If n = (33)^43 + (43)^33, what is the unit digit of n?

Solution: Units digit of any exponential expression depends on the exponent / power / index of the unit digit only.

So, here, in 33^43, the units digit depends on 3^43 only.
Now, power cycle of 3 is 4, i.e. every after 4th power the units digit repeats.

In short, to find the unit digit, just divide the exponent 43 by 4. The remainder is 3.

So, now it becomes simply 3^3 and 3^3 = 27. Units digit is 7.
Similarly, units digit of 43^33 is 3^1 = 3.

Finally, units digit of (33)^43 +(43)^33 is simply 7+3 = 10 where 0 is in units place.

Hence, ans. is 0.

*** MORE ABOUT CYCLICITY (Power Cycle):

Let’s start with cyclicity of 2.
2^1=2
2^2=4
2^3=8
2^4=16
2^5=32
2^6=64
2^7=128
2^8=256
2^9=512
And so on…

If we observe, we see that the unit digit of powers of 2 repeats as 2, 4, 8, 6 again 2, 4, 8, 6 again 2, 4, 8, 6 and the process goes on.
So, the cyclicity of power of 2 is said to be 4.

Similarly, the cyclicity of 3 is also 4.

How, let’s see:
3^1=3
3^2=9
3^3=27
3^4=81
3^5=243 [we can get it by multiplying 81 by 3]
3^6= unit digit is 9 [we need not the full number. We need only the unit digit to check. And unit digit can be obtained by multiplying 3 with the unit digit of previous result i.e. 243. 3*3 is 9]
3^7=unit digit is 7 [9*3]
3^8=unit digit is 1 [7*3]
3^9=unit digit is 3 [1*3]
And so on…

So we can see that the unit digit of powers of 3 repeats as 3, 9, 7, 1 again 3, 9, 7, 1 again 3, 9, 7, 1 and the process goes on.

So, the cyclicity of power of 3 is said to be 4.

We can find the cyclicity of any digit by going through this process.

However, if you want the cyclicity of all the the digits, here it is for your ready reference:
Digit ---------- Cyclicity
0 -------------- 1
1 -------------- 1
2 -------------- 4
3 -------------- 4
4 -------------- 2
5 -------------- 1
6 -------------- 1
7 -------------- 4
8 -------------- 4
9 -------------- 2
From 0 to 9, it goes like, 11442-11442