Math Shortcuts

Sum

1. The sum of first n natural numbers = n (n+1)/2
2. The sum of squares of first n natural numbers is n (n+1)(2n+1)/6
3. The sum of first n even numbers= n (n+1)
4. The sum of first n odd numbers= n^2
5. sum of the cubes of first 'n' natural numbers – n^2*(n+1)^2/4

Square

1. To find the squares of numbers near numbers of which squares are known

To find 41^2 , Add 40+41 to 1600 =1681
To find 59^2 , Subtract 60^2-(60+59) =3481
2. To find the squares of numbers from 50 to 59

For 5X^2 , use the formula (5X)^2 = 5^2 +X / X^2

Eg ; (55^2) = 25+5,25=3025

(56)^2 = 25+6, 36=3136

(59)^2 = 25+9, 81=3481

Number Properties

1. when a three digit number is reversed and the difference of these two numbers is taken , the middle number is always 9 and the sum of the other two numbers is always 9 .
2. To find the number of factors of a given number, express the number as a product of powers of prime numbers.In this case, 48 can be written as 16 * 3 = (2^4 * 3)
   Now, increment the power of each of the prime numbers by 1 and multiply the result.

   In this case it will be (4 + 1)*(1 + 1) = 5 * 2 = 10 (the power of 2 is 4 and the power of 3 is 1)

   Therefore, there will 10 factors including 1 and 48. Excluding, these two numbers, you will have 10 – 2 = 8 factors.

3. Product of any two numbers = Product of their HCF and LCM .
   Hence product of two numbers = LCM of the numbers if they are prime to each other.
4. If n is even , n(n+1)(n+2) is divisible by 24If n is any integer , n^2 + 4 is not divisible by 4
5. (m+n)! is divisible by m! * n! .
6. **1) 2^2n-1 is always divisible by 3**
   2^2n-1 = (3-1)^2n -1
   = 3M +1 -1
   = 3M, thus divisible by 3
7. **How many times the digit 0 will appear from 1 to 10000**
   ANS: In 2 digit numbers : 9,
   In 3 digit numbers : 18 + 162 = 180,
   In 4 digit numbers : 2187 + 486 + 27 = 2700,
   total = 9 + 180 + 2700 + 4 = 2893

Equation

1. For a cubic equation $ax^3+bx^2+cx+d=0$sum of the roots = – b/a
   sum of the product of the roots taken two at a time = c/a
   product of the roots = -d/a
1. For a biquadratic equation $ax^4+bx^3+cx^2+dx+e = 0$sum of the roots = – b/a
   sum of the product of the roots taken three at a time = c/a
   sum of the product of the roots taken two at a time = -d/a
   product of the roots = e/a

Coordinates

1. The coordinates of the centroid of a triangle with vertices (a,b) (c,d) (e,f) is((a+c+e)/3 , (b+d+f)/3) .
2. Given the coordinates (a,b) (c,d) (e,f) (g,h) of a parallelogram , the coordinates of the meeting point of the diagonals can be found out by solving for
   [(a+e)/2,(b+f)/2] =[(c+g)/2 , (d+h)/2]

Geometry

Circle

1. The ratio of the radii of the circumcircle and incircle of an equilateral triangle is 2:1 .

Triangle

1. APPOLLONIUS THEOREM: In a triangle, if AD be the median to the side BC, then
   $AB^2 + AC^2 = 2(AD^2 + BD^2)$ or $2(AD^2 + DC^2)$.
2. In an isosceles triangle, the perpendicular from the vertex to the base or the angular bisector from vertex to base bisects the base.
3. In any triangle the angular bisector of an angle bisects the base in the ratio of the other two sides.
4. Some Pythagorean triplets:3,4,5 ($3^2=4+5$)
   5,12,13 ($5^2=12+13$)
   7,24,25 ($7^2=24+25$)
   8,15,17 ($8^2 / 2 = 15+17$)
   9,40,41 ($9^2=40+41$)
   11,60,61 ($11^2=60+61$)
   12,35,37 ($12^2 / 2 = 35+37$)
   16,63,65 ($16^2 /2 = 63+65$)
   20,21,29(EXCEPTION)
5. Let 'a' be the side of an equilateral triangle. then if three circles be drawn inside this triangle touching each other than each's radius = a/ {2*(√ (3) +1)}
6. Area of a triangle
   1/2*base*altitude = 1/2*a*b*sinC = 1/2*b*c*sinA = 1/2*c*a*sinB = root(s*(s-a)*(s-b)*(s-c))
   where s=a+b+c/2
   =a*b*c/(4*R) where R is the CIRCUMRADIUS of the triangle = r*s ,where r is the inradius of the triangle .In any triangle
   a=b*CosC + c*CosB
   b=c*CosA + a*CosC
   c=a*CosB + b*CosA

Quadrilateral

1. The quadrilateral formed by joining the angular bisectors of another quadrilateral is always a rectangle.
2. Let W be any point inside a rectangle ABCD .
   Then
   $WD^2 + WB^2 = WC^2 + WA^2$
3. If any parallelogram can be inscribed in a circle , it must be a rectangle.

4. If a trapezium can be inscribed in a circle it must be an isosceles trapezium (i:e oblique sides equal).
5. For an isosceles trapezium, sum of a pair of opposite sides is equal in length to the sum of the other pair of opposite sides. (i: e AB+CD = AD+BC, taken in order).
6. For any quadrilateral whose diagonals intersect at right angles, the area of the quadrilateral is 0.5*d1*d2, where d1, d2 are the lengths of the diagonals.
7. For a cyclic quadrilateral, area = root((s-a) * (s-b) * (s-c) * (s-d)), where s=(a+b+c+d)/2
8. For a cyclic quadrilateral, the measure of an external angle is equal to the measure of the internal opposite angle.
9. If a quadrilateral circumscribes a circle, the sum of a pair of opposite sides is equal to the sum of the other pair.

Cone

1. For similar cones, ratio of radii = ratio of their bases. The HCF and LCM of two nos. are equal when they are equal .

Area & Volume

1. Area of a parallelogram = base * height
2. Volume of a pyramid = 1/3 * base area * height
3. Area of a trapezium = 1/2 * (sum of parallel sids) * height = median * height
   where median is the line joining the midpoints of the oblique sides.
4. Area of a regular hexagon :

Polygon

1. For any regular polygon , the sum of the exterior angles is equal to 360 degrees
   hence measure of any external angle is equal to 360/n. (where n is the number of sides)

Clock

1. Problems on clocks can be tackled as assuming two runners going round a circle , one 12 times as fast as the other . That is ,
   the minute hand describes 6 degrees /minute
   the hour hand describes 1/2 degrees /minute .

So, **Angle of the hour hand = (30H+M/2)**

Angle of the minute hand= 6M

Thus the minute hand describes 5(1/2) degrees more than the hour hand per minute .

2. The hour and the minute hand meet each other after every 65(5/11) minutes after being together at midnight.
   (This can be derived from the above) .
3. If you are looking for the angle between the hands, Angle = 6M -(30H+M/2)