq$ 0.12 s	$\geq$ 0.12 s	atrial enlargement	[1]

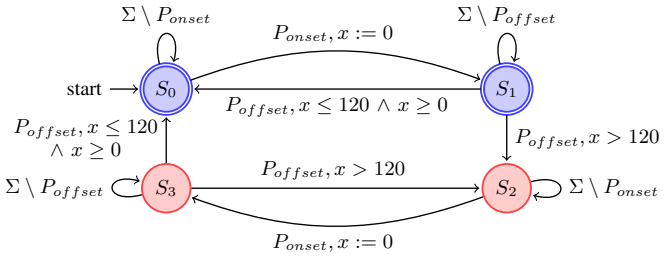

Fig. 4: Policy φ_{ECG5} specified as TA.

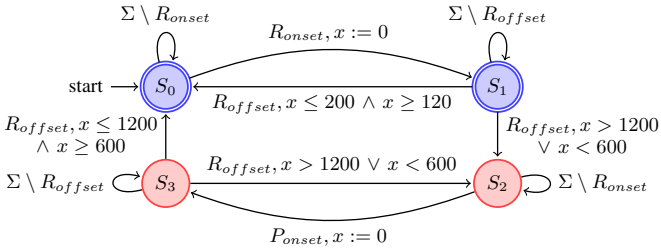

Fig. 5: Policy φ_{ECG4} specified as TA.

D. Capturing wide P-wave as TA

In addition, when the P-wave duration is longer than normal, it usually indicates that one or both atria are enlarged (hypertrophied). P waves can broaden or amplify as a result of atrial enlargements. The shape of the P waves can be altered by ectopic atrial beats. An absence of P waves is a characteristic of many abnormal heart rhythms, such as atrial fibrillation and junctional arrhythmias. Short RP can occur when the P waves are obscured by the end of the QRS complex, as can happen in atrioventricular reentrant tachycardia. The wide P-wave interval is depicted by the TA in the Fig. 5. When the specified duration is exceeded, the RV monitor will sound the alarm to alert the user to the fact that there is a potential for cardiac irregularities as a result of the longer P-wave interval.

E. Capturing variation in heartbeat as TA

Variations in the RR interval have been extensively researched in the context of irregular cardiac rhythm. The fluctuations in the time intervals between individual heartbeats (R-peaks) are measured by heart rate variability (HRV). The HRV can provide insights into autonomic neural function as well as sympathetic-parasympathetic autonomic balance and cardiovascular health ([13]). The extended RR interval is captured by the TA in Fig. 4. Every time the normal RR interval is violated, the RV monitor sounds the alarm, potentially alerting the patient to the likelihood of cardiac problems caused by the prolonged RR interval.

APPENDIX B
EXAMPLE

Let us consider the policy

$$\varphi_{\text{ECG}} \;=\; \varphi_{\text{ECG}_1} \cap \varphi_{\text{ECG}_2} \cap \varphi_{\text{ECG}_3} \cap \varphi_{\text{ECG}_4} \cap \varphi_{\text{ECG}_5}$$

To monitor with a sample ECG trace, let

$$(P_{\text{onset}},\, 270) \cdot (P_{\text{offset}},\, 350) \cdot (QRS_{\text{onset}},\, 400) \cdot (R_{\text{peak}},\, 420) \cdot (QRS_{\text{offset}},\, 510) \cdot (T_{\text{offset}},\, 890) \cdot (R_{\text{peak}},\, 1120)$$

be the sample ECG input event sequence with occurrence times (in ms). The RV monitor receives the trace, and each event is associated with a delay that represents the amount of time since the preceding event or the system initialization. Table II displays the monitor's step-by-step behaviour.

Observe how the RV monitor processes increasingly complete prefixes of the input ECG trace. When the first event $(P_{\text{onset}}, 270)$ arrives at $t = 270$ ms, none of the interval constraints can yet be evaluated, so all five sub-monitors—and thus their conjunction—report CT (true). Next, at $t = 350$ ms, the event $(P_{\text{offset}}, 350)$ is received; because the P-wave duration remains within its allowable range, φ_{ECG_5} continues to hold and the overall verdict stays CT.

As the trace grows, the QRS onset $(QRS_{\text{onset}}, 400)$ at $t = 400$ ms yields another CT, since the PR interval

$$QRS_{\text{onset}} - P_{\text{onset}} \;=\; 400 - 270 \;=\; 130 \text{ ms}$$

is under the 200 ms limit, keeping φ_{ECG_1} satisfied. Upon seeing the R peak $(R_{\text{peak}}, 420)$ at $t = 420$ ms, φ_{ECG_2} remains true and the conjunction again outputs CT. When the QRS offset $(QRS_{\text{offset}}, 510)$ occurs at $t = 510$ ms, the QRS complex duration

$$QRS_{\text{offset}} - QRS_{\text{onset}} \;=\; 510 - 400 \;=\; 110 \text{ ms}$$

still falls inside the safe window, so all sub-monitors continue to emit CT.

However, at $t = 890$ ms, the T-wave offset $(T_{\text{offset}}, 890)$ produces a QT interval of

$$T_{\text{offset}} - QRS_{\text{onset}} \;=\; 890 - 400 \;=\; 490 \text{ ms},$$

TABLE II: Step-by-step behaviour of RV monitor verifying φ_{ECG}.

Timed word σ	φ_{ECG_1}	φ_{ECG_2}	φ_{ECG_3}	φ_{ECG_4}	φ_{ECG_5}	$\varphi_{\mathrm{ECG}_1} \cap \cdots \cap \varphi_{\mathrm{ECG}_5}$
(P_{onset} , 270)	-	–	–	–	–	CT
(P_{onset} , 270) (P_{offset} , 350)	–	–	–	–	CT	CT
(P_{onset} , 270) (P_{offset} , 350) (QRS_{onset}, 400)	CT	–	–	–	CT	CT
(P_{onset} , 270) (P_{offset} , 350) (QRS_{onset}, 400) (R_{peak}, 420)	CT	–	–	–	CT	CT
(P_{onset} , 270) (P_{offset} , 350) (QRS_{onset}, 400) (R_{peak}, 420) (QRS_{offset}, 510)	CT	CT	–	–	CT	CT
(P_{onset} , 270) (P_{offset} , 350) (QRS_{onset}, 400) (R_{peak}, 420) (QRS_{offset}, 510) (T_{offset}, 890)	CT	CT	CF	–	CT	CF
(P_{onset} , 270) (P_{offset} , 350) (QRS_{onset}, 400) (R_{peak}, 420) (QRS_{offset}, 510) (T_{offset}, 890) (R_{peak}, 1120)	CT	CT	CF	CT	CT	CF

which exceeds the allowable maximum of 480 ms. Consequently, φ_{ECG_3} fires CF (false) while the other sub-monitors still report CT, and the overall conjunction switches to CF, indicating a violation. Finally, even when a late R peak $(R_{\mathrm{peak}}, 1120)$ is appended, the false verdict for φ_{ECG_3} persists and the monitor continues to report CF for the remainder of the trace.

Appendix C
Related work

The book "ECG Made Easy" [1] provides a comprehensive overview of ECGs. It explains what an ECG is, how to identify its peaks, their significance, and the normal durations and ranges for each peak. Abnormalities arise when these ranges are not within the given range. The book has two sections: the first, 'The Basics', covers ECG fundamentals, including recording, reporting, and classical abnormalities. The second section, 'Making the Most of ECG,' underscores that an ECG is a diagnostic tool and the interpretation of the ECG should consider the patient's history and physical examination.

At the same time, A. Bauer et al. [9] presented a runtime verification approach for properties expressed either in linear-time temporal logic (LTL) or timed linear-time temporal logic (TLTL), suitable for monitoring discrete-time and real-time systems, respectively.

Using ECG and Runtime verification, Srinivas Pinisetty et al. [7] show how to synthesize a runtime verification monitor for any regular timed property given by a deterministic timed automata and provide algorithms to implement these mechanisms.

Using that, Abhinandan Pand et al. [6] propose a formal runtime monitor for monitoring hypertension, which is explainable and clinically interpretable. Abhinandan Pand et al. [3] also implemented a data mining model from the ECG features to infer ECG policies and synthesize a formal verification monitor based on the policies.

Similarly, Abhinandan Pand et al. [4] also showed that the RV monitor of timed ECG policies can detect anomalies in the blood glucose level efficiently and monitor the security of the underlying insulin infusion system.

From [6] [3] [4], we can conclude that runtime verification monitors (RV) have proved to be dependable and explainable tools for online health monitoring applications, like diabetes detection, a safe insulin infusion system, and detecting hypertension.

In Pacemaker safety policies, Abhinandan Pand et al. [5] validate the developed correlation of ECG and PPG signals and the proposed runtime monitoring approach. Security vulnerabilities in pacemakers are also an increasing concern. S. Pinisetty et al. [8] propose tackling this using runtime verification (RV). [5] [8] shows that RV monitors are lightweight and secure.

References

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