UNIT-2 » PROPOSITIONAL LOGIC AND PREDICATE LOGIC

PART-I PROPOSITIONAL LOGIC

❖ INTRODUCTION

- ✓ One of the main aim of logic is to provide rules by which one can determine whether any particular argument or reasoning is valid (correct).
- ✓ Logic is concerned with all kinds of reasoning, whether they be legal arguments or mathematical proofs or conclusions in a scientific theory based upon a set of hypothesis.
- ✓ Because of the diversity of their application, these rules, called rules of inference, must be stated in general terms and must be independent of any particular argument or discipline involved.

❖ STATEMENTS AND NOTIFICATION

- ✓ In this section we introduce certain basic units of our object language called primary (primitive or atomic) statements.
- ✓ We begin by assuming that the object language contains a set of declarative sentences which cannot be further broken down or analyzed into simpler sentences. These are primary statements.
- ✓ Only those declarative sentences will be admitted in the object language which have one and only one of two values called "truth values".
- ✓ The two truth values are true and false denoted by T and F respectively. Occasionally, they are also denoted by the symbols 1 and 0.
- ✓ Declarative sentences in the object language are of two types. The first type includes those sentences which are considered to be primitive in the object language. These will be denoted by distinct symbols selected from the capital letters A, B, C, …, P, Q, …, while declarative sentences of the second type are obtained from the primitive ones by using certain symbols, called connectives, and certain punctuation marks, such as parentheses, to join primitive sentences.
- ✓ In any case, all the declarative sentences to which it is possible to assign one and only one of the two possible truth values are called statements. These statements which do not contain any of the connectives are called atomic (primary, primitive) statements.

- ✓ Examples
  - Canada is a country. (This statement has truth value true.)
  - Moscow is the capital of Spain. (This statement has truth value false.)
  - This statement is false. (This sentence has no truth value. So, it is not a statement.)
  - 1+101=110. (Its truth value depends upon context. In decimal system it is false. But, in binary it is true.)
  - Close the door. (This is not a statement because it is a command.)
  - Toronto is an old city. (This statement may have different truth values in the world.)

❖ CONNECTIVES

- ✓ It is possible to construct complicated statements from simple statements by using certain connecting words or expressions known as "sentential connectives".
- ✓ The statements that we consider initially are simple statements, called atomic or primary (simple) statements. As already indicated, new statements can be formed from atomic statements through the use of sentential connectives. This resulting statements are called molecular or compound (composite) statements. Thus, the atomic statements are those which do not have any connectives.
- ✓ NEGATION: The negation of statement is generally formed by introducing the word "not" at a proper place in the statement or by prefixing the statement with the phrase "it is not the case that". If "P" denotes a statement, then the negation of "P" is written as "$\neg P$" or $\sim P$ or $\overline{P}$ and read as "not P". If the truth value of "P" is T, then the truth value of "$\neg P$" is F. Also, if the truth value of "P" is F, then the truth value of "$\neg P$" is T.
- ✓ Examples
  - P : London is a city. $\Longrightarrow$ $\neg P$: London is not a city.
  - P : I went to my class yesterday. $\Longrightarrow$ $\neg P$: I did not go to my class yesterday.
- ✓ Truth table for negation is as follows.

P	$\neg P$
T	F
F	T

- CONJUNCTION: The conjunction of two statements P and Q is the statement P ∧ Q which is read as "P and Q". The statement P ∧ Q has the truth value T whenever both P and Q have the truth value T, otherwise it has the truth value F.
- Example

  P : It is raining today.

  Q : There is a wind storm.

  P ∧ Q : It is raining today and there is a wind storm.
- Truth table for conjunction is as follows.

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

- Note: The truth values of P ∧ Q and Q ∧ P are same.
- DISJUNCTION: The disjunction of two statements P and Q is the statement P ∨ Q which is read as "P or Q". The statement P ∨ Q has the truth value F only when both P and Q have the truth value F, otherwise it has the truth value T.
- Example

  P : There is something wrong with the bulb.

  Q : There is something wrong with the wiring.

  P ∨ Q : There is something wrong with the bulb or with the wiring.
- Truth table for disjunction is as follows.

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

- Note: The statement "Twenty or thirty animals were killed in the fire today." is not disjunction because here "or" is used to indicate approximate numbers.

❖ STATEMENT FORMULAS

- ✓ Let P and Q be any two statements. Then, some of compound statements formed by using P and Q are $\neg P, P \wedge Q, (P \vee Q) \wedge (\neg P)$ etc. The compound statements given here are statement formulas derived from the statement variables P and Q. Therefore, P and Q may be called the components of the statement formulas.

❖ CONDITIONAL

- ✓ If P and Q are any two statements, then the statement $P \rightarrow Q$ which is read as "If P, then Q" is called a conditional statement.
- ✓ The statement $P \rightarrow Q$ has a truth value F, "when Q has truth value F and P has T", otherwise it has the truth value T.
- ✓ Truth table for conditional is as follows.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

- ✓ Here, the statement P is called the antecedent and Q the consequent.

❖ BICONDITIONAL

- ✓ If P and Q are any two statements, then the statement $P \rightleftarrows Q$ which is read as "P if and only if Q"(sometimes written as "P iff Q") is called a biconditional statement.
- ✓ The statement $P \rightleftarrows Q$ has the truth value T whenever both P and Q have identical truth value.
- ✓ Truth table for biconditional is as follows.

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

METHOD-1: BASIC EXAMPLES ON PROPOSITIONAL LOGIC

H	1	Write truth table for Negation, Conjuction, Disjunction, Conditional and Biconditional.	
H	2	Form the conjunction of P : It is raining today. and Q : There are 20 tables in this room.	
C	3	Translate the statement “Jack and Jill went up the hill.” into symbolic form.	
C	4	Consider the statements P : Mark is rich. and Q : Mark is happy. Write the following statements into symbolic form. a) Mark is poor but happy. b) Mark is rich or unhappy. c) Mark is neither rich nor happy. d) Mark is poor or he is both rich and unhappy.	
H	5	Construct the truth table for the statement formula $P \vee \neg Q$.	
H	6	Construct the truth table for $P \wedge \neg P$.	
C	7	Construct the truth table for $(P \vee Q) \wedge R$.	
H	8	Construct the truth table for the following formulas. a) $\neg(\neg P \vee \neg Q)$ b) $\neg(\neg P \wedge \neg Q)$ c) $P \wedge (P \vee Q)$ d) $P \wedge (Q \wedge P)$ e) $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$ f) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$	
C	9	Given the truth values of P and Q as T and those of R and S as F, find the truth values of the following. a) $P \vee (Q \wedge R)$ b) $\big(P \wedge (Q \wedge R)\big) \vee \neg\big((P \vee Q) \wedge (R \vee S)\big)$ c) $\big(\neg(P \wedge Q) \vee (\neg R)\big) \vee \Big(\big((\neg P \wedge Q) \vee (\neg R)\big) \wedge S\Big)$	
C	10	Write the statement “If either Jerry takes Calculus or Ken takes Sociology, then Larry will take English.” in symbolic form.	

H	**11**	Write the statement “The crop will be destroyed if there is a flood.” in symbolic form.	
C	**12**	Construct the truth table for $(P \to Q) \wedge (Q \to P)$.	
C	**13**	Construct the truth table for $7(P \wedge Q) \rightleftarrows (7P \vee 7Q)$.	
H	**14**	Construct the truth table for the following formulas. a) $\big(Q \wedge (P \to Q)\big) \to P$ b) $7\big(P \vee (Q \wedge R)\big) \rightleftarrows (P \vee Q) \wedge (P \vee R)$	
H	**15**	Given the truth values of P and Q as T and those of R and S as F, find the truth values of the following. a) $\big(7(P \wedge Q) \vee (7R)\big) \vee \big((Q \rightleftarrows 7P) \to (R \vee 7S)\big)$ b) $(P \rightleftarrows R) \wedge (7Q \to S)$ c) $\Big(P \vee \big(Q \to (R \wedge 7P)\big)\Big) \rightleftarrows (Q \vee 7S)$	

❖ WELL-FORMED FORMULA

- ✓ A statement formula is an expression which is a string consisting of variables (capital letters with or without subscripts), parentheses and connective symbols. Not every string of these symbol is a formula. We shall now give a recursive definition of a statement formula, often called a well-formed formula.
- ✓ A well-formed formula can be generated by the following rules.
  - ➢ A statement variable standing alone is a well-formed formula.
  - ➢ If A is a well-formed formula, then $7A$ is a well-formed formula.
  - ➢ If A and B are well-formed formulas, then $(A \wedge B)$, $(A \vee B)$, $(A \to B)$ and $(A \rightleftarrows B)$ are well-fromed formulas.
- ✓ A string of symbols containing the statement variables, connectives, and parentheses is a well-formed formula, if and only if it can be obtained by finitely many applications of these rules.
- ✓ The following are well-formed formulas.
  - ➢ $7(P \wedge Q)$

- $\neg(P \vee Q)$
- $\big(P \rightarrow (P \wedge Q)\big)$
- $\big(((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightleftarrows (P \rightarrow R)\big)$

✓ The following are not well-formed formulas.

- $\neg P \wedge Q$
- $(P \rightarrow Q) \rightarrow (\wedge Q)$
- $(P \rightarrow Q$
- $(P \rightarrow Q) \rightarrow Q)$

❖ TAUTOLOGIES

✓ If the final column of a truth table of a given statement formula is true regardless then it is called a universally valid formula or a tautology or a logical truth.

✓ If the final column of a truth table of a given statement formula is false regardless then it is called a contradiction.

✓ So, the negation of a contradiction is a tautology.

✓ We may say that a statement formula which is tautology, is identically true and a formula which is a contradiction, is identically false.

✓ Example: $\neg P \vee P$ is a tautology.

❖ EQUIVALENCE FORMULA

✓ Let A and B be two statement formulas. If truth values of A are equal to the truth values of B, then A and B are said to be equivalent.

✓ It is denoted by $A \Leftrightarrow B$.

✓ Example: $\neg\neg P$ is equivalent to P.

✓ Equivalence is a symmetric relation. (i.e. A is equivalence to B, then B is equivalence to A.)

❖ DUALITY LAW

✓ The formula A and A^* are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$ (also T by F and F by T). The connectives $\wedge$ and $\vee$ are also called duals of each other.

❖ TAUTOLOGICAL IMPLICATIONS

- ✓ Statement A is said to tautologically imply statement B if and only if $A \rightarrow B$ is a tautology.
- ✓ It is denoted by $A \Rightarrow B$.

METHOD-2: EXAMPLES ON TAUTOLOGIES AND EQUIVALENCES

C	1	Check whether the given formulas are well-formed formulas. Also, indicate which ones are tautologies or contradictions. a) $\big(P \rightarrow (P \vee Q)\big)$ b) $\Big(\big(P \rightarrow (\neg P)\big) \rightarrow \neg P\Big)$ c) $\big((\neg Q \wedge P) \wedge Q\big)$ d) $(\big(P \rightarrow (Q \rightarrow R)\big) \rightarrow \big((P \rightarrow Q) \rightarrow (P \rightarrow R)\big)$ e) $\big((\neg P \rightarrow Q) \rightarrow (Q \rightarrow P)\big))$ f) $\big((P \wedge Q) \leftrightarrows P\big)$	
H	2	Show that $\Big((P \vee Q) \wedge \neg\big(\neg P \wedge (\neg Q \vee \neg R)\big)\Big) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.	
C	3	Show the following implications. a) $(P \wedge Q) \Rightarrow (P \rightarrow Q)$ b) $P \Rightarrow (Q \rightarrow P)$ c) $\big(P \rightarrow (Q \rightarrow R)\big) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$	
C	4	Prove that $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$.	
H	5	Show that $(P \vee Q) \wedge \big(\neg P \wedge (\neg P \wedge Q)\big) \Leftrightarrow (\neg P \wedge Q)$.	
H	6	Show that $\big(\neg P \wedge (\neg Q \wedge R)\big) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.	
H	7	Show that $\neg(P \wedge Q) \rightarrow \big(\neg P \vee (\neg P \vee Q)\big) \Leftrightarrow (\neg P \vee Q)$.	
C	8	Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \rightarrow R$.	
C	9	If $A(P, Q, R)$ is $\neg P \wedge \neg(Q \vee R)$, then verify $\neg A(P, Q, R) \Leftrightarrow A^{*}(\neg P, \neg Q, \neg R)$.	

H	**10**	Show the following equivalences. a) $P \rightarrow (Q \rightarrow P) \Leftrightarrow \neg P \rightarrow (P \rightarrow Q)$ b) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$ c) $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$ d) $\neg(P \rightleftarrows Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$	
H	**11**	Show that P is equivalent to the following formulas. a) $\neg\neg P$ b) $P \wedge P$ c) $P \vee P$ d) $P \vee (P \wedge Q)$ e) $P \wedge (P \vee Q)$ f) $(P \wedge Q) \vee (P \wedge \neg Q)$ g) $(P \vee Q) \wedge (P \vee \neg Q)$	
H	**12**	Show the following equivalences. a) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ b) $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$ c) $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$ d) $\neg(P \rightleftarrows Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$	

PART-II PREDICATE LOGIC

❖ PREDICATES

- ✓ Let us consider two statements.
  - ➢ John is a bachelor.
  - ➢ Smith is a bachelor.
- ✓ Obviously, if we express these statements by symbols, we require two different symbols to denote them. Such symbols do not reveal the common features of these two statements; viz, both are statements about two different individuals who are bachelors.
- ✓ If we introduce some symbol to denote, "is a bachelor" and a method to join it with symbols denoting the names of individuals, then we will have a symbolism to denote statements about any individual's being a bachelor. The part "is a bachelor" is called a predicate.
- ✓ Now, denote the predicate "is a bachelor" symbolically by the predicate letter B, "John" by j, and "Smith" by s. Then, statements 1 and 2 can be written as B(j) and B(s) respectively. In general, any statement of the type "P is Q" where Q is a predicate and P is the subject can be denoted by Q(P).
- ✓ Example: "Jack is taller than Jill."
  - ➢ For this example, we consider G symbolizes "is taller than", j_1 denotes "Jack", and j_2 denotes "Jill", then statement can be translated as $G(j_1, j_2)$.
- ✓ "Canada is to the north of the United states" can be translated by N(C, S), where N denotes the predicate "is to the north of ", C for "Canada " and S for "United states".

❖ STATEMENT FUNCTION, VARIABLES, AND QUANTIFIERS

- ✓ Let H be the predicate "is a mortal", b the name "Jack", c "Canada", and s "A shirt". Then, H(b), H(c) and H(s) all denote statements. In fact, these statements have a common form. If we write H(x) for "x is a mortal", then H(b), H(c) and H(s) and others having the same form can be obtained from H(x) by replacing x by an appropriate name. The letter x used here is a placeholder, called statement variable.
- ✓ A simple statement function of one variable is defined to be an expression consisting of a predicate symbol and an individual variable. Such a statement function becomes a statement when the variable is replaced by the name of any object. The statement resulting

from a replacement is called a substitution instance of the statement function and is a formula of statement calculus.

- ✓ Example: Let $G(x, y)$ denotes "x is taller than y.".
  - ➢ Now, if x and y both are replaced by the names of the objects, we get a statement. If m represents Jack and n Jill, then we have

    $G(m, n)$: Jack is taller than Jill.

    $G(n, m)$: Jill is taller than Jack.
- ✓ $x + 2 = 5$ is a statement function for universe of real numbers.
- ✓ $x^2 + 1 = 0$ is a statement function for universe of complex numbers not real numbers.
- ✓ Note: $x^2 - 1 = (x - 1)\,(x + 1)$ is not a statement function, in fact it is a statement because it is true for all x. Similarly, following are also such examples.
  - ➢ "All men are mortal."
  - ➢ "Any integer is either positive or negative."
- ✓ Now, for the statement "All men are mortal." the symbolize statement is

$$(x)\big(M(x) \to H(x)\big) \text{ or } (\forall x)\big(M(x) \to H(x)\big)$$

  Where $M(x)$: x is a man. & $H(x)$: x is a mortal.
- ✓ Here, the symbols (x) or $(\forall x)$ are called universal quantifiers. Strictly, the quantification symbol is "( )" or "(∀)", and it contains the variable which is to be quantified.
- ✓ Also, for the statement "Some real numbers are rational." the symbolize statement is

$$(\exists x)\big(M(x) \land H(x)\big)$$

  Where $M(x)$: x is a real number. & $H(x)$: x is a rational.
- ✓ Then, the symbol "$(\exists x)$" is called the existential quantifier, which symbolizes expressions such as "there is at least one x such that" or "there exists an x such that" or "for some x".

❖ PREDICATE FORMULAS

- ✓ Consider the n-place predicate, capital letter is followed by n individual variables which are enclosed in parentheses and separated by commas.

- Example: $P(x_1, x_2, \dots, x_n)$ denotes an n-place predicate formula in which the letter P is an n-place predicate and $x_1, x_2, \dots, x_n$ are individual variables. In general, $P(x_1, x_2, \dots, x_n)$ will be called an atomic formula of predicate calculus. It may be noted that our symbolism includes the atomic formulas of the statement calculus as special cases $(n = 0)$.
- A well-formed formula of predicate calculus is obtained by using the following rules.
  - An atomic formula is a well-formed formula.
  - If A is a well formed formula, then $\urcorner A$ is a well-formed formula.
  - If A and B are well-formed formulas, then $(A \wedge B)$, $(A \vee B)$, $(A \to B)$ and $(A \rightleftarrows B)$ are also well-formed formulas.
  - If A is a well-formed formula and x is any variable, then $(x)A$ and $(\exists x)A$ are well-formed formulas.
- Only those formulas obtained by using these rules are well-formed formulas.

FREE AND BOUND VARIABLES

- Given a formula containing a part of the form $(x)P(x)$ and $(\exists x)P(x)$, such a part is called an x-bound part of the formula. Any occurrence of x in x-bound part of a formula is called bound occurrence of x, while any occurrence of x or of any variable that is not a bound occurrence is called free occurrence.
- Further, the formula $P(x)$ either in $(x)P(x)$ or in $(\exists x)P(x)$ is described as the scope of the quantifier. In other word, the scope of the quantifier is the formula immediately following the quantifier.
- Examples
  - In $(x)P(x, y)$, $P(x, y)$ is the scope of the quantifier, occurrence of x is bound occurrence and occurrence of y is free occurrence.
  - In $(x)\big(P(x) \to Q(x)\big)$, the scope of the universal quantifier is $P(x) \to Q(x)$, and all occurrence of x are bound.
  - Also, in $(x)\big(P(x) \to (\exists y)R(x, y)\big)$, the scope of (x) is $P(x) \to (\exists y)R(x, y)$, while scope of $(\exists y)$ is $R(x, y)$. All occurrence of both x and y are bound occurrences.

❖ THE UNIVERSE OF DISCOURSE

- ✓ The process of symbolizing a statement in predicate calculus can be quite complicated. However, some specification can be introduced by limiting the class of individuals or objects under consideration.
- ✓ This limitation means that the variables which are quantified stand for only those objects which are members of a particular set or class. Such a restricted class is called the universe of discourse or the domain of individuals or simply the universe.
- ✓ If the discussion refers to human beings only, then the universe of discourse is the class of human beings.
- ✓ In elementary algebra or number theory, the universe of discourse could be numbers (real, complex, rational, etc.).
- ✓ Examples
  - ➢ Let us symbolize the statement "all men are giants.".
  - ➢ Let $G(x)$: x is a giant and $M(x)$: x is a man. Then, given statement can be symbolized as $(x)\big(M(x) \rightarrow G(x)\big)$. Now, if we restrict the variable x to the universe which is class of men, then the statement is $(x)G(x)$.
  - ➢ Let us symbolize the statement "all cats are animals.".
  - ➢ Then, it is true for any universe of discourse. Let $E = \{\text{cuddle}, \text{ginger}, 0, 1\}$ (first two elements are name of the cats). Obviously, statement is true over E.
  - ➢ Now, if we take $C(x)$: x is a cat. and $A(x)$: x is an animal. Then, the statement $(x)C(x) \rightarrow A(x)$ is rue over E. But, the statement $(x)\big(C(x) \wedge A(x)\big)$ is false over E because statement is not true for elements 0 and 1 of the set E.

❖ VALID FORMULAS AND EQUIVALENCES

- ✓ Let A and B be any two predicate formulas defined over a common universe denoted by the symbol E. If, for every assignment of object names from the universe of discourse E to each of the variables appearing in A and B, the resulting statements have the same truth values, then the predicate formulas A and B are said to be equivalent to each other over E.
- ✓ This idea is symbolized by writing $A \Leftrightarrow B$ over E. The definition of implication can extend in the same way. It is assumed that the same object names are assigned to the same variables throughout both A and B.

- ✓ Similarly, a formula A is said to be valid in E written ‡ A in E if, for every assignment of object names of E to the corresponding variables in A and for every assignment of statements to statement variables, the resulting statement have the truth value T.
- ✓ As before, if a formula is valid for an arbitrary E, then it is written as ‡ A.
- ✓ Note: $A \Leftrightarrow B$ requires that the equivalence of A and B be examined over all universe, and the same is true for ‡ A, since these statements are made for any arbitrary universe.

METHOD-3: BASIC EXAMPLES ON PREDICATE LOGIC

C	**1**	Let $P(x) : x$ is a person. $F(x, y) : x$ is the father of y. $M(x, y) : x$ is the mother of y. Write the predicate "x is the father of the mother of y.".	
C	**2**	Symbolize the expression "All the world loves a lover.".	
H	**3**	Which of the following are statements? a) $(x)\big(P(x) \vee Q(x)\big) \wedge R$ b) $(x)\big(P(x) \vee Q(x)\big) \wedge (\exists x)S(x)$ c) $(x)\big(P(x) \vee Q(x)\big) \wedge S(x)$	
C	**4**	Indicate the variables that are free and bound. Also, show the scope of the quantifiers. a) $(x)\big(P(x) \wedge R(x)\big) \rightarrow (x)P(x) \wedge Q(x)$ b) $(x)\big(P(x) \wedge (\exists x)Q(x)\big) \vee \big((x)P(x) \rightarrow Q(x)\big)$ c) $(x)\big(P(x) \rightleftarrows Q(x) \wedge (\exists x)R(x)\big) \wedge S(x)$	
C	**5**	Find the truth value of a) $(x)\big(P(x) \vee Q(x)\big)$, where $P(x) : x = 1,\ Q(x) : x = 2$ and the universe of discourse is $\{1, 2\}$. b) $(x)\big(P \rightarrow Q(x)\big) \vee R(a)$, where $P :\ 2 > 1, Q(x) : x \leq 3, R(x) : x > 5,$ and $a : 5$, with the universe being $\{-2, 3, 6\}$. c) $(\exists x)\big(P(x) \rightarrow Q(x)\big) \wedge T$, Where $P(x) : x > 2,\ Q(x) : x = 0$, and T is any tautology, with the universe of discourse as $\{1\}$.	

H	**6**	Show that $(\exists z)\big(Q(z) \wedge R(z)\big)$ is not implied by the formulas $(\exists x)\big(P(x) \wedge Q(x)\big)$ and $(\exists y)\big(P(y) \wedge R(y)\big)$, by assuming the universe of discourse which has two elements.	

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