Basic Discussion:

If a number can be expressed as a product of two whole numbers, then the whole numbers are called factors of that number.

In other words, a factor is a whole number which divides exactly into a whole number, leaving no remainder.

For example, 13 is a factor of 52 because 13 divides exactly into 52 (52÷13 = 4 leaving no remainder).

52 = 1 x 52 = 2 x 26 = 4 x 13

So, the complete list of factors of 52 is: 1, 2, 4, 13, 26, and 52 (all these divide exactly into 52).

A simple technique to find the number of factors of a given number is to express the number as a product of powers of prime numbers or prime factors.

To illustrate let’s find the numbers of factors of our example 52.

Note that, 52 can be expressed as 4 x 13 = 2^2 x 13

So, the prime factors of 52 are 2 and 13.

Now, increment the power of each of the prime numbers by 1 and multiply the result.

In this case it will be (2+1) x (1+1) = 3 x 2 = 6 (power of 2 is 2 and power of 13 is 1)

Therefore, there will 6 factors including 1 and 52.

Also note that, all numbers have a factor of 1 since 1 multiplied by any number equals that number. All numbers can be divided by themselves to produce the number 1. Therefore, we normally ignore 1 and the number itself as useful factors.

So, excluding, these two numbers, you will have (6 – 2) = 4 factors.

To be certain the factors are: 2, 4, 13 and 26.

To further illustrate let’s find the numbers of factors of 48.

48 can be written as 16 x 3 = 2^4 x 3

So, the prime factors of 48 are 2 and 3.

Now, increment the power of each of the prime numbers by 1 and multiply the result.

In this case it will be (4+1) x (1+1) = 5 x 2 = 10 (the power of 2 is 4 and the power of 3 is 1)

Therefore, there will be 10 factors including 1 and 48. Excluding, these two numbers, you will have (10 – 2) = 8 factors. And the factors are: 2, 3, 4, 6, 8, 12, 16 and 24

Another example: 216 = 8 x 27 = 2^3 x 3^3

Here also, the prime factors of 216 are 2 and 3.

Now, increment the power of each of the prime numbers by 1 and multiply the result.

(3+1) x (3+1) = 4 x 4 = 16 (power of both 2 and 3 is 3)

Therefore, there will be 16 factors including 1 and 216.

Excluding, these two numbers, 16 – 2 = 14 factors which are 2, 3, 4, 6, 8, 9, 12, 18, 24, 27, 36, 54, 72 and 108.

Related Questions:

120 is natural number. Find the following:

a) No. of Total factors:

120 = 2^3 x 3 x 5

So, no. of total factors = (3+1) x (1+1) x (1+1) = 4 x 2 x 2 = 16

b) No. of Even factors:

120 = 2 x 60

Now, find No. of factors of 60.

60 => 2^2 x 3 x 5

So, no. of total factors of 60 => (2+1) x (1+1) x (1+1) = 3 x 2 x 2 = 12

Then, total No. of Even factors of 120 is 12.

c) No. of Odd factors:

Consider the power of odd prime factors of 120.

120 = 2^3 x 3 x 5

Odd prime factors are 3 and 5.

So, total No. of Odd factors of 120 is: (1+1) x (1+1) = 2 x 2 = 4

d) No. of Factors divisible by 3:

120 = 3 x 40

Now, find No. of factors of 40.

40 => 2^3 x 5

So, no. of factors of 40 => (3+1) x (1+1) = 4 x 2 = 8

Then, No. of factors of 120 that are divisible by 3 is 8.

e) No. of Factors divisible by 4:

120 = 4 x 30

Now, find No. of factors of 30.

30 => 2 x 3 x 5

So, no. of factors of 30 => (1+1) x (1+1) x (1+1) = 2 x 2 x 2 = 8

Then, No. of factors of 120 that are divisible by 4 is 8.

f) No. of Factors divisible by 5:

120 = 5 x 24

Now, find No. of factors of 24.

24 => 2^3 x 3

So, no. of factors of 24 => (3+1) x (1+1) = 4 x 2 = 8

Then, No. of factors of 120 that are divisible by 5 is 8.

g) No. of Factors divisible by 6:

120 = 6 x 20

Now, find No. of factors of 20.

20 => 2^2 x 5

So, no. of factors of 20 => (2+1) x (1+1) = 3 x 2 = 6

Then, No. of factors of 120 that are divisible by 6 is 6.

h) No. of Prime factors:

120 = 2^3 x 3 x 5

So, No. of Prime factors of 120 is 3 [and these are 2, 3 and5]

i) No. of Composite factors:

Total number of factors = 1 + (Prime factors) + (Composite Factors)

So, No. of Composite factors = No. of Total factor – (No. of prime factor + 1)

=> 16 – (3+1) = 12

So, No. of composite factors of 120 is 12.

j) Sum of all the factors:

120 = 2^3 x 3 x 5

Sum of all the factors => (2^0+2^1+2^2+2^3) x (3^0+3^1) x (5^0+5^1)

=> (1+2+4+8) x (1+3) x (1+5)

=> 15 x 4 x 6

=> 360

So, Sum of all the factors of 120 is 360.

k) Sum of Even factors:

120 = 2^3 x 3 x 5

Sum of all the Even factors => (2^1+2^2+2^3) x (3^0+3^1) x (5^0+5^1)

=> (2+4+8) x (1+3) x (1+5)

=> 14 x 4 x 6

=> 336

So, Sum of all the even factors of 120 is 336.

l) Sum of Odd factors:

120 = 2^3 x 3 x 5

Sum of all the Odd factors => (3^0+3^1) x (5^0+5^1)

=> (1+3) x (1+5)

=> 4 x 6

=> 24

So, Sum of all the Odd factors of 120 is 24.

m) Average of factors:

Average of factors = Sum of factors / No. of total factors

=> 360/16 = 22.5

So, Average of factors of 120 is 22.5

n) No. of ways 120 can be expressed as product of 2 integers:

120 = 2^3 x 3 x 5

So, no. of total factors = (3+1) x (1+1) x (1+1) = 4 x 2 x 2 = 16

Then, No. of ways to be expressed as product of 2 integers => 16/2 = 8