Assignment

Name :- Tanishq Chauhan

Roll No. :- 21C3184

Subject :- Applied Mathematics - III (ACR3C1)

Semester :- 3rd Semester

Branch :- Computer Science & Engineering (CS-B)

Chauhan

Tanishq Chauhan
21C3184
CS-B

Applied Mathematics - III (ACR3C1)

*** Answer of Q. No. 1 (a)**

We have the equation,

$$x - \ln(x+1) - \cos x = 0$$

$$x_{k+1} = \frac{a_k \cdot f(b_k) - b_k f(a_k)}{f(b_k) - f(a_k)}$$

$f(0) = -1$, $f(1) = -0.2334$, $f(2) = 1.3175$

The interval will be $[1, 2]$

So, $\boxed{a = 1}$ and $\boxed{b = 2}$

k	a_k	b_k	$f(a_k)$	$f(b_k)$	x_{k+1}	$f(x_{k+1})$
0	1	2	-0.2334	1.3175	1.1505	-0.0232
1	1	1.1505	-0.2334	-0.0232	1.1671	-0.0009
2	1.1671	1.1505	0.0009	-0.0232	1.1664	-0.0001
3	1.1671	1.1664	0.0009	-0.0001	1.1664	-0.0001

Hence, the required ~~negative~~ root of the given equation is $x = 1.1664$.

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No. 1 (b)

We have,

$$-3x - 5y + 7z = 1,$$
$$6x + 4y + 2z = 7,$$
$$2x - 6y + 3z = 5$$

By partial Pivoting

$$6x + 4y + 2z = 7$$
$$2x - 6y + 3z = 5$$
$$-3x - 5y + 7z = 1$$

$$x = \frac{1}{6}\left[7 - 4y - 2z\right], \quad y = \frac{1}{6}\left[2x + 3z - 5\right]$$

$$z = \frac{1}{7}\left[1 + 3x + 5y\right]$$

Let $x^{(0)} = y^{(0)} = z^{(0)} = 0$

K	$x^{(K)}$	$y^{(K)}$	$z^{(K)}$	$x^{(K+1)}$	$y^{(K+1)}$	$z^{(K+1)}$
0	0	0	0	1.1667	-0.8333	0.1428
1	1.1667	-0.8333	0.1428	1.6746	-0.2037	0.7150
2	1.6746	-0.2037	0.7150	1.0641	-0.1211	0.5124
3	1.0641	-0.1211	0.5124	1.0766	-0.2183	0.4483
4	1.0766	-0.2183	0.4483	1.1628	-0.2216	0.4829
5	1.1628	-0.2216	0.4829	1.1534	-0.2074	0.4890

Tanishq Chauhan
21(3184
CS-B

Chauhan

So,

$$x = 1.1534$$

$$y = -0.2074$$

$$z = 0.4890$$

Tanishq Chauhan
2103184
CS-B

Chauhan

Date
Page 04

Answer of Q. No. 1 (c)

$x = 0.60$, $n = 2$, $\frac{dy}{dx} = \frac{xy^2 + \ln x}{xe^y}$;

$y(0.30) = 0.65$

$x_0 - 0.30$, $y_0 - 0.65$, $n = 2$, $f(x,y) = \frac{xy^2 + \ln x}{xe^y}$

$h = \frac{0.60 - 0.30}{2} = 0.15$

$x_0 = 0.30$, $x_1 = 0.45$, $x_2 = 0.60$

$y_0 = 0.65$, $y_1 = ?$, $y_3 = ?$

$K_1 = hf(x_{n-1}, y_{n-1})$

$K_2 = hf\left(x_{n-1} + \frac{h}{2}, y_{n-1} + \frac{K_1}{2}\right)$

$K_3 = hf\left(x_{n-1} + \frac{h}{2}, y_{n-1} + \frac{K_2}{2}\right)$

$K_4 = hf\left(x_{n-1} + h, y_{n-1} + K_3\right)$

$K = \frac{K + 2K_2 + 2K_3 + K_4}{6}$

$y_n = y_{n-1} + K$

Tanishq Chauhan
21C3184
CS-B

T Chauhan

Step 1

$n=1, \quad K_1 = 0.15 \; f(0.30, 0.65)$

$$= \frac{0.15 \times (0.30)(0.65)^2 + \ln(0.30)}{(0.30)\, e^{0.65}}$$

$$= \frac{-0.0211}{0.5747}$$

$$= -0.2812$$

$$K2 = 0.15 \left[f(0.375, +0.5094) \right]$$

$$= 0.15 \left[\frac{0.0973 - 0.9808}{0.6241} \right]$$

$$= 0.15 \left[\frac{-0.8835}{0.6241} \right]$$

$$= -0.2123$$

$$K3 = 0.15 \left[f(0.375, 0.5438) \right]$$

$$= 0.15 \left[\frac{-0.8699}{0.6450} \right]$$

$$= -0.2020$$

Tanishq Chauhan
21C3184
CS-B

Chauhan

$K_4 = 0.15 \left[f(0.45, 0.448) \right]$

$= 0.15 \left[\dfrac{0.0903 - 0.7985}{0.7043} \right]$

$= -0.1508$

$K = \dfrac{-0.2812 - 0.4246 - 0.4040 - 0.1508}{6}$

$K = -0.2101$

$y_1 = 0.65 - 0.2101 = 0.4399$

$\boxed{y_1 = 0.4399}$

Step 2

$n = 2 \quad K_1 = 0.15 f(0.45, 0.4399)$

$K_1 = 0.15 \left[\dfrac{-0.7114}{0.6986} \right]$

$K_1 = -0.1527$

$K_2 = 0.15 f(0.525, 0.3636)$

Tanishq Chauhan
21C3184
CS-B

Chauhan

$$K_2 = 0.15 \left[\frac{-0.5749}{1.2103} \right]$$

$$K_2 = -0.1142$$

$$K_3 = 0.15\, f(0.525, 0.3828)$$

$$K_3 = 0.15 \left[\frac{-0.5674}{0.7698} \right]$$

$$K_3 = -0.1106$$

$$K_4 = 0.15\, f(0.6, 0.3293)$$

$$K_4 = 0.15 \left[\frac{-0.4457}{0.8334} \right]$$

$$K_4 = 0.0802$$

$$K = \frac{-0.1527 - 0.2284 - 0.2212 - 0.0802}{6}$$

$$K = -0.1138$$

$$y_2 = 0.4399 - 0.1138$$

$$\boxed{y_2 = 0.3261}$$

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No. 2 (a)

W	50	70	110	130	160
P	12	15	21	23	26

W = 105 kg

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x-x_3)(x_0-x_4)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$x = 105$

$$f(x) = \frac{(105-70)(105-110)(105-130)(105-160)}{(50-70)(50-110)(50-130)(50-160)} \times 12 +$$

$$\frac{(105-50)(105-110)(105-130)(105-160)}{(70-50)(70-110)(70-130)(70-160)} \times 15 +$$

Tanishq Chauhan
21C3184
CS-B

$$\frac{(105-50)(105-70)(105-130)(105-160)}{(110-50)(110-70)(110-130)(110-160)} \times 21 +$$

$$\frac{(105-50)(105-70)(105-110)(105-160)}{(130-50)(130-70)(130-110)(130-160)} \times 23 +$$

$$\frac{(105-50)(105-70)(105-110)(105-130)}{(160-50)(160-70)(160-110)(160-130)} \times 26 -$$

$$f(n) = \frac{-2887500}{10560000} + \frac{(-5671875)}{(-5280000)} + \frac{(55584375)}{2400000} + \frac{12175625}{-2880000}$$

$$+ \frac{6256250}{14850000}$$

$$f(n) = -0.2734 + 1.0742 + 23.1602 - 4.2276 + \cancel{20}\,0.4213$$

$$= 20.1547$$

The pull P required to lift a load $W = 105$ kg is $\boxed{P = 20.1547}$

Tanishq Chauhan
21CS3184
CS-B

* Answer of Q. No. 2 (b)

t	40	50	60	70	80
x	1069.1	1063.6	1058.2	1052.7	1049.3

Find $\Rightarrow \dfrac{dx}{dt}$ at $t = 76$

$t_p = 76$, $t_5 = 80$

$p = \dfrac{x_p - x_0}{h} = \dfrac{76 - 80}{10} = -0.4$

Difference Table :-

t	x	1st Diff	2nd Diff	3rd Diff	4th Diff
40	1069.1				
		-5.5			
50	1063.6		0.1		
		-5.4		-0.2	
60	1058.2		-0.1		1.8 $\nabla^4 y_5$
		-5.5		2.0 $\nabla^3 y_5$	
70	1052.7		2.1 $\nabla^2 y_5$		
		-3.4 ∇y_5			
80 (t_5)	1049.3 (y_5)				

Tanishq Chauhan
21C3184
CS - B

Using Newton's Backward Interpolation Formula

$$f'(t_5+ph) = \frac{1}{h}\left[\nabla y_5 + \left(\frac{2p+1}{2!}\right)\nabla^2 y_5 + \left(\frac{3p^2+6p+2}{3!}\right)\nabla^3 y_5 + \left(\frac{4p^3+18p^2+22p+6}{4!}\right)\nabla^4 y_5\right]$$

$$= \frac{1}{10}\left[-3.4 + \left(\frac{1-0.8}{2}\right)2.1 + \left(\frac{0.08}{6}\right)2 + \left(\frac{-0.256+2.88-8.8+6}{24}\right)1.8\right]$$

$$= \frac{1}{10}\left[-3.4 + 0.21 + 0.027 - 0.013\right]$$

$$= -0.3176$$

The rate of change of latent heat at $t=76$ is -0.3176.

Tanishq Chauhan
21C3184
CS-B

Chauhan

12

* Answer of Q. No. 2 (c)

h (ft)	10	11	12	13	14	15	16
A (sq. ft)	950	1070	1200	1350	1530	1575	1615

$$\frac{dh}{dt} = -48 \cdot \sqrt{h} / A$$

16ft to 10 ft

$$\frac{dh}{dt} = \frac{-48\sqrt{h}}{\sqrt{A}}$$

$$dt = \frac{1}{48}\frac{\sqrt{A}}{\sqrt{h}} dh$$

$$t = \frac{-1}{48}\int_{10}^{16} \frac{\sqrt{A}}{\sqrt{h}} dh$$

Let $n = 6$

$$h = \frac{16-10}{6} = 1$$

h	10	11	12	13	14	15	16
A	950	1070	1200	1350	1530	1575	1615
$\sqrt{\frac{A}{h}}$	9.75	9.86	10	10.19	10.45	10.25	10.05

Tanishq Chauhan
21C3184
CS-B

Simpson's 1/3 Rule

$$t = \frac{-1}{48} \int_{10}^{12} \sqrt{\frac{A}{h}}\, dh$$

$$t = \frac{-1}{48} \times \frac{h}{3} \left[(t_0 + t_6) + 4(t_1 + t_3 + t_5) + 2(t_2 + t_4) \right]$$

$$t = \frac{-1}{48} \times \frac{1}{3} \left[(9.75 + 10.05) + 4(9.86 + 10.19 + 10.25) + 2(10 + 10.45) \right]$$

$$t = \frac{-1}{48} \times \frac{1}{3} \left[19.8 + 121.2 + 40.9 \right]$$

$$\boxed{t = -1.2632}$$

Simpson's 3/8 Rule

$$t = \frac{-1}{48} \times \frac{3h}{8} \left[(t_0 + t_6) + 3(t_1 + t_2 + t_4 + t_5) + 4(t_3) \right]$$

$$t = \frac{-1}{48} \times \frac{3h}{8} \left[(9.75 + 10.05) + 3(9.86 + 10 + 10.45 + 10.25) + 4(10.19) \right]$$

$$t = -0.0078 \left[19.8 + 121.68 + 40.76 \right]$$

$$\boxed{t = -1.4215}$$

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No. 3 (a)

Suppose,

$$-6x + 4y = 2 \rightarrow y \text{ on } x$$
$$4x - 5y = 7 \rightarrow x \text{ on } y$$

$\rightarrow$
$$-6x + 4y = 2$$
$$4y = 2 + 6x$$
$$y = \frac{1}{2} + \frac{3}{2}x$$

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$\boxed{b_{yx} = \frac{3}{2}}$$

$\rightarrow$
$$4x - 5y = 7$$
$$5y = 4x - 7$$

$$y = \frac{4x}{5} - \frac{7}{5}$$

$$(y - \bar{y}) = \frac{1}{b_{xy}}(x - \bar{x})$$

$$\frac{1}{b_{xy}} = \frac{4}{5}$$

$$\boxed{b_{xy} = \frac{5}{4}}$$

Tanishq Chauhan
21C3184
CS-B

$$b_{xy} \cdot b_{yx} = \left(\frac{3}{2}\right)\left(\frac{5}{4}\right) = \frac{15}{8} > 1$$

i.e wrong assumption

→ Therefore

→ $-6x + 4y = 2$ → x on y
$4x - 5y = 7$ → y on x

$$b_{xy} = \frac{2}{3}$$

$$b_{yx} = \frac{4}{5}$$

(i) The correlation coefficient

We have $r = \sqrt{b_{xy} \cdot b_{yx}}$

$$r = \sqrt{\frac{8}{15}}$$

$$\boxed{r = 0.73}$$

(ii) The angle between the lines

We have $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Tanishq Chauhan
21CS3184
CS-B

$m_1 = \frac{3}{2}$ and $m_2 = \frac{4}{5}$

$$\theta = \tan^{-1} \left| \frac{\frac{4}{5} - \frac{3}{2}}{1 + \frac{4}{5} \times \frac{3}{2}} \right| =$$

$\theta = 0.32\,rad$

$$\boxed{\theta = 18.33^\circ.}$$

(iii) y for $x = -5$ and x for $y = 7$

Using $x = -5$

$$y = \frac{4x - 7}{5} = \frac{-27}{5}$$

$$\boxed{y = -5.4}$$

Using $y = 7$

$$x = \frac{7 - 44}{6} = \frac{13}{8}$$

$$\boxed{x = 1.625}$$

Tanishq Chauhan
21C3184
CS-B

Chauhan

* Answer of Q. No. 3. (b)

$r_{12} = 0.59$

$r_{13} = 0.46$

$r_{23} = 0.77$

$$W = \begin{vmatrix} 1 & 0.54 & 0.46 \\ 0.59 & 1 & 0.77 \\ 0.46 & 0.77 & 1 \end{vmatrix}$$

$$r_{23\cdot 1} = \frac{-w_{ij}}{\sqrt{w_{ii}\, w_{jj}}} = \frac{-w_{23}}{\sqrt{w_{22}\, w_{33}}}$$

$w_{22} = [1 - 0.46^2] = 0.7884$

$w_{33} = [1 - 0.59^2] = 0.6519$

$w_{23} = -[0.77 - (0.46)(0.59)]$

$w_{23} = -0.4986$

$$r_{23\cdot 1} = \frac{0.4986}{\sqrt{0.51395796}}$$

$$\boxed{r_{23\cdot 1} = 0.6955}$$

Tanishq Chauhan
21C3184
CS-B

$$R_{1.32} = \sqrt{1 - \frac{w}{w_{11}}}$$

$$w_{11} = 1 - 0.77^2 = 0.4071$$

$$w = 1[0.4071] - (0.59)(0.2358) + (0.46)(-0.0057)$$

$$w = 0.4071 - 0.139122 - 0.002622$$

$$w = 0.265356$$

$$R_{1.32} = \sqrt{1 - \frac{0.265356}{0.4071}}$$

$$R_{1.32} = \sqrt{0.3482}$$

$$\boxed{R_{1.32} = 0.59}$$

So, The Answer is,

$\boxed{r_{23.1} = 0.6955}$ and $\boxed{R_{1.32} = 0.59}$

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No. 3 (c)

Since $f(x)$ is a pdf,

$$\int_0^1 f(t)\,dt = 1$$

$$\Rightarrow 10c\int_0^{0.6} t^2 dt + 9c\int_{0.6}^1 (1-t)\,dt = 1$$

$$\Rightarrow 1.44c = 1$$

$$\Rightarrow c = \frac{1}{1.44} = 0.694$$

Therefore

$$f(t) = \begin{cases} 6.94t^2, & 0 \leq t < 0.6 \\ 6.246(1-t), & 0.6 < t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

The cdf is given by

$$F(t) = P(T \leq t) = \int_0^t f(x)\,dx, \quad t \geq 0$$

For $t<0$

$$F(t) = \int_{-\infty}^t f(x)\,dx = 0$$

Tanishq Chauhan
21C3184
CS-B

Chauhan

For $0 \leq t < 0.6$

$$F(t) = P(T \leq t) = \int_0^t f(x)\,dx = 6.94 \int_0^t x^2\,dx = \frac{6.94}{3} t^3$$

For $0.6 \leq t < 1$

$$F(t) = P(T \leq t) = \int_0^t f(x)\,dx$$

$$= \int_0^{0.6} f(x)\,dx + \int_{0.6}^t 6.246(1-x)\,dx$$

$$= F(0.6) + \int_{0.6}^t 6.246(1-x)\,dx$$

$$= \frac{6.94}{3}(0.6)^3 - 3.123t^2 + 6.246t - 2.6233$$

$$= -3.123t^2 + 6.246t - 2.1236$$

For $t \geq 1$

$$F(t) = P(T \leq t) = \int_0^t f(x)\,dx$$

$$= \int_{-\infty}^0 f(x)\,dx + 6.94 \int_0^{0.6} x^2\,dx + 6.246 \int_{0.6}^1 (1-x)\,dx$$

$$+ \int_1^{\infty} f(x)\,dx$$

Tanishq Chauhan
21C3184
CS-B

$$= 6.94 \int_0^{0.6} x^2 dx + 6.246 \int_{0.6}^{1} (1-x) dx$$

$$= 1$$

Hence

$$F(t) = \begin{cases} 0, & t<0 \\ 2.3133t^3, & 0 \leq 0.6 \\ -3.123t^2 + 6.246t - 2.1236, & 0.6 \leq t \leq 1 \\ 1, & t \geq 1 \end{cases}$$

(ii) Probability that the journey time will be more than 48 mins

$$P(T>48) = 6.246 \int_{0.8}^{1} (1-t) dt = 0.1249$$

$$= F(1) - F(0.8)$$

$$= 1 + 3.123(0.8)^2 - 6.246(0.8) + 2.1236$$

$$= 0.1255$$

(iii) Probability that the journey time will be between 24 and 48 mins

$$P(0.4 \leq T \leq 0.8) = 6.94 \int_{0.4}^{0.6} t^2 dt + 6.246 \int_{0.6}^{0.8} (1-t) dt = 0.7264$$

Tanishq Chauhan
21CS3184
CS-B

$$= F(0.8) - F(0.4)$$

$$= -3.123(0.8)^2 + 6.246(0.8) - 2.1236 - 2.3133(0.4)^3$$

$$= 0.7264$$

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No. 4 (a)

Given,

$n_1 = 250$ & $n_2 = 400$
$\bar{x}_1 = 120$ & $\bar{x}_2 = 124$
$\sigma_1 = 12$ & $\sigma_2 = 14$.

$\alpha = 2\%$

Now,
Standard Error of Difference $\sigma_{\bar{x}_1 - \bar{x}_2}$;

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}$$

$$= \sqrt{\frac{(12)^2}{250} + \frac{(14)^2}{400}}$$

$$= \sqrt{\frac{144}{250} + \frac{196}{400}}$$

$$\therefore \boxed{\sigma_{\bar{x}_1 - \bar{x}_2} = 1.0324}$$

Now;

$$Z_\alpha = Z_{0.02} = 2.326$$

Now, calculating Z;

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{120 - 124}{1.0324}$$

Tanishq Chauhan
21C3184
CS-B

Chauhan

24

$$z = \frac{-4}{1.0324}$$

$$|z| = 3.87$$

Null Hypothesis:

$H_0 : \mu_1 = \mu_2$; Difference not Significant

Alternative Hypothesis

$H_1 : \mu_1 \neq \mu_2$; Difference Significant

Now since

$$|z| > Z_{0.02}$$

i.e $3.87 > 2.326$

∴ We reject Null Hypothesis & The Difference is significant

Now : 99% confidence interval :

$$Z_{0.01} = 2.58$$

$$\therefore C.I = (\bar{x}_1 - \bar{x}_2) \pm Z_{0.01} \times \sigma_{\bar{x}_1 - \bar{x}_2}$$

$$= (4) \pm 2.58 \times (1.0324)$$

Confidence Interval $= (4 \pm 2.66)$

$$= [1.34, 6.66]$$

Tanishq Chauhan
21CS3184
CS-B

* Answer of Q. No. 4(b)

Given,

Drug A	3.6	5.5	5.9	4.1	1.4		
Drug B	4.5	3.6	5.5	6.8	2.7	3.6	5.0

Null Hypothesis H_0 : Both are equally effective
$\mu_1 = \mu_2$

Alternative Hypothesis H_1 : $\mu_1 < \mu_2$, B is more effective

Here, $n_1 = 5$; $n_2 = 7$

$A(x_1)$	$(x_1 - \bar{x}_1)$	$(x_1 - \bar{x}_1)^2$
3.6	-0.5	0.25
5.5	1.4	1.96
5.9	1.8	3.24
4.1	0	0
1.4	-2.7	7.29
$\Sigma x_1 = 4.1$		$\Sigma = 12.74$

let $\bar{x}_2 = 4.5$

$B(x_2)$	$(x_2 - \bar{x}_2)$	$(x_2 - \bar{x}_2)^2$
4.5	0	0
3.6	-0.9	0.81
5.5	1.0	1
6.8	2.3	5.29
2.7	-1.8	3.24
3.6	-0.9	0.81
5.0	0.5	0.25
$\Sigma x_2 = 4.528$		$\Sigma = 11.4$

Tanishq Chauhan
21C3184
CS-B

Combined, S.D. $= \sqrt{\dfrac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$

$= \sqrt{\dfrac{12.74+11.4}{5+7-2}}$

$= \sqrt{\dfrac{24.14}{10}}$

$S = 1.553$

∴ Applying Student -t test

$t = \dfrac{\bar{x}_1-\bar{x}_2}{S}\sqrt{\dfrac{n_1 n_2}{n_1+n_2}}$

$= \dfrac{(4.1-4.528)}{1.553}\sqrt{\dfrac{35}{12}}$

$= \dfrac{-0.428}{1.553} \times 1.707$

∴ $t = -0.470$

Degree of Freedom $\quad v = n_1+n_2-2$

$= 10$

∴ Critical value of t at $\alpha = 0.02$ & $v = 10$

$t_{0.02} = 2.3598$

Tanishq Chauhan
21C3184
CS-B

Since,

$|t| < t_{critical}$

$\therefore$ Null Hypothesis is not rejected & B Drug is not more effective than A.

Tanishq Chauhan
2163184
CS-B

* Answer of Q. No. 4 (c)

Given,

Total no. of individuals = 500

Religious Affiliation

Martial Status	A	B	C	D	None	
Single	39 $\boxed{49}$	19 $\boxed{19}$	12 $\boxed{13}$	28 $\boxed{23}$	18 $\boxed{13}$	$\boxed{116}$
Married	172 $\boxed{162}$	61 $\boxed{61}$	44 $\boxed{43}$	70 $\boxed{75}$	37 $\boxed{42}$	$\boxed{384}$
	$\boxed{211}$	$\boxed{80}$	$\boxed{56}$	$\boxed{98}$	$\boxed{55}$	

Now;

$$E_{11} = \frac{R_1 C_1}{n} = \frac{116 \times 211}{500} = 49$$

$$E_{12} = \frac{R_1 C_2}{n} = \frac{116 \times 80}{500} = 19$$

$$E_{13} = \frac{R_1 C_3}{n} = \frac{116 \times 56}{500} = 13$$

$$E_{21} = \frac{R_2 C_1}{n} = \frac{384 \times 211}{500} = 162$$

$$E_{22} = \frac{R_2 C_2}{n} = \frac{384 \times 80}{500} = 61$$

$$E_{23} = \frac{R_2 \times C_3}{n} = \frac{384 \times 56}{500} = 43$$

Tanishq Chauhan,
21CS3184
CS-B

Chauhan

Date
Page 29

$$E_{14} = \frac{R_1 C_4}{n} = \frac{116 \times 98}{500} = 23$$

$$E_{15} = \frac{R_1 C_5}{n} = \frac{116 \times 55}{500} = 13$$

$$E_{24} = \frac{R_2 C_4}{n} = \frac{384 \times 98}{500} = 75$$

$$E_{25} = \frac{R_2 C_5}{n} = \frac{384 \times 55}{500} = 42$$

Now, Applying χ^2 test :-

$$\chi^2 = \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Null Hypothesis H_0 :
Independent Cases

Alternative Hypothesis H_1 :
Dependent

$$\chi^2 = \frac{(39-49)^2}{49} + \frac{(19-19)^2}{19} + \frac{(12-13)^2}{13} + \frac{(28-23)^2}{23}$$

$$+ \frac{(18-13)^2}{13} + \frac{(172-162)^2}{162} + \frac{(61-61)^2}{61} + \frac{(44-43)^2}{43}$$

$$+ \frac{(70-75)^2}{75} + \frac{(37-42)^2}{42}$$

$$= \frac{100}{49} + 0 + \frac{1}{13} + \frac{25}{23} + \frac{25}{13} + \frac{100}{162} + 0$$

$$\frac{1}{43} + \frac{25}{75} + \frac{25}{42}$$

Tanishq Chauhan
21C3184
CS-B

$\chi^2 = 6.696$

Hence the Degree of Freedom in $v = (2-1)(5-1)$
$= 4$

$\chi^2_{0.01} = +13.277$

Since,

$\chi^2_{0.01} > \chi^2_{calculated}$

Null Hypothesis is not rejected.

Hence, Martial Status & Regligious affliation are independent.

Tanishq Chauhan
21C3184
CS-B

Chauhan

* Answer of Q. No. 5 (a)

If X and Y present the same stochastic process then the correlation function becomes the speial case called autocorrelation.

$$R_X(\tau) = E[X(t)X^*(t+\tau)]$$

(i) R(t) is an even function of time difference τ

$R(\tau) = R(-\tau) \rightarrow$ we need to prove

Proof :- $R(\tau) = E[(X(t) \cdot X(t+\tau))]$

$$R(-\tau) = E[X(t) \cdot X(t+\tau)]$$

put $t+\tau = t'$
$t = t' - \tau$

$$R(-\tau = E\{X(t'-\tau) \cdot X(t')\}$$

$$= E[X(t') \cdot X(t'-\tau)$$

$\therefore R(-\tau) = R(\tau)$

Hence Proved

Tanishq Chauhan
2103184
CS-B

(ii) $R(\tau)$ is maximum at $\tau = 0$.

i.e, $|R(\tau)| \leq R(0)$

Proof :- If we use cauchy schwarz inequality

$$E(xy)^2 \leq E(x^2) \times E(y^2)$$

put

$x = x(t)$ and $y = x(t-\tau)$

Then

$$[(E(x(t)\, x(t-\tau))^2 \leq E(x^2(t)) \times E(x^2(t-\tau))]$$

i.e., $\{R(\tau)\}^2 \leq \{E(x^2(t))\}^2$ --- (i)

We know, $R(\tau) = E\{x(t) \cdot x(t-\tau)\}$

$\tau = 0 \Rightarrow R(0) = \{x^2(t)\}$

$\therefore$ from eq (i)

$$\{R(\tau)\}^2 \leq \{R(0)\}^2$$

Taking Underoot

$$\boxed{R(\tau) \leq R(0)}$$

Hence Proved

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No. 5 (b)

If P is the tpm of a homogeneous Markov chain, then, n step tpm is given by

$$P^{(n)} = P^n = P^{n-1} \times P. \quad \text{[To Prove]}$$

Proof :-

$$P_{ij}^{(2)} = P\{X_2 = x_j \mid X_0 = x_i\}$$

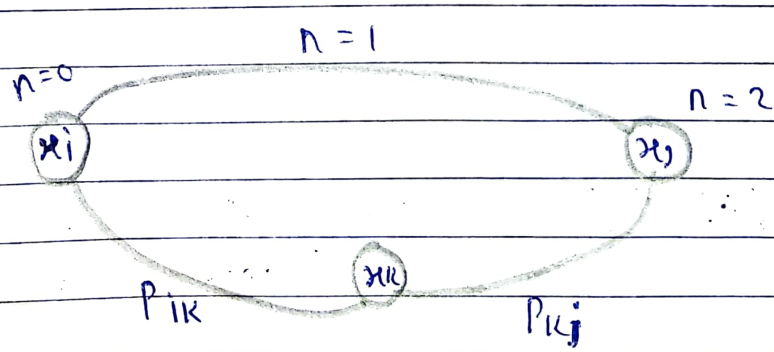
$$= P\{X_2 = x_j / X_1 = x_K, X_0 = x_i\}$$

This means at $n=1$ it assume state k_i and we are going from $x_i \rightarrow x_K \rightarrow x_j$ in two steps.

$$= P\{x_2 = x_j \mid x = x_K\} \; P\{X_1 = x_K \mid x_0 = x_i\}$$

We have broken it into two parts

(i) n(0) to (n(1)) and n(1) → (n(2)



Tanishq Chauhan
21C3184
CS-B

$\therefore \quad P_{kj}^{(1)} \quad P_{ik}^{(1)}$

$= P_{ik} P_{kj}$

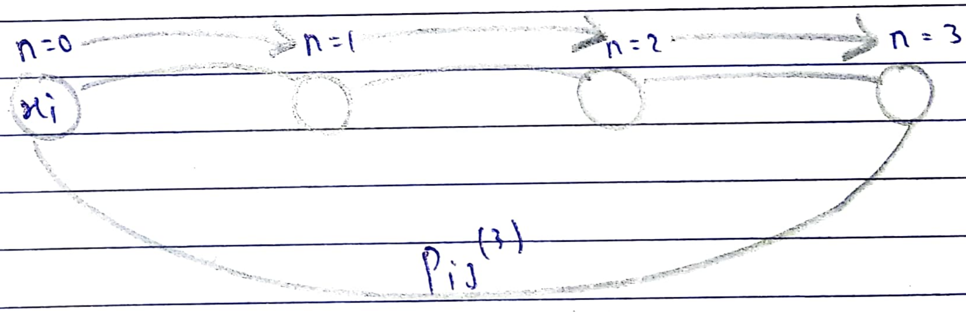
$\because \quad k - 1, 2, 3 \text{ - - - - } m,$

$$\therefore \quad P_{ij}^{(2)} = \sum_{k=1}^{m} P_{ik} P_{kj}$$

$$P^{(2)} = P^2$$

Two step tpm is P^2

Now, $P_{ij}^{(3)} = P(X_3 = x_j \mid X_0 = x_i)$



We will break in two parts

(i) $n=0$ to $n=2$ (calculated Already)
(ii) $n=2$ to $n=3$ (Given)

$= P(X_3 = x_j \mid X_2 = x_k) \; \underline{P(x_2 = x_k \mid x_0 = x_i)}$
(Already calculated)

$= P_{kj}^{(1)} P_{ik}^{(2)}$

$= P_{ik}^{(2)} P_{kj}$

Tanishq Chauhan
21C3184
CS-B

$\because$ $K = 1, 2, 3 \ldots m$

$\therefore$ $P_{ij}^{(3)} = \sum_{K=1}^{m} P_{iK} P_{Kj}^{(2)}$

$$P^{(3)} = P^2 P = PP^2$$

$\therefore$ Proceeding in same manner we will get

$$P(n) = P^{(n)} = P^{n-1} \times P$$

$\therefore$ Hence Proved

Tanishq Chauhan
21C3184
CS-B

* Answer of Q. No 5 (c)

From the given information, we have the following:

$$R_0 = \begin{array}{ccc} A & B & C \\ \end{array}$$
$$R_0 = \begin{bmatrix} 0.4 & 0.4 & 0.2 \end{bmatrix}$$

and Transition probabilitics matrix as

$$P = \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.10 \\ 0.10 & 0.07 & 0.83 \end{bmatrix}$$

Wherein the three rows represent the states of A, B and C, respectively. We may notice that the sum of transition probabilities in each of rows is equal to 1

Now, to determine each backery's share one year hence, we calcolate.

$$R_1 = R_0 \times P'$$

$$= \begin{bmatrix} 0.4 & 0.4 & 0.2 \end{bmatrix} \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.85 & 0.10 \\ 0.10 & 0.07 & 0.83 \end{bmatrix}$$

$$= \begin{bmatrix} 0.40 & 0.374 & 0.226 \end{bmatrix}$$

Tanishq Chauhan
2103184
CS-B

Thus, on January 1 next year, market share for Bakeries A, B and C would be 40%, 37.4% and 22.6% respectively.

To calculate shares of three bakeries at equilibrium, A, B and C respectively, we first formulate the linear equations as follow.

$$A = 0.90A + 0.05B + 0.10C$$
$$B = 0.05A + 0.85B + 0.07C$$
$$C = 0.05A + 0.10B + 0.83C$$
and
$$A + B + C = 1$$

Considering the first two the above equations and the fourth one, and re-arranging we get

$$0.10A - 0.05B - 0.10C = 0$$
$$-0.05A + 0.15B - 0.07C = 0$$

$$A + B + C = 1$$

In matrix notation,

$$\begin{bmatrix} 0.10 & -0.05 & -0.10 \\ -0.05 & 0.15 & -0.07 \\ 1.00 & 1.00 & 1.00 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Tanishq Chauhan
2103184
CS-B

Chauhan

Using the method of determinants, we get

$$\Delta = 0.0430$$
$$\Delta_1 = 0.0185$$
$$\Delta_2 = 0.0120$$
$$\Delta_3 = 0.0125$$

$$A = \frac{\Delta_1}{\Delta} = \frac{0.0185}{0.0430} = 0.430$$

$$B = \frac{\Delta_2}{\Delta} = \frac{0.0120}{0.0430} = 0.279$$

$$C = \frac{\Delta_3}{\Delta} = \frac{0.0125}{0.0430} = 0.291$$

Thus at equilibrium, market shares would be

$A = 43\%$

$B = 27.9\%$

$C = 29.1\%$