

Applied Mathematics - III (ACR3C1)

Mid Semester Test - 3

* Answer of Q. No. 3

Given process

$$x(t) = 4 \cos(3t + \alpha)$$

x is distributed in α interval $(-\pi, \pi)$

~~This~~ If x is uniformly distributed, then density function is given by :-

$$f(\alpha) = \frac{1}{\pi - (-\pi)}, \quad -\pi < \alpha < \pi$$

$$f(\alpha) = \frac{1}{2\pi}$$

To prove that the value above random process is WSS we need two conditions :-

1. Expectation of $x(t)$ $\{x(t)\}$ should be independent of time
2. Expectation of $(x(t) x(t-\tau))$ should be dependent on time

1)

$$\therefore E(x(t)) = E(4 \cos(3t + \alpha))$$

$$= \int_{-\pi}^{\pi} f(\alpha) 4 \cos(3t + \alpha) \cdot d\alpha$$

$$\begin{aligned}
 &= \frac{4}{2\pi} \int_{-\pi}^{\pi} \cos(3t + x) dx \\
 &= \frac{4}{2\pi} [\sin(3t + x)]_{-\pi}^{\pi} \\
 &= \frac{2}{\pi} [\sin(3t + \pi) - \sin(3t - \pi)] \\
 &= \frac{2}{\pi} \left[\frac{2 \cos \left(\frac{3t + \pi + 3t - \pi}{2} \right) \sin \left(\frac{3t + \pi - 3t + \pi}{2} \right)}{2} \right] \\
 &= \frac{4}{\pi} \left[\cos \left(\frac{6t}{2} \right) \sin(\pi) \right] \\
 &= 0
 \end{aligned}$$

$$2) E\{x(t)x(t-\tau)\} = 16E\{\cos(3t + \pi)\cos(3(t-\tau) + \pi)\}$$

$$\Rightarrow \frac{16}{2} E[\cos(3(2t - \tau) + 2\pi) + \cos 3\tau] dx$$

$$\Rightarrow \frac{16}{4\pi} \int_{-\pi}^{\pi} [\cos(3(2t - \tau) + 2\pi) + \cos 3\tau] d\pi$$

$$= \frac{4}{\pi} \left[\sin \left(\frac{6t - 3\tau}{2} + 2\pi \right) + \sin 3\tau \pi \right]_{-\pi}^{\pi}$$

$$= \frac{4}{\pi} \left[\sin(6t - 3\tau + 2\pi) + (\sin 3\tau)(\pi) - \sin(6t - 3\tau - 2\pi) \right]$$

$$= \frac{4}{\pi} [\sin(3\tau)]\pi = 4 \sin 3\tau$$

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It is the function of time T .

Hence, above given Random process is WSS process.

* Answer of Q. No 1

Let p_m = proportion of male iphone users.

from sample

$$p_m = \frac{337}{1343} = 0.25$$

p_f = proportion of female iphone users

$$p_f = \frac{52}{232} = 0.22$$

H_0 : $p_m > p_f$; proportion of male iphone users is greater than of female

H_1 : $p_m \leq p_f$; proportion of male iphone users is less than or equal to that of

$$\text{i.e. } |Z| < |Z|_{0.05} \quad (1.11 < 1.645)$$

The proportion of male iphone users is greater than that of female

now, for 95% confidence limit

$$Z \leq |Z|_{0.05}$$

$$\frac{p_m - p_f}{\sqrt{\frac{p_f(1-p_f)}{n_f}}} \leq 1.645$$

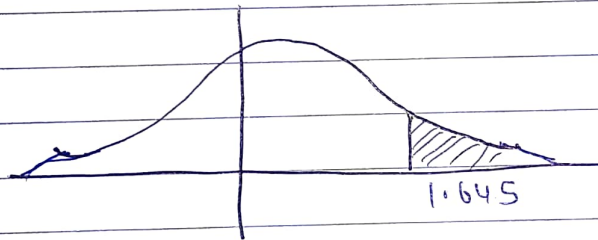
$$p_m - p_f \leq 1.645 \times 0.027$$
$$\leq 0.009$$

$$\Rightarrow (0 - \infty, 0.004)$$

female, i.e

The Los, α is 5%

$$|Z_{0.05}| = 1.645$$



$$Z = \frac{p_m - p_f}{\sqrt{\frac{p_f q_f}{n_f}}}$$

$$Z = \frac{0.25 - 0.22}{\sqrt{\frac{0.22 \times 0.78}{232}}}$$

$$Z = \frac{0.03}{\sqrt{0.00074}} = \frac{0.03}{0.027}$$

$$= 1.11$$

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* Answer of Q. No-2

$$n_1 = 274, \bar{X}_1 = 3.95, \bar{X}_2 = 4.10$$

$$n_2 = 320, s_1 = 0.63, s_2 = 0.47$$

$$SE(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= \sqrt{\frac{(0.63)^2}{274} + \frac{(0.47)^2}{320}}$$

$$= \sqrt{0.0021}$$

$$= 0.045$$

Let; $H_0: \bar{X}_1 = \bar{X}_2$, Both are equally satisfied

$H_1: \bar{X}_1 < \bar{X}_2$, B's customer are more satisfied from
A's customer (left tailed)

$$LOS: = 1\% = 0.01$$

$$Z_{0.01} = -2.33$$

$$\text{Now; } Z = \frac{\bar{X}_1 - \bar{X}_2}{SE(\bar{X}_1 - \bar{X}_2)}$$

$$Z = \frac{3.95 - 4.10}{0.045} = -3.33$$

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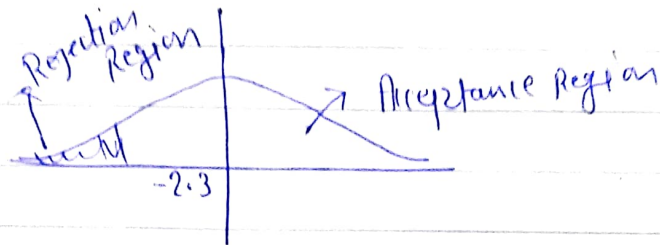
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Since;

$$Z = -3.33 < -2.33 = Z_{0.01}$$

Hence it falls in rejection region area, H_0 is rejected & H_1 is Accepted.

i.e. B's customer are more satisfied than A's customer.

\Rightarrow 98% confidence interval :-

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{SE(\bar{X}_1 - \bar{X}_2)} \geq Z_{0.02}$$

$$\frac{\bar{X}_1 - \bar{X}_2}{0.045} \geq -2.055$$

$$\bar{X}_1 - \bar{X}_2 \geq -2.055 \times 0.045$$

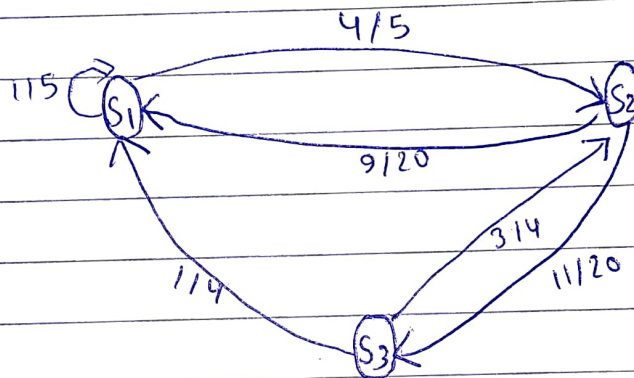
$$\boxed{\bar{X}_1 - \bar{X}_2 \geq 0.0924}$$

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* Answer of Q. No. 4

$$P = \begin{bmatrix} 1/5 & 4/5 & 0 \\ 9/20 & 0 & 11/20 \\ 1/4 & 3/4 & 0 \end{bmatrix}$$


\Rightarrow State S_2 is recurrent because from wherever we can go, there is a way of returning to S_2