CAPSTONE

Mathematics: Algebra

Lecture 01

Overview	Next Lecture
◆ Number	◆ Fraction
◆ Factor	◆ Approximation
◆ Multiple	◆ Exponent
	◆ Equation

Name: **Batch:**

IBA Regular Batch

Important Terms:

Integer → পূর্ণসংখ্যা

Sum → যোগ

Difference → বিয়োগ

Product/ Multiplication → গুণ

Quotient → ভাগফল

Remainder → ভাগশেষ

Divide → ভাগ করা

Even → জোড়

Odd → বিজোড়

Prime number → মৌলিক সংখ্যা

Factor → উৎপাদক

Multiple → গুণিতক

Numerator → লব

Denominator → হর

Consecutive → ধারাবাহিক

L.C.M (Least Common Multiple) → ল.সা.গু (লঘিষ্ঠ সাধারণ গুণিতক)

H.C.F/ G.C.F (Greatest / Highest Common Factor) → গ.সা.গু (গরিষ্ঠ সাধারণ গুণনীয়ক)

Elementary concept of number system:

1. Natural numbers: Counting numbers are called natural numbers. Thus, 1, 2, 3, 4, ... are all natural numbers.

2. Whole numbers: All the natural numbers and 0 constitute whole numbers. Thus, 0, 1, 2, 3, 4, ... are all whole numbers.

3. Integers: Integers are 'whole' numbers, such as 0, 1, 2, 3, 4, ... that have no fractional part. Integers can be positive (1, 2, 3, … … …), negative (−1, −2, −3, … … …) or the number 0.

4. Even number: A number exactly divisible by 2 is called an even number. Thus, 0, 2, 4, 6, … … … etc. are all even numbers.

5. Odd number: A number not divisible by 2 is called an odd number. Thus, 1, 3, 5, 7, … … … etc. are all odd numbers.

6. Consecutive numbers: Number that follow each other in order, in a set of consecutive numbers, the difference between consecutive two numbers is same.
Example: 1, 2, 3, 4, … … … … … …; 5, 10, 15, 20, … … … … … …

Test of Divisibility:

2 = If the integer is even or zero.

3 = If the sum of the integers digit is divisible by 3.

4 = If the integer is divisible by 2 TWICE or if the LAST Two digits are divisible by 4.

5 = If the integer ends in 0 or 5.

6 = If the integer is divisible by BOTH 2 and 3.

7 = Take the last digit of the number, double it and subtract the double number from the remaining number. If the result is divisible by 7 (eg. 14, 7, 0, −7, etc.) then the number is divisible by 7.

8 = If the integer is divisible by 2 three times, or if the LAST THREE digits are divisible by 8.

9 = If the sum of the integers DIGITS is divisible by 9.

11 = If the difference between the sum of its digits at odd places and the sum of it digits at even places is either 0 or a number divisible by 11.

Notes:

→ 1 (*and* − 1) are division of every integer.

→ Every integer is a divisor of itself.

→ Every integer is a divisor of 0, except, by convention, 0 itself.

→ Number divisible by 2 are called even and number not divisible by 2 are called odd.

→ Zero is a neutral number.

→ Zero is an even number.

Prime numbers:

A number which has no factor except itself and unity is a prime number.

→ The number 1 is not considered prime, as it has only one factor (itself). 1 is a unique number.

→ The smallest prime number is 2.

→ The only even prime number is 2.

→ The first odd number is 3.

→ The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

→ Prime numbers from 1 to 100 are:

2, 3, 5, 7	11, 13, 17, 19	23, 29	31, 37	41, 43, 47	53, 59	61, 67	71, 73, 79	83, 89	97
4	4	2	2	3	2	2	3	2	1

That is <u>44 22 32 23 21</u> , this is how we can easily remember first 25 prime numbers upto 100.

Example: How many prime numbers are there between 60 and 100?

Answer: That is $2 + 3 + 2 + 1 = 8$ [$61, 67 \rightarrow 2$; $71, 73, 79 \rightarrow 3$; $83, 89 \rightarrow 2$; $97 \rightarrow 1$]

Test for a number to be prime:

Step 1: Find the nearest approximate square root of the number.

Step 2: Divide the number by all the prime numbers lower than the approximate square root number.

Step 3: If divisible by any of them, then it isn't a prime number. If not divisible, then it is a PRIME number.

Example 01: Test, which of the following is a prime number?

i. 137   ii. 319

Solution: i. We know that $(12)^2 > 137$

Prime number less than 12 are 2, 3, 5, 7, 11.

Clearly none of them divides 137.

∴ 137 is a prime number.

ii. We know that $(18)^2 > 319$

Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17. Out of these prime number 11 divides 319 completely.

∴ 319 is not a prime number.

Example Math's:

02. The subtraction between the largest and the smallest number of 4 digits comprised of 0, 1, 2, 3 is:

    A. 3147   B. 2287   C. 2987   D. 2187   E. None of these

Solution: The largest number comprised of $0, 1, 2, 3 = 3210$

The smallest number comprised of $0, 1, 2, 3 = 1023$

Subtraction result $= (3210 - 1023) = 2187$

Answer: D. 2187

03. If one-third of one-fourth of a number is 15, then three-tenth of that number is:

    A. 35   B. 36   C. 45   D. 54   E. None of these

Solution: Let the number be x.

Then, $\frac{1}{3}$ of $\frac{1}{4}$ of x = 15

$\Rightarrow x \times \frac{1}{12} = 15$

$\Rightarrow x = 180$

$$\therefore \frac{3}{10} \text{ of } x = \frac{3}{10} \times 180 = 54$$

Answer: D. 54

04. Which of the following is divisible by 2 & 7?
    A. 524   B. 461   C. 385   D. 350   E. 550

Solution: The number must be divisible by 2 & 7.
461 & 385 are odd. So, they are not divisible by 2.
524 & 550 are not divisible by 7.
So, only option left is 350 which is divisible by 2 & 7.

05. What is the next highest prime number after 67? [BBA 15-16]
    A. 68   B. 69   C. 71   D. 73   E. 76

Solution: 68 is divisible by 2
69 is divisible by 3
71 is divisible by none but 1 & 71.
∴ the answer is 71.

06. What is the value of z in the sequence of 2, 5, 10, 17, 28, z ? [MBA June '15]
A. 15 B. 26 C. 37 D. 41 E. None of these

Solution:
Here, difference between the numbers (2~5) or 3, (5~10) or 5, (10~7) or 7, (17~28) or 11
That is 3, 5, 7, 11, 13, 17, ...
It is a sequence of prime numbers.
So, z will be, $28 + 13 = 41$

Answer: D. 41

Factors & Multiples:

If a number 'a' divides another number 'b' exactly, we say that 'a' is a factor of 'b'. In this case, 'b' is called multiple of 'a'.

Example: 5 divides 10 completely. So, 5 is a factor of 10 and 10 is a multiple of 5.

Highest/ Greatest Common Factor (HCF or GCF):

The greatest common factor of two or more whole numbers is the largest whole number that divides evenly into each of the numbers.

There are two methods of determining of H.C.F:

1. Prime factorization method:

We can determine the H.C.F of 144, 180 and 108 form following process.

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

In prime factorization of the above mentioned three , the common factorization is $2 \times 2 \times 3 \times 3 = 36$

Thus, required H.C.F of 144, 180 and 108 is 36

2. Division method:

```
2 | 108, 144, 180
  2 | 54, 72, 90
    3 | 27, 36, 45
      3 | 9, 12, 15
          3, 4, 5
```

So, H.C.F is = $2 \times 2 \times 3 \times 3 = 36$

Least Common Multiple (LCM):

The least common multiple of two or more non-zero whole numbers are exactly the smallest whole number that is divisible by each of the numbers.

We can determine L.C.M of two given numbers by the following two methods:

1. Prime factorization method:

Suppose we have to find the L.C.M of 12, 16, 8, 30, than

$12 = 2 \times 2 \times 3$

$16 = 2 \times 2 \times 2 \times 2$

$8 = 2 \times 2 \times 2$

$30 = 2 \times 3 \times 5$

Thus, required L.C.M of the given numbers = $2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240$

2. Division method:

We can determine the L.C.M of above mentioned number from the following process:

```
2 | 12, 16, 8, 30
  2 | 6, 8, 4, 15
    2 | 3, 4, 2, 15
      3 | 3, 2, 1, 15
          1, 2, 1, 5
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Thus, required L.C.M of the given number = $2 \times 2 \times 2 \times 3 \times 1 \times 2 \times 1 \times 5 = 240$

Notes:

1. H.C.F of two or more non-zero integers, is the largest positive integer that divides the numbers without a remainder. So, H.C.F can only be a positive integer. It should be obvious as greatest factor of two integers cannot be negative. For example if -3 is a factor of two integer then 3 is also a factor of these two integers.

2. L.C.M of two integers a and b is the smallest positive integer that is a multiple of both a and b.

3. Divisor of a positive integer cannot be more than that integer. From this it follows that the greatest common divisor of two positive integers x and y cannot be more than x or y.

Shortcut formulas:

1. HCF of fraction $= \dfrac{\text{HCF of numerator}}{\text{LCM of denominator}}$

2. LCM of fraction $= \dfrac{\text{LCM of numerator}}{\text{HCF of denominator}}$

3. Product of the numbers $= \text{LCM} \times \text{HCF}$

Example:

06. Find the HCF and LCM of $\dfrac{2}{3}, \dfrac{8}{9}$ and $\dfrac{10}{27}$

Solution: $\text{HCF} = \dfrac{\text{HCF of 2,8,10}}{\text{LCM of 3,9,27}} = \dfrac{2}{27}$

$\text{LCM} = \dfrac{\text{LCM of 2,8,10}}{\text{HCF of 3,9,27}} = \dfrac{40}{3}$

07. The HCF of two numbers is 11 and their LCM is 693. If one of the numbers is 77, find the other.

Solution: Product of two number $= \text{LCM} \times \text{HCF}$

$\Rightarrow 77 \times$ the other number $= 693 \times 11$

$\Rightarrow$ the other number $= \dfrac{11 \times 693}{77} = 99$

Practice Test

01. If n is even, which of the following cannot be odd?

    A. $n + 3$ B. $3n + 1$ C. $n^2 - 1$ D. $2(n + 3)$ E. None of these

02. If the sum of three consecutive integers is odd, then the first and the last integers must be:

    A. odd, even   B. odd, odd   C. even, odd   D. even, even   E. cannot be determined

03. Which of the following cannot be a sum of two prime integers?

    A. 7   B. 19   C. 23   D. 31   E. 55

04. What is the smallest number that should be added to 100 so that it can be completely divided by all the prime numbers between 10 to 15?

    A. 3   B. 6   C. 9   D. 43   E. 11

05. One third of the sum of 13 and a certain number is the same as one more than twice the number. Find out the number.

    A. 6   B. 2   C. 5   D. 3   E. 7

06. How many odd, positive divisor does 540 have?

    A. 6   B. 8   C. 12   D. 15   E. 7

07. If x is divided by 9, the remainder is 3. What will be the remainder when 4x is divided by 9?

    A. 1   B. 2   C. 3   D. Cannot be determined   E. None of these

08. If X is an odd integer and Y is an even integer, which of the following statements is (are) always true? [MBA 01-02]

    I. X + Y is odd   II. X is odd   III. 2X + Y is even

    A. I only   B. II & III only   C. III only   D. I & III only   E. None of these

09. The reciprocal of $\frac{p}{n}$ is $\frac{n}{p}$ and vice versa. Which of the following is the reciprocal of $\frac{2}{\sqrt{18}}$? [BBA 06-07]

    A. $\frac{\sqrt{2}}{3}$ B. $\frac{2}{\sqrt{3}}$ C. $\frac{12}{9}$ D. $\frac{4\sqrt{3}}{3}$ E. None of these

10. If y is between 0 and 1, which of the following increases as y increases? [BBA 09-10]

    I. $(1 - y^2)$ II. $(y - 1)$ III. $\frac{1}{y^2}$

    A. I & II   B. II & III   C. II only   D. I, II and III   E. None of these

11. If a, b and c are odd integers, which of the following expression must be an even integer? [MBA 13-14]

    A. $ab + bc + ca$ B. $a(b + c - 1)$ C. $a^2 - b^2 + c^2$ D. $3(ac - bc)$ E. None of these

12. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

    A. 4   B. 7   C. 9   D. 13   E. Cannot be determined

13. If $\frac{y}{3}, \frac{y}{4}$ and $\frac{y}{7}$ represent integers, then y can be: [MBA 03-04]

    A. 42   B. 56   C. 70 & 84   D. 126   E. None of these

14. The LCM of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the number is:

    A. 28   B. 40   C. 32   D. 64   E. 80

15. If $\frac{x}{y} = \frac{3}{7}$, then which of the following cannot be a possible value of $(y - x)$? [BBA 94-95]

    A. 4   B. 21   C. 24   D. 84   E. None of these

16. When a positive integer x is divided by 5 the remainder is 1. When x is divided by 8, the remainder is 4. What is the smallest positive integer y, such that $(x + y)$ is divisible by 40? [MBA 2017]

    A. 3   B. 4   C. 9   D. 13   E. None of these

17. 3 and 5 are factor of F. From the information we can conclude that- [BBA '94]

    A. 8 is a factor of F   B. F is multiple of 15   C. $F = 3 \times 5$

    D. 15 is a multiple of F   E. 3 & 5 are the only factor of F

18. A department of motor vehicles asks visitor to draw numbered tickets from a dispenser so that they can be served in order by number. Six friends have graduated from truck driving school and go to the department to get commercial driving licenses. They draw tickets and find that their numbers are a set of evenly spaced integers with a range of 10. Which of the following could not be the sum of their numbers?

    A. 1254   B. 1428   C. 3972   D. 4316   E. 8010

19. A red light flashes 3 times per minute and green light flashes 5 times in two minutes at regular intervals. If both light start flashing at the same time, how many times do they flash together in each hour?

    A. 30   B. 24   C. 20   D. 60   E. 40

20. If x and y are integers and $x = 32y + 15$, which of the following must be an odd integer?

    A. xy   B. $x + y$ C. $x + 2y$ D. None of these   E. Cannot be determined

21. A number consists of 3 digits whose sum is 10. The middle digit equal to sum of the other two and the number will be increased by 99, if its digits are reverse. What is the number?

    A. 145   B. 253   C. 353   D. 370   E. 352

22. The sum of three integer is 40. The largest integer is 3 times the middle integer and the smallest integer is 23 less than the largest integer. What is the product of the three integers?

    A. 1104   B. 972   C. 672   D. 294   E. None of these

23. Which one of the following is the minimum value of the sum of two integers whose product is 36? [BBA 15-16]

    A. 37   B. 20   C. 15   D. 12   E. None of these

24. If x, y, z are consecutive positive integers and if $x < y < z$, which of the following must be a positive even integer? [MBA 2016]

    A. $x + y - z$ B. $x - y + z$ C. $yz - x$ D. $(x - z)y$ E. None of these

25. What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30?

    A. 196   B. 630   C. 330   D. 1260   E. 2520

HOME WORK

01. If a and b are odd numbers, then which of the following is even?

    A. $a + b$ B. $a + b + 1$ C. ab D. $ab + 2$ E. None of these

02. If a number exceed another number by 13 and the larger number is 1.5 time the smaller number. Then the smaller number is:

    A. 13   B. 26   C. 31   D. 42   E. None of these

03. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?

    A. 4   B. 2   C. 6   D. 10   E. 16

04. The product of two numbers x and y is twice the sum of the numbers. What is the sum of the reciprocals of x and y?

    A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{3}{7}$ D. $\frac{1}{2}$ E. None of these

05. If m and n are negative integers, which of the following must be true?

    A. $m + n < 0$ B. $m - n < 0$ C. $mn < 0$ D. $\frac{m}{n} < 0$ E. None of these

06. If m and p are positive integer and $m(m + p)$ is even. Which of the following must be true?

    A. If m is odd, then p is odd   B. If m is odd, then p is even

    C. If m is even, then p is even   D. If m is even, then p is odd

    E. None of these

07. The ratio of cupcakes to children at a particular birthday party is 104 to 7. Each child at the birthday party eats exactly x cupcakes (where x is a positive integer) and the adults attending the birthday party do not eat anything. If the number of cupcakes that remain uneaten is less than the number of children at the birthday party, what must be true about the number of uneaten cupcakes?

    I. It is a multiple of 2.   II. It is a multiple of 3.   III. It is a multiple of 7.

    A. I only   B. II only   C. III only   D. I and II only   E. I, II and III

08. A group of n students can be divided into equal groups of 4 with 1 student left over or equal groups of 5 with 3 students left over. What is the sum of the two smallest possible values of n?

    A. 33   B. 46   C. 49   D. 53   E. 86

09. The sum of two number is 22. Five times one number is equal to 6 times the other. The bigger of the two numbers is:

    A. 10   B. 12   C. 22   D. 15   E. 16

10. If 3 less than twice the number is equal to 2 more than 3 times the number, then 5 less than 5 times the number is:

    A. -30 B. -15 C. 0   D. 30   E. 15

11. In a division sum, the remainder is zero. A student mistook the division by 12 instead of 21 and obtained 35 as quotient. What is the correct quotient?

    A. 0   B. 12   C. 13   D. 20   E. 22

12. If n is an integer divisible by 6 but not by 4, then which of the following cannot be an integer?

    A. $\frac{n}{2}$ B. $\frac{n}{3}$ C. $\frac{n}{12}$ D. $\frac{n}{10}$ E. $\frac{n}{30}$

13. How many positive integers less than 100 have a remainder 2 when divided by 13?

    A. 0   B. 4   C. 6   D. 8   E. 10

14. What is the remainder when 6^3 is divided by 8?

    A. 5   B. 3   C. 2   D. 0   E. Cannot be determined

15. What is the maximum number of $1\frac{1}{4}$ or $\frac{5}{4}$ foot pieces of wire that can be cut from a wire that is 24 feet long?

    A. 11   B. 18   C. 19   D. 20   E. 21

16. On a radio tower, a red light flashes every 6 seconds and a blue light flasher every 10 seconds. If both lights flash together at a certain time how many seconds later will both lights flash together the next time?

    A. 6   B. 10   C. 15   D. 30   E. 60

17. If x and y are positive integer and xy is divisible by 4, which of the following must be true?

    A. if x is even then y is odd   B. if x is odd then y is multiple of 4

    C. if $x + y$ is odd then $\frac{y}{x}$ is not an integer   D. if $x + y$ is even then $\frac{x}{y}$ is an integer

    E. None of these

18. Six balls commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 second respectively. In 30 minutes, how many times do they toll together?

    A. 4   B. 10   C. 15   D. 18   E. 16

19. Five consecutive integers are given. If the sum of the first three integer is 24. What is the sum of the last three? [MBA 01-02]

    A. 27   B. 28   C. 29   D. 30   E. None of these

20. If x is a positive even number, then each of following is odd except: [MBA 05-06]

    A. $(x + 3)(x + 5)$ B. $x^2 + 5$ C. $x^2 + 6x + 9$ D. $3x^2 + 4$ E. None of these

21. When positive integer X is divided by positive integer y, the remainder is 9. If $\frac{x}{y} = 96.12$, what is the value of y? [BBA 08-09]

    A. 96   B. 75   C. 48   D. 25   E. 12

22. If y is an integer divisible by 3 but not by 2, then which of the following will never be an integer? [BBA 09-10]

    A. $\frac{y-1}{2}$ B. $\frac{y}{7}$ C. $\frac{y}{24}$ D. $\frac{y^3}{3}$ E. None of these

23. If x and y are two distinct positive integers divisible by 4, then which of the following is necessarily divisible by 8? [MBA '13]

    A. $x + y$ B. $x - y$ C. $x^2 + y^2$ D. $2x + y$ E. None of these

24. If x is decreased from 2 to −2, which of the following must increase? [MBA 2016-17]

    A. $2 + x$ B. $5 - x^2$ C. $1 + \frac{1}{x}$ D. $\frac{1}{x-3}$ E. None of these

25. Kishore can buy apples from two stores: a supermarket that sells apples only in bundles of 4, and a convenience store that sells single, unbundled apples. If Kishore wants to ensure that the total number of apples he buys is a multiple of 5, what is the minimum number of apples he must buy from the convenience store?

    A. 0   B. 1   C. 2   D. 3   E. 4