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CS 325 Project 3: Linear Programming

Problem 1

Part A: *Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.*

i. Formulate the problem as a linear program with an objective function and all constraints.

The variables involved are defined as follows:

P_i = Plant i where $0 < i \leq 4$.	cp(i,j) = Cost of shipping from P_i to W_j.
W_j = Warehouse j where $0 < j \leq 3$.	cw(j,k) = Cost of shipping from W_j to R_k.
R_k = Retailer k where $0 < k \leq 7$.	s_i = max supply of plant i.
n = number of plants.	d_k= demand of retailer k.
q = number of warehouses.	$v(i,j)$ = fridges sent from P_i to W_j
m = number of retailers.	$b(j,k)$ = fridges sent from W_j to R_k

The **objective function** is defined as follows:

$Z = min\{\sum_{i=1}^{n}\sum_{j=1}^{q} cp(i,j)v(i,j) + \sum_{j=1}^{q}\sum_{k=1}^{m} cp(j,k)b(j,k)\}$, where Z is sum of travel costs incurred by all the refrigerators.

Constraints:

$\sum_{j=1}^{q} b(j,k) \geq d_k,\ 1 \leq k \leq 7,$ *The sum of fridges sent to retailer R_k must at least equal the demand.*

$\sum_{i=1}^{n} v(i,j) = \sum_{k=1}^{m} b(j,k)\ ,\ 1 \leq j \leq 3,$ *All fridges going in must equal fridges going out from a warehouse j.*

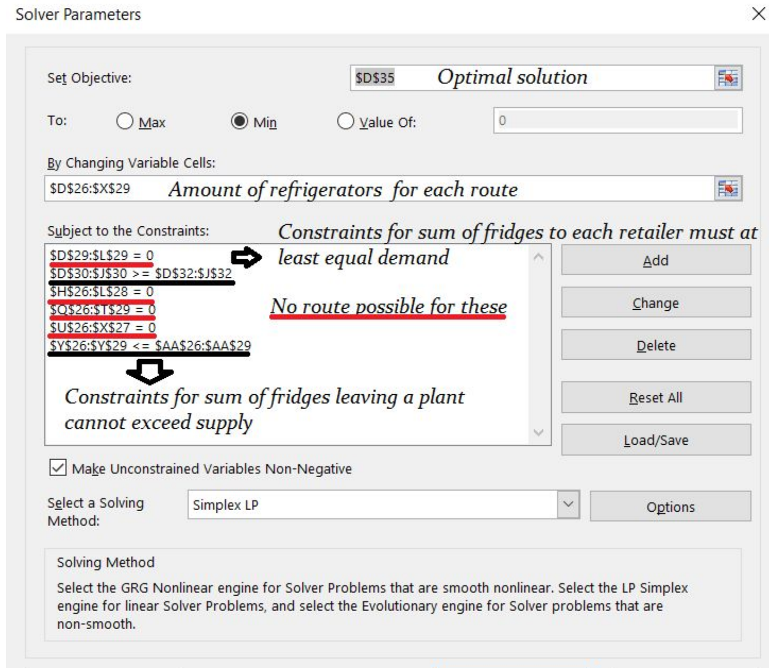
$\sum_{j=1}^{q} v(i,j) \leq s_i\ ,\ 1 \leq i \leq 4,$ *The sum of fridges sent from a given plant cannot exceed that plant's supply*

$v(1,3),\ v(2,3),\ v(4,1) = 0,$ *No fridges can be shipped on these routes.*

$b(1,5),\ b(1,6),\ b(1,7),\ b(2,1),\ b(2,2),\ b(2,7),\ b(3,1),\ b(3,2),\ b(3,3) = 0,$ *No fridges can be shipped on these routes.*

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The excel solver was used for this problem. The optimal solution for minimizing the cost was found to be 17100 dollhairs. Infeasible routes were given a cost of 999 per unit to differentiate them for personal use. However these did not affect the overall solution due to the constraints forcing those routes to 0. Below can be found the spreadsheet used to create the values and constraints set in the excel solver window.



cost	W1	W2	W3
P1	10	15	999
P2	11	8	999
P3	13	8	9
P4	999	14	8

Cost from each plant to warehouse

cost	R1	R2	R3	R4	R5	R6	R7
W1	5	6	7	10	999	999	999
W2	999	999	12	8	10	14	999
W3	999	999	999	14	12	12	6

Cost to each retailer from each warehouse

supply	150	450	250	150			
	R1	R2	R3	R4	R5	R6	R7
demand	100	150	100	200	200	150	100

	w1r1	w1r2	w1r3	w1r4	w1r5	w1r6	w1r7
p1	15	16	17	20	1009	1009	1009
p2	16	17	18	21	1010	1010	1010
p3	18	19	20	23	1012	1012	1012
p4	1004	1005	1006	1009	1998	1998	1998

	w2r1	w2r2	w2r3	w2r4	w2r5	w2r6	w2r7
p1	1009	1014	27	23	25	29	1014
p2	1010	1007	20	16	18	22	1007
p3	1012	1007	20	16	18	22	1007
p4	1998	1013	26	22	24	28	1013

	w3r1	w3r2	w3r3	w3r4	w3r5	w3r6	w3r7
p1	1998	1998	1998	1013	1011	1011	1005
p2	1998	1998	1998	1013	1011	1011	1005
p3	1008	1008	1008	23	21	21	15
p4	1007	1007	1007	22	20	20	14

Calculated cost of each Plant, warehouse, retailer combination

	w1r1	w1r2	w1r3	w1r4	w1r5	w1r6	w1r7
p1	50	0	100	0	0	0	0
p2	50	150	0	0	0	0	0
p3	0	0	0	0	0	0	0
p4	0	0	0	0	0	0	0
	100	150	100	200	200	150	100
	>=	>=	>=	>=	>=	>=	>=
	100	150	100	200	200	150	100

Constraints for demand

	w2r1	w2r2	w2r3	w2r4	w2r5	w2r6	w2r7
p1	0	0	0	0	0	0	0
p2	0	0	0	50	200	0	0
p3	0	0	0	150	0	0	0
p4	0	0	0	0	0	0	0
				All shipments to warehouse 1,2,3			
				350	400	250	
				=	=	=	

Constraints for supply

	w3r1	w3r2	w3r3	w3r4	w3r5	w3r6	w3r7			
p1	0	0	0	0	0	0	0	150	<=	150
p2	0	0	0	0	0	0	0	450	<=	450
p3	0	0	0	0	0	0	100	250	<=	250
p4	0	0	0	0	0	150	0	150	<=	150

Obj Value 17100

iii. What are the optimal shipping routes and minimum cost.

The minimum cost is $17100. The optimum routes are arranged by plant and then warehouse:

- Plant 1 ships 150 to warehouse 1, 50 goes to retailer 1 and 100 to 3.
- Plant 2 ships 200 to warehouse 1, 50 goes to retailer 1 and 150 to retailer 2

- Plant 2 ships 250 to warehouse 2, 50 goes to retailer 4 and 200 to retailer 5
- Plant 3 ships 150 to warehouse 2 and it all goes to retailer 4.
- Plant 3 ships 100 to warehouse 3 and it all goes to retailer 7.
- Plant 4 ships 150 to warehouse 3 and it all goes to retailer 6.

In other words the optimal routes are:

From plant i to warehouse j:
v(1,1), v(2,1), v(2,2), v(3,2), v(3,3), v(4,3)

From warehouse j to retailer k:
b(1,1), b(1,3), b(1,2), b(2,4), b(2,5), b(3,7), b(3,6).

Part B: *Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes.What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?*

Running it in the excel solver resulted, the program was able to satisfy the request of every retailer until the seventh one as shown below (green box). From this detail the following was extracted:

By removing warehouse 2, only plants 3 and 4 are capable of reaching retailers 5, 6, and 7.
Plants 3 and 4 have a combined capacity of 400. However, the combined demand of the retailers is 450.
Therefore a solution is impossible.

	w1r1	w1r2	w1r3	w1r4	w1r5	w1r6	w1r7	w2r1	w2r2	w2r3	w2r4	w2r5	w2r6	w2r7	w3r1	w3r2	w3r3	w3r4	w3r5	w3r6	w3r7				
p1	0	0	0	150	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150	<=		150
p2	100	150	100	50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	400	<=		450
p3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50	150	50	250	<=		250
p4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150	0	0	150	<=		150
	100	150	100	200	200	150	50																		
	>=	>=	>=	>=	>=	>=	>=				All shipments to warehouse 1,2,3														
	100	150	100	200	200	150	100				550	0	400												

Part C: *Instead of closing Warehouse 2 management has decided to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?*

Yes there is a feasible solution with a cost of 18300. The routes are arranged by plant and then warehouse:

- Plant 1 ships 150 to warehouse 1 and it all goes to retailer 4.
- Plant 2 ships 350 to warehouse 1, 100 goes to retailer 1, 150 to retailer 2, 100 to retailer 3.
- Plant 2 ships 100 to warehouse 2, 50 goes to retailer 4, 50 to retailer 5.
- Plant 3 ships 250 to warehouse 3, 150 goes to retailer 6, 100 to retailer 7.
- Plant 4 ships 150 to warehouse 3 and it all goes to retailer 5

	w1r1	w1r2	w1r3	w1r4	w1r5	w1r6	w1r7	w2r1	w2r2	w2r3	w2r4	w2r5	w2r6	w2r7	w3r1	w3r2	w3r3	w3r4	w3r5	w3r6	w3r7				
p1	0	0	0	150	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150	<=		150
p2	100	150	100	0	0	0	0	0	0	0	50	50	0	0	0	0	0	0	0	0	0	450	<=		450
p3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150	100	250	<=		250
p4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150	0	0	150	<=		150
	100	150	100	200	200	150	100																		
	>=	>=	>=	>=	>=	>=	>=				All shipments to warehouse 1,2,3														
	100	150	100	200	200	150	100				500	100	400												
											=	=	=												
Obj Value	18300								For part C			100		*Constraint added for part C, sum of all that passes through warehouse 2 cannot exceed 100.*											
												<=													
												100													

Part D: *Formulate a generalized linear programming model for the transshipment problem. Give the objective function and constraints as mathematical formulas.*

Objective Function:

$Z \ = \ min\{\sum_{i=1}^{n}\sum_{j=1}^{q} cp(i,j)v(i,j) \ + \sum_{j=1}^{q}\sum_{k=1}^{m} cp(j,k)b(j,k) \ \}$, where Z is sum of travel costs incurred by all the refrigerators.

Constraints:

$\sum_{j=1}^{q} b(j,k) \ \geq \ d_k, \ 1 \leq \ k \ \leq 7, \ \ The\ sum\ of\ fridges\ sent\ to\ retailer\ R_k\ must\ at\ least\ equal\ the\ demand.$

$\sum_{i=1}^{n} v(i,j) = \sum_{k=1}^{m} b(j,k) \ , \ 1 \leq \ j \ \leq 3, \ \ All\ fridges\ going\ in\ must\ equal\ fridges\ going\ out\ from\ a\ warehouse\ j.$

$\sum_{j=1}^{q} v(i,j) \ \leq \ s_i \, , \ 1 \leq \ i \ \leq 4, \ The\ sum\ of\ fridges\ sent\ from\ a\ given\ plant\ cannot\ exceed\ that\ plant's\ supply$

Problem 2

Part A*: Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.*

Allow for the following variable definitions:

v_1 = 100 grams of tomato
v_2 = 100 grams of lettuce
v_3 = 100 grams of spinach
v_4 = 100 grams of carrot
v_5 = 100 grams of sunflower seed
v_6 = 100 grams of smoked tofu
v_7 = 100 grams of chickpeas
v_8 = 100 grams of oil

i. Formulate the problem as a linear program with an objective function and all constraints.

Objective Function:
$K = min\{21v_1 + 16v_2 + 40v_3 + 41v_4 + 585v_5 + 120v_6 + 164v_7 + 884v_8\}$, where K is the minimum number of calories in a salad that meets all the nutritional requirements.

Constraints:
$0.85v_1 + 1.62v_2 + 2.86v_3 + 0.93v_4 + 23.4v_5 + 16v_6 + 9v_7 \geq 15$ *grams of protein*
2 *grams of fat* $\leq 0.33v_1 + 0.2v_2 + 0.39v_3 + 0.24v_4 + 48.7v_5 + 5v_6 + 2.6v_7 + 100v_8 \leq 8$ *grams of fat*
$4.64v_1 + 2.37v_2 + 3.63v_3 + 9.58v_4 + 15v_5 + 3v_6 + 27v_7 \geq 4$ *grams of carbohydrates*
$9v_1 + 28v_2 + 65v_3 + 69v_4 + 3.8v_5 + 120v_6 + 78v_7 \geq 200$ *milligrams of sodium*
$(v_1 + v_2) / (v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8) \geq 40\%$ *by mass*
$v_i \geq 0$ *for all integers* $1 \leq i \leq 8$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The following equations were input into LINDO:

```
MIN 21 v1 + 16 v2 + 40 v3 + 41 v4 + 585 v5 + 120 v6 + 164 v7 + 884 v8
ST
        0.85 v1 + 1.62 v2 + 2.86 v3 + 0.93 v4 + 23.4 v5 + 16.0 v6 + 9.00 v7 > 15
        0.33 v1 + 0.20 v2 + 0.39 v3 + 0.24 v4 + 48.7 v5 + 5.00 v6 + 2.60 v7 + 100 v8 > 2
        0.33 v1 + 0.20 v2 + 0.39 v3 + 0.24 v4 + 48.7 v5 + 5.00 v6 + 2.60 v7 + 100 v8 < 8
        4.64 v1 + 2.37 v2 + 3.63 v3 + 9.58 v4 + 15.0 v5 + 3.00 v6 + 27.0 v7 > 4
        9.00 v1 + 28.0 v2 + 65.0 v3 + 69.0 v4 + 3.80 v5 + 120  v6 + 78.0 v7 < 200
         0.4 v1 - 0.4 v2 - 0.4 v3 - 0.4 v4 - 0.4 v5 - 0.4 v6 - 0.4 v7 - 0.4 v8 + v2 + v3 > 0
        v1  > 0
        v2  > 0
        v3  > 0
        v4  > 0
        v5  > 0
        v6  > 0
        v7  > 0
        v8  > 0
END
```

The optimal solution generated by LINDO:

Variable (ingredient)	**Value (grams)**
v1 (tomato)	$100 \times 0 = 0$
v2 (lettuce)	$100 \times 0.58548 = 58.548$

v3 (spinach)	$100 \times 0 = 0$
v4 (carrot)	$100 \times 0 = 0$
v5 (sunflower seeds)	$100 \times 0 = 0$
v6 (smoked tofu)	$100 \times 0.87822 = 87.822$
v7 (chickpeas)	$100 \times 0 = 0$
v8 (oil)	$100 \times 0 = 0$

The objective function returns a value of 114.7541. This represents the minimum number of calories needed to create a salad, a concoction containing 57 grams of lettuce and 87 grams of smoked tofu, that meets the nutritional requirements.

iii. What is the cost of the low calorie salad?

Since there are ≈ 57 grams of lettuce and ≈ 88 grams of smoked tofu in the salad, and both numbers represent a equivalent percentage of 100, we can sum the product of each weight with its price and derive the cost of the salad.
$(0.57 \times 0.75) + (0.88 \times 2.15) \approx \$2.32$

Part B: *Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately, some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.*

Allow for the following variable definitions:
v_1 = 100 grams of tomato
v_2 = 100 grams of lettuce
v_3 = 100 grams of spinach
v_4 = 100 grams of carrot
v_5 = 100 grams of sunflower seed
v_6 = 100 grams of smoked tofu
v_7 = 100 grams of chickpeas
v_8 = 100 grams of oil

i. Formulate the problem as a linear program with an objective function and all constraints.

Objective Function:
$C = min\{v_1 + 0.75v_2 + 0.5v_3 + 0.5v_4 + 0.45v_5 + 2.15v_6 + 0.95v_7 + 2v_8\}$, where C is the minimum cost of a salad that meets all the nutritional requirements.

Constraints:

$0.85v_1 + 1.62v_2 + 2.86v_3 + 0.93v_4 + 23.4v_5 + 16v_6 + 9v_7 \geq 15$ *grams of protein*

2 *grams of fat* $\leq 0.33v_1 + 0.2v_2 + 0.39v_3 + 0.24v_4 + 48.7v_5 + 5v_6 + 2.6v_7 + 100v_8 \leq 8$ *grams of fat*

$4.64v_1 + 2.37v_2 + 3.63v_3 + 9.58v_4 + 15v_5 + 3v_6 + 27v_7 \geq 4$ *grams of carbohydrates*

$9v_1 + 28v_2 + 65v_3 + 69v_4 + 3.8v_5 + 120v_6 + 78v_7 \geq 200$ *milligrams of sodium*

$(v_1 + v_2) / (v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8) \geq 40\%$ *by mass*

$v_i \geq 0$ *for all* $1 \leq i \leq 8$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The following equations were input into LINDO:

```
MIN v1 + 0.75 v2 + 0.5 v3 + 0.5 v4 + 0.45 v5 + 2.15 v6 + 0.95 v7 + 2 v8
ST
        0.85 v1 + 1.62 v2 + 2.86 v3 + 0.93 v4 + 23.4 v5 + 16.0 v6 + 9.00 v7 > 15
        0.33 v1 + 0.20 v2 + 0.39 v3 + 0.24 v4 + 48.7 v5 + 5.00 v6 + 2.60 v7 + 100 v8 > 2
        0.33 v1 + 0.20 v2 + 0.39 v3 + 0.24 v4 + 48.7 v5 + 5.00 v6 + 2.60 v7 + 100 v8 < 8
        4.64 v1 + 2.37 v2 + 3.63 v3 + 9.58 v4 + 15.0 v5 + 3.00 v6 + 27.0 v7 > 4
        9.00 v1 + 28.0 v2 + 65.0 v3 + 69.0 v4 + 3.80 v5 + 120  v6 + 78.0 v7 < 200
         0.4 v1 - 0.4 v2 - 0.4 v3 - 0.4 v4 - 0.4 v5 - 0.4 v6 - 0.4 v7 - 0.4 v8 + v2 + v3 > 0
        v1  > 0
        v2  > 0
        v3  > 0
        v4  > 0
        v5  > 0
        v6  > 0
        v7  > 0
        v8  > 0
END
```

The optimal solution generated by LINDO:

Variable (ingredient)	**Value (grams)**
v1 (tomato)	$100 \times 0 = 0$
v2 (lettuce)	$100 \times 0 = 0$
v3 (spinach)	$100 \times 0.832298 = 83.2298$
v4 (carrot)	$100 \times 0 = 0$

v5 (sunflower seeds)	$100 \times 0.096083 = 9.6083$
v6 (smoked tofu)	$100 \times 0 = 0$
v7 (chickpeas)	$100 \times 1.152364 = 115.2364$
v8 (oil)	$100 \times 0 = 0$

The objective function returns a value of 1.554133 which is $\approx$ \$1.55 . This represents the minimum cost of a salad that meets the nutritional requirements. It contains 83 grams of lettuce, 9.6 grams of sunflower seeds and 115 grams of chickpeas.

iii. How many calories are in the low cost salad?

The number of calories can be calculated by get the summing the products of each weight and its associated caloric value.
$(0.832298 \times 40) + (0.096083 \times 585) + (1.152364 \times 164) \approx 278$ calories.

Part C: *Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However if she can advertise that the salad has under 250 calories then she may be able to sell more.*

i. Suggest some possible ways that she can select a combination of ingredients that is "near optimal" for both objectives. This is a type of multi-objective optimization.

There seem to be three approaches Veronica can take to optimize her salad recipe:

1. She could derive the minimum number of ingredients by gram that her salad can contain while meeting all the nutritional requirements, with a cost no greater than $2.00 and a calorie count no more than 250.
2. She could deduce the minimum cost of a salad that meets all the nutritional requirements and is no more than 250 calories.
3. She could determine the minimum number of calories in a salad that meets all the nutritional requirements and costs no more than $2.00 to produce.

ii. What combination of ingredient would you suggest and what is the associated cost and calorie.

If she wants to maximize her profits, then she needs to find a salad recipe that is under 250 calories and is cheap to produce. In this case, her best bet is to take the second approach. Using LINDO to derive the optimal recipe we get:

Variable (ingredient)	**Value (grams)**
v1 (tomato)	$100 \times 0 = 0$
v2 (lettuce)	$100 \times 0 = 0$
v3 (spinach)	$100 \times 0.761996 = 76.1996$
v4 (carrot)	$100 \times 0 = 0$
v5 (sunflower seeds)	$100 \times 0.09383 = 9.383$
v6 (smoked tofu)	$100 \times 0.168941 = 16.8941$
v7 (chickpeas)	$100 \times 0.880222 = 88.0222$
v8 (oil)	$100 \times 0 = 0$

If we round the previous values up, Veronica's optimal salad recipe will require 76 grams of spinach, 9 grams of sunflower seeds, 17 grams of smoked tofu and 88 grams of chickpeas. Such a salad will contain

$$(0.76 \times 40) + (0.09 \times 585) + (0.17 \times 120) + (0.88 \times 164) = 247.77 \textit{ calories}$$

and the production cost will be

$$(0.76 \times 0.5) + (0.09 \times 0.45) + (0.17 \times 2.15) + (0.88 \times 0.95) = \$1.62$$

If she chooses to sell this salad at a price of $5.00, then she will make a profit of $3.38 for ever salad sold. Furthermore, she will make more sales since the salad contains less than 250 calories.

iii. Note: There is not one "right" answer. Discuss how you derived your solution.

The previous solution was derived by applying each optimization approach and comparing the results. Since each approach produces a salad that meets all the necessary constraints, the "best" salad will be the one that creates the most profit. Below is a table of the salads created using each approach (LINDO):

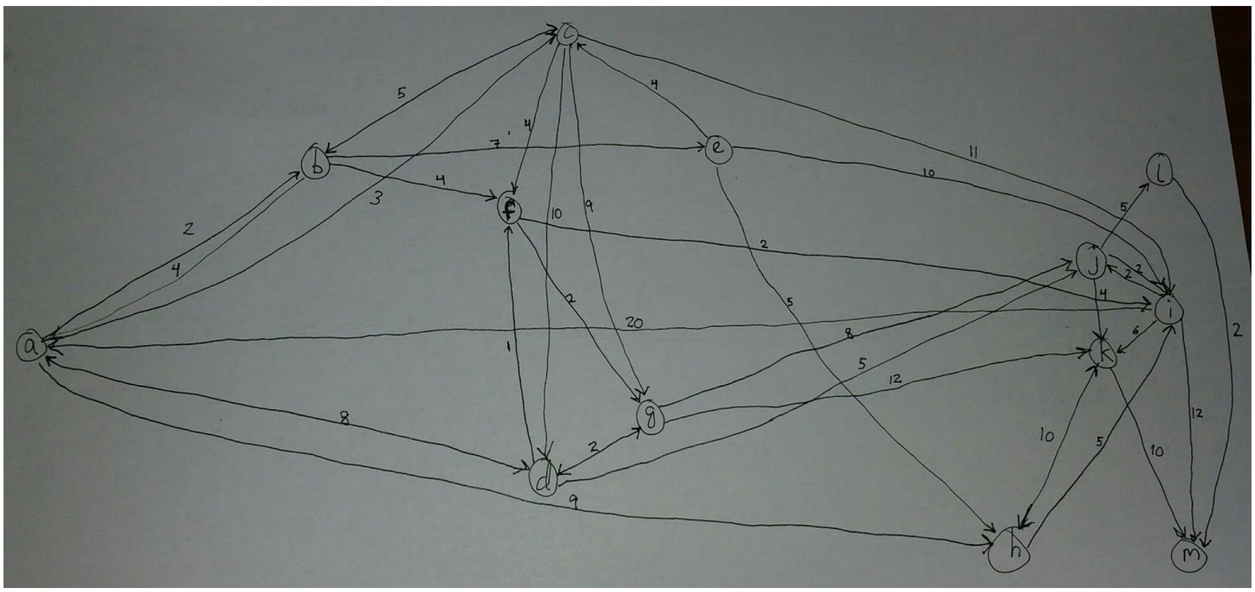
Approach	**Salad Description (grams rounded up)**	**Calories**	**Cost**	**Profit**
1	53g spinach, 9g sunflower seeds, 72g smoked tofu	160.52 kcal	$1.85	$3.15
2	76g spinach, 9g sunflower seeds, 17g smoked tofu, 88g chickpeas	247.77 kcal	$1.62	$3.38

3	55g spinach, 3g sunflower seeds, 80g smoked tofu	135.55 kcal	$2.00	$3.00

Clearly, the recipe derived from approach 2 will net Veronica the largest profits.

Problem 3

Rough sketch of the directed graph:



Part A: *What are the lengths of the shortest paths from vertex a to all other vertices?*

The problem:

Maximize d(x)

Subject to d(a) = 0

d(v)-d(u) <= l(u→v) for every edge u→v

Code (replace “max db” with “max dc”, “max dd”, etc to find the distance from a to each vertex):

```
max db
ST
   da = 0
   db - da <= 2
   dc - da <= 3
   dd - da <= 8
   dh - da <= 9
   da - db <= 4
   dc - db <= 5
```

de - db <= 7
df - db <= 4
dd - dc <= 10
db - dc <= 5
dg - dc <= 9
di - dc <= 11
df - dc <= 4
da - dd <= 8
dg - dd <= 2
dj - dd <= 5
df - dd <= 1
dh - de <= 5
dc - de <= 4
di - de <= 10
di - df <= 2
dg - df <= 2
dd - dg <= 2
dj - dg <= 8
dk - dg <= 12
di - dh <= 5
dk - dh <= 10
da - di <= 20
dk - di <= 6
dj - di <= 2
dm - di <= 12
di - dj <= 2
dk - dj <= 4
dl - dj <= 5
dh - dk <= 10
dm - dk <= 10
dm - dl <= 2
END

Shortest lengths from vertex a to each other vertex:

Start Vertex	End Vertex	Distance
a	a	0

a	b	2
a	c	3
a	d	8
a	e	9
a	f	6
a	g	8
a	h	9
a	i	8
a	j	10
a	k	14
a	l	15
a	m	17

Part B: *If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a)?*

An "UNBOUNDED" error will occur, and the distance will approach infinity.

Part C: *What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?*

To find a path from all vertices to m using one linear program, we can reverse the directions of the edges that represent the distances between vertices. Swap v and u for every statement after ST (such that). This reverses the direction the path travels, going from m to each vertex, but this method still gives the correct distances because although the directions are reversed, the order of visited vertices and their respective distances stay the same.

Example:
Maximize d(x)
Subject to d(m) = 0
    d(u)-d(v) <= l(v⟶u) for every edge v⟶u

Code (replace "max da" with "max db", "max dc", etc to find the distance from each vertex to m):
max da

```
ST
        dm = 0
        da - db <= 2
        da - dc <= 3
        da - dd <= 8
        da - dh <= 9
        db - da <= 4
        db - dc <= 5
        db - de <= 7
        db - df <= 4
        dc - dd <= 10
        dc - db <= 5
        dc - dg <= 9
        dc - di <= 11
        dc - df <= 4
        dd - da <= 8
        dd - dg <= 2
        dd - dj <= 5
        dd - df <= 1
        de - dh <= 5
        de - dc <= 4
        de - di <= 10
        df - di <= 2
        df - dg <= 2
        dg - dd <= 2
        dg - dj <= 8
        dg - dk <= 12
        dh - di <= 5
        dh - dk <= 10
        di - da <= 20
        di - dk <= 6
        di - dj <= 2
        di - dm <= 12
        dj - di <= 2
        dj - dk <= 4
        dj - dl <= 5
        dk - dh <= 10
        dk - dm <= 10
        dl - dm <= 2
END
```

Shortest lengths from each vertex to vertex m:

Start Vertex	End Vertex	Distance
a	m	17
b	m	15
c	m	15
d	m	12
e	m	19
f	m	11
g	m	14
h	m	14
i	m	9
j	m	7
k	m	10
l	m	2
m	m	0

Part D: *Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).*

To find the length of the shortest path from any vertex to vertex y that passes through vertex i, you can use two linear programs. One linear program finds the distance from x to i, and the second linear program finds the distance from i to y. The results of each linear program can be combined to give us the total distance.

Code (replace da=0 with db=0, dc=0, etc to find the shortest length from each vertex to i):
From a to i:

```
max di
ST
	da = 0
	db - da <= 2
```

```
    dc - da <= 3
    dd - da <= 8
    dh - da <= 9
    da - db <= 4
    dc - db <= 5
    de - db <= 7
    df - db <= 4
    dd - dc <= 10
    db - dc <= 5
    dg - dc <= 9
    di - dc <= 11
    df - dc <= 4
    da - dd <= 8
    dg - dd <= 2
    dj - dd <= 5
    df - dd <= 1
    dh - de <= 5
    dc - de <= 4
    di - de <= 10
    di - df <= 2
    dg - df <= 2
    dd - dg <= 2
    dj - dg <= 8
    dk - dg <= 12
    di - dh <= 5
    dk - dh <= 10
    da - di <= 20
    dk - di <= 6
    dj - di <= 2
    dm - di <= 12
    di - dj <= 2
    dk - dj <= 4
    dl - dj <= 5
    dh - dk <= 10
    dm - dk <= 10
    dm - dl <= 2
END
```

Code (replace max da with max db, max dc, etc to find the shortest length from i to each vertex):

From i to a:

```
max da
ST
    di = 0
```

```
        db - da <= 2
        dc - da <= 3
        dd - da <= 8
        dh - da <= 9
        da - db <= 4
        dc - db <= 5
        de - db <= 7
        df - db <= 4
        dd - dc <= 10
        db - dc <= 5
        dg - dc <= 9
        di - dc <= 11
        df - dc <= 4
        da - dd <= 8
        dg - dd <= 2
        dj - dd <= 5
        df - dd <= 1
        dh - de <= 5
        dc - de <= 4
        di - de <= 10
        di - df <= 2
        dg - df <= 2
        dd - dg <= 2
        dj - dg <= 8
        dk - dg <= 12
        di - dh <= 5
        dk - dh <= 10
        da - di <= 20
        dk - di <= 6
        dj - di <= 2
        dm - di <= 12
        di - dj <= 2
        dk - dj <= 4
        dl - dj <= 5
        dh - dk <= 10
        dm - dk <= 10
        dm - dl <= 2
END
```

We combine the distance from the starting vertex to the middle vertex i with the distance from vertex i to the end vertex, giving us the total distance as shown in the tables below.

Start Vertex	Mid Vertex	Distance Start to Mid	End Vertex	Distance Mid to End	Total Distance
a	i	8	a	20	28
a	i	8	b	22	30
a	i	8	c	23	31
a	i	8	d	28	36
a	i	8	e	29	37
a	i	8	f	26	34
a	i	8	g	28	36
a	i	8	h	16	24
a	i	8	i	0	8
a	i	8	j	2	10
a	i	8	k	6	14
a	i	8	l	7	15
a	i	8	m	9	17

Start Vertex	Mid Vertex	Distance Start to Mid	End Vertex	Distance Mid to End	Total Distance
b	i	6	a	20	26
b	i	6	b	22	28
b	i	6	c	23	29
b	i	6	d	28	34
b	i	6	e	29	35
b	i	6	f	26	32

b	i	6	g	28	34
b	i	6	h	16	22
b	i	6	i	0	6
b	i	6	j	2	8
b	i	6	k	6	12
b	i	6	l	7	13
b	i	6	m	9	15

Start Vertex	Mid Vertex	Distance Start to Mid	End Vertex	Distance Mid to End	Total Distance
c	i	6	a	20	26
c	i	6	b	22	28
c	i	6	c	23	29
c	i	6	d	28	34
c	i	6	e	29	35
c	i	6	f	26	32
c	i	6	g	28	34
c	i	6	h	16	22
c	i	6	i	0	6
c	i	6	j	2	8
c	i	6	k	6	12
c	i	6	l	7	13
c	i	6	m	9	15

Start Vertex	Mid Vertex	Distance Start to Mid	End Vertex	Distance Mid to End	Total Distance
d	i	3	a	20	23
d	i	3	b	22	25
d	i	3	c	23	26
d	i	3	d	28	31
d	i	3	e	29	32
d	i	3	f	26	29
d	i	3	g	28	31
d	i	3	h	16	19
d	i	3	i	0	3
d	i	3	j	2	5
d	i	3	k	6	9
d	i	3	l	7	10
d	i	3	m	9	12

Start Vertex	Mid Vertex	Distance Start to Mid	End Vertex	Distance Mid to End	Total Distance
e	i	10	a	20	30
e	i	10	b	22	32
e	i	10	c	23	33
e	i	10	d	28	38
e	i	10	e	29	39
e	i	10	f	26	36
e	i	10	g	28	38

e	i	10	h	16	26
e	i	10	i	0	10
e	i	10	j	2	12
e	i	10	k	6	16
e	i	10	l	7	17
e	i	10	m	9	19

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**
f	i	2	a	20	22
f	i	2	b	22	24
f	i	2	c	23	25
f	i	2	d	28	30
f	i	2	e	29	31
f	i	2	f	26	28
f	i	2	g	28	30
f	i	2	h	16	18
f	i	2	i	0	2
f	i	2	j	2	4
f	i	2	k	6	8
f	i	2	l	7	9
f	i	2	m	9	11

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**

g	i	5	a	20	25
g	i	5	b	22	27
g	i	5	c	23	28
g	i	5	d	28	33
g	i	5	e	29	34
g	i	5	f	26	31
g	i	5	g	28	33
g	i	5	h	16	21
g	i	5	i	0	5
g	i	5	j	2	7
g	i	5	k	6	11
g	i	5	l	7	12
g	i	5	m	9	14

Start Vertex	Mid Vertex	Distance Start to Mid	End Vertex	Distance Mid to End	Total Distance
h	i	5	a	20	25
h	i	5	b	22	27
h	i	5	c	23	28
h	i	5	d	28	33
h	i	5	e	29	34
h	i	5	f	26	31
h	i	5	g	28	33
h	i	5	h	16	21

h	i	5	i	0	5
h	i	5	j	2	7
h	i	5	k	6	11
h	i	5	l	7	12
h	i	5	m	9	14

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**
i	i	0	a	20	20
i	i	0	b	22	22
i	i	0	c	23	23
i	i	0	d	28	28
i	i	0	e	29	29
i	i	0	f	26	26
i	i	0	g	28	28
i	i	0	h	16	16
i	i	0	i	0	0
i	i	0	j	2	2
i	i	0	k	6	6
i	i	0	l	7	7
i	i	0	m	9	9

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**
j	i	2	a	20	22

j	i	2	b	22	24
j	i	2	c	23	25
j	i	2	d	28	30
j	i	2	e	29	31
j	i	2	f	26	28
j	i	2	g	28	30
j	i	2	h	16	18
j	i	2	i	0	2
j	i	2	j	2	4
j	i	2	k	6	8
j	i	2	l	7	9
j	i	2	m	9	11

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**
k	i	15	a	20	35
k	i	15	b	22	37
k	i	15	c	23	38
k	i	15	d	28	43
k	i	15	e	29	44
k	i	15	f	26	41
k	i	15	g	28	43
k	i	15	h	16	31
k	i	15	i	0	15

k	i	15	j	2	17
k	i	15	k	6	21
k	i	15	l	7	22
k	i	15	m	9	24

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**
l	i	Unbounded	a	20	Unbounded
l	i	Unbounded	b	22	Unbounded
l	i	Unbounded	c	23	Unbounded
l	i	Unbounded	d	28	Unbounded
l	i	Unbounded	e	29	Unbounded
l	i	Unbounded	f	26	Unbounded
l	i	Unbounded	g	28	Unbounded
l	i	Unbounded	h	16	Unbounded
l	i	Unbounded	i	0	Unbounded
l	i	Unbounded	j	2	Unbounded
l	i	Unbounded	k	6	Unbounded
l	i	Unbounded	l	7	Unbounded
l	i	Unbounded	m	9	Unbounded

Start Vertex	**Mid Vertex**	**Distance Start to Mid**	**End Vertex**	**Distance Mid to End**	**Total Distance**
m	i	Unbounded	a	20	Unbounded

m	i	Unbounded	b	22	Unbounded
m	i	Unbounded	c	23	Unbounded
m	i	Unbounded	d	28	Unbounded
m	i	Unbounded	e	29	Unbounded
m	i	Unbounded	f	26	Unbounded
m	i	Unbounded	g	28	Unbounded
m	i	Unbounded	h	16	Unbounded
m	i	Unbounded	i	0	Unbounded
m	i	Unbounded	j	2	Unbounded
m	i	Unbounded	k	6	Unbounded
m	i	Unbounded	l	7	Unbounded
m	i	Unbounded	m	9	Unbounded