A Course Material on

PRINCIPLES OF DIGITAL SIGNAL PROCESSING

By

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QUALITY CERTIFICATE

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Subject Code : **EC6501**

Subject : **PRINCIPLES OF DIGITAL SIGNAL PROCESSING**

Class **:** III Year ECE

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SEAL

EC6502 DIGITAL SIGNAL PROCESSING

UNIT I DISCRETE FOURIER TRANSFORM 9

DFT and its properties, Relation between DTFT and DFT, FFT computations using Decimation in time and Decimation in frequency algorithms, Overlap-add and save methods

UNIT II INFINITE IMPULSE RESPONSE DIGITAL FILTERS 9

Review of design of analogue Butterworth and Chebyshev Filters, Frequency transformation in analogue domain – Design of IIR digital filters using impulse invariance technique – Design of digital filters using bilinear transform – pre warping – Realization using direct, cascade and parallel forms.

UNIT III FINITE IMPULSE RESPONSE DIGITAL FILTERS 9

Symmetric and Antisymmetric FIR filters – Linear phase FIR filters – Design using Hamming, Hanning and Blackmann Windows – Frequency sampling method – Realization of FIR filters – Transversal, Linear phase and Polyphase structures.

UNIT IV FINITE WORD LENGTH EFFECTS 9

Fixed point and floating point number representations – Comparison – Truncation and Rounding errors - Quantization noise – derivation for quantization noise power – coefficient quantization error – Product quantization error - Overflow error – Roundoff noise power - limit cycle oscillations due to product roundoff and overflow errors - signal scaling

UNIT V MULTIRATE SIGNAL PROCESSING 9

Introduction to Multirate signal processing-Decimation-Interpolation-Polyphase implementation of FIR filters for interpolator and decimator -Multistage implementation of sampling rate conversion- Design of narrow band filters - Applications of Multirate signal processing.

TOTAL= 60 PERIODS

TEXT BOOKS

1. John G Proakis and Manolakis, " Digital Signal Processing Principles, Algorithms and Applications", Pearson, Fourth Edition, 2007.
2. S.Salivahanan, A. Vallavaraj, C. Gnanapriya, Digital Signal Processing, TMH/McGraw Hill International, 2007

REFERENCES

1. E.C.Ifeachor and B.W.Jervis,"Digital signal processing–A practical approach " Second edition, Pearson, 2002.
2. S.K. Mitra, Digital Signal Processing, A Computer Based approach, Tata Mc GrawHill, 1998.
3. P.P.Vaidyanathan, Multirate Systems & Filter Banks, Prentice Hall, Englewood cliffs, NJ, 1993.
4. Johny R. Johnson, Introduction to Digital Signal Processing, PHI, 2006.

UNIT I
FREQUENCY TRANSFORMATIONS

PRE REQUISITE DISCUSSION:

The Discrete Fourier transform is performed for Finite length sequence whereas DTFT is used to perform transformation on both finite and also Infinite length sequence. The DFT and the DTFT can be viewed as the logical result of applying the standard continuous Fourier transform to discrete data. From that perspective, we have the satisfying result that it's not the transform that varies.

1.1 INTRODUCTION

Any signal can be decomposed in terms of sinusoidal (or complex exponential) components. Thus the analysis of signals can be done by transforming time domain signals into frequency domain and vice-versa. This transformation between time and frequency domain is performed with the help of Fourier Transform(FT) But still it is not convenient for computation by DSP processors hence Discrete Fourier Transform(DFT) is used.

Time domain analysis provides some information like amplitude at sampling instant but does not convey frequency content & power, energy spectrum hence frequency domain analysis is used.

For Discrete time signals x(n) , Fourier Transform is denoted as x(ω) & given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \qquad \text{FT.........(1)}$$

DFT is denoted by x(k) and given by (ω= 2 ∏ k/N)

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2 \prod kn / N} \qquad \text{DFT.......(2)}$$

IDFT is given as

N-1

$$x(n) = 1/N \sum_{k=0} X(k)\, e^{j2\prod kn/N} \qquad \text{IDFT......(3)}$$

1.2 DIFFERENCE BETWEEN FT & DFT

Sr No	Fourier Transform (FT)	Discrete Fourier Transform (DFT)
1	FT x(ω) is the continuous function of x(n).	DFT x(k) is calculated only at discrete values of ω. Thus DFT is discrete in nature.
2	The range of ω is from - ∏ to ∏ or 0 to 2∏.	Sampling is done at N equally spaced points over period 0 to 2∏. Thus DFT is sampled version of FT.
3	FT is given by equation (1)	DFT is given by equation (2)
4	FT equations are applicable to most of infinite sequences.	DFT equations are applicable to causal, finite duration sequences
5	In DSP processors & computers applications of FT are limited because x(ω) is continuous function of ω.	In DSP processors and computers DFTs are mostly used. **APPLICATION** a) Spectrum Analysis b) Filter Design

Q) Prove that FT x(ω) is periodic with period 2∏.

Q) Determine FT of $x(n)= a^n u(n)$ for $-1< a < 1$.

Q) Determine FT of x(n)= A for $0 \le n \le L-1$.
Q) Determine FT of x(n)= u(n)
Q) Determine FT of x(n)= δ(n)
Q) Determine FT of $x(n)= e^{-at} u(t)$

1.3 CALCULATION OF DFT & IDFT

For calculation of DFT & IDFT two different methods can be used. First method is using mathematical equation & second method is 4 or 8 point DFT. If x(n) is the sequence of N samples then consider $W_N= e^{-j2\prod/N}$ (twiddle factor)

Four POINT DFT (4-DFT)

Sr No	$W_N=W_4=e^{-j\prod/2}$	Angle	Real	Imaginary	Total
1	W_4^{0}	0	1	0	1
2	W_4^{1}	- ∏/2	0	-j	-j
3	W_4^{2}	- ∏	-1	0	-1
4	W_4^{3}	- 3 ∏ /2	0	J	J

$$[W_N] = \begin{array}{c|cccc} & n=0 & n=1 & n=2 & n=3 \\ k=0 & W_4^{0} & W_4^{0} & W_4^{0} & W_4^{0} \\ k=1 & W_4^{0} & W_4^{1} & W_4^{2} & W_4^{3} \\ k=2 & & & & \end{array}$$

W_4^0 W_4^2 W_4^4 W_4^6

k=3 W_4^0 W_4^3 W_4^6 W_4^9

Thus 4 point DFT is given as $X_N = [W_N]\, X_N$

$$[W_N] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

EIGHT POINT DFT (8-DFT)

Sr No	$W_N = W_8 = e^{-j\prod/4}$	Angle	Magnitude	Imaginary	Total
1	W_8^0	0	1	----	1
2	W_8^1	$-\prod/4$	$1/\sqrt{2}$	$-j\, 1/\sqrt{2}$	$1/\sqrt{2} - j\, 1/\sqrt{2}$
3	W_8^2	$-\prod/2$	0	$-j$	$-j$
4	W_8^3	$-3\prod/4$	$-1/\sqrt{2}$	$-j\, 1/\sqrt{2}$	$-1/\sqrt{2} - j\, 1/\sqrt{2}$
5	W_8^4	$-\prod$	-1	----	-1
6	W_8^5	$-5\prod/4$	$-1/\sqrt{2}$	$+j\, 1/\sqrt{2}$	$-1/\sqrt{2} + j\, 1/\sqrt{2}$
7	W_8^6	$-7\prod/4$	0	J	J
8	W_8^7	$-2\prod$	$1/\sqrt{2}$	$+j\, 1/\sqrt{2}$	$1/\sqrt{2} + j\, 1/\sqrt{2}$

Remember that $W_8^0 = W_8^8 = W_8^{16} = W_8^{24} = W_8^{32} = W_8^{40}$ (Periodic Property)

Magnitude and phase of x(k) can be obtained as,

$|x(k)| = \text{sqrt}(X_r(k)^2 + X_I(k)^2)$ Angle

$x(k) = \tan^{-1}(X_I(k) / X_R(k))$

Examples:

Q) Compute DFT of x(n) = {0,1,2,3} Ans: x4=[6, -2+2j, -2, -2-2j]

Q) Compute DFT of x(n) = {1,0,0,1} Ans: x4=[2, 1+j, 0, 1-j]

Q) Compute DFT of x(n) = {1,0,1,0} Ans: x4=[2, 0, 2, 0]

Q) Compute IDFT of x(k) = {2, 1+j, 0, 1-j } Ans: x4=[1,0,0,1]

1.4 DIFFERENCE BETWEEN DFT & IDFT

Sr No	DFT (Analysis transform)	IDFT (Synthesis transform)
1	DFT is finite duration discrete frequency sequence that is obtained by sampling one period of FT.	IDFT is inverse DFT which is used to calculate time domain representation (Discrete time sequence) form of x(k).
2	DFT equations are applicable to causal finite duration sequences.	IDFT is used basically to determine sample response of a filter for which we know only transfer function.

3	Mathematical Equation to calculate DFT is given by $$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\prod kn / N}$$	Mathematical Equation to calculate IDFT is given by $$x(n) = 1/N \sum_{n=0}^{N-1} X(k) e^{j2\prod kn / N}$$
4	Thus DFT is given by **$X(k)= [W_N][x_n]$**	In DFT and IDFT difference is of factor 1/N & sign of exponent of twiddle factor. Thus **$x(n)= 1/N\, [W_N]^{-1}[X_K]$**

1.5 PROPERTIES OF DFT

$$x(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x(k)$$

1. Periodicity

Let x(n) and x(k) be the DFT pair then if

$$x(n+N) = x(n) \quad \text{for all n then}$$

$$X(k+N) = X(k) \quad \text{for all k}$$

Thus periodic sequence xp(n) can be given as

$$xp(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

2. Linearity

The linearity property states that if

$$x1(n) \overset{DFT}{\longleftrightarrow} X1(k) \text{ And}$$

$$x_2(n) \xleftrightarrow[N]{DFT} X_2(k) \quad \text{Then}$$

Then

$$a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow[N]{DFT} a_1 X_1(k) + a_2 X_2(k)$$

DFT of linear combination of two or more signals is equal to the same linear combination of DFT of individual signals.

3. Circular Symmetries of a sequence

A) A sequence is said to be circularly even if it is symmetric about the point zero on the circle. Thus $X(N-n) = x(n)$

B) A sequence is said to be circularly odd if it is anti symmetric about the point zero on the circle. Thus $X(N-n) = - x(n)$

C) A circularly folded sequence is represented as $x((-n))_N$ and given by $x((-n))_N = x(N-n)$.

D) Anticlockwise direction gives delayed sequence and clockwise direction gives advance sequence. Thus delayed or advances sequence x`(n) is related to x(n) by the circular shift.

4. Symmetry Property of a sequence

A) Symmetry property for real valued x(n) i.e $x_I(n)=0$

This property states that if x(n) is real then $X(N-k) = X^*(k)=X(-k)$

B) Real and even sequence x(n) i.e $x_I(n)=0$ & $X_I(K)=0$

This property states that if the sequence is real and even $x(n)= x(N-n)$ then DFT becomes

$$X(k) = \sum_{n=0}^{N-1} x(n) \cos (2\prod kn/N)$$

C) Real and odd sequence x(n) i.e $x_I(n)=0$ & $X_R(K)=0$

This property states that if the sequence is real and odd $x(n)=-x(N-n)$ then DFT becomes

$$X(k) = -j \sum_{n=0}^{N-1} x(n) \sin (2\prod kn/N)$$

D) Pure Imaginary x(n) i.e $x_R(n)=0$

This property states that if the sequence is purely imaginary $x(n)=j\ X_I(n)$ then DFT becomes

$$X_R(k) = \sum_{n=0}^{N-1} x_I(n) \sin (2\prod kn/N)$$

$$X_I(k) = \sum_{n=0}^{N-1} x_I(n) \cos(2\prod kn/N)$$

5. Circular Convolution

The Circular Convolution property states that if

$$x1(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X1(k) \quad \text{And}$$

$$x2(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X2(k) \quad \text{Then}$$

$$\text{Then } x1(n) \; N \; x2(n) \overset{\text{DFT}}{\longleftrightarrow} x1(k)\ x2(k)\ N$$

It means that circular convolution of x1(n) & x2(n) is equal to multiplication of their DFTs.

Thus circular convolution of two periodic discrete signal with period N is given by

$$\mathbf{y(m) = \sum_{n=0}^{N-1} x1(n)\, x2(m-n)_N} \qquad \text{……….(4)}$$

Multiplication of two sequences in time domain is called as Linear convolution while Multiplication of two sequences in frequency domain is called as circular convolution. Results of both are totally different but are related with each other.

There are two different methods are used to calculate circular convolution

1) Graphical representation form
2) Matrix approach

DIFFERENCE BETWEEN LINEAR CONVOLUTION & CIRCULAR CONVOLUTION

Sr No	Linear Convolution	Circular Convolution
1	In case of convolution two signal sequences input signal x(n) and impulse response h(n) given by the same system, output y(n) is calculated	Multiplication of two DFTs is called as circular convolution.
2	Multiplication of two sequences in time domain is called as Linear convolution	Multiplication of two sequences in frequency domain is called as circular convolution.
3	Linear Convolution is given by the equation y(n) = x(n) * h(n) & calculated as $y(n) = \sum_{k}^{\infty} x(k)\ h(n-k)$	Circular Convolution is calculated as $y(m) = \sum_{n=0}^{N-1} x1(n)\ x2(m-n)_N$

$k = -\infty$

4	Linear Convolution of two signals returns N-1 elements where N is sum of elements in both sequences.	Circular convolution returns same number of elements that of two signals.

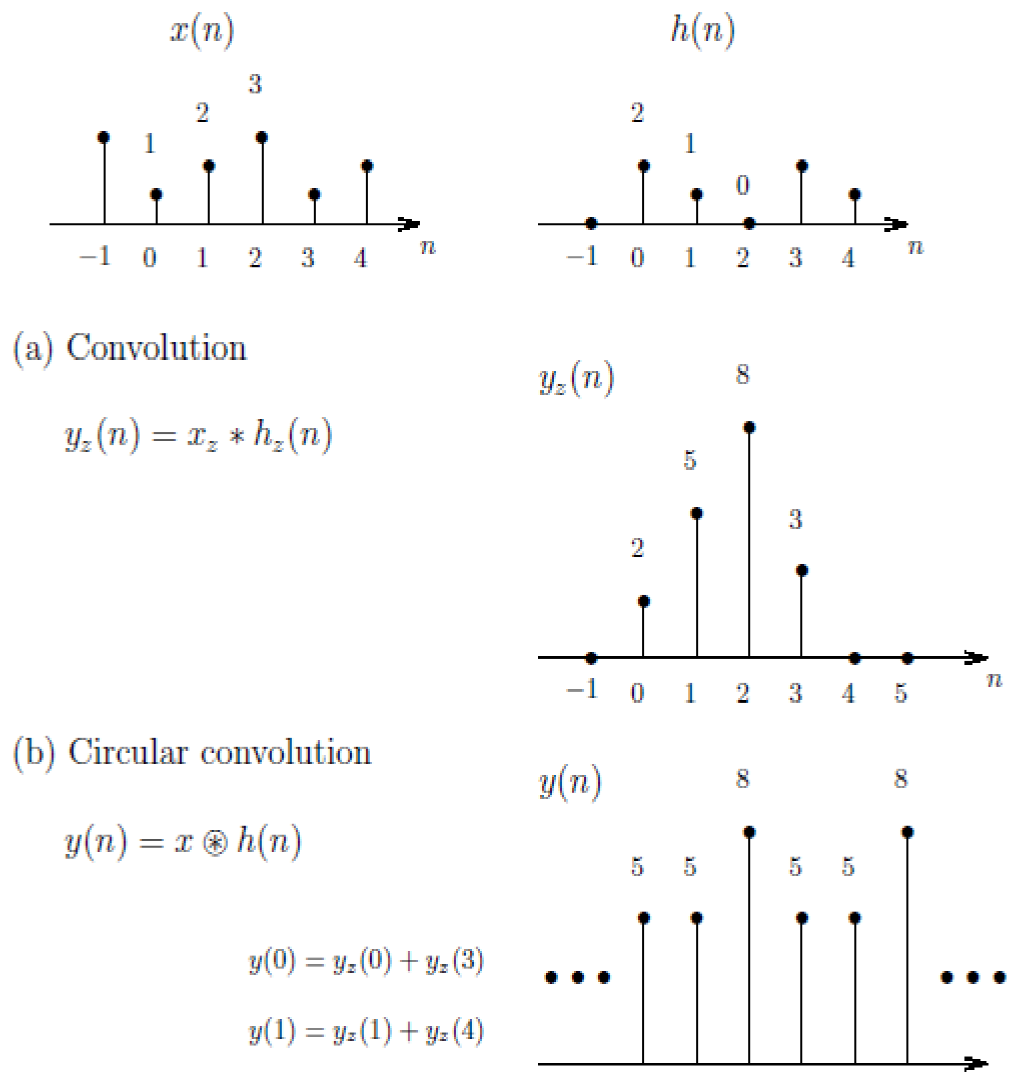
Q) The two sequences x1(n)={2,1,2,1} & x2(n)={1,2,3,4}. Find out the sequence x3(m) which is equal to circular convolution of two sequences. Ans: X3(m)={14,16,14,16}

Q) x1(n)={1,1,1,1,-1,-1,-1,-1} & x2(n)={0,1,2,3,4,3,2,1}. Find out the sequence x3(m) which is equal to circular convolution of two sequences. Ans: X3(m)={-4,-8,-8,-4,4,8,8,4}

Q) Perform Linear Convolution of x(n)={1,2} & h(n)={2,1} using DFT & IDFT.

Q) Perform Linear Convolution of x(n)={1,2,2,1} & h(n)={1,2,3} using 8 Pt DFT & IDFT.

DIFFERENCE BETWEEN LINEAR CONVOLUTION & CIRCULAR CONVOLUTION



6. Multiplication

The Multiplication property states that if

$$X1(n) \xleftrightarrow[N]{DFT} x1(k) \quad \text{And}$$

$$X2(n) \xleftrightarrow[N]{DFT} x2(k) \quad \text{Then}$$

Then $\quad x1(n)\ x2(n) \xleftrightarrow{DFT} 1/N \quad x1(k)\ N\ \textcircled{N}\ x2(k)$

It means that multiplication of two sequences in time domain results in circular convolution of their DFT□s in frequency domain.

7. Time reversal of a sequence

The Time reversal property states that if

$$X(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x(k) \quad \text{And}$$

Then $x((-n))_N = x(N-n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x((-k))_N = x(N-k)$

It means that the sequence is circularly folded its DFT is also circularly folded.

8. Circular Time shift

The Circular Time shift states that if

$$X(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x(k) \quad \text{And}$$

Then $x((n-l))_N \underset{N}{\overset{DFT}{}} x(k)\ e^{-j2 \prod k\, l / N}$

Thus shifting the sequence circularly by „l□ samples is equivalent to multiplying its DFT by $e^{-j2 \prod k\, l / N}$

9. Circular frequency shift

The Circular frequency shift states that if

$$X(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x(k) \quad \text{And}$$

Then $x(n)\, e^{j2 \prod l\, n / N} \underset{N}{\overset{DFT}{\longleftrightarrow}} x((n-l))_N$

Thus shifting the frequency components of DFT circularly is equivalent to multiplying its time domain sequence by $e^{-j2 \prod k\, l / N}$

10. Complex conjugate property

The Complex conjugate property states that if

$$X(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x(k) \quad \text{then}$$

$$x^*(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} x^*((-k))_N = x^*(N-k) \quad \text{And}$$

$$x^*((-n))_N = x^*(N-k) \underset{N}{\overset{DFT}{\longleftrightarrow}} x^*(k)$$

11. Circular Correlation

The Complex correlation property states

$$r_{xy}(l) \underset{N}{\overset{DFT}{\longleftrightarrow}} R_{xy}(k)= x(k)\ Y^*(k)$$

Here $r_{xy}(l)$ is circular cross correlation which is given as

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\ y^*((n-l))_N$$

This means multiplication of DFT of one sequence and conjugate DFT of another sequence is equivalent to circular cross-correlation of these sequences in time domain.

12.Parseval’sTheorem

The Parseval□s theorem states

$$\sum_{n=0}^{N-1} X(n)\ y^*(n) = 1/N \sum_{n=0}^{N-1} x(k)\ y^*(k)$$

This equation give energy of finite duration sequence in terms of its frequency components.

1.6 APPLICATION OF DFT

1. DFT FOR LINEAR FILTERING

Consider that input sequence x(n) of Length L & impulse response of same system is h(n) having M samples. Thus y(n) output of the system contains N samples where N=L+M-1. If DFT of y(n) also contains N samples then only it uniquely represents y(n) in time domain. Multiplication of two DFT□s is equivalent to circular convolution of corresponding time domain sequences. But the length of x(n) & h(n) is less than N. Hence these sequences are appended with zeros to make their length N called as “Zero padding”. The N point circular convolution and linear convolution provide the same sequence. Thus linear convolution can be obtained by circular convolution. Thus linear filtering is provided by DFT.

When the input data sequence is long then it requires large time to get the output sequence. Hence other techniques are used to filter long data sequences. Instead of finding the output of complete input sequence it is broken into small length sequences. The output due to these small length sequences are computed fast. The outputs due to these small length sequences are fitted one after another to get the final output response.

METHOD 1: OVERLAP SAVE METHOD OF LINEAR FILTERING

Step 1> In this method L samples of the current segment and M-1 samples of the previous segment forms the input data block. Thus data block will be

X1(n) ={0,0,0,0,0,………………… ,x(0),x(1),…………….x(L-1)}
X2(n) ={x(L-M+1), …………….x(L-1),x(L),x(L+1),,,,,,,,,,,,,x(2L-1)} X3(n) ={x(2L-M+1), …………….x(2L-1),x(2L),x(2L+2),,,,,,,,,,,,,x(3L-1)}

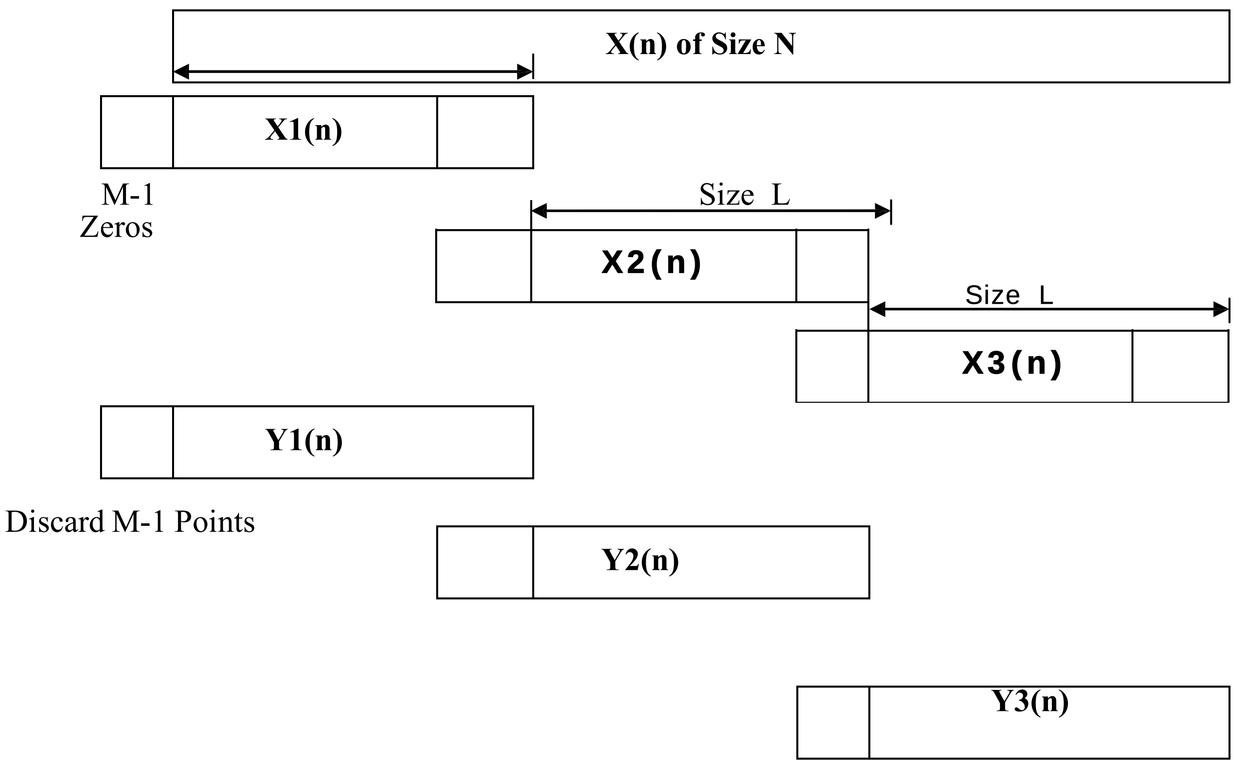
Step2> Unit sample response h(n) contains M samples hence its length is made N by padding zeros. Thus h(n) also contains N samples.

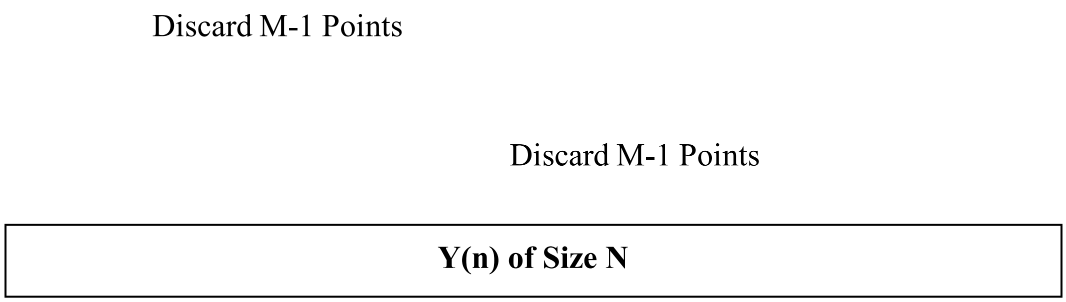
h(n)={ h(0), h(1), …………….h(M-1), 0,0,0,……………………(L-1 zeros)}

Step3> The N point DFT of h(n) is H(k) & DFT of m^{th} data block be $x_m(K)$ then corresponding DFT of output be $Y`_m(k)$

$$Y`_m(k)= H(k)\ x_m(K)$$

Step 4> The sequence $y_m(n)$ can be obtained by taking N point IDFT of $Y`_m(k)$. Initial (M-1) samples in the corresponding data block must be discarded. The last L samples are the correct output samples. Such blocks are fitted one after another to get the final output.





METHOD 2: OVERLAP ADD METHOD OF LINEAR FILTERING

Step 1> In this method L samples of the current segment and M-1 samples of the previous segment forms the input data block. Thus data block will be

X1(n) ={x(0),x(1),…………….x(L-1),0,0,0,……….}
X2(n) ={x(L),x(L+1),x(2L-1),0,0,0,0}
X3(n) ={x(2L),x(2L+2),,,,,,,,,,,,x(3L-1),0,0,0,0}

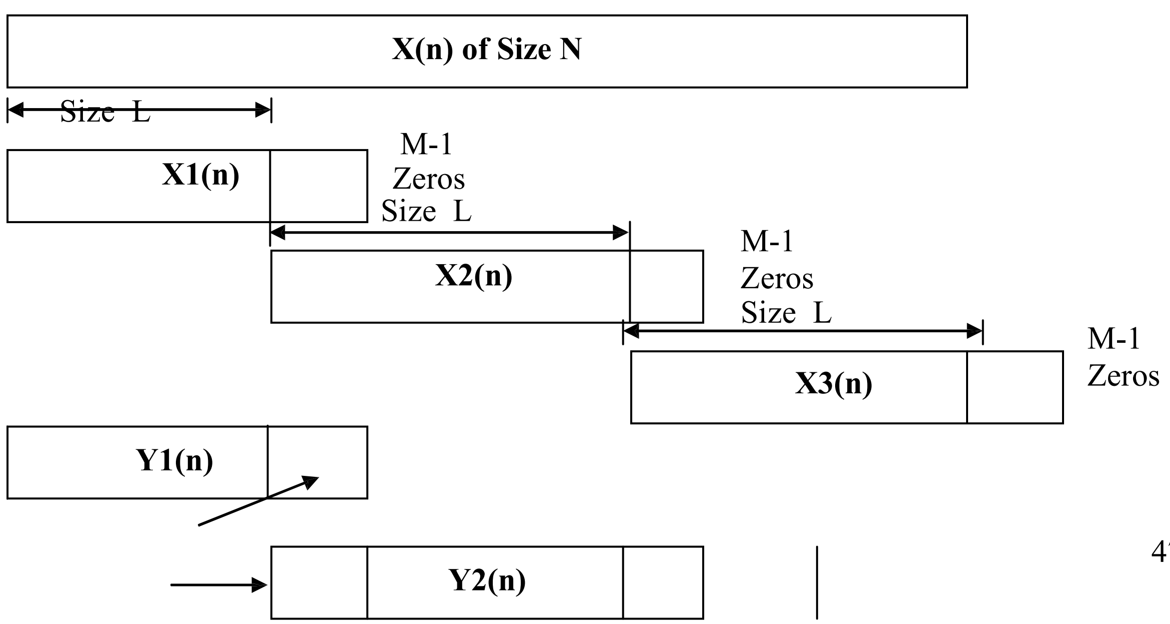
Step2> Unit sample response h(n) contains M samples hence its length is made N by padding zeros. Thus h(n) also contains N samples.

h(n)={ h(0), h(1), …………….h(M-1), 0,0,0,……………………(L-1 zeros)}

Step3> The N point DFT of h(n) is H(k) & DFT of m^{th} data block be $x_m(K)$ then corresponding DFT of output be $Y`_m(k)$

$$Y`_m(k)= H(k)\, x_m(K)$$

Step 4> The sequence $y_m(n)$ can be obtained by taking N point IDFT of $Y`_m(k)$. Initial (M-1) samples are not discarded as there will be no aliasing. The last (M-1) samples of current output block must be added to the first M-1 samples of next output block. Such blocks are fitted one after another to get the final output.



M-1
Points add
together

Y(n) of Size N

DIFFERENCE BETWEEN OVERLAP SAVE AND OVERLAP ADD METHOD

Sr No	OVERLAP SAVE METHOD	OVERLAP ADD METHOD
1	In this method, L samples of the current segment and (M-1) samples of the previous segment forms the input data block.	In this method L samples from input sequence and padding M-1 zeros forms data block of size N.
2	Initial M-1 samples of output sequence are discarded which occurs due to aliasing effect.	There will be no aliasing in output data blocks.
3	To avoid loss of data due to aliasing last M-1 samples of each data record are saved.	Last M-1 samples of current output block must be added to the first M-1 samples of next output block. Hence called as overlap add method.

2. SPECTRUM ANALYSIS USING DFT

DFT of the signal is used for spectrum analysis. DFT can be computed on digital computer or digital signal processor. The signal to be analyzed is passed through anti-aliasing filter and samples at the rate of $F_s \geq 2\ F_{max}$. Hence highest frequency component is $F_s/2$.

Frequency spectrum can be plotted by taking N number of samples & L samples of waveforms. The total frequency range $2\prod$ is divided into N points. Spectrum is better if we take large value of N & L But this increases processing time. DFT can be computed quickly using FFT algorithm hence fast processing can be done. Thus most accurate resolution can be obtained by increasing number of samples.

1.7 FAST FOURIER ALGORITHM (FFT)

1. Large number of the applications such as filtering, correlation analysis, spectrum analysis require calculation of DFT. But direct computation of DFT require large number of computations and hence processor remain busy. Hence special algorithms are developed to compute DFT quickly called as Fast Fourier algorithms (FFT).

2. The radix-2 FFT algorithms are based on divide and conquer approach. In this method, the N-point DFT is successively decomposed into smaller DFT□s. Because of this decomposition, the number of computations are reduced.

1.8 RADIX-2 FFT ALGORITHMS

1. DECIMATION IN TIME (DITFFT)

There are three properties of twiddle factor W_N

1) $W_N^{k+N} = W_N^K$ (Periodicity Property)
2) $W_N^{k+N/2} = -W_N^K$ (Symmetry Property)
3) $W_N^2 = W_{N/2}$.

N point sequence x(n) be splitted into two N/2 point data sequences f1(n) and f2(n). f1(n) contains even numbered samples of x(n) and f2(n) contains odd numbered samples of x(n). This splitted operation is called decimation. Since it is done on time domain sequence it is called "**Decimation in Time**". Thus

$$f1(m)=x(2m) \qquad \text{where } n=0,1,\ldots\ldots\ldots\ldots N/2-1$$
$$f2(m)=x(2m+1) \qquad \text{where } n=0,1,\ldots\ldots\ldots\ldots N/2-1$$

N point DFT is given as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad (1)$$

Since the sequence x(n) is splitted into even numbered and odd numbered samples, thus

$$X(k) = \sum_{m=0}^{N/2-1} x(2m) W_N^{2km} + \sum_{m=0}^{N/2-1} x(2m+1) W_N^{k(2m+1)} \qquad (2)$$
$$X(k) = F1(k) + W_N^k F2(k) \qquad (3)$$
$$X(k+N/2) = F1(k) - W_N^k F2(k) \quad \text{(Symmetry property)} \qquad (4)$$

Fig 1 shows that 8-point DFT can be computed directly and hence no reduction in computation.

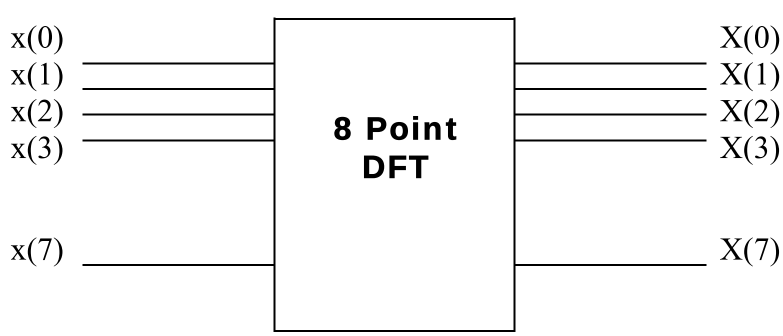


Fig 1. DIRECT COMPUTATION FOR N=8

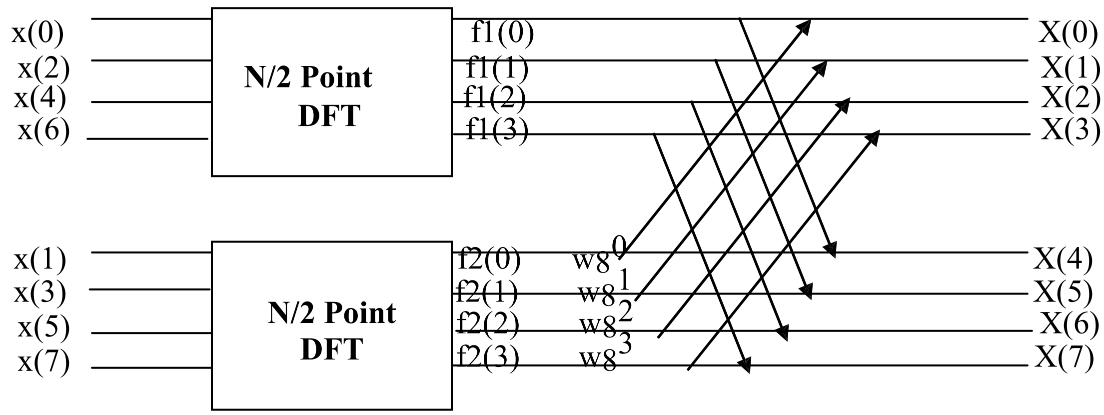


Fig 2. FIRST STAGE FOR FFT COMPUTATION FOR N=8

Fig 3 shows N/2 point DFT base separated in N/4 boxes. In such cases equations become

$$g1(k) = P1(k) + W_N^{2k}\, P2(k) \quad (5)$$

$$g1(k+N/2) = p1(k) - W_N^{2k}\, P2(k) \quad (6)$$

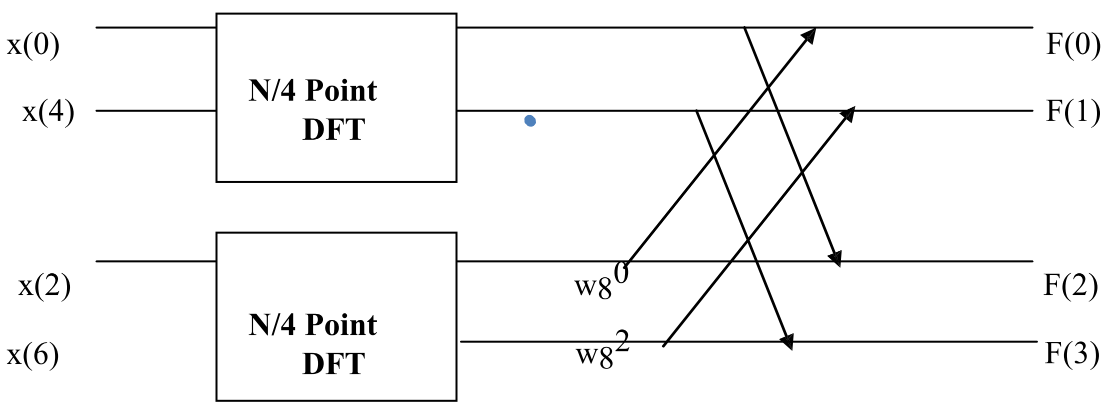


Fig 3. SECOND STAGE FOR FFT COMPUTATION FOR N=8

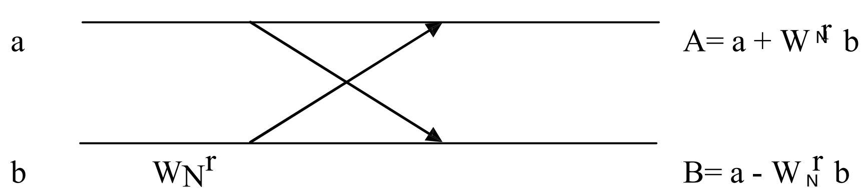


Fig 4. BUTTERFLY COMPUTATION (THIRD STAGE)

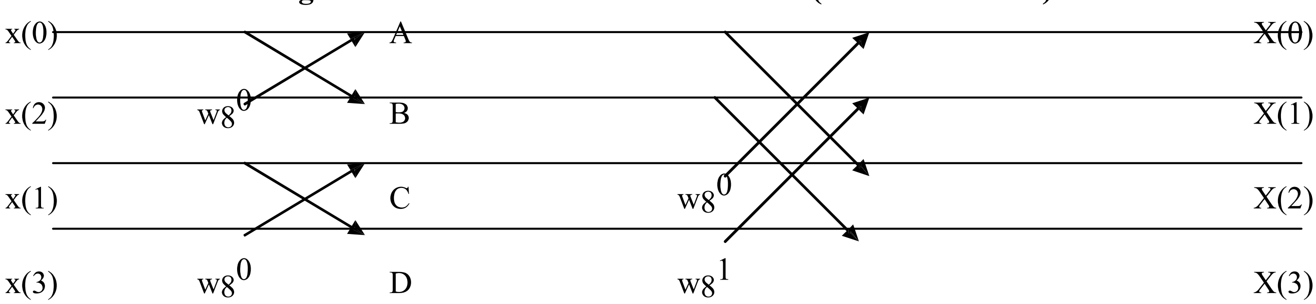


Fig 5. SIGNAL FLOW GRAPH FOR RADIX- DIT FFT N=4

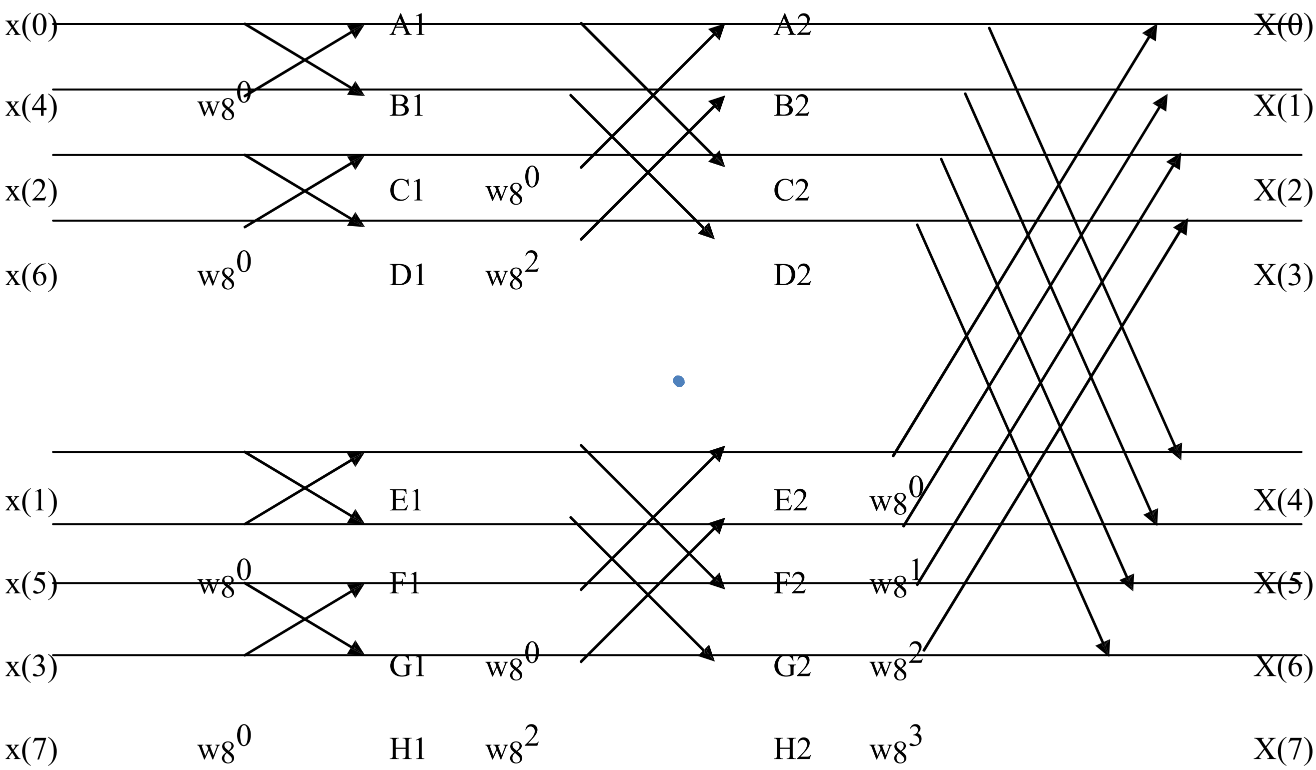


Fig 6. SIGNAL FLOW GRAPH FOR RADIX- DIT FFT N=8

1.9 COMPUTATIONAL COMPLEXITY FFT V/S DIRECT COMPUTATION

For Radix-2 algorithm value of N is given as $N= 2^V$
Hence value of v is calculated as

$$V= \log_{10} N / \log_{10} 2$$
$$= \log_2 N$$

Thus if value of N is 8 then the value of v=3. Thus three stages of decimation. Total number of butterflies will be Nv/2 = 12.
If value of N is 16 then the value of v=4. Thus four stages of decimation. Total number of butterflies will be Nv/2 = 32.

Each butterfly operation takes two addition and one multiplication operations. Direct computation requires N^2 multiplication operation & N^2 – N addition operations.

N	**Direct computation**		**DIT FFT algorithm**		**Improvement in processing speed for multiplication**
	Complex Multiplication N^2	Complex Addition N^2 - N	Complex Multiplication N/2 $\log_2$ N	Complex Addition N $\log_2$ N	
8	64	52	12	24	5.3 times
16	256	240	32	64	8 times
256	65536	65280	1024	2048	64 times

MEMORY REQUIREMENTS AND IN PLACE COMPUTATION

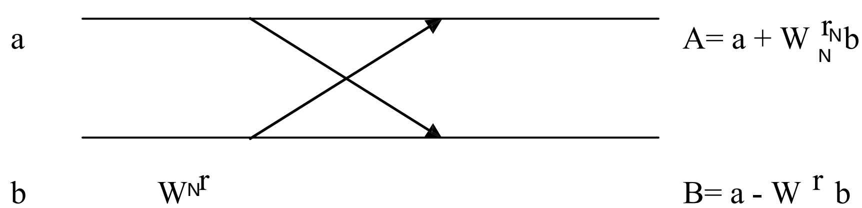


Fig. BUTTERFLY COMPUTATION

From values a and b new values A and B are computed. Once A and B are computed, there is no need to store a and b. Thus same memory locations can be used to store A and B where a and b were stored hence called as In place computation. The advantage of in place computation is that it reduces memory requirement.

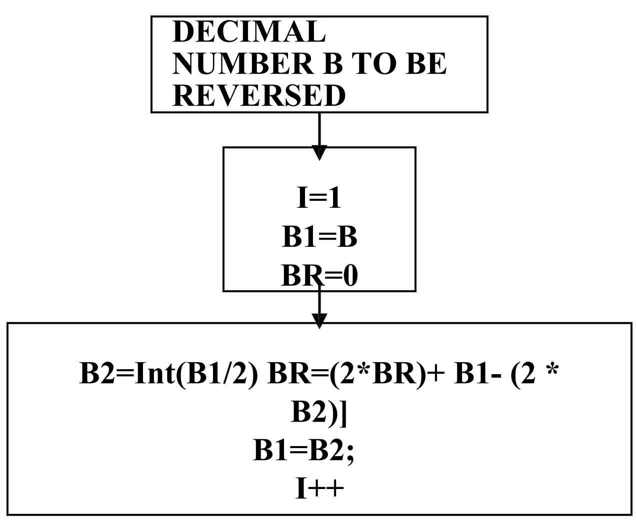
Thus for computation of one butterfly, four memory locations are required for storing two complex numbers A and B. In every stage there are N/2 butterflies hence total 2N memory locations are required. 2N locations are required for each stage. Since stages are computed successively these memory locations can be shared. In every stage N/2 twiddle factors are required hence maximum storage requirements of N point DFT will be (2N + N/2).

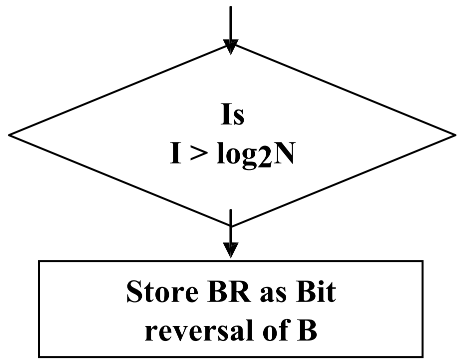
1.10 BIT REVERSAL

For 8 point DIT DFT input data sequence is written as x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7) and the DFT sequence X(k) is in proper order as X(0), X(1), X(2), X(3), X(4), x(5), X(6), x(7). In DIF FFT it is exactly opposite. This can be obtained by bit reversal method.

Decimal	**Memory Address x(n) in binary (Natural Order)**			**Memory Address in bit reversed order**			**New Address in decimal**
0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	4
2	0	1	0	0	1	0	2
3	0	1	1	1	1	0	6
4	1	0	0	0	0	1	1
5	1	0	1	1	0	1	5
6	1	1	0	0	1	1	3
7	1	1	1	1	1	1	7

Table shows first column of memory address in decimal and second column as binary. Third column indicates bit reverse values. As FFT is to be implemented on digital computer simple integer division by 2 method is used for implementing bit reversal algorithms. Flow chart for Bit reversal algorithm is as follows





1.11. DECIMATION IN FREQUENCY (DIFFFT)

In DIF N Point DFT is splitted into N/2 points DFT□s. X(k) is splitted with k even and k odd this is called Decimation in frequency(DIF FFT).

N point DFT is given as

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \quad (1)$$

Since the sequence x(n) is splitted N/2 point samples, thus

$$X(k) = \sum_{m=0}^{N/2-1} x(n)\, W_N + \sum_{m=0}^{N/2-1} x(n + N/2)\, W_N \quad (2)$$

$$X(k) = \sum_{m=0}^{N/2-1} x(n)\, W_N^{kn} + W_N^{kN/2} \sum_{m=0}^{N/2-1} x(n + N/2)\, W_N^{kn}$$

$$X(k) = \sum_{m=0}^{N/2-1} x(n)\, W_N^{kn} + (-1)^k \sum_{m=0}^{N/2-1} x(n + N/2)\, W_N^{kn}$$

$$X(k) = \sum_{m=0}^{N/2-1} \left[x(n) + (-1)^k\, x(n + N/2)\, W_N^{kn} \right] \quad (3)$$

Let us split X(k) into even and odd numbered samples

$$X(2k) = \sum_{m=0}^{N/2-1} \left[x(n) + (-1)^{2k}\, x(n + N/2)\, W_N^{2kn} \right] \quad (4)$$

$$X(2k+1) = \sum_{m=0}^{N/2-1} \left[x(n) + (-1)^{(2k+1)} x(n+N/2) W_N^{(2k+1)n} \right] \qquad (5)$$

Equation (4) and (5) are thus simplified as

$$g1(n) = x(n) + x(n+N/2)$$
$$g2(n) = x(n) - x(n+N/2)\, W_N^{\;n}$$

Fig 1 shows **Butterfly computation** in DIF FFT.

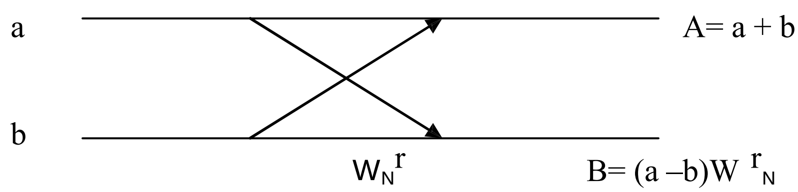


Fig 1. BUTTERFLY COMPUTATION

Fig 2 shows signal flow graph and stages for computation of radix-2 DIF FFT algorithm of N=4

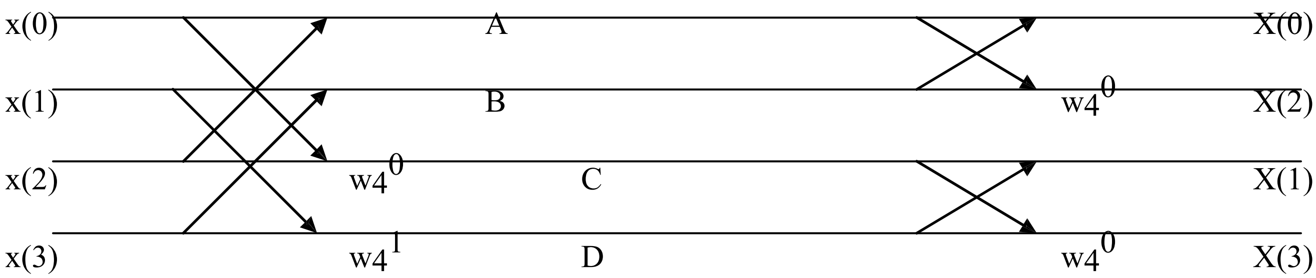
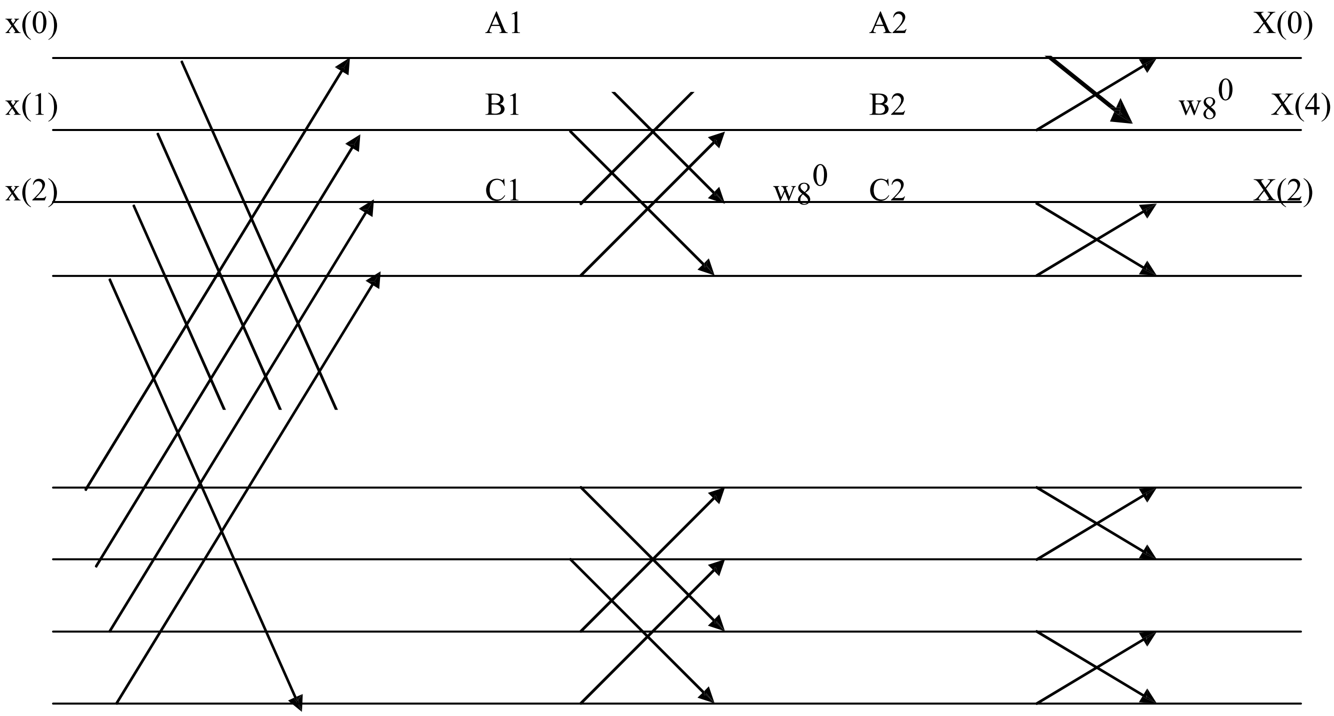


Fig 2. SIGNAL FLOW GRAPH FOR RADIX- DIF FFT N=4

Fig 3 shows signal flow graph and stages for computation of radix-2 DIF FFT algorithm of N=8



x(3)		D1	w_8^2	D2	w_8^0	X(6)
x(4)	w_8^0	E1		E2		X(1)
x(5)	w_8^1	F1		F2	w_8^0	X(5)
x(6)	w_8^2	G1	w_8^0	G2		X(3)
x(7)	w_8^3	H1	w_8^2	H2	w_8^0	X(7)

Fig 3. SIGNAL FLOW GRAPH FOR RADIX- DIF FFT N=8

DIFFERENCE BETWEEN DITFFT AND DIFFFT

Sr No	DIT FFT	DIF FFT
1	DITFFT algorithms are based upon decomposition of the input sequence into smaller and smaller sub sequences.	DIFFFT algorithms are based upon decomposition of the output sequence into smaller and smaller sub sequences.
2	In this input sequence x(n) is splitted into even and odd numbered samples	In this output sequence X(k) is considered to be splitted into even and odd numbered samples
3	Splitting operation is done on time domain sequence.	Splitting operation is done on frequency domain sequence.
4	In DIT FFT input sequence is in bit reversed order while the output sequence is in natural order.	In DIFFFT, input sequence is in natural order. And DFT should be read in bit reversed order.

DIFFERENCE BETWEEN DIRECT COMPUTATION & FFT

Sr No	Direct Computation	Radix -2 FFT Algorithms
1	Direct computation requires large number of computations as compared with FFT algorithms.	Radix-2 FFT algorithms requires less number of computations.
2	Processing time is more and more for large number of N hence processor remains busy.	Processing time is less hence these algorithms compute DFT very quickly as compared with direct computation.

3	Direct computation does not requires splitting operation.	Splitting operation is done on time domain basis (DIT) or frequency domain basis (DIF)
4	As the value of N in DFT increases, the efficiency of direct computation decreases.	As the value of N in DFT increases, the efficiency of FFT algorithms increases.
5	In those applications where DFT is to be computed only at selected values of k(frequencies) and when these values are less than $\log_2 N$ then direct computation becomes more efficient than FFT.	Applications 1) Linear filtering 2) Digital filter design

Q) x(n)={1,2,2,1} Find X(k) using DITFFT. Q) x(n)={1,2,2,1} Find X(k) using DIFFFT.

Q) x(n)={0.3535,0.3535,0.6464,1.0607,0.3535,-1.0607,-1.3535,-0.3535} Find X(k) using DITFFT.

Q) Using radix 2 FFT algorithm, plot flow graph for N=8.

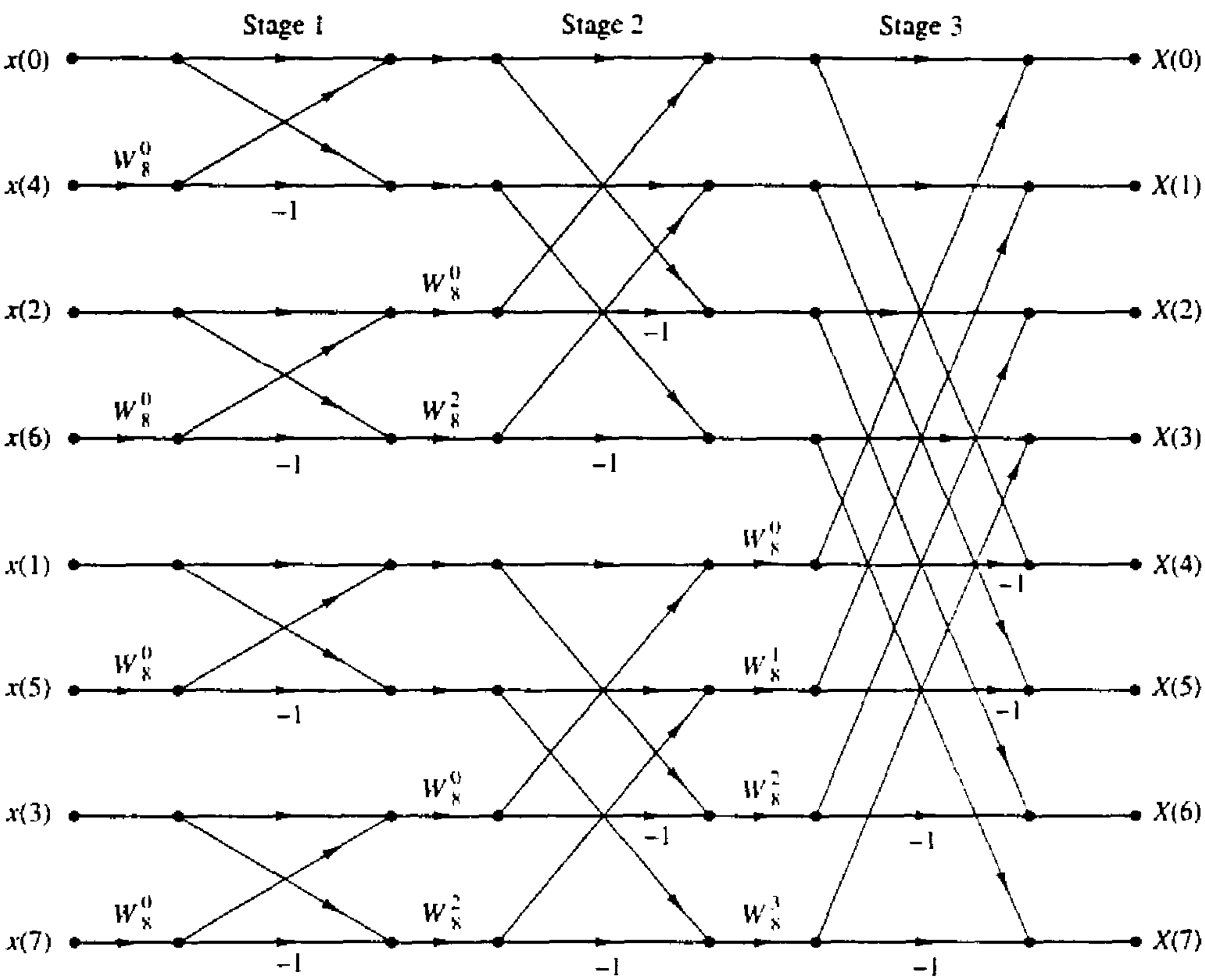
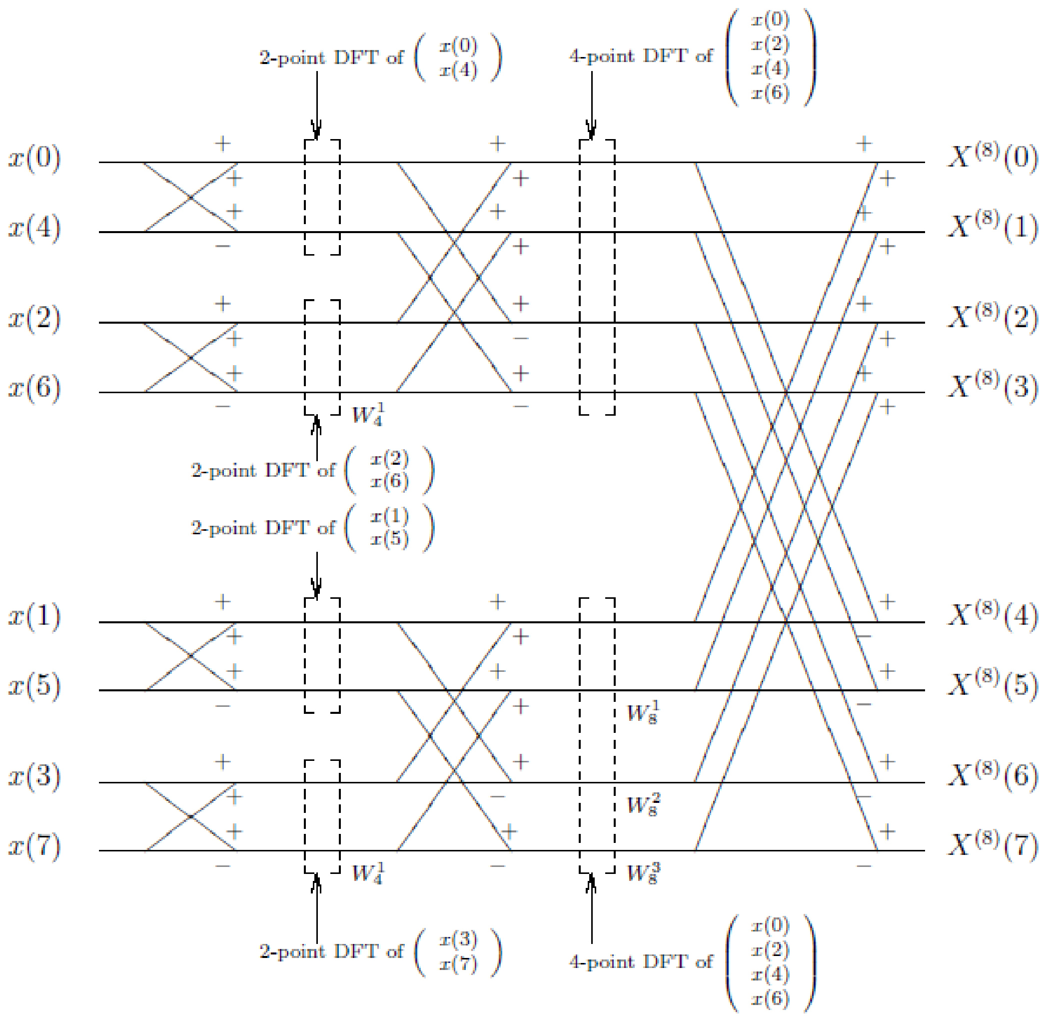


Figure 6.6 Eight-point decimation-in-time FFT algorithm.



$$
\begin{aligned}
W_2 &= e^{-j\frac{2\pi}{2}} = -1, \\
W_4 &= e^{-j\frac{2\pi}{4}} = -j, \\
W_8 &= e^{-j\frac{2\pi}{8}}. \quad \blacksquare
\end{aligned}
$$

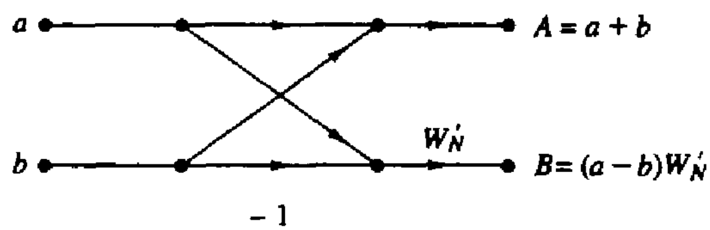


Figure 6.10 Basic butterfly computation in the decimation-in-frequency FFT algorithm.

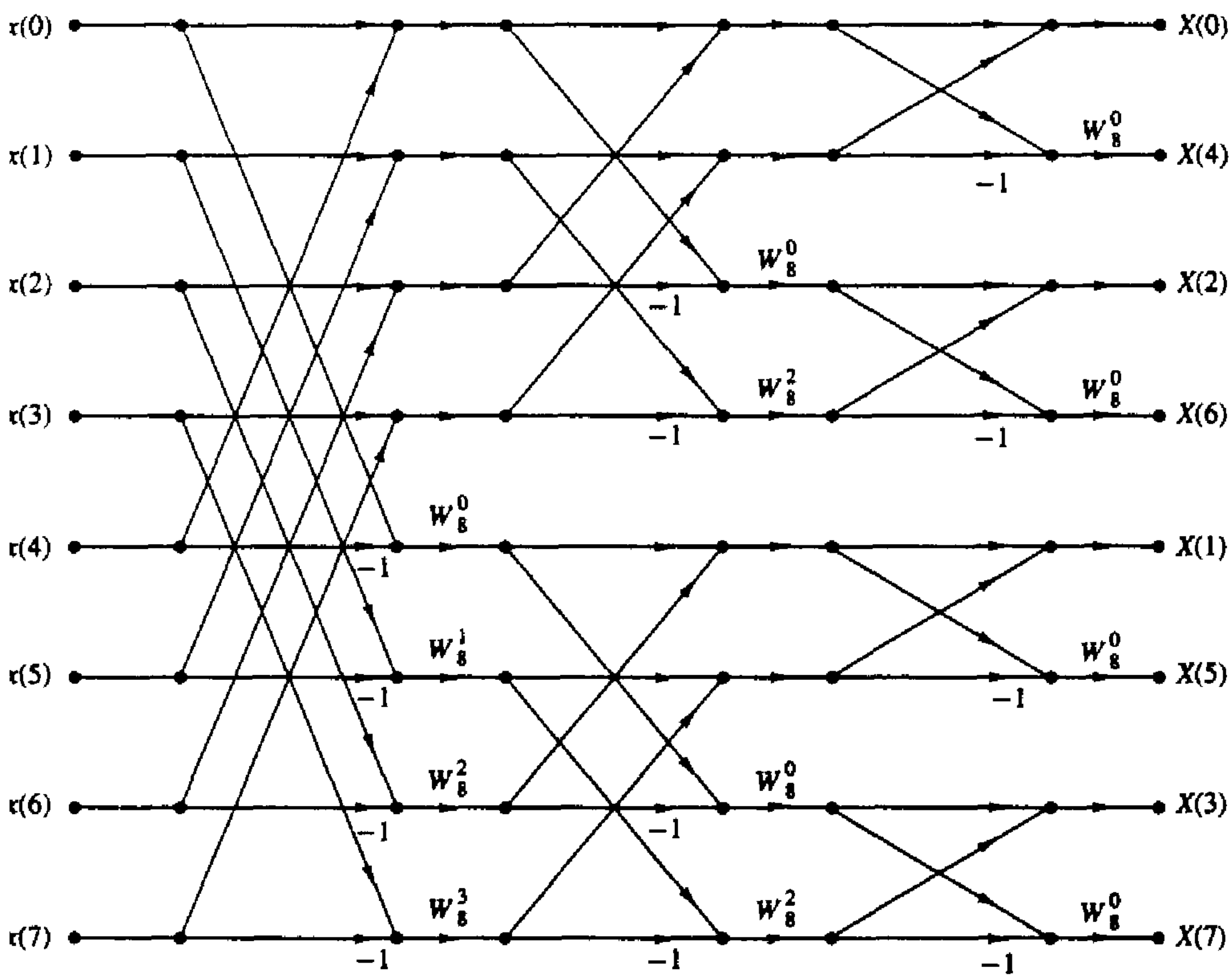


Figure 6.11 N = 8-point decimation-in-frequency FFT algorithmn.

1.12 GOERTZEL ALGORITHM:

FFT algorithms are used to compute N point DFT for N samples of the sequence x(n). This requires N/2 $\log_2 N$ number of complex multiplications and N $\log_2 N$ complex additions. In some applications DFT is to be computed only at selected values of frequencies and selected values are less than $\log_2 N$, then direct computations of DFT becomes more efficient than FFT. This direct computations of DFT can be realized through linear filtering of x(n). Such linear filtering for computation of DFT can be implemented using Goertzel algorithm.

By definition N point DFT is given as

$$X(k) = \sum_{m=0}^{N-1} x(m)\, W_N^{km} \qquad (1)$$

Multiplying both sides by W_N^{-kN} (which is always equal to 1).

$$X(k) = \sum_{m=0}^{N-1} x(m)\, W_N^{k(N-m)} \qquad (2)$$

Thus for LSI system which has input x(n) and having unit sample response

$$h_k(n) = W_N^{-kn}\, u(n)$$

$$X(n) \longrightarrow \boxed{h_k(n) = W_N^{-kn} \quad u(n)} \longrightarrow y_k(n)$$

Linear convolution is given by

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y_k(n) = \sum_{m=-\infty}^{\infty} x(m)\, W_N \qquad u(n-m) \qquad (3)$$

As x(m) is given for N values

$$y_k(n) = \sum_{m=0}^{N-1} x(m)\, W_N^{-} \qquad (4)$$

The output of LSI system at n=N is given by

$$y_k(n)|_{n=N} = \sum_{m=-\infty}^{\infty} x(m)\, W_N^{-k(N-m)} \qquad (5)$$

Thus comparing equation (2) and (5), X(k)

$$= y_k(n)|_{n=N}$$

Thus DFT can be obtained as the output of LSI system at n=N. Such systems can give X(k) at selected values of k. Thus DFT is computed as linear filtering operations by Goertzel Algorithm.

GLOSSARY:

Fourier Transform:

The Transform that used to analyze the signals or systems Characteristics in frequency domain , which is difficult in case of Time Domain.

Laplace Transform:

Laplace Transform is the basic continuous Transform. Then it is developed to represent the continuous signals in frequency domain.

Discrete Time Fourier Transform:

For analyzing the discrete signals, the DTFT (Discrete Time Fourier Transform) is used. The output, that the frequency is continuous in DTFT. But the Transformed Value should be discrete. Since the Digital Signal Processors cannot work with the continuous frequency signals. So the DFT is developed to represent the discrete signals in discrete frequency domain.

Discrete Fourier Transform:

Discrete Fourier Transform is used for transforming a discrete time sequence of finite length “N” into a discrete frequency sequence of the same finite length “N”.

Periodicity:

If a discrete time signal is periodic then its DFT is also periodic. i.e. if a signal or sequence is repeated after N Number of samples, then it is called periodic signal.

Symmetry:

If a signal or sequence is repeated its waveform in a negative direction after “N/2” number of Samples, then it is called symmetric sequence or signal.

Linearity:

A System which satisfies the superposition principle is said to be a linear system. The DFT have the Linearity property. Since the DFT of the output is equal to the sum of the DFT’s of the Inputs.

Fast Fourier Transform:

Fast Fourier Transform is an algorithm that efficiently computes the discrete fourier transform of a sequence x(n). The direct computation of the DFT requires $2N^2$ evaluations of trignometric functions. $4N^2$ real multiplications and 4N(N-1) real additions.

UNIT II
IIR FILTER DESIGN

PREREQUISITE DISCUSSION:

Basically a digital filter is a linear time –invariant discrete time system. The terms Finite Impulse response (FIR) and Infinite Impulse Response (IIR) are used to distinguish filter types. The FIR filters are of Non-Recursive type whereas the IIR Filters are of recursive type.

2.1 INTRODUCTION

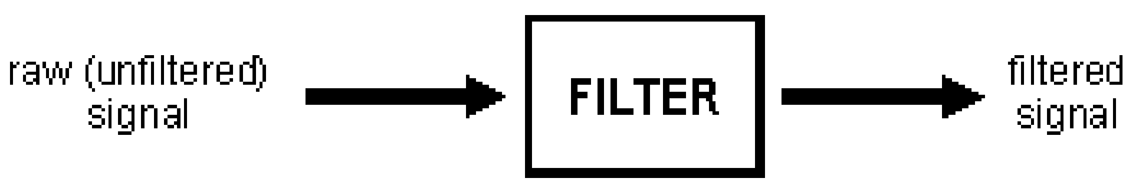
To remove or to reduce strength of unwanted signal like noise and to improve the quality of required signal filtering process is used. To use the channel full bandwidth we mix up two or more signals on transmission side and on receiver side we would like to separate it out in efficient way. Hence filters are used. Thus the digital filters are mostly used in

1. Removal of undesirable noise from the desired signals
2. Equalization of communication channels
3. Signal detection in radar, sonar and communication
4. Performing spectral analysis of signals.

Analog and digital filters

In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within a certain frequency range.

The following block diagram illustrates the basic idea.



There are two main kinds of filter, *analog* and *digital*. They are quite different in their physical makeup and in how they work.

An analog filter uses analog electronic circuits made up from components such as resistors, capacitors and op amps to produce the required filtering effect. Such filter circuits are widely used in such applications as noise reduction, video signal enhancement, graphic equalizers in hi-fi systems, and many other areas.

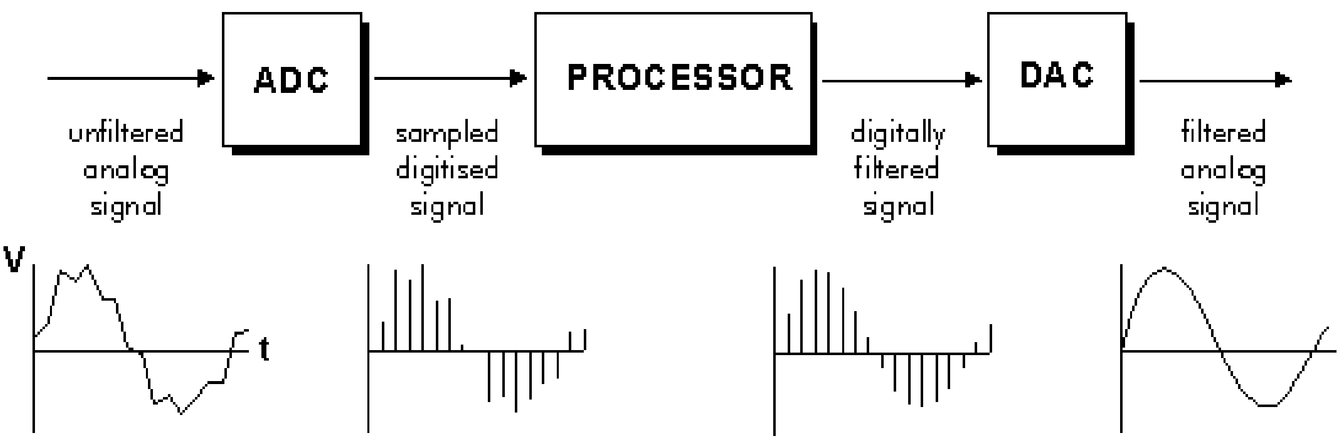
In analog filters the signal being filtered is an electrical voltage or current which is the direct analogue of the physical quantity (e.g. a sound or video signal or transducer output) involved.

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialized DSP (Digital Signal Processor) chip.

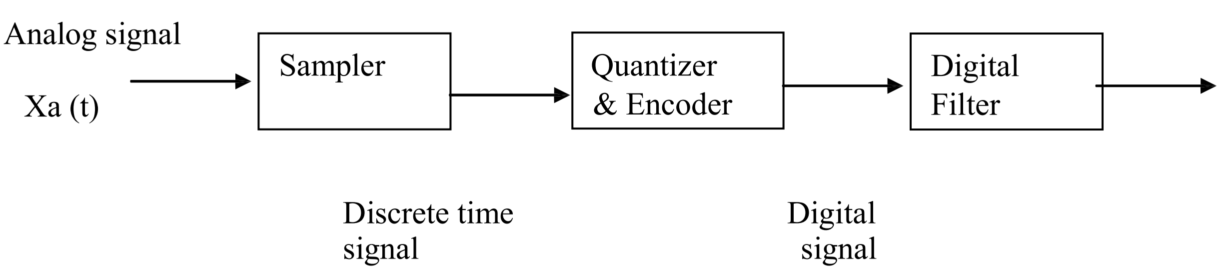
The analog input signal must first be sampled and digitized using an ADC (analog to digital converter). The resulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calculations on them. These calculations typically involve multiplying the input values by constants and adding the products together. If necessary, the results of these calculations, which now represent sampled values of the filtered signal, are output through a DAC (digital to analog converter) to convert the signal back to analog form.

In a digital filter, the signal is represented by a sequence of numbers, rather than a voltage or current.

The following diagram shows the basic setup of such a system.



BASIC BLOCK DIAGRAM OF DIGITAL FILTERS



1. Samplers are used for converting continuous time signal into a discrete time signal by taking samples of the continuous time signal at discrete time instants.

2. The Quantizer are used for converting a discrete time continuous amplitude signal into a digital signal by expressing each sample value as a finite number of digits.

3. In the encoding operation, the quantization sample value is converted to the binary equivalent of that quantization level.

4. The digital filters are the discrete time systems used for filtering of sequences. These digital filters performs the frequency related operations such as low pass, high pass, band pass and band reject etc. These digital Filters are designed with digital hardware and software and are represented by difference equation.

DIFFERENCE BETWEEN ANALOG FILTER AND DIGITAL FILTER

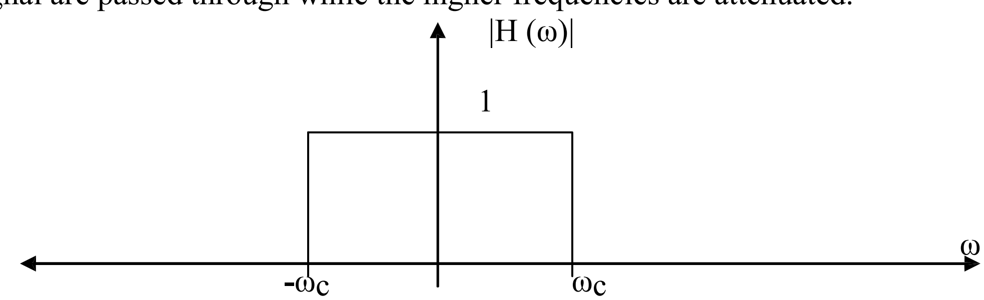
Sr No	Analog Filter	Digital Filter
1	Analog filters are used for filtering analog signals.	Digital filters are used for filtering digital sequences.
2	Analog filters are designed with various components like resistor, inductor and capacitor	Digital Filters are designed with digital hardware like FF, counters shift registers, ALU and software□s like C or assembly language.
3	Analog filters less accurate & because of component tolerance of active components & more sensitive to environmental changes.	Digital filters are less sensitive to the environmental changes, noise and disturbances. Thus periodic calibration can be avoided. Also they are extremely stable.
4	Less flexible	These are most flexible as software programs & control programs can be easily modified. Several input signals can be filtered by one digital filter.
5	Filter representation is in terms of system components.	Digital filters are represented by the difference equation.
6	An analog filter can only be changed by redesigning the filter circuit.	A digital filter is programmable, i.e. its operation is determined by a program stored in the processor's memory. This means the digital filter can easily be changed without affecting the circuitry (hardware).

FILTER TYPES AND IDEAL FILTER CHARACTERISTIC:

Filters are usually classified according to their frequency-domain characteristic as lowpass, highpass, bandpass and bandstop filters.

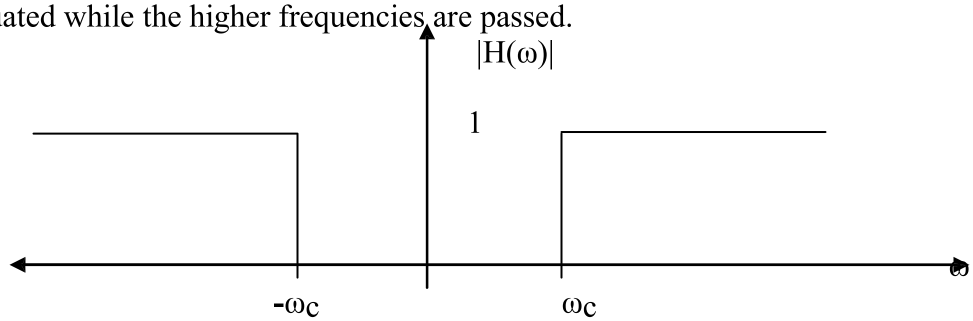
1. Lowpass Filter

A lowpass filter is made up of a passband and a stopband, where the lower frequencies Of the input signal are passed through while the higher frequencies are attenuated.



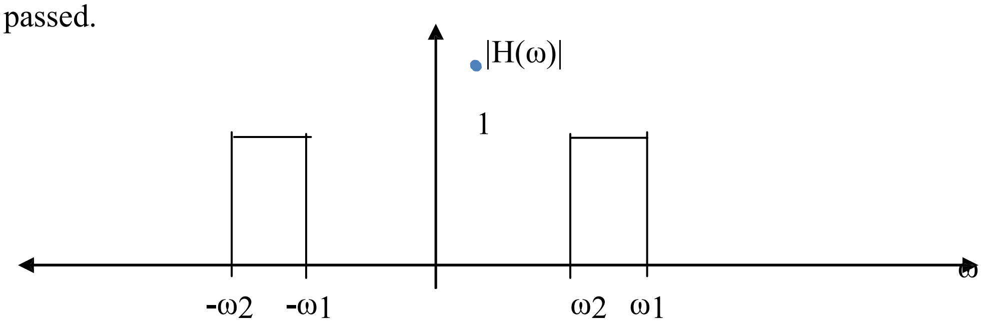
2. Highpass Filter

A highpass filter is made up of a stopband and a passband where the lower frequencies of the input signal are attenuated while the higher frequencies are passed.



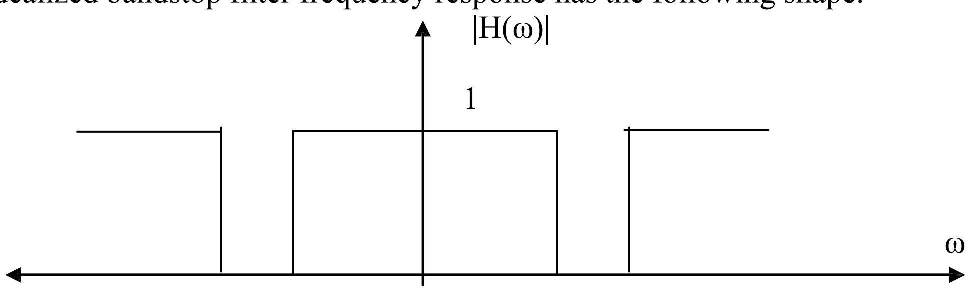
3. Bandpass Filter

A bandpass filter is made up of two stopbands and one passband so that the lower and higher frequencies of the input signal are attenuated while the intervening frequencies are passed.



4. Bandstop Filter

A bandstop filter is made up of two passbands and one stopband so that the lower and higher frequencies of the input signal are passed while the intervening frequencies are attenuated. An idealized bandstop filter frequency response has the following shape.



5. Multipass Filter

A multipass filter begins with a stopband followed by more than one passband. By default, a multipass filter in Digital Filter Designer consists of three passbands and four stopbands. The frequencies of the input signal at the stopbands are attenuated while those at the passbands are passed.

6. Multistop Filter

A multistop filter begins with a passband followed by more than one stopband. By default, a multistop filter in Digital Filter Designer consists of three passbands and two stopbands.

7. All Pass Filter

An all pass filter is defined as a system that has a constant magnitude response for all frequencies.

$$|H(\omega)| = 1 \qquad \text{for } 0 \le \omega < \prod$$

The simplest example of an all pass filter is a pure delay system with system function $H(z) = Z^{-k}$. This is a low pass filter that has a linear phase characteristic.

All Pass filters find application as phase equalizers. When placed in cascade with a system that has an undesired phase response, a phase equalizers is designed to compensate for the poor phase characteristic of the system and therefore to produce an overall linear phase response.

IDEAL FILTER CHARACTERISTIC

1. Ideal filters have a constant gain (usually taken as unity gain) passband characteristic and zero gain in their stop band.
2. Ideal filters have a linear phase characteristic within their passband.
3. Ideal filters also have constant magnitude characteristic.
4. Ideal filters are physically unrealizable.

2.2 TYPES OF DIGITAL FILTER

Digital filters are of two types. Finite Impulse Response Digital Filter & Infinite Impulse Response Digital Filter

DIFFERENCE BETWEEN FIR FILTER AND IIR FILTER

Sr No	FIR Digital Filter	IIR Digital Filter
1	FIR system has finite duration unit sample response. i.e $h(n) = 0$ for $n<0$ and $n \ge M$ Thus the unit sample response exists for the duration from 0 to M-1.	IIR system has infinite duration unit sample response. i. e $h(n) = 0$ for $n<0$ Thus the unit sample response exists for the duration from 0 to ∞.
2	FIR systems are non recursive. Thus output of FIR filter depends upon present and past inputs.	IIR systems are recursive. Thus they use feedback. Thus output of IIR filter depends upon present and past inputs as well as past outputs
3	Difference equation of the LSI system for FIR filters becomes $y(n)=\sum_{k=0}^{M} b_k\ x(n-k)$	Difference equation of the LSI system for IIR filters becomes $y(n)=-\sum_{k=1}^{N} a_k\ y(n-k)+\sum_{k=0}^{M} b_k\ x(n-k)$
4	FIR systems has limited or finite memory requirements.	IIR system requires infinite memory.

5	FIR filters are always stable	Stability cannot be always guaranteed.
6	FIR filters can have an exactly linear phase response so that no phase distortion is introduced in the signal by the filter.	IIR filter is usually more efficient design in terms of computation time and memory requirements. IIR systems usually requires less processing time and storage as compared with FIR.
7	The effect of using finite word length to implement filter, noise and quantization errors are less severe in FIR than in IIR.	Analogue filters can be easily and readily transformed into equivalent IIR digital filter. But same is not possible in FIR because that have no analogue counterpart.
8	All zero filters	Poles as well as zeros are present.
9	FIR filters are generally used if no phase distortion is desired. Example: System described by Y(n) = 0.5 x(n) + 0.5 x(n-1) is FIR filter. h(n)={0.5,0.5}	IIR filters are generally used if sharp cutoff and high throughput is required. Example: System described by Y(n) = y(n-1) + x(n) is IIR filter. $h(n)=a^n\ u(n)$ for $n\geq 0$

2. 3 STRUCTURES FOR FIR SYSTEMS

FIR Systems are represented in four different ways

1. Direct Form Structures
2. Cascade Form Structure
3. Frequency-Sampling Structures
4. Lattice structures.

1. DIRECT FORM STRUCTURE OF FIR SYSTEM

The convolution of h(n) and x(n) for FIR systems can be written as

$$y(n)=\sum_{k=0}^{M-1} h(k)\, x(n-k) \qquad (1)$$

The above equation can be expanded as,

$$Y(n)= h(0)\, x(n) + h(1)\, x(n-1) + h(2)\, x(n-2) + \ldots\ldots\ldots\ldots + h(M-1)\, x(n-M+1) \qquad (2)$$

Implementation of direct form structure of FIR filter is based upon the above equation.

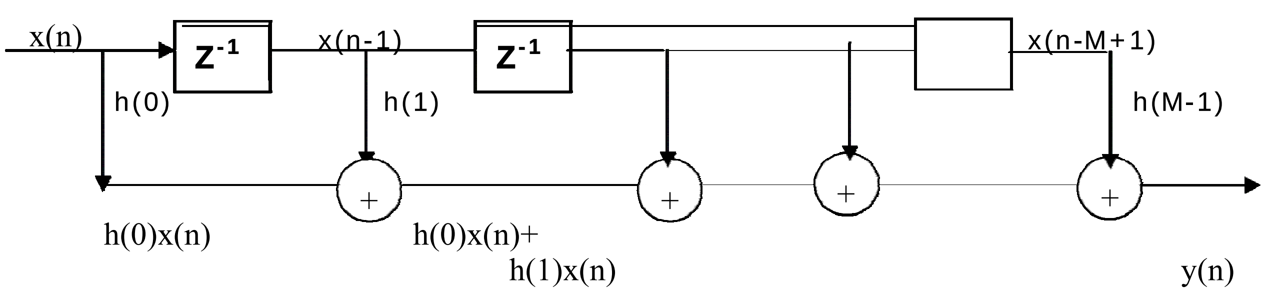


FIG - DIRECT FORM REALIZATION OF FIR SYSTEM

1) There are M-1 unit delay blocks. One unit delay block requires one memory location. Hence direct form structure requires M-1 memory locations.

2) The multiplication of h(k) and x(n-k) is performed for 0 to M-1 terms. Hence M multiplications and M-1 additions are required.

3) Direct form structure is often called as transversal or tapped delay line filter.

2. CASCADE FORM STRUCTURE OF FIR SYSTEM

In cascade form, stages are cascaded (connected) in series. The output of one system is input to another. Thus total K number of stages are cascaded. The total system function 'H' is given by

$$H = H_1(z) \cdot H_2(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots H_k(z) \qquad (1)$$

$$H = Y_1(z)/X_1(z).\ \ Y_2(z)/X_2(z).\ \ldots\ldots\ldots\ldots\ldots Y_k(z)/X_k(z) \qquad (2)$$

$$H(z) = \prod_{k=1}^{k} H_k(z) \qquad (3)$$

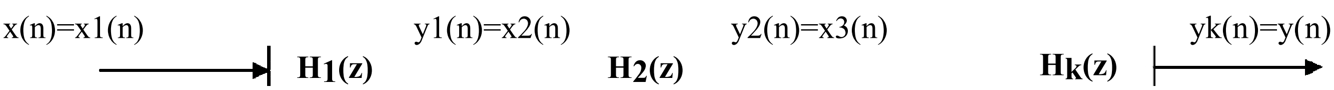


FIG- CASCADE FORM REALIZATION OF FIR SYSTEM

Each H1(z), H2(z)… etc is a second order section and it is realized by the direct form as shown in below figure.

System function for FIR systems

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k} \qquad (1)$$

Expanding the above terms we have

$$H(z) = H_1(z) \cdot H_2(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots H_k(z)$$

where $H_K(z) = b_{k0} + b_{k1}\, z^{-1} + b_{k2}\, z^{-2}$ (2)

Thus Direct form of second order system is shown as

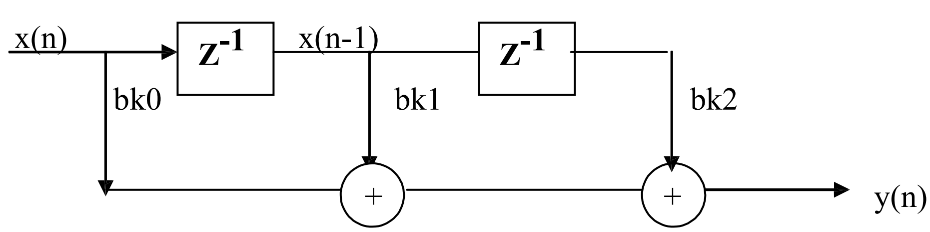


FIG - DIRECT FORM REALIZATION OF FIR SECOND ORDER SYSTEM

2.4 STRUCTURES FOR IIR SYSTEMS

IIR Systems are represented in four different ways

1. Direct Form Structures Form I and Form II
2. Cascade Form Structure
3. Parallel Form Structure
4. Lattice and Lattice-Ladder structure.

DIRECT FORM STRUCTURE FOR IIR SYSTEMS

IIR systems can be described by a generalized equations as

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k) \qquad (1)$$

Z transform is given as

$$H(z) = \sum_{K=0}^{M} b_k\, z^{-k} \,/\, 1 + \sum_{k=1}^{N} a_k\, z^{-k} \qquad (2)$$

Here $H1(z) = \sum_{K=0}^{M} b_k\, z^{-k}$ And $H2(z) = 1 + \sum_{k=0}^{N} a_k\, z^{-k}$

Overall IIR system can be realized as cascade of two function H1(z) and H2(z). Here H1(z) represents zeros of H(z) and H2(z) represents all poles of H(z).

DIRECT FORM - I

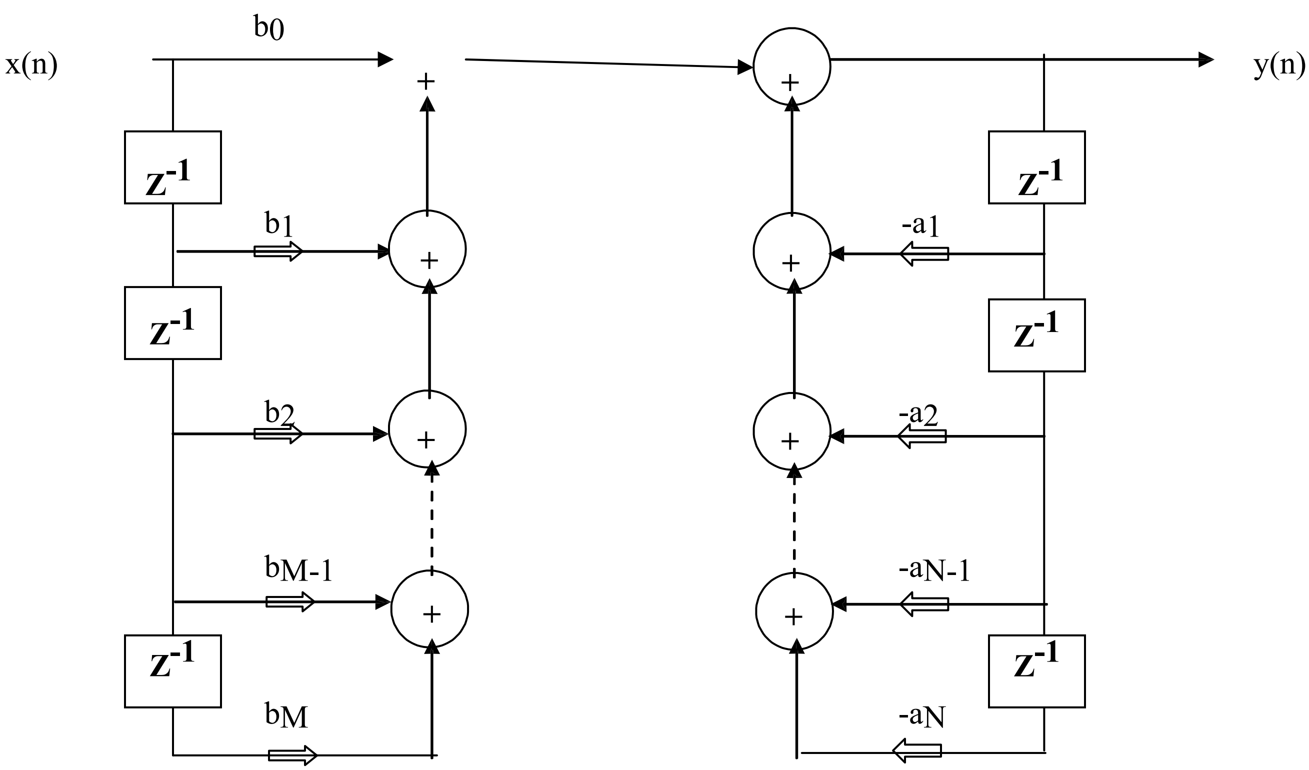


FIG - DIRECT FORM I REALIZATION OF IIR SYSTEM

1. Direct form I realization of H(z) can be obtained by cascading the realization of H1(z) which is all zero system first and then H2(z) which is all pole system.

2. There are M+N-1 unit delay blocks. One unit delay block requires one memory location. Hence direct form structure requires M+N-1 memory locations.

3. Direct Form I realization requires M+N+1 number of multiplications and M+N number of additions and M+N+1 number of memory locations.

DIRECT FORM - II

1. Direct form realization of H(z) can be obtained by cascading the realization of H1(z) which is all pole system and H2(z) which is all zero system.

2. Two delay elements of all pole and all zero system can be merged into single delay element.

3. Direct Form II structure has reduced memory requirement compared to Direct form I structure. Hence it is called canonic form.

4. The direct form II requires same number of multiplications(M+N+1) and additions (M+N) as that of direct form I.

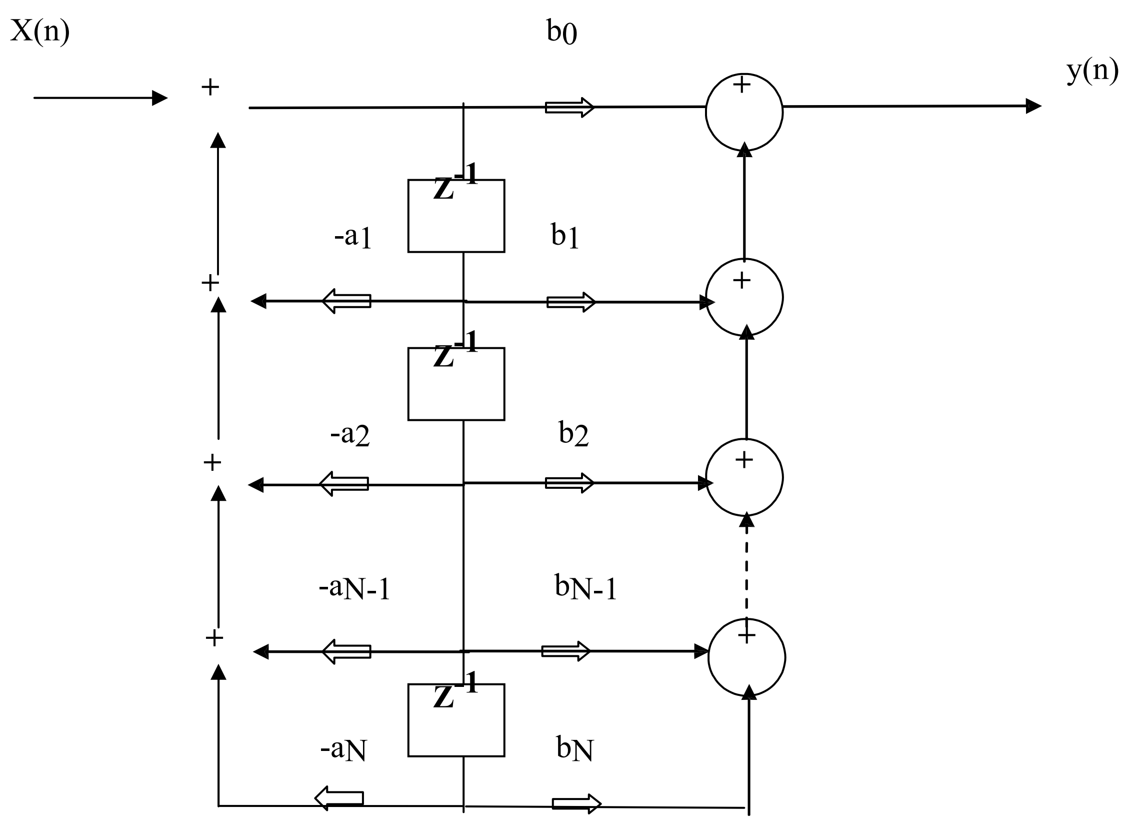


FIG - DIRECT FORM II REALIZATION OF IIR SYSTEM

CASCADE FORM STRUCTURE FOR IIR SYSTEMS

In cascade form, stages are cascaded (connected) in series. The output of one system is input to another. Thus total K number of stages are cascaded. The total system function 'H' is given by

$$H= H_1(z) \,.\, H_2(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. H_k(z) \qquad (1)$$

$$H= Y_1(z)/X_1(z). \;\; Y_2(z)/X_2(z). \ldots\ldots\ldots\ldots\ldots Y_k(z)/X_k(z) \qquad (2)$$

$$H(z)=\pi_{k=1}^{k} H_k(z) \qquad (3)$$

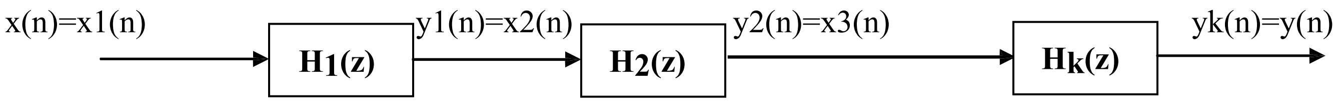


FIG - CASCADE FORM REALIZATION OF IIR SYSTEM

Each H1(z), H2(z)… etc is a second order section and it is realized by the direct form as shown in below figure.

System function for IIR systems

$$H(z) = \sum_{K=0}^{M} b_k \, z^{-k} \; / \; 1+ \sum_{k=1}^{N} a_k \, z^{-k} \qquad (1)$$

Expanding the above terms we have

$$H(z)= H_1(z) \,.\, H_2(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. H_k(z)$$

where $H_K(z) = b_{k0} + b_{k1} \, z^{-1} + b_{k2} \, z^{-2} \; / \; 1 + a_{k1} \, z^{-1} + a_{k2} \, z^{-2}$ (2)

Thus Direct form of second order IIR system is shown as

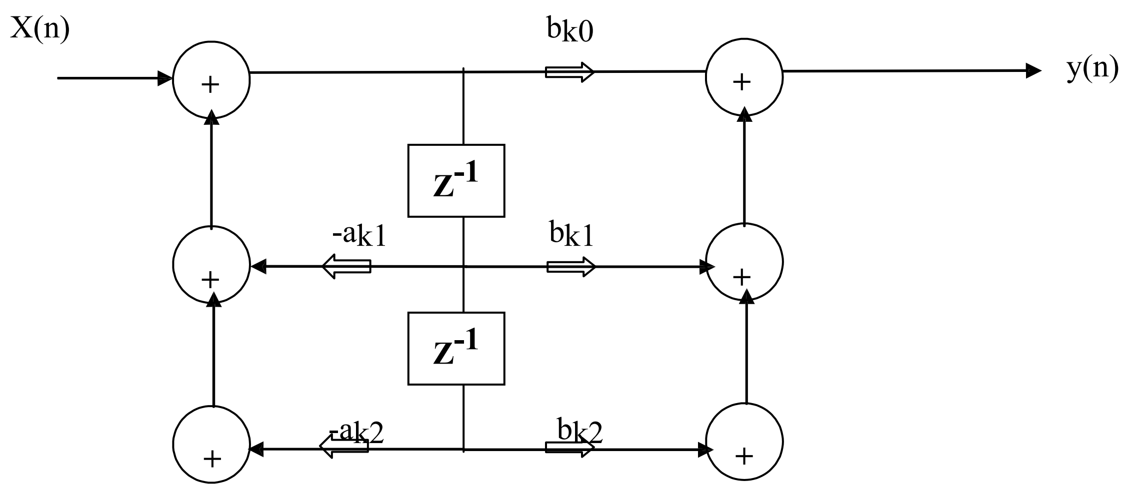


FIG - DIRECT FORM REALIZATION OF IIR SECOND ORDER SYSTEM (CASCADE)

PARALLEL FORM STRUCTURE FOR IIR SYSTEMS

System function for IIR systems is given as

$$H(z) = \sum_{K=0}^{M} b_k z^{-k} \Big/ 1 + \sum_{k=1}^{N} a_k z^{-k} \quad (1)$$

$$= b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots\ldots + b_M z^{-M} / 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots\ldots + a_N z^{-N} \quad (2)$$

The above system function can be expanded in partial fraction as follows

$$H(z) = C + H_1(z) + H_2(z)\ldots\ldots\ldots\ldots + H_k(z) \quad (3)$$

Where C is constant and Hk(z) is given as

$$H_k(z) = b_{k0} + b_{k1} z^{-1} / 1 + a_{k1} z^{-1} + a_{k2} z^{-2} \quad (4)$$

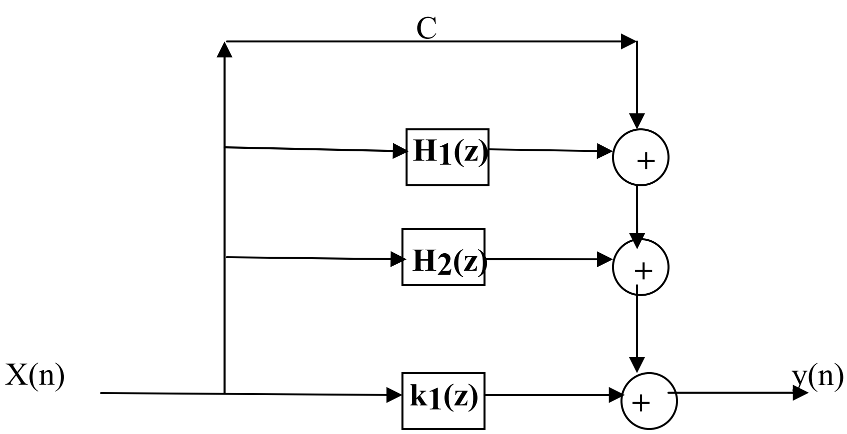


FIG - PARALLEL FORM REALIZATION OF IIR SYSTEM

IIR FILTER DESIGN

1. **IMPULSE INVARIANCE**
2. **BILINEAR TRANSFORMATION**
3. **BUTTERWORTH APPROXIMATION**

4. IIR FILTER DESIGN - IMPULSE INVARIANCE METHOD

Impulse Invariance Method is simplest method used for designing IIR Filters. Important Features of this Method are

1. In impulse variance method, Analog filters are converted into digital filter just by replacing unit sample response of the digital filter by the sampled version of impulse response of analog filter. Sampled signal is obtained by putting t=nT hence

   $h(n) = h_a(nT)$ $\qquad n=0,1,2. \ldots\ldots\ldots\ldots$

   where h(n) is the unit sample response of digital filter and T is sampling interval.
2. But the main disadvantage of this method is that it does not correspond to simple algebraic mapping of S plane to the Z plane. Thus the mapping from analog frequency to digital frequency is many to one. The segments $(2k-1)\prod/T \le \Omega \le (2k+1)\prod/T$ of $j\Omega$ axis are all mapped on the unit circle $\prod\le\omega\le\prod$. This takes place because of sampling.

3. Frequency aliasing is second disadvantage in this method. Because of frequency aliasing, the frequency response of the resulting digital filter will not be identical to the original analog frequency response.

4. Because of these factors, its application is limited to design low frequency filters like LPF or a limited class of band pass filters.

RELATIONSHIP BETWEEN Z PLANE AND S PLANE

Z is represented as $re^{j\omega}$ in polar form and relationship between Z plane and S plane is given as $Z=e^{ST}$ where $s= \sigma + j\,\Omega$.

$Z= e^{ST}$ (Relationship Between Z plane and S plane)

$Z= e^{(\sigma + j\,\Omega)\,T}$

$= e^{\sigma\,T} \,.\, e^{j\,\Omega\,T}$

Comparing Z value with the polar form we have.

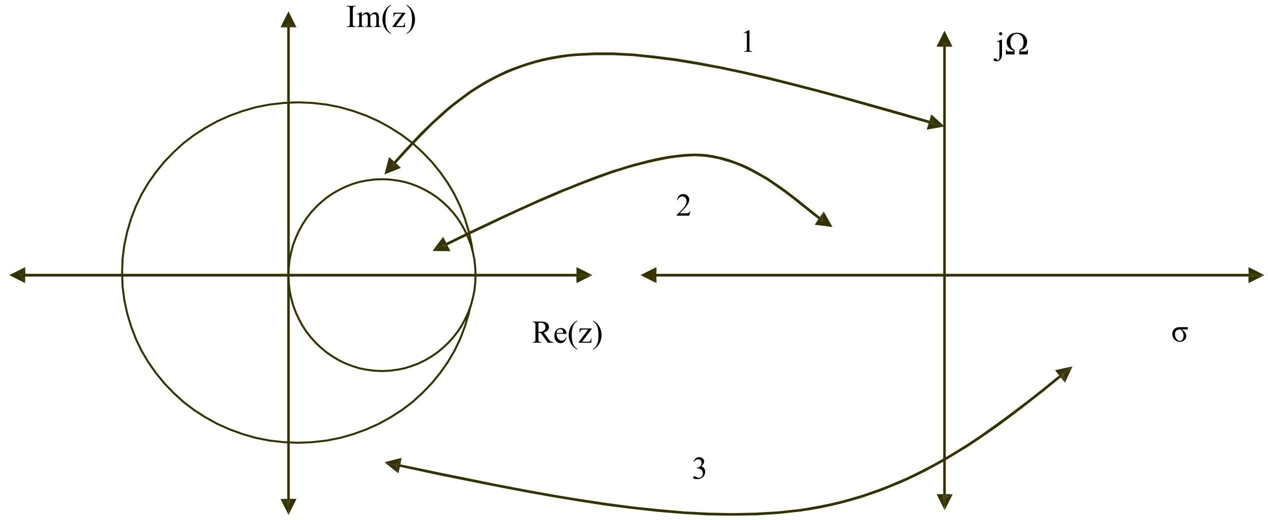
$$\mathbf{r= e^{\sigma\,T} \quad and \quad \omega = \Omega\,T}$$

Here we have three condition

1) If $\sigma = 0$ then $r=1$
2) If $\sigma < 0$ then $0 < r < 1$
3) If $\sigma > 0$ then $r> 1$

Thus

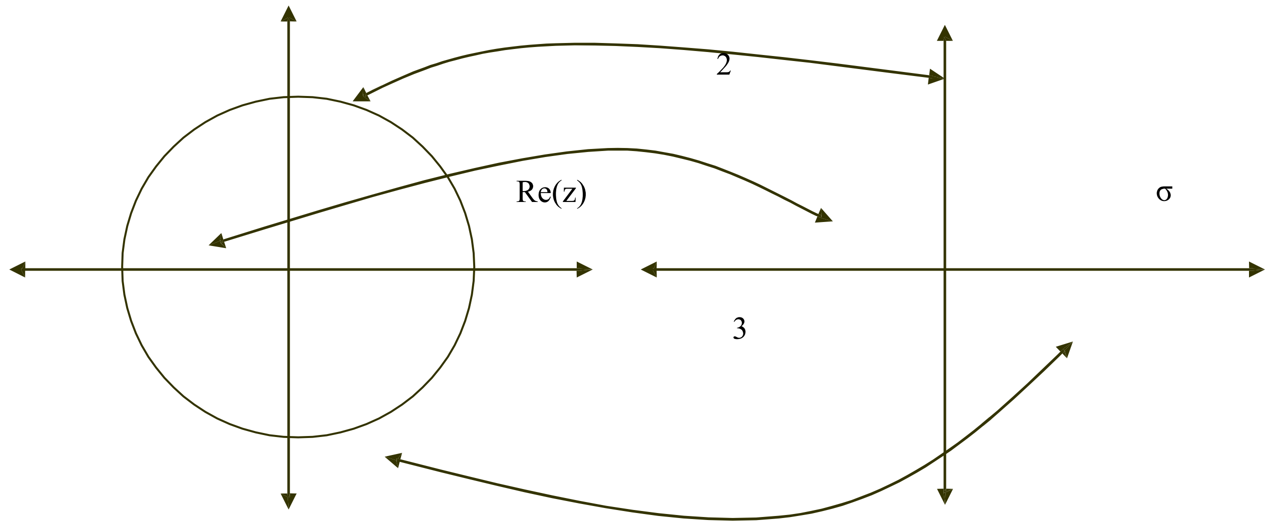
1) Left side of s-plane is mapped inside the unit circle.
2) Right side of s-plane is mapped outside the unit circle.
3) jΩ axis is in s-plane is mapped on the unit circle.



Im(z)

1

jΩ



2.5 CONVERSION OF ANALOG FILTER INTO DIGITAL FILTER

Let the system function of analog filter is

$$Ha(s)= \sum_{k=1}^{n} C_k / s-p_k \qquad (1)$$

where pk are the poles of the analog filter and ck are the coefficients of partial fraction expansion. The impulse response of the analog filter ha(t) is obtained by inverse Laplace transform and given as

$$ha(t) = \sum_{k=1}^{n} C_k \; e^{pkt} \qquad (2)$$

The unit sample response of the digital filter is obtained by uniform sampling of ha(t). h(n) = $h_a(nT)$
n=0,1,2. …………

$$h(n) = \sum_{k=1}^{n} C_k \; e^{pknT} \qquad (3)$$

System function of digital filter H(z) is obtained by Z transform of h(n).

$$H(z) = \sum_{k=1}^{N} C_k \sum_{n=0}^{\infty} \left[e^{pkT} z^{-1} \right]^n \qquad (4)$$

Using the standard relation and comparing equation (1) and (4) system function of digital filter is given as

1 1

$$\frac{}{s - p_k} \longleftrightarrow \frac{}{1 - e^{p_k T} z^{-1}}$$

STANDARD RELATIONS IN IIR DESIGN

Sr No	Analog System Function	Digital System function
1	$\frac{1}{s - a}$	$\frac{1}{1 - e^{aT} z^{-1}}$
2	$\frac{s + a}{(s+a)^2 + b^2}$	$\frac{1 - e^{-aT} (\cos bT) z^{-1}}{1 - 2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}}$
3	$\frac{b}{(s+a)^2 + b^2}$	$\frac{e^{-aT} (\sin bT) z^{-1}}{1 - 2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}}$

EXAMPLES - IMPULSE INVARIANCE METHOD

Sr No	Analog System Function	Digital System function
1	$\frac{s + 0.1}{(s+0.1)^2 + 9}$	$\frac{1 - (e^{-0.1T} \cos 3T) z^{-1}}{1 - 2e^{-0.1T} (\cos 3T) z^{-1} + e^{-0.2T} z^{-2}}$
2	$\frac{1}{(s+1)(s+2)}$ (for sampling frequency of 5 samples/sec)	$\frac{0.148 z}{z^2 - 1.48 z + 0.548}$
3	$\frac{10}{(s+2)}$ (for sampling time is 0.01 sec)	$\frac{10}{1 - z^{-1}}$

2.6 IIR FILTER DESIGN - BILINEAR TRANSFORMATION METHOD (BZT)

The method of filter design by impulse invariance suffers from aliasing. Hence in order to overcome this drawback Bilinear transformation method is designed. In analogue domain frequency axis is an infinitely long straight line while sampled data z plane it is unit circle radius. The bilinear transformation is the method of squashing the infinite straight analog frequency axis so that it becomes finite.

Important Features of Bilinear Transform Method are

1. Bilinear transformation method (BZT) is a mapping from analog S plane to digital Z plane. This conversion maps analog poles to digital poles and analog zeros to digital zeros. Thus all poles and zeros are mapped.

2. This transformation is basically based on a numerical integration techniques used to simulate an integrator of analog filter.

3. There is one to one correspondence between continuous time and discrete time frequency points. Entire range in Ω is mapped only once into the range $-\prod \leq \omega \leq \prod$.

4. Frequency relationship is non-linear. Frequency warping or frequency compression is due to non-linearity. Frequency warping means amplitude response of digital filter is expanded at the lower frequencies and compressed at the higher frequencies in comparison of the analog filter.

5. But the main disadvantage of frequency warping is that it does change the shape of the desired filter frequency response. In particular, it changes the shape of the transition bands.

CONVERSION OF ANALOG FILTER INTO DIGITAL FILTER

Z is represented as $re^{j\omega}$ in polar form and relationship between Z plane and S plane in BZT method is given as

$$S = \frac{2}{T}\,\frac{z-1}{z+1}$$

$$S = \frac{2}{T}\,\frac{re^{j\omega}-1}{re^{j\omega}+1}$$

$$S = \frac{2}{T}\,\frac{r(\cos\omega + j\sin\omega) - 1}{r(\cos\omega + j\sin\omega) + 1}$$

$$S = \frac{2}{T}\left[\frac{r^2 - 1}{1+r^2+2r\cos\omega} + \frac{j\,2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

Comparing the above equation with $S = \sigma + j\,\Omega$. We have

$$\boldsymbol{\sigma = \frac{2}{T}\,\frac{r^2-1}{1+r^2+2r\cos\omega}}$$

$$\boldsymbol{\Omega = \frac{2}{T}\,\frac{2r\sin\omega}{1+r^2+2r\cos\omega}}$$

Here we have three condition

1) If $\sigma < 0$ then $0 < r < 1$
2) If $\sigma > 0$ then $r > 1$
3) If $\sigma = 0$ then $r=1$

When $r = 1$

$$\Omega = 2 \sin\omega$$

$T \quad \overline{1+\cos\omega}$

$\Omega =$ **(2/T) tan (ω/2)**

$\omega = 2 \tan^{-1} (\Omega T/2)$

The above equations shows that in BZT frequency relationship is non-linear. The frequency relationship is plotted as

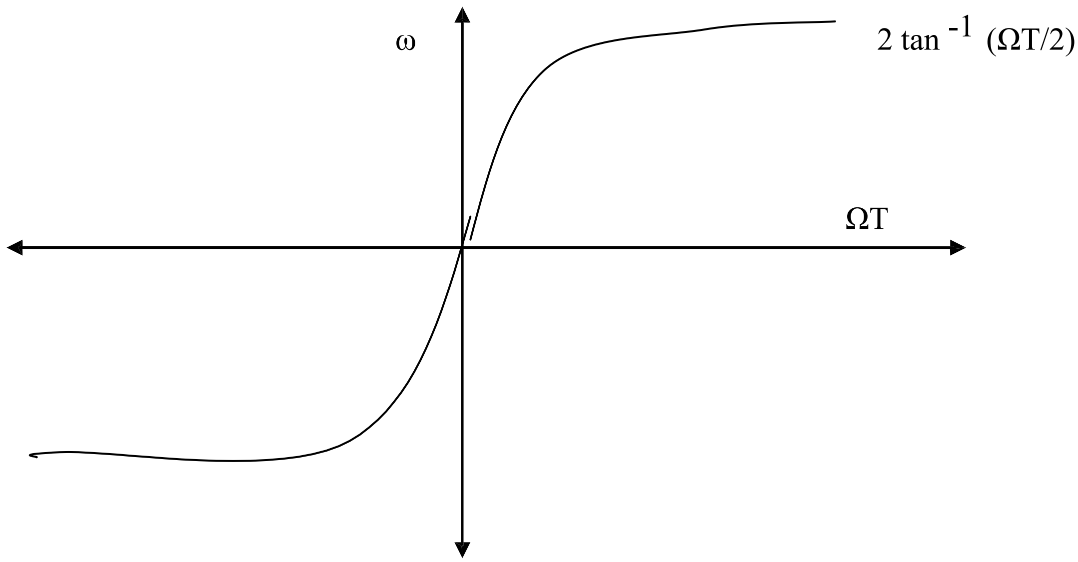


FIG - MAPPING BETWEEN FREQUENCY VARIABLE ω AND Ω IN BZT METHOD.

DIFFERENCE - IMPULSE INVARIANCE Vs BILINEAR TRANSFORMATION

Sr No	Impulse Invariance	Bilinear Transformation
1	In this method IIR filters are designed having a unit sample response h(n) that is sampled version of the impulse response of the analog filter.	This method of IIR filters design is based on the trapezoidal formula for numerical integration.
2	In this method small value of T is selected to minimize the effect of aliasing.	The bilinear transformation is a conformal mapping that transforms the j Ω axis into the unit circle in the z plane only once, thus avoiding aliasing of frequency components.

3	They are generally used for low frequencies like design of IIR LPF and a limited class of bandpass filter	For designing of LPF, HPF and almost all types of Band pass and band stop filters this method is used.
4	Frequency relationship is linear.	Frequency relationship is non-linear. Frequency warping or frequency compression is due to non-linearity.
5	All poles are mapped from the s plane to the z plane by the relationship $Z^k = e^{pkT}$. But the zeros in two domain does not satisfy the same relationship.	All poles and zeros are mapped.

2.7 LPF AND HPF ANALOG BUTTERWORTH FILTER TRANSFER FUNCTION

Sr No	Order of the Filter	Low Pass Filter	High Pass Filter
1	1	$1 / s+1$	$s / s+1$
2	2	$1 / s^2 + \sqrt{2}\, s + 1$	$s^2 / s^2 + \sqrt{2}\, s + 1$
3	3	$1 / s^3 + 2 s^2 + 2s + 1$	$s^3 / s^3 + 2 s^2 + 2s + 1$

2.8 METHOD FOR DESIGNING DIGITAL FILTERS USING BZT

step 1. Find out the value of ω_c^*.

$$\omega_c^* = (2/T) \tan (\omega_c T_s/2)$$

step 2. Find out the value of frequency scaled analog transfer function

Normalized analog transfer function is frequency scaled by replacing s by s/ω_p^*.

step 3. Convert into digital filter

Apply BZT. i.e Replace s by the ((z-1)/(z+1)). And find out the desired transfer function of digital function.

Example:

Q) Design first order high pass butterworth filter whose cutoff frequency is 1 kHz at sampling frequency of 10^4 sps. Use BZT Method

Step 1. To find out the cutoff frequency

$\omega c = 2\prod f$
$= 2000$ rad/sec

Step 2. To find the prewarp frequency

$\omega c^* = \tan (\omega c\ Ts/2)$
$= \tan(\prod/10)$

Step 3. Scaling of the transfer function

For First order HPF transfer function H(s) = s/(s+1) Scaled transfer function $H^{*}(s) = H(s)\,|_{s=s/\omega c^{*}}$

$$H^*(s) = s/(s + 0.325)$$

Step 4. Find out the digital filter transfer function. Replace s by (z-1)/(z+1)

$$H(z) = \frac{z-1}{1.325z - 0.675}$$

Q) Design second order low pass butterworth filter whose cutoff frequency is 1 kHz at sampling frequency of 10^4 sps.

Q) First order low pass butterworth filter whose bandwidth is known to be 1 rad/sec . Use BZT method to design digital filter of 20 Hz bandwidth at sampling frequency 60 sps.

Q) Second order low pass butterworth filter whose bandwidth is known to be 1 rad/sec . Use BZT method to obtain transfer function H(z) of digital filter of 3 DB cutoff frequency of 150 Hz and sampling frequency 1.28 kHz.

Q) The transfer function is given as s^2+1 / s^2+s+1 The function is for Notch filter with frequency 1 rad/sec. Design digital Notch filter with the following specification

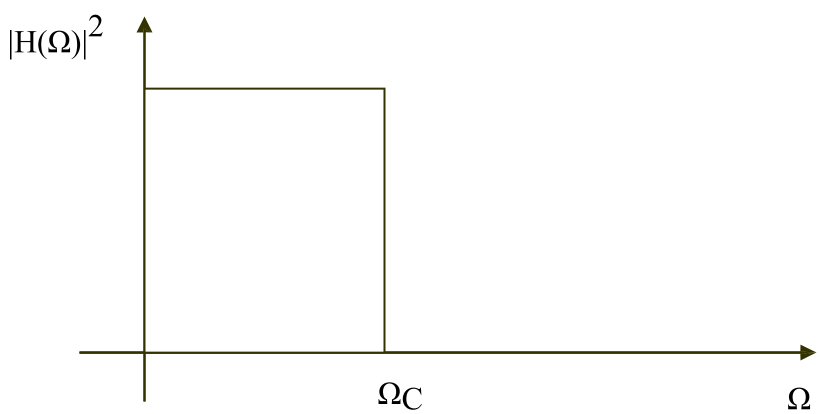
(1) Notch Frequency= 60 Hz
(2) Sampling frequency = 960 sps.

2.9 BUTTERWORTH FILTER APPROXIMATION

The filter passes all frequencies below Ω_C. This is called passband of the filter. Also the filter blocks all the frequencies above Ω_C. This is called stopband of the filter. Ω_C is called cutoff frequency or critical frequency.

No Practical filters can provide the ideal characteristic. Hence approximation of the ideal characteristic are used. Such approximations are standard and used for filter design.
Such three approximations are regularly used. a)
Butterworth Filter Approximation
b) Chebyshev Filter Approximation
c) Elliptic Filter Approximation

Butterworth filters are defined by the property that the magnitude response is maximally flat in the passband.



$$|H_a(\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}$$

The squared magnitude function for an analog butterworth filter is of the form.

$$|H_a(\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}$$

N indicates order of the filter and Ω_c is the cutoff frequency (-3DB frequency).
At s = j Ω magnitude of H(s) and H(-s) is same hence

$$H_a(s)\, H_a(-s) = \frac{1}{1 + (-s^2/\Omega_c^2)^N}$$

To find poles of H(s). H(-s) , find the roots of denominator in above equation.

$$\frac{-s^2}{\Omega_c^2} = (-1)^{1/N}$$

As $e^{j(2k+1)\Pi} = -1$ where k = 0,1,2,…….. N-1.

$$\frac{-s^2}{\Omega_c^2} = (e^{j(2k+1)\Pi})^{1/N}$$

$$s^2 = (-1)\, \Omega_c^2\, e^{j(2k+1)\Pi / N}$$

Taking the square root we get poles of s. $p_k =$

$$+ \sqrt{-1}\, \Omega_c \, [\, e^{j(2k+1)\Pi / N}\,]^{1/2}$$

$$P_k = + j\, \Omega_c\, e^{j(2k+1)\Pi / 2N}$$

As $e^{j\Pi/2} = j$

$$P_k = + \Omega_c\, e^{j\Pi/2}\, e^{j(2k+1)\Pi / 2N}$$

$$\mathbf{P_k = + \Omega_c\, e^{j(N+2k+1)\Pi / 2N}} \qquad \textbf{(1)}$$

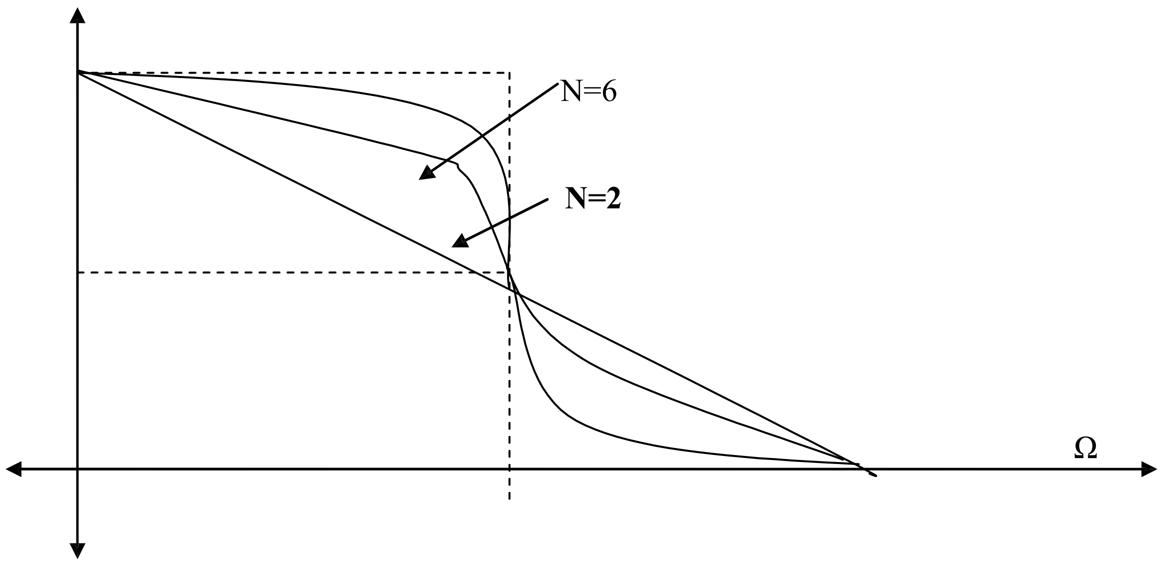
This equation gives the pole position of H(s) and H(-s).

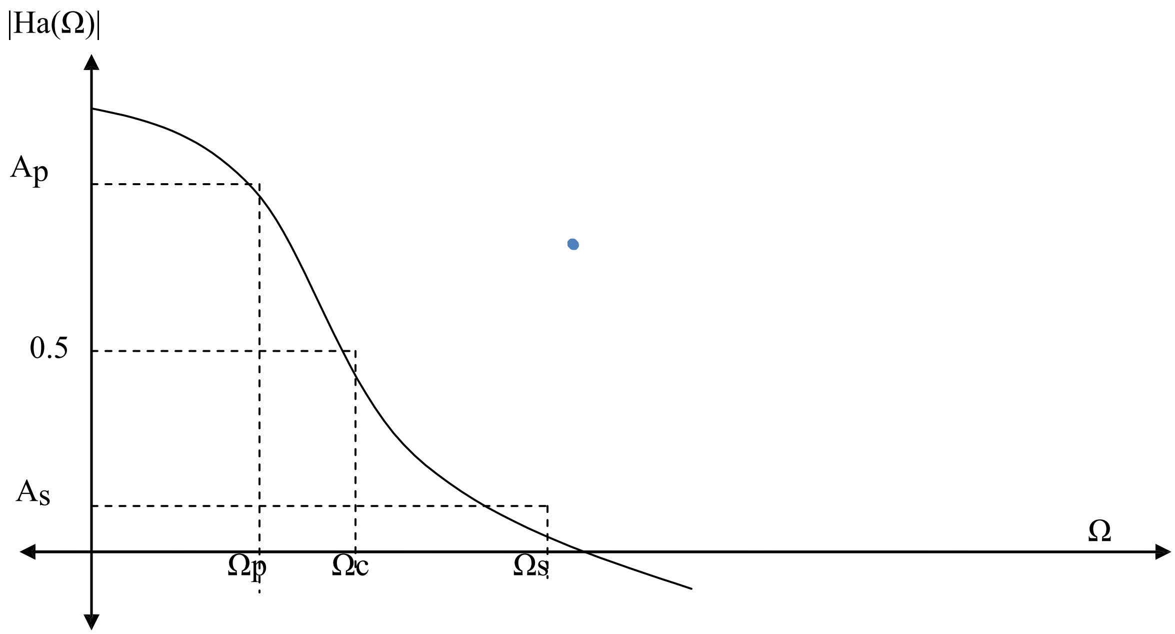
2.10 FREQUENCY RESPONSE CHARACTERISTIC

The frequency response characteristic of $|H_a(\Omega)|^2$ is as shown. As the order of the filter N increases, the butterworth filter characteristic is more close to the ideal characteristic. Thus at higher orders like N=16 the butterworth filter characteristic closely approximate ideal filter characteristic. Thus an infinite order filter (N □ ∞) is required to get ideal characteristic.

$|Ha(\Omega)|^2$

N=18





Ap= attenuation in passband. As= attenuation in stopband. Ω_p = passband edge frequency Ω_s = stopband edge frequency

Specification for the filter is
$|Ha(\Omega)| \geq Ap$ for $\Omega \leq \Omega p$ and $|Ha(\Omega)| \leq As$ for $\Omega \geq \Omega s$. Hence we have

$$\frac{1}{1 + (\Omega p/\Omega c)^{2N}} \geq Ap^2$$

$$\frac{1}{1 + (\Omega s/\Omega c)^{2N}} \leq As^2$$

To determine the poles and order of analog filter consider equalities.

$$(\Omega p/\Omega c)^{2N} = (1/Ap^2) - 1 \qquad (\Omega s/\Omega c)^{2N} = (1/As^2) - 1$$

$$\left[\frac{\Omega s}{\Omega p}\right]^{2N} = \frac{(1/As^2)-1}{(1/Ap^2)-1}$$

Hence order of the filter (N) is calculated as

$$\mathbf{N= 0.5} \; \frac{\mathit{log}\left[\dfrac{(1/As^2)-1}{(1/Ap^2)-1}\right]}{\mathit{log}\,(\Omega s/ \Omega p)} \qquad \mathbf{(2)}$$

$$\mathbf{N= 0.5} \; \frac{\mathit{log}((1/As2) -1)}{\mathit{log}\,(\Omega s/ \Omega c)} \qquad \mathbf{(2A)}$$

And cutoff frequency Ωc is calculated as

$$\mathbf{\Omega c =} \frac{\Omega p}{[(1/Ap^2) -1]^{1/2N}} \qquad \mathbf{(3)}$$

If As and Ap values are given in DB then

$$As\ (DB) = -\ 20 \log As$$

$$\log As = -As /20$$

$$As = 10^{-As/20}$$

$$(As)^{-2} = 10^{As/10}$$

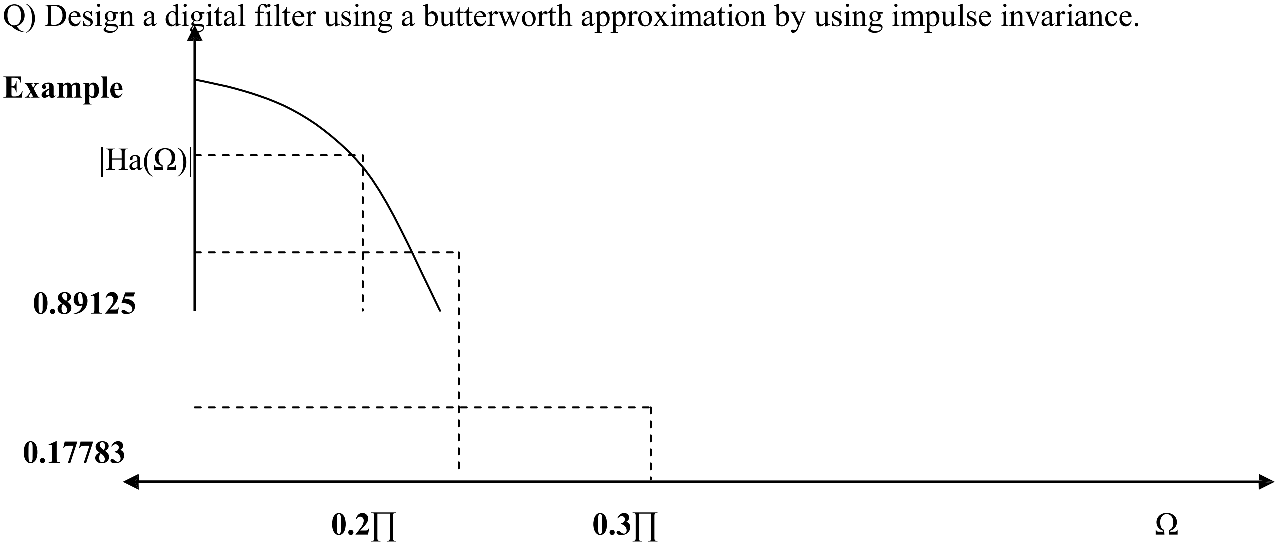
$$(As)^{-2} = 10^{0.1\ As} \quad DB$$

Hence equation (2) is modified as

$$\mathbf{N= 0.5} \; \frac{\mathit{log}\left[\dfrac{10^{0.1\ As} -1}{10^{0.1\ Ap} -1}\right]}{\mathit{log}\,(\Omega s/ \Omega p)} \qquad \mathbf{(4)}$$

Q) Design a digital filter using a butterworth approximation by using impulse invariance.

Example



Filter Type - Low Pass Filter
Ap - 0.89125
As - 0.17783
Ωp - 0.2∏
Ωs - 0.3∏

Step 1) To convert specification to equivalent analog filter.
(In impulse invariance method frequency relationship is given as $\omega = \Omega T$ while in Bilinear transformation method frequency relationship is given as $\Omega = (2/T) \tan(\omega/2)$ If Ts is not specified consider as 1)

$|Ha(\Omega)| \geq 0.89125$ for $\Omega \leq 0.2\prod/T$ and $|Ha(\Omega)| \leq 0.17783$ for $\Omega \geq 0.3\prod/T$.

Step 2) To determine the order of the filter.

$$N = 0.5 \frac{\log \dfrac{(1/As^2)-1}{(1/Ap^2)-1}}{\log(\Omega s/\Omega p)}$$

N= 5.88

A) Order of the filter should be integer.
B) Always go to nearest highest integer vale of N.

Hence N=6

Step 3) To find out the cutoff frequency (-3DB frequency)

$$\Omega c = \frac{\Omega p}{[(1/Ap^2)-1]^{1/2N}}$$

cutoff frequency $\Omega c = 0.7032$

Step 4) To find out the poles of analog filter system function.

$$P_k = +\Omega_c \; e^{j(N+2k+1)\Pi / 2N}$$

As N=6 the value of k = 0,1,2,3,4,5.

K	Poles	
0	$P0= + 0.7032\, e^{j7\Pi/12}$	$-0.182 + j\,0.679$ $0.182 - j\,0.679$
1	$P1= + 0.7032\, e^{j9\Pi/12}$	$-0.497 + j\,0.497$ $0.497 - j\,0.497$
2	$P2= + 0.7032\, e^{j11\Pi/12}$	$-0.679 + j\,0.182$ $0.679 - j\,0.182$
3	$P3= + 0.7032\, e^{j13\Pi/12}$	$-0.679 - j\,0.182$ $0.679 + j\,0.182$
4	$P4= + 0.7032\, e^{j15\Pi/12}$	$-0.497 - j\,0.497$ $0.497 + j\,0.497$
5	$P5= + 0.7032\, e^{j17\Pi/12}$	$-0.182 - j\,0.679$ $0.182 + j\,0.679$

For stable filter all poles lying on the left side of s plane is selected. Hence

$S1 = -0.182 + j\,0.679$ $\qquad S1^* = -0.182 - j\,0.679$
$S2 = -0.497 + j\,0.497$ $\qquad S2^* = -0.497 - j\,0.497$
$S3 = -0.679 + j\,0.182$ $\qquad S3^* = -0.679 - j\,0.182$

Step 5) To determine the system function (Analog Filter)

$$\mathbf{Ha(s)} = \frac{\Omega c^6}{(s-s1)(s-s1^*)\,(s-s2)(s-s2^*)\,(s-s3)(s-s3^*)}$$

Hence

$$\mathbf{Ha(s)} = \frac{(0.7032)^6}{(s+0.182-j0.679)(s+0.182+j0.679)\,(s+0.497-j0.497)\,(s+0.497+j0.497)\,(s+0.679-j0.182)(s+0.679-j0.182)}$$

$$\mathbf{Ha(s)} = \frac{0.1209}{[(s+0.182)^2 +(0.679)^2]\,[(s+0.497)^2+(0.497)^2]\,[(s+0.679)^2-(0.182)^2]}$$

$$\mathbf{Ha(s)} = \frac{1.97 \times 0.679 \times 0.497 \times 0.182}{[(s+0.182)^2 +(0.679)^2]\,[(s+0.497)^2+(0.497)^2]\,[(s+0.679)^2-(0.182)^2]}$$

Step 6) To determine the system function (Digital Filter)
(In Bilinear transformation replace s by the term ((z-1)/(z+1)) and find out the transfer function of digital function)

Step 7) Represent system function in cascade form or parallel form if asked.

Q) Given for low pass butterworth filter
Ap= -1 db at 0.2∏
As= -15 db at 0.3∏
1) Calculate N and Pole location
2) Design digital filter using BZT method.

Q) Obtain transfer function of a lowpass digital filter meeting specifications
Cutoff 0-60Hz
Stopband > 85Hz
Stopband attenuation > 15 db
Sampling frequency= 256 Hz . use butterworth characteristic.

Q) Design second order low pass butterworth filter whose cutoff frequency is 1 kHz at sampling frequency of 10^4 sps. Use BZT and Butterworth approximation.

2.11 FREQUENCY TRANSFORMATION

When the cutoff frequency Ωc of the low pass filter is equal to 1 then it is called normalized filter. Frequency transformation techniques are used to generate High pass filter, Bandpass and bandstop filter from the lowpass filter system function.

2.11.1 FREQUENCY TRANSFORMATION (ANALOG FILTER)

Sr No	Type of transformation	Transformation (Replace s by)
1	Low Pass	$\frac{s}{\omega_{lp}}$ ω_{lp} - Password edge frequency of another LPF
2	High Pass	$\frac{\omega_{hp}}{s}$ ω_{hp} = Password edge frequency of HPF
3	Band Pass	$\frac{(s^2 + \omega_l \omega_h)}{s(\omega_h - \omega_l)}$ ω_h - higher band edge frequency ω_l - Lower band edge frequency
4	Band Stop	$\frac{s(\omega_h - \omega_l)}{s^2 + \omega_h \omega_l}$ ω_h - higher band edge frequency ω_l - Lower band edge frequency

2.11.2 FREQUENCY TRANSFORMATION (DIGITAL FILTER)

Sr No	Type of transformation	Transformation (Replace z^{-1} by)
1	Low Pass	$\frac{z^{-1} - a}{1 - az^{-1}}$
2	High Pass	$\frac{-(z^{-1} + a)}{1 + az^{-1}}$
3	Band Pass	$\frac{-(z^{-2} - a_1 z^{-1} + a_2)}{a_2 z^{-2} - a_1 z^{-1} + 1}$
4	Band Stop	$\frac{z^{-2} - a_1 z^{-1} + a_2}{a_2 z^{-2} - a_1 z^{-1} + 1}$

Example:

Q) Design high pass butterworth filter whose cutoff frequency is 30 Hz at sampling frequency of 150 Hz. Use BZT and Frequency transformation.

Step 1. To find the prewarp cutoff frequency

$$\omega c^* = \tan(\omega c Ts/2)$$
$$= 0.7265$$

Step 2. LPF to HPF transformation

For First order LPF transfer function H(s) = 1/(s+1) Scaled transfer function $H^*(s) = H(s)\,|_{s=\omega c^*/s}$

$$H^*(s) = s/(s + 0.7265)$$

Step 4. Find out the digital filter transfer function. Replace s by (z-1)/(z+1)

$$H(z) = \frac{z-1}{1.7265z - 0.2735}$$

Q) Design second order band pass butterworth filter whose passband of 200 Hz and 300 Hz and sampling frequency is 2000 Hz. Use BZT and Frequency transformation.

Q) Design second order band pass butterworth filter which meet following specification

Lower cutoff frequency = 210 Hz
Upper cutoff frequency = 330 Hz
Sampling Frequency = 960 sps

Use BZT and Frequency transformation.

GLOSSARY:

System Design:

Usually, in the IIR Filter design, Analog filter is designed, then it is transformed to a digital filter the conversion of Analog to Digital filter involves mapping of desired digital filter specifications into equivalent analog filter.

Warping Effect:

The analog Frequency is same as the digital frequency response. At high frequencies, the relation between ω and Ω becomes Non-Linear. The Noise is introduced in the Digital Filter as in the Analog Filter. Amplitude and Phase responses are affected by this warping effect.

Prewarping:

The Warping Effect is eliminated by prewarping of the analog filter. The analog frequencies are prewarped and then applied to the transformation.

Infinite Impulse Response:

Infinite Impulse Response filters are a Type of Digital Filters which has infinite impulse response. This type of Filters are designed from analog filters. The Analog filters are then transformed to Digital Domain.

Bilinear Transformation Method:

In Bilinear transformation method the transform of filters from Analog to Digital is carried out in a way such that the Frequency transformation produces a Linear relationship between Analog and Digital Filters.

Filter:

A filter is one which passes the required band of signals and stops the other unwanted band of frequencies.

Pass band:

The Band of frequencies which is passed through the filter is termed as passband.

Stopband:

The band of frequencies which are stopped are termed as stop band.

UNIT III
FIR FILTER DESIGN

PREREQUISITE DISCUSSION:

The FIR Filters can be easily designed to have perfectly linear Phase. These filters can be realized recursively and Non-recursively. There are greater flexibility to control the Shape of their Magnitude response. Errors due to round off noise are less severe in FIR Filters, mainly because Feed back is not used.

3.1 Features of FIR Filter

1. FIR filter always provides linear phase response. This specifies that the signals in the pass band will suffer no dispersion Hence when the user wants no phase distortion, then FIR filters are preferable over IIR. Phase distortion always degrade the system performance. In various applications like speech processing, data transmission over long distance FIR filters are more preferable due to this characteristic.

2. FIR filters are most stable as compared with IIR filters due to its non feedback nature.

3. Quantization Noise can be made negligible in FIR filters. Due to this sharp cutoff FIR filters can be easily designed.

4. Disadvantage of FIR filters is that they need higher ordered for similar magnitude response of IIR filters.

FIR SYSTEM ARE ALWAYS STABLE. Why?

Proof:

Difference equation of FIR filter of length M is given as

$$y(n)=\sum_{k=0}^{M-1} b_k\, x(n-k) \qquad (1)$$

And the coefficient b_k are related to unit sample response as

$$H(n) = b_n \text{ for } 0 \le n \le M-1$$
$$= 0 \text{ otherwise.}$$

We can expand this equation as

$$Y(n)= b_0\, x(n) + b_1\, x(n-1) + \ldots\ldots + b_{M-1}\, x(n-M+1) \qquad (2)$$

System is stable only if system produces bounded output for every bounded input. This is stability definition for any system.

Here h(n)={b0, b1, b2, } of the FIR filter are stable. Thus y(n) is bounded if input x(n) is bounded. This means FIR system produces bounded output for every bounded input. Hence FIR systems are always stable.

3.2 Symmetric and Anti-symmetric FIR filters

1. Unit sample response of FIR filters is symmetric if it satisfies following condition.

$$h(n)= h(M-1-n) \qquad n=0,1,2\ldots\ldots\ldots M-1$$

2. Unit sample response of FIR filters is Anti-symmetric if it satisfies following condition

$$h(n)= -h(M-1-n) \qquad n=0,1,2\ldots\ldots\ldots M-1$$

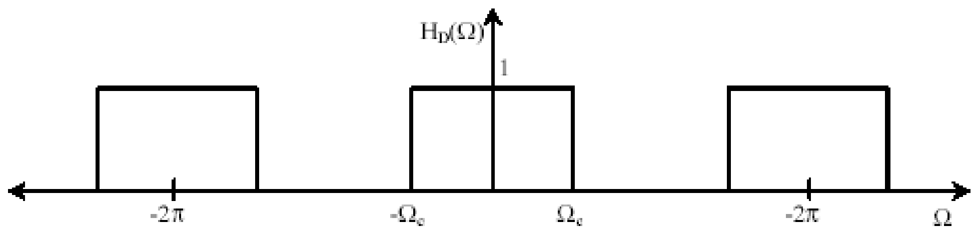
FIR Filter Design Methods

The various method used for FIR Filer design are as follows

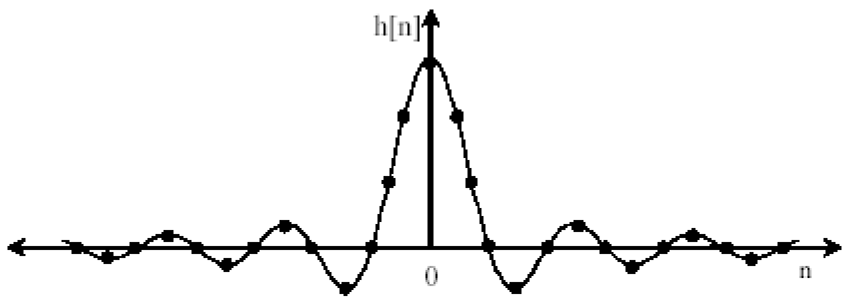
1. Fourier Series method
2. Windowing Method
3. DFT method
4. Frequency sampling Method. (IFT Method)

3.3 GIBBS PHENOMENON

Consider the ideal LPF frequency response as shown in Fig 1 with a normalizing angular cut off frequency Ω_c.



Impulse response of an ideal LPF is as shown in Fig 2.

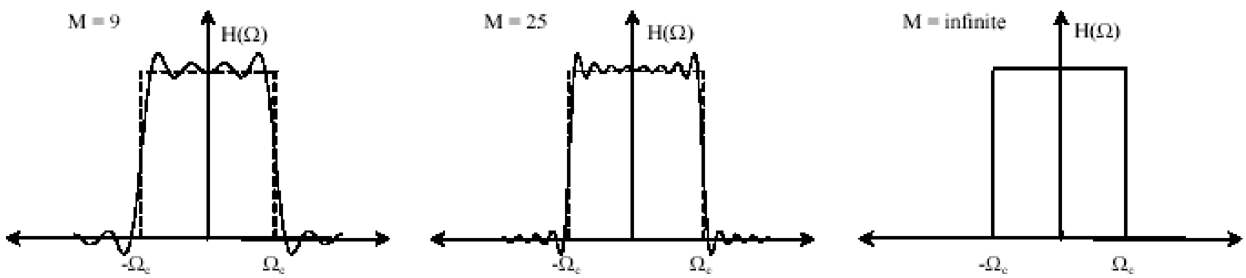


1. In Fourier series method, limits of summation index is -∞ to ∞. But filter must have finite terms. Hence limit of summation index change to -Q to Q where Q is some finite integer. But this type of truncation may result in poor convergence of the series. Abrupt truncation of infinite series is equivalent to multiplying infinite series with rectangular sequence. i.e at the point of discontinuity some oscillation may be observed in resultant series.

2. Consider the example of LPF having desired frequency response $H_d(\omega)$ as shown in figure. The oscillations or ringing takes place near band-edge of the filter.

3. This oscillation or ringing is generated because of side lobes in the frequency response $W(\omega)$ of the window function. This oscillatory behavior is called "Gibbs Phenomenon".

Truncated response and ringing effect is as shown in fig 3.



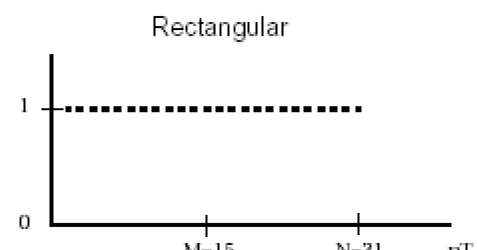
WINDO WING TECHNIQUE

W[n]

Windowing is the quickest method for designing an FIR filter. A windowing function simply truncates the ideal impulse response to obtain a causal FIR approximation that is non causal and infinitely long. Smoother window functions provide higher out-of band rejection in the filter response. However this smoothness comes at the cost of wider stopband transitions.

Various windowing method attempts to minimize the width of the main lobe (peak) of the frequency response. In addition, it attempts to minimize the side lobes (ripple) of the frequency response.

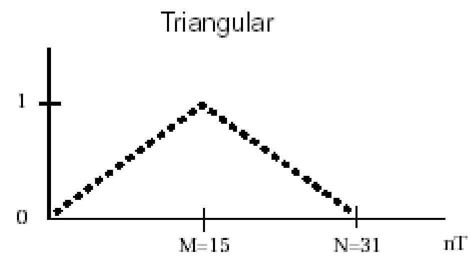
Rectangular Window: Rectangular This is the most basic of windowing methods. It does not require any operations because its values are either 1 or 0. It creates an abrupt discontinuity that results in sharp roll-offs but large ripples.



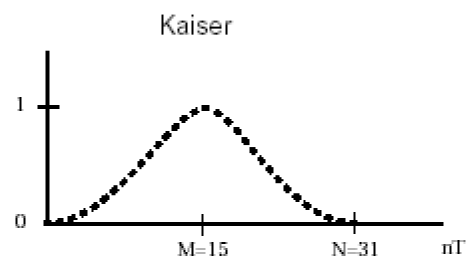
Rectangular window is defined by the following equation.

= 1	for $0 \leq n \leq N$
= 0	otherwise

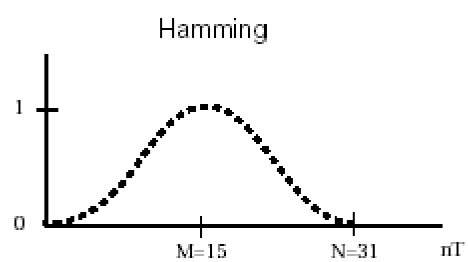
Triangular Window: The computational simplicity of this window, a simple convolution of two rectangle windows, and the lower sidelobes make it a viable alternative to the rectangular window.



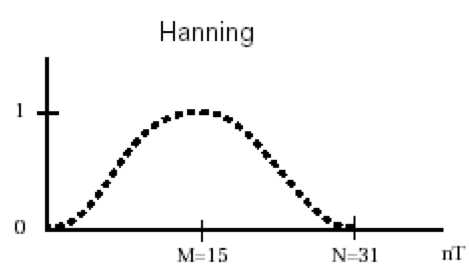
Kaiser Window: This windowing method is designed to generate a sharp central peak. It has reduced side lobes and transition band is also narrow. Thus commonly used in FIR filter design.



Hamming Window: This windowing method generates a moderately sharp central peak. Its ability to generate a maximally flat response makes it convenient for speech processing filtering.



Hanning Window: This windowing method generates a maximum flat filter design.



Name of window function $w(n)$	Mathematical definition
Rectangular	1
Hanning	$0.5-0.5\cos\left[\frac{2\pi n}{N-1}\right]$
Hamming	$0.54-0.46\cos\left[\frac{2\pi n}{N-1}\right]$
Blackman	$0.42-0.5\cos\left[\frac{2\pi n}{N-1}\right]+0.08\cos\left[\frac{2\pi n}{N-1}\right]$

3.4 DESIGNING FILTER FROM POLE ZERO PLACEMENT

Filters can be designed from its pole zero plot. Following two constraints should be imposed while designing the filters.

1. All poles should be placed inside the unit circle on order for the filter to be stable. However zeros can be placed anywhere in the z plane. FIR filters are all zero filters hence they are always stable. IIR filters are stable only when all poles of the filter are inside unit circle.

2. All complex poles and zeros occur in complex conjugate pairs in order for the filter coefficients to be real.

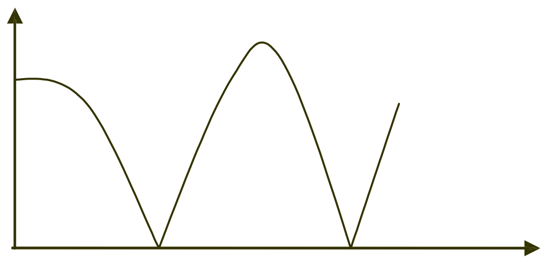
In the design of low pass filters, the poles should be placed near the unit circle at points corresponding to low frequencies (near ω=0)and zeros should be placed near or on unit circle at points corresponding to high frequencies (near ω=∏). The opposite is true for high pass filters.

3.5 NOTCH AND COMB FILTERS

A notch filter is a filter that contains one or more deep notches or ideally perfect nulls in its frequency response characteristic. Notch filters are useful in many applications where specific frequency components must be eliminated. Example Instrumentation and recording systems required that the power-line frequency 60Hz and its harmonics be eliminated.

To create nulls in the frequency response of a filter at a frequency ω_0, simply introduce a pair of complex-conjugate zeros on the unit circle at an angle ω_0.

comb filters are similar to notch filters in which the nulls occur periodically across the frequency band similar with periodically spaced teeth. Frequency response characteristic of notch filter |H(ω)| is as shown



3.6 DIGITAL RESONATOR

A digital resonator is a special two pole bandpass filter with a pair of complex conjugate poles located near the unit circle. The name resonator refers to the fact that the filter has a larger magnitude response in the vicinity of the pole locations. Digital resonators are useful in many applications, including simple bandpass filtering and speech generations.

IDEAL FILTERS ARE NOT PHYSICALLY REALIZABLE. Why?

Ideal filters are not physically realizable because Ideal filters are anti-causal and as only causal systems are physically realizable.

Proof:
Let take example of ideal lowpass filter.

$$H(\omega) = 1 \text{ for } -\omega_c \leq \omega \leq \omega_c$$
$$= 0 \text{ elsewhere}$$

The unit sample response of this ideal LPF can be obtained by taking IFT of $H(\omega)$.

$$h(n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega)\, e^{j\omega n}\, d\omega \qquad (1)$$

$$h(n) = \frac{1}{2\pi} \int_{-\omega c}^{\omega c} 1\, e^{j\omega n}\, d\omega \qquad (2)$$

$$h(n) = \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\omega c}^{\omega c}$$

$$\frac{1}{2\pi jn} \left[e^{j\omega cn} - e^{-j\omega cn} \right]$$

Thus $h(n) = \sin \omega_c n / \pi n$ for $n \neq 0$

Putting n=0 in equation (2) we have

$$h(n) = \frac{1}{2\pi} \int_{-\omega c}^{\omega c} 1\, d\omega \qquad (3)$$

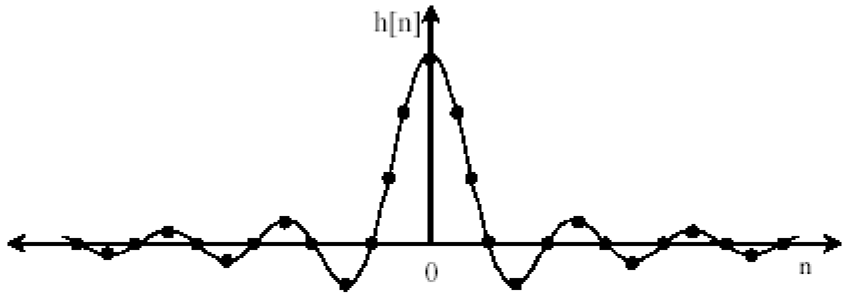
$\frac{1}{2\prod} \; [\omega]_{-\omega c}^{\omega c}$

and $\quad h(n) = \omega_c \; / \prod \quad$ for n=0

i.e

$$h(n) = \begin{cases} \dfrac{\sin(\omega c n)}{\prod n} & \text{for } n \neq 0 \\ \dfrac{\omega c}{n} & \text{for } n = 0 \end{cases}$$

Hence impulse response of an ideal LPF is as shown in Fig



LSI system is causal if its unit sample response satisfies following condition. h(n) = 0 for n<0

In above figure h(n) extends -∞ to ∞. Hence h(n) ≠0 for n<0. This means causality condition is not satisfied by the ideal low pass filter. Hence ideal low pass filter is non causal and it is not physically realizable.

EXAMPLES OF SIMPLE DIGITAL FILTERS:

The following examples illustrate the essential features of digital filters.

1. **UNITY GAIN FILTER:** $y_n = x_n$
   Each output value y_n is exactly the same as the corresponding input value x_n:

2. **SIMPLE GAIN FILTER:** $y_n = Kx_n$ (K = constant) Amplifier or attenuator) This simply applies a gain factor *K* to each input value:

3. **PURE DELAY FILTER:** $y_n = x_{n-1}$
   The output value at time *t = nh* is simply the input at time *t = (n-1)h*, i.e. the signal is delayed by time *h*:

4. **TWO-TERM DIFFERENCE FILTER:** $y_n = x_n - x_{n-1}$
   The output value at *t = nh* is equal to the difference between the current input x_n and the previous input x_{n-1}:

5. **TWO-TERM AVERAGE FILTER:** $y_n = (x_n + x_{n-1}) / 2$
   The output is the average (arithmetic mean) of the current and previous input:

6. **THREE-TERM AVERAGE FILTER:** $y_n = (x_n + x_{n-1} + x_{n-2}) / 3$
This is similar to the previous example, with the average being taken of the current and two previous inputs.

7. **CENTRAL DIFFERENCE FILTER:** $y_n = (x_n - x_{n-2}) / 2$
This is similar in its effect to example (4). The output is equal to half the change in the input signal over the previous two sampling intervals:

ORDER OF A DIGITAL FILTER

The order of a digital filter can be defined as the *number of previous inputs* (stored in the processor's memory) used to calculate the current output.
This is illustrated by the filters given as examples in the previous section.

Example (1): $y_n = x_n$
This is a *zero order* filter, since the current output y_n depends only on the current input x_n and not on any previous inputs.

Example (2): $y_n = Kx_n$
The order of this filter is again *zero*, since no previous outputs are required to give the current output value.

Example (3): $y_n = x_{n-1}$
This is a *first order* filter, as one previous input (x_{n-1}) is required to calculate y_n. (Note that this filter is classed as first-order because it uses one *previous* input, even though the current input is not used).

Example (4): $y_n = x_n - x_{n-1}$
This is again a *first order* filter, since one previous input value is required to give the current output.

Example (5): $y_n = (x_n + x_{n-1}) / 2$
The order of this filter is again equal to 1 since it uses just one previous input value.

Example (6): $y_n = (x_n + x_{n-1} + x_{n-2}) / 3$
To compute the current output y_n, two previous inputs (x_{n-1} and x_{n-2}) are needed; this is therefore a *second-order* filter.

Example (7): $y_n = (x_n - x_{n-2}) / 2$
The filter order is again 2, since the processor must store two previous inputs in order to compute the current output. This is unaffected by the absence of an explicit x_{n-1} term in the filter expression.

Q) For each of the following filters, state the order of the filter and identify the values of its coefficients:

(a) $yn = 2x_n - x_{n-1}$	A) Order = 1: $a_0 = 2$, $a_1 = -1$
(b) $yn = x_{n-2}$	B) Order = 2: $a_0 = 0$, $a_1 = 0$, $a_2 = 1$
(c) $yn = x_n - 2x_{n-1} + 2x_{n-2} + x_{n-3}$	C) Order = 3: $a_0 = 1$, $a_1 = -2$, $a_2 = 2$, $a_3 = 1$

Of these, the linear phase property is probably the most important. A filter is said to have a generalised linear phase response if its frequency response can be expressed in the form

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega + j\beta}$$

where α and β are constants, and $A(e^{j\omega})$ is a real function of ω. If this is the case, then

- If A is positive, then the phase is

  $$\sphericalangle H(e^{j\omega}) = \beta - \alpha\omega.$$

  If A is negative, then

  $$\sphericalangle H(e^{j\omega}) = \pi + \beta - \alpha\omega.$$

In either case, the phase is a linear function of ω.

It is common to restrict the filter to having a real-valued impulse response $h[n]$, since this greatly simplifies the computational complexity in the implementation of the filter.

A FIR system has linear phase if the impulse response satisfies either the even symmetric condition

$$h[n] = h[N - 1 - n],$$

or the odd symmetric condition

$$h[n] = -h[N - 1 - n].$$

The system has different characteristics depending on whether N is even or odd. Furthermore, it can be shown that all linear phase filters must satisfy one of these conditions. Thus there are exactly four types of linear phase filters.

Consider for example the case of an odd number of samples in $h[n]$, and even symmetry. The frequency response for $N = 7$ is

$$
\begin{aligned}
H(e^{j\omega}) &= \sum_{n=0}^{6} h[n]e^{-j\omega n} \\
&= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} + h[4]e^{-j4\omega} \\
&\qquad\qquad + h[5]e^{-j5\omega} + h[6]e^{-j6\omega} \\
&= e^{-j3\omega}(h[0]e^{j3\omega} + h[1]e^{j2\omega} + h[2]e^{j\omega} + h[3] + h[4]e^{-j\omega} \\
&\qquad\qquad + h[5]e^{-j2\omega} + h[6]e^{-j3\omega}).
\end{aligned}
$$

The specified symmetry property means that $h[0] = h[6]$, $h[1] = h[5]$, and $h[2] = h[4]$, so

$$
\begin{aligned}
H(e^{j\omega}) &= e^{-j3\omega}(h[0](e^{j3\omega} + e^{-j3\omega}) + h[1](e^{j2\omega} + e^{-j2\omega}) \\
&\qquad\qquad + h[2](e^{j\omega} + e^{-j\omega}) + h[3]) \\
&= e^{-j3\omega}(2h[0]\cos(3\omega) + 2h[1]\cos(2\omega) + 2h[2]\cos(\omega)) \\
&= e^{-j3\omega} \sum_{n=0}^{3} a[n]\cos(\omega n),
\end{aligned}
$$

where $a[0] = h[3]$, and $a[n] = 2h[3-n]$ for $n = 1, 2, 3$. The resulting filter clearly has a linear phase response for real $h[n]$. It is quite simple to show that in general for odd values of N the frequency response is

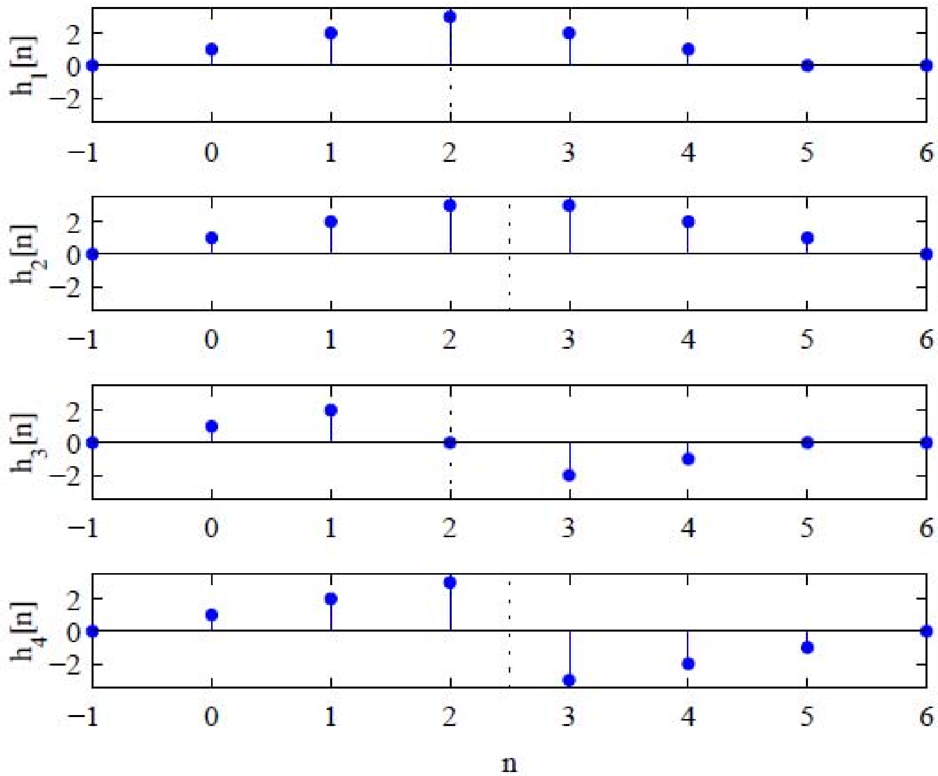
$$
H(e^{j\omega}) = e^{-j\omega(N-1)/2} \sum_{n=0}^{(N-1)/2} a[n]\cos(\omega n),
$$

for a set of real-valued coefficients $a[0], \ldots, a[(N-1)/2]$. As different values for $a[n]$ are selected, different linear-phase filters are obtained.

The cases of N odd and $h[n]$ antisymmetric are similar to that presented, and the frequency responses are summarised in the following table:

Symmetry	**N**	$H(e^{j\omega})$	**Type**
Even	Odd	$e^{-j\omega(N-1)/2} \sum_{n=0}^{(N-1)/2} a[n]\cos(\omega n)$	1
Even	Even	$e^{-j\omega(N-1)/2} \sum_{n=1}^{N/2} b[n]\cos(\omega(n-1/2))$	2
Odd	Odd	$e^{-j[\omega(N-1)/2-\pi/2]} \sum_{n=0}^{(N-1)/2} a[n]\sin(\omega n)$	3
Odd	Even	$e^{-j[\omega(N-1)/2-\pi/2]} \sum_{n=1}^{N/2} b[n]\sin(\omega(n-1/2))$	4

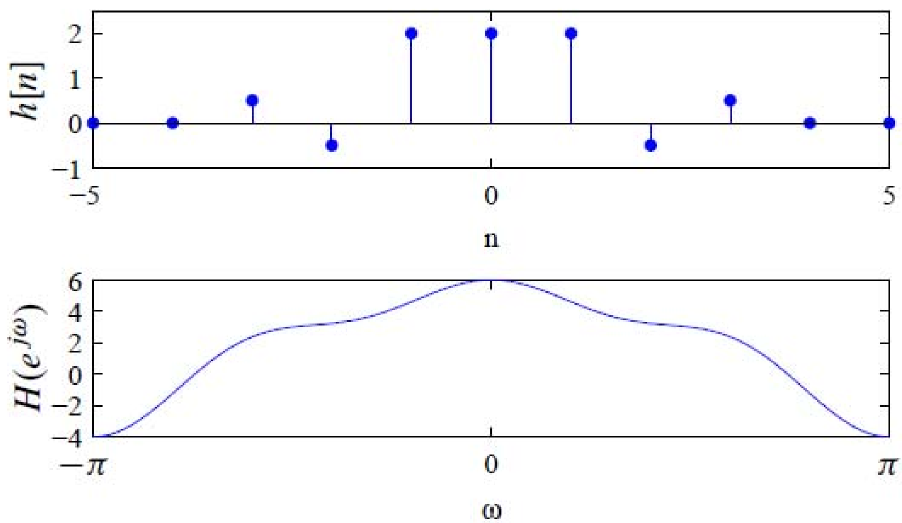
Recall that even symmetry implies $h[n] = h[N-1-n]$ and odd symmetry $h[n] = -h[N-1-n]$. Examples of filters satisfying each of these symmetry conditions are:



The center of symmetry is indicated by the dotted line.

The process of linear-phase filter design involves choosing the $a[n]$ values to obtain a filter with a desired frequency response. This is not always possible, however — the frequency response for a type II filter, for example, has the property that it is *always* zero for $\omega = \pi$, and is therefore not appropriate for a highpass filter. Similarly, filters of type 3 and 4 introduce a 90° phase shift, and have a frequency response that is always zero at $\omega = 0$ which makes them unsuitable for as lowpass filters. Additionally, the type 3 response is always zero at $\omega = \pi$, making it unsuitable as a highpass filter. The type I filter is the most versatile of the four.

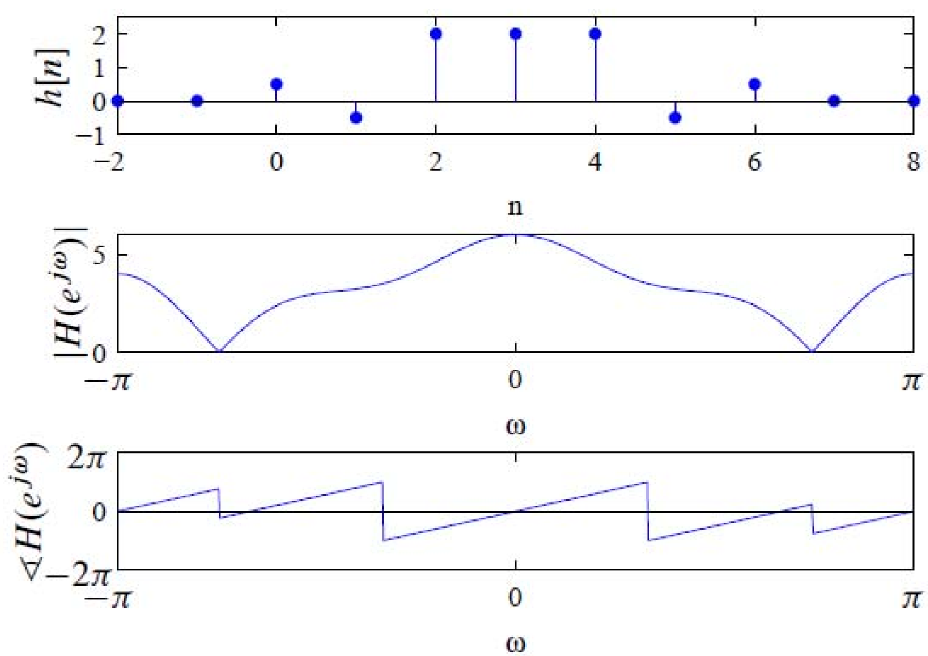
Linear phase filters can be thought of in a different way. Recall that a linear phase characteristic simply corresponds to a time shift or delay. Consider now a real FIR filter with an impulse response that satisfies the even symmetry condition $h[n] = h[-n]$:



Recall from the properties of the Fourier transform this filter has a real-valued frequency response $A(e^{j\omega})$. Delaying this impulse response by $(N-1)/2$ results in a causal filter with frequency response

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega(N-1)/2}.$$

This filter therefore has linear phase.



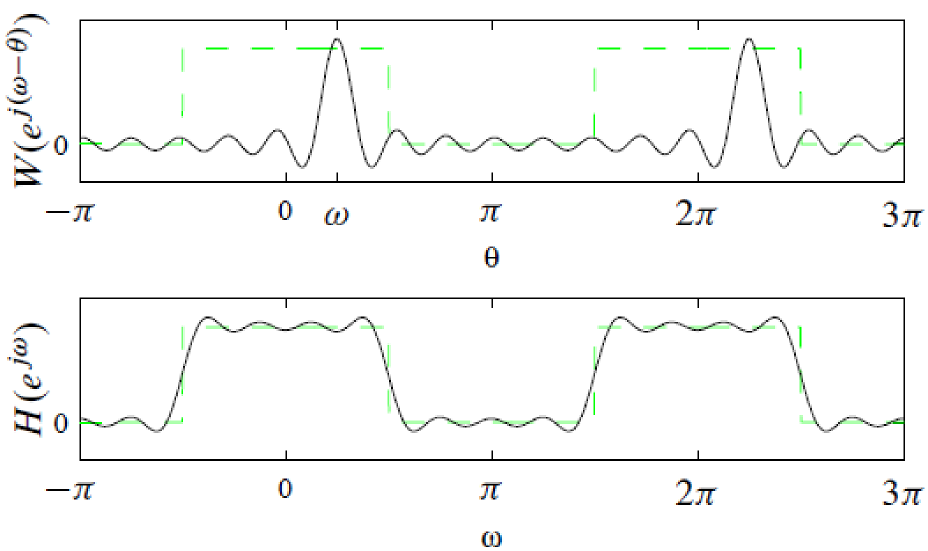
Window method for FIR filter design

Assume that the desired filter response $H_d(e^{j\omega})$ is known. Using the inverse Fourier transform we can determine $h_d[n]$, the desired unit sample response. In the window method, a FIR filter is obtained by multiplying a window $w[n]$ with $h_d[n]$ to obtain a finite duration $h[n]$ of length N. This is required since $h_d[n]$ will in general be an infinite duration sequence, and the corresponding filter will therefore not be realisable. If $h_d[n]$ is even or odd symmetric and $w[n]$ is even symmetric, then $h_d[n]w[n]$ is a linear phase filter.

Two important design criteria are the *length* and *shape* of the window $w[n]$. To see how these factors influence the design, consider the multiplication operation in the frequency domain: since $h[n] = h_d[n]w[n]$,

$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega}).$$

The following plot demonstrates the convolution operation. In each case the dotted line indicates the desired response $H_d(e^{j\omega})$.



From this, note that

- The *mainlobe* width of $W(e^{j\omega})$ affects the *transition* width of $H(e^{j\omega})$. Increasing the length N of $h[n]$ reduces the mainlobe width and hence the transition width of the overall response.

- The *sidelobes* of $W(e^{j\omega})$ affect the passband and stopband tolerance of $H(e^{j\omega})$. This can be controlled by changing the shape of the window. Changing N does not affect the sidelobe behaviour.

Some commonly used windows for filter design are

- **Rectangular:**

$$w[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

- **Bartlett (triangular):**

$$w[n] = \begin{cases} 2n/N & 0 \le n \le N/2 \\ 2 - 2n/N & N/2 < n \le N \\ 0 & \text{otherwise} \end{cases}$$

- **Hanning:**

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/N) & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

- **Hamming:**

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/N) & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

- **Kaiser:**

$$w[n] = \begin{cases} I_0[\beta(1 - [(n-\alpha)/\alpha]^2)^{1/2}] & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

Examples of five of these windows are shown below:

Window	Peak sidelobe amplitude (dB)	Mainlobe transition width	Peak approximation error (dB)
Rectangular	-13	$4\pi/(N+1)$	-21
Bartlett	-25	$8\pi/N$	-25
Hanning	-31	$8\pi/N$	-44
Hamming	-41	$8\pi/N$	-53

The Kaiser window has a number of parameters that can be used to explicitly tune the characteristics.

In practice, the window shape is chosen first based on passband and stopband tolerance requirements. The window size is then determined based on transition width requirements. To determine $h_d[n]$ from $H_d(e^{j\omega})$ one can sample $H_d(e^{j\omega})$ closely and use a large inverse DFT.

2.2 Frequency sampling method for FIR filter design

In this design method, the desired frequency response $H_d(e^{j\omega})$ is sampled at equally-spaced points, and the result is inverse discrete Fourier transformed. Specifically, letting

$$H[k] = H_d(e^{j\omega})\big|_{\omega=\frac{2\pi k}{N}}, \qquad k = 0, \ldots, N-1,$$

the unit sample response of the filter is $h[n] = \mathrm{IDFT}(H[k])$, so

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k]e^{j2\pi nk/N}.$$

The resulting filter will have a frequency response that is exactly the same as the original response at the sampling instants. Note that it is also necessary to specify the *phase* of the desired response $H_d(e^{j\omega})$, and it is usually chosen to be a linear function of frequency to ensure a linear phase filter. Additionally, if a filter with real-valued coefficients is required, then additional constraints have to be enforced.

The *actual* frequency response $H(e^{j\omega})$ of the filter $h[n]$ still has to be determined. The z-transform of the impulse response is

$$H(z) = \sum_{n=0}^{N-1} h[n]z^{-n} = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} H[k]e^{j2\pi nk/N} \right] z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H[k] \sum_{n=0}^{N-1} e^{j2\pi nk/N} z^{-n}$$

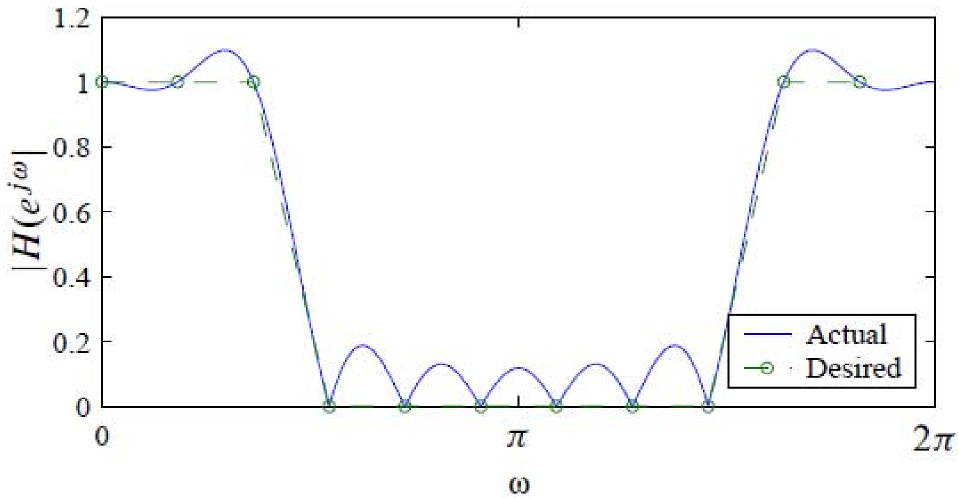
$$= \frac{1}{N} \sum_{k=0}^{N-1} H[k] \left[\frac{1 - z^{-N}}{1 - e^{j2\pi k/N} z^{-1}} \right].$$

Evaluating on the unit circle $z = e^{j\omega}$ gives the frequency response

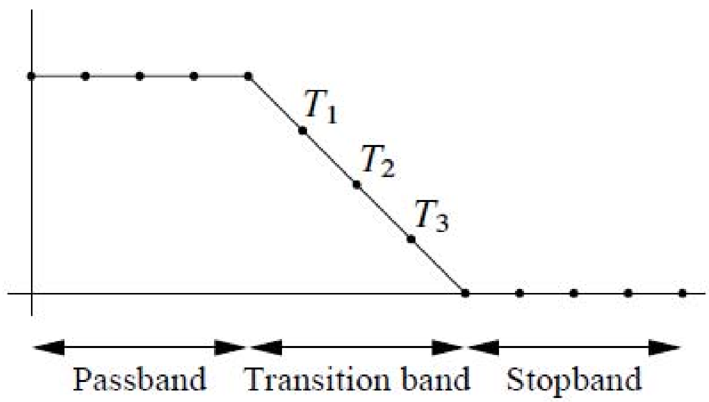
$$H(e^{j\omega}) = \frac{1 - e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{H[k]}{1 - e^{j2\pi k/N} e^{-j\omega}}.$$

This expression can be used to find the actual frequency response of the filter obtained, which can be compared with the desired response.

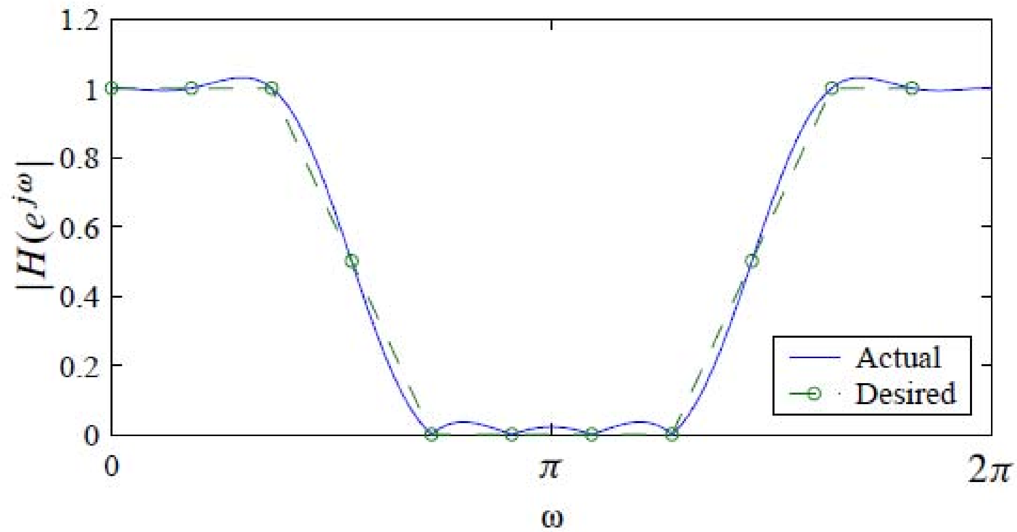
The method described only guarantees correct frequency response values at th points that were sampled. This sometimes leads to excessive ripple at intermediate points:



One way of addressing this problem is to allow **transition samples** in the region where discontinuities in $H_d(e^{j\omega})$ occur:



This effectively increases the transition width and can decrease the ripple, as observed below:



By leaving the value of the transition sample unconstrained, one can to some extent optimise the filter to minimise the ripple. Empirically, with three transition samples a stopband attenuation of 100dB is achievable. Recall however that for $h[n]$ real we require even or odd symmetry in the impulse response, so the values are not entirely unconstrained.

GLOSSARY:

FIR Filters:

In the Finite Impulse Response Filters the No.of. Impulses to be considered for filtering are finite. There are no feed back Connections from the Output to the Input. There are no Equivalent Structures of FIR filters in the Analog Regime.

Symmetric FIR Filters:

Symmetric FIR Filters have their Impulses that occur as the mirror image in the first quadrant and second quadrant or Third quadrant and fourth quadrant or both.

Anti Symmetric FIR Filters:

The Antisymmetric FIR Filters have their impulses that occur as the mirror image in the first quadrant and third quadrant or second quadrant and Fourth quadrant or both.

Linear Phase:

The FIR Filters are said to have linear in phase if the filter have the impulses that increases according to the Time in digital domain.

Frequency Response:

The Frequency response of the Filter is the relationship between the angular frequency and the Gain of the Filter.

Gibbs Phenomenon:

The abrupt truncation of Fourier series results in oscillation in both passband and stop band. These oscillations are due to the slow convergence of the fourier series. This is termed as Gibbs Phenomenon.

Windowing Technique:

To avoid the oscillations instead of truncating the fourier co-efficients we are multiplying the fourier series with a finite weighing sequence called a window which has non-zero values at the required interval and zero values for other Elements.

UNIT – IV
FINITE WORDLENGTH EFFECTS

4.1 Number Representation

In digital signal processing, $(B + 1)$-bit fixed-point numbers are usually represented as two's- complement signed fractions in the format

bo b-ib-2 …… b-B

The number represented is then

$$X = -b_0 + b_{-1}2^{-1} + b_{-2}2^{-2} + \cdots + b_{-B}\,2^{-B} \qquad (3.1)$$

where bo is the sign bit and the number range is —1 <X < 1. The advantage of this representation is that the product of two numbers in the range from — 1 to 1 is another number in the same range. Floating-point numbers are represented as

$$X = (-1)^s m 2^c \qquad (3.2)$$

where s is the *sign bit*, m is the mantissa, and c is the *characteristic* or *exponent*. To make the representation of a number unique, the mantissa is *normalized* so that 0.5 <m < 1.

Although floating-point numbers are always represented in the form of (3.2), the way in which this representation is actually *stored* in a machine may differ. Since $m > 0.5$, it is not necessary to store the 2^{-1} -weight bit of m, which is always set. Therefore, in practice numbers are usually stored as

$$X = (-1)^s(0.5 + f)2^c \qquad (3.3)$$

where f is an unsigned fraction, 0 <f < 0.5.

Most floating-point processors now use the IEEE Standard 754 32-bit floating-point format for storing numbers. According to this standard the exponent is stored as an unsigned integer p where

$$p = c + 126 \qquad (3.4)$$

Therefore, a number is stored as

$$X = (-1)^s(0.5 + f)2^{p-126} \qquad (3.5)$$

where s is the sign bit, f is a 23-b unsigned fraction in the range 0 <f < 0.5, and p is an 8-b unsigned integer in the range 0 <p < 255. The total number of bits is 1 + 23 + 8 = 32. For example, in IEEE format 3/4 is written $(-1)^0(0.5 + 0.25)2^\circ$ so $s = 0$, $p = 126$, and $f = 0.25$. The value $X = 0$ is a unique case and is represented by all bits zero (i.e., s = 0, $f = 0$, and $p = 0$). Although the 2^{-1}-weight mantissa bit is not actually stored, it does exist so the mantissa has 24 b plus a sign bit.

4.1.1 Fixed-Point Quantization Errors

In fixed-point arithmetic, a multiply doubles the number of significant bits. For example, the product of the two 5-b numbers 0.0011 and 0.1001 is the 10-b number 00.000 110 11. The extra bit to the left of the decimal point can be discarded without introducing any error. However, the least significant four of the remaining bits must ultimately be discarded by some form of quantization so that the result can be stored to 5 b for use in other calculations. In the example above this results in 0.0010 (quantization by rounding) or 0.0001 (quantization by truncating). When a sum of products calculation is performed, the quantization can be performed either after each multiply or after all products have been summed with double-length precision.

We will examine three types of fixed-point quantization—rounding, truncation, and magnitude truncation. If X is an exact value, then the rounded value will be denoted $Q_r(X)$, the truncated value $Q_t(X)$, and the magnitude truncated value $Q_{mt}(X)$. If the quantized value has B bits to the right of the decimal point, the quantization step size is

$$A = 2^{-B} \tag{3.6}$$

Since rounding selects the quantized value nearest the unquantized value, it gives a value which is never more than ± A /2 away from the exact value. If we denote the rounding error by

$$fr = Qr(X) - X \tag{3.7}$$

then

$$\frac{AA}{2} < f_r < \frac{}{- 2} \tag{3.8}$$

Truncation simply discards the low-order bits, giving a quantized value that is always less than or equal to the exact value so

$$- A < f_t < 0 \tag{3.9}$$

Magnitude truncation chooses the nearest quantized value that has a magnitude less than or equal to the exact value so

$$- A < f_{mt} < A \tag{3.10}$$

The error resulting from quantization can be modeled as a random variable uniformly distributed over the appropriate error range. Therefore, calculations with roundoff error can be considered error-free calculations that have been corrupted by additive white noise. The mean of this noise for rounding is

$$m_{(r} — E\{fr\} = \frac{1}{A} \int_{-A/2}^{A/2} fr\,dfr — 0 \tag{3.11}$$

where $E\{\}$ represents the operation of taking the expected value of a random variable. Similarly, the variance of the noise for rounding is

$$a_{€}^2 — E\{(fr - m_{€r})^2\} — \frac{1}{A} \int_{-A/2}^{A/2} (fr - m_{€r})^2\, dfr = \frac{A^2}{12} \tag{3.12}$$

Likewise, for truncation,

$$\begin{aligned} m_{f_t} &= E\{f_t\} = \frac{A}{y} \\ a^2 &= E\{(ft - mft)^2\} = \frac{A^2}{} \\ m_{f_{mt}} &— E\{f_{mt}\} — 0 \end{aligned} \tag{3.13}$$

and, for magnitude truncation

$$\sigma^2_{f\text{-}mt} = E\{(f_{mt} - m_m)^2\} = \frac{A^2}{\ }\qquad (3.14)$$

4.1.2 Floating-Point Quantization Errors

With floating-point arithmetic it is necessary to quantize after both multiplications and additions. The addition quantization arises because, prior to addition, the mantissa of the smaller number in the sum is shifted right until the exponent of both numbers is the same. In general, this gives a sum mantissa that is too long and so must be quantized.

We will assume that quantization in floating-point arithmetic is performed by rounding. Because of the exponent in floating-point arithmetic, it is the relative error that is important. The relative error is defined as

$$e_r = \frac{Q_r(X) - X}{X} = \frac{e_r}{X} \qquad (3.15)$$

Since $X = (-1)^s m 2^c$, $Q_r(X) = (-1)^s Q_r(m) 2^c$ *and*

$$e_r = \frac{Q_r(m) - m}{m} = \frac{e}{m} \qquad (3.16)$$

If the quantized mantissa has B bits to the right of the decimal point, $|e| < A/2$ where, as before, $A = 2^{-B}$. Therefore, since $0.5 < m < 1$,

$$|e_r| < A \qquad (3.17)$$

If we assume that e is uniformly distributed over the range from $-A/2$ to $A/2$ and m is uniformly distributed over 0.5 to 1,

$$m_{e_r} = E\left\{\frac{e}{m}\right\} = 0$$

$$\sigma^2_{e_r} = E\left\{\left(\frac{e}{m}\right)^2\right\} = \frac{2}{A}\int_{1/2}^{1}\int_{-A/2}^{A/2} \frac{e^2}{m^2}\, de\, dm$$

$$= \frac{A^2}{6} = (0.167)2^{-2B} \qquad (3.18)$$

In practice, the distribution of m is not exactly uniform. Actual measurements of roundoff noise in [1] suggested that

$$\sigma^2_{e_r} \approx 0.23A^2 \qquad (3.19)$$

while a detailed theoretical and experimental analysis in [2] determined

$$\sigma^2 \approx 0.18A^2 \qquad (3.20)$$

From (3.15) we can represent a quantized floating-point value in terms of the unquantized value and the random variable e_r using

$$Q_r(X) = X(1 + e_r) \qquad (3.21)$$

Therefore, the finite-precision product X_1X_2 and the sum $X_1 + X_2$ can be written

$$fl(X_1X_2) = X_1X_2(1 + e_r) \qquad (3.22)$$

and

$$fl(X1 + X2) = (X1 + X2)(1 + e_r) \quad (3.23)$$

where e_r is zero-mean with the variance of (3.20).

4.2 Roundoff Noise:

To determine the roundoff noise at the output of a digital filter we will assume that the noise due to a quantization is stationary, white, and uncorrelated with the filter input, output, and internal variables. This assumption is good if the filter input changes from sample to sample in a sufficiently complex manner. It is not valid for zero or constant inputs for which the effects of rounding are analyzed from a limit cycle perspective.

To satisfy the assumption of a sufficiently complex input, roundoff noise in digital filters is often calculated for the case of a zero-mean white noise filter input signal $x(n)$ of variance a^1. This simplifies calculation of the output roundoff noise because expected values of the form $E\{x(n)x(n - k)\}$ are zero for $k = 0$ and give a^2 when $k = 0$. This approach to analysis has been found to give estimates of the output roundoff noise that are close to the noise actually observed for other input signals.

Another assumption that will be made in calculating roundoff noise is that the product of two quantization errors is zero. To justify this assumption, consider the case of a 16-b fixed-point processor. In this case a quantization error is of the order 2^{-15}, while the product of two quantization errors is of the order 2^{-30}, which is negligible by comparison.

If a linear system with impulse response $g(n)$ is excited by white noise with mean m_x and variance a^2, the output is noise of mean [3, pp.788-790]

$$my = mx \sum_{n=-TO}^{TO} g(n) \quad (3.24)$$

and variance

$$ay = al \sum_{n=-TO}^{TO} g^2(n) \quad (3.25)$$

Therefore, if $g(n)$ is the impulse response from the point where a roundoff takes place to the filter output, the contribution of that roundoff to the variance (mean-square value) of the output roundoff noise is given by (3.25) with a^2 replaced with the variance of the roundoff. If there is more than one source of roundoff error in the filter, it is assumed that the errors are uncorrelated so the output noise variance is simply the sum of the contributions from each source.

4.3 Roundoff Noise in FIR Filters:

The simplest case to analyze is a finite impulse response (FIR) filter realized via

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$$al = \frac{1}{} 2^{-2B} \quad (3.28)$$

the convolution summation

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n - k) \quad (3.26)$$

When fixed-point arithmetic is used and quantization is performed after each multiply, the result of the N multiplies is N-times the quantization noise of a single multiply. For example, rounding after each multiply gives, from (3.6) and (3.12), an output noise variance of

$$a^2 = N\frac{2^{-2B}}{} \quad (3.27)$$

Virtually all digital signal processor integrated circuits contain one or more double-length accumulator registers which permit the sum-of-products in (3.26) to be accumulated without quantization. In this case only a single quantization is necessary following the summation and

For the floating-point roundoff noise case we will consider (3.26) for $N = 4$ and then generalize the result to other values of N. The finite-precision output can be written as the exact output plus an error term $e(n)$. Thus,

$$\begin{aligned} y(n) + e(n) = (\{[h(0)x(n)[1 + E1(n)] \\ + h(1)x(n-1)[1 + £2(n)]][1 + S3(n)] \\ + h(2)x(n-2)[1 + £4(n)]\}\{1 + s_5(n)\} \\ + h(3)x(n-3)[1 + £6(n)])[1 + £j(n)] \end{aligned} \quad (3.29)$$

In (3.29), $£1(n)$ represents the errorin the first product, $£2(n)$ the error in the second product, $£3(n)$ the error in the firstaddition, etc.Notice that it has been assumed that the products are summed in the order implied by the summation of (3.26).

Expanding (3.29), ignoring products of error terms, and recognizing $y(n)$ gives

$$\begin{aligned} e(n) = h(0)x(n)[£1(n) + £3(n) + £\$(n) + £i(n)] \\ + h(1)x(n-1)[£2(n) + £3(n) + £_5(n) + £j(n)] \\ + h(2)x(n-2)[£4(n) + £_5(n) + £i(n)] \\ + h(3)x(n-3)[£6(n) + £j(n)] \end{aligned} \quad (3.30)$$

Assuming that the input is white noise of variance $a^{\wedge}$ so that $E\{x(n)x(n - k)\}$ is zero for $k = 0$, and assuming that the errors are uncorrelated,

$$E\{e^2(n)\} = [4h^2(0) + 4h^2(1) + 3h^2(2) + 2h^2(3)]a^2a^2 \quad (3.31)$$

In general, for any N,

$$aO = E\{e^2(n)\} = Nh^2(0) + \sum_{k=1}^{N-1} (N + 1 - k)h^2(k) \; a^{\circ}a^{\circ}_r \quad (3.32)$$

Notice that if the order of summation of the product terms in the convolution summation is changed, then the order in which the $h(k)$'s appear in (3.32) changes. If the order is changed so that the $h(k)$ with smallest magnitude is first, followed by the next smallest, etc., then the roundoff noise variance is minimized. However, performing the convolution summation in nonsequential order greatly complicates data indexing and so may not be worth the reduction obtained in roundoff noise.

4.4 Roundoff Noise in Fixed-Point IIR Filters

To determine the roundoff noise of a fixed-point infinite impulse response (IIR) filter realization, consider a causal first-order filter with impulse response

$$h(n) = a^n u(n) \qquad (3.33)$$

realized by the difference equation

$$y(n) = ay(n - 1) + x(n) \qquad (3.34)$$

Due to roundoff error, the output actually obtained is

$$y(n) = Q\{ay(n - 1) + x(n)\} = ay(n - 1) + x(n) + e(n) \qquad (3.35)$$

where $e(n)$ is a random roundoff noise sequence. Since $e(n)$ is injected at the same point as the input, it propagates through a system with impulse response $h(n)$. Therefore, for fixed-point arithmetic with rounding, the output roundoff noise variance from (3.6), (3.12), (3.25), and (3.33) is

$$\sigma_o^2 = \frac{\Delta^2}{12}\sum_{n=-\infty}^{\infty} h^2(n) = \frac{\Delta^2}{12}\sum_{n=0}^{\infty} a^{2n} = \frac{2^{-2B}}{12}\,\frac{1}{1-a^2} \tag{3.36}$$

With fixed-point arithmetic there is the possibility of overflow following addition. To avoid overflow it is necessary to restrict the input signal amplitude. This can be accomplished by either placing a *scaling* multiplier at the filter input or by simply limiting the maximum input signal amplitude. Consider the case of the first-order filter of (3.34). The transfer function of this filter is

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{e^{j\omega} - a} \tag{3.37}$$

so

$$|H(e^{j\omega})|^2 = \frac{1}{1 + a^2 - 2a\cos\omega} \tag{3.38}$$

and

$$|H(e^{j\omega})|_{max} = \frac{1}{1-|a|} \tag{3.39}$$

The peak gain of the filter is $1/(1-|a|)$ so limiting input signal amplitudes to $|x(n)| < 1-|a|$ will make overflows unlikely.

An expression for the output roundoff noise-to-signal ratio can easily be obtained for the case where the filter input is white noise, uniformly distributed over the interval from $-(1-|a|)$ to $(1-|a|)$ [4,5]. In this case

$$\sigma_x^2 = \frac{1}{2(1-|a|)}\int_{-(1-|a|)}^{1-|a|} x^2\,dx = \frac{1}{3}(1-|a|)^2 \tag{3.40}$$

so, from (3.25),

$$\sigma_y^2 = \frac{1}{3}\,\frac{(1-|a|)^2}{1-a^2} \tag{3.41}$$

Combining (3.36) and (3.41) then gives

$$\frac{\sigma_o^2}{\sigma_y^2} = \left(\frac{2^{-2B}}{12}\,\frac{1}{1-a^2}\right)\left(\frac{3(1-a^2)}{(1-|a|)^2}\right) = \frac{3\cdot 2^{-2B}}{12\,(1-|a|)^2} \tag{3.42}$$

Notice that the noise-to-signal ratio increases without bound as $|a| \to 1$.

Similar results can be obtained for the case of the causal second-order filter realized by the difference equation

$$y(n) = 2r\cos(0)y(n-1) - r^2y(n-2) + x(n) \quad (3.43)$$

This filter has complex-conjugate poles at $re^{\pm j0}$ and impulse response

$$h(n) = r^n \frac{\sin[(n+1)0]}{\sin(0)} u(n) \quad (3.44)$$

Due to roundoff error, the output actually obtained is

$$y(n) = 2r\cos(0)y(n-1) - r^2y(n-2) + x(n) + e(n) \quad (3.45)$$

There are two noise sources contributing to $e(n)$ if quantization is performed after each multiply, and there is one noise source if quantization is performed after summation. Since

$$\sum_{n=-\infty} \quad 2 \quad \frac{1}{1 - r^2 \; (1 + r^2)^2 - 4r^2 \cos^2(9)} \tag{3.46}$$

the output roundoff noise is

$$a^2 = v \; \frac{2^{-2B}}{12} \; \frac{1 + r^2}{1 - r^2} \; \frac{1}{(1 + r^2)^2 - 4r^2 \cos^2(9)} \tag{3.47}$$

where $V = 1$ for quantization after summation, and $V = 2$ for quantization after each multiply. To obtain an output noise-to-signal ratio we note that

$$H(e^{jw}) = \frac{1}{1 - 2r\cos(9)e^{-jm} + r^2 e^{-j2m}} \tag{3.48}$$

and, using the approach of [6],

$$iH(emmax = \frac{1}{4r^2 \quad \mathrm{sat}\left(\cos(9)\wedge - \cos(9)\right)^2 + {}^2 \sin(9)^2} \tag{3.49}$$

where

$$sat(i) = \begin{matrix} I > 1 \\ -1 < I < 1 \end{matrix} \tag{3.50}$$

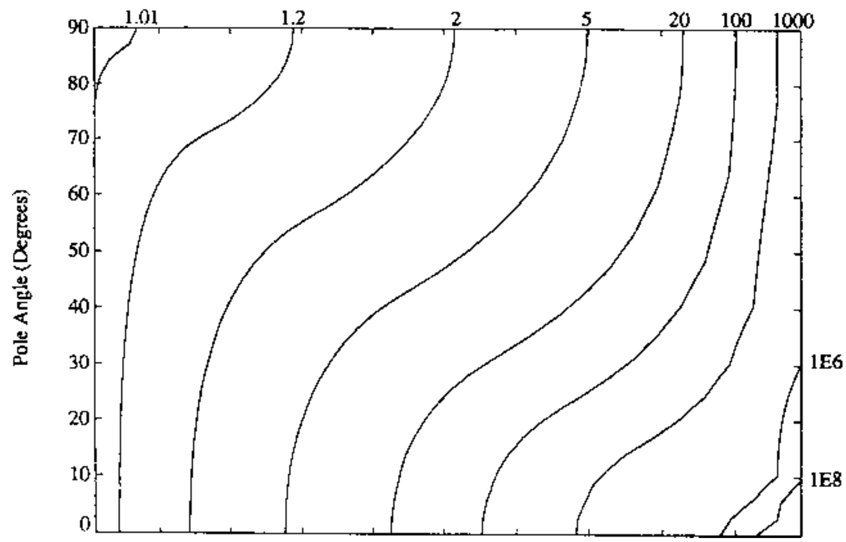
Following the same approach as for the first-order case then gives

$$\frac{2}{y} \; V \; \frac{2^{-2B}}{12} \; \frac{1 + r^2}{1 - r^2} \; \frac{3}{(1 + r^2)^2 - 4r^2 \cos^2(9)}$$

$$X \; \frac{1}{4r^2 \quad \mathrm{sat}\left(\wedge\cos(9)\right) - \cos(9)\right)^2 + \left(\frac{1 - r^2}{2r}\sin(9)\right)^2} \tag{3.51}$$

Figure3.1 is a contour plot showing the noise-to-signal ratio of (3.51) for $v = 1$ in units of the noise variance of a single quantization, $2^{-2B}/12$. The plot is symmetrical about $9 = 90°$, so only the range from 0° to 90° is shown. Notice that as $r \wedge 1$, the roundoff noise increases without bound. Also notice that the noise increases as $9 \wedge 0°$.

It is possible to design state-space filter realizations that minimize fixed-point roundoff noise [7] - [10]. Depending on the transfer function being realized, these structures may provide a roundoff noise level that is orders-of-magnitude lower than for a nonoptimal realization. The price paid for this reduction in roundoff noise is an increase in the number of computations required to implement the filter. For an Nth-order filter the increase is from roughly $2N$ multiplies for a direct form realization to roughly $(N + 1)^2$ for an optimal realization. However, if the filter is realized by the parallel or cascade connection of first- and second-order optimal subfilters, the increase is only to about $4N$ multiplies. Furthermore, near-optimal realizations exist that increase the number of multiplies to only about 3N [10].



0.010.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 0.99

Normalized fixed-point roundoff noise variance.

4.5 Limit Cycle Oscillations:

A limit cycle, sometimes referred to as a multiplier roundoff limit cycle, is a low-level oscillation that can exist in an otherwise stable filter as a result of the nonlinearity associated with rounding (or truncating) internal filter calculations [11]. Limit cycles require recursion to exist and do not occur in nonrecursive FIR filters. As an example of a limit cycle, consider the second-order filter realized by

$$y(n) = Q_r\{\tfrac{7}{8}y(n-1) - \tfrac{5}{8}y(n-2) + x(n)$$

where Q_r {} represents quantization by rounding. This is stable filter with poles at $0.4375 \pm j0.6585$. Consider the implementation of this filter with 4-b (3-b and a sign bit) two's complement fixed-point arithmetic, zero initial conditions ($y(-1) = y(-2) = 0$), and an input sequence $x(n) = |S(n),$ where $S(n)$ is the unit impulse or unit sample. The following sequence is obtained;

Notice that while the input is zero except for the first sample, the output oscillates with amplitude $1/8$ and period 6.

Limit cycles are primarily of concern in fixed-point recursive filters. As long as floating-point filters are realized as the parallel or cascade connection of first- and second-order subfilters, limit cycles will generally not be a problem since limit cycles are practically not observable in first- and second-order systems implemented with 32-b floating-point arithmetic [12]. It has been shown that such systems must have an extremely small margin of stability for limit cycles to exist at anything other than underflow levels, which are at an amplitude of less than 10^{-38} [12]. There are at least three ways of dealing with limit cycles when fixed-point arithmetic is used. One is to determine a bound on the maximum limit cycle amplitude, expressed as an integral number of quantization steps [13]. It is then possible to choose a word length that makes the limit cycle amplitude acceptably low. Alternately, limit cycles can be prevented by randomly rounding calculations up or down [14]. However, this approach is complicated to implement. The third approach is to properly choose the filter realization structure and then quantize the filter calculations using magnitude truncation [15,16]. This approach has the disadvantage of producing more roundoff noise than truncation or rounding [see (3.12)—(3.14)].

4.6 Overflow Oscillations:

With fixed-point arithmetic it is possible for filter calculations to overflow. This happens when two numbers of the same sign add to give a value having magnitude greater than one. Since numbers with magnitude greater than one are not representable, the result overflows. For example, the two's complement numbers 0.101 (5/8) and 0.100 (4/8) add to give 1.001 which is the two's complement representation of -7/8.

The overflow characteristic of two's complement arithmetic can be represented as $R\{\}$ where

$$R\{X\} = \begin{cases} X - 2 & X \geq 1 \\ X & -1 \leq X < 1 \\ X + 2 & X < -1 \end{cases} \tag{3.71}$$

For the example just considered, $R\{9/8\} = -7/8$.

An overflow oscillation, sometimes also referred to as an *adder overflow limit cycle,* is a high- level oscillation that can exist in an otherwise stable fixed-point filter due to the gross nonlinearity associated with the overflow of internal filter calculations [17]. Like limit cycles, overflow oscillations require recursion to exist and do not occur in nonrecursive FIR filters. Overflow oscillations also do not occur with floating-point arithmetic due to the virtual impossibility of overflow.

As an example of an overflow oscillation, once again consider the filter of (3.69) with 4-b fixed-point two's complement arithmetic and with the two's complement overflow characteristic of (3.71):

$$y(n) = Q_r\left[R\left\{\frac{7}{8}y(n-1) - \frac{5}{8}y(n-2) + x(n)\right\}\right] \tag{3.72}$$

In this case we apply the input

35

$$x(n) = -4^{\&}(n) - \wedge^{\&}(n - 1) \qquad (3.73)$$

3 5 , ,

0, 0, ■

4 8

s to scale the filter calculations so as to render overflow impossible. However, this may unacceptably restrict the filter dynamic range. Another method is to force completed sums-of- products to saturate at ±1, rather than overflowing [18,19]. It is important to saturate only the completed sum, since intermediate overflows in two's complement arithmetic do not affect the accuracy of the final result. Most fixed-point digital signal processors provide for automatic saturation of completed sums if their *saturation arithmetic* feature is enabled. Yet another way to avoid overflow oscillations is to use a filter structure for which any internal filter transient is guaranteed to decay to zero [20]. Such structures are desirable anyway, since they tend to have low roundoff noise and be insensitive to coefficient quantization [21].

4.7 Coefficient Quantization Error:

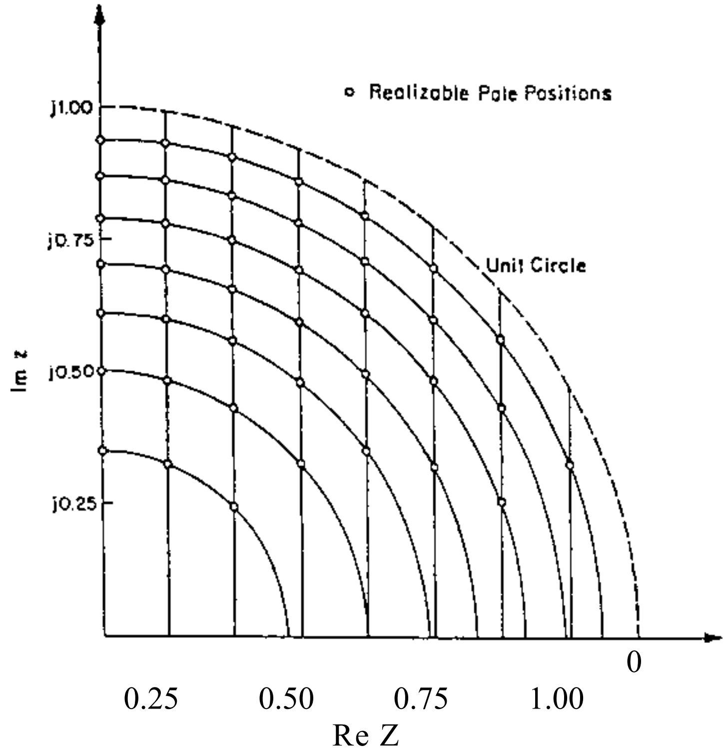


FIGURE: Realizable pole locations for the difference equation of (3.76).

The sparseness of realizable pole locations near z = ± 1 will result in a large coefficient quantization error for poles in this region.
Figure3.4 gives an alternative structure to (3.77) for realizing the transfer function of (3.76). Notice that quantizing the coefficients of this structure corresponds to quantizing X_r and Xi. As shown in Fig.3.5 from [5], this results in a uniform grid of realizable pole locations. Therefore, large coefficient quantization errors are avoided for all pole locations.
It is well established that filter structures with low roundoff noise tend to be robust to coefficient quantization, and visa versa [22]- [24]. For this reason, the uniform grid structure of Fig.3.4 is also popular because of its low roundoff noise. Likewise, the low-noise realizations of [7]- [10] can be expected to be relatively insensitive to coefficient quantization, and digital wave filters and lattice filters that are derived from low-sensitivity analog structures tend to have not only low coefficient sensitivity, but also low roundoff noise [25,26].
It is well known that in a high-order polynomial with clustered roots, the root location is a very sensitive function of the polynomial coefficients. Therefore, filter poles and zeros can be much more accurately controlled if higher order filters are realized by breaking them up into the parallel or cascade connection of first- and second-order subfilters. One exception to this rule is the case of linear-phase FIR filters in which the symmetry of the polynomial coefficients and the spacing of the filter zeros around the unit circle usually permits an acceptable direct realization using the convolution summation.
Given a filter structure it is necessary to assign the ideal pole and zero locations to the realizable locations. This is generally done by simplyrounding or truncatingthe filter coefficients to the available number of bits, or by assigning the ideal pole and zero locations to the nearest realizable locations. A more complicated alternative is to consider the original filter design problem as a problem in discrete

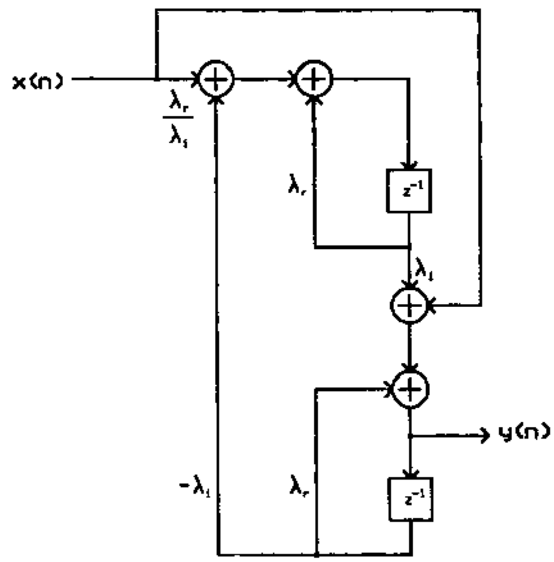


FIGURE 3.4: Alternate realization structure.

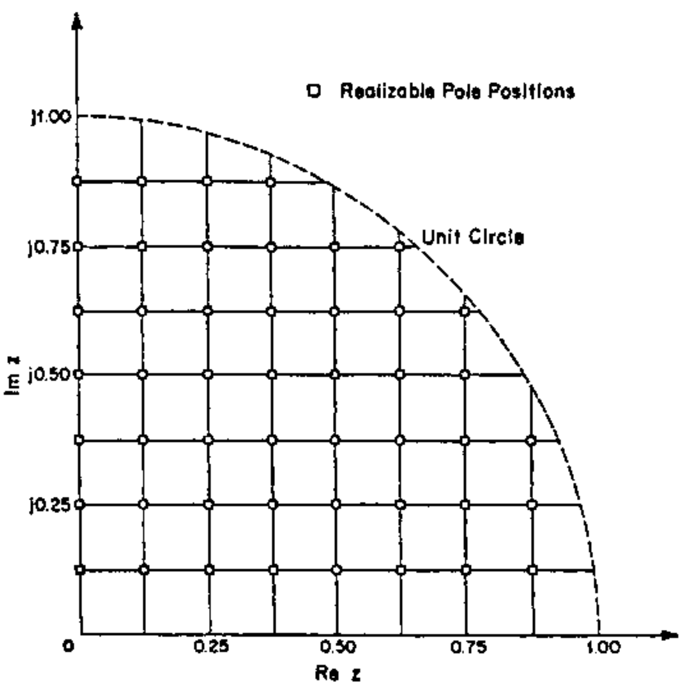


FIGURE 3.5: Realizable pole locations for the alternate realization structure.

optimization, and choose the realizable pole and zero locations that give the best approximation to the desired filter response [27]- [30].

4.8 Realization Considerations:

Linear-phase FIR digital filters can generally be implemented with acceptable coefficient quantization sensitivity using the direct convolution sum method. When implemented in this way on a digital signal processor, fixed-point arithmetic is not only acceptable but may actually be preferable to floating-point arithmetic. Virtually all fixed-point digital signal processors accumulate a sum of products in a double-length accumulator. This means that only a single quantization is necessary to compute an output. Floating-point arithmetic, on the other hand, requires a quantization after every multiply and after every add in the convolution summation. With 32-b floating-point arithmetic these quantizations introduce a small enough error to be insignificant for many applications.

When realizing IIR filters, either a parallel or cascade connection of first- and second-order subfilters is almost always preferable to a high-order direct-form realization. With the availability of very low-cost floating-point digital signal processors, like the Texas Instruments TMS320C32, it is highly recommended that floating-point arithmetic be used for IIR filters. Floating-point arithmetic simultaneously eliminates most concerns regarding scaling, limit cycles, and overflow oscillations. Regardless of the arithmetic employed, a low roundoff noise structure should be used for the second- order sections. Good choices are given in [2] and [10]. Recall that realizations with low fixed-point roundoff noise also have low floating-point roundoff noise. The use of a low roundoff noise structure for the second-order sections also tends to give a realization with low coefficient quantization sensitivity. First-order sections are not as critical in determining the roundoff noise and coefficient sensitivity of a realization, and so can generally be implemented with a simple direct form structure.

GLOSSARY:

Quantization:

Total number of bits in x is reduced by using two methods namely Truncation and Rounding. These are known as quantization Processes.

Input Quantization Error:

The Quantized signal are stored in a b bit register but for nearest values the same digital equivalent may be represented. This is termed as Input Quantization Error.

Product Quantization Error:

The Multiplication of a b bit number with another b bit number results in a 2b bit number but it should be stored in a b bit register. This is termed as Product Quantization Error.

Co-efficient Quantization Error:

The Analog to Digital mapping of signals due to the Analog Co-efficient Quantization results in error due to the Fact that the stable poles marked at the edge of the $j\Omega$ axis may be marked as an unstable pole in the digital domain.

Limit Cycle Oscillations:

If the input is made zero, the output should be made zero but there is an error occur due to the quantization effect that the system oscillates at a certain band of values.

Overflow limit Cycle oscillations:

Overflow error occurs in addition due to the fact that the sum of two numbers may result in overflow. To avoid overflow error saturation arithmetic is used.

Dead band:

The range of frequencies between which the system oscillates is termed as Deadband of the Filter. It may have a fixed positive value or it may oscillate between a positive and negative value.

Signal scaling:

The inputs of the summer is to be scaled first before execution of the addition operation to find for any possibility of overflow to be occurred after addition. The scaling factor s_0 is multiplied with the inputs to avoid overflow.

UNIT V
APPLICATIONS OF DSP

PRE REQUISITE DISCUSSION:
The time domain waveform is transformed to the frequency domain using a filter bank. The strength of each frequency band is analyzed and quantized based on how much effect they have on the perceived decompressed signal.

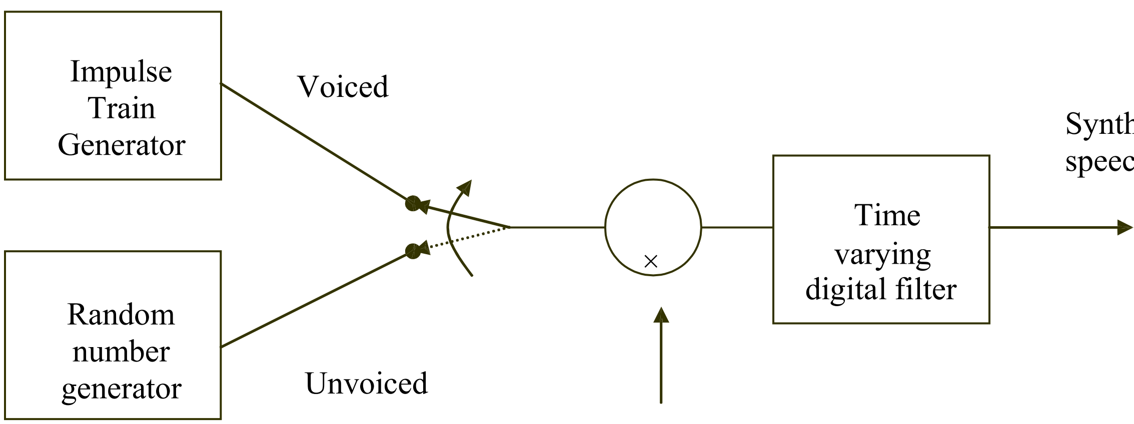
5.1. SPEECH RECOGNITION:

Basic block diagram of a speech recognition system is shown in Fig 1

1. In speech recognition system using microphone one can input speech or voice. The analog speech signal is converted to digital speech signal by speech digitizer. Such digital signal is called digitized speech.

2. The digitized speech is processed by DSP system. The significant features of speech such as its formats, energy, linear prediction coefficients are extracted. The template of this extracted features are compared with the standard reference templates. The closed matched template is considered as the recognized word.

3. Voice operated consumer products like TV, VCR, Radio, lights, fans and voice operated telephone dialing are examples of DSP based speech recognized devices.



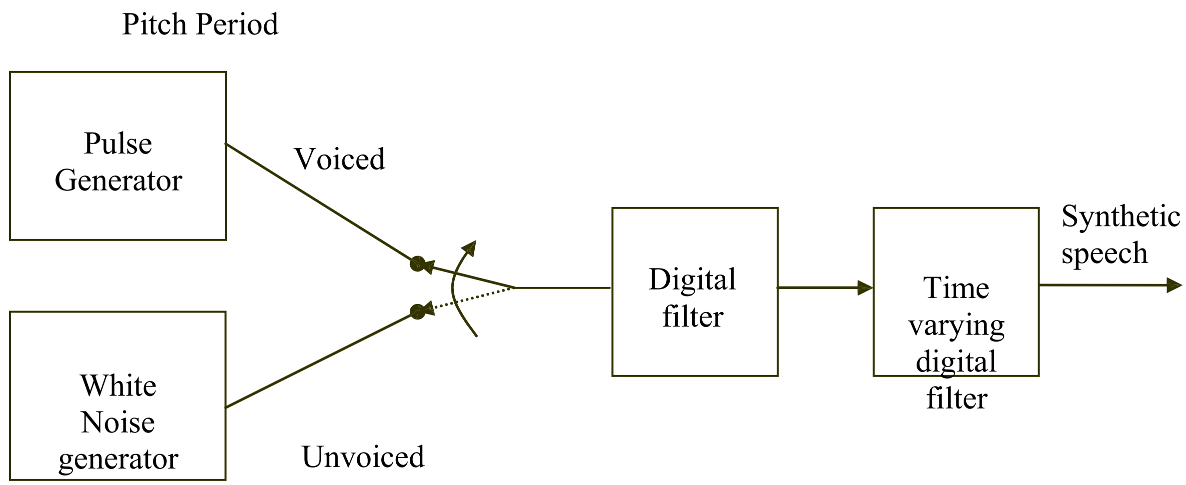
5.2. LINEAR PREDICTION OF SPEECH SYNTHESIS

Fig shows block diagram of speech synthesizer using linear prediction.

1. For voiced sound, pulse generator is selected as signal source while for unvoiced sounds noise generator is selected as signal source.

2. The linear prediction coefficients are used as coefficients of digital filter. Depending upon these coefficients , the signal is passed and filtered by the digital filter.

3. The low pass filter removes high frequency noise if any from the synthesized speech. Because of linear phase characteristic FIR filters are mostly used as digital filters.


Pitch Period
Pulse Generator
Voiced
White Noise generator
Unvoiced
Digital filter
Time varying digital filter
Synthetic speech

Filter Coefficients

5.3. SOUND PROCESSING:

1. In sound processing application, Music compression(MP3) is achieved by converting the time domain signal to the frequency domain then removing frequencies which are no audible.

2. The time domain waveform is transformed to the frequency domain using a filter bank. The strength of each frequency band is analyzed and quantized based on how much effect they have on the perceived decompressed signal.

3. The DSP processor is also used in digital video disk (DVD) which uses MPEG-2 compression, Web video content application like Intel Indeo, real audio.

4. Sound synthesis and manipulation, filtering, distortion, stretching effects are also done by DSP processor. ADC and DAC are used in signal generation and recording.

5. 4. ECHO CANCELLATION

In the telephone network, the subscribers are connected to telephone exchange by two wire circuit. The exchanges are connected by four wire circuit. The two wire circuit is bidirectional and carries signal in both the directions. The four wire circuit has separate paths for transmission and reception. The hybrid coil at the exchange provides the interface between two wire and four wire circuit which also provides impedance matching between two wire and four wire circuits. Hence there are no echo or reflections on the lines. But this impedance matching is not perfect because it is length dependent. Hence for echo cancellation, DSP techniques are used as follows.

1. An DSP based acoustic echo canceller works in the following fashion: it records the sound going to the loudspeaker and substract it from the signal coming from the microphone. The sound going through the echo-loop is transformed and delayed, and noise is added, which complicate the substraction process.

2. Let ■ be the input signal going to the loudspeaker; let ■ be the signal picked up by the microphone, which will be called the desired signal. The signal after

substraction will be called the error signal and will be denoted by . The adaptive filter will try to identify the equivalent filter seen by the system from the loudspeaker to the microphone, which is the transfer function of the room the loudpeaker and microphone are in.

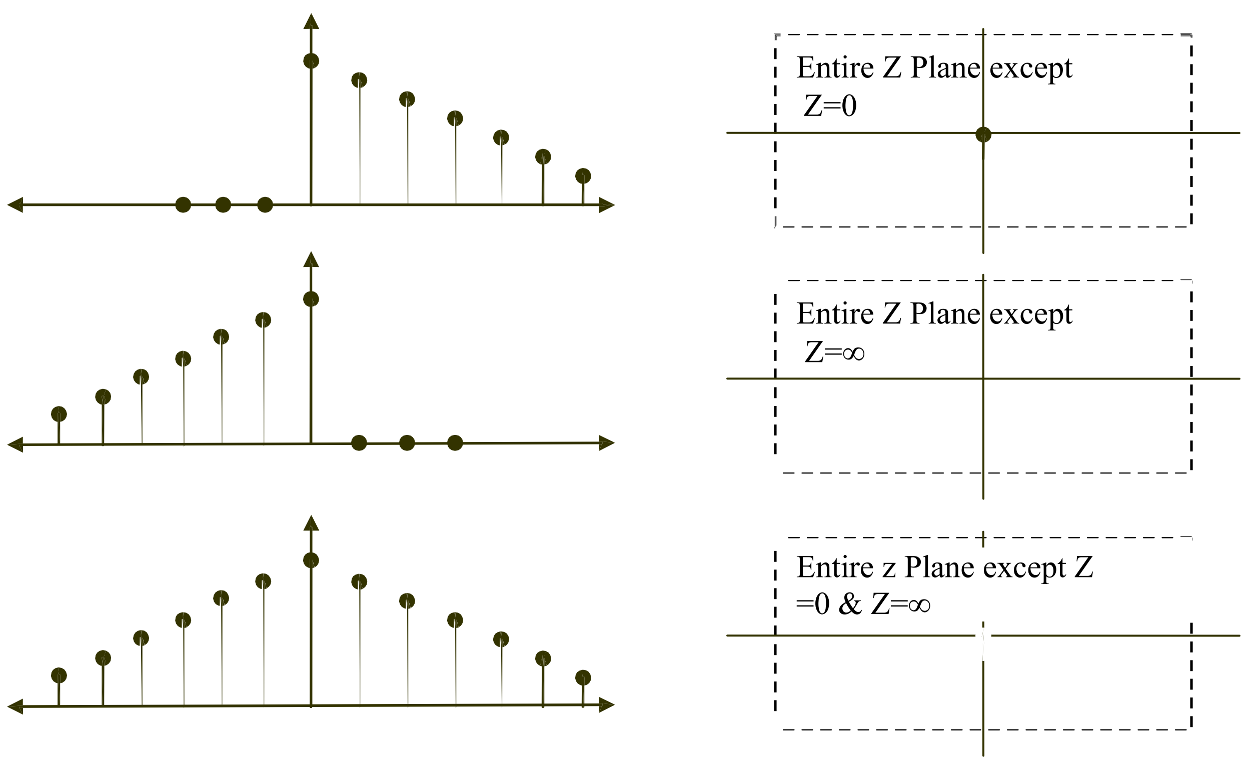
3. This transfer function will depend heavily on the physical characteristics of the environment. In broad terms, a small room with absorbing walls will origninate just a few, first order reflections so that its transfer function will have a short impulse response. On the other hand, large rooms with reflecting walls will have a transfer function whose impulse response decays slowly in time, so that echo cancellation will be much more difficult.

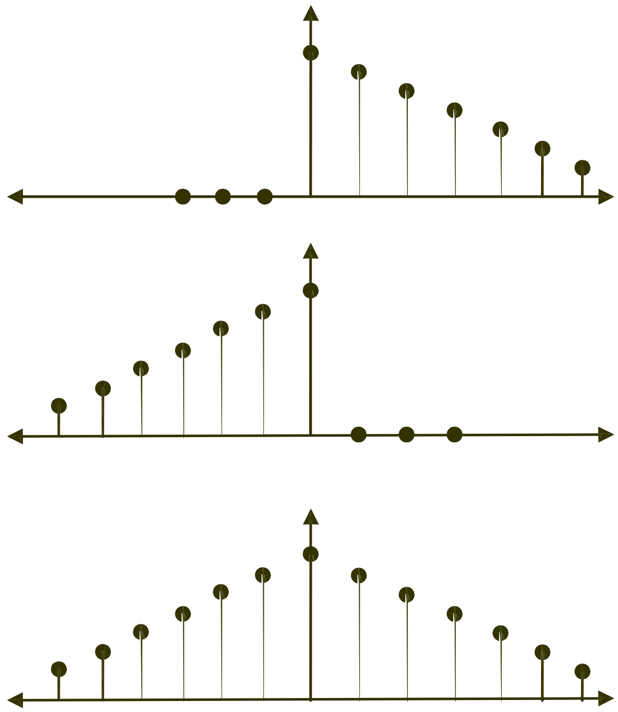
5. 5 VIBRATION ANALYSIS:

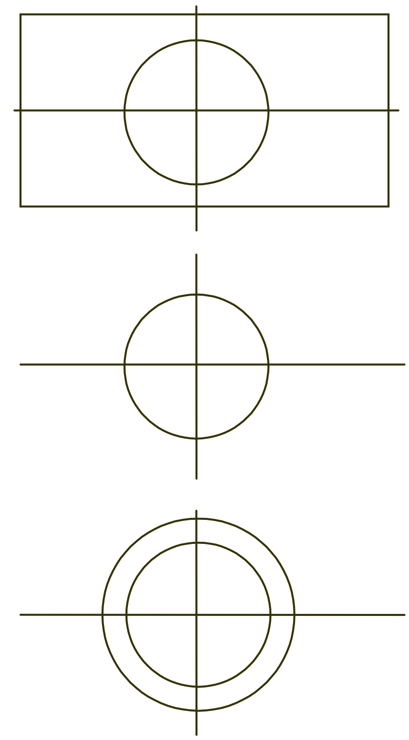
1. Normally machines such as motor, ball bearing etc systems vibrate depending upon the speed of their movements.

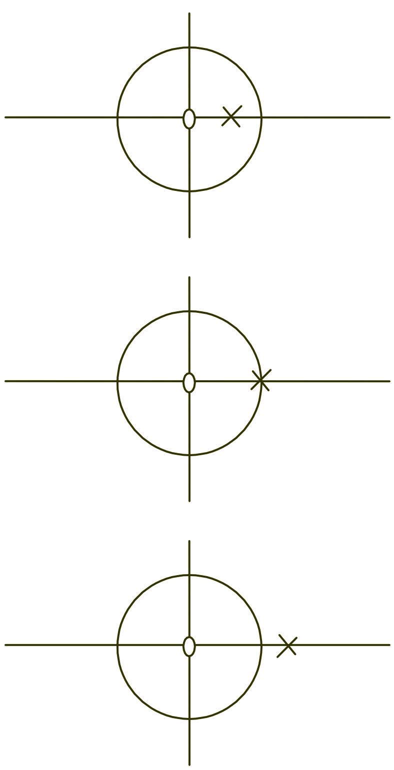
2. In order to detect fault in the system spectrum analysis can be performed. It shows fixed frequency pattern depending upon the vibrations. If there is fault in the machine, the predetermined spectrum is changes. There are new frequencies introduced in the spectrum representing fault.

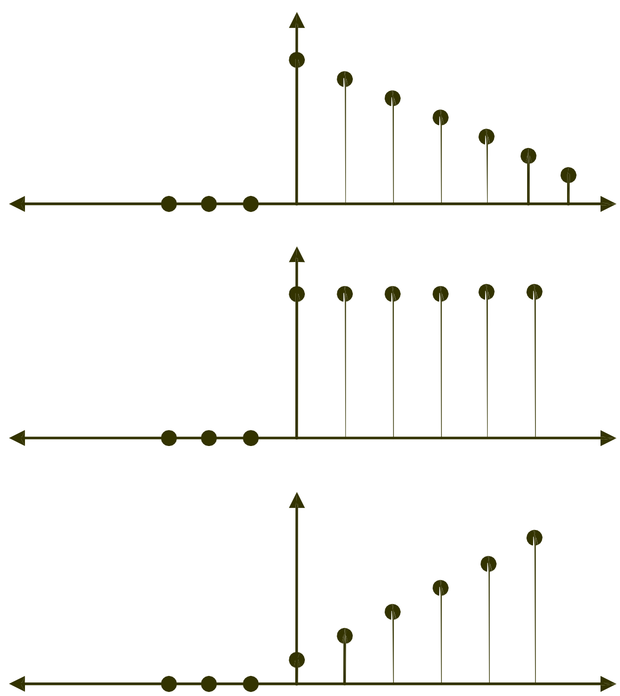
3. This spectrum analysis can be performed by DSP system. The DSP system can also be used to monitor other parameters of the machine simultaneously.











5.6. Multistage Implementation of Digital Filters:
In some applications we want to design filters where the bandwidth is just a small fraction of the overall sampling rate. For example, suppose we want to design a lowpass filter with bandwidth of the order of a few hertz and a sampling frequency of the order of several kilohertz. This filter would require a very sharp transition region in the digital frequency $a>$, thus requiring a high-complexity filter.
Example <

As an example of application, suppose you want to design a Filter with the fallowing specifications:
Passband F_p = 450 Hz
Stopband F_s = 500 Hz
Sampling frequency F_s~96 kHz
Notice that the stopband is several orders of magnitude smaller than the sampling frequency. This leads to a filter with a very short transition region of high complexity. In

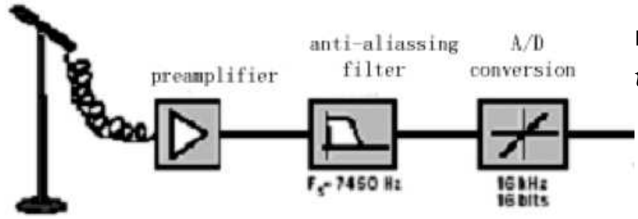
Speech signals

© From prehistory to the new media of the future, speech has been and will be a primary form of communication between humans.
© Nevertheless, there often occur conditions under which we measure and then transform the speech to another form, speech signal, in order to enhance our ability to communicate.
© The speech signal is extended, through technological media such as telephony, movies, radio, television, and now Internet. This trend reflects the primacy of speech communication in human psychology.
© "Speech will become the next major trend in the personal computer market in the near future."

5.7.Speech signal processing:

© The topic of speech signal processing can be loosely defined as the manipulation of sampled speech signals by a digital processor to obtain a new signal with some desired properties.

Speech signal processing is a diverse field that relies on knowledge of language at the levels of Signal processing
Acoustics ***(P¥)***
Phonetics **(^ ^ ^) Language-independent**
Phonology **(^ ^)**
Morphology **(i ^ ^ ^)**
Syntax **(^ , £) Language-dependent**
Semantics *(\%X¥)*
Pragmatics **(i f , f f l ^)**
7 layers for describing speech From Speech to Speech Signal, in terms of Digital Signal Processing



At-*

■ Acoustic (and perceptual) features *{traits)*

- fundamental freouency (FO) (pitch)
- **amplitude (loudness)**
- **spectrum (timber)**

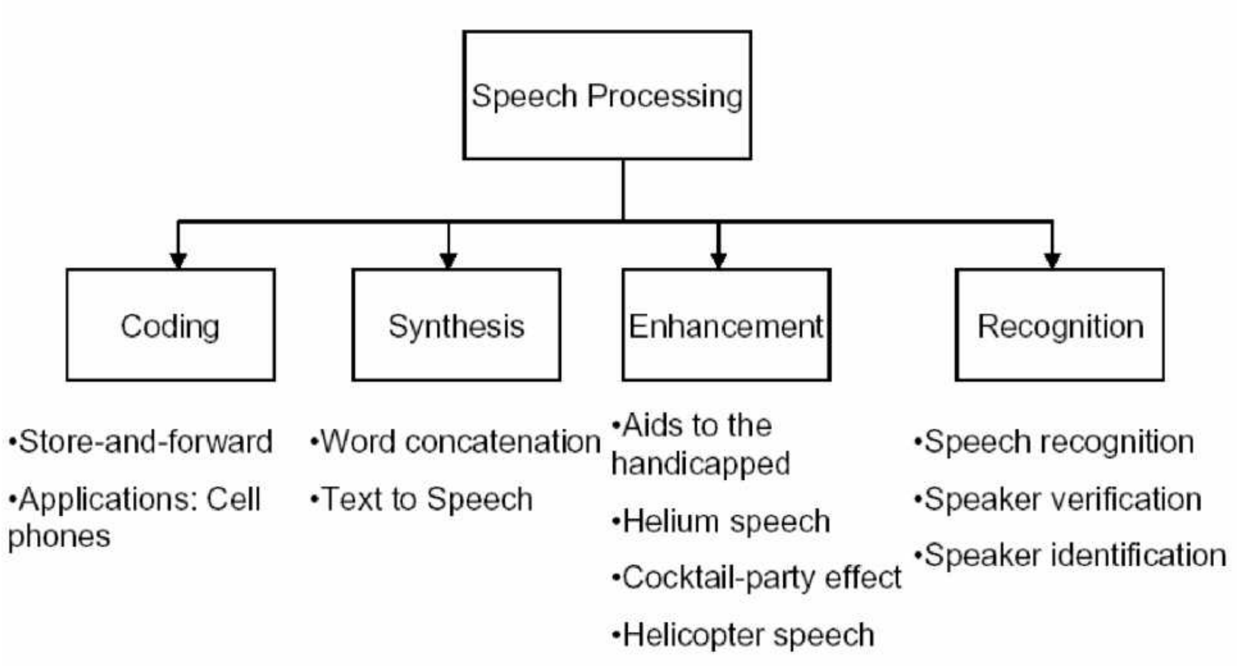
It is based on the fact that

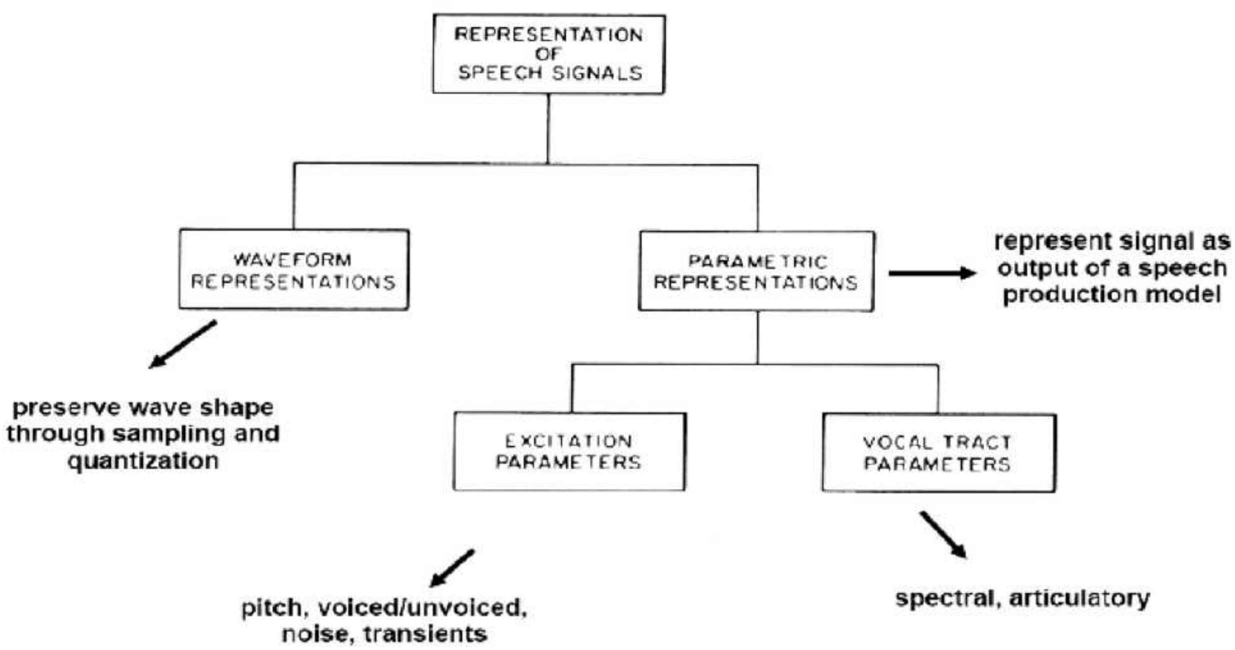
- Most of energy between 20 Hz to about 7KHz ,
- Human ear sensitive to energy between 50 Hz and 4KHz

© In terms of acoustic or perceptual, above features are considered.

© From Speech to Speech Signal, in terms of Phonetics (Speech production), the digital model of Speech Signal will be discussed in Chapter 2.

© Motivation of converting speech to digital signals:



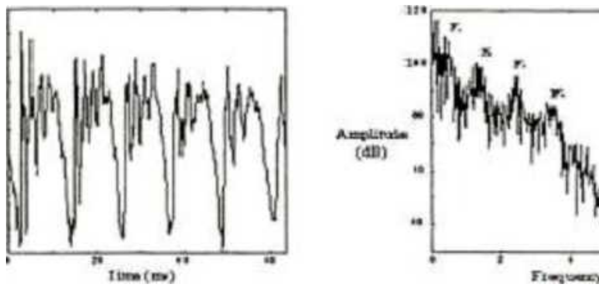


pitch,
fine structure

Speech coding, A PC and SBC

Adaptive predictive coding (APC) is a technique used for speech coding, that is data compression of spccch signals APC assumes that the input speech signal is repetitive with a period significantly longer than the average frequency content. Two predictors arc used in APC. The high frequency components (up to 4 kHz) are estimated using a 'spectral' or 'formant' prcdictor and the low frequency components (50-200 Hz) by a 'pitch' or 'fine structure' prcdictor (see figure 7.4). The spcctral estimator may he of order 1- 4 and the pitch estimator about order 10. The low-frequency components of the spccch signal are due to the movement of the tongue, chin and spectral

Typical Voiced **speech**



envelope, formants

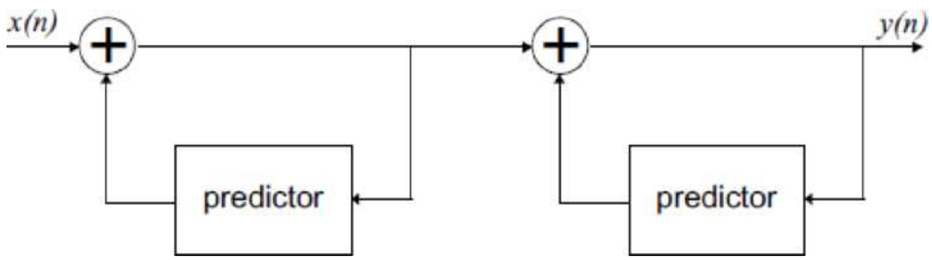


***Figure 7.4* Encoder for adaptive, predictive coding of speech signals. The decoder is mainly a mirrored version of the encoder**

The high-frequency components originate from the vocal chords and the noise-like sounds (like in 's') produced in the front of the mouth.

The output signal $y(n)$together with the predictor parameters, obtained adaptively in the encoder, are transmitted to the decoder, where the spcech signal is reconstructed. The decoder has the same structure as the encoder but the predictors arc not adaptive and arc invoked in the reverse order. The prediction parameters are adapted for blocks of data corresponding to for instance 20 ms time periods.

A PC' is used for coding spcech at 9.6 and 16 kbits/s. The algorithm works well in noisy environments, but unfortunately the quality of the processed speech is not as good as for other methods like CELP described below.

5..8 Subband Coding:

Another coding method is *sub-band coding (SBC)* (see Figure 7.5) which belongs to the class of *waveform coding* methods, in which the frequency domain properties of the input signal arc utilized to achieve data compression.

The basic idea is that the input speech signal is split into *sub-bands* using band-pass filters. The sub-band signals arc then encoded using ADPCM or other techniques. In this way, the available data transmission capacity can be allocated between bands according to pcrccptual criteria, enhancing the speech quality as pcrceivcd by listeners. A sub-band that is more 'important' from the human listening point of view can be allocated more bits in the data stream, while less important sub-bands will use fewer bits.

A typical setup for a sub-band codcr would be a bank of N(digital) bandpass filters followed by dccimators, encoders (for instance ADPCM) and a multiplexer combining the data bits coming from the sub-band channels. The output of the multiplexer is then transmitted to the sub-band dccodcr having a demultiplexer splitting the multiplexed data stream back into N sub-band channels. Every sub-band channel has a dccodcr (for instance ADPCM), followed by an interpolator and a band-pass filter. Finally, the outputs of the band-pass filters are summed and a reconstructed output signal results.

Sub-band coding is commonly used at bit rates between 9.6 kbits/s and 32 kbits/s and performs quite well. The complexity of the system may however be considerable if the number of sub-bands is large. The design of the band-pass filters is also a critical topic when working with sub-band coding systems.

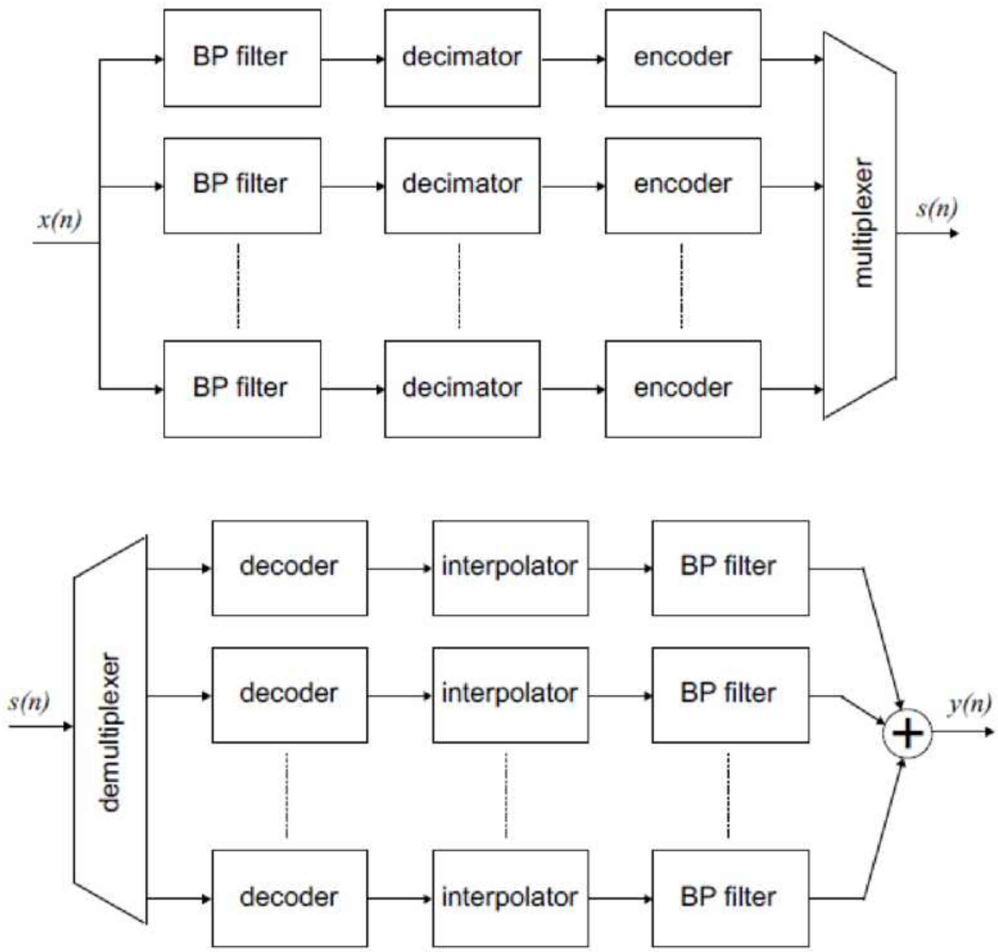


Figure 7.5 An example of a sub-band coding system

Vocoders and LPC

In the methods described above (APC, SBC and ADPCM), speech signal applications have been used as examples. By modifying the structure and parameters of the predictors and filters, the algorithms may also be used for other signal types. The main objective was to achieve a reproduction that was as faithful as possible to the original signal. Data compression was possible by removing redundancy in the time or frequency domain.

The class of vocoders (also called source coders) is a special class of data compression devices aimed only at spcech signals. The input signal is analysed and described in terms of speech model parameters. These parameters are then used to synthesize a voice pattern having an acceptable level of perceptual quality. Hence, waveform accuracy is not the main goal, as in the previous methods discussed.

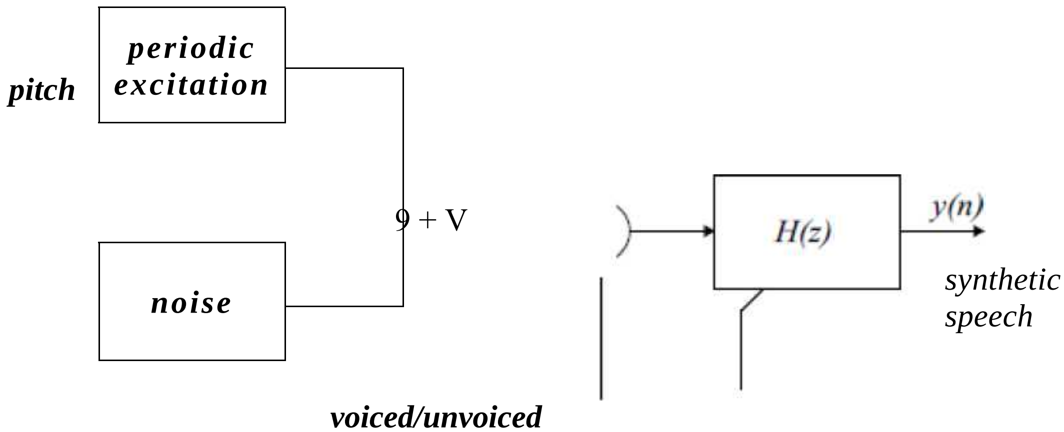


Figure 7.6 **The LPC model**

The first vocoder was designed by H. Dudley in the 1930s and demonstrated at the ‘New York Fair’ in 1939* Vocoders have become popular as they achieve reasonably good speech quality at low data rates, from *2A* kbits/s to 9,6 kbits/s. There arc many types of vocoders (Marvcn and Ewers, 1993), some of the most common techniques will be briefly presented below.

Most vocoders rely on a few basic principles. Firstly, the characteristics of the spccch signal is assumed to be fairly constant over a time of approximately 20 ms, hcncc most signal processing is performed on (overlapping) data blocks of 20 40 ms length. Secondly, the spccch model consists of a time varying filter corresponding to the acoustic properties of the mouth and an excitation signal. The cxeilalion signal is cither a periodic waveform, as crcatcd by the vocal chords, or a random noise signal for production of ‘unvoiced' sounds, for example ‘s’ and T. The filter parameters and excitation parameters arc assumed to be independent of each other and are commonly coded separately.

Linear predictive coding (LPC) is **a** popular method, which has however been replaced by newer approaches in many applications. LPC works exceedingly well **at** low bit rates and the **LPC** parameters contain sufficient information of the spccch signal to be used in spccch recognition applications. The **LPC** model is shown in Figure 7*6.

LPC is basically an **autn-regressive** model (sec Chapter 5) and the vocal tract is modelled as a time-varying all-pole filter (HR filter) having the transfer function $H(z)$

(7*17)

-k

***k*=I**

where p is the order of the filter. The excitation signal *?(«), being either noise or a periodic waveform, is fed to the filter via a variable gain factor G. The output signal can be expressed in the time domain as

$$y(n) \sim Ge(ri) - a_{,}y(n-1) - a_2 y(n-2) \qquad \%y\{n-p) \qquad (1.\ \mathrm{IK})$$

The output sample at time n is a linear combination of p previous samples and the excitation signal (linear predictive coding). The filter coefficients $a_k$arc time varying.

The model above describes how to *synthesize* the speech given the pitch information (if noise or pet iodic excitation should be used), the gain and the filter parameters. These parameters must be determined by the cncoder or the analyser, taking the original spccch signal $x(n)$ as input.

The analyser windows the spccch signal in blocks of 20-40 ms. usually with a Hamming window (see Chapter 5). These blocks or 'frames' arc repeated every 10—30 ms, hence there is a certain overlap in time. Every frame is then analysed with respect to the parameters mentioned above.

Firstly, the pitch frequency is determined. This also tells whether we arc dealing with a voiced or unvoiccd spccch signal. This is a crucial part of the system and many pitch detection algorithms have been proposed. If the segment of the spccch signal is voiced and has a dear periodicity or if it is unvoiced and not pet iodic, things arc quite easy* Segments having properties in between these two extremes are difficult to analyse. No algorithm has been found so far that is [1]perfect* for all listeners.

Now, the second step of the analyser is to determine the gain and the filter parameters. This is done by estimating the spccch signal using an adaptive predictor. The predictor has the same structure and order as the filter in the synthesizer, Hencc, the output of the predictor is

$$-i(n) — -tf]jt(/7— 1) — a_2x(n—2) —...— OpX(n—p) \qquad (7\text{-}19)$$

where i(rt) is the predicted input spcech signal and jc(rt) is the actual input signal. The filter coefficients $a_k$are determined by minimizing the square error

$$\sum_n \left(x(n) - \hat{x}(n)\right)^2 = \sum_n r^2(n)$$

This can be done in different ways, cither by calculating the auto-corrc- lation coefficients and solving the Yule Walker equations (see Chapter 5) or by using some recursive, adaptive filter approach (see Chapter 3),

So, for every frame, all the parameters above arc determined and irans- mittcd to the synthesiser, where a synthetic copy of the spccch is generated.

An improved version of LPC is **residual excited linear prediction (RELP).** Let us take a closer look at **the** error or residual signal r(fi) resulting from the prediction in the analyser (equation (7.19)). The residual signal (wc arc try ing to minimize) can be expressed as

$$r(n)= *(«) -i(rt)$$

$$= jf(rt) + a\^x(n— 1) + a_2x(n— 2) -h *.. + a_px(ft—p) \qquad <7\text{-}21)$$

From this it is straightforward to find out that the corresponding expression using the z-transforms is

$$R(z) = \frac{X(z)}{H(z)} = X(z)H^{-1}(z)$$

Hcncc, the prcdictor can be regarded as an 'inverse' filter to the LPC model filter. If we now pass this residual signal to the synthesizer and use it to excite the LPC filter, that is $E(z)$ - $R(z)$, instead of using the noise or periodic waveform sources we get

$$Y(z) = E(z)H(z) = R(z)H(z) = X(z)H\sim\backslash z)H(z) = X(z) \quad (7.23)$$

In the ideal case, we would hence get the original speech signal back. When minimizing the variance of the residual signal (equation (7.20)), we gathered as much information about the spccch signal as possible using this model in the filter coefficients a_k. The residual signal contains the remaining information. If the model is well suited for the signal type (speech signal), the residual signal is close to white noise, having a flat spectrum. In such a case we can get away with coding only a small range of frequencies, for instance 0-1 kHz of the residual signal. At the synthesizer, this baseband is then repeated to generate higher frequencies. This signal is used to excite the LPC filter

Vocoders using RELP are used with transmission rates of 9.6 kbits/s. The advantage of RELP is a better speech quality compared to LPC for the same bit rate. However, the implementation is more computationally demanding.

Another possible extension of the original LPC approach is to use **multipulse excited linear predictive coding (MLPC).** This extension is an attempt to make the synthesized spcech less 'mechanical', by using a number of different pitches of the excitation pulses rather than only the two (periodic and noise) used by standard LPC.

The MLPC algorithm sequentially detects k pitches in a speech signal. As soon as one pitch is found it is subtracted from the signal and detection starts over again, looking for the next pitch. Pitch information detection is a hard task and the complexity of the required algorithms is often considerable. MLPC however offers a better spcech quality than LPC for a given bit rate and is used in systems working with 4.S-9.6 kbits/s.

Yet another extension of LPC is the **code excited linear prediction (CELP).** The main feature of the CELP compared to LPC is the way in which the filter coefficients are handled. Assume that we have a standard LPC system, with a filter of the order p. If every coefficient a_k requires N bits, we need to transmit N-p bits per frame for the filter parameters only. This approach is all right if all combinations of filter coefficients arc equally probable. This is however not the case. Some combinations of coefficients are very probable, while others may never occur. In CELP, the coefficient combinations arc represented by p dimensional vectors. Using vector quantization techniques, the most probable vectors are determined. Each of these vectors are assigned an **index** and stored in a **codebook.** Both the analyser and synthesizer of course have identical copies of the codebook, typically containing 256-512 vectors. Hcncc, instead of transmitting N-p bits per frame for the filter parameters only 8-9 bits arc needed.

This method offers high-quality spcech at low-bit rates but requires considerable computing power to be able to store and match the incoming spcech to the 'standard' sounds stored in the codebook. This is of course especially true if the codebook is large. Speech quality degrades as the codebook size decreases.

Most CELP systems do not perform well with respect to higher frequency components of the spccch signal at low hit rates. This is countcractcd in

There is also a variant of CELP called **vector sum excited linear prediction (VSELP).** The main difference between CELP and $VSELP$ is the way the codebook is organized. Further, since VSELP uses fixed point arithmetic algorithms, **it** is possible to implement using cheaper DSP chips than

Adaptive Filters

The signal degradation in some physical systems is time varying, unknown, or possibly both. For example,consider a high-speed modem for transmitting and receiving data over telephone channels. It employs a filter called a channel equalizer to compensate for the channel distortion. Since the dial-up communication channels have different and time-varying characteristics on each connection, the equalizer must be an adaptive filter.

5.9 Adaptive Filter:

Adaptive filters modify their characteristics to achieve certain objectives by automatically updating their coefficients. Many adaptive filter structures and adaptation algorithms have been developed for different applications. This chapter presents the most widely used adaptive filters based on the FIR filter with the least-mean-square (LMS) algorithm. These adaptive filters are relatively simple to design and implement. They are well understood with regard to stability, convergence speed, steady-state performance, and finite-precision effects.

Introduction to Adaptive Filtering

An adaptive filter consists of two distinct parts - a digital filter to perform the desired filtering, and an adaptive algorithm to adjust the coefficients (or weights) of the filter. A general form of adaptive filter is illustrated in Figure 7.1, where d(n) is a desired (or primary input) signal, y(n) is the output of a digital filter driven by a reference input signal $x(n)$, and an error signal e(n) is the difference between d(n) and y(n). The adaptive algorithm adjusts the filter coefficients to minimize the mean-square value of e(n). Therefore, the filter weights are updated so that the error is progressively minimized on a sample-bysample basis.

In general, there are two types of digital filters that can be used for adaptive filtering: FIR and IIR filters. The FIR filter is always stable and can provide a linear-phase response. On the other hand, the IIR

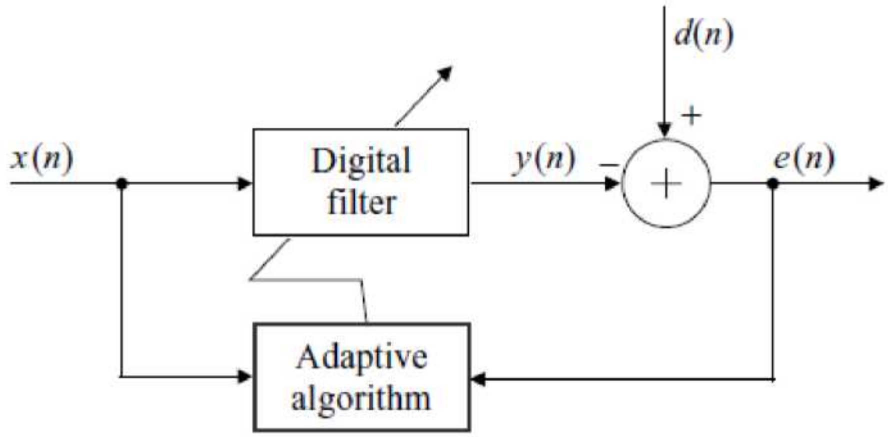


Figure 7.1 Block diagram of adaptive filter

filter involves both zeros and poles. Unless they are properly controlled, the poles in the filter may move outside the unit circle and result in an unstable system during the adaptation of coefficients. Thus, the adaptive FIR filter is widely used for practical real-time applications. This chapter focuses on the class of adaptive FIR filters.

The most widely used adaptive FIR filter is depicted in Figure 7.2. The filter output signal is computed

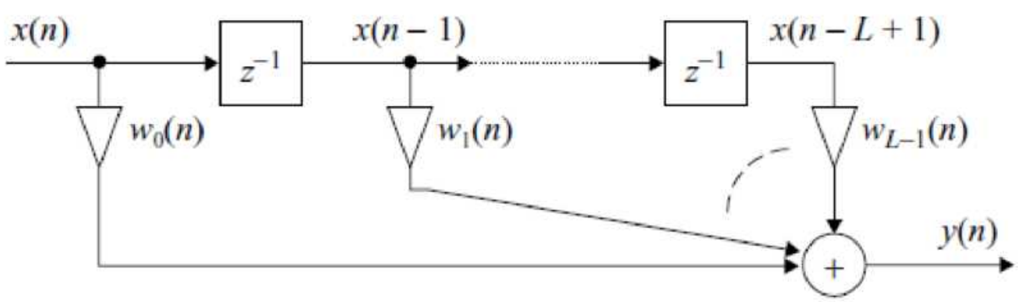


Figure 7.2 Block diagram of FIR filter for adaptive filtering

$$y(n) = \sum_{l=0}^{L-1} w_l(n)x(n-1),$$

(7.13)

, where the filter coefficients w_l (n) are time varying and updated by the adaptive algorithms that will be discussed next.
We define the input vector at time n as
x(n) = [x(n)x(n - 1) . . . $x(n - L + 1)$]T , (7.14)
and the weight vector at time n as
w(n) = [w0(n)w1(n) . . . wL-1(n)]T . (7.15)
Equation (7.13) can be expressed in vector form as
$y(n) = wT$ (n)x(n) = xT (n)w(n). (7.16)
The filter outputy(n) is compared with the desired $d(n)$ to obtain the error signal $e(n) =$
$d(n) - y(n) = d(n) - wT (n)x(n)$. (7.17)
Our objective is to determine the weight vector w(n) to minimize the predetermined performance (or cost) function.

Performance Function:

The adaptive filter shown in Figure 7.1 updates the coefficients of the digital filter to optimize some predetermined performance criterion. The most commonly used performance function is based on the mean-square error (MSE).

5.10 Audio Processing:

The two principal human senses are vision and hearing. Correspondingly,much of DSP is related to image and audio processing. People listen toboth *music* and *speech*. DSP has made revolutionary changes in both these areas.

5.10.1 Music Sound processing:

The path leading from the musician's microphone to the audiophile's speaker is remarkably long. Digital data representation is important to prevent the degradation commonly associated with analog storage and manipulation. This is very familiar to anyone who has compared the musical quality of cassette tapes with compact disks. In a typical scenario, a musical piece is recorded in a sound studio on multiple channels or tracks. In some cases, this even involves recording individual instruments and singers separately. This is done to give the sound engineer greater flexibility in creating the final product. The complex process of combining the individual tracks into a final product is called *mix down*. DSP can provide several important functions during mix down, including: filtering, signal addition and subtraction, signal editing, etc. One of the most interesting DSP applications in music preparation is *artificial reverberation*. If the individual channels are simply added together, the resulting piece sounds frail and diluted, much as if the musicians were playing outdoors. This is because listeners are greatly influenced by the echo or reverberation content of the music, which is usually minimized in the sound studio. DSP allows artificial echoes and reverberation to be added during mix down to simulate various ideal listening environments. Echoes with delays of a few hundred milliseconds give the impression of cathedral likelocations. Adding echoes with delays of 10-20 milliseconds provide the perception of more modest size listening rooms.

5.10.2 Speech generation:

Speech generation and recognition are used to communicate between humans and machines. Rather than using your hands and eyes, you use your mouth and ears. This is very convenient when your hands and eyes should be doing something else, such as: driving a car, performing surgery, or (unfortunately) firing your weapons at the enemy. Two approaches are used for computer generated speech: *digital recording* and *vocal tract simulation*. In digital recording, the voice of a human speaker is digitized and stored, usually in a compressed form. During playback, the stored data are uncompressed and converted back into an analog signal. An entire hour of recorded speech requires only about three me gabytes of storage, well within the capabilities of even small computer systems. This is the most common method of digital speech generation used today. Vocal tract simulators are more complicated, trying to mimic the physical mechanisms by which humans create speech. The human vocal tract is an acoustic cavity with resonate frequencies determined by the size and shape of the chambers. Sound originates in the vocal tract in one of two basic ways, called *voiced* and *fricative* sounds. With voiced sounds, vocal cord vibration produces near periodic pulses of air into the vocal cavities. In comparison, fricative sounds originate from the noisy air turbulence at narrow constrictions, such as the teeth and lips. Vocal tract simulators operate by generating digital signals that resemble these two types of excitation. The characteristics of the resonate chamber are simulated by passing the excitation signal through a digital filter with similar resonances. This approach was used in one of the very early DSP success stories, the *Speak & Spell,* a widely sold electronic learning aid for children.

5.10.3 Speech recognition:

The automated recognition of human speech is immensely more difficult than speech generation. Speech recognition is a classic example of things that the human brain does well, but digital computers do poorly. Digital computers can store and recall vast amounts of data, perform mathematical calculations at blazing speeds, and do repetitive tasks without becoming bored or inefficient. Unfortunately, present day computers perform very poorly when faced with raw sensory data. Teaching a computer to send you a monthly electric bill is easy. Teaching the same computer to understand your voice is a major undertaking. Digital Signal Processing generally approaches the problem of voice recognition in two steps: *feature extraction* followed by *feature matching.* Each word in the incoming audio signal is isolated and then analyzed to identify the type of excitation and resonate frequencies. These parameters are then compared with previous examples of spoken words to identify the closest match. Often, these systems are limited to only a few hundred words; can only accept speech with distinct pauses between words; and must be retrained for each individual speaker. While this is adequate for many commercialapplications, these limitations are humbling when compared to the abilities of human hearing. There is a great deal of work to be done in this area, with tremendous financial rewards for those that produce successful commercial products.

5.11 Image Enhancement:

5.11.1 Spatial domain methods:

Suppose we have a digital image which can be represented by a two dimensional random field ^{A,y} ^ .
t *r*

An image processing operator in the spatial domain may be expressed as a mathematical function L applied to the image *f* to produce a new image -^v^ “ ^ ^ 'X' -^v^ - as follows.

$g\{x,y) = T\backslash\{x,y)\_$

The operator ^{T} applied on $f(x, y)$ may be defined over:

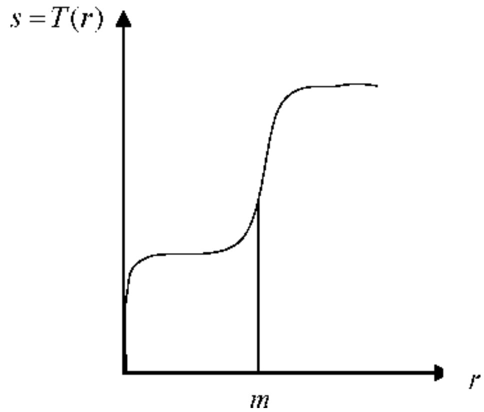
(i) A single pixel $^{(x,y)}$. In this case ^{T} is a grey level transformation (or mapping) function.

(ii) Some neighbourhood of $^{(x, y)}$.

(iii) ^{T} may operate to a set of input images instead of a single image.

EXample 1

The result of the transformation shown in the figure below is to produce an image of higher contrast than the original, by darkening the levels below *m* and brightening the levels above *m* in the original image. This technique is known as contrast stretching.



Example 2
The result of the transformation shown in the figure below is to produce a binary image.

$$s = T(r)$$

Frequency domain methods
Let g(x, y) be a desired image formed by the convolution of an image f (x, y) and a linear, position invariant operator h(x,y), that is:
$g(x,y) = h(x,y)2f(x,y)$
The following frequency relationship holds:

$$G(u, i') = H (a, v)F(u, i')$$

We can select H (u, v) so that the desired image

$$g(x,y) = 3^{-1} i\$(ii,v)F(u,v)$$

exhibits some highlighted features of f (x,y) . For instance, edges in f (x,y) can be accentuated by using a function H(u,v) that emphasises the high frequency components of $F(u,v)$.

Glossary:

Sampling Rate:

The No. of samples per cycle given in the signal is termed as sampling rate of the signal .The samples occur at T equal intervals of Time.

Sampling Theorem:

Sampling Theorem states that the no. of samples per cycle should be greater than or equal to twice that of the frequency of the input message signal.

Sampling Rate Conversion:

The Sampling rate of the signal may be increased or decreased as per the requirement and application. This is termed as sampling rate Conversion.

Decimation:

The Decrease in the Sampling Rate are termed as decimation or Downsampling. The No. of Samples per Cycle is reduced to M-1 no. of terms.

Interpolation:

The Increase in the Sampling rate is termed as Interpolation or Up sampling. The No. of Samples per Cycle is increased to L-1 No. of terms.

Polyphase Implementation:

If the Length of the FIR Filter is reduced into a set of smaller filters of length k. Usual upsampling process Inserts I-1 zeros between successive Values of x(n). If M Number of Inputs are there, Then only K Number of Outputs are non-zero. These k Values are going to be stored in the FIR Filter.

Narrow band Filters:

If we want to design a narrow passband and a narrow transition band, then a lowpass linear phase FIR filters are more efficiently implemented in a Multistage Decimator – Interpolator.

EC 6502 DIGITAL SIGNAL PROCESSING

UNIT – I DISCRETE FOURIER TRANSFORM

1. Define DTFT. APRIL/MAY2008

The discrete-time Fourier transform (or DTFT) of $x[n]$ is usually written:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-i\omega n}.$$

2. Define Periodicity of DTFT.

Sampling $x(t)$ causes its spectrum (DTFT) to become periodic. In terms of ordinary frequency, (cycles per second), the period is the sample rate, f_s. In terms of normalized frequency, f/f_s (cycles per sample), the period is 1. And in terms of ω(radians per sample), the period is 2π, which also follows directly from the periodicity of . That is:

$$e^{-i(\omega+2\pi k)n} = e^{-i\omega n}$$

Where both n and k are arbitrary integers. Therefore:

$$X(\omega + 2\pi k) = X(\omega)$$

3. Difference between DTFT and other transform. NOV/DEC 2010

The DFT and the DTFT can be viewed as the logical result of applying the standard continuous Fourier transform to discrete data. From that perspective, we have the satisfying result that it's not the transform that varies; it's just the form of the input:

If it is discrete, the Fourier transform becomes a DTFT.
If it is periodic, the Fourier transform becomes a Fourier series.
If it is both, the Fourier transform becomes a DFT.

4. Write about symmetry property of DTFT. APRIL/MAY2009

The Fourier Transform can be decomposed into a real and imaginary or into even and odd.

$$X(e^{i\omega}) = X_R(e^{i\omega}) + iX_I(e^{i\omega})$$

or

$$X(e^{i\omega}) = X_E(e^{i\omega}) + X_O(e^{i\omega})$$

Time Domain	Frequency Domain
$x[n]$	$X(e^{i\omega})$

$x^*[n]$	$X^*(e^{-i\omega})$
$x^*[-n]$	$X^*(e^{i\omega})$

5. Define DFT pair.

The sequence of N complex numbers $x_0, ..., x_{N-1}$ is transformed into the sequence of N complex numbers $X_0, ..., X_{N-1}$ by the DFT according to the formula:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} kn} \qquad k = 0, \dots, N-1$$

where $e^{\frac{2\pi i}{N}}$ is a primitive N'th root of unity.

The transform is sometimes denoted by the symbol $\mathcal{F}$, as in $\mathbf{X} = \mathcal{F}\left\{\mathbf{x}\right\}$ or $\mathcal{F}\left(\mathbf{x}\right)$ or $\mathcal{F}\mathbf{x}$.

The inverse discrete Fourier transform (IDFT) is given by

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{\frac{2\pi i}{N} kn} \qquad n = 0, \dots, N-1.$$

6. How will you express IDFT interms of DFT. MAY/JUNE 2010

A useful property of the DFT is that the inverse DFT can be easily expressed in terms of the (forward) DFT, via several well-known "tricks". (For example, in computations, it is often convenient to only implement a fast Fourier transform corresponding to one transform direction and then to get the other transform direction from the first.) First, we can compute the inverse DFT by reversing the inputs:

$$\mathcal{F}^{-1}(\{x_n\}) = \mathcal{F}(\{x_{N-n}\})/N$$

(As usual, the subscripts are interpreted modulo N; thus, for $n = 0$, we have $x_{N-0} = x_0$.)

Second, one can also conjugate the inputs and outputs:

$$\mathcal{F}^{-1}(\mathbf{x}) = \mathcal{F}(\mathbf{x}^*)^*/N$$

Third, a variant of this conjugation trick, which is sometimes preferable because it requires no modification of the data values, involves swapping real and imaginary parts (which can be done on a computer simply by modifying pointers). Define swap(x_n) as x_n with its real and imaginary parts swapped—that is, if $x_n = a + bi$ then swap(x_n) is $b + ai$. Equivalently, swap(x_n) equals ix_n^*. Then

$$\mathcal{F}^{-1}(\mathbf{x}) = \mathrm{swap}(\mathcal{F}(\mathrm{swap}(\mathbf{x})))/N$$

7. Write about Bilateral Z transform. APRIL/MAY2009

The *bilateral* or *two-sided* Z-transform of a discrete-time signal *x[n]* is the function *X(z)* defined as

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

where *n* is an integer and *z* is, in general, a complex number:

$z = Ae^{j\varphi}$ (OR)

$z = A(cosj\varphi + sinj\varphi)$

where *A* is the magnitude of *z*, and φ is the *complex argument* (also referred to as *angle* or *phase*) in radians

8. Write about Unilateral Z transforms.

Alternatively, in cases where $x[n]$ is defined only for $n \geq 0$, the *single-sided* or *unilateral* Z-transform is defined as

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=0}^{\infty} x[n]z^{-n}$$

In signal processing, this definition is used when the signal is causal.

9. Define Region Of Convergence.

The region of convergence (ROC) is the set of points in the complex plane for which the Z-transform summation converges.

$$ROC = \left\{ z : \left| \sum_{n=-\infty}^{\infty} x[n]z^{-n} \right| < \infty \right\}$$

10.Write about the output response of Z transform APRIL/MAY2008

If such a system $H(z)$ is driven by a signal $X(z)$ then the output is $Y(z) = H(z)X(z)$. By performing partial fraction decomposition on $Y(z)$ and then taking the inverse Z-transform the output $y[n]$ can be found. In practice, it is often useful to fractionally decompose $\frac{Y(z)}{z}$ before multiplying that quantity by z to generate a form of $Y(z)$ which has terms with easily computable inverse Z-transforms.

11. Define Twiddle Factor. MAY/JUNE 2010

A twiddle factor, in fast Fourier transform (FFT) algorithms, is any of the trigonometric constant coefficients that are multiplied by the data in the course of the algorithm.

12. State the condition for existence of DTFT? MAY/JUNE 2007

The conditions are, If x(n)is absolutely summable then |x(n)|< If x(n) is not absolutely summable then it should have finite energy for DTFT to exit.

13. List the properties of DTFT.

Periodicity, Linearity, Time shift, Frequency shift, Scaling, Differentiation in frequency domain, Time reversal, Convolution, Multiplication in time domain, Parseval's theorem

14. What is the DTFT of unit sample? NOV/DEC 2010

The DTFT of unit sample is 1 for all values of w.

15. Define Zero padding.

The method of appending zero in the given sequence is called as Zero padding.

16. Define circularly even sequence.

A Sequence is said to be circularly even if it is symmetric about the point zero on the circle. x(N-n)=x(n),1<=n<=N-1.

17. Define circularly odd sequence.

A Sequence is said to be circularly odd if it is anti symmetric about point x(0) on the circle

18. Define circularly folded sequences.

A circularly folded sequence is represented as x((-n))N. It is obtained by plotting x(n) in clockwise direction along the circle.

19. State circular convolution. NOV/DEC 2009

This property states that multiplication of two DFT is equal to circular convolution of their sequence in time domain.

20. State parseval's theorem. NOV/DEC 2009

Consider the complex valued sequences x(n) and y(n).If

x(n) → X(k), y(n)→Y(k) then x(n)y*(n)=1/N X(k)Y*(k)

21. Define Z transform.

The Z transform of a discrete time signal x(n) is denoted by X(z) and is given by X(z)= x(n)Z-n.

22. Define ROC.

The value of Z for which the Z transform converged is called region of convergence.

23. Find Z transform of x(n)={1,2,3,4}

x(n)= {1,2,3,4}

X(z)= x(n)z-n

= 1+2z-1+3z-2+4z-3.

= 1+2/z+3/z2+4/z3.

24. State the convolution property of Z transforms. APRIL/MAY2010

The convolution property states that the convolution of two sequences in time domain is equivalent to multiplication of their Z transforms.

25. What z transform of (n-m)?

By time shifting property

Z[A (n-m)]=AZ-m sinZ[(n)] =1

26. State initial value theorem.

If x(n) is causal sequence then its initial value is given by x(0)=lim X(z)

27. List the methods of obtaining inverse Z transform.

Partial fraction expansion.

Contour integration
Power series expansion
Convolution.

28. Obtain the inverse z transform of X(z)=1/z-a,|z|>|a| APRIL/MAY2010

Given X(z)=z-1/1-az-1

By time shifting property X(n)=an.u(n-1)

16-MARKS

1, Determine the 8-Point DFT of the Sequence x(n)={1,1,1,1,1,1,0,0}
(Nov / 2013)

Soln:

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N},\ k=0,1,........N-1.$$

$$X(0)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,...7.$$

X(0) = 6

$$X(1)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,...7.$$

X(1)=-0.707-j1.707.

$$X(2)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,....7$$

X(2)=1-j

$$X(3)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,.....7.$$

X(3)=0.707+j0.293

$$X(4)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,.......7$$

X(4)=0

$$X(5)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,........7.$$

X(5)=0.707-j0.293

$$X(6)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,........7.$$

X(6)=1+j

$$X(7)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,........7.$$

X(7)= -0.707+j1.707.

X(k)={6,-0.707-j1.707,1-j, 0.707+j0.293, 0, 0.707-j0.293, 1+j, -0.707+j1.707}

2, Determine the 8-Point DFT of the Sequence x(n)={1,1,1,1,1,1,1,1}
(April 2014)

Soln:

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, k=0,1,........N-1.$$

$$X(0)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,...7.$$

X(0) = 8

$$X(1)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,...7.$$

X(1)=0.

$$X(2)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,....7$$

X(2)=0.

$$X(3)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,.....7.$$

X(3)=0

$$X(4)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,.......7$$

X(4)=0

$$X(5)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8}, k=0,1,........7.$$

X(5)=0

$$X(6)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,........7.$$

X(6)=0

$$X(7)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,........7.$$

X(7)= 0.

X(k)={8, 0, 0, 0, 0, 0, 0, 0}

3, Determine the 8-Point DFT of the Sequence x(n)={1,2,3,4,4,3,2,1}

(Nov / 2014)

Soln:

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N},\ k=0,1,........N-1.$$

$$X(0)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,...7.$$

X(0) = 20

$$X(1)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,...7.$$

X(1)=-5.828-j2.414

$$X(2)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,....7$$

X(2)=0.

$$X(3)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,.....7.$$

X(3)=-0.172-j0.414.

$$X(4)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,.......7$$

X(4)=0

$$X(5)=\sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,........7.$$

n=0

X(5)= -0.172+j0.414

$$X(6)= \sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,........7.$$

X(6)=0

$$X(7)= \sum_{n=0}^{7} x(n)e^{-j2\pi kn/8},\ k=0,1,........7.$$

X(7)= -5.828+j2.414

X(k)={20,-5.828-j2.414, 0, -0.172-j0.414, 0, -0.172+j0.414, 0, -5.828+j2.414}

4, Determine the 8-Point IDFT of the Sequence x(n)={5,0,1-j,0,1,0,1+j,0}
(Nov / 2014)

Soln:

$$x(n)=1/N \sum_{k=0}^{N-1} X(k)e^{-j2\pi kn/N},\ n=0,1,........N-1.$$

$$x(0)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8},\ n=0,1,.......7.$$

x(0) = 1

$$x(1)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8},\ n=0,1,.......7.$$

x(1)=0.75

$$x(2)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8},\ n=0,1,.......7.$$

x(2)=0.5

$$x(3)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8},\ n=0,1,.......7.$$

x(3)=0.25

$$x(4)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8},\ n=0,1,.......7.$$

x(4)=1

$$x(5)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8}, n=0,1,.......7.$$

x(5)= 0.75

$$x(6)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8}, n=0,1,.......7.$$

x(6)=0.5

$$x(7)=1/8 \sum_{k=0}^{7} X(k)e^{-j2\pi kn/8}, n=0,1,.......7.$$

x(7)= 0.25

x(n)={1,0.75, 0.5, 0.25,1,0.75,0.5,0.25}

5. Derive and draw the radix -2 DIT algorithms for FFT of 8 points. (16) **DEC 2009**

RADIX-2 FFT ALGORITHMS

The /V-point DFT of an fV-point sequence J is

jV-l
rj=fl

Because rv(/i) may be either real or complex, evaluating -¥(A) requires on the order of jV complex multiplications and N complex additions for each value of jt. Therefore, because there are Nvalues of $X(kh$ computing an N-point DFT requires N^1 complex multiplications and additions.

The basic strategy thaiis used in the FFT algorithm is one of "divide and conquer." which involves decomposing an /V-point DFT inlo successively smaller DFTs. To see how this works, suppose lhat the length of is even (i.e,,N is divisible by 2). If jr(n) is *decimated* into two sequences of length $N/2$, computing the jV/2-point DFT of each of these sequences requires approximatelymultiplications and thesame number of additions, Thus, ihe two DFTs require $2(iV/2)^2$ = ^ /V^2multiplies and adds, Therefore, if it is possible to find the fy-poim DFT of jt(h) from these two /V/2-point DFTs in fewer ihan N^2f2operations, a savings has been realized.

Deetmation-tn-Tlme FFT

The decimation'in-time FFT algorilhm is based on splitting (decimating) jt{a) into smaller sequences and finding $X(k)$from the DFTsof these decimated sequences This section describes how this decimation leads to an efficient algorithm when the sequence length is a power of 2.

Lei .r(n) be a sequence of length N =2', and suppose thal v(n) is splil (decimated) into iwo subsequences, each of length $N/2$. As illustrated in Fig. the first sequence, %(n). is formed frum the cven-index tcrmx,

N

$g(n) = x(2n)$ $n = 0$. I— - I

and ihe second, $h(ri)$, is formed from ihe odd-index terms,

N

$h\{n) = x(2n+1)$ $n=0$. I - - I

2

In terms of these sequences, the JV-point DFT of is

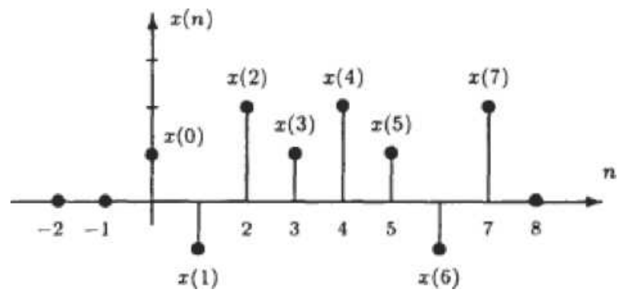
N - I

X(k) = ^x(n)Wtf = x{n)Wtf 4- x{n)Wj?

$h$even odd

*** _ i iV _ n**

1=0 I=\)



Odd-Index Ttraii

Because $W\$^k =$

y—| 1|

$X\{t)$ =£ amiin 4- w*J2$W)K_{fl}$

1=0 l=to

Note that the firs! $term$ is the A//2-poim DFT of and (ho second is Ihe $N/2$-point DFT of $h(n);$

$X\{k)$ = Gik) + W^{l}„H(k) $k = 0, IN - I$

Although the N/2-point DFTs of $g(n)$and $h(n)$are sequences of length $Nt[2$, the periodicity of the complex exponentials allows us to write

G(*)=G(*+y) H(*>=/m-+y)

Therefore, X(k) may be computed from the A[1]'/2-point DFTs G(k) and H(k), Note that because

IV* _ U/* LV'V^ —— IV*

IT $_N$ —iy $_{iV}w_{iV}$— vv_{iV}

then IV.*'4* H(it + ‘j} = - W* H $[k)$

and it is only necessarv to form the products $W\#H$(Jt) tor k =0, 1 ... $N/2$ —!. The complex exponentials multiplying$ti\{k$ $)$ are called $twiddle$ $factors$. A block diagram showing the computations thaiare

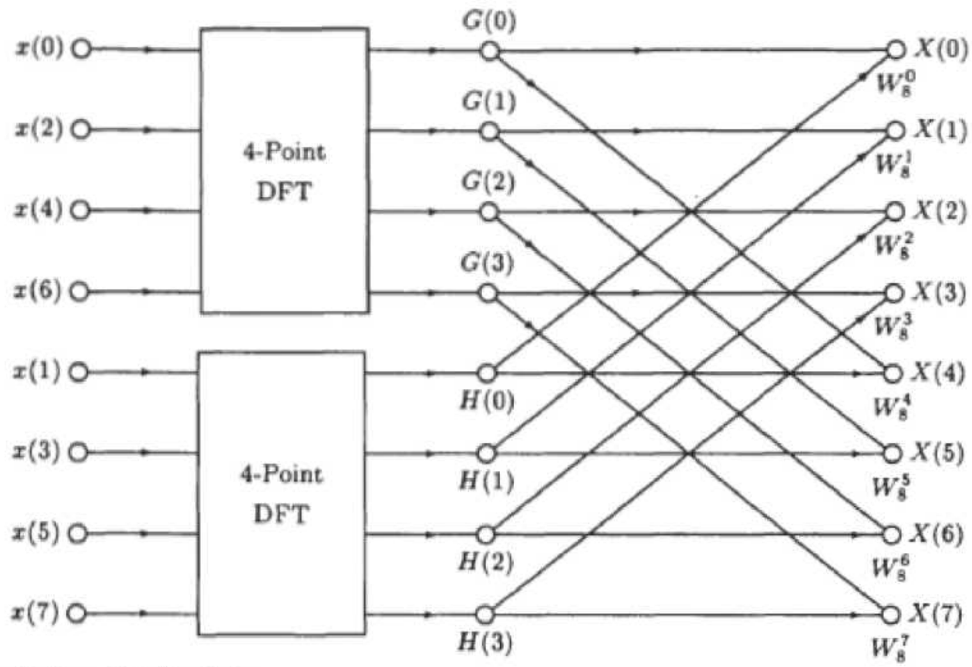
necessaryfor the first $stage$of an eight-point dedmation-in-time FFT is shown in Fig.

If $N/2$ is even, #tft) andA(rt) may again be decimated. For example, G(A) may be evaluated as follows:

: -1 ^ i ■■ -1

$G(k)=$ = V + V *gi«W$2*

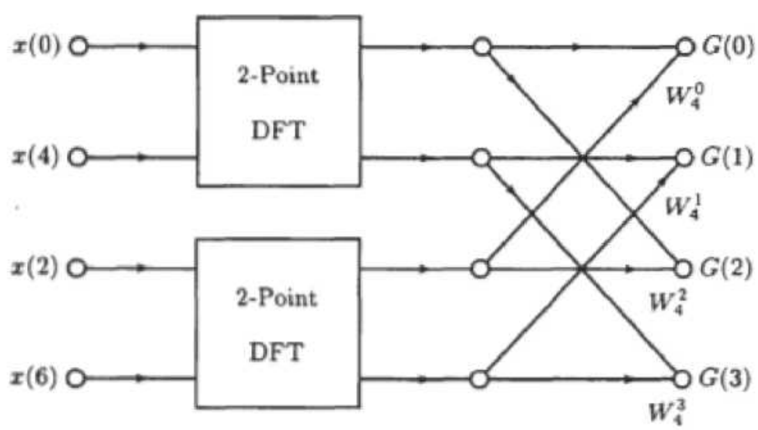
ir—(I Ji even n J.II Id

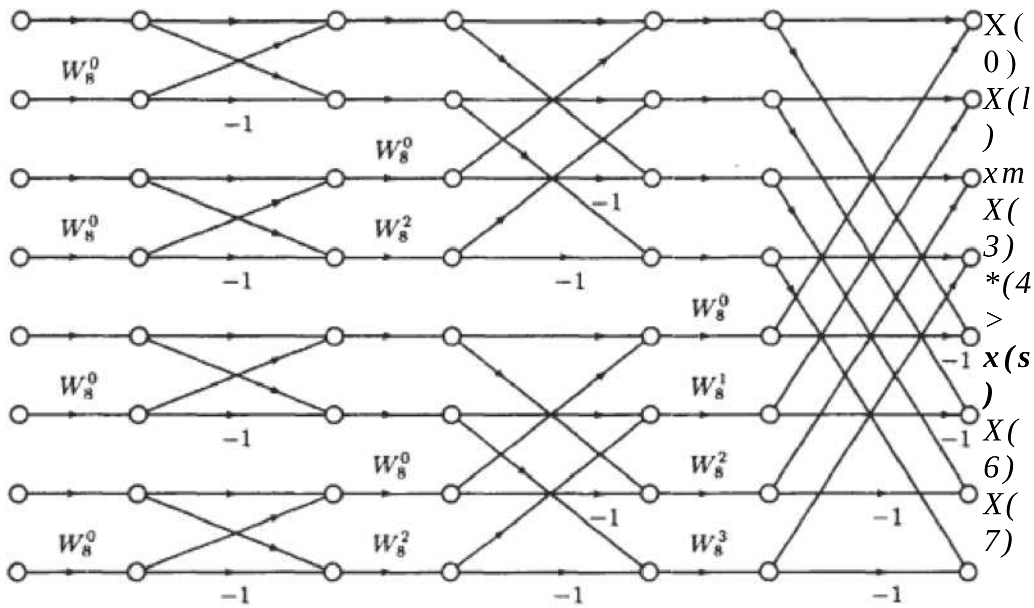


As before, this leads to

$$G(k) = \sum_{n=0}^{\frac{N}{4}-1} g(2n) W_{N/4}^{nk} + W_{N/2}^{k} \sum_{n=0}^{\frac{N}{4}-1} g(2n+1) W_{N/4}^{nk}$$

where the first term is the $N/4$-point DFT of the even samples of $g(n)$, and the second is the $N/4$-point DFT of the odd samples. A block diagram illustrating this decomposition is shown in Fig. . If N is a power of 2, the decimation may be continued until there are only two-point DFTs of the form shown in Fig.





Computing an /V-point DFT using a radix-2 decimation-in-lime FFT is much more efficient lhancalculating the DFTdirectly. For example, if $N = 2'\$ there are log, $X = v$ *stages* of compulation. Bee a use each stage requires $N/2$ complex multiplies by the twiddle factors $W^r{}_N$ and N complex additions, there are a total of $\backslash X \log_{;} N$ complex multiplications[1] and Xlog-i N complex additions.

From the structure of the dec i mat ion-in-time FFT algorithm, note that once a butterfly operation has been performed on a pair of complex numbers, there is no need to save the input pair. Therefore, ihe output pair may be stored in (he same registers as the input. Thus, only one airay of size X is required, and il is said that the computations may be performed *in place*. To perform the compulations in place, however, the input sequence a(h) must he stored (or accessed) in nonsequential order as seen in Fig. The *shuffling* of the inpul sequence that takes place is due to ihe successive decimations of The ordering that results corresponds to a bil-reversed indexing of Ihe original sequence. In oilier words, if the index n is written in binary fonn. Ihe order in which in the inpul sequence musi be accessed is found by reading the binary representation for /? In reverse order as illustrated in ihe table below for N = 8;

N	Binary	Bil-Reversed Binary	
0	000	000	0
1	001	100	4
2	010	010	2
3	Oil	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

Alternate forms nFFFT algorithms may he derived from the decimalion-in-time FFT hy manipulating the flowgrapli and rearranging the order in which ihe results of each stage of Ihe computation are stored. For example, the ntxJes of the fluwgraph may be rearranged su that the input sequenuc ,r(/i) is in normal order. What is lost with this reordering, however, is the .ibililv iti perform the computations in place.

6. Derive DIF radix 2 FFT algorithm

Decimation-in-Frequency FFT

Another class of FFT algorithms may be derived by decimating the output sequence $X\ (k)$ into smaller and smaller subsequences. These algorithms are called *decimation-in-frequency* FFTs and may be derived as follows. Let N be a power of 2, /V = 2[1]'. and consider separately evaluating the even-index and odd-indcx samples of $X(k)$. The even samples are

fi~ *I*

X(2k)=

fl=0

Separating this sum into the first $N\ /\ 2$ points and the last $N\ /\ 2$ points, and using the fact that Wjf^k= W^, this

becomes

S-i ftf-r

X(2k)= '£*(n)W*/2+£ x(n)W*$_N{}^k$,$_2$

n=0 n=A//2

With a change in the indexing on the second sum we have

X(2k)= J2$^{x(n)W}$N/2$^+$ £^{x}(*ll* $^+$7)^/2^

n—l) n-0 >• ^

Finally, because ^"2^5 ** = W'Jv/2’

X(2*)=£f N\l
x(n) HK
L+ x 1 K).
N
/2

which is the $N/2$-point DFT of the sequence that is formed by adding the first $N/2$ points of.*(n) to the last $N/2$. Proceeding in the same way for the odd samples of $X(k)$ leads to

X(2k+1)= '-1 x(n) -H). nK N/2 4).
()-

A flowgraph illustrating this first stage of decimation is shown in Fig. 7-7. As with the decimation-in-time FFT, the decimation may be continued until only two-point DFTs remain. A completeeight-point decimation-in-frequency FFT is shown in Fig. 7-8. The complexity of the decimation-in-frequency FFT is the same as the decimation-in-time, and the computations may be performed in place. Finally, note that although the input sequence $x(n)$ is in normal order, the frequency samples $X(k)$ are in bit-reversed order.

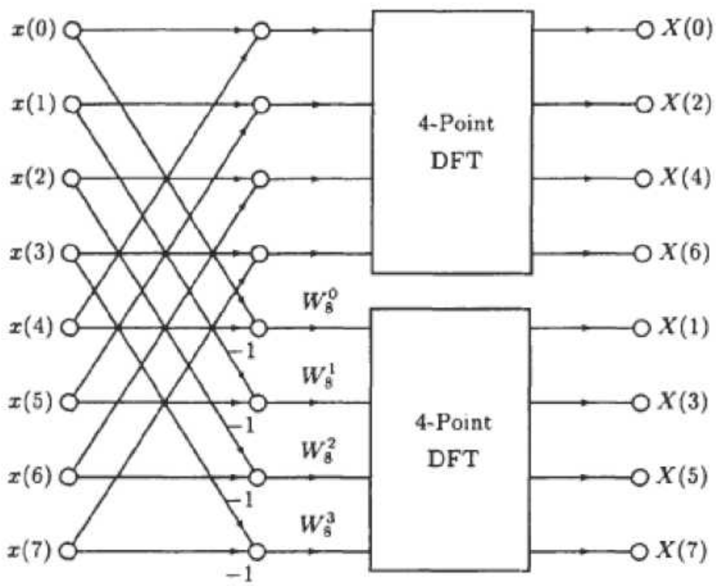


Fig. 7-7. An eight-point decimation-in-frequency FFT algorithm after the first stage of decimation.

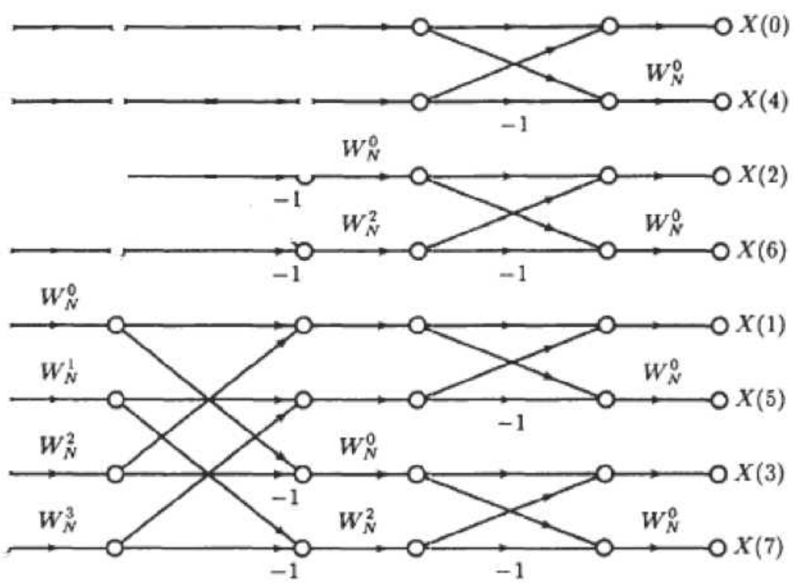


Fig. 7-8. Eight-point radix-2 decimation-in-frequency FFT.

7, State and prove properties of DFT.

PROPERTIES OF DFT

DFT

x(n) +--- ► x(k)

N

1. Periodicity

Let x(n) and x(k) be the DFT pair then if

x(n + N) = x(n) for all n then

X(k+N) = X(k) for all k

Thus periodic sequence xp(n) can be given as

ra

xp(n) = Z x(n-lN) l=-ra

N

DFT

x2(n) * -- ^ X2(k) Then

N

Then DFT

a1 x1(n) + a2 x2(n) ^^ a1 X1(k) + a2 X2(k)

N

DFT of linear combination of two or more signals is equal to the same linear combination of DFT of individual signals.

1. Circular Symmetries of a sequence

A) A sequence is said to be circularly even if it is symmetric about the point zero on the circle. Thus X(N-n) = x(n)

B) A sequence is said to be circularly odd if it is anti symmetric about the point zero on the circle. Thus X(N-n) = - x(n)

C) A circularly folded sequence is represented as $x((-n))_N$ and given by $x((-n))_N = x(N-n)$.

D) Anticlockwise direction gives delayed sequence and clockwise direction gives advance sequence. Thus delayed or advances sequence x'(n) is related to x(n) by the circular shift.

2. Symmetry Property of a sequence

A) Symmetry property for real valued x(n) i.e xi(n)=0

This property states that if x(n) is real then X(N-k) = X (k)=X(-k)

B) Real and even sequence x(n) i.e xI(n)=0 & X*i*(K)=0

This property states that if the sequence is real and even x(n)= x(N-n) then DFT becomes

N-1

X(k) = I x(n) cos (2nkn/N) n=*0*

C) Real and odd sequence x(n) i.e xI(n)=0 & X*r*(K)=0

This property states that if the sequence is real and odd x(n)=-x(N-n) then DFT becomes

N-1

X(k) = -j I x(n) sin (2nkn/N) n=*0*

D) Pure Imaginary x(n) i.e $x_R(n)=0$

This property states that if the sequence is purely imaginary x(n)=j Xi(n) then DFT becomes

N-1

XR(k) = I xi(n) sin (2nkn/N) n=*0*

d x(n) i.

N-1

$$Xi(k) = I\ xi(n) \cos (2nkn/N)\ n=0$$

3. Circular Convolution

The Circular Convolution property states that if

DFT

x1(n) ^+ X1(k) And

N

DFT

x2(n)X2(k) Then

N

DFT

Then x1(n) © x2(n) -^1(k) x2(k)

N

It means that circular convolution of x1(n) & x2(n) is equal to multiplication of their DFT's. Thus circular convolution of two periodic discrete signal with period N is given by

$$\mathbf{y(m) = \sum_{n=0}^{N-1} x1\,(n)\; x2\,(m\text{-}n)N} \qquad \textbf{...........(4)}$$

Multiplication of two sequences in time domain is called as Linear convolution while Multiplication of two sequences in frequency domain is called as circular convolution. Results of both are totally different but are related with each other.

UNIT-II
INFINITE IMPULSE RESPONSE DIGITAL FILTERS

1.Give the expression for location of poles of normalized Butterworth filter(May07,Nov10)

The poles of the Butterworth filter is given by (S-S1)(S-S2)……..(S-SN)

Where N is the order of filter.

2.What are the parameters(specifications) of a Chebyshev filter?(May/june-07)

From the given chebyshev filter specifications we can obtain the parameters like the order of the filter N, ε, transition ratio k, and the poles of the filter.

3.Find the digital transfer function H(z) by using impulse invariant method for the analog transfer function H(s)=1/s+2.Assume T=0.5sec(May/june-07)

$H(z)= 1/ 1-e^{-1}z^{-1}$

4.What is Warping effect?(Nov-07,May-09)

The relation between the analog and digital frequencies in bilinear transformation is given by $\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$. For smaller values of ω there exist linear relationship between ω and **Ω**. But for large values of ω the relationship is non-linear. This non-linearity introduces distortion in the frequency axis. This is known as warping effect. This effect compresses the magnitude and phase response at high frequencies.

5.Compare Butterworth & Chebyshev filter.(Nov-07,May-09)

S.no	**Butterworth**	**Chebyshev Type-1**
1	All pole design	All pole design
2	The poles lie on a circle in	The poles lie on ellipse in S-plane.

	S-plane.	
3	The magnitude response is maximally flat at the origin and monotonically decreasing function of Ω	The magnitude response is equiriple in passband and monotonically decreasing in the stopband.
4	The normalized magnitude response has a value of 1/√2 at the cutoff frequency Ωc	The normalized magnitude response has a value of 1/√1+ε↑2 at the cutoff frequency Ωc.
5	Only few parameters has to be calculated to determine the transfer function	A large number of parameter has to be calculated to determine the transfer function

6.What is the relationship between analog & digital freq. in impulse invariant transformation?(Apr/May-08)

If H(s)= Σ Ck / S-Pk then H(z) = Σ Ck / 1-$e^{PkT}Z^{-1}$

7.What is bilinear transformation?(Nov/Dec-08)

The bilinear transformation is a mapping that transforms the left half of s plane into the unit circle in the z-plane only once, thus avoiding aliasing of frequency components. The mapping from the s-plane to the z-plane in bilinear transformation is $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$.

8.What is the main disadvantage of direct form-I realization?(Nov/Dec-08)

The direct form realization is extremely sensitive to parameter quantization. When the order of the system N is large, a small change in a filter coefficient due to parameter quantization, results in a large change in the location of the pole and zeros of the system.

9.What is Prewarping?(May-09,Apr-10,May-11)

The effect of the non-linear compression at high frequencies can be compensated. When the desired magnitude response is piece-wise constant over frequency, this compression can be compensated by introducing a suitable prescaling, or prewarping the critical frequencies by using the **formula, Ω=2/T tan ω/2.**

10.List the features that make an analog to digital mapping for IIR filter design coefficient.(May/june-2010)

- The bilinear transformation provides one-to-one mapping.
- Stable continuous systems can be mapped into realizable, stable digital systems.
- There is no aliasing.

11.State the limitations of impulse invariance mapping technique.(Nov-09)

In impulse invariant method, the mapping from s-plane to z-plane is many to one i.e., all the poles in the s-plane between the intervals **[(2k-1)π]/T** to **[(2k+1)π]/T (** for k=0,1,2……**)** map into the entire z-plane. Thus, there are an infinite number of poles that map to the same location in the z-plane, producing aliasing effect. Due to spectrum aliasing the impulse invariance method is inappropriate for designing high pass filters. That is why the impulse invariance method is not preferred in the design of IIR filter other than low pass filters.

12.Find digital transfer function using approximate derivative technique for the analog transfer function H(s)=1/s+3.Assume T=0.1sec (May-10)

$$H(z) = 1/\ Z+e^{-0.3}$$

13.Give the square magnitude function of Butterworth filter.(Nov-2010)

The magnitude function of the butter worth filter is given by

$$\left|H(j\Omega)\right| = \frac{1}{\left[1+\left[\frac{\Omega}{\Omega_c}\right]^{2N}\right]^{\frac{1}{2}}} \quad N = 1,2,3,...$$

Where N is the order of the filter and Ω_c is the cutoff frequency. The magnitude response of the butter worth filter closely approximates the ideal response as the order N increases. The phase response becomes more non-linear as N increases.

14. Find the digital transfer function H(z) by using impulse invariant method for the analog transfer function H(s)=1/s+1.Assume T=1sec.(Apr-11)

$$H(z)= 1/ 1-e^{-1}z^{-1}$$

15.Give the equation for the order of N and cut-off frequency Ωc of butter worth filter.(Nov-06)

The order of the filter $N = \dfrac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\dfrac{\Omega_s}{\Omega_p}}$

Where α_s = stop band attenuation at stop band frequency Ω_s

α_p = pass band attenuation at pass band frequency Ω_p

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha}-1)^{\frac{1}{2N}}}$$

16.What are the properties of the bilinear transformation?(Apr-06)

- The mapping for the bilinear transformation is a one-to-one mapping; that is for every point z, there is exactly one corresponding point s, and vice versa.
- The jΩ-axis maps on to the unit circle |z|=1, the left half of the s-plane maps to the interior of the unit circle |z|=1 and the right half of the s-plane maps on to the exterior of the unit circle |z|=1.

17.Write a short note on pre-warping.(Apr-06)

The effect of the non-linear compression at high frequencies can be compensated. When the desired magnitude response is piece-wise constant over frequency, this compression can be compensated by introducing a suitable pre-scaling, or pre-warping the critical frequencies by using the formula. $\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$

18.What are the different types of structure for realization of IIR systems?(Nov-05)

The different types of structures for realization of IIR system are

i. Direct-form-I structure

ii. Direct-form-II structure

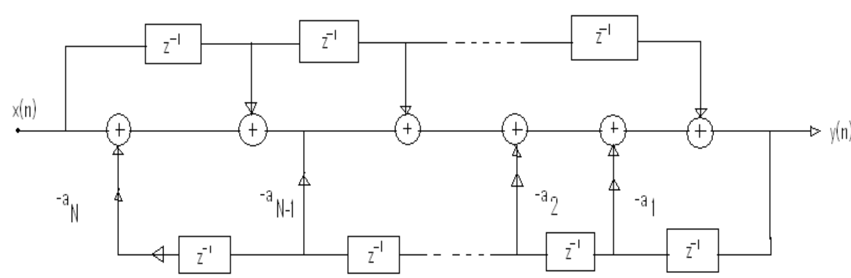
iii. Transposed direct-form II structure

iv. Cascade form structure

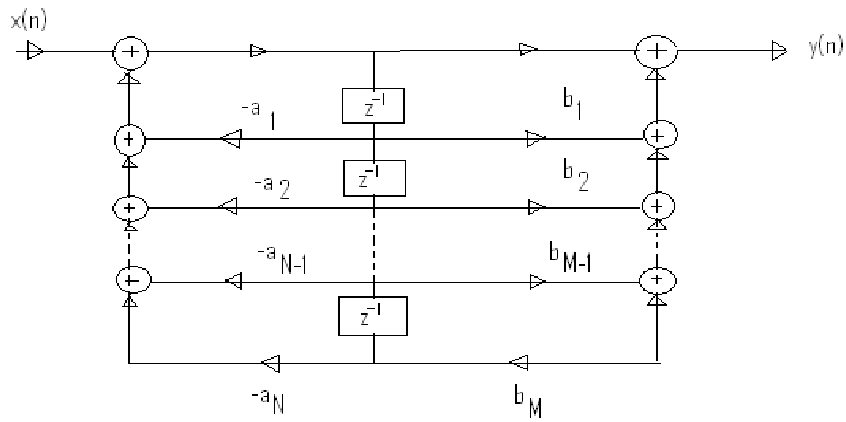
v. Parallel form structure

vi. Lattice-Ladder structure

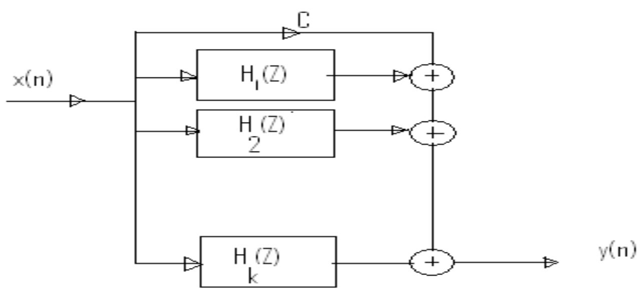
19.Draw the general realization structure in direct-form I of IIR system.(May-04)



20.Give direct form II structure.(Apr-05)



21.Draw the parallel form structure of IIR filter.(May-06)



22.Mention any two techniques for digitizing the transfer function of an analog filter. (Nov11)

The two techniques available for digitizing the analog filter transfer function are **Impulse invariant transformation** and **Bilinear transformation.**

23.Write a brief notes on the design of IIR filter. (Or how a digital IIR filter is designed?)

For designing a digital IIR filter, first an equivalent analog filter is designed using any one of the approximation technique for the given specifications. The result of the analog filter design will be an analog filter transfer function Ha(s). The analog filter transfer function is transformed to digital filter transfer function H(z) using either Bilinear or Impulse invariant transformation.

24.Define an IIR filter

The filters designed by considering all the infinite samples of impulse response are called IIR filers. The impulse response is obtained by taking inverse Fourier transform of ideal frequency response.

25. Compare IIR and FIR filters.

S.No	**IIR Filter**	**FIR Filter**
i.	All the infinite samples of impulse response are considered.	Only N samples of impulse response are considered.
ii.	The impulse response cannot be directly converted to digital filter transfer function.	The impulse response can be directly converted to digital filter transfer function.
iii.	The design involves design of analog filter and then transforming analog filter to digital filter.	The digital filter can be directly designed to achieve the desired specifications.
iv.	The specifications include the desired characteristics for magnitude response only.	The specifications include the desired characteristics for both magnitude and phase response.
v.	Linear phase characteristics cannot be achieved.	Linear phase filters can be easily designed.

PART- B(16 MARKS)

1. **Explain Frequency Transformation in Analog domain and frequency transformation in digital domain. (Nov/Dec-07) (12)**

i. Frequency transformation in analog domain

In this transformation technique normalized Low Pass filter with cutoff freq of Ω_p= 1 rad /sec is designed and all other types of filters are obtained from this prototype. For example, normalized LPF is transformed to LPF of specific cutoff freq by the following transformation formula,

Normalized LPF to LPF of specific cutoff:

$$s \to \frac{s}{\Omega_p'}$$

$$H_1(s) = H_p(\frac{\Omega_p}{\Omega_p'} s)$$

Where,
Ω_p= normalized cutoff freq=1 rad/sec
Ω'_p= Desired LP cutoff freq

at $\Omega = \Omega' p$ it is H(j1)

The other transformations are shown in the below table.

Type of Transformation	Transformation	Band edge frequencies of new filter
LP	$S \to \frac{\Omega_p}{\Omega_p^!} S$	Ω'_p
HP	$S \to \frac{\Omega_p \Omega'_p}{S}$	Ω'_p
BP	$S \to \Omega_p \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_l - \Omega_u)}$	Ω_l, Ω_u
BS	$S \to \Omega_p \frac{s(\Omega_u - \Omega_c)}{s^2 + \Omega_u \Omega_l}$	Ω_l, Ω_u

ii. Frequency Transformation in Digital Domain:

This transformation involves replacing the variable Z^{-1} by a rational function $g(z^{-1})$, while doing this following properties need to be satisfied:

1. Mapping Z^{-1} to $g(z^{-1})$ must map points inside the unit circle in the Z- plane onto the unit circle of z- plane to preserve causality of the filter.

2. For stable filter, the inside of the unit circle of the Z - plane must map onto the inside of the unit circle of the z-plane.

The general form of the function g(.) that satisfy the above requirements of " all-pass " type is

$$g(z^{-1}) = \pm \prod_{k=1}^{n} \frac{z^{-1} - \alpha_k}{1 - \alpha_k z^{-1}}$$

The different transformations are shown in the below table.

Type of Transformation	Transformation	Parameters
Lowpass	$z^{-1} \rightarrow \dfrac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$	ω_C=cutoff frequency of new filter $\alpha = \dfrac{\sin[(\omega'_c - \omega_c)/2]}{\sin[(\omega'_c + \omega_c)/2]}$
Highpass	$z^{-1} \rightarrow -\dfrac{z^{-1} + \alpha}{1 + \alpha z^{-1}}$	ω_C=cutoff frequency of new filter $\alpha = -\dfrac{\cos[(\omega'_c + \omega_c)/2]}{\cos[(\omega'_c - \omega_c)/2]}$

Bandpass	$z^{-1} \rightarrow -\dfrac{z^{-2} - \alpha_1 z^{-1} + \alpha_2}{\alpha_2 z^{-2} - \alpha_1 z^{-1} + 1}$	ω_e = lower cutoff frequency ω_u = upper cutoff frequency $\alpha_1 = -2\beta K/(K+1)$ $\alpha_2 = (K-1)/(K+1)$ $\beta = \dfrac{\cos[(\omega_u + \omega_e)/2]}{\cos[(\omega_u - \omega_e)/2]}$ $K = \cot \dfrac{\omega_u - \omega_e}{2} \tan \dfrac{\omega'_c}{2}$
Bandstop	$z^{-1} \rightarrow \dfrac{z^{-2} - \alpha_1 z^{-1} + \alpha_2}{\alpha_2 z^{-2} - \alpha_1 z^{-1} + 1}$	ω_e = lower cutoff frequency ω_u = upper cutoff frequency $\alpha_1 = -2\beta K/(K+1)$ $\alpha_2 = (K-1)/(K+1)$ $\beta - \dfrac{\cos[(\omega_u + \omega_e)/2]}{\cos[(\omega_u - \omega_e)/2]}$ $K = \tan \dfrac{\omega_u - \omega_e}{2} \tan \dfrac{\omega'_c}{2}$

2. **Represents the transfer function of a low-pass filter (not butterworth) with a pass-band of 1 rad/sec. Use freq transformation to find the transfer function of the following filters:**

Let $$H(s) = \frac{1}{s^2 + s + 1}$$

Represents the transfer function of a low pass filter (not butterworth) with a passband of 1 rad/sec. Use freq transformation to find the transfer function of the following filters: (Apr/May-08) (12)

1. **A LP filter with a pass band of 10 rad/sec**
2. **A HP filter with a cutoff freq of 1 rad/sec**
3. **A HP filter with a cutoff freq of 10 rad/sec**
4. **A BP filter with a pass band of 10 rad/sec and a corner freq of 100 rad/sec**

5. A BS filter with a stop band of 2 rad/sec and a center freq of 10 rad/sec

Solution:

Given

$$H(s)=\frac{1}{s^2+s+1}$$

a. LP – LP Transform

replace

$$s \rightarrow \frac{s}{\Omega'_p}=\frac{s}{10}$$

$$sub \quad H_a(s)=H(s)|_{s\rightarrow\frac{s}{10}}=\frac{1}{\left((\frac{s}{10})^2+(\frac{s}{10})+1\right)}$$

$$=\frac{100}{s^2+10s+100}$$

b. LP – HP(normalized) Transform

$$s \rightarrow \frac{\Omega_u}{s}=\frac{1}{s}$$

$$sub \quad H_a(s)=H(s)|_{s\rightarrow\frac{1}{s}}=\frac{1}{\left((\frac{1}{s})^2+(\frac{1}{s})+1\right)}$$

$$=\frac{s^2}{s^2+s+1}$$

c. LP – HP(specified cutoff) Transform

$$s \rightarrow \frac{\Omega_u}{s}=\frac{10}{s}$$

$$sub \quad H_a(s)=H(s)|_{s\rightarrow\frac{10}{s}}=\frac{1}{\left((\frac{10}{s})^2+(\frac{10}{s})+1\right)}$$

$$=\frac{s^2}{s^2+10s+100}$$

replace

d. LP – BP Transform

replace

$$s \to \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)} = \frac{s^2 + \Omega_o^2}{sB_0} \quad where \quad \Omega_o = \sqrt{\Omega_u \Omega_l}$$

$$and \quad B_o = (\Omega_u - \Omega_l)$$

$$sub \quad H_a(s) = H(s)|_{s \to \frac{s^2+10^4}{10s}}$$

$$= \frac{100s^2}{s^4 + 10s^3 + 20100s^2 + 10^5 s + 10^8}$$

e. LP – BS Transform

replace

$$s \to \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l} = \frac{sB_0}{s^2 + \Omega_o^2} \quad where \quad \Omega_o = \sqrt{\Omega_u \Omega_l}$$

$$and \quad B_o = (\Omega_u - \Omega_l)$$

$$sub \quad H_a(s) = H(s)|_{s \to \frac{2s}{s^2+100}}$$

$$= \frac{(s^2+100)^2}{s^4 + 2s^3 + 204s^2 + 200s + 10^4}$$

3. **Convert single pole LP Bufferworth filter with system function** $H(z) = \dfrac{0.245(1+z^{-1})}{1+0.509z^{-1}}$ **into BPF with upper & lower cutoff frequency** ω_u & ω_l **respectively , The LPF has 3-dB bandwidth** $\omega_p = 0.2\pi$. **(Nov-07) (8)**

Solution:

We have the transformation formula given by,

$$z^{-1} \rightarrow \frac{z^{-2} - \alpha_1 z^{-1} + \alpha_2}{\alpha_z z^{-2} - \alpha_1 z^{-1} + 1}$$

applying this to the given transfer function,

$$H(Z) = \frac{0.245\ (1 + \dfrac{z^{-2} - \alpha_1 z^{-1} + \alpha_2}{\alpha_2 z^{-2} - \alpha_1 z^{-1} + 1}}{1 + 0.509 \left(\dfrac{z^{-2} - \alpha_1 z^{-1} + \alpha_2}{\alpha_2 z^{-2} - \alpha_1 z^{-1} + 1} \right)}$$

$$H[z] = \frac{0.245(1 - \alpha_2)(1 - z^{-2})}{(1 + 0.509\alpha_2) - 1.509\alpha_1 z^{-1} + (\alpha_2 + 0.509)z^{-2}}$$

Note that the resulting filter has zeros at z=±1 and a pair of poles that depend on the choice of ωl and ωu

Ex : $\omega_u = \frac{3\pi}{5} \qquad \omega_l = \frac{2\pi}{5}$

$\omega_p = 0.2\pi$

Then k=1, $\alpha_2 = 0, \alpha_1 = 0$

$$\therefore\ H[z] = \frac{0.245\ (1 - z^{-2})}{1 + 0.509\ z^{-2}}$$

This filter has poles at z=±j0.713 and hence resonates at ω=π/2

The following observations are made,

- It is shown here that how easy to convert one form of filter design to another form.
- What we require is only prototype low pass filter design steps to transform to any other form.

4. Explain the different structures for IIR Filters. (May-09) (16)

The IIR filters are represented by system function;

$$H(Z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

And corresponding difference equation given by,

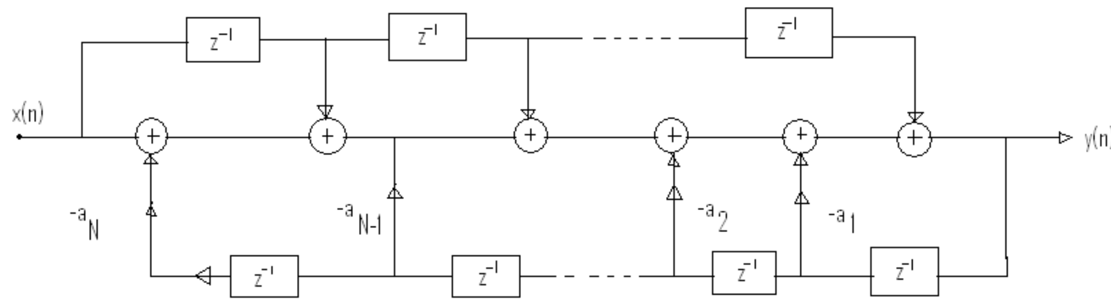
$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{N} b_k x(n-k)$$

Different realizations for IIR filters are,

1. Direct form-I
2. Direct form-II
3. Cascade form
4. Parallel form
5. Lattice form

- **Direct form-I**

This is a straight forward implementation of difference equation which is very simple. Typical Direct form – I realization is shown below. The upper branch is forward path and lower branch is feedback path. The number of delays depends on presence of most previous input and output samples in the difference equation.



- **Direct form-II**

The given transfer function H(z) can be expressed as,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{V(z)}{X(z)} \cdot \frac{Y(z)}{V(z)}$$

where V(z) is an intermediate term. We identify,

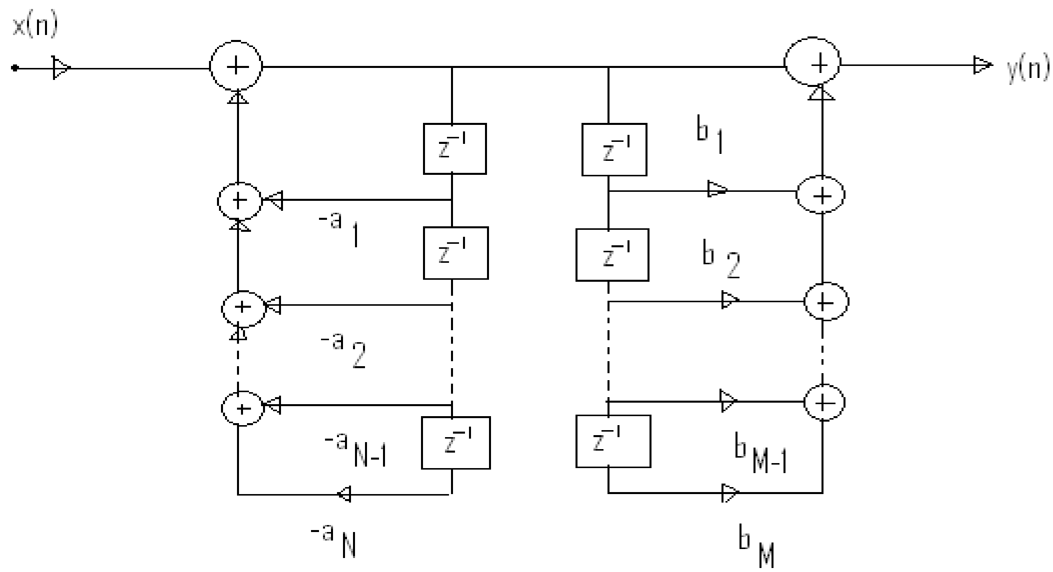
$$\frac{V(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$ -------------------all poles

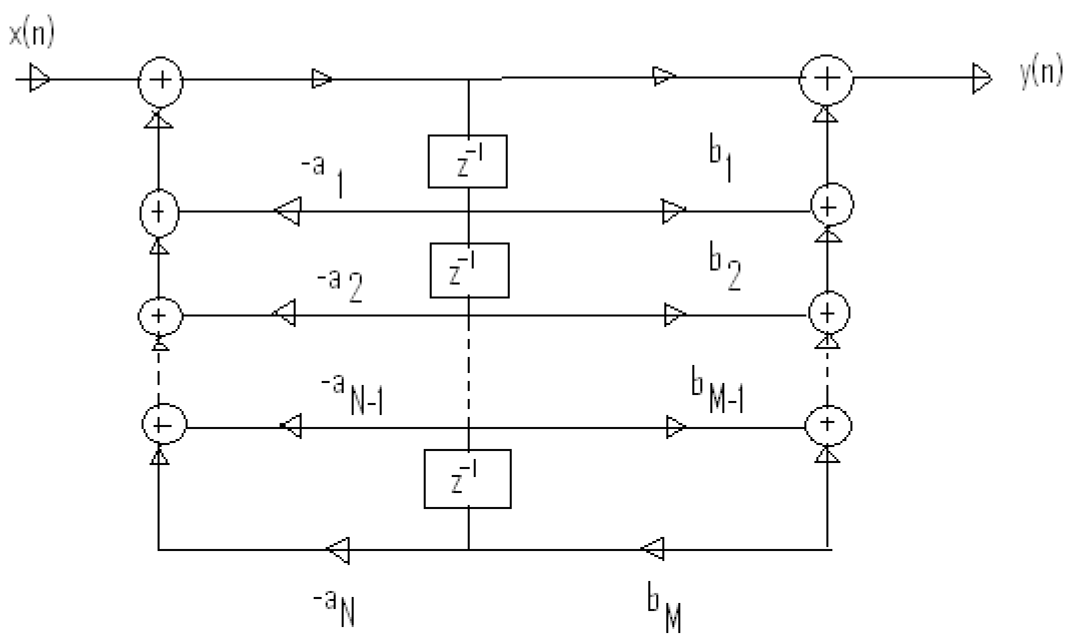
$$\frac{Y(z)}{V(z)} = \left(1 + \sum_{k=1}^{M} b_k z^{-k}\right)$$ -------------------all zeros

The corresponding difference equations are,

$$v(n) = x(n) - \sum_{k=1}^{N} a_k v(n-k)$$

$$y(n) = v(n) + \sum_{k=1}^{M} b_k v(n-1)$$


x(n)
y(n)
z⁻¹
-a 1
-a 2
-a N-1
-a N
b 1
b 2
b M-1
b M


x(n)
y(n)
z⁻¹
-a 1
-a 2
-a N-1
-a N
b 1
b 2
b M-1
b M

This realization requires M+N+! multiplications, M+N addition and the maximum of {M, N} memory location

- **Cascade Form**

The transfer function of a system can be expressed as,

$$H(z) = H_1(z)H_2(z)....H_k(z)$$

Where $H_k(Z)$ could be first order or second order section realized in Direct form – II form i.e.,

$$H_k(Z) = \frac{b_{k0} + b_{k1}Z^{-1} + b_{k2}Z^{-2}}{1 + a_{k1}Z^{-1} + a_{k2}Z^{-2}}$$

Where K is the integer part of (N+1)/2

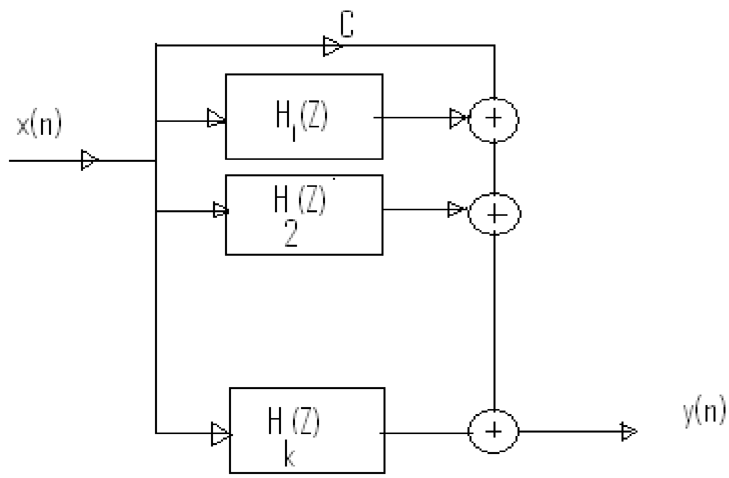
Similar to FIR cascade realization, the parameter b_0 can be distributed equally among the k filter section B_0 that $b_0 = b_{10}b_{20}.....b_{k0}$. The second order sections are required to realize section which has complex-conjugate poles with real co-efficients. Pairing the two complex-conjugate poles with a pair of complex-conjugate zeros or real-valued zeros to form a subsystem of the type shown above is done arbitrarily. There is no specific rule used in the combination. Although all cascade realizations are equivalent for infinite precision arithmetic, the various realizations may differ significantly when implemented with finite precision arithmetic.

- **Parallel form structure**

In the expression of transfer function, if $N \geq M$ we can express system function

$$H(Z) = C + \sum_{k=1}^{N} \frac{A_k}{1 - p_k Z^{-1}} \qquad = C + \sum_{k=1}^{N} H_k(Z)$$

Where $\{p_k\}$ are the poles, $\{A_k\}$ are the coefficients in the partial fraction expansion, and the constant C is defined as $C = b_N / a_N$, The system realization of above form is shown below.



Where $H_k(Z) = \dfrac{b_{k0} + b_{k1}Z^{-1}}{1 + a_{k1}Z^{-1} + a_{k2}Z^{-2}}$

Once again choice of using first- order or second-order sections depends on poles of the denominator polynomial. If there are complex set of poles which are conjugative in nature then a second order section is a must to have real coefficients.

- **Lattice Structure for IIR System:**

Consider an All-pole system with system function.

$$H(Z) = \frac{1}{1 + \sum_{k=1}^{N} a_N(k) Z^{-k}} = \frac{1}{A_N(Z)}$$

The corresponding difference equation for this IIR system is,

$$y(n) = -\sum_{k=1}^{N} a_N(k)y(n-k) + x(n)$$

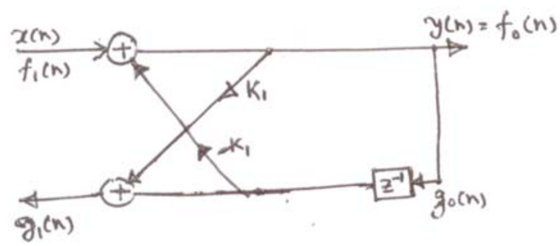
OR

$$x(n) = y(n) + \sum_{k=1}^{N} a_N(k)y(n-k)$$

For N=1

$$x(n) = y(n) + a_1(1)y(n-1)$$

Which can realized as,



We observe

$$x(n) = f_1(n)$$

$$y(n) = f_0(n) = f_1(n) - k_1 g_0(n-1)$$

$$= x(n) - k_1 y(n-1)$$

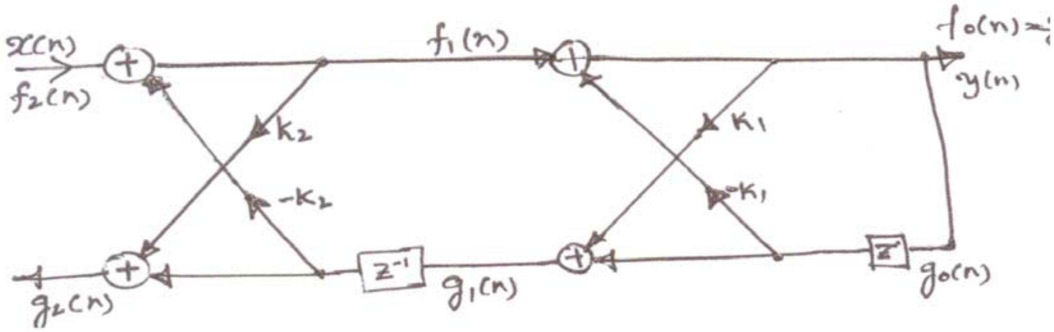
$$k_1 = a_1(1)$$

$$g_1(n) = k_1 f_0(n) + g_0(n-1) = k_1 y(n) + y(n-1)$$

For N=2, then

$$y(n) = x(n) - a_2(1)y(n-1) - a_2(2)y(n-2)$$

This output can be obtained from a two-stage lattice filter as shown in below fig



$$f_2(n) = x(n)$$

$$f_1(n) = f_2(n) - k_2 g_1(n-1)$$

$$g_2(n) = k_2 f_1(n) + g_1(n-1)$$

$$f_0(n) = f_1(n) - k_1 g_0(n-1)$$

$$g_1(n) = k_1 f_0(n) + g_0(n-1)$$

$$y(n) = f_0(n) = g_0(n) = f_1(n) - k_1 g_0(n-1)$$
$$= f_2(n) - k_2 g_1(n-1) - k_1 g_0(n-1)$$

$$= f_2(n) - k_2\left[k_1 f_0(n-1) + g_0(n-2)\right] - k_1 g_0(n-1)$$

$$= x(n) - k_2\left[k_1 y(n-1) + y(n-2)\right] - k_1 y(n-1)$$

$$= x(n) - k_1(1+k_2) y(n-1) - k_2 y(n-2)$$

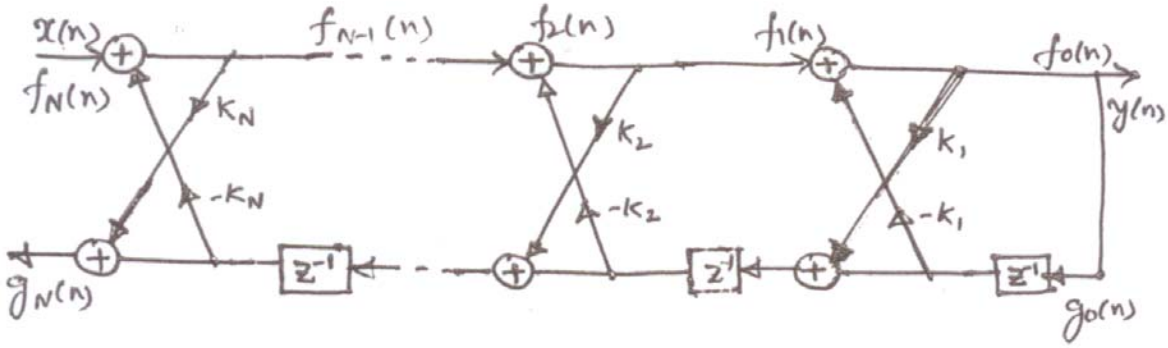
Similarly

$$g_2(n) = k_2 y(n) + k_1(1+k_2)y(n-1) + y(n-2)$$

We observe

$$a_2(0) = 1; a_2(1) = k_1(1+k_2); a_2(2) = k_2$$

N-stage IIR filter realized in lattice structure is,



$$f_N(n) = x(n)$$

$$f_{m-1}(n) = f_m(n) - k_m g_{m-1}(n-1) \qquad \text{m=N, N-1,---1}$$

$$g_m(n) = k_m f_{m-1}(n) + g_{m-1}(n-1) \qquad \text{m=N, N-1,---1}$$

$$y(n) = f_0(n) = g_0(n)$$

5. **Realize the following system functions (Dec-08) (12)**

$$H(Z) = \frac{10\left(1-\frac{1}{2}Z^{-1}\right)\left(1-\frac{2}{3}Z^{-1}\right)\left(1+2Z^{-1}\right)}{\left(1-\frac{3}{4}Z^{-1}\right)\left(1-\frac{1}{8}Z^{-1}\right)\left(1-\left(\frac{1}{2}+j\frac{1}{2}\right)Z^{-1}\right)\left(1-\left(\frac{1}{2}-j\frac{1}{2}\right)Z^{-1}\right)}$$

a). Direct form –I

b). Direct form-II

c). Cascade
d). Parallel
Solution:

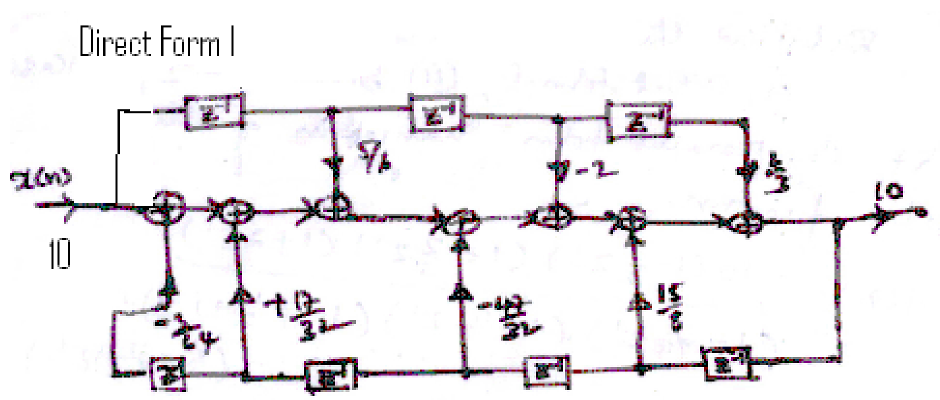
$$H(Z)=\frac{10\left(1-\frac{1}{2}Z^{-1}\right)\left(1-\frac{2}{3}Z^{-1}\right)\left(1+2Z^{-1}\right)}{\left(1-\frac{3}{4}Z^{-1}\right)\left(1-\frac{1}{8}Z^{-1}\right)\left(1-\left(\frac{1}{2}+j\frac{1}{2}\right)Z^{-1}\right)\left(1-\left(\frac{1}{2}-j\frac{1}{2}\right)Z^{-1}\right)}$$

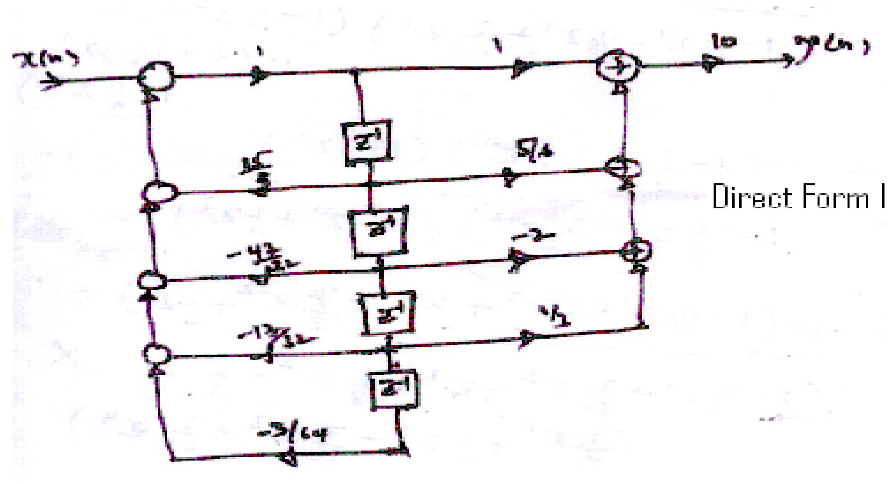
$$=\frac{10\left(1-\frac{7}{6}Z^{-1}+\frac{1}{3}Z^{-2}\right)\left(1+2Z^{-1}\right)}{\left(1+\frac{7}{8}Z^{-1}+\frac{3}{32}Z^{-2}\right)\left(1-Z^{-1}+\frac{1}{2}Z^{-2}\right)}$$

$$H(Z)=\frac{10\left(1+\frac{5}{6}Z^{-1}-2Z^{-2}+\frac{2}{3}Z^{-3}\right)}{\left(1-\frac{15}{8}Z^{-1}+\frac{47}{32}Z^{-2}-\frac{17}{32}Z^{-3}+\frac{3}{64}Z^{-4}\right)}$$

$$H(z)=\frac{(-14.75-12.90z^{-1})}{(1+\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2})}+\frac{(24.50+26.82z^{-1})}{(1-z^{-1}+\frac{1}{2}z^{-2})}$$

a). Direct form-I





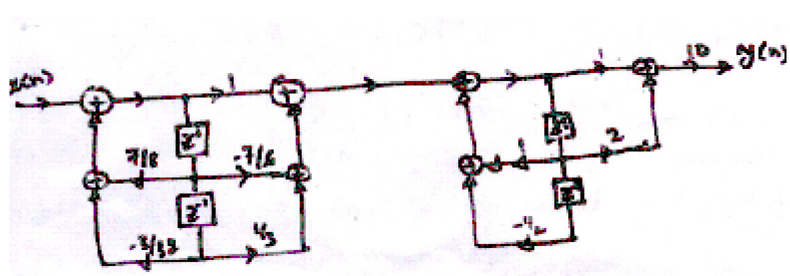
c). Cascade Form

H(z) = $H_1(z)\ H_2(z)$

Where

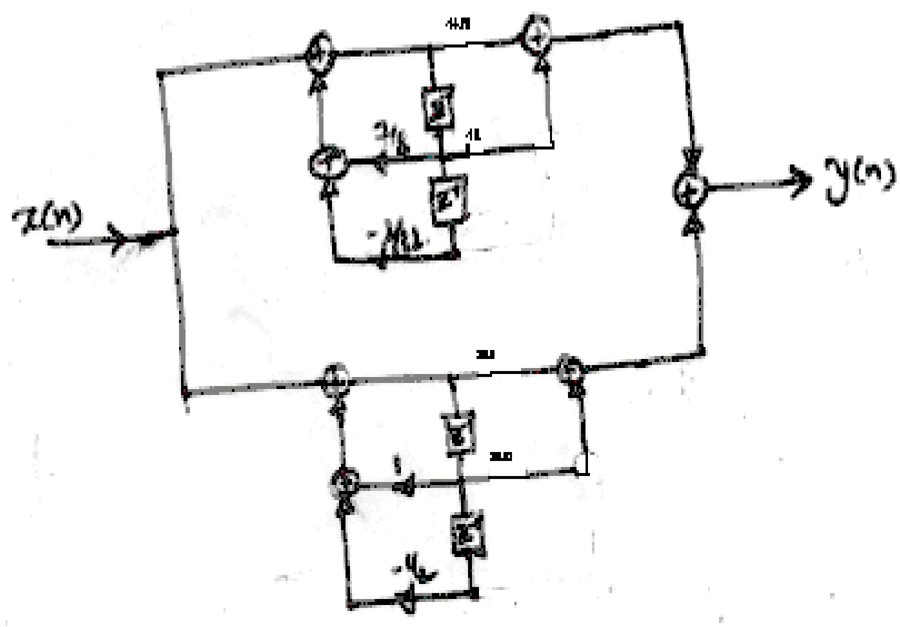
$$H_1(z) = \frac{1 - \frac{7}{6}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{7}{8}z^{-1} + \frac{3}{32}z^{-2}}$$

$$H_1(z) = \frac{10(1 + 2z^{-1})}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$



Parallel Form

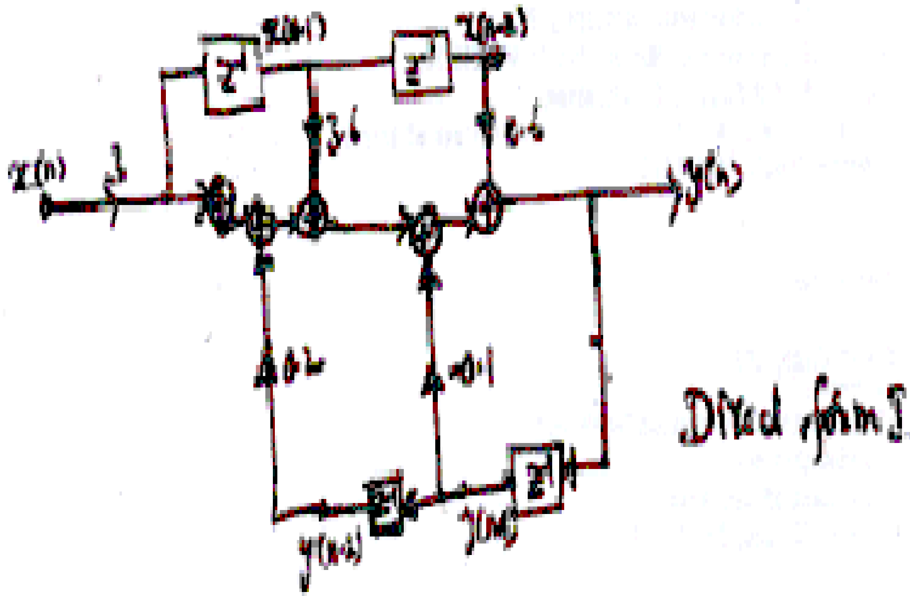
$$H(z) = H_1(z) + H_2(z) \quad ; \quad H(z) = \frac{(-14.75 - 12.90z^{-1})}{(1 + \frac{7}{8}z^{-1} + \frac{3}{32}z^{-2})} + \frac{(24.50 + 26.82z^{-1})}{(1 - z^{-1} + \frac{1}{2}z^{-2})}$$



6. **Obtain the direct form – I, direct form-II, Cascade and parallel form realization for the following system, y(n)=-0.1 y(n-1)+0.2y(n-2)+3x(n)+3.6 x(n-1)+0.6 x(n-2) (12). (May/june-07)**

Solution:

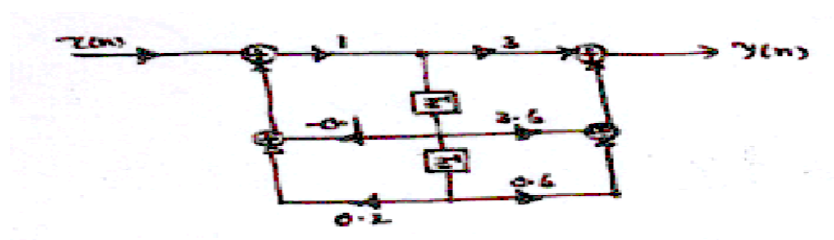
The Direct form realization is done directly from the given i/p – o/p equation, show in below diagram



Direct form –II realization

Taking ZT on both sides and finding H(z)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$



Cascade form realization

The transformer function can be expressed as:

$$H(z) = \frac{(3 + 0.6z^{-1})(1 + z^{-1})}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})}$$

which can be re written as

where $H_1(z) = \dfrac{3 + 0.6z^{-1}}{1 + 0.5z^{-1}}$ and $H_2(z) = \dfrac{1 + z^{-1}}{1 - 0.4z^{-1}}$

Parallel Form realization

The transfer function can be expressed as

$H(z) = C + H_1(z) + H_2(z)$ where $H_1(z)$ & $H_2(z)$ is given by,

$$H(z) = -3 + \frac{7}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$

7. **Convert the following pole-zero IIR filter into a lattice ladder structure,(Apr-10)**

$$H(Z) = \frac{1 + 2Z^{-1} + 2Z^{-2} + Z^{-3}}{1 + \frac{13}{24}Z^{-1} + \frac{5}{8}Z^{-2} + \frac{1}{3}Z^{-3}}$$

Solution:

Given $b_M(Z) = 1 + 2Z^{-1} + 2Z^{-2} + Z^{-3}$

And $A_N(Z) = 1 + \frac{13}{24}Z^{-1} + \frac{5}{8}Z^{-2} + \frac{1}{3}Z^{-3}$

$a_3(0) = 1; \quad a_3(1) = \frac{13}{24}; \quad a_3(2) = \frac{5}{8}; \quad a_3(3) = \frac{1}{3}$

$$k_3 = a_3(3) = \frac{1}{3}$$

Using the equation

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)a_m(m-k)}{1 - a^2 m(m)}$$

For m=3, k=1

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1 - a_3^2(3)} = \frac{\frac{13}{24} - \frac{1}{3}\cdot\frac{5}{8}}{1 - \left(\frac{1}{3}\right)^2} = \frac{3}{8}$$

For m=3, & k=2

$$a_2(2) = k_2 = \frac{a_3(2) - a_3(3)a_3(1)}{1 - a_3^2(3)}$$

$$\frac{\frac{5}{8} - \frac{1}{3}\cdot\frac{13}{24}}{1 - \frac{1}{9}} = \frac{\frac{45-13}{72}}{\frac{8}{9}} = \frac{1}{2}$$

for m=2, & k=1

$$a_1(1) = k_1 = \frac{a_2(1) - a_2(2)a_2(1)}{1 - a_2^2(2)}$$

$$\frac{\frac{3}{8} - \frac{1}{2}\cdot\frac{3}{8}}{1 - \left(\frac{1}{2}\right)^2} = \frac{\frac{3}{8} - \frac{3}{16}}{1 - \frac{1}{4}} = \frac{1}{4}$$

for lattice structure $k_1 = \frac{1}{4}, \quad k_2 = \frac{1}{2}, \quad k_3 = \frac{1}{3}$

For ladder structure

$$C_m = b_m - \sum_{i=m+1}^{M} C_1.a_1(1-m) \qquad \text{m=M, M-1,1,0}$$

M=3
$$C_3 = b_3 = 1; \quad C_2 = b_2 - C_3 a_3(1)$$
$$= 2 - 1.(\tfrac{13}{24}) = 1.4583$$

$$C_1 = b_1 - \sum_{i=2}^{3} c_1 a_1(i-m) \qquad \text{m=1}$$

$$= b_1 - \left[c_2 a_2(1) + c_3 a_{3(2)}\right]$$

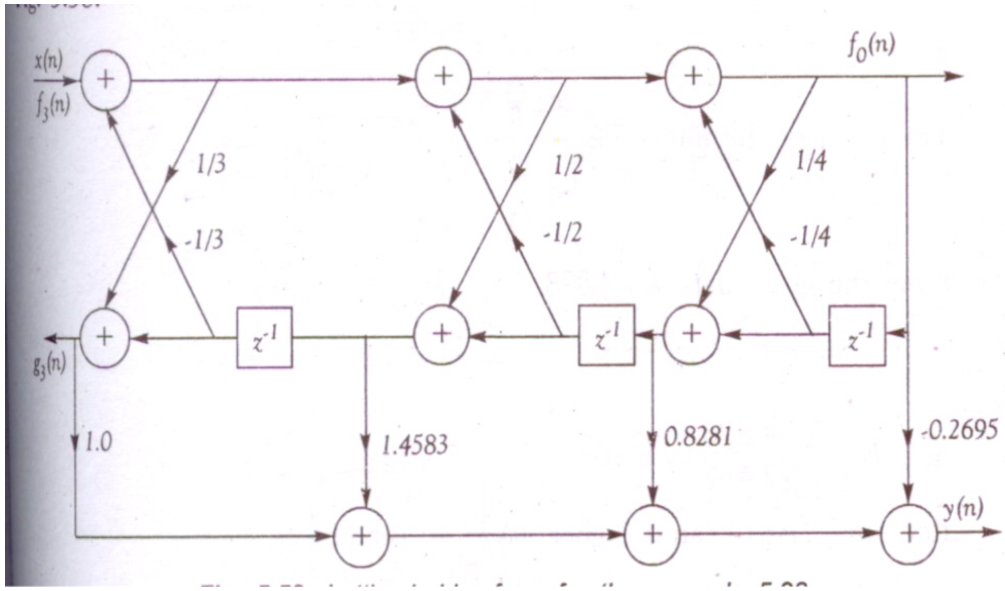
$$= 2 - \left[(1.4583)(\tfrac{3}{8}) + \tfrac{5}{8}\right] = 0.8281$$

$$c_0 = b_0 - \sum_{i=1}^{3} c_1 a_1(i-m)$$

$$= b_0 - \left[c_1 a_1(1) + c_2 a_2(2) + c_3 a_3(3)\right]$$
$$= 1 - \left[08281(\tfrac{1}{4}) + 1.4583(\tfrac{1}{2}) + \tfrac{1}{3}\right] = -02695$$

To convert a lattice- ladder form into a direct form, we find an equation to obtain

$a_N(k)$ From k_m (m=1,2,………N) then equation for c_m is recursively used to compute b_m (m=0,1,2,………M).



UNIT III

FINITE IMPULSE RESPONSE DIGITAL FILTERS

1.What is the condition satisfied by linear phase FIR filter?(May/june-09)

The condition for constant phase delay are

Phase delay, α = (N-1)/2 (i.e., phase delay is constant)

Impulse response, h(n) = h(N-1-n) (i.e., impulse response is symmetric)

2.What are the desirable characteristics of the frequency response of window function?(Nov-07,08)(nov-10)

Advantages:

1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non recursive structure.

4. Filters with any arbitrary magnitude response can be tackled using FIR sequency.

Disadvantages:

1. For the same filter specifications the order of FIR filter design can be as high as 5 to n10 times that of IIR design.
2. Large storage requirements needed.

3.Powerful computational facilities required for the implementation**.**

3.What is meant by optimum equiripple design criterion.(Nov-07)

The Optimum Equiripple design Criterion is used for designing FIR Filters with Equal level filteration throughout the Design.

4.What are the merits and demerits of FIR filters?(Ap/may-08)

Merits: 1. FIR Filter is always stable.
2. FIR Filter with exactly linear phase can easily be designed.
Demerits: 1. High Cost.
2.Require more Memory.

5.For what type of filters frequency sampling method is suitable?(nov/dec-08)

FIR FIlters

6.What are the properties of FIR filters?(Nov/Dec-09)

1. FIR Filter is always stable.
2. A Realizable filter can always be obtained.

7.What is known as Gibbs phenomenon?(ap/may-10,11)

In the filter design by Fourier series method the infinite duration impulse response is truncated to finite duration impulse response at **n=± (N-1/2)**. The abrupt truncation of impulse introduces oscillations in the pass band and stop band. This effect is known as Gibb's phenomenon.

8.Mention various methods available for the design of FIR filter.Also list a few window for the design of FIR filters.(May/june-2010)

There are three well known method of design technique for linear phase FIR filter. They are

1. Fourier series method and window method
2. Frequency sampling method
3. Optimal filter design methods.

Windows: i.Rectangular ii.Hamming iii.Hanning iv.Blackman v.Kaiser

9.List any two advantages of FIR filters.(Nov/dec-10)

1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non recursive structure.

10.Mention some realization methods available to realize FIR filter(Nov/dec-10)

i.Direct form. ii.Cascade form iii.Linear phase realization.

11.Mention some design methods available to design FIR filter.(Nov/dec-10)

There are three well known method of design technique for linear phase FIR filter. They are

1. Fourier series method and window method
2. Frequency sampling method
3. Optimal filter design methods.

Windows: i.Rectangular ii.Hamming iii.Hanning iv.Blackman v.Kaiser

12.What are FIR filters?(nov/dec-10)

The specifications of the desired filter will be given in terms of ideal frequency response **Hd(ω).** The impulse response **hd(n)** of desired filter can be obtained by inverse Fourier transform of **hd(ω),** which consists of infinite samples. The filters designed by selecting finite number of samples of impulse response are called FIR filters.

13.What are the conditions to be satisfied for constant phase delay in linear phase FIR filter?(Apr-06)

The condition for constant phase delay are

Phase delay, α = (N-1)/2 (i.e., phase delay is constant)

Impulse response, h(n) = h(N-1-n) (i.e., impulse response is symmetric)

14.What is the reason that FIR filter is always stable?(nov-05)

FIR filter is always stable because all its poles are at the origin.

15.What are the possible types of impulse response for linear phase FIR filter?(Nov-11)

There are four types of impulse response for linear phase FIR filters

1. Symmetric impulse response when N is odd.

2. Symmetric impulse response when N is even.

3. Antisymmetric impulse response when N is odd

4. Antisymmetric impulse response when is even.

16.Write the procedure for designing FIR filter using window.(May-06)

1. Choose the desired frequency response of the filter Hd(ω).

2. Take inverse Fourier transform of Hd(ω) to obtain the desired impulse response hd (n).

3.Choose a window sequence w(n) and multiply hd(n) by w(n) to convert the infinite duration impulse response to finite duration impulse response h(n).

4. The Transfer function H(z) of the filter is obtained by taking z-transform of h(n).

17.Write the procedure for FIR filter design by frequency sampling method.(May-05)

1. Choose the desired frequency response Hd(ω).

2. Take N-samples of Hd (ω) to generate the sequence H (K) (Here H bar of k should come) 3. Take inverse of DFT of H (k) to get the impulse response h (n).

4. The transfer function H (z) of the filter is obtained by taking z-transform of impulse response.

18.List the characteristic of FIR filter designed using window.(nov-04)

1. The width of the transition band depends on the type of window.

2. The width of the transition band can be made narrow by increasing the value of N where N is the length of the window sequence.

3. The attenuation in the stop band is fixed for a given window, except in case of Kaiser Window where it is variable.

19.List the features of hanning window spectrum.(nov-04)

1. The mainlobe width is equal to 8π/N. **2**. The maximum sidelobe magnitude is -31db.

3. The sidelobe magnitude decreases with increasingω.

20. Compare the rectangular window and hanning window.(Dec-07)

Rectangular window	**Hamming window**
1. The width of mainlobe in window spectrum is 4π/N.	1. The width of mainlobe in window spectrum is 8π/N.
2.The maximum sidelobe magnitude in window spectrum is -13db	2.The maximum sidelobe magnitude in window spectrum is -41db
3. In window spectrum the sidelobe magnitude slightly decreases with increasing ω.	3. In window spectrum the sidelobe magnitude remains constant.
4. In FIR filter designed using rectangular window the minimum	4. In FIR filter designed using hamming window the minimum stopband

stopband attenuation is 22db.	attenuation is 51db.

21.Compare Hamming window with Kaiser Window.(Nov-06)

Hamming window	**Kaiser window**
1.The main lobe width is equal to $8\pi/N$ and the peak side lobe level is –41dB.	The main lobe width ,the peak side lobe level can be varied by varying the parameter α and N.
2.The low pass FIR filter designed will have first side lobe peak of –53 dB	The side lobe peak can be varied by varying the parameter α.

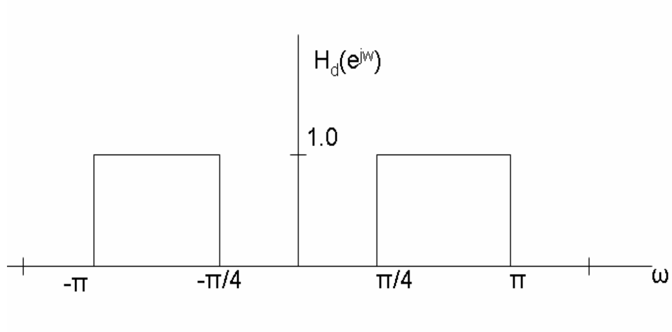
Part –B(16 MARKS)

1. Design an ideal high-pass filter with a frequency response using a hanning window with M = 11 and plot the frequency response. (12)(Nov-07,09)

$$H_d(e^{j\omega}) = 1 \qquad \text{for } \frac{\pi}{4} \le |\omega| \le \pi$$

$$= 0 \qquad |\omega| < \frac{\pi}{4}$$

Solution:



$$h_d(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n} d\omega + \int_{\pi/4}^{\pi} e^{j\omega n} d\omega\right]$$

$$h_d(n) = \frac{1}{\pi n}\left[\sin \pi n - \sin\frac{\pi n}{4}\right] \quad for \quad -\infty \le n \le \infty \quad and \quad n \ne 0$$

$$h_d(0) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} d\omega + \int_{\pi/4}^{\pi} d\omega\right] = \frac{3}{4} = 0.75$$

$h_d(1) = h_d(-1)=-0.225$

$h_d(2) = h_d(-2)= -0.159$

$h_d(3) = h_d(-3)= -0.075$

$h_d(4) = h_d(-4)= 0$

$h_d(5) = h_d(-5) = 0.045$

The hamming window function is given by

$$w_{hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{M-1} \qquad -(\frac{M-1}{2}) \le n \le (\frac{M-1}{2})$$
$$= 0 \qquad otherwise$$

$$for \quad N = 11$$
$$w_{hn}(n) = 0.5 + 0.5\cos\frac{\pi n}{5} \qquad -5 \le n \le 5$$

$w_{hn}(0) = 1$

$w_{hn}(1) = w_{hn}(-1)=0.9045$

$w_{hn}(2)= w_{hn}(-2)=0.655$

$w_{hn}(3)= w_{hn}(-3)= 0.345$

$w_{hn}(4)= w_{hn}(-4)=0.0945$

$w_{hn}(5)= w_{hn}(-5)=0$

$h(n)= w_{hn}(n)h_d(n)$

h(n)=[0 0 -0.026 -0.104 -0.204 0.75 -0.204 -0.104 -0.026 0 0]

2. **Design a filter with a frequency response: using a Hanning window with M = 7**

$$H_d(e^{j\omega}) = e^{-j3\omega} \qquad for -\frac{\pi}{4} \le \omega \le \frac{\pi}{4}$$

(8)(Apr/08)

$$= 0 \quad \frac{\pi}{4} < |\omega| \le \pi$$

Solution:

The freq resp is having a term $e^{-j\omega(M-1)/2}$ which gives h(n) symmetrical about n = M-1/2 = 3 i.e we get a causal sequence.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j3\omega} e^{j\omega n} d\omega$$

$$= \frac{\sin\frac{\pi}{4}(n-3)}{\pi(n-3)}$$

$$this \quad gives \quad h_d(0) = h_d(6) = 0.075$$
$$h_d(1) = h_d(5) = 0.159$$
$$h_d(2) = h_d(4) = 0.22$$
$$h_d(3) = 0.25$$

The Hanning window function values are given by

$w_{hn}(0) = w_{hn}(6) = 0$

$w_{hn}(1) = w_{hn}(5) = 0.25$

$w_{hn}(2) = w_{hn}(4) = 0.75$

$w_{hn}(3) = 1$

$h(n) = h_d(n)\, w_{hn}(n)$

h(n)=[0 0.03975 0.165 0.25 0.165 0.3975 0]

3. Design a LP FIR filter using Freq sampling technique having cutoff freq of π/2 rad / sample. The filter should have linear phase and length of 17. (12)(May-07)

The desired response can be expressed as

$$H_d(e^{j\omega}) = e^{-j\omega(\frac{M-1}{2})} \quad for \quad |\omega| \le \omega c$$
$$= 0 \quad otherwise$$
$$with \quad M = 17 \quad and \quad \omega c = \pi/2$$

$$H_d(e^{j\omega}) = e^{-j\omega 8} \quad for \quad 0 \le \omega \le \pi/2$$
$$= 0 \quad for \quad \pi/2 \le \omega \le \pi$$

Selecting $\omega_k = \frac{2\pi k}{M} = \frac{2\pi k}{17} \quad for \quad k = 0,1,......16$

$$H(k) = H_d(e^{j\omega})\,\Big|_{\omega = \frac{2\pi k}{17}}$$

$$H(k) = e^{-j\frac{2\pi k}{17}8} \qquad for \quad 0 \le \frac{2\pi k}{17} \le \frac{\pi}{2}$$

$$= 0 \qquad for \qquad \pi/2 \le \frac{2\pi k}{17} \le \pi$$

$$H(k) = e^{-j\frac{16\pi k}{17}} \qquad for \quad 0 \le k \le \frac{17}{4}$$

$$= 0 \qquad for \quad \frac{17}{4} \le k \le \frac{17}{2}$$

The range for “k” can be adjusted to be an integer such as

$$0 \le k \le 4$$
$$and \quad 5 \le k \le 8$$

The freq response is given by

$$H(k) = e^{-j\frac{2\pi k}{17}8} \qquad for \quad 0 \le k \le 4$$
$$= 0 \qquad for \qquad 5 \le k \le 8$$

Using these value of H(k) we obtain h(n) from the equation

$$h(n) = \frac{1}{M}(H(0) + 2\sum_{k=1}^{(M-1)/2} \mathrm{Re}(H(k)e^{j2\pi kn/M}))$$

i.e.,

$$h(n) = \frac{1}{17}(1 + 2\sum_{k=1}^{4} \mathrm{Re}(e^{-j16\pi k/17}e^{j2\pi kn/17}))$$

$$h(n) = \frac{1}{17}(H(0) + 2\sum_{k=1}^{4} \cos(\frac{2\pi k(8-n)}{17}) \quad for \quad n = 0,1,........16$$

- Even though k varies from 0 to 16 since we considered ω varying between 0 and π/2 only k values from 0 to 8 are considered
- While finding h(n) we observe symmetry in h(n) such that n varying 0 to 7 and 9 to 16 have same set of h(n)

4.Design an Ideal Differentiator using

a) Rectangular window
(6)

b) Hamming window
(6)

with length of the system = 7.(Nov-09)

Solution:

From differentiator frequency characteristics

$H(e^{j\omega}) = j\omega$ between $-\pi$ to $+\pi$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} j\omega\, e^{j\omega n}\, d\omega = \frac{\cos \pi n}{n} \qquad -\infty \le n \le \infty \quad and \quad n \ne 0$$

The $h_d(n)$ is an add function with $h_d(n)=-h_d(-n)$ and $h_d(0)=0$

a) rectangular window

$h(n)=h_d(n)w_r(n)$

h(1)=-h(-1)=hd(1)=-1

h(2)=-h(-2)=hd(2)=0.5

h(3)=-h(-3)=hd(3)=-0.33

h'(n)=h(n-3) for causal system

thus,

$$H'(z) = 0.33 - 0.5z^{-1} + z^{-2} - z^{-4} + 0.5z^{-5} - 0.33z^{-6}$$

Also from the equation $H_r(e^{j\omega}) = 2\sum_{n=0}^{(M-3)/2} h(n)\sin\omega(\frac{M-1}{2} - n)$

For M=7 and h'(n) as found above we obtain this as

$$H_r(e^{j\omega}) = 0.66\sin 3\omega - \sin 2\omega + 2\sin \omega$$

$$H(e^{j\omega}) = jH_r(e^{j\omega}) = j(0.66\sin 3\omega - \sin 2\omega + 2\sin \omega)$$

b) Hamming window

$h(n)=h_d(n)w_h(n)$

where $w_h(n)$ is given by

$$w_h(n) = 0.54 + 0.46\cos\frac{2\pi n}{(M-1)} \quad -(M-1)/2 \le n \le (M-1)/2$$
$$= 0 \quad otherwise$$

For the present problem

$$w_h(n) = 0.54 + 0.46\cos\frac{\pi n}{3} \quad -3 \le n \le 3$$

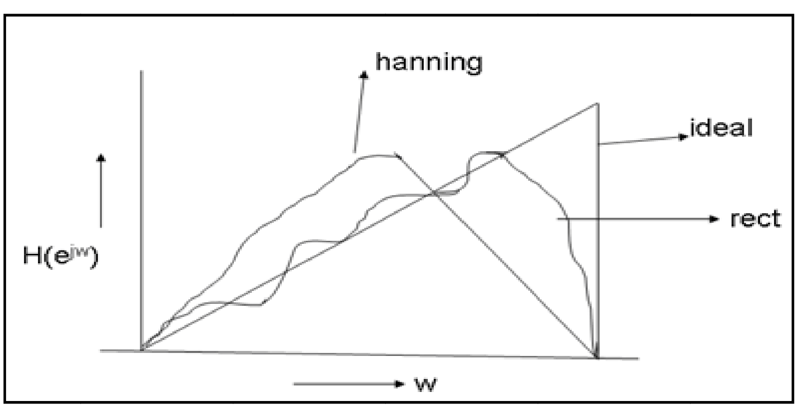
The window function coefficients are given by for n=-3 to +3

Wh(n)= [0.08 0.31 0.77 1 0.77 0.31 0.08]

Thus h'(n) = h(n-5) = [0.0267, -0.155, 0.77, 0, -0.77, 0.155, -0.0267]

Similar to the earlier case of rectangular window we can write the freq response of differentiator as

$$H(e^{j\omega}) = jH_r(e^{j\omega}) = j(0.0534\sin 3\omega - 0.31\sin 2\omega + 1.54\sin \omega)$$



- With rectangular window, the effect of ripple is more and transition band width is small compared with hamming window
- With hamming window, effect of ripple is less whereas transition band is more

5.Justify Symmetric and Anti-symmetric FIR filters giving out Linear Phase characteristics.(Apr-08) (10)

Symmetry in filter impulse response will ensure linear phase

An FIR filter of length M with i/p x(n) & o/p y(n) is described by the difference equation:

$$y(n)= b_0\, x(n) + b_1\, x(n-1)+\ldots\ldots+b_{M-1}\, x(n-(M-1)) = \sum_{k=0}^{M-1} b_k x(n-k) \qquad -(1)$$

Alternatively, it can be expressed in convolution form

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k) \qquad -(2)$$

i.e bk= h(k), k=0,1,…..M-1

Filter is also characterized by

$H(z) = \sum_{k=0}^{M-1} h(k)z^{-k}$ -(3) polynomial of degree M-1 in the variable z^{-1}. The roots of this polynomial constitute zeros of the filter.

An FIR filter has linear phase if its unit sample response satisfies the condition

$$h(n)= \pm\, h(M-1-n) \quad n=0,1,\ldots\ldots M-1 \qquad -(4)$$

Incorporating this symmetry & anti symmetry condition in eq 3 we can show linear phase characteristics of FIR filters

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + ... + h(M-2)z^{-(M-2)} + h(M-1)z^{-(M-1)}$$

If M is odd

$$H(z) = h(0) + h(1)z^{-1} + ... + h(\frac{M-1}{2})z^{-(\frac{M-1}{2})} + h(\frac{M+1}{2})z^{-(\frac{M+1}{2})} + h(\frac{M+3}{2})z^{-(\frac{M+3}{2})} + ...$$
$$+ h(M-2)z^{-(M-2)} + h(M-1)z^{-(M-1)}$$

$$= z^{-(\frac{M-1}{2})}\left[h(0)z^{(\frac{M-1}{2})} + h(1)z^{(\frac{M-3}{2})} + ... + h(\frac{M-1}{2}) + h(\frac{M+1}{2})z^{-1} + h(\frac{M+3}{2})z^{-2} + ...h(M-1)z^{-(\frac{M-1}{2})}\right]$$

Applying symmetry conditions for M odd

$$h(0) = \pm h(M-1)$$
$$h(1) = \pm h(M-2)$$
.
.
$$h(\frac{M-1}{2}) = \pm h(\frac{M-1}{2})$$
$$h(\frac{M+1}{2}) = \pm h(\frac{M-3}{2})$$
.
.
$$h(M-1) = \pm h(0)$$

$$H(z) = z^{-(\frac{M-1}{2})}\left[h(\frac{M-1}{2}) + \sum_{n=0}^{\frac{M-3}{2}} h(n)\{z^{(M-1-2n)/2} \pm z^{-(M-1-2n)/2}\}\right]$$

similarly for M even

$$H(z) = z^{-(\frac{M-1}{2})}\left[\sum_{n=0}^{\frac{M}{2}-1} h(n)\{z^{(M-1-2n)/2} \pm z^{-(M-1-2n)/2}\}\right]$$

6.What are the structure of FIR filter systems and explain it. (Dec-10) (12)

FIR system is described by,

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

Or equivalently, the system function

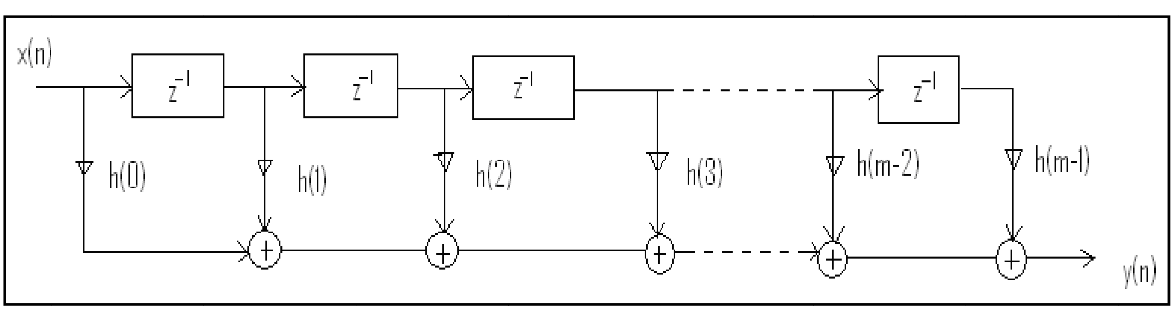
$$H(Z) = \sum_{k=0}^{M-1} b_k Z^{-k}$$

Where we can identify $h(n) = \begin{cases} b_n & 0 \le n \le n-1 \\ 0 & otherwise \end{cases}$

Different FIR Structures used in practice are,

1. Direct form
2. Cascade form
3. Frequency-sampling realization
4. Lattice realization

- **Direct form**
  - It is Non recursive in structure



- As can be seen from the above implementation it requires M-1 memory locations for storing the M-1 previous inputs
- It requires computationally M multiplications and M-1 additions per output point
- It is more popularly referred to as tapped delay line or transversal system
- Efficient structure with linear phase characteristics are possible where $h(n) = \pm h(M-1-n)$

- **Cascade form**

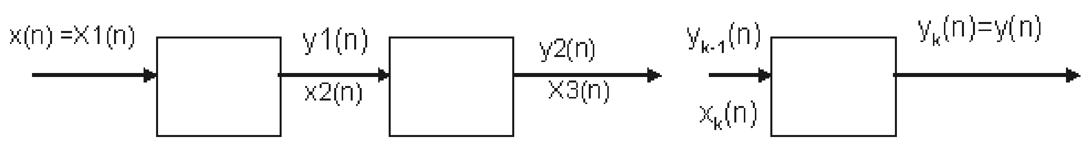
The system function H (Z) is factored into product of second – order FIR system

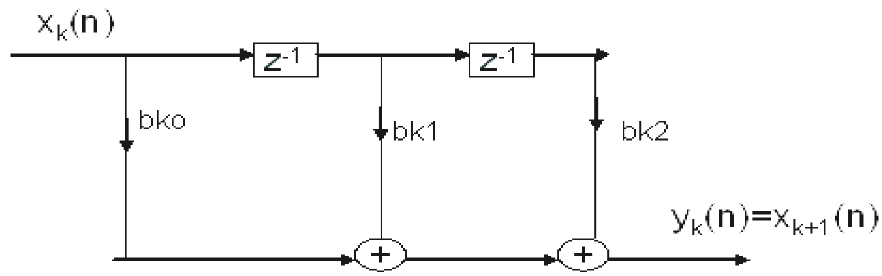
$$H(Z) = \prod_{k=1}^{K} H_k(Z)$$

Where $H_k(Z) = b_{k0} + b_{k1}Z^{-1} + b_{k2}Z^{-2}$ k = 1, 2 ….. K

and K = integer part of (M+1) / 2

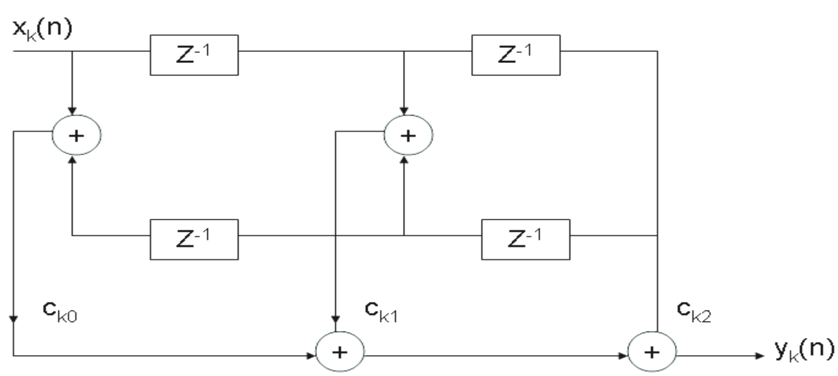
The filter parameter b_0 may be equally distributed among the K filter section, such that $b_0 = b_{10}\ b_{20}\ \ldots\ b_{k0}$ or it may be assigned to a single filter section. The zeros of H (z) are grouped in pairs to produce the second – order FIR system. Pairs of complex-conjugate roots are formed so that the coefficients $\{b_{ki}\}$ are real valued.





In case of linear –phase FIR filter, the symmetry in h(n) implies that the zeros of H(z) also exhibit a form of symmetry. If zk and zk* are pair of complex – conjugate zeros then 1/zk and 1/zk* are also a pair complex –conjugate zeros. Thus simplified fourth order sections are formed. This is shown below,

$$H_k(z) = C_{k0}(1 - z_k z^{-1})(1 - z_k * z^{-1})(1 - z^{-1} / z_k)(1 - z^{-1} / z_{k*})$$
$$= C_{k0} + C_{k1} z^{-1} + C_{k2} z^{-2} + C_{k1} z^{-3} + z^{-4}$$



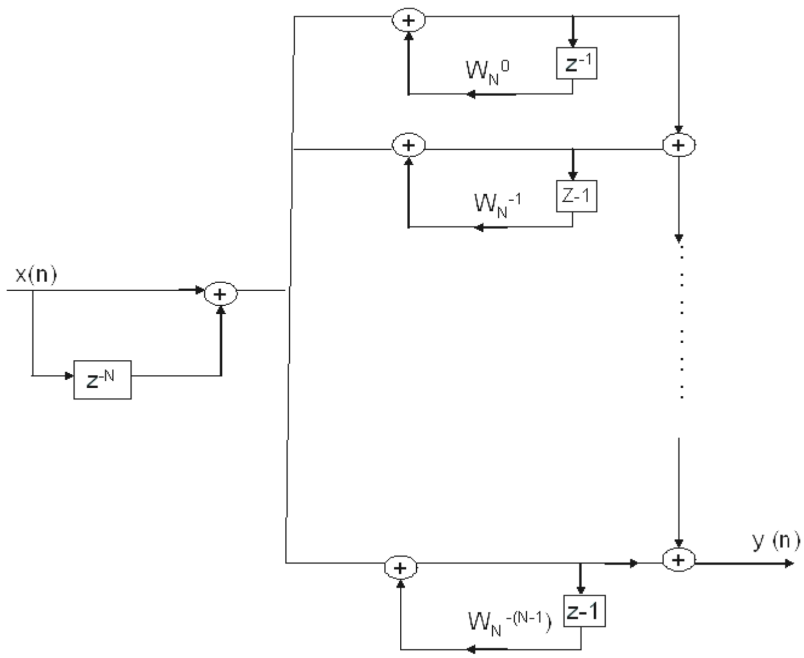
- **Frequency sampling method**
  We can express system function H(z) in terms of DFT samples H(k) which is given by

$$H(z) = (1 - z^{-N})\frac{1}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1 - W_N^{-k}z^{-1}}$$

This form can be realized with cascade of FIR and IIR structures. The term $(1\text{-}z^{-N})$ is realized as FIR and the term $\frac{1}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1 - W_N^{-k}z^{-1}}$ as IIR structure.

The realization of the above freq sampling form shows necessity of complex arithmetic.

Incorporating symmetry in h (n) and symmetry properties of DFT of real sequences the realization can be modified to have only real coefficients.



- **Lattice realization**
  Lattice structures offer many interesting features:

1. Upgrading filter orders is simple. Only additional stages need to be added instead of redesigning the whole filter and recalculating the filter coefficients.
2. These filters are computationally very efficient than other filter structures in a filter bank applications (eg. Wavelet Transform)

3. Lattice filters are less sensitive to finite word length effects.

Consider

$$H(z) = \frac{Y(z)}{X(z)} = 1 + \sum_{i=1}^{m} a_m(i) z^{-i}$$

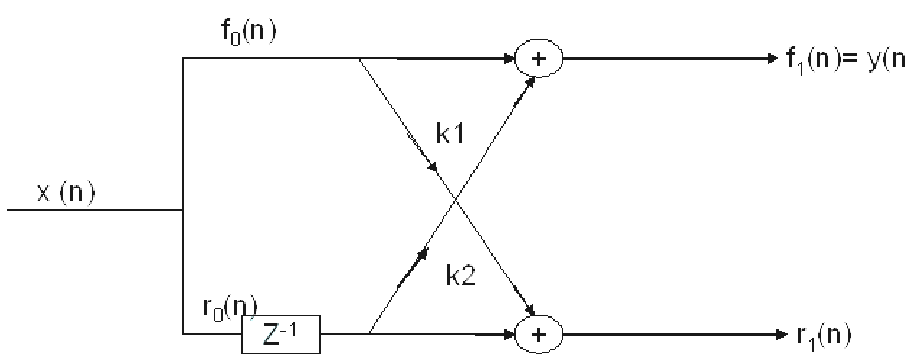
m is the order of the FIR filter and $a_m(0)=1$

when m = 1 $Y(z)/X(z) = 1 + a_1(1)\, z^{-1}$

$y(n) = x(n) + a_1(1)x(n-1)$

f1(n) is known as upper channel output and r1(n)as lower channel output.

$f_0(n) = r_0(n) = x(n)$



The outputs are

$$f_1(n) = f_0(n) + k_1 r_0(n-1) \qquad 1a$$
$$r_1(n) = k_1 f_0(n) + r_0(n-1) \qquad 1b$$
$$if \quad k_1 = a_1(1), then \quad f_1(n) = y(n)$$

If m=2

$$\frac{Y(z)}{X(z)} = 1 + a_2(1)z^{-1} + a_2(2)z^{-2}$$
$$y(n) = x(n) + a_2(1)x(n-1) + a_2(2)x(n-2)$$
$$y(n) = f_1(n) + k_2 r_1(n-1) \qquad (2)$$

Substituting 1a and 1b in (2)

$$\begin{aligned} y(n) &= f_0(n) + k_1 r_0(n-1) + k_2[k_1 f_0(n-1) + r_0(n-2)] \\ &= f_0(n) + k_1 r_0(n-1) + k_2 k_1 f_0(n-1) + k_2 r_0(n-2)] \end{aligned}$$
$$\sin ce \quad f_0(n) = r_0(n) = x(n)$$
$$\begin{aligned} y(n) &= x(n) + k_1 x(n-1) + k_2 k_1 x(n-1) + k_2 x(n-2)] \\ &= x(n) + (k_1 + k_1 k_2) x(n-1) + k_2 x(n-2) \end{aligned}$$

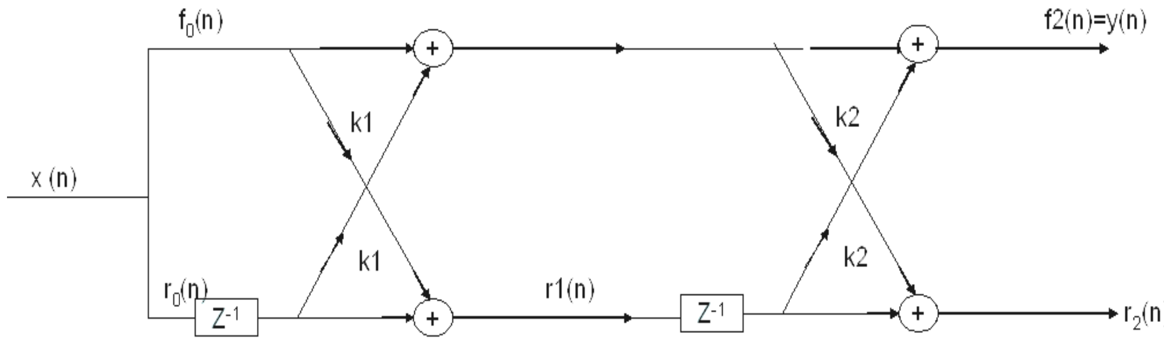
We recognize

$$a_2(1) = k_1 + k_1 k_2$$
$$a_2(1) = k_2$$

Solving the above equation we get

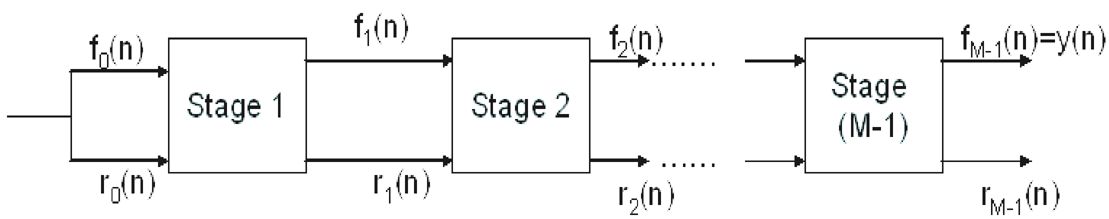
$$k_1 = \frac{a_2(1)}{1 + a_2(2)} \quad and \quad k_2 = a_2(2) \qquad (4)$$

Equation (3) means that, the lattice structure for a second-order filter is simply a cascade of two first-order filters with k1 and k2 as defined in eq (4)



Similar to above, an M^{th} order FIR filter can be implemented by lattice structures with

M – Stages



7.Realize the following system function using minimum number of multiplication

$$H(Z) = 1 + \frac{1}{3}Z^{-1} + \frac{1}{4}Z^{-2} + \frac{1}{4}Z^{-3} + \frac{1}{3}Z^{-4} + Z^{-5}$$ **(6)(Dec-10)**

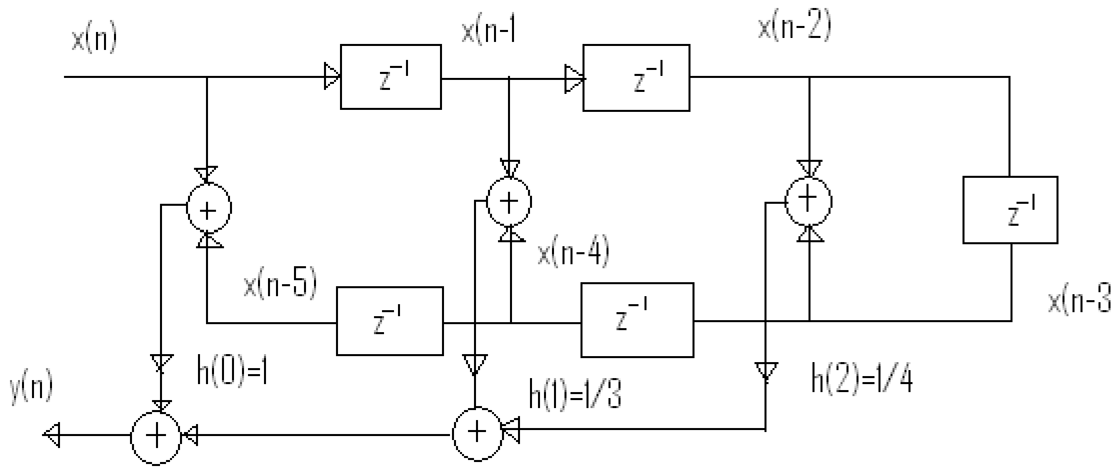
Solution:

We recognize $h(n) = \left[1, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{3}, 1\right]$

M is even = 6, and we observe h (n) = h (M-1-n) h (n) = h (5-n)

i.e h (0) = h (5) h (1) = h (4) h (2) = h (3)

Direct form structure for linear phase FIR can be realized



8. Realize the following using system function using minimum number of multiplication. (8)(May-09)

$$H(Z) = 1 + \frac{1}{4}Z^{-1} + \frac{1}{3}Z^{-2} + \frac{1}{2}Z^{-3} - \frac{1}{2}Z^{-5} - \frac{1}{3}Z^{-6} - \frac{1}{4}Z^{-7} - Z^{-8}$$

Solution:

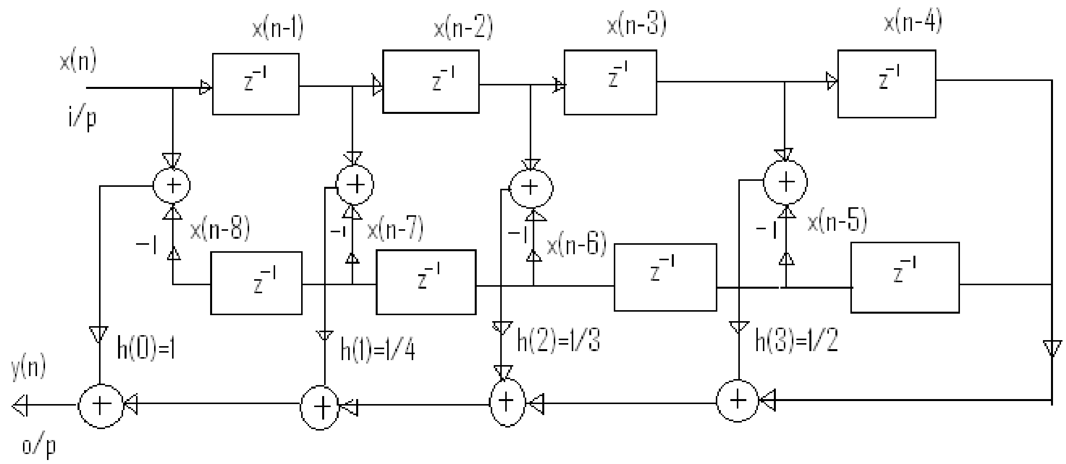
m=9

$$h(n) = \left[1, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -1\right]$$

Odd symmetry

h (n) = -h(M-1-n); h (n) = -h (8-n); h (m-1/2) = h (4) = 0

h (0) = -h (8); h (1) = -h (7); h (2) = -h (6); h (3) = -h (5)

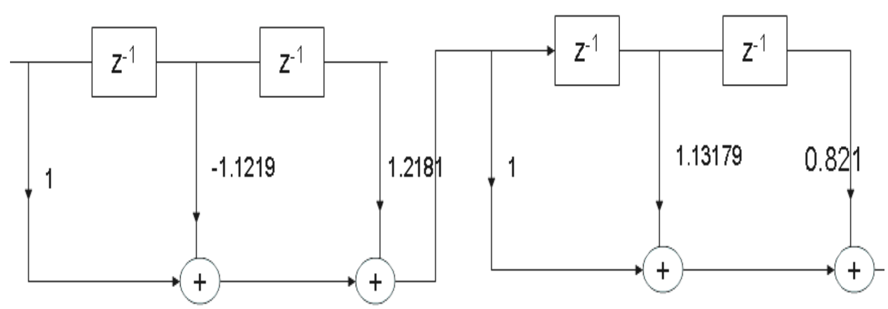


9. Realize the difference equation in cascade form (May-07) (6)

$$y(n) = x(n) + 0.25x(n-1) + 0.5x(n-2) + 0.75x(n-3) + x(n-4)$$

Solution:

$$Y(z) = X(z)\{1 + 0.25z^{-1} + 0.5z^{-2} + 0.75z^{-3} + z^{-4})$$
$$H(z) = 1 + 0.25z^{-1} + 0.5z^{-2} + 0.75z^{-3} + z^{-4}$$
$$H(z) = (1 - 1.1219z^{-1} + 1.2181z^{-2})(1 + 1.3719z^{-1} + 0.821z^{-2})$$
$$H(z) = H_1(z)H_2(z)$$



10.Given FIR filter $H(Z) = 1 + 2Z^{-1} + \frac{1}{3}Z^{-2}$ obtain lattice structure for the same (4)(Apr-08)

Solution:

Given $a_1(1) = 2$, $a_2(2) = \frac{1}{3}$

Using the recursive equation for

m = M, M-1… 2, 1

Here M=2 therefore m = 2, 1

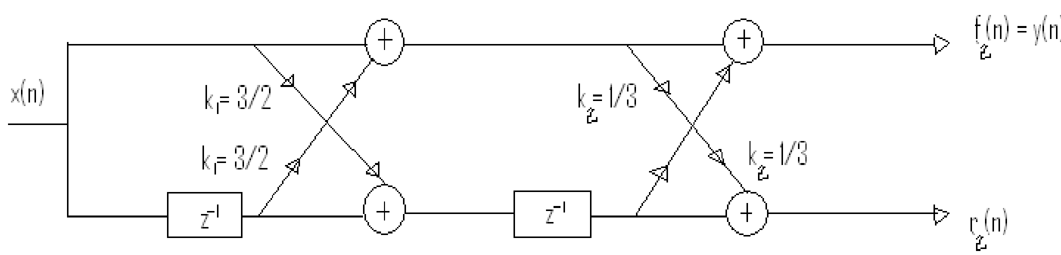
If m=2 $k_2 = a_2(2) = \frac{1}{3}$

If m=1 $k_1 = a_1(1)$

Also, when m=2 and i=1

$$a_1(1) = \frac{a_2(1)}{1 + a_2(2)} = \frac{2}{1 + \frac{1}{3}} = \frac{3}{2}$$

Hence $k_1 = a_1(1) = \frac{3}{2}$



UNIT IV

FINITE WORDLENGTH EFFECTS

1.Determine 'Dead band' of the filter.(May-07,Dec-07,Dec-09)

The limit cycle occur as a result of quantization effects in multiplications. The amplitudes of output during a limit cycle are confined to a range of values that is called the dead band of the filter.

2.Why rounding is preferred to truncation in realizing digital filter?(May-07)

In digital system the product quantization is performed by rounding due to the following desirable characteristics of rounding.

1. ***The rounding error is independent of the type of arithmetic.***
2. ***The mean value of rounding error signal is zero.***
3. ***The variance of the rounding error signal is least.***

3.What is Sub band coding?(May-07)

Sub band coding is a method by which the signal (speech signal) is sub divided in to several frequency bands and each band is digitally encoded separately.

4.Identify the various factors which degrade the performance of the digital filter implementation when finite word length is used.(May-07,May-2010)

5.What is meant by truncation & rounding?(Nov/Dec-07)

The truncation is the process of reducing the size of binary number by discarding all bits less significant than the least significant bit that is retained.

Rounding is the process of reducing the size of a binary number to finite word sizes of b-bits such that, the rounded b-bit number is closest to the original unquantized number.

6.What is meant by limit cycle oscillation in digital filters?(May-07,08,Nov-10)

In recursive system when the input is zero or some nonzero constant value, the nonlinearities due to finite precision arithmetic operation may cause periodic oscillations in the output. These oscillations are called limit cycles.

7.What are the three types of quantization error occurred in digital systems?(Apr-08,Nov-10)

i. Input quantization error ii. Product quantization error iii. Coefficient quantization error.

8.What are the advantages of floating point arithmetic(Nov-08)

i. Larger dynamic range.

ii. Overflow in floating point representation is unlikely.

9.Give the IEEE754 standard for the representation of floating point numbers.(May-09)

The IEEE-754 standard for 32-bit single precision floating point number is given by Floating point number,

$$\mathbf{N_f = (-1)^* \, (2^\wedge E\text{-}127)^* M}$$

0 1 8 9 31

S	E	M

S = 1-bit field for sign of numbers

E = 8-bit field for exponent

M = 23-bit field for mantissa

10.Compare the fixed point & floating point arithmetic.(May/june-09)

Fixed point arithmetic	*Floating point arithmetic*
1. The accuracy of the result is less due to smaller dynamic range.	1. The accuracy of the result will be higher due to larger dynamic range.
2. Speed of processing is high.	2. Speed of processing is low.
3. Hardware implementation is cheaper.	3. Hardware implementation is costlier.
4. Fixed point arithmetic can be used for real; time computations.	4. Floating point arithmetic cannot be used for real time computations
5. quantization error occurs only in multiplication	5. Quantization error occurs in both multiplication and addition.

11.Define Zero input limit cycle oscillation.(Dec-09)

In recursive system, the product quantization may create periodic oscillations in the output. These oscillations are called limit cycles. If the system output enters a limit cycle, it will continue to remain in limit cycle even when the input is made zero. Hence these limit cycles are called zero input limit cycles.

12.What is the effect of quantization on pole locations?(Apr-2010)

Quantization of coefficients in digital filters lead to slight changes in their value. These changes in value of filter coefficients modify the pole-zero locations. Some times the pole locations will be changed in such a way that the system may drive into instability.

13.What is the need for signal scaling?(Apr-2010)

To prevent overflow, the signal level at certain points in the digital filters must be scaled so that no overflow occurs in the adder.

14.What are the results of truncation for positive & negative numbers?(Nov-06)

consider the real numbers 5.6341432543653654 ,32.438191288 ,−6.3444444444444

To *truncate* these numbers to 4 decimal digits, we only consider the 4 digits to the right of the decimal point.

The result would be: 5.6341 ,32.4381 ,−6.3444 Note that in some cases, truncating would yield the same result as rounding, but truncation does not round up or round down the digits; it merely cuts off at the specified digit. The truncation error can be twice the maximum error in rounding.

15.What are the different quantization methods?(Nov-2011)

The two types of quantization are: i. Truncation and ii. Rounding.

16. List out some of the finite word length effects in digital filter.(Apr-06)

1. Errors due to quantization of input data.
2. Errors due to quantization of filter coefficients.

3. Errors due to rounding the product in multiplications.

4. Limit cycles due to product quantization and overflow in addition.

17. What are the different formats of fixed point's representation?(May-05)

In fixed point representation, there are three different formats for representing binary numbers.

1. Signed-magnitude format

2. One's-complement format

3. Two's-complement format.

In all the three formats, the positive number is same but they differ only in representing negative numbers.

18. Explain the floating point representation of binary number.(Dec-06)

The floating numbers will have a mantissa part and exponent part. In a given word size the bits allotted for mantissa and exponent are fixed. The mantissa is used to represent a binary fraction number and the exponent is a positive or negative binary integer. The value of the exponent can be adjusted to move the position of binary point in mantissa. Hence this representation is called floating point. The floating point number can be expressed as,

Floating point number, $N_f = M*2^E$

Where M = Mantissa and E=Exponent.

19. What is quantization step size?(Apr-07,11)

In digital system, the number is represented in binary. With b-bit binary we can generate 2^b different binary codes. Any range of analog value to be represented in binary should be divided into 2^b levels with equal increment. The **2^b** level are called quantization levels and the increment in each level is called quantization step size. It R is the range of analog signal then.

***Quantization step size*, q = R/2^b**

20. What is meant by product quantization error?(Nov-11)

In digital computation, the output of multipliers i.e., the products is quantized to finite word length in order to store them in register and to be used in subsequent calculation. The error due to the quantization of the output of multipliers is referred to as product quantization error.

22. What is overflow limit cycle?(May/june-10)

In fixed point addition the overflow occurs when the sum exceeds the finite word length of the register used to store the sum. The overflow in addition may be lead to oscillation in the output which is called overflow limit cycle.

23. What is input quantization error?(Nov-04)

The filter coefficients are computed to infinite precision in theory. But in digital computation the filter coefficients are represented in binary and are stored in registers. If a b bit register is used the filter coefficients must be rounded or truncated to b bits, which produces an error.

24.What are the different types of number representation?(Apr-11)

There are three forms to represent numbers in digital computer or any other digital hardware.

i. Fixed point representation

ii. Floating point representation

iii. Block floating point representation.

25. Define white noise?(Dec-06)

A stationary random process is said to be white noise if its power density spectrum is constant. Hence the white noise has flat frequency response spectrum.

$$S_X(w) = \sigma_x^2, -\pi \leq w\pi$$

28. What are the methods used to prevent overflow?(May-05)

There are two methods to prevent overflow

i) ***saturation arithmetic***
ii) ***scaling***

29. What is meant by A/D conversion noise?(MU-04)

A DSP contains a device, A/D converter that operates on the analog input **x(t)** to

produce **xq(t)** which is binary sequence of 0s and 1s. At first the signal **x(t)** is sampled at regular intervals to produce a sequence **x(n)** is of infinite precision. Each sample **x(n)** is expressed in terms of a finite number of bits given the sequence xq(n). The difference signal **e(n) =xq (n)-x(n)** is called ***A/D conversion noise.***

16 MARKS

1, Explain in detail about Number Representation.

Number Representation

In digital signal processing, $(B + 1)$-bit fixed-point numbers are usually represented as two's- complement signed fractions in the format

bo b-ib-2 …… b-B

The number represented is then

$$X = -b_0 + b_{-1}2^{-1} + b_{-2}2^{-2} + \cdots + b_{-B}\, 2^{-B} \qquad (3.1)$$

where bo is the sign bit and the number range is —1 <X < 1. The advantage of this representation is that the product of two numbers in the range from — 1 to 1 is another number in the same range. Floating-point numbers are represented as

$$X = (-1)^s m 2^c \qquad (3.2)$$

where s is the *sign bit,* m is the mantissa, and c is the *characteristic* or *exponent*. To make the representation of a number unique, the mantissa is *normalized* so that 0.5 <m < 1.

Although floating-point numbers are always represented in the form of (3.2), the way in which this representation is actually *stored* in a machine may differ. Since m > 0.5, it is not necessary to store the 2^{-1} -weight bit of m, which is always set. Therefore, in practice numbers are usually stored as

$$X = (-1)^s(0.5 + f)2^c \qquad (3.3)$$

where f is an unsigned fraction, 0 <f < 0.5.

Most floating-point processors now use the IEEE Standard 754 32-bit floating-point format for storing numbers. According to this standard the exponent is stored as an unsigned integer p where

$$p = c + 126 \qquad (3.4)$$

Therefore, a number is stored as

$$X = (-1)^s(0.5 + f)2^{p-126} \qquad (3.5)$$

where s is the sign bit, f is a 23-b unsigned fraction in the range $0 < f < 0.5$, and p is an 8-b unsigned integer in the range $0 < p < 255$. The total number of bits is 1 + 23 + 8 = 32. For example, in IEEE format 3/4 is written $(-1)^0(0.5 + 0.25)2^0$ so $s = 0$, $p = 126$, and $f = 0.25$. The value $X = 0$ is a unique case and is represented by all bits zero (i.e., s = 0, $f = 0$, and $p = 0$). Although the 2^{-1}-weight mantissa bit is not actually stored, it does exist so the mantissa has 24 b plus a sign bit.

2, Explain about the Fixed-Point Quantization Errors and Floating Point Quantization Errors

Fixed-Point Quantization Errors

In fixed-point arithmetic, a multiply doubles the number of significant bits. For example, the product of the two 5-b numbers 0.0011 and 0.1001 is the 10-b number 00.000 110 11. The extra bit to the left of the decimal point can be discarded without introducing any error. However, the least significant four of the remaining bits must ultimately be discarded by some form of quantization so that the result can be stored to 5 b for use in other calculations. In the example above this results in 0.0010 (quantization by rounding) or 0.0001 (quantization by truncating). When a sum of products calculation is performed, the quantization can be performed either after each multiply or after all products have been summed with double-length precision.

We will examine three types of fixed-point quantization—rounding, truncation, and magnitude truncation. If X is an exact value, then the rounded value will be denoted $Q_r(X)$, the truncated value $Q_t(X)$, and the magnitude truncated value $Q_{mt}(X)$. If the quantized value has B bits to the right of the decimal point, the quantization step size is

$$A = 2^{-B} \qquad (3.6)$$

Since rounding selects the quantized value nearest the unquantized value, it gives a value which is never more than ± A /2 away from the exact value. If we denote the rounding error by

$$f_r = Q_r(X) - X \qquad (3.7)$$

then

$$-\frac{A}{2} < f_r < \frac{A}{2} \qquad (3.8)$$

Truncation simply discards the low-order bits, giving a quantized value that is always less than or equal to the exact value so

$$-A < f_t < 0 \qquad (3.9)$$

Magnitude truncation chooses the nearest quantized value that has a magnitude less than or equal to the exact value so

$$-A < f_{mt} < A \qquad (3.10)$$

The error resulting from quantization can be modeled as a random variable uniformly distributed over the appropriate error range. Therefore, calculations with roundoff error can be considered error-free calculations that have been corrupted by additive white noise. The mean of this noise for rounding is

$$m_{\epsilon_r} = E\{\epsilon_r\} = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} \epsilon_r \, d\epsilon_r = 0 \quad (3.11)$$

where $E\{\}$ represents the operation of taking the expected value of a random variable. Similarly, the variance of the noise for rounding is

$$\sigma_{\epsilon_r}^2 = E\{(\epsilon_r - m_{\epsilon_r})^2\} = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} (\epsilon_r - m_{\epsilon_r})^2 \, d\epsilon_r = \frac{\Delta^2}{12} \quad (3.12)$$

Likewise, for truncation,

$$m_{\epsilon_t} = E\{\epsilon_t\} = -\frac{\Delta}{2}$$

$$\sigma_{\epsilon_t}^2 = E\{(\epsilon_t - m_{\epsilon_t})^2\} = \frac{\Delta^2}{12} \quad (3.13)$$

and, for magnitude truncation

$$m_{\epsilon_{mt}} = E\{\epsilon_{mt}\} = 0$$

$$\sigma_{\epsilon_{mt}}^2 = E\{(\epsilon_{mt} - m_{mt})^2\} = \frac{\Delta^2}{3} \quad (3.14)$$

Floating-Point Quantization Errors

With floating-point arithmetic it is necessary to quantize after both multiplications and additions. The addition quantization arises because, prior to addition, the mantissa of the smaller number in the sum is shifted right until the exponent of both numbers is the same. In general, this gives a sum mantissa that is too long and so must be quantized.

We will assume that quantization in floating-point arithmetic is performed by rounding. Because of the exponent in floating-point arithmetic, it is the relative error that is important. The relative error is defined as

$$\varepsilon_r = \frac{Q_r(X) - X}{X} = \frac{\epsilon_r}{X} \quad (3.15)$$

Since $X = (-1)^s m 2^c$, $Q_r(X) = (-1)^s Q_r(m) 2^c$ *and*

$$\varepsilon_r = \frac{Q_r(m) - m}{m} = \frac{\epsilon}{m} \quad (3.16)$$

If the quantized mantissa has B bits to the right of the decimal point, $|\epsilon| < \Delta/2$ where, as before, $\Delta = 2^{-B}$. Therefore, since $0.5 \le m < 1$,

$$|\varepsilon_r| < \Delta \quad (3.17)$$

If we assume that ϵ is uniformly distributed over the range from $-\Delta/2$ to $\Delta/2$ and m is uniformly distributed over 0.5 to 1,

$$m_{\varepsilon_r} = E\left\{\frac{\epsilon}{m}\right\} = 0$$

$$\sigma_{\varepsilon_r}^2 = E\left\{\left(\frac{\epsilon}{m}\right)^2\right\} = \frac{2}{\Delta}\int_{1/2}^{1}\int_{-\Delta/2}^{\Delta/2} \frac{\epsilon^2}{m^2} \, d\epsilon \, dm$$

$$= \frac{}{6} = (0.167)2^{-2B} \quad (3.18)$$

In practice, the distribution of m is not exactly uniform. Actual measurements of roundoff noise in [1] suggested that

$$al_r \ll 0.23A^2 \quad (3.19)$$

while a detailed theoretical and experimental analysis in [2] determined

$$a^2 \ll 0.18A^2 \quad (3.20)$$

From (3.15) we can represent a quantized floating-point value in terms of the unquantized value and the random variable er using

$$Qr(X) = X(1 + er) \quad (3.21)$$

Therefore, the finite-precision product $X1X2$ and the sum $X1 + X2$ can be written

$$f\ IX1X2) = X1X2U + e_r) \quad (3.22)$$

and

$$fl(X1 + X2) = (X1 + X2)(1 + er) \quad (3.23)$$

where e_r is zero-mean with the variance of (3.20).

4, Explain about Roundoff Noise.

Roundoff Noise:

To determine the roundoff noise at the output of a digital filter we will assume that the noise due to a quantization is stationary, white, and uncorrelated with the filter input, output, and internal variables. This assumption is good if the filter input changes from sample to sample in a sufficiently complex manner. It is not valid for zero or constant inputs for which the effects of rounding are analyzed from a limit cycle perspective.

To satisfy the assumption of a sufficiently complex input, roundoff noise in digital filters is often calculated for the case of a zero-mean white noise filter input signal $x(n)$ of variance a^1. This simplifies calculation of the output roundoff noise because expected values of the form $E\{x(n)x(n — k)\}$ are zero for $k = 0$ and give a^2 when $k = 0$. This approach to analysis has been found to give estimates of the output roundoff noise that are close to the noise actually observed for other input signals.

Another assumption that will be made in calculating roundoff noise is that the product of two quantization errors is zero. To justify this assumption, consider the case of a 16-b fixed-point processor. In this case a quantization error is of the order $2^{—15}$, while the product of two quantization errors is of the order $2^{—30}$, which is negligible by comparison.

If a linear system with impulse response $g(n)$ is excited by white noise with mean m_x and variance a^2, the output is noise of mean [3, pp.788-790]

$$al = \quad {}_{1} \quad 2^{-2B} \quad (3.28)$$

TO

$$my = mx \,\hat{}\, 2g(n) \qquad (3.24)$$

n=—TO

and variance

TO

$$ay = al \,\hat{}\, g^2(n) \qquad (3.25)$$

n=—TO

Therefore, if $g(n)$ is the impulse response from the point where a roundoff takes place to the filter output, the contribution of that roundoff to the variance (mean-square value) of the output roundoff noise is given by (3.25) with a^2 replaced with the variance of the roundoff. If there is more than one source of roundoff error in the filter, it is assumed that the errors are uncorrelated so the output noise variance is simply the sum of the contributions from each source.

Roundoff Noise in FIR Filters

The simplest case to analyze is a finite impulse response (FIR) filter realized via the convolution summation

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n - k) \qquad (3.26)$$

When fixed-point arithmetic is used and quantization is performed after each multiply, the result of the N multiplies is N-times the quantization noise of a single multiply. For example, rounding after each multiply gives, from (3.6) and (3.12), an output noise variance of

$$a^2 = N\frac{2^{-2B}}{} \qquad (3.27)$$

Virtually all digital signal processor integrated circuits contain one or more double-length accumulator registers which permit the sum-of-products in (3.26) to be accumulated without quantization. In this case only a single quantization is necessary following the summation and

For the floating-point roundoff noise case we will consider (3.26) for $N = 4$ and then generalize the result to other values of N. The finite-precision output can be written as the exact output plus an error term $e(n)$. Thus,

$$\begin{aligned} y(n) + e(n) = (\{[&h(0)x(n)[1 + E1(n)] \\ &+ h(1)x(n-1)[1 + £2(n)]][1 + S3(n)] \\ &+ h(2)x(n-2)[1 + £4(n)]\}\{1 + s_5(n)\} \\ &+ h(3)x(n-3)[1 + £6(n)])[1 + £j(n)] \end{aligned} \qquad (3.29)$$

In (3.29), $£1(n)$ represents the errorin the first product, $£2(n)$ the error in the second product, $£3(n)$ the error in the firstaddition, etc.Notice that it has been assumed that the products are summed in the order implied by the summation of (3.26).

Expanding (3.29), ignoring products of error terms, and recognizing $y(n)$ gives

$$e(n) = h(0)x(n)[£1(n) + £3(n) + £\$(n) + £i(n)]$$

+ h(1)x(n-*1)[£2*(n) + £3(n) + £5(n) + £j(n)]
+ h(2)x(n-*2)[£4(n)* + £5(n) + £i(n)]
+ *h(3)x(n*- 3)[£6(n)+ *£j(n)]*
(3.30)

Assuming that the input is white noise of variance $a\wedge$ so that $E\{x(n)x(n-k)\}$ is zero for $k = 0$, and assuming that the errors are uncorrelated,

$$E\{e^2(n)\} = [4h^2(0)+ 4h^2(1) +3h^2(2) + 2h^2(3)]a^2a^2 \qquad (3.31)$$

In general, for any N,

$$aO = E\{e^{\sim}(n)\} = Nh^2(0) + \sum_{k=1}^{N-1} (N + 1 - k)h^2(k) \quad a^{\circ}a^{\circ}_r \qquad (\cdot.\cdot\cdot)$$

Notice that if the order of summation of the product terms in the convolution summation is changed, then the order in which the *h(k)'s* appear in (3.32) changes. If the order is changed so that the *h(k)*with smallest magnitude is first, followed by the next smallest, etc., then the roundoff noise variance is minimized. However, performing the convolution summation in nonsequential order greatly complicates data indexing and so may not be worth the reduction obtained in roundoff noise.

Roundoff Noise in Fixed-Point IIR Filters

To determine the roundoff noise of a fixed-point infinite impulse response (IIR) filter realization, consider a causal first-order filter with impulse response

$$h(n) = a^n u(n) \qquad (3.33)$$

realized by the difference equation

$$y(n) = ay(n - 1) + x(n) \qquad (3.34)$$

Due to roundoff error, the output actually obtained is

$$y(n) = Q\{ay(n - 1) + x(n)\} = ay(n - 1) + x(n) + e(n) \qquad (3.35)$$

where $e(n)$ is a random roundoff noise sequence. Since $e(n)$ is injected at the same point as the input, it propagates througha system with impulse response $h(n)$. Therefore, forfixed-point arithmetic with rounding,the outputroundoff noise variance from (3.6), (3.12), (3.25), and (3.33) is

$$a_o^2 = \frac{A^2}{12} \sum_{n=-\infty}^{\infty} h^2(n) = \frac{A^2}{12} \sum_{n=0}^{\infty} a^{2n} = \frac{2^{-2B}}{12} \frac{1}{1-a^2} \quad (3.36)$$

With fixed-point arithmetic there is the possibility of overflow following addition. To avoid overflow it is necessary to restrict the input signal amplitude. This can be accomplished by either placing a *scaling* multiplier at the filter input or by simply limiting the maximum input signal amplitude. Consider the case of the first-order filter of (3.34). The transfer function of this filter is

$$H(e^{jm}) = \frac{Y(e^{jm})}{X(e^{jm})} = \frac{1}{e^{jm} - a} \quad (3.37)$$

so

$$|H(e^{jm})|^2 = \frac{1}{1 + a^2 - 2a \cos,)} \quad (3.38)$$

and

$$|H(e^{j\omega})|_{max} = \frac{1}{1 - |a|} \quad (3.39)$$

The peak gain of the filter is $1/(1 - |a|)$ so limiting input signal amplitudes to $|x(n)| < 1 - |a|$ will make overflows unlikely.

An expression for the output roundoff noise-to-signal ratio can easily be obtained for the case where the filter input is white noise, uniformly distributed over the interval from $-(1 - |a|)$ to $(1 - |a|)$ [4,5]. In this case

$$a_x^2 = \frac{1}{2(1-|a|)} \int_{-(1-|a|)}^{1-|a|} x^2 \, dx = \frac{1}{3}(1-|a|)^2 \quad (3.40)$$

so, from (3.25),

$$a_y^2 = \frac{1}{3} \frac{(1-|a|)^2}{1-a^2} \quad (3.41)$$

Combining (3.36) and (3.41) then gives

$$\frac{a_o^2}{a_y^2} = \left(\frac{2^{-2B}}{12} \frac{1}{1-a^2}\right)\left(\frac{3(1-a^2)}{(1-|a|)^2}\right) = \frac{2^{-2B}}{12} \frac{3}{(1-|a|)^2} \quad (3.42)$$

Notice that the noise-to-signal ratio increases without bound as $|a| \to 1$.

Similar results can be obtained for the case of the causal second-order filter realized by the difference equation

$$y(n) = 2r\cos(\theta)y(n-1) - r^2 y(n-2) + x(n) \quad (3.43)$$

This filter has complex-conjugate poles at $re^{\pm j\theta}$ and impulse response

$$h(n) = r^n \frac{\sin[(n+1)0]}{\sin(0)} u(n) \quad (3.44)$$

Due to roundoff error, the output actually obtained is

$$y(n) = 2r\cos(0)y(n-1) - r^2y(n-2) + x(n) + e(n) \qquad (3.45)$$

There are two noise sources contributing to $e(n)$ if quantization is performed after each multiply, and there is one noise source if quantization is performed after summation. Since

$$\sum_{n=-\infty} \quad \frac{1}{\cos^2(9)} \; \frac{2}{1 - r^2} \; \frac{1}{(1 + r^2)^2 - 4r^2} \tag{3.46}$$

the output roundoff noise is

$$a^2 = \frac{2^{-2B}}{12} \, V \, \frac{1 + r^2}{1 - r^2} \; \frac{1}{(1 + r^2)^2 - 4r^2 \cos^2(9)} \tag{3.47}$$

where $V = 1$ for quantization after summation, and $V = 2$ for quantization after each multiply. To obtain an output noise-to-signal ratio we note that

$$H(e^{j\omega}) = \frac{1}{1 - 2r\cos(9)e^{-j\omega} + r^2 e^{-j2\omega}} \tag{3.48}$$

and, using the approach of [6],

$$|H(e^{j\omega})|_{max} = \frac{1}{4r^2 \left[\mathrm{sat}\left(\cos(9)\right) - \cos(9) \right]^2 + \ \sin(9)^2} \tag{3.49}$$

where

$$\mathrm{sat}(I) = \begin{cases} 1 & I > 1 \\ I & -1 < I < 1 \end{cases} \tag{3.50}$$

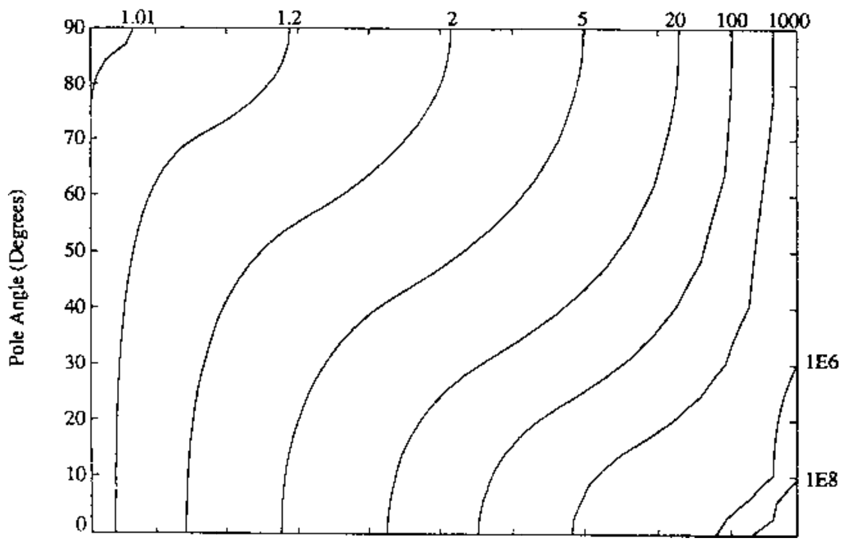
Following the same approach as for the first-order case then gives

$$y^2 \quad V \frac{2^{-2B}}{12} \, \frac{1 + r^2}{1 - r^2} \; \frac{3}{(1 + r^2)^2 - 4r^2 \cos^2(9)}$$

$$\times \frac{1}{4r^2 \left[\mathrm{sat}\left(\frac{1+r^2}{2r}\cos(9)\right) - \cos(9) \right]^2 + \left(\frac{1 - r^2}{2r}\sin(9)\right)^2} \tag{3.51}$$

Figure3.1 is a contour plot showing the noise-to-signal ratio of (3.51) for $v = 1$ in units of the noise variance of a single quantization, $2^{-2B}/12$. The plot is symmetrical about $9 = 90°$, so only the range from 0° to 90° is shown. Notice that as $r \wedge 1$, the roundoff noise increases without bound. Also notice that the noise increases as $9 \wedge 0°$.

It is possible to design state-space filter realizations that minimize fixed-point roundoff noise [7] - [10]. Depending on the transfer function being realized, these structures may provide a roundoff noise level that is orders-of-magnitude lower than for a nonoptimal realization. The price paid for this reduction in roundoff noise is an increase in the number of computations required to implement the filter. For an Nth-order filter the increase is from roughly $2N$ multiplies for a direct form realization to roughly $(N + 1)^2$ for an optimal realization. However, if the filter is realized by the parallel or cascade connection of first- and second-order optimal subfilters, the increase is only to about $4N$ multiplies. Furthermore, near-optimal realizations exist that increase the number of multiplies to only about 3N [10].



0.010.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 0.99

Normalized fixed-point roundoff noise variance.

5. Explain about Limit Cycle Oscillations.

Limit Cycle Oscillations:

A limit cycle, sometimes referred to as a multiplier roundoff limit cycle, is a low-level oscillation that can exist in an otherwise stable filter as a result of the nonlinearity associated with rounding (or truncating) internal filter calculations [11]. Limit cycles require recursion to exist and do not occur in nonrecursive FIR filters. As an example of a limit cycle, consider the second-order filter realized by

$$y(n) = Q_r\{\tfrac{7}{8}y(n-1) - \tfrac{5}{8}y(n-2) + x(n)\}$$

where Q_r {} represents quantization by rounding. This is stable filter with poles at $0.4375 \pm j0.6585$. Consider the implementation of this filter with 4-b (3-b and a sign bit) two's complement fixed-point arithmetic, zero initial conditions ($y(-1) = y(-2) = 0$), and an input sequence $x(n) = |S(n)$, where $S(n)$ is the unit impulse or unit sample. The following sequence is obtained;

Notice that while the input is zero except for the first sample, the output oscillates with amplitude $1/8$ and period 6.

Limit cycles are primarily of concern in fixed-point recursive filters. As long as floating-point filters are realized as the parallel or cascade connection of first- and second-order subfilters, limit cycles will generally not be a problem since limit cycles are practically not observable in first- and second-order systems implemented with 32-b floating-point arithmetic [12]. It has been shown that such systems must have an extremely small margin of stability for limit cycles to exist at anything other than underflow levels, which are at an amplitude of less than 10^{-38} [12]. There are at least three ways of dealing with limit cycles when fixed-point arithmetic is used. One is to determine a bound on the maximum limit cycle amplitude, expressed as an integral number of quantization steps [13]. It is then possible to choose a word length that makes the limit cycle amplitude acceptably low. Alternately, limit cycles can be prevented by randomly rounding calculations up or down [14]. However, this approach is complicated to implement. The third approach is to properly choose the filter realization structure and then quantize the filter calculations using magnitude truncation [15,16]. This approach has the disadvantage of producing more roundoff noise than truncation or rounding [see (3.12)—(3.14)].

6, Explain about Overflow Oscillations.

With fixed-point arithmetic it is possible for filter calculations to overflow. This happens when two numbers of the same sign add to give a value having magnitude greater than one. Since numbers with magnitude greater than one are not representable, the result overflows. For example, the two’s complement numbers 0.101 (5/8) and 0.100 (4/8) add to give 1.001 which is the two’s complement representation of -7/8.

The overflow characteristic of two’s complement arithmetic can be represented as $R\{\}$ where

$$R\{X\} = \begin{cases} X - 2 & X > 1 \\ X & -1 \le X < 1 \\ X + 2 & X < -1 \end{cases} \qquad (3.71)$$

For the example just considered, $R\{9/8\} = -7/8$.

An overflow oscillation, sometimes also referred to as an *adder overflow limit cycle,* is a high- level oscillation that can exist in an otherwise stable fixed-point filter due to the gross nonlinearity associated with the overflow of internal filter calculations [17]. Like limit cycles, overflow oscillations require recursion to exist and do not occur in nonrecursive FIR filters. Overflow oscillations also do not occur with floating-point arithmetic due to the virtual impossibility of overflow.

As an example of an overflow oscillation, once again consider the filter of (3.69) with 4-b fixed-point two’s complement arithmetic and with the two’s complement overflow characteristic of (3.71):

$$y(n) = Q_r\left[R\left\{\tfrac{7}{8}y(n-1) - \tfrac{5}{8}y(n-2) + x(n)\right\}\right] \qquad (3.72)$$

In this case we apply the input

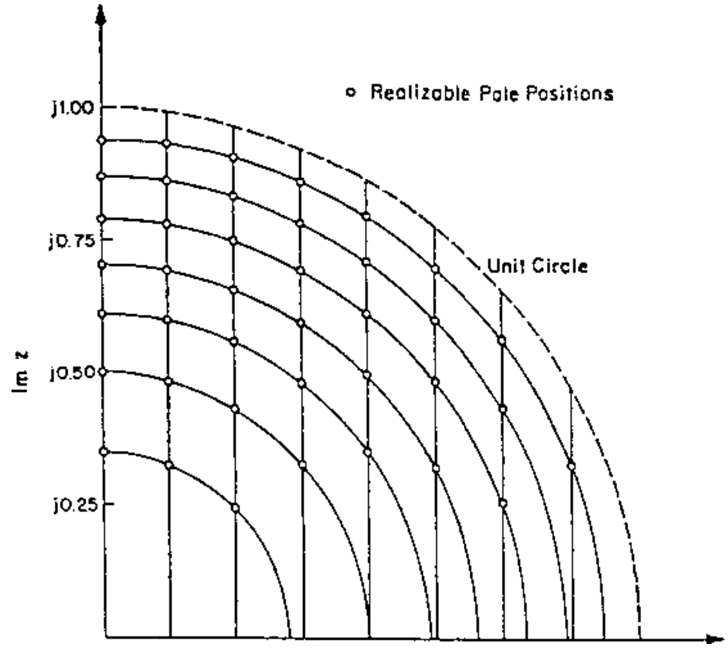
$$y(4) \quad x(n) = -4^{\&}(n) \overset{35}{-} \wedge^{\&}(n - 1) \quad (3.74)$$

$$= Qr \quad R \quad 0, \tfrac{3}{4} 0, \blacksquare \tfrac{5}{8} = Qr , , \quad (3.73)$$

s to scale the filter calculations so as to render overflow impossible. However, this may unacceptably restrict the filter dynamic range. Another method is to force completed sums-of- products to saturate at ±1, rather than overflowing [18,19]. It is important to saturate only the completed sum, since intermediate overflows in two's complement arithmetic do not affect the accuracy of the final result. Most fixed-point digital signal processors provide for automatic saturation of completed sums if their *saturation arithmetic* feature is enabled. Yet another way to avoid overflow oscillations is to use a filter structure for which any internal filter transient is guaranteed to decay to zero [20]. Such structures are desirable anyway, since they tend to have low roundoff noise and be insensitive to coefficient quantization [21].

7, Explain about Coefficient Quantization Error.

Coefficient Quantization Error:



0 0.25 0.50 0.75 1.00

Re Z

FIGURE: Realizable pole locations for the difference equation of (3.76).

The sparseness of realizable pole locations near $z = \pm 1$ will result in a large coefficient quantization error for poles in this region.

Figure3.4 gives an alternative structure to (3.77) for realizing the transfer function of (3.76). Notice that quantizing the coefficients of this structure corresponds to quantizing X_r and Xi. As shown in Fig.3.5 from [5], this results in a uniform grid of realizable pole locations. Therefore, large coefficient quantization errors are avoided for all pole locations.

It is well established that filter structures with low roundoff noise tend to be robust to coefficient quantization, and visa versa [22]- [24]. For this reason, the uniform grid structure of Fig.3.4 is also popular because of its low roundoff noise. Likewise, the low-noise realizations of [7]- [10] can be expected to be relatively insensitive to coefficient quantization, and digital wave filters and lattice filters that are derived from low-sensitivity analog structures tend to have not only low coefficient sensitivity, but also low roundoff noise [25,26].

It is well known that in a high-order polynomial with clustered roots, the root location is a very sensitive function of the polynomial coefficients. Therefore, filter poles and zeros can be much more accurately controlled if higher order filters are realized by breaking them up into the parallel or cascade connection of first- and second-order subfilters. One exception to this rule is the case of linear-phase FIR filters in which the symmetry of the polynomial coefficients and the spacing of the filter zeros around the unit circle usually permits an acceptable direct realization using the convolution summation.

Given a filter structure it is necessary to assign the ideal pole and zero locations to the realizable locations. This is generally done by simplyrounding or truncatingthe filter coefficients to the available number of bits, or by assigning the ideal pole and zero locations to the nearest realizable locations. A more complicated alternative is to consider the original filter design problem as a problem in discrete

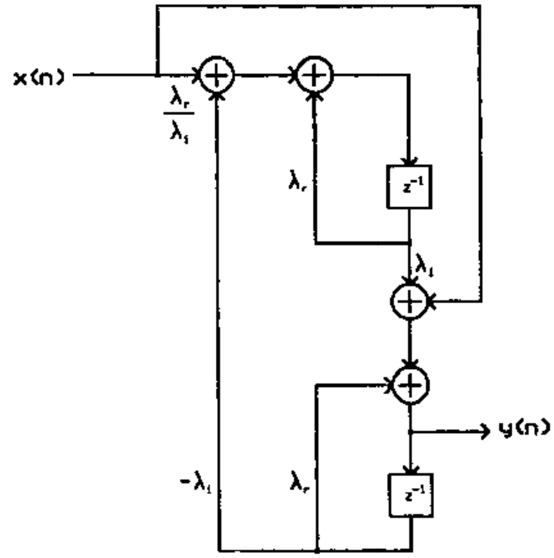


FIGURE 3.4: Alternate realization structure.

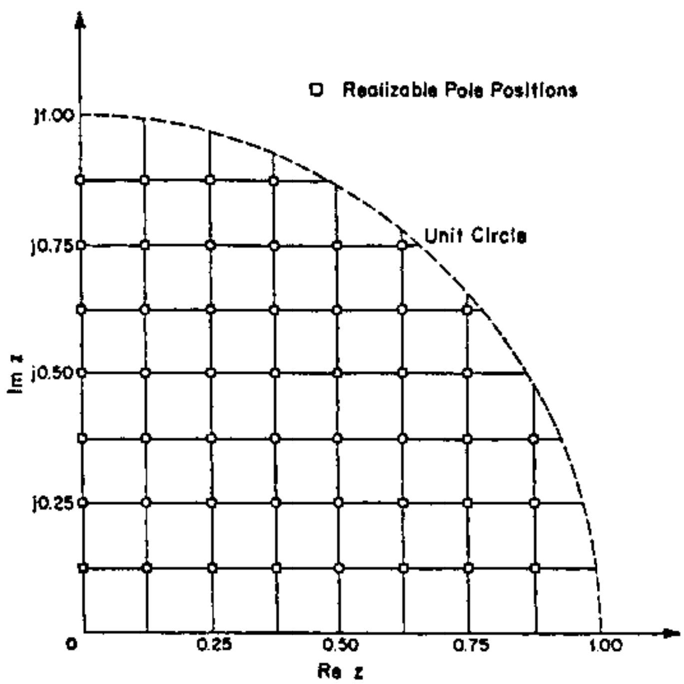


FIGURE 3.5: Realizable pole locations for the alternate realization structure.

optimization, and choose the realizable pole and zero locations that give the best approximation to the desired filter response [27]- [30].

1.6 Realization Considerations

Linear-phase FIR digital filters can generally be implemented with acceptable coefficient quantization sensitivity using the direct convolution sum method. When implemented in this way on a digital signal processor, fixed-point arithmetic is not only acceptable but may actually be preferable to floating-point arithmetic. Virtually all fixed-point digital signal processors accumulate a sum of products in a double-length accumulator. This means that only a single quantization is necessary to compute an output. Floating-point arithmetic, on the other hand, requires a quantization after every multiply and after every add in the convolution summation. With 32-b floating-point arithmetic these quantizations introduce a small enough error to be insignificant for many applications.

When realizing IIR filters, either a parallel or cascade connection of first- and second-order subfilters is almost always preferable to a high-order direct-form realization. With the availability of very low-cost floating-point digital signal processors, like the Texas Instruments TMS320C32, it is highly recommended that floating-point arithmetic be used for IIR filters. Floating-point arithmetic simultaneously eliminates most concerns regarding scaling, limit cycles, and overflow oscillations. Regardless of the arithmetic employed, a low roundoff noise structure should be used for the second- order sections. Good choices are given in [2] and [10]. Recall that realizations with low fixed-point roundoff noise also have low floating-point roundoff noise. The use of a low roundoff noise structure for the second-order sections also tends to give a realization with low coefficient quantization sensitivity. First-order sections are not as critical in determining the roundoff noise and coefficient sensitivity of a realization, and so can generally be implemented with a simple direct form structure.

UNIT V MULTIRATE SIGNAL PROCESSING

2 MARKS

1. What is the need for multirate signal processing?

In real time data communication we may require more than one sampling rate for processing data in such a cases we go for multi-rate signal processing which increase and/or decrease the sampling rate.

2. Give some examples of multirate digital systems.

Decimator and interpolator

3. Write the input output relationship for a decimator.

$$Fy = Fx/D$$

4. Write the input output relationship for an interpolator.

$$Fy = IFx$$

5. What is meant by aliasing?

The original shape of the signal is lost due to under sampling. This is called aliasing.

6. How can aliasing be avoided?

Placing a LPF before down sampling.

7. How can sampling rate be converted by a factor I/D.

Cascade connection of interpolator and decimator.

8. What is meant by sub-band coding?

It is an efficient coding technique by allocating lesser bits for high frequency signals and more bits for low frequency signals.

9. What is meant by up sampling?

Increasing the sampling rate.

10. What is meant by down sampling?

Decreasing the sampling rate.

11. What is meant by decimator?

Down sampling and a anti-aliasing filter.

12. What is meant by interpolator?

An anti-imaging filters and Up sampling.

13. What is meant by sampling rate conversion?

Changing one sampling rate to other sampling rate is called sampling rate conversion.

14. What are the sections of QMF.
Analysis section and synthesis section.

15. Define mean.
Mxn=E[xn]=intg xpxn(x,n) dx

16. Define variance.
Zxn2=E[{xn=mxn}2]

17. Define cross correlation of random process.
R xy (n.m)= intxy*pxn,ym(x,n,y,m)dxdy.

18. Define DTFT of cross correlation
Txy(e jw) = x rxy(l) e jwl

19. What is the cutoff frequency of Decimator?
Pi/M where M is the down sampling factor

20. What is the cutoff frequency of Interpolator?
Pi/L where L is the UP sampling factor.

21. What is the difference in efficient transversal structure?
Number of delayed multiplications are reduced.

22. What is the shape of the white noise spectrum?
Flat frequency spectrum.

16 MARKS

1, Explain about Decimation and Interpolation.

Definition.
Given an integer D, we define the *downsampling* operator S_{iJD}, shown in Figure by the following relationship:

>{»] - " *[«£>]

The operator S_{IW} decreases the sampling frequency by a factor of D, by keeping one sample out of D samples.

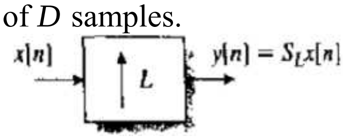


Figure Upsampling

jctn]

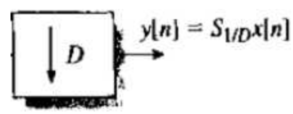


Figure Downsampling

Example

Let x = [..., 1,2,3,4,5,6,...], then j[n] = S(/2x[n] is given by

?[«] = [— 1.3,5f7—]

bamplmg Kate Conversion by a Rational t* actor

the problem of designing an algorithm for resampling a digital signal x[n] from the original rate F_x (in Hz) into a rate $F_v = (UD)F_{zf}$ with L and D integers. For example, we have a signal at telephone quality, F_x = 8 kHz, and we want to resample it at radio quality, F_y = 22 kHz. In this case, clearly L = 11 and D = 4.

First consider two particular cases, interpolation and decimation, where we upsample and downsample by an integer factor without creating aliasing or image frequencies.

Decimation by an Integer Factor D

We have seen in the previous section that simple downsampling decreases the sampling frequency by a factor D. In this operation, all frequencies of X(tu) = DTFT{jr[n]} above 7tID cause aliasing, and therefore they have to be filtered out ***before*** downsampling the signal. Thus we have the scheme in Figure where the signal is downsampled by a factor D, without aliasing.

Interpolation by an Integer Factor L

As in the case of decimation, upsampling by a factor L alone introduces artifacts in the frequency domain as ***image frequencies.*** Fortunately, as seen in the previous section, all image frequencies are outside the interval ***[-ttIL,*** +t***tIL],*** and thev can be eliminated by filtering the signal ***after*** downsampling. This is shown in Figure **i**

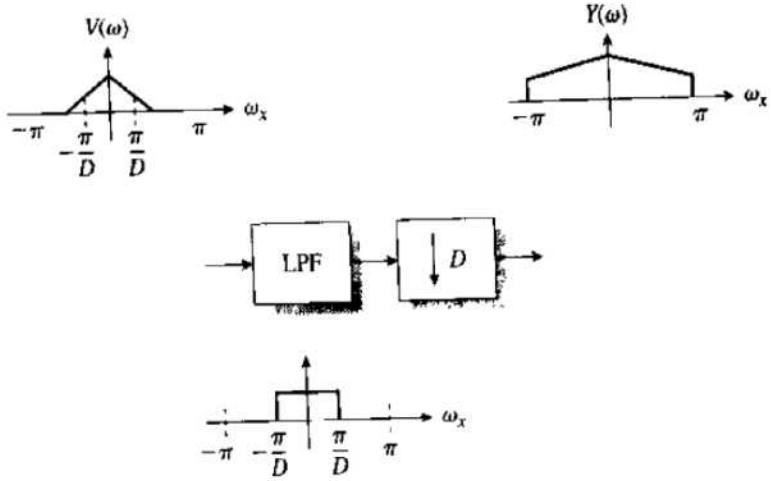


Figure Decimation

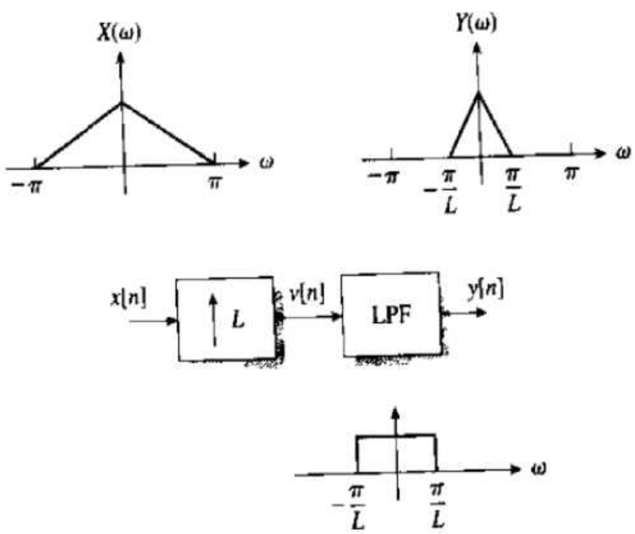


Figure Interpolation

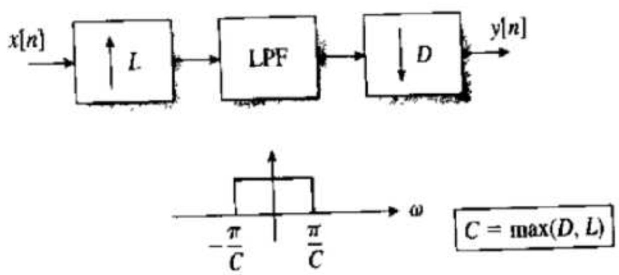


Figure Rational frequency change

frequency spectrum in Figure . there is some loss of information since we decrease the sampling frequency and all frequencies above 4 kHz have to be eliminated.

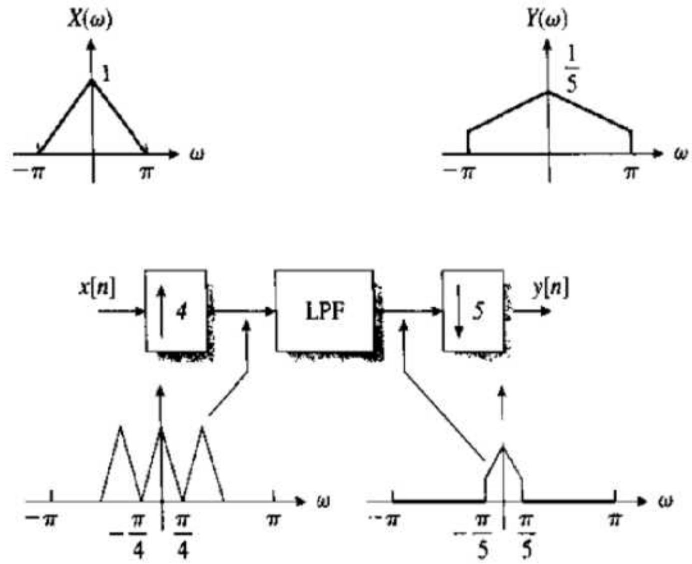


Figure Example of resampling by a lower sampling frequency: loss of information

2, Explain about Multistage Implementation of Digital Filters.

I Multistage Implementation of Digital filters

In some applications we want to design filters where the bandwidth is just a small fraction of the overall sampling rate. For example, suppose we want to design a lowpass filter with bandwidth of the order of a few hertz and a sampling frequency of the order of several kilohertz. This filter would require a very sharp transition region in the digital frequency *a* >, thus requiring a high-complcxity filter.

Example <

As an example of application, suppose you want to design a Filter with the fallowing specifications:

Passband F_p = 450 Hz

Stopband $F_S = 500$ Hz

Sampling frequency F_S~96 kHz

Notice that the stopband is several orders of magnitude smaller than the sampling frequency. This leads to a filter with a very short transition region of high complexity. In

Speech signals

© From prehistory to the new media of the future, speech has been and will be a primary form of communication between humans.

© Nevertheless, there often occur conditions under which we measure and then transform the speech to another form, speech signal, in order to enhance our ability to communicate.

© The speech signal is extended, through technological media such as telephony, movies, radio, television, and now Internet. This trend reflects the primacy of speech communication in human psychology.

© "Speech will become the next major trend in the personal computer market in the near future."

Speech signal processing

© The topic of speech signal processing can be loosely defined as the manipulation of sampled speech signals by a digital processor to obtain a new signal with some desired properties.

Speech signal processing is a diverse field that relies on knowledge of language at the levels of Signal processing

Acoustics ***(P¥)***

Phonetics

(^^^) Language-independent Phonology

(^^)

Morphology **(i^^^)**

Syntax

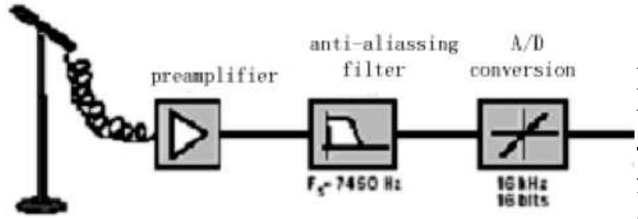
(^,£) Languag e-dependent

Semantics

(\%X¥)

Pragmatics **(if,ffl^)**

7 layers for describing speech From Speech to Speech Signal, in terms of Digital Signal Processing



At-*

■ Acoustic (and perceptual) features *{traits)*

- fundamental freouency (FO) (pitch)

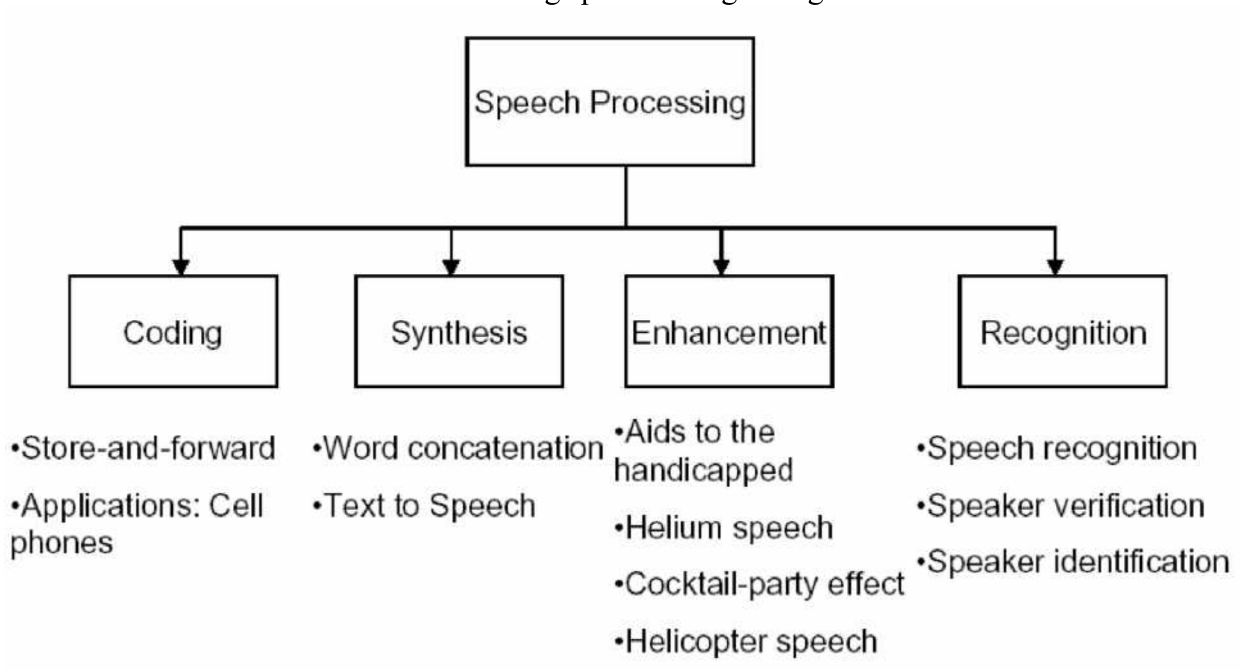
- amplitude (loudness)

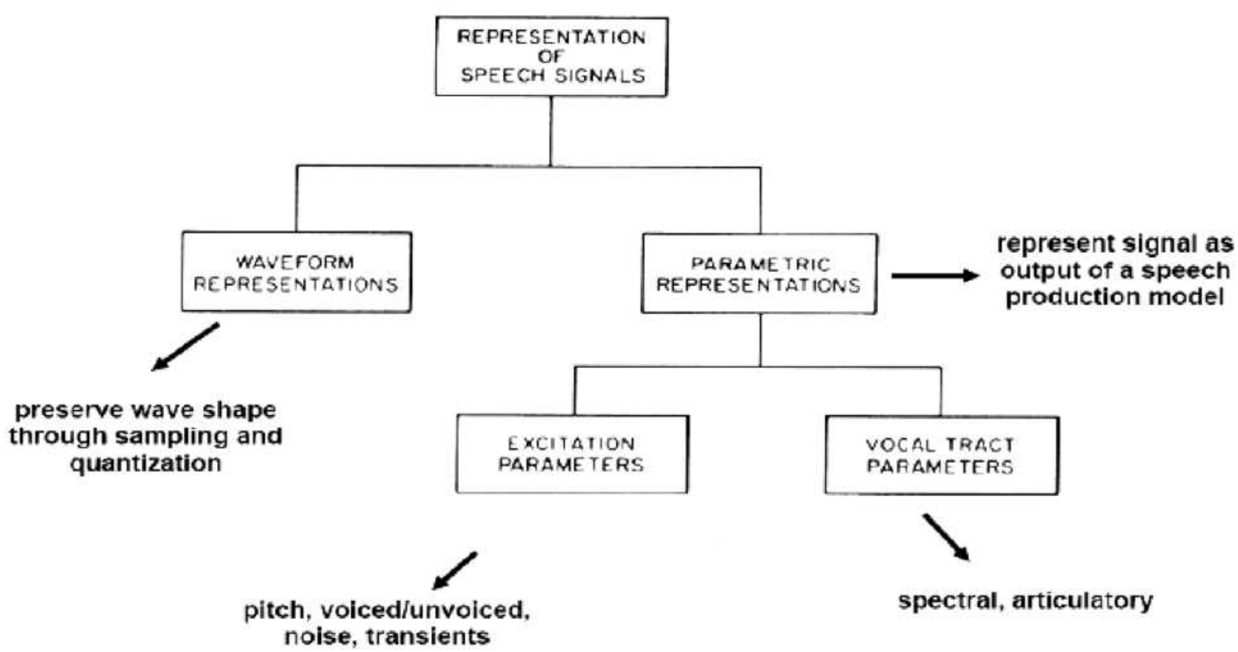
- spectrum (timber)

It is based on the fact that

- Most of energy between 20 Hz to about 7KHz ,
- Human ear sensitive to energy between 50 Hz and 4KHz

© In terms of acoustic or perceptual, above features are considered.
© From Speech to Speech Signal, in terms of Phonetics (Speech production), the digital model of Speech Signal will be discussed in Chapter 2.
© Motivation of converting speech to digital signals:



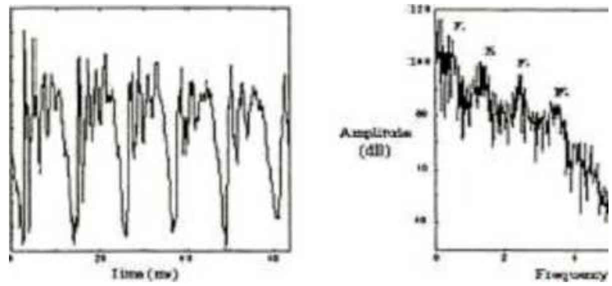


Speech coding, A PC and SBC

pitch,
fine structure

Adaptive predictive coding (APC) is a technique used for speech coding, that is data compression of spccch

Typical Voiced speech



signals APC assumes that the input speech signal is repetitive with a period significantly longer than the average frequency content. Two predictors arc used in APC. The high frequency components (up to 4 kHz) are estimated using a 'spectral' or 'formant' prcdictor and the low frequency components (50-200 Hz) by a 'pitch' or 'fine structure' prcdictor (see figure 7.4). The spcctral estimator may he of order 1- 4 and the pitch estimator about order 10. The low-frequency components of the spccch signal are due to the movement of the tongue, chin and

spectral
envelope, formants

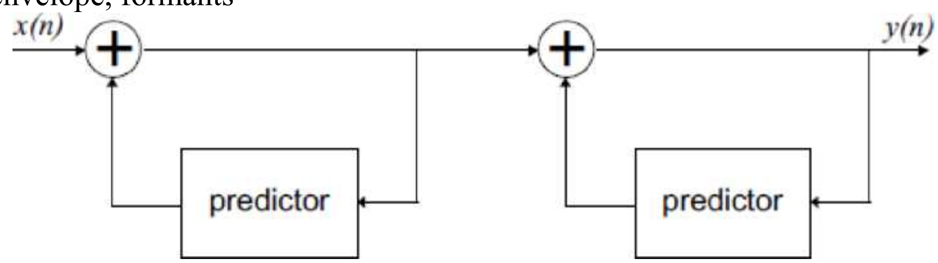


Figure 7.4 Encoder for adaptive, predictive coding of speech signals. The decoder is mainly a mirrored version of the encoder

The high-frequency components originate from the vocal chords and the noise-like sounds (like in 's') produced in the front of the mouth.

The output signal $y(n)$together with the predictor parameters, obtained adaptively in the encoder, are transmitted to the decoder, where the spcech signal is reconstructed. The decoder has the same structure as the encoder but the predictors arc not adaptive and arc invoked in the reverse order. The prediction parameters are adapted for blocks of data corresponding to for instance 20 ms time periods.

A PC' is used for coding spcech at 9.6 and 16 kbits/s. The algorithm works well in noisy environments, but unfortunately the quality of the processed speech is not as good as for other methods like CELP described below.

3, Explain about Subband Coding.

Another coding method is *sub-band coding (SBC)* (see Figure 7.5) which belongs to the class of *waveform coding* methods, in which the frequency domain properties of the input signal arc utilized to achieve data compression.

The basic idea is that the input speech signal is split into *sub-bands* using band-pass filters. The sub-band signals arc then encoded using ADPCM or other techniques. In this way, the available data transmission capacity can be allocated between bands according to pcrccptual criteria, enhancing the speech quality as pcrceivcd by listeners. A sub-band that is more 'important' from the human listening point of view can be allocated more bits in the data stream, while less important sub-bands will use fewer bits.

A typical setup for a sub-band codcr would be a bank of N(digital) bandpass filters followed by dccimators, encoders (for instance ADPCM) and a multiplexer combining the data bits coming from the sub-band channels. The output of the multiplexer is then transmitted to the sub-band dccodcr having a demultiplexer splitting the multiplexed data stream back into Nsub-band channels. Every sub-band channel has a dccodcr (for instance ADPCM), followed by an interpolator and a band-pass filter. Finally, the outputs of the band-pass filters are summed and a reconstructed output signal results.

Sub-band coding is commonly used at bit rates between 9.6 kbits/s and 32 kbits/s and performs quite well. The complexity of the system may however be considerable if the number of sub-bands is large. The design of the band-pass filters is also a critical topic when working with sub-band coding systems.

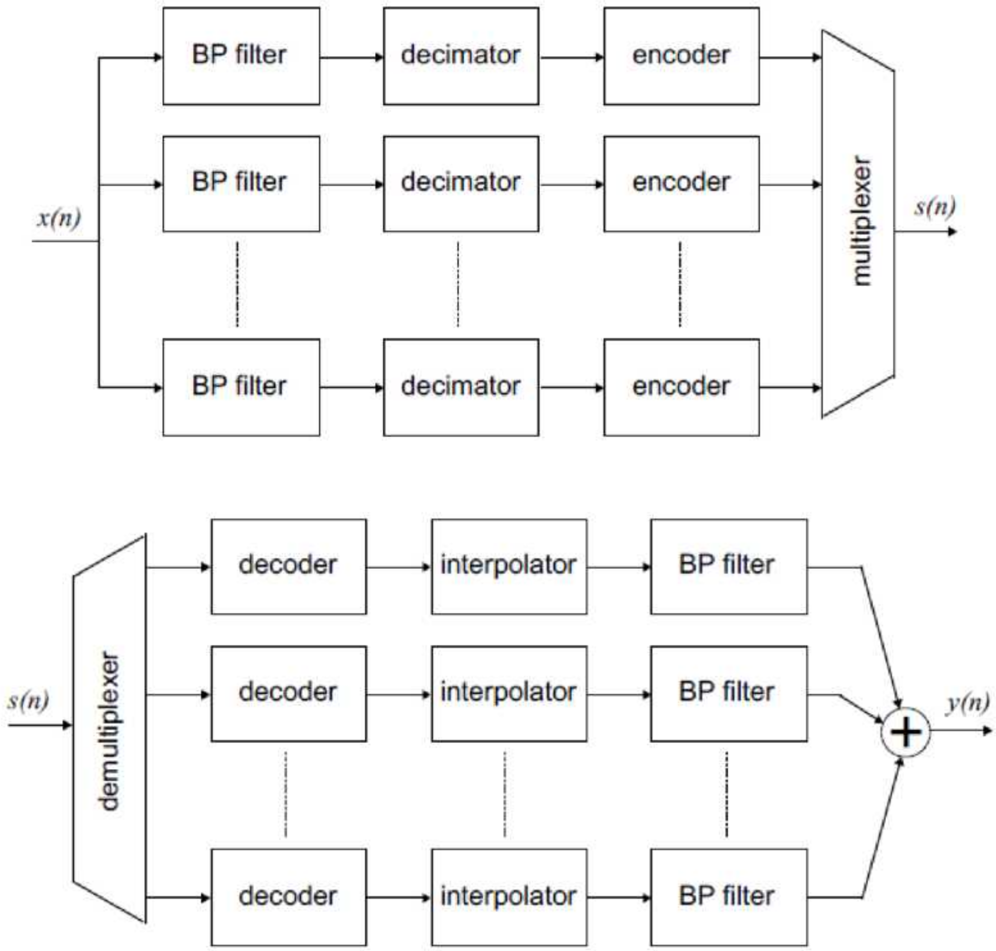


Figure 7.5 An example of a sub-band coding system

~~Vocoders and LPC~~

In the methods described above (APC, SBC and ADPCM), speech signal applications have been used as examples. By modifying the structure and parameters of the predictors and filters, the algorithms may also be used for other signal types. The main objective was to achieve a reproduction that was as faithful as possible to the original signal. Data compression was possible by removing redundancy in the time or frequency domain.

~~The~~ class of ~~vocoders~~ (also called source coders) is a special class of data compression devices aimed only at spcech signals. ~~The~~ input signal is analysed and described in terms of speech model parameters. These parameters are then used to synthesize a voice pattern having an acceptable level of perceptual quality. Hence, waveform accuracy is not the main goal, as in the previous methods discussed.

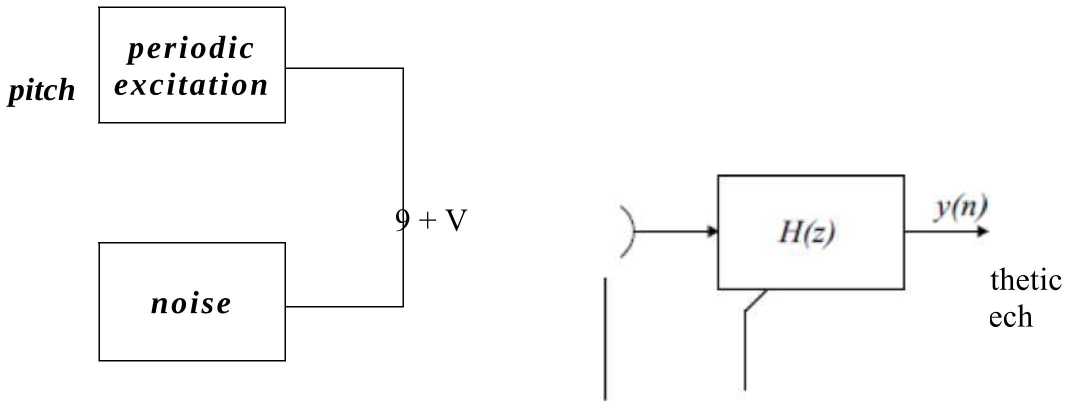


Figure 7.6 The LPC model

The first vocoder was designed by H. Dudley in the 1930s and demonstrated at the ‘New York Fair’ in 1939* Vocoders have become popular as they achieve reasonably good speech quality at low data rates, from *2A* kbits/s to 9,6 kbits/s. There arc many types of vocoders (Marvcn and Ewers, 1993), some of the most common techniques will be briefly presented below.

Most vocoders rely on a few basic principles. Firstly, the characteristics of the spccch signal is assumed to be fairly constant over a time of approximately 20 ms, hcncc most signal processing is performed on (overlapping) data blocks of 20 40 ms length. Secondly, the spccch model consists of a time varying filter corresponding to the acoustic properties of the mouth and an excitation signal. The cxeilalion signal is cither a periodic waveform, as crcatcd by the vocal chords, or a random noise signal for production of ‘unvoiced' sounds, for example ‘s’ and T. The filter parameters and excitation parameters arc assumed to be independent of each other and are commonly coded separately.

Linear predictive coding (LPC) is **a** popular method, which has however been replaced by newer approaches in many applications. LPC works exceedingly well **at** low bit rates and the **LPC** parameters contain sufficient information of the spccch signal to be used in spccch recognition applications. The **LPC** model is shown in Figure 7*6.

LPC is basically an **autn-regressive** model (sec Chapter 5) and the vocal tract is modelled as a time-varying all-pole filter (HR filter) having the transfer function $H(z)$

(7*17)

$$-k \qquad k=1$$

where p is the order of the filter. The excitation signal *?(«), being either noise or a periodic waveform, is fed to the filter via a variable gain factor G. The output signal can be expressed in the time domain as

$$y(n) \sim Ge(ri) - a_1 y(n-1) - a_2 y(n-2) \qquad \% y\{n-p) \qquad (1.\ \mathrm{IK})$$

The output sample at time n is a linear combination of p previous samples and the excitation signal (linear predictive coding). The filter coefficients $a_k$arc time varying.

The model above describes how to *synthesize* the speech given the pitch information (if noise or pet iodic excitation should be used), the gain and the filter parameters. These parameters must be determined by the cncoder or the analyser, taking the original spccch signal $x(n)$ as input.

The analyser windows the spccch signal in blocks of 20-40 ms. usually with a Hamming window (see Chapter 5). These blocks or 'frames' arc repeated every 10—30 ms, hence there is a certain overlap in time. Every frame is then analysed with respect to the parameters mentioned above.

Firstly, the pitch frequency is determined. This also tells whether we arc dealing with a voiced or unvoiccd spccch signal. This is a crucial part of the system and many pitch detection algorithms have been proposed. If the segment of the spccch signal is voiced and has a dear periodicity or if it is unvoiced and not pet iodic, things arc quite easy* Segments having properties in between these two extremes are difficult to analyse. No algorithm has been found so far that is [1]perfect* for all listeners.

Now, the second step of the analyser is to determine the gain and the filter parameters. This is done by estimating the spccch signal using an adaptive predictor. The predictor has the same structure and order as the filter in the synthesizer, Hencc, the output of the predictor is

-i(n) — -*tf]jt(/7— 1)* — a2x(n—*2)* —...— *OpX(n—p)* *(7-19)*

where i(rt) is the predicted input spcech signal and jc(rt) is the actual input signal. The filter coefficients $a_k$are determined by minimizing the square error

$$\sum_n (x(n) - \hat{x}(n))^2 = \sum_n r^2(n)$$

(7,20)

This can be done in different ways, cither by calculating the auto-corrc- lation coefficients and solving the Yule Walker equations (see Chapter 5) or by using some recursive, adaptive filter approach (see Chapter 3),

So, for every frame, all the parameters above arc determined and irans- mittcd to the synthesiser, where a synthetic copy of the spccch is generated.

An improved version of LPC is **residual excited linear prediction (RELP).** Let us take a closer look at **the** error or residual signal r(fi) resulting from the prediction in the analyser (equation (7.19)). The residual signal (wc arc try ing to minimize) can be expressed as

r(n)= *(«) -i(rt)

= jf(rt) + *a^x(n*— 1) + *a₂x(n*— 2) -h *.. + a_p*x(ft—p)* <7-21)

From this it is straightforward to find out that the corresponding expression using the z-transforms is

$$R(z) = \frac{X(z)}{H(z)} = X(z)H^{-1}(z)$$

(7,22)

Hcncc, the prcdictor can be regarded as an 'inverse' filter to the LPC model filter. If we now pass this residual signal to the synthesizer and use it to excite the LPC filter, that is $E(z)$ - $R(z)$, instead of using the noise or periodic waveform sources we get

$$Y(z) = E(z)H(z) = R(z)H(z) = X(z)H\sim\backslash z)H(z) = X(z) \quad (7.23)$$

In the ideal case, we would hence get the original speech signal back. When minimizing the variance of the residual signal (equation (7.20)), we gathered as much information about the spccch signal as possible using this model in the filter coefficients a_k. The residual signal contains the remaining information. If the model is well suited for the signal type (speech signal), the residual signal is close to white noise, having a flat spectrum. In such a case we can get away with coding only a small range of frequencies, for instance 0-1 kHz of the residual signal. At the synthesizer, this baseband is then repeated to generate higher frequencies. This signal is used to excite the LPC filter

Vocoders using RELP are used with transmission rates of 9.6 kbits/s. The advantage of RELP is a better speech quality compared to LPC for the same bit rate. However, the implementation is more computationally demanding.

Another possible extension of the original LPC approach is to use **multipulse excited linear predictive coding (MLPC).** This extension is an attempt to make the synthesized spcech less ‘mechanical’, by using a number of different pitches of the excitation pulses rather than only the two (periodic and noise) used by standard LPC.

The MLPC algorithm sequentially detects k pitches in a speech signal. As soon as one pitch is found it is subtracted from the signal and detection starts over again, looking for the next pitch. Pitch information detection is a hard task and the complexity of the required algorithms is often considerable. MLPC however offers a better spcech quality than LPC for a given bit rate and is used in systems working with 4.S-9.6 kbits/s.

Yet another extension of LPC is the **code excited linear prediction (CELP).** The main feature of the CELP compared to LPC is the way in which the filter coefficients are handled. Assume that we have a standard LPC system, with a filter of the order p. If every coefficient a_k requires N bits, we need to transmit N-p bits per frame for the filter parameters only. This approach is all right if all combinations of filter coefficients arc equally probable. This is however not the case. Some combinations of coefficients are very probable, while others may never occur. In CELP, the coefficient combinations arc represented by p dimensional vectors. Using vector quantization techniques, the most probable vectors are determined. Each of these vectors are assigned an **index** and stored in a **codebook.** Both the analyser and synthesizer of course have identical copies of the codebook, typically containing 256-512 vectors. Hcncc, instead of transmitting N-p bits per frame for the filter parameters only 8-9 bits arc needed.

This method offers high-quality spcech at low-bit rates but requires considerable computing power to be able to store and match the incoming spcech to the ‘standard’ sounds stored in the codebook. This is of course especially true if the codebook is large. Speech quality degrades as the codebook size decreases.

Most CELP systems do not perform well with respect to higher frequency components of the spccch signal at low hit rates. This is countcractcd in

There is also a variant of CELP called **vector sum excited linear prediction (VSELP).** The main difference between CELP and $VSELP$ is the way the codebook is organized. Further, since VSELP uses fixed point arithmetic algorithms, **it** is possible to implement using cheaper DSP chips than

Adaptive Filters

The signal degradation in some physical systems is time varying, unknown, or possibly both. For example,consider a high-speed modem for transmitting and receiving data over telephone channels. It employs a filter called a channel equalizer to compensate for the channel distortion. Since the dial-up communication channels have different and time-varying characteristics on each connection, the equalizer must be an adaptive filter.

4. Explain about Adaptive Filter4. Explain about Adaptive Filter

Adaptive filters modify their characteristics to achieve certain objectives by automatically updating their coefficients. Many adaptive filter structures and adaptation algorithms have been developed for different applications. This chapter presents the most widely used adaptive filters based on the FIR filter with the least-mean-square (LMS) algorithm. These adaptive filters are relatively simple to design and implement. They are well understood with regard to stability, convergence speed, steady-state performance, and finite-precision effects.

Introduction to Adaptive Filtering

An adaptive filter consists of two distinct parts - a digital filter to perform the desired filtering, and an adaptive algorithm to adjust the coefficients (or weights) of the filter. A general form of adaptive filter is illustrated in Figure 7.1, where d(n) is a desired (or primary input) signal, y(n) is the output of a digital filter driven by a reference input signal $x(n)$, and an error signal e(n) is the difference between d(n) and y(n). The adaptive algorithm adjusts the filter coefficients to minimize the mean-square value of e(n). Therefore, the filter weights are updated so that the error is progressively minimized on a sample-bysample basis.

In general, there are two types of digital filters that can be used for adaptive filtering: FIR and IIR filters. The FIR filter is always stable and can provide a linear-phase response. On the other hand, the IIR

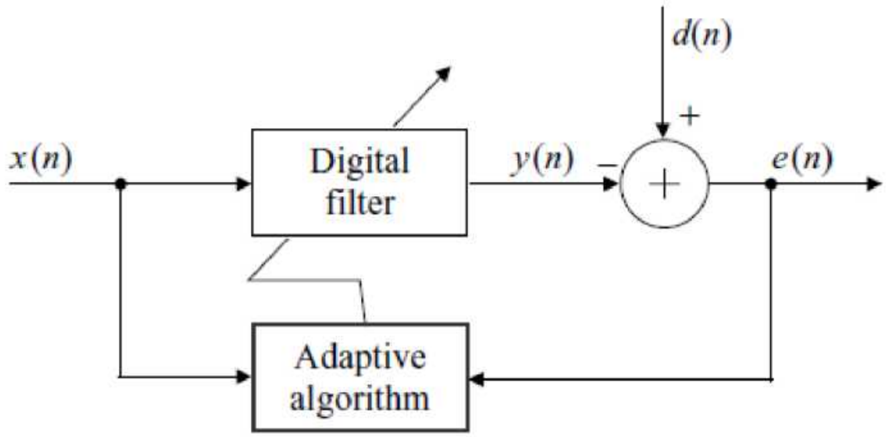


Figure 7.1 Block diagram of adaptive filter

filter involves both zeros and poles. Unless they are properly controlled, the poles in the filter may move outside the unit circle and result in an unstable system during the adaptation of coefficients. Thus, the adaptive FIR filter is widely used for practical real-time applications. This chapter focuses on the class of adaptive FIR filters.

The most widely used adaptive FIR filter is depicted in Figure 7.2. The filter output signal is computed

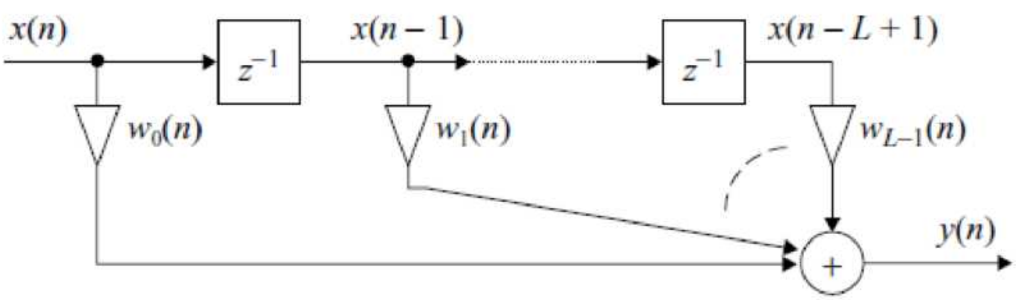


Figure 7.2 Block diagram of FIR filter for adaptive filtering

$$y(n) = \sum_{l=0}^{L-1} w_l(n)x(n-l),$$

(7.13)

, where the filter coefficients wl (n) are time varying and updated by the adaptive algorithms that will be discussed next.
We define the input vector at time n as
x(n) = [x(n)x(n - 1) . . . $x(n - L$ + 1)]T , (7.14)
and the weight vector at time n as
w(n) = [w0(n)w1(n) . . . wL-1(n)]T . (7.15)
Equation (7.13) can be expressed in vector form as
$y(n) = wT$ (n)x(n) = xT (n)w(n). (7.16)
The filter outputy(n) is compared with the desired $d(n)$ to obtain the error
signal $e(n) = d(n) - y(n) = d(n) - wT (n)x(n).$ (7.17)
Our objective is to determine the weight vector w(n) to minimize the predetermined performance (or cost) function.

Performance Function:

The adaptive filter shown in Figure 7.1 updates the coefficients of the digital filter to
optimize some predetermined performance criterion. The most commonly used performance
function is
based on the mean-square error (MSE) defined as

5, Explain about Audio Processing.

The two principal human senses are vision and hearing. Correspondingly,much of DSP is related to image and audio processing. People listen toboth *music* and *speech*. DSP has made revolutionary changes in both these areas.

Music Sound processing

The path leading from the musician's microphone to the audiophile's speaker is remarkably long. Digital data representation is important to prevent the degradation commonly associated with analog storage and manipulation. This is very familiar to anyone who has compared the musical quality of cassette tapes with compact disks. In a typical scenario, a musical piece is recorded in a sound studio on multiple channels or tracks. In some cases, this even involves recording individual instruments and singers separately. This is done to give the sound engineer greater flexibility in creating the final product. The complex process of combining the individual tracks into a final product is called *mix down*. DSP can provide several important functions during mix down, including: filtering, signal addition and subtraction, signal editing, etc. One of the most interesting DSP applications in music preparation is *artificial reverberation*. If the individual channels are simply added together, the resulting piece sounds frail and diluted, much as if the musicians were playing outdoors. This is because listeners are greatly influenced by the echo or reverberation content of the music, which is usually minimized in the sound studio. DSP allows artificial echoes and reverberation to be added during mix down to simulate various ideal listening environments. Echoes with delays of a few hundred milliseconds give the impression of cathedral likelocations. Adding echoes with delays of 10-20 milliseconds provide the perception of more modest size listening rooms.

Speech generation

Speech generation and recognition are used to communicate between humans and machines. Rather than using your hands and eyes, you use your mouth and ears. This is very convenient when your hands and eyes should be doing something else, such as: driving a car, performing surgery, or (unfortunately) firing your weapons at the enemy. Two approaches are used for computer generated speech: *digital recording* and *vocal tract simulation.* In digital recording, the voice of a human speaker is digitized and stored, usually in a compressed form. During playback, the stored data are uncompressed and converted back into an analog signal. An entire hour of recorded speech requires only about three me gabytes of storage, well within the capabilities of even small computer systems. This is the most common method of digital speech generation used today. Vocal tract simulators are more complicated, trying to mimic the physical mechanisms by which humans create speech. The human vocal tract is an acoustic cavity with resonate frequencies determined by the size and shape of the chambers. Sound originates in the vocal tract in one of two basic ways, called *voiced* and *fricative* sounds. With voiced sounds, vocal cord vibration produces near periodic pulses of air into the vocal cavities. In comparison, fricative sounds originate from the noisy air turbulence at narrow constrictions, such as the teeth and lips. Vocal tract simulators operate by generating digital signals that resemble these two types of excitation. The characteristics of the resonate chamber are simulated by passing the excitation signal through a digital filter with similar resonances. This approach was used in one of the very early DSP success stories, the *Speak & Spell,* a widely sold electronic learning aid for children.

Speech recognition

The automated recognition of human speech is immensely more difficult than speech generation. Speech recognition is a classic example of things that the human brain does well, but digital computers do poorly. Digital computers can store and recall vast amounts of data, perform mathematical calculations at blazing speeds, and do repetitive tasks without becoming bored or inefficient. Unfortunately, present day computers perform very poorly when faced with raw sensory data. Teaching a computer to send you a monthly electric bill is easy. Teaching the same computer to understand your voice is a major undertaking. Digital Signal Processing generally approaches the problem of voice recognition in two steps: *feature extraction* followed by *feature matching.* Each word in the incoming audio signal is isolated and then analyzed to identify the type of excitation and resonate frequencies. These parameters are then compared with previous examples of spoken words to identify the closest match. Often, these systems are limited to only a few hundred words; can only accept speech with distinct pauses between words; and must be retrained for each individual speaker. While this is adequate for many commercialapplications, these limitations are humbling when compared to the abilities of human hearing. There is a great deal of work to be done in this area, with tremendous financial rewards for those that produce successful commercial products.

DEPARTMENT OF INFORMATION TECHNOLOGY

QUESTION BANK

Subject Name: Digital Signal Processing **Year/ Sem: II/IV**

UNIT – 1 SIGNAL AND SYSTEMS

PART-A (2 MARKS)

1. What is a continuous and discrete time signal?
2. Give the classification of signals?
3. What are the types of systems?
4. What are even and odd signals?
5. What are deterministic and random signals?
6. What are energy and power signal?
7. What are the operations performed on a signal?
8. What are elementary signals and name them?
9. What are the properties of a system?
10. What is memory system and memory less system?
11. What is an invertible system?
12. What are time invariant systems?
13. Is a discrete time signal described by the input output relation?

$y[n]= r^n x[n]$ time invariant.

14. Show that the discrete time system described by the input-output relationship

$y[n] =nx[n]$ is linear?

15. What is SISO system and MIMO system?
16. What is the output of the system with system function H1 and H2 when connected in cascade and parallel?
17. What do you mean by periodic and non-periodic signals?
18. Determine the convolution sum of two sequences x(n) = {3, 2, 1, 2} and h (n) = {1, 2, 1, 2}
19. Find the convolution of the signals?

20. Detemine the solution of the difference equation.

21. Determine the response y (n), n>=0 of the system described by the second order difference equation.

PART B(16 MARKS)

1. i) Find the inverse z –transform of X(z),

$$X(z) = \frac{1}{1-az^{-1}} \quad \text{where a constant.} \qquad (8)$$

ii) Find the z- transform of autocorrelation function. (8)

2. i) Explain the method of approximation of derivatives for digitizing the analog filter into a digital filter. (8)

ii) Determine H(z) using impulse invariance method for the given transfer function,

$$H(S) = \frac{3}{(S+1)(S+3)}$$

Assume T=1 Sec (8)

3. i) Find the convolution of X(n) and h(n)

$$X(n) = (1/2)^n \ u(n)$$
$$h(n) = (1/3)^n \ u(n)$$ (8)

ii) Find the z-transform of x (n), $x(n) = (1/2)^{n-5} \ u(n-2.) + 8(n-5)$. (8)

4. i) Find the inverse Z- transform of X(z),

$$X(z) = \frac{1}{1+az^{-1}}, \text{ Where a constant.} \qquad (8)$$

(ii) Find the z-transform of autocorrelation function. (8)

5. State and prove important properties of the z-transforms.

UNIT II- FAST FOURIER TRANSFORMS

PART-A (2 MARKS)

1. Differentiate DTFT and DFT.

2. Differentiate between DIT and DIF algorithm.

3. How many stages are there for 8 point DFT?

4 How many multiplication terms are required for doing DFT by expressional method and FFT method?

5. Distinguish IIR and FIR filters.

6. Distinguish analog and digital filters.

7. Write the expression for order of Butterworth filter?

8. Write the expression for the order of chebyshev filter?

9. Write the various frequency transformations in analog domain?

10. Write the steps in designing chebyshev filter?

11. Write down the steps for designing a Butterworth filter?

12. State the equation for finding the poles in chebyshev filter.

PART B(16 MARKS)

1. Determine the DFT of the sequence
   x (n) =1/4, for 0<=n <=2
   0, otherwise
2. Derive the DFT of the sample data sequence x (n) = {1, 1, 2, 2, 3, 3} and compute the corresponding amplitude and phase spectrum.
3. Given x(n) = {0,1,2,3,4,5,6,7} find X(k) using DIT FFT algorithm.
4. Given X (k) = {28,-4+j9.656,-4+j4,-4+j1.656,-4,-4-j1.656,-4-j4,-4-j9.656}, find x (n) using inverse DIT FFT algorithm.
5. Find the inverse DFT of X (k) = {1,2,3,4}

UNIT III- IIR FILTER DESIGN
PART-A (2 MARKS)

1. State the steps to design digital IIR filter using bilinear method.

2. What is warping effect?

3. Write a note on pre warping.

4. Give the bilinear transform equation between s plane and z plane.

5. Why impulse invariant method is not preferred in the design of IIR filters other than low pass filter?

6. What is meant by impulse invariant method?

7. What do you understand by backward difference?

8. What are the properties of chebyshev filter?

9. Give the magnitude function of Butterworth filter?

10. Give the equation for the order N, major, minor axis of an ellipse in case of chebyshev filter?

11. Give the expression for poles and zeroes of a chebyshev type 2 filters.

12. How can you design a digital filter from analog filter?

13. Write down bilinear transformation.

14. Differentiate Butterworth and Chebyshev filter.

15. What is filter?

16. What are the types of digital filter according to their impulse response?

17. How phase distortion and delay distortion are introduced?

PART B(16 MARKS)

1) i) Describe impulse invariant mapping technique for designing IIR filter. (8)
   ii) Develop cascade and parallel realization of the system described by the difference equation
   $y(n) + (3/8)\, y(n-1) - (3/32)\, y(n-2) - (1/64)\, y(n-3) = x(n) + 3x(n-1) + 2x(n-2)$ (8)

2) Design a Butter worth digital filter to meet the following constraints.

$$0.9 \leq |H(\omega)| \leq 1,\ 0\leq \omega\leq \Pi/2$$

$$|H(\omega)| \leq 0.2,\ 3\Pi/4\leq \omega\leq \Pi$$

Use bilinear transformation mapping technique. Assume T=1 Sec

3) Consider the system described by
   $y(n)-(3/4)y(n-1)+(1/8)y(n-2)=x(n)+(1/3)x(n-1)$
   Determine and draw all possible realization structures

4) Explain the following terms briefly.
   i.Frequency sampling structure. (4)
   ii.Lattice structure for IIR filter (4)
   iii.Perturbation error (4)
   iv.Limit cycles. (4)

UNIT IV FIR FILTER DESIGN
PART-A (2 MARKS)

1. What is mean by FIR filter?

2. What is mean by FIR filter?

3. Write the steps involved in FIR filter design.

4. What are advantages of FIR filter?

5. What are the disadvantages of FIR FILTER?

6. What is the necessary and sufficient condition for the linear phase characteristic of a FIR filter?

7. List the well known design technique for linear phase FIR filter design?

8. Define IIR filter?

9. For what kind of application, the antisymmetrical impulse response can be used?

10. For what kind of application, the symmetrical impulse response can be used?

11. What is the reason that FIR filter is always stable?

12. What condition on the FIR sequence h(n) are to be imposed n order that this filter can be called a liner phase filter?

13. Under what conditions a finite duration sequence h(n) will yield constant group delay in its frequency response characteristics and not the phase delay?

14. State the condition for a digital filter to be causal and stable?

15. What are the properties of FIR filter?

16. When cascade from realization is preferred in FIR filters?

17. What are the disadvantages of Fourier series method?

18. What are the desirable characteristics of the windows?

19. Compare Hamming window with Kaiser Window.

20. What is the necessary and sufficient condition for linear phase characteristics in FIR filter?

PART B(16 MARKS)

1. Derive the condition of FIR filter to be linear in phase.
2. Describe briefly the different methods of power spectral estimation?
   i. Bartlett method (6)
   ii.Welch method (6)
   iii.Blackman-Tukey method and its derivation. (4)
3 Design a digital low pass filter FIR filter of length 11,with cut off frequency of 0.5 kHz and sampling rate 2 kHz using hamming window.
4. Explain the design of FIR filter using frequency sampling technique.

UNIT V FINITE WORD LENGTH EFFECTS
PART-A (2 MARKS)

1. Define white noise?

2. What do you understand by a fixed-point number?

3. What is the objective of spectrum estimation?

4. List out the addressing modes supported by C5X processors?

5. What is meant by block floating point representation? What are its advantages?

6. What are the advantages of floating point arithmetic?

7. How the multiplication & addition are carried out in floating point arithmetic?

8. What do you understand by input quantization error?

9. List the on-chip peripherals in 5X.

10. What is the relationship between truncation error e and the bits b for representing a decimal into binary?

11. What is meant rounding? Discuss its effect on all types of number representation?

12. What is meant by A/D conversion noise?

13. What is the effect of quantization on pole location?

14. What is meant by quantization step size?

15. How would you relate the steady-state noise power due to quantization and the b bits representing the binary sequence?

16. What is overflow oscillation?

17. What are the methods used to prevent overflow?

18. What are the two kinds of limit cycle behavior in DSP?

19. Determine "dead band" of the filter.

20. Explain briefly the need for scaling in the digital filter implementation.

21. What are the different buses of TMS320C5X and their functions?

PART B(16 MARKS)

1. Derive the expression for steady state I/P Noise Power and Steady state O/P Noise power
2. Draw the product quantatization model for first order and second order filter
   Write the difference equation and draw the noise model.
3. For the second order filter draw the direct form II realization and find the scaling factor S0 to avoid over flow
   Find the scaling factor from the formula

$$I = \frac{1+r2}{(1-r2)(1-2r2\cos 2ø =r4)}$$

4. Explain briefly about various number representation in digital computer.
5. Consider the transfer function H (Z) =H1 (Z) H2 (Z) where H1 (Z) =1/1-a1Z-1
   H2(z) =1/ 1-a2Z-1
   Find the o/p Round of noise power Assume a1=0.5 and a2= 0.6 and find o.p round off noise power.
6. What is meant by A/D conversion noise? Explain in detail?

DEPARTMENT OF ECE
QUESTION BANK
DIGITAL SIGNAL PROCESSING

BRANCH/SEM/SEC:CSE/IV/A& B

UNIT I

SIGNALS AND SYSTEMS

Part – A

1. What do you understand by the terms : signal and signal processing
2. Determine which of the following signals are periodic and compute their fundamental period (AU DEC 07)
   a) sin√2 Лt b)sin20 Лt+sin5Лt
3. What are energy and power signals? (MU Oct. 96)
4. State the convolution property of Z transform (AU DEC 06)
5. Test the following systems for time invariance: (DEC 03)
   a) $y(n)=n x^2 (n)$ b)$a^{x(n)}$
6. Define symmetric and antisymmetric signals. How do you prevent alaising while sampling a CT signal? (AU MAY 07)(EC 333, May '07)
7. What are the properties of region of convergence(ROC) ?(AU MAY 07)
8. Differentiate between recursive and non recursive difference equations (AU APR 05)
9. What are the different types of signal representation?
10. Define correlation (AU DEC 04)
11. what is the causality condition for LTI systems? (AU DEC 04)
12. define linear convolution of two DT signals (AU APR 04)
13. Define system function and stability of DT system (AU APR 04)
14. Define the following (a) System (b) Discrete-time system
15. What are the classifications of discrete-time system?
16. What is the property of shift-invariant system?
17. Define (a) Static system (b) Dynamic system? (AU DEC 03)
18. define cumulative and associative law of convolution (AU DEC 03)
19. Define a stable and causal system
20. What is the necessary and sufficient condition on the impulse response for stability? (MU APR.96)
21. What do you understand by linear convolution? (MU APR. 2000)
22. What are the properties of convolution? (AU IT Dec. 03)
23. State Parseval's energy theorem for discrete-time aperiodic signals(AU DEC 04)
24. Define DTFT pair (MU Apr. 99)
25. What is aliasing effect? (AU MAY 07) (EC 333 DEC 03)
26. State sampling theorem

27. What is an anti-aliasing filter?
28. What is the necessary and sufficient condition on the impulse response for stability? (EC 333, May '07)
22. State the condition for a digital filter to be causal and stable

Part-B

1. a) Compute the convolution y(n) of the signals (AU DEC 07)

   $x(n)= a^n \quad -3\le n\le 5$
   $\quad 0 \quad$ elsewhere and
   $h(n)= 1 \quad 0\le n\le 4$
   $\quad 0 \quad$ elsewhere

   b) A discrete time system can be static or dynamic, linear or non-linear, Time variant or time invariant, causal or non causal, stable or unstable. Examine the following system with respect to the properties also (AU DEC 07)

1) $y(n)=\cos(x(n))$
2) $y(n)=x(-n+2)$
3) $y(n)=x(2n)$
4) $y(n)=x(n) \cos \omega n$

2. a) Determine the response of the causal system
   $y(n)-y(n-1)=x(n)+x(n-1)$ to inputs $x(n)=u(n)$ and $x(n)=2^{-n}u(n)$.Test its stability
   b) Determine the IZT of $X(Z)=1/(1-Z^{-1})(1-Z^{-1})^2$ (AU DEC 07)

Determine whether each of the following systems defined below is (i) casual (ii) linear (iii) dynamic (iv) time invariant (v) stable
   (a) $y(n) = \log_{10}[\{x(n)\}]$
   (b) $y(n) = x(-n-2)$
   (c) $y(n) = \cosh[nx(n) + x(n-1)]$

3. Compute the convolution of the following signals

   $x(n) = \{1,0,\underset{\uparrow}{2},5,4\}$ $h(n) = \{\underset{\uparrow}{1},-1,1,-1\}$

   $h(n) = \{1,\underset{\uparrow}{0},1\}$ $x(n) = \{1,\underset{\uparrow}{-2},-2,3,4\}$

4. Find the convolution of the two signals
   $x(n) = 3^n u(-n)$; $h(n) = (1/3)^n u(n-2)$
   $x(n) = (1/3)^{-n} u(-n-1)$; $h(n) = u(n-1)$
   $x(n) = u(n) - u(n-5)$; $h(n) = 2[u(n) - u(n-3)]$

5. Find the discrete-time Fourier transform of the following
   $x(n) = 2^{-2n}$ for all n
   $x(n) = 2^{n}u(-n)$
   x(n) = n [1/2] (n)

6. Determine and sketch the magnitude and phase response of the following systems
   (a) y(n) = 1/3 [x(n) + x(n-1) + x(n-2)]
   (b) y(n) = ½[x(n) – x(n-1)]
   (c) y(n) - 1/2y(n-1)=x(n)

7. a) Determine the impulse response of the filter defined by y(n)=x(n)+by(n-1)
   b) A system has unit sample response h(n) given by
   h(n)=-1/δ(n+1)+1/2δ(n)-1-1/4 δ(n-1). Is the system BIBO stable? Is the filter causal? Justify your answer (DEC 2003)

8. Determine the Fourier transform of the following two signals(CS 331 DEC 2003)
   a) a^{n} u(n) for a<1
   b) cos ωn u(n)

9. Check whether the following systems are linear or not (AU APR 05)
   a) $y(n)=x^{2}(n)$ b) y(n)=n x(n)

10. For each impulse response listed below, dtermine if the corresponding system is
   i) causal ii) stable (AU MAY 07)
      1) 2^{n} u(-n)
      2) sin nЛ/2 (AU DEC 04)
      3) δ(n)+sin nЛ
      4) e^{2n} u(n-1)

11. Explain with suitable block diagram in detail about the analog to digital conversion and to reconstruct the analog signal (AU DEC 07)

12. Find the cross correlation of two sequences
   x(n)={1,2,1,1} y(n)={1,1,2,1} (AU DEC 04)

13. Determine whether the following systems are linear , time invariant
      1) y(n)=A x(n)+B
      2) y(n)=x(2n)
   Find the convolution of the following sequences: (AU DEC 04)
      1) x(n)=u(n) h(n)=u(n-3)
      2) x(n)={1,2,-1,1} h(n)={1,0,1,1}

UNIT II

FAST FOURIER TRANSFORMS

1) THE DISCRETE FOURIER TRANSFORM

PART A

1. Find the N-point DFT of a sequence x(n) ={1 ,1, 2, 2}
2. Determine the circular convolution of the sequence x1(n)={1,2,3,1} and x2(n)={4,3,2,1} (AU DEC 07)
3. Draw the basic butterfly diagram for radix 2 DIT-FFT and DIF-FFT(AU DEC 07)
4. Determine the DTFT of the sequence $x(n)=a^{n} u(n)$ for a<1 (AU DEC 06)
5. Is the DFT of the finite length sequence periodic? If so state the reason (AU DEC 05)
6. Find the N-point IDFT of a sequence X(k) ={1 ,0 ,0 ,0} (Oct 98)
7. what do you mean by 'in place' computation of FFT? (AU DEC 05)
8. What is zero padding? What are its uses? (AU DEC 04)
9. List out the properties of DFT (MU Oct 95,98,Apr 2000)
10. Compute the DFT of $x(n)=\partial(n-n_0)$
11. Find the DFT of the sequence of $x(n)= \cos(n\prod/4)$ for $0\leq n\geq 3$ (MU Oct 98)
12. Compute the DFT of the sequence whose values for one period is given by x(n)={1,1,-2,-2}. (AU Nov 06,MU Apr 99)
13. Find the IDFT of Y(k)={1,0,1,0} (MU Oct 98)
14. What is zero padding? What are its uses?
15. Define discrete Fourier series.
16. Define circular convolution
17. Distinguish between linear convolution and Circular Convolution. (MU Oct 96,Oct 97,Oct 98)
18. Obtain the circular convolution of the following sequences x(n)={1, 2, 1} and h(n)={1, -2, 2}
19. Distinguish between DFT and DTFT (AU APR 04)
20. Write the analysis and synthesis equation of DFT (AU DEC 03)
21. Assume two finite duration sequences $x_1(n)$ and $x_2(n)$ are linearly combined. What is the DFT of x3(n)?(x3(n)=Ax1(n)+Bx2(n)) (MU Oct 95)
22. If X(k) is a DFT of a sequence x(n) then what is the DFT of real part of x(n)?
23. Calculate the DFT of a sequence x(n)=(1/4)^n for N=16 (MU Oct 97)
24. State and prove time shifting property of DFT (MU Oct 98)
25. Establish the relation between DFT and Z transform (MU Oct 98,Apr 99,Oct 00)
26. What do you understand by Periodic convolution? (MU Oct 00)
27. How the circular convolution is obtained using concentric circle method? (MU Apr 98)
28. State the circular time shifting and circular frequency shifting properties of DFT
29. State and prove Parseval's theorem
30. Find the circular convolution of the two sequences using matrix method X1(n)={1, 2, 3, 4} and x2(n)={1, 1, 1, 1}

31. State the time reversal property of DFT
32. If the DFT of x(n) is X(k) then what is the DFT of x*(n)?
33. State circular convolution and circular correlation properties of DFT
34. Find the circular convolution of the following two sequences using concentric circle method
    $x_1(n)=\{1, 2, 3, 4\}$ and $x_2(n)=\{1, 1, 1, 1\}$
35. The first five coefficients of X(K)={1, 0.2+5j, 2+3j, 2 ,5 }Find the remaining coefficients

PART B

1. Find 4-point DFT of the following sequences
(a) x(n)={1,-1,0,0}
(b) x(n)={1,1,-2,-2} (AU DEC 06)
(c) $x(n)=2^n$
(d) $x(n)=\sin(n\prod/2)$

2. Find 8-point DFT of the following sequences
(a) x(n)={1,1,1,1,0,0,0,0}
(b) x(n)={1,2,1,2}

3. Determine IDFT of the following
(a)X(k)={1,1-j2,-1,1+j2}
(b)X(k)={1,0,1,0}
(c)X(k)={1,-2-j,0,-2+j}

4. Find the circular convolution of the following using matrix method and concentric circle method
(a) $x_1(n)=\{1,-1,2,3\}$; $x_2(n)=\{1,1,1\}$;
(b) $x_1(n)=\{2,3,-1,2\}$; $x_2(n)=\{-1,2,-1,2\}$;
(c) $x_1(n)=\sin n\prod/2$; $x_2(n)=3^n \quad 0\leq n\geq 7$

5.Calculate the DFT of the sequence x(n)={1,1,-2,-2}
Determine the response of the LTI system by radix2 DIT-FFT? (AU Nov 06).
If the impulse response of a LTI system is h(n)=(1,2,3,-1)
$\Uparrow$

6. Determine the impulse response for the cascade of two LTI systems having impulse responses $h_1(n)=(1/2)$^n* u(n),$h_2(n)=(1/4)$^n*u(n) (AU May 07)

7. Determine the circular convolution of the two sequences $x_1(n)=\{1, 2, 3, 4\}$ $x_2(n)=\{1, 1, 1, 1\}$ and prove that it is equal to the linear convolution of the same.

8. Find the output sequence y(n)if h(n)={1,1,1,1} and x(n)={1,2,3,1} using circular convolution (AU APR 04)

9. State and prove the following properties of DFT (AU DEC 03)
   1) Cirular convolution 2) Parseval's relation
   2) Find the circular convolution of x1(n)={1,2,3,4} x2(n)={4,3,2,1}

2) FAST FOURIER TRANSFORM

PART A

1. Why FFT is needed? (AU DEC 03) (MU Oct 95,Apr 98)
2. What is FFT? (AU DEC 06)
3. Obtain the block diagram representation of the FIR filter (AU DEC 06)
4. Calculate the number of multiplications needed in the calculation of DFT and FFT with 64 point sequence. (MU Oct 97, 98).
5. What is the main advantage of FFT?
6. What is FFT? (AU Nov 06)
7. How many multiplications and additions are required to compute N-point DFT using radix 2 FFT? (AU DEC 04)
8. Draw the direct form realization of FIR system (AU DEC 04)
9. What is decimation-in-time algorithm? (MU Oct 95).
10. What do you mean by 'in place' computation in DIT-FFT algorithm? (AU APR 04)
11. What is decimation-in-frequency algorithm? (MU Oct 95,Apr 98).
12. Mention the advantage of direct and cascade structures (AU APR 04)
13. Draw the direct form realization of the system y(n)=0.5x(n)+0.9y(n-1) (AU APR 05)
14. Draw the flow graph of a two point DFT for a DIT decomposition.
15. Draw the basic butterfly diagram for DIT and DIF algorithm. (AU 07).
16. How do we can calculate IDFT using FFT algorithm?
17. What are the applications of FFT algorithms?
18. Find the DFT of sequence x(n)={1,2,3,0} using DIT-FFT algorithms
19. Find the DFT of sequence x(n)={1,1, 1, 1} using DIF-FFT algorithms (AU DEC 04)

PART B

1. Compute an 8-point DFT of the following sequences using DIT and DIF algorithms
   (a)x(n)={1,-1,1,-1,0,0,0,0}
   (b)x(n)={1,1,1,1,1,1,1,1} (AU APR 05)
   (c)x(n)={0.5,0,0.5,0,0.5,0,0.5,0}
   (d)x(n)={1,2,3,2,1,2,3,2}
   (e)x(n)={0,0,1,1,1,1,0,0} (AU APR 04)

2. Compute the 8 point DFT of the sequence x(n)={0.5, 0.5 ,0.5,0.5,0,0,0,0} using radix 2 DIF and DIT algorithm (AU DEC 07)

3. a) Discuss the properties of DFT
   b) Discuss the use of FFT algorithm in linear filtering (AU DEC 07)

4. How do you linear filtering by FFT using save-add method (AU DEC 06)

5. Compute the IDFT of the following sequences using (a)DIT algorithm (b)DIF algorithms
   (a)X(k)={1,1+j,1-j2,1,0,1+j2,1+j}
   (b)X(k)={12,0,0,0,4,0,0,0}
   (c)X(k)={5,0,1-j,0,1,0,1+j,0}
   (d)X(k)={8,1+j2,1-j,0,1,0,1+j,1-j2}
   (e)X(k)={16,1-j4.4142,0,1+j0.4142,0,1-j0.4142,0,1+j4.4142}

6. Derive the equation for DIT algorithm of FFT.
   How do you do linear filtering by FFT using Save Add method? (AU Nov 06)

7. a) From first principles obtain the signal flow graph for computing 8 point DFT using radix 2 DIT-FFT algorithm.

   b) Using the above signal flow graph compute DFT of x(n)=cos(n*Л)/4 ,0<=n<=7 (AU May 07).

8. Draw the butterfly diagram using 8 pt DIT-FFT for the following sequences x(n)={1,0,0,0,0,0,0,0} (AU May 07).

9. a) From first principles obtain the signal flow graph for computing 8 point DFT using radix 2 DIF-FFT algorithm.
   b) Using the above signal flow graph compute DFT of x(n)=cos(n*Л)/4 ,0<=n<=7

10. State and prove circular time shift and circular frequency shift properties of DFT

11. State and prove circular convolution and circular conjugate properties of DFT

12. Explain the use of FFT algorithms in linear filtering and correlation

13. Determine the direct form realization of the following system
    y(n)=-0.1y(n-1)+0.72y(n-2)+0.7x(n)-0.252x(n-2) (AU APR 05)

14. Determine the cascade and parallel form realization of the following system
y(n)=-0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2)
    Expalin in detail about the round off errors in digital filters (AU DEC 04)

UNIT-III

IIR FILTER DESIGN

PART-A

1. Distinguish between Butterworth and Chebyshev filter
2. What is prewarping? (AU DEC 03)
3. Distinguish between FIR and IIR filters (AU DEC 07)
4. Give any two properties of Butterworth and chebyshev filters (AU DEC 06)
5. Give the bilinear transformation (AU DEC 03)
6. Determine the order of the analog butterworth filter that has a -2 dB pass band attenuation at a frequency of 20 rad/sec and atleast -10 dB stop band attenuation at 30 rad/sec (AU DEC 07)
7. By impulse invariant method obtain the digital filter transfer function and differential equation of the analog filter H(S)=1/S+1 (AU DEC 07)
8. Give the expression for location of poles of normalized butterworth filter (EC 333, May '07)
9. What are the parameters(specifications) of a chebyshev filter (EC 333, May '07)
10. Why impulse invariance method is not preferred in the design of IIR filter other than low pass filter?
11. What are the advantages and disadvantages of bilinear transformation?(AU DEC 04)
12. Write down the transfer function of the first order butterworth filter having low pass behavior (AU APR 05)
13. What is warping effect? What is its effect on magnitude and phase response?
14. Find the digital filter transfer function H(Z) by using impulse invariance method for the analog transfer function H(S)= 1/S+2 (MAY AU '07)
15. Find the digital filter transfer function H(Z) by using bilinear transformation method for the analog transfer function H(S)= 1/S+3
16. Give the equation for converting a normalized LPF into a BPF with cutoff frequencies Ω_l and Ω_u
17. Give the magnitude function of Butterworth filter. What is the effect of varying order of N on magnitude and phase response?
18. Give any two properties of Butterworth low pass filters. (MU NOV 06).
19. What are the properties of Chebyshev filter? (AU NOV 06).
20. Give the equation for the order of N and cut off frequency Ω_c of Butterworth filter.
21. Give the Chebyshev filter transfer function and its magnitude response.
22. Distinguish between the frequency response of Chebyshev Type I filter for N odd and N even.
23. Distinguish between the frequency response of Chebyshev Type I & Type II filter.
24. Give the Butterworth filter transfer function and its magnitude characteristics for different order of filters.
25. Give the equations for the order N, major, minor and axis of an ellipse in case of Chebyshev filter.
26. What are the parameters that can be obtained from the Chebyshev filter specification? (AU MAY 07).

27. Give the expression for the location of poles and zeros of a Chebyshev Type II filter.
28. Give the expression for location of poles for a Chebyshev Type I filter. (AU MAY 07)
29. Distinguish between Butterworth and Chebyshev Type I filter.
30. How one can design Digital filters from Analog filters.
31. Mention any two procedures for digitizing the transfer function of an analog filter. (AU APR 04)
32. What are properties that are maintained same in the transfer of analog filter into a digital filter.
33. What is the mapping procedure between s-plane and z-plane in the method of mapping of differentials? What is its characteristics?
34. What is mean by Impulse invariant method of designing IIR filter?
35. What are the different types of structures for the realization of IIR systems?
36. Write short notes on prewarping.
37. What are the advantages and disadvantages of Bilinear transformation?
38. What is warping effect? What is its effect on magnitude and phase response?
39. What is Bilinear Transformation?
40. How many numbers of additions, multiplications and memory locations are required to realize a system H(z) having M zeros and N poles in direct form-I and direct form –II realization?
41. Define signal flow graph.
42. What is the transposition theorem and transposed structure?
43. Draw the parallel form structure of IIR filter.
44. Give the transposed direct form –II structure of IIR second order system.
45. What are the different types of filters based on impulse response? (AU 07)
46. What is the most general form of IIR filter?

PART B

1. a) Derive bilinear transformation for an analog filter with system function H(S)=b/S+a (AU DEC 07)
   b) Design a single pole low pass digital IIR filter with-3 Db bandwidth of 0.2Л by using bilinear transformation
2. a) Obtain the direct form I, Direct form II, cascade and parallel realization for the following Systems
   y(n)=-0.1x(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2)
   b) Discuss the limitation of designing an IIR filetr using impulse invariant method (AU DEC 07)

3. Determine H(Z) for a Butterworth filter satisfying the following specifications:

$$0.8 \leq |H(e^{j\omega})| \leq 1, \text{ for } 0 \leq \omega \leq \pi/4$$
$$|H(e^{j\omega})| \leq 0.2, \text{ for } \pi/2 \leq \omega \leq \pi$$

Assume T= 0.1 sec. Apply bilinear transformation method (AU MAY 07)

4.Determine digital Butterworth filter satisfying the following specifications:

$$0.707 \leq |H(e^{j\omega})| \leq 1, \text{ for } 0 \leq \omega \leq \pi/2$$
$$|H(e^{j\omega})| \leq 0.2, \text{ for } 3\pi/4 \leq \omega \leq \pi$$

Assume T= 1 sec. Apply bilinear transformation method. Realize the filter in mose convenient form (AU DEC 06)

5. Design a Chebyshev lowpass filter with the specifications α_p=1 dB ripple in the pass band $0\leq\omega\leq0.2\pi$, α_s=15 dB ripple in the stop band $0.3\pi\leq\omega\leq\pi$ using impulse invariance method(AU DEC 06)

6. Design a Butterworth high pass filter satisfying the following specifications.
   α_p =1 dB; α_s=15 dB
   Ω_p =0.4Π; Ω_s =0.2Π

7. Design a Butterworth low pass filter satisfying the following specifications.
   (AU DEC 04)
   f_p=0.10 Hz;α_p=0.5 dB
   f_s=0.15 HZ;α_s=15 dB:F=1Hz.

8. Design (a) a Butterworth and (b) a Chebyshev analog high pass filter that will pass all radian frequencies greater than 200 rad/sec with no more that 2 dB attuenuation and have a stopband attenuation of greater than 20 dB for all Ω less than 100 rad/sec.

9. Design a digital filter equivalent to this using impulse invariant method
   $H(S)=10/S^2+7S+10$ (AU DEC 03)(AU DEC 04)

10. Use impulse invariance to obtain H(Z) if T= 1 sec and H(s) is
    $1/(s^3+3s^2+4s+1)$
    $1/(s^2+\sqrt{2}\,s+1)$

11. Use bilinear transformation method to obtain H(Z) if T= 1 sec and H(s) is
    $1/(s+1)(S+2)$ (AU DEC 03)
    $1/(s^2+\sqrt{2}\,s+1)$

12. Briefly explain about bilinear transformation of digital filter design(AU APR 05)

13. Use bilinear transform to design a butterworth LPF with 3 dB cutoff frequeny of 0.2Π (AU APR 04)

14. Compare bilinear transformation and impulse invariant mapping

15. a) Design a chebyshev filter with a maxmimum pass band attenuation of 2.5 Db; at Ωp=20 rad/sec and the stop band attenuation of 30 Db at Ωs=50 rad/sec.
    b)Realize the system given by difference equation
    y(n)=-0.1 y(n-1)+0.72y(n-2)+0.7x(n)-0.25x(n-2) in parallel form
    (EC 333 DEC '07)

UNIT IV

FIR FILTER DESIGN

PART A

1. What are the desirable and undesirable features of FIR filter?
2. Discuss the stability of the FIR filters (AU APR 04) (AU DEC 03)
3. What are the main advantages of FIR over IIR (AU APR 04)
4. What is the condition satisfied by Linear phase FIR filter? (DEC 04) (EC 333 MAY 07)
5. What are the design techniques of designing FIR filters?
6. What condition on the FIR sequence h(n) are to be imposed in order that this filter can be called a Linear phase filter? (AU 07)
7. State the condition for a digital filter to be a causal and stable. (AU 06)
8. What is Gibbs phenomenon? (AU DEC 04) (AU DEC 07)
9. Show that the filter with h(n)={-1, 0, 1} is a linear phase filter
10. Explain the procedure for designing FIR filters using windows. (MU 02)
11. What are desirable characteristics of windows?
12. What is the principle of designing FIR filters using windows?
13. What is a window and why it is necessary?
14. Draw the frequency response of N point rectangular window. (MU 03)
15. Give the equation specifying Hanning and Blackman windows.
16. Give the expression for the frequency response of
17. Draw the frequency response of N point Bartlett window
18. Draw the frequency response of N point Blackman window
19. Draw the frequency response of N point Hanning window. (AU DEC 03)
20. What is the necessary and sufficient condition for linear phase characteristics in FIR filter. (MU Nov 03)
21. Give the equation specifying Kaiser window.
22. Compare rectangular and hanning window functions
23. Briefly explain the frequency sampling method of filter design
24. Compare frequency sampling and windowing method of filter design

PART-B

1. Use window method with a Hamming window to design a 13-tap differentiator (N=13). (AU '07)

2. i) Prove that FIR filter has linear phase if the unit impulse responsesatisfies the condition h(n)=h(N-1-n), n=0,1,……M-1. Also discuss symmetric and antisymmetric cases of FIR filter (AU DEC 07)

3. What are the issues in designing FIR filter using window method?(AU APR 04, DEC 03)

4. ii) Explain the need for the use of window sequences in the design of FIR filter. Describe the window sequences generally used and compare their properties

5. Derive the frequency response of a linear phase FIR filter when impulse responses symmetric & order N is EVEN and mention its applications

6. i) Explain the type I design of FIR filter using frequency sampling method
   ii) A low pass filter has the desired response as given below

   $Hd(e^{j\omega}) = e^{-j3\omega},\ 0 \le \omega \le \pi/2$

   $0 \qquad \pi/2 \le \omega \le \pi$

   Determine the filter coefficients h(n) for M=7 using frequency sampling technique (AU DEC 07)

7. i) Derive the frequency response of a linear phase FIR filter when impulse responses antisymmetric & order N is odd
   ii) Explain design of FIR filter by frequency sampling technique (AU MAY 07)

7. Design an approximation to an ideal bandpass filter with magnitude response

   $H(e^{j\omega}) = 1 \ ; \ \pi/4 \le |\omega| \le 3\pi/4$

   $0 \ ; \text{otherwise}$

   Take N=11. (AU DEC 04)

8. Design a 15-tap linear phase filter to the following discrete frequency response (N=15) using frequency sampling method (MU 03)

   $H(k) = 1 \qquad 0 \le k \le 4$

   $= 0.5 \qquad k=5$

   $= 0.25 \qquad k=6$

   $= 0.1 \qquad k=7$

   $= 0 \qquad \text{elsewhere}$

9. Design an ideal band pass digital FIR filter with desired frequency response

   $H(e^{j\omega}) = 1 \text{ for } 0.25\pi \le |\omega| \le 0.75\pi$

   $0 \text{ for } |\omega| \le 0.25\pi \text{ and } 0.75\pi \le |\omega| \le \pi$

   by using rectangular window function of length N=11. (AU DEC 07)

10. Design an Ideal Hilbert transformer using hanning window and Blackman window for N=11. Plot the frequency response in both Cases

11. a) How is the design of linear phase FIR filter done by frequency sampling method? Explain.
    b) Determine the coefficients of a linear phase FIR filter of length N=15 which has Symmetric unit sample response and a frequency response that satisfies the following conditions

    $H_r(2\pi k/15) = 1 \text{ for } k=0,1,2,3$

0 for k=4
0 for k=5,6,7

12. An FIR filter is given by the difference equation
$y(n)=2x(n)+4/5\ x(n-1)+3/2\ x(n-2)+2/3\ x(n-3)$ Determine its lattice form(EC 333 DEC 07)

13. Using a rectangular window technique design a low pass filter with pass band gain of unity cut off frequency of 1000 Hz and working at a sampling frequency of 5 KHz. The length of the impulse response should be 7.(EC 333 DEC 07)

16. Design an Ideal Hilbert transformer using rectangular window and Black man window for N=11. Plot the frequency response in both Cases (EC 333 DEC '07)

9. 17. Design an approximation to an ideal lowpass filter with magnitude response
$H(e^{j\omega}) = 1$; $0\le|\omega|\le\prod/4$
0 ; otherwise
Take N=11.Use hanning and hamming window (AU DEC 04)

UNIT V

FINITE WORD LENGTH EFFECTS

PART –A

1. What do you understand by a fixed point number? (MU Oct'95)
2. Express the fraction 7/8 and -7/8 in sign magnitude, 2's complement and 1's complement (AU DEC 06)
3. What are the quantization errors due to finite word length registers in digital filters? (AU DEC 06)
4. What are the different quantization methods? (AU DEC 07)
5. What are the different types of fixed point number representation?
6. What do you understand by sign-magnitude representation?
7. What do you understand by 2's complement representation?
8. Write an account on floating point arithmetic? (MU Apr 2000)
9. What is meant by block floating point representation? What are its advantages?
10. what are advantages of floating point arithmetic?
11. Compare the fixed point and floating point arithmetic. (MU Oct'96)
12. What are the three quantization errors due to finite word length registers in digital filters? (MU Oct'98)
13. How the multiplication and addition are carried out in floating point arithmetic?
14. Brief on co-efficient inaccuracy.
15. What do you understand by input quantization error?
16. What is product quantization error?
17. What is meant by A/D conversion mode?
18. What is the effect of quantization on pole locations?

19. What are the assumptions made concerning the statistical independence of various noise sources that occur in realizing the filter? (M.U. Apr 96)
20. What is zero input limit cycle overflow oscillation (AU 07)
21. What is meant by limit cycle oscillations?(M.U Oct 97, 98, Apr 2000) (AU DEC 07)
29. Explain briefly the need for scaling digital filter implementation? (M.U Oct 98)(AU-DEC 07)
30. Why rounding is preferred than truncation in realizing digital filter? (M.U. Apr 00)
31. Define the deadband of the filter? (AU 06)
25. Determine the dead band of the filter with pole at 0.5 and the number of bits used for quantization is 4(including sign bit)
26. Draw the quantization noise model for a first order IIR system
27. What is meant by rounding? Draw the pdf of round off error
28. What is meant by truncation? Draw the pdf of round off error
29. What do you mean by quantization step size?
30. Find the quantization step size of the quantizer with 3 bits
31. Give the expression for signal to quantization noise ratio and calculate the improvement with an increase of 2 bits to the existing bit.
32. Express the following binary numbers in decimal
    A) $(100111.1110)_2$ (B) $(101110.1111)_2$ C $(10011.011)_2$
33.Why rounding is preferred to truncation in realizing digital filter? (EC 333, May '07)
34. List the different types of frequency domain coding (EC 333 MAY 07)
35. What is subband coding? (EC 333 MAY 07)

PART-B

1. Draw the quantization noise model for a second order system and explain $H(z)=1/(1-2r\cos\theta z^{-1}+r^2z^{-2})$ and find its steady state output noise variance (ECE AU' 05)

2. Consider the transfer function $H(z)=H_1(z)H_2(z)$ where $H_1(z)=1/(1-a_1z^{-1})$, $H_2(z)=1/(1-a_2z^{-2})$.Find the output round off noise power. Assume $a_1=0.5$ and $a_2=0.6$ and find out the output round off noise power. (ECE AU' 04)(EC 333 DEC 07)

3. Find the effect of coefficient quantiztion on pole locations of the given second order IIR system when it is realized in direct form –I and in cascade form. Assume a word length of 4-bits through truncation.
   $H(z)= 1/(1-0.9z^{-1}+0.2z^{-2})$ (AU' Nov 05)

4. Explain the characteristics of Limit cycle oscillations with respect to the system described by the differential equations.
   $y(n)=0.95y(n-1)+x(n)$ and
   determine the dead band of the filter (AU' Nov 04)
5. i) Describe the quantization errors that occur in rounding and truncation in two's complement
   ii) Draw a sample/hold circuit and explain its operation
   iii) What is a vocoder? Expalin with a block diagram (AU DEC 07)

6. Two first order low pass filter whose system functions are given below are connected in cascade. Determine the overall output noise power
   $H1(Z)=1/(1-0.9Z^{-1})$ $H2(Z)=1/(1-0.8Z^{-1})$ (AU DEC 07)

7. Consider a Butterworth lowpass filter whose transfer function is
$H(z)=0.05(1+z^{-1})^2 /(1-1.2z^{-1}+0.8 z^{-2})$.
Compute the pole positions in z-plane and calculate the scale factor S_o to prevent overflow in adder 1.

8. Express the following decimal numbers in binary form

A) 525 B) 152.1875 C) 225.3275

10. Express the decimal values 0.78125 and -0.1875 in

One's complement form

sign magnitude form

Two's complement form.

11. Express the decimal values -6/8 and 9/8 in (i) Sign magnitude form (ii) One's complement form (iii) Two's complement form

12. Study the limit cycle behavior of the following systems

i. $y(n) = 0.7y(n-1) + x(n)$

ii. $y(n) = 0.65y(n-2) + 0.52y(n-1) + x(n)$

13. For the system with system function H (z) =1+0.75z-1 / 1-0.4z-1 draw the signal flow graph
14. and find scale factor s0 to prevent overflow limit cycle oscillations
15. Derive the quantization input nose power and determine the signal to noise ratio of the system
16. Derive the truncation error and round off error noise power and compare both errors
17. Explain product quantization error and coefficient quantization error with examples
18. Derive the scaling factor So that prevents the overflow limit cycle oscillations in a second order IIR system.
19. The input to the system $y(n)=0.999y(n-1)+x(n)$ is applied to an ADC. What is the power produced by the quantization noise at the output of the filter if the input is quantized to

1) 8 bits 2) 16 bits (EC 333 DEC 07)

19. Convert the following decimal numbers into binary: (EC 333 DEC 07)

1) $(20.675)_{10}$ 2) $(120.75)_{10}$

20. Find the steady state variance of the noise in the output due to quantization of input for the first order filter $y(n)=ay(n-1)+x(n)$ (EC 333 DEC 07)

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DEPARTMENT OF ECE

Date: 15-05-2009

PART-A QUESTIONS AND ANSWERS

Subject : Digital signal Processing **Sub Code : IT1252**

Staff Name: Robert Theivadas.J Class : VII Sem/CSE A&B

UNIT-1 - SIGNALS AND SYSTEMS

PART A

1. Determine which of the following sinusoids are periodic and compute their fundamental period

(a) Cos 0.01πn
(b) sin (π62n/10) **Nov/Dec 2008 CSE**

a) Cos 0.01 πn

Wo=0.01 π the fundamental frequency is multiply of π .Therefore the signal is periodic

Fundamental period

N=2π [m/wo]

=2π(m/0.01π)

Choose the smallest value of m that will make N an integer

M=0.1

N=2π(0.1/0.01π)

N=20

Fundamental period N=20

b) sin (π62n/10)

Wo=0.01 π the fundamental frequency is multiply of π .Therefore the signal is periodic

Fundamental period

N=2π [m/wo]

=2π(m/(π62/10))

Choose the smallest value of m that will make N an integer

M=31

N=2π(310/62π)

N=10

Fundamental period N=10

2. State sampling theorem **Nov/Dec 2008 CSE**

A band limited continuous time signal, with higher frequency f_{max} Hz can be uniquely recovered from its samples provided that the sampling rate $Fs>2f_{max}$ samples per second

3.State sampling theorem , and find Nyquist rate of the signal

x(t)=5 sin250 πt + 6cos300 πt **April/May2008 CSE**

A band limited continuous time signal, with higher frequency f_{max} Hz can be uniquely recovered from it's samples provided that the sampling rate $Fs>2f_{max}$ samples per second.

Nyquist rate

$x(t)=5\sin 250\pi t+6\cos 300\pi t$

Frequency present in the signals

$F1=125Hz$ $F2=150Hz$
$Fmax=150Hz$
$Fs>2Fmax=300$ Hz
The Nyquist rate is $FN= 300Hz$

4. State and prove convolution property of Z transform. April/May2008 CSE
Convolution Property (MAY 2006 ECESS)

$$x1(n) \xleftrightarrow{Z} X1(z)$$

$$x2(n) \xleftrightarrow{Z} X2(z)$$

Then

$$x(n)= x1(n) * x2(n) \xleftrightarrow{Z} X(z)=X1(z)X2(z).$$

The ROC of X(z) is ,at least the intersection of that for X1(z) And X2(z).

5. Determine which of the following signals are periodic and compute their fundamental period. Nov/Dec 2007 CSE

(a) sin √2πt
(b) sin 20πt + sin 5πt

(a) sin √2πt

wo=√2π .The Fundamental frequency is multiply of π.Therefore, the signal is Periodic .

Fundamental period

$N=2\pi\,[m/wo]$
$= 2\pi\,[m/\sqrt{2}\pi]$
$m=\sqrt{2}$
$=2\pi\,[\sqrt{2}/\sqrt{2}\pi]$
$N=2$

(b) sin 20πt + sin 5πt

wo=20π, 5π .The Fundamental frequency is multiply of π.Therefore, the signal is Periodic .

Fundamental period of signal sin 20πt

$N1=2\pi\,[m/wo]$
$=2\pi\,[m/20\pi]$ $m=1$
$=1/10$

Fundamental period of signal sin 5πt

$N2=2\pi\,[m/wo]$
$=2\pi\,[m/5\pi]$ $m=1$
$=2/5$
$N1/N2=(1/10)/(2/5)$
$=1/4$
$4N1=N2$

N= 4N1=N2
N=2/5

6. **Determine the circular convolution of the sequence x1(n)={1,2,3,1} and x2(n)={4,3,2,1}.** **Nov/Dec 2007 CSE**

Soln:

x1(n)={1,2,3,1}
x2(n)={4,3,2,1}.

$$\begin{vmatrix} x1(0) & x1(3) & x1(2) & x1(1) \\ x1(1) & x1(0) & x1(3) & x1(2) \\ x1(2) & x1(1) & x1(0) & x1(3) \\ x1(3) & x1(2) & x1(1) & x1(0) \end{vmatrix} \begin{vmatrix} x2(0) \\ x2(1) \\ x2(2) \\ x2(3) \end{vmatrix} = \begin{vmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 4 \\ 3 \\ 2 \\ 1 \end{vmatrix} = \begin{vmatrix} 15 \\ 16 \\ 21 \\ 15 \end{vmatrix}$$

Y(n)= $\{15,16,21,15\}$

7. **Define Z transform for x(n)=-na^n u(-n-1)** **April/May 2008 IT**

X(n) =-na^n u(-1-n)

X (z)=$\sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

=$\sum_{n=-\infty}^{\infty} -\mathrm{na}^n\, \mathrm{u}(-1-\mathrm{n}) z^{-n}$ u(-n-1)=0for n>1

=$\sum_{n=-\infty}^{\infty} -\mathrm{na}^n z^{-n}$
= -$\sum_{n=-\infty}^{\infty} \mathrm{na}^n z^{-n}$
= -z d/dz X(z)
=z d/dz($\frac{1}{1-az^{-1}}$)= $az^{-1}/(1-az^{-1})^2$

8. **Find whether the signal y= n^2 x(n) is linear** **April/May 2008 IT**

Y=n^2 x(n)

Y1(n)=T[x1(n)]= n^2 x1(n)

Y2(n)= T[x2(n)]= n^2 x2(n)

The weighted sum of input is

$a1\ T[x1(n)]+a2\ T[x2(n)]=a1n^2\ x1(n)+a2n^2\ x2(n)$-----------1

the output due to weighted sum of input is

$y3(n)=T[a1X1(n)+a2X2(n)]$

$= a1n^2\ x1(n)+a2n^2\ x2(n)$---------------------------------2

9. Is the system y(n)=ln[x(n)] is linear and time invariant? (MAY 2006 IT)

The system y(n)=ln[x(n)] is non-linear and time invariant

$a\ln x_1(n)+b\ln x_2(n) \neq \ln(ax_1(n)+bx_2(n)) \rightarrow$ Non-linear system

$\ln x\ (n)=\ln x\ (n-n_0) \rightarrow$ Time invariant system

10. Write down the expression for discrete time unit impulse and unit step function. (APR 2005 IT).

Discrete time unit impulse function

$\delta(n) =1,\ n=0$

$=0,\ n\neq 0$

Discrete time step impulse function.

$u(n) = 1$, for $n\geq 0$

$= 0$ for $n<0$

11. List the properties of DT sinusoids. (NOV 2005 IT)

- DT sinusoid is periodic only if its frequency f is a rational number.
- DT sinusoid whose frequencies are separated by an integer multiple of 2π are identical.

12. Determine the response a system with y(n)=x(n-1) for the input signal

$x(n) = |n|$ for $-3\leq n\leq 3$

$= 0$ otherwise (NOV 2005 IT)

$x(n)= \{3,2,1,\underset{\uparrow}{0},1,2,3\}$

$y(n) = x(n-1) =\{3,2,1,0,1,2,3\}$

13. Define linear convolution of two DT signals. (APR 2004 IT)

$y(n)=x(n)*h(n)$, * represent the convolution operator

$y(n)$, $x(n)$&$h(n)$, $\rightarrow$Output, Input and response of the system respectively.

14. Define system function and stability of a DT system. (APR 2004 IT)

$H(z)=Y(z)/X(z)$

$H(z)$,$Y(z)$ & $X(z)\rightarrow$z-transform of the system impulse, output and input respectively.

15. What is the causality condition for an LTI system? (NOV 2004 IT)

Conditions for the causality

$h(n)=0$ for $n<0$

16. What are the different methods of evaluating inverse z transform. (NOV 2004 IT)

- Long division method
- Partial fraction expansion method
- Residue method
- Convolution method

UNIT-II - FAST FOURIER TRANSFORMS

1. Find out the DFT of the signal X(n)= δ (n) Nov/Dec 2008 CSE

$$\delta(n) = \begin{cases} 1 \; for \; n = 0 \\ 0 \; for \; n \neq 0 \end{cases}$$

X(n)={1,0,0,0}

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \qquad k = 0,1,2,3 \ldots\ldots N-1$$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi nk/4} \qquad k = 0,1,2,3$$

$$X(k) = x(0) + x(1)e^{-\frac{jk\pi}{2}} + x(2)e^{-jk\pi} + x(3)e^{-j3k\pi/4} \qquad k = 0,1,2,3.$$

X(k)={1,1,1,1}

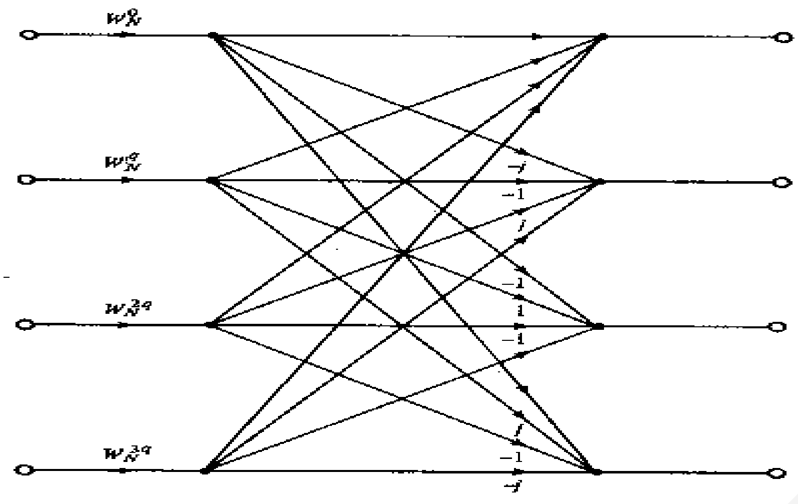
2. What is meant by bit reversal and in place commutation as applied to FFT?

Nov/Dec 2008 CSE

"Bit reversal" is just what it sounds like: reversing the bits in a binary word from left to write. Therefore the MSB's become LSB's and the LSB's become MSB's.The data ordering required by radix-2 FFT's turns out to be in "bit reversed" order, so bit-reversed indexes are used to combine FFT stages.

Input sample index	Binary Representation	Bit reversed binary	Bit reversal sample index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

3. **Draw radix 4 butterfly structure for (DIT) FFT algorithm April/May2008 CSE**



4. Find DFT for {1,0,0,1}. **April/May2008 CSE /April/May 2008 IT**

$$x(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N} \qquad K=0,1,2,3,\ldots N-1$$

$$x(k)=\sum_{n=0}^{3} x(n)e^{-j2\pi nk/N} \qquad K=0,1,2,3$$

$$N=4$$

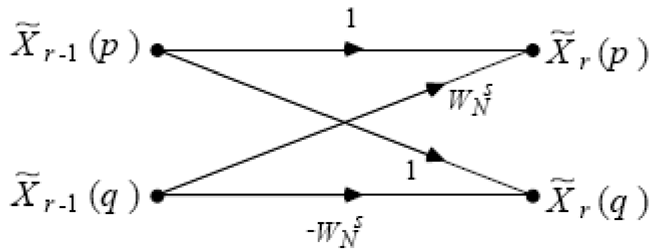
$$=x(0)+x(1)e^{-jk\pi/2}+x(2)e^{-jk\pi}+x(3)e^{-j3k\pi/2}$$

$$X(k)=1+e^{-j3k\pi/2} \qquad K=0,1,2,3$$

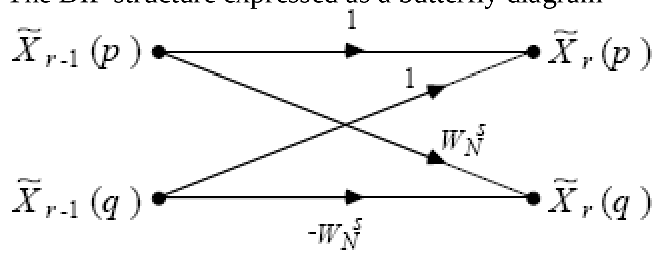
5. Draw the basic butterfly diagram for radix 2 DIT-FFT and DIF-FFT. **Nov/Dec 2007 CSE**

Butterfly Structure for DIT FFT **MAY 2006 ECESS &(NOV 2006 ITSS)**

The DIT structure can be expressed as a butterfly diagram



The DIF structure expressed as a butterfly diagram



6. What are the advantages of Bilinear mapping **April/May 2008 IT**

- Aliasing is avoided
- Mapping the S plane to the Z plane is one to one
- The closed left half of the S plane is mapped onto the unit disk of the Z plane

7. How may multiplication and addition is needed for radix-2 FFT? April/May 2008 IT

Number of complex addition is given by $N\log_2 N$

Number of complex multiplication is given by $N/2 \log_2 N$

8. Define DTFT pair? (May/June 2007)-ECE

The DTFT pairs are **(MAY 2006 IT)**

$X(k) = \quad x(n)e^{-j2\pi kn/N}$

$X(n) = \quad x(k)e^{j2\pi kn/N}$

9. Define Complex Conjugate of DFT property. (May/Jun 2007)-ECE

If $x(n) \underset{N}{\overset{DFT}{\leftrightarrow}} X(k)$ then

$X^*(n) \leftrightarrow (X^*(-k))_N = X^*(N-K)$

10.Differentiate between DIT and DIF FFT algorithms. (MAY 2006 IT)

S.No	DIT FFT algorithm	DIF FFT algorithm
1	Decimation in time FFT algorithm	Decimation in frequency FFT algorithm
2	Twiddle factor $k=(Nt/2^m)$	Twiddle factor $k=(Nt/2^{M-m+1})$

11.**Give any two properties of DFT** **(APR 2004 IT SS)**

Linearity : DFT [ax(n)+b y(n)]=a X(K)+bX(K)

Periodicity: x(n+N)=x(n) for all n

X(K+N)=X(K) for all n

12.What are the advantages of FFT algorithm over direct computation of DFT?
(May/June 2007)-ECE

The complex multiplication in the FFT algorithm is reduced by (N/2) log2N times.

Processing speed is very high compared to the direct computation of DFT.

13. What is FFT? (Nov/Dec 2006)-ECE

The fast Fourier transform is an algorithm is used to calculate the DFT. It is based on fundamental principal of decomposing the computation of DFT of a sequence of the length N in to successively smaller discrete Fourier Transforms. The FFT algorithm provides speed increase factor when compared with direct computation of the DFT.

14.Determine the DIFT of a sequence $x(n) = a^n u(n)$ (Nov/Dec 2006)-ECE

$X(K) = \quad x(n)\, e^{j2\pi kn/N}$

The given sequence $x(n) = a^n u(n)$

$DTFT\{x(n)\} = \quad x(n)\, e^{j2\pi kn/N}$
$= \quad (a\, e^{j2\pi k/N})^n$

Where $a^n = 1-a^n/(1-a)$
$X(K) = (1 - a^N e^{j2\pi k}) / (1-ae^{j2\pi k/N})$

15. What do you mean by in place computation in FFT. (APR 2005 IT)

FFT algorithms, for computing the DFT when the size N is a power of 2 and when it is a power of 4

16.Is the DFT is a finite length sequence periodic. Then state the reason (APR 2005 ITDSP)

DFT is a finite length sequence periodic.

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$$

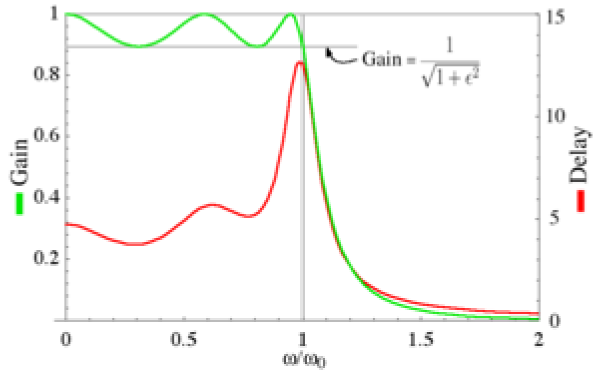
$X(e^{j\omega})$ is continuous & periodic in ω, with period 2π.

UNIT-III - IIR FILTER DESIGN

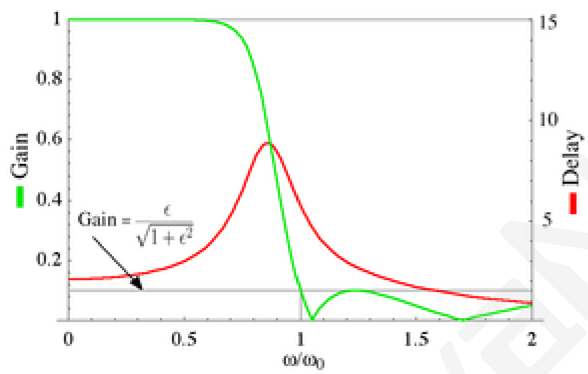
1. What are the requirements for converting a stable analog filter into a stable digital filter? Nov/Dec 2008 CSE

- The JΩ axis in the s plane should be map into the unit circle in the Z plane .thus there will be a direct relationship between the two frequency variables in the two domains
- The left half plane of the s plane should be map into the inside of the unit circle in the z – plane .thus the stable analog filter will be converted to a stable digital filter

2. Distinguish between the frequency response of chebyshev type I and Type II filter Nov/Dec 2008 CSE



Type I chebyshev filter



Type II chebyshev filter

Type I chebyshev filters are all pole filters that exhibit equirpple behavior in the pass band and monotonic in stop band .Type II chebyshev filters contain both poles and zeros and exhibits a monotonic behavior in the pass band and an equiripple behavior in the stop band

3. What is the need for prewraping in the design of IIR filter Nov/Dec 2008 CSE

The warping effect can be eliminated by prewarping the analog filter .This can be done by finding prewarping analog frequencies using the formula

$$\Omega = 2\tan^{-1}\Omega T/2$$

4.Write frequency translation for BPF from LPF April/May2008 CSE

Low pass with cut – off frequency Ω_C to band –pass with lower cut-off frequency Ω_1 and higher cut-off frequency Ω_2:

S ------------- $\Omega_C\ (s^2 + \Omega 1\ \Omega 2) / s\ (\Omega_2 - \Omega_1)$

The system function of the high pass filter is then

$$H(s) = H_p\ \{ \Omega_C\ (s^2 + \Omega 1\ \Omega 2) / s\ (\Omega_2 - \Omega_1)\}$$

5.Compare Butterworth, Chebyshev filters April/May2008 CSE

Butter Worth Filter	Chebyshev filters.
Magnitude response of Butterworth filter decreases monotonically, as frequency	Magnitude response of chebyshev filter exhibits ripple in pass band

increases from 0 ∞	
Poles on the butter worth lies on the circle	Poles of the chebyshev filter lies on the ellipse

6. Determine the order of the analog Butterworth filter that has a -2 db pass band attenuation at a frequency of 20 rad/sec and atleast -10 db stop band attenuation at 30 rad/sec.

Nov/Dec 2007CSE

α_p =2 dB; Ωp =20 rad/sec
α_s = 10 dB; Ωs = 30 rad/sec

$$N \geq \frac{\log\sqrt{10^{0.1\,\alpha s}-1/\,10^{0.1\,\alpha p}-1}}{\text{Log } \alpha_{s/}\,\alpha_p}$$

$$N \geq \frac{\log\sqrt{10}-1/\,10^{0.2}-1}{\text{Log } 30_{/}\,20}$$

≥ 3.37

Rounding we get N=4

7. By Impulse Invariant method, obtain the digital filter transfer function and differential equation of the analog filter H(s)=1 / (s+1) Nov/Dec 2007 CSE

H(s) =1/(s+1)

Using partial fraction

H(s) =A/(s+1)

= 1/(s-(-1)

Using impulse invariance method

$H(z) = 1/1-e^{-T}z^{-1}$

AssumeT=1sec

$H(z)=1/1-e^{-1}z^{-1}$

$H(z)=1/1-0.3678z^{-1}$

8.Distinguish between FIR and IIR filters. Nov/Dec 2007 CSE

Sl.No	IIR	FIR
1	H(n) is infinite duration	H(n) is finite duration
2	Poles as well as zeros are present. Sometimes all pole filters are also designed.	These are all zero filters.
3	These filters use feedback from output. They are recursive filters.	These filters do not use feedback. They are nonrecursive.
4	Nonlinear phase response. Lin phase is obtained if $H(z) = \pm Z^{-1}H(Z^{-1})$	Linear phase response for $h(n) = \pm h(m-1-n)$
5	These filters are to be designed stability	These are inherently stable filters

6	Number of multiplication requirement is less.	More
7	More complexity of implementation	Less complexity of implementation
8	Less memory is required	More memory is requied
9	Design procedure is complication	Less complicated
10	Design methods: 1. Bilinear Transform 2. Impulse invariance.	Design methods: 1. Windowing 2. Frequency sampling
11	Can be used where sharp cutoff characteristics with minimum order are required	Used where linear phase characteristic is essential.

9.Define Parsevals relation **April/May 2008 IT**

If X1(n) and X2(n) are complex valued sequences ,then

$$\sum_{-\infty}^{\infty} x1(n)x2^*(n) = 1/2\prod j \oint_c X1(v)X2*(\frac{1}{v}*)v^{-1}\,dv$$

10.What are the advantages and disadvantages of bilinear transformation? (May/June 2006)-ECE **Advantages:**

1. Many to one mapping.
2. linear frequency relationship between analog and its transformed digital frequency,

Disadvantage:

Aliasing

11.What is frequency warping? (MAY 2006 IT DSP)

The bilinear transform is a method of compressing the infinite, straight analog frequency axis to a finite one long enough to wrap around the unit circle only once. This is also sometimes called frequency warping. This introduces a distortion in the frequency. This is undone by pre-warping the critical frequencies of the analog filter (cutoff frequency, center frequency) such that when the analog filter is transformed into the digital filter, the designed digital filter will meet the desired specifications**.**

12. Give any two properties of Butterworth filter and chebyshev filter. (Nov/Dec 2006)

a. The magnitude response of the Butterworth filter decreases monotonically as the frequency increases (Ω) from 0 to ∞.
b. The magnitude response of the Butterworth filter closely approximates the ideal response as the order N increases.
c. The poles on the Butterworth filter lies on the circle.
d. The magnitude response of the chebyshev type-I filter exhibits ripple in the pass band.
e. The poles of the Chebyshev type-I filter lies on an ellipse.

S = (2/T) (Z-1) (Z+1)

13.Find the transfer function for normalized Butterworth filter of order 1 by determining the pole values. **(MAY 2006 IT DSP)**

Poles = 2N

N=1

Poles = 2

14..Differentiate between recursive and non-recursive difference equations.
(APR 2005 ITDSP)

The FIR system is a non-recursive system, described by the difference equation M-1

$$y(n) = \sum_{k=0} b_k x(n-k)$$

The IIR system is a non-recursive system, described by the difference equation

$$y(n) = \sum_{k=0}^{N} b_k x(n-k) - \sum_{k=1}^{M} a_k y(n-k)$$

15.Find the order and poles of Butterworth LPF that has -3dB bandwidth of 500 Hz and an attenuation of -40 dB at 1000 Hz. (NOV 2005 ITDSP)

α_p = -3dB α_s = -40dB Ω_s = 1000*2π rad/sec Ω_p=500*2π

The order of the filter $N \geq (\log(\lambda/\varepsilon))/(\log(\Omega_s/\Omega_p))$

$\lambda = (10^{0.1\alpha s}-1)^{1/2} = 99.995$

$\varepsilon = (10^{0.1\alpha p}-1)^{1/2} = 0.9976$

$N = (\log(99.995/0.9976))/(\log(2000\pi/1000\pi)) = 2/0.3 = 6.64$

$N \geq 6.64 = 7$

Poles=2N=14

16.What is impulse invariant mapping? What is its limitation? (Apr/May 2005)-ECE

The philosophy of this technique is to transform an analog prototype filter into an IIR discrete time filter whose impulse response [h(n)] is a sampled version of the analog filter's impulse response, multiplied by T.This procedure involves choosing the response of the digital filter as an equi-spaced sampled version of the analog filter.

17.Give the bilinear transformation. (Nov/Dec 2003)-ECE

The bilinear transformation method overcomes the effect of aliasing that is caused due to the analog frequency response containing components at or beyond the nyquist frequency. The bilinear transform is a method of compressing the infinite, straight analog frequency axis to a finite one long enough to wrap around the unit circle only once.

18.Mention advantages of direct form II and cascade structures. (APR 2004 ITDSP)

(i) The main advantage direct form-II structure realization is that the number of delay elements is reduced by half. Hence, the system complexity drastically reduces the number of memory elements .

(ii) Cascade structure realization, the system function is expressed as a product of several sub system functions. Each sum system in the cascade structure is realized in direct form-II. The order of each sub system may be two or three (depends) or more.

19. What is prewarping? (Nov/Dec 2003)-ECE

When bilinear transformation is applied, the discrete time frequency is related continuous time frequency as,

$$\Omega = 2\tan^{-1}\Omega T/2$$

This equation shows that frequency relationship is highly nonlinear. It is also called frequency warping. This effect can be nullified by applying prewarping. The specifications of equivalent analog filter are obtained by following relationship,

$$\Omega = 2/T \tan \omega/2$$

This is called prewarping relationship.

UNIT-IV - FIR FILTER DESIGN

1.What is gibb's Phenomenon. April/May2008 CSE

The oscillatory behavior of the approximation XN(W) to the function X(w) at a point of discontinuity of X(w) is called Gibb's Phenomenon

2.**Write procedure for designing FIR filter using windows. April/May2008 CSE**

1. Begin with the desired frequency response specification Hd(w)

2. Determine the corresponding unit sample response hd(n)
3. Indeed hd(n) is related to Hd(w) by the Fourier Transform relation.

3.**What are Gibbs oscillations?** **Nov/Dec 2007 CSE**

Oscillatory behavior observed when a square wave is reconstructed from finite number of harmonics.

The unit cell of the square wave is given by

$$Y(\nu) = \mathrm{rect}\left(\frac{\nu}{2\nu_c}\right)$$

Its Fourier series representation is

$$Y(\nu) = 2\nu_c \sum_{n=-\infty}^{\infty} \mathrm{sinc}(2n\nu_c) e^{-j2\pi n\nu}$$

4. **Explain briefly the need for scaling in the digital filter realization** **Nov/Dec 2007 CSE**

To prevent overflow, the signal level at certain points in the digital filters must be scaled so that no overflow occur in the adder

5. **What are the advantages of FIR filters?** **April/May 2008 IT**

1.FIR filter has exact linear phase

2.FIR filter always stable

3.FIR filter can be realized in both recursive and non recursive structure

4.Filters wit h any arbitrary magnitude response can be tackled using FIR sequency

6. Define Phase Dealy **April/May 2008 IT**

When the input signal X(n) is applied which has non zero response $\theta(w) = \arg\left[H\left(e^{jw}\right)\right]$ the output signal y(n) experience a delay with respect to the input signal .Let the input signal be

X(n)=A $\cos[won + \emptyset]$, $+\infty > n > -\infty$

Where A= Maximum Amplitude of the signal

Wo=Frequency in radians

f=phase angle

Due to the delay in the system response ,the output signal lagging in phase $\theta(wo)$ but the frequency remain the same

Y(n)=$\left|H(e^{jw})\right|$A $\cos[won + \theta(wo) + \emptyset]$,

In This equation that the output is the time delayed signal and is more commonly known as phase delayed at w=wo $\tau_{p(wo)} = -\theta(wo)/wo$ Is called phase delay

7. State the advantages and disadvantages of FIR filter over IIR filter. **(MAY 2006 IT DSP) & (NOV 2004 ECEDSP)**

Advantages of FIR filter over IIR filter

- It is a stable filter
- It exhibit linear phase, hence can be easily designed.
- It can be realized with recursive and non-recursive structures
- It is free of limit cycle oscillations when implemented on a finite word length digital system

Disadvantages of FIR filter over IIR filter

- Obtaining narrow transition band is more complex.
- Memory requirement is very high
- Execution time in processor implementation is very high.

8. List out the different forms of structural realization available for realizing a FIR system. (MAY 2006 IT DSP)

The different types of structures for realization of FIR system are

1.Direct form-I 2. Direct form-II

9. **What are the desirable and undesirable features of FIR Filters? (May/June 2006)-ECE**

The width of the main lobe should be small and it should contain as much of total energy as possible.The side lobes should decease in energy rapidly as w tends to π

10. Define Hanning and Blackman window functions. (May/June 2006)-ECE

The window function of a causal hanning window is given by

$W_{Hann}(n) = 0.5 – 0.5\cos 2\pi n/ (M-1)$, $0 \le n \le M-1$

0, Otherwise

The window function of non-causal Hanning window I s expressed by

$W_{Hann}(n) = 0.5 + 0.5\cos 2\pi n/ (M-1)$, $0 \le |n| \le (M-1)/2$

0, Otherwise

The width of the main lobe is approximately $8\pi/M$ and thee peak of the first side lobe is at -32dB**.**

The window function of a causal Blackman window is expressed by

$W_B(n) = 0.42 – 0.5 \cos 2\pi n/ (M-1) +0.08 \cos 4\pi n/(M-1)$, $0 \le n \le M-1$

= 0, otherwise

The window function of a non causal Blackman window is expressed by

$W_B(n) = 0.42 + 0.5 \cos 2\pi n/ (M-1) +0.08 \cos 4\pi n/(M-1)$, $0 \le |n| \le (M-1)/2$

= 0, otherwise

The width of the main lobe is approximately $12\pi/M$ and the peak of the first side lobe is at -58dB.

11. What is the condition for linear phase of a digital filter? (APR 2005 ITDSP)

$h(n) = h(M-1-n)$ → Linear phase FIR filter with a nonzero response at $\omega=0$

$h(n) = -h(M-1-n)$ →Low pass Linear phase FIR filter with a nonzero response at $\omega=0$

12. Define backward and forward predictions in FIR lattice filter. (NOV 2005 IT)

The reflection coefficient in the lattice predictor is the negative of the cross correlation coefficients between forward and backward prediction errors in the lattice.

13. List the important characteristics of physically realizable filters. (NOV 2005 ITDSP)

Symmetric and anti- symmetric

- Linear phase frequency response
- Impulse invariance

14. **Write the magnitude function of Butterworth filter. What is the effect of varying order of N on magnitude and phase response? (Nov/Dec2005) -ECE**

$|H(j\Omega)|^2 = 1 / [1 + (\Omega/\Omega_C)^{2N}]$ where N= 1,2,3,….

15. **List the characteristics of FIR filters designed using window functions. NOV 2004 ITDSP**

- the Fourier transform of the window function $W(e^{jw})$ should have a small width of main lobe containing as much of the total energy as possible
- the fourier transform of the window function $W(e^{jw})$ should have side lobes that decrease in energy rapidly as w to π. Some of the most frequently used window functions are described in the following sections

16. **Give the Kaiser Window function.** **(Apr/May 2004)-ECE**

The Kaiser Window function is given by

$W_K(n) = I_0(\beta) / I_0(\alpha)$, for $|n| \leq (M-1)/2$

Where α is an independent variable determined by Kaiser.

$B = \alpha[\ 1 - (2n/M-1)^2]$

17. What is meant by FIR filter? And why is it stable? (APR 2004 ITDSP)

FIR filter → Finite Impulse Response. The desired frequency response of a FIR filter can be represented as

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n}$$

If h(n) is absolutely summable(i.e., Bounded Input Bounded Output Stable). So, it is in stable.

18. Mention two transformations to digitize an analog filter. (APR 2004 ITDSP)

(i) Impulse-Invariant transformation techniques

(ii) Bilinear transformation techniques

19. Draw the direct form realization of FIR system. (NOV 2004 ITDSP)

20.Give the equation specifying Barlett and hamming window. (NOV 2004 ITDSP)

The transfer function of Barlett window

$w_B(n) = 1-(2|n|)/(N-1),\ ((N-1)/2) \geq n \geq -((N-1)/2)$

The transfer function of Hamming window

$w_{hm}(n) = 0.54+0.46\cos((2\pi n)/(N-1),\ ((N-1)/2) \geq n \geq -((N-1)/2)\quad \alpha = 0.54$

UNIT-V - FINITE WORD LENGTH EFFECTS

1. Compare fixed point and floating point arithmetic. Nov/Dec 2008 CSE&MAY 2006 IT

Fixed Point Arithmetic	Floating Point Arithmetic
• It covers only the dynamic range. • Compared to FPA, accuracy is poor • Compared to FPA it is low cost and easy to design • It is preferred for real time operation system • Errors occurs only for multiplication • Processing speed is high • Overflow is rare phenomenon	• It covers a large range of numbers • It attains its higher accuracy • Hardware implementation is costlier and difficult to design • It is not preferred for real time operations. • Truncation and rounding errors occur both for multiplication and addition • Processing speed is low • Overflow is a range phenomenon

2.What are the errors that arise due to truncation in floating point numbers **Nov/Dec 2008** **CSE**

1.Quantization error
2.Truncation error
Et=Nt-N

3.What are the effects of truncating an infinite flourier series into a finite series? **Nov/Dec 2008** **CSE**

4. Draw block diagram to convert a 500 m/s signal to 2500 m/s signal and state the problem due to this conversion **April/May2008** **CSE**

5.List errors due to finite world length in filter design **April/May2008** **CSE**

- Input quantization error
- Product quantization error
- Coefficient quantization error

5. What do you mean by limit cycle oscillations in digital filter? Nov/Dec 2007 CSE

In recursive system the nonlinearities due to the finite precision arithmetic operations often cause periodic oscillations to occur in the output ,even when the input sequence is zero or some non zero constant value .such oscillation in recursive system are called limit cycle oscillation

7.Define truncation error for sign magnitude representation and for 2's complement Representation April/May 2008 IT&APR 2005 IT

Truncation is a process of discarding all bits less significant than least significant bit that is retained For truncation in floating point system the effect is seen only in mantissa.if the mantissa is truncated to b bits ,then the error satisfies

$0 \geq \epsilon > -2.2^{-b}$ for $x > 0$ and

$0 \leq \epsilon < -2.2^{-b}$ for $x < 0$

8. What are the types of limit cycle oscillation? April/May 2008 IT

i.Zero input limit cycle oscillation

ii.overflow limit cycle oscillation

9. What is meant by overflow limit cycle oscillations? (May/Jun 2006)

In fixed point addition, overflow occurs due to excess of results bit, which are stored at the registers. Due to this overflow, oscillation will occur in the system. Thus oscillation is called as an overflow limit cycle oscillation.

10. How will you avoid Limit cycle oscillations due to overflow in addition(MAY 2006 IT DSP)

Condition to avoid the Limit cycle oscillations due to overflow in addition

$|a_1|+|a_2|<1$

a_1 and a_2 are the parameter for stable filter from stability triangle.

11.**What are the different quantization methods? (Nov/Dec 2006)-ECE**

- amplitude quantization
- vector quantization
- scalar quantization

12.List the advantages of floating point arithmetic. (Nov/Dec 2006)-ECE

- Large dynamic range
- Occurrence of overflow is very rare
- Higher accuracy

13.Give the expression for signal to quantization noise ratio and calculate the improvement with an increase of 2 bits to the existing bit.

(Nov/Dec 2006, Nov/Dec 2005)-ECE

$SNR_{A/D} = 16.81+6.02b-20\log_{10}(R_{FS}/\sigma_x)$ dB.

With b = 2 bits increase, the signal to noise ratio will increase by 6.02 X 2 = 12dB.

14. What is truncation error? (APR 2005 ITDSP)

Truncation is an approximation scheme wherein the rounded number or digits after the pre-defined decimal position are discarded.

15. What are decimators and interpolators? (APR 2005 ITDSP)

Decimation is a process of reducing the sampling rate by a factor D, i.e., down-sampling. Interpolation is a process of increasing the sampling rate by a factor I, i.e., up-sampling.

16.What is the effect of down sampling on the spectrum of a signal?

(APR 2005 ITDSP) & (APR 2005 ITDSP)

The signal (n) with spectrum X(ω) is to be down sampled by the factor D. The spectrum X(ω) is assumed to be non-zero in the frequency interval $0 \leq |\omega| \leq \pi$.

17.Give the rounding errors for fixed and floating point arithmetic.

(APR 2004 ITDSP)

A number x represented by b bits which results in b_R after being Rounded off. The quantized error ε_R due to rounding is given by

$$\varepsilon_R = Q_R(x) - x$$

where $Q_R(x)$ = quantized number(rounding error)

The rounding error is independent of the types of fixed point arithmetic, since it involves the magnitude of the number. The rounding error is symmetric about zero and falls in the range.

$$-((2^{-bT}-2^{-b})/2) \leq \varepsilon_R \leq ((2^{-bT}-2^{-b})/2)$$

ε_R may be +ve or –ve and depends on the value of x.

The error ε_R incurred due to rounding off floating point number is in the range

$$-2^E.2^{-bR/2}) \leq \varepsilon_R \leq 2^E.2^{-bR/2}$$

18.Define the basic operations in multirate signal processing.

(APR 2004 ITDSP)

The basic operations in multirate signal processing are

(i)Decimation

(ii)Interpolation

Decimation is a process of reducing the sampling rate by a factor D, i.e., down-sampling. Interpolation is a process of increasing the sampling rate by a factor I, i.e., up-sampling.

19. Define sub band coding of speech. (APR 2004 ITDSP) & (NOV 2003 ECEDSP) & (NOV 2005 ECEDSP)

Sub band coding of speech is a method by which the speech signal is subdivided into several frequency bands and each band is digitally encode separately. In the case of speech signal processing, most of its energy is contained in the low frequencies and hence can be coded with more bits then high frequencies.

20.What is the effect of quantization on pole locations? (NOV 2004 ITDSP)

$$D(z) = \prod_{k=1}^{N} (1-p_k z^{-1})$$

▲pk is the error or perturbation resulting from the quantization of the filter coefficients

21.What is an anti-imaging filter? (NOV 2004 ITDSP)

The image signal is due to the aliasing effect. In caseof decimation by M, there will be M-1 additional images of the input spectrum. Thus, the input spectrum X(ω) is band limited to the low pass frequency response. An anti-aliasing filter eliminates the spectrum of X(ω) in the range $(\pi/D \leq \omega \leq \pi$.

The anti-aliasing filter is LPF whose frequency response $H_{LPF}(\omega)$ is given by

$H_{LPF}(\omega) = 1, \ |\omega| \leq \pi/M$

$= 0$, otherwise.

D → Decimator

22.What is a decimator? If the input to the decimator is x(n)={1,2,-1,4,0,5,3,2}, What is the output? (NOV 2004 ITDSP)

Decimation is a process of reducing the sampling rate by a factor D, I.e., down-sampling.
x(n)={1,2,-1,4,0,5,3,2}
D=2
Output y(n) = {1,-1,0,3}

23.What is dead band? (Nov/Dec 2004)-ECE

In a limit cycle the amplitude of the output are confined to a range of value, which is called dead band.

24.How can overflow limit cycles be eliminated? (Nov/Dec 2004)-ECE

- Saturation Arithmetic
- Scaling

25.What is meant by finite word length effects in digital filters? (Nov/Dec 2003)-ECE

The digital implementation of the filter has finite accuracy. When numbers are represented in digital form, errors are introduced due to their finite accuracy. These errors generate finite precision effects or finite word length effects.

When multiplication or addition is performed in digital filter, the result is to be represented by finite word length (bits). Therefore the result is quantized so that it can be represented by finite word register. This quantization error can create noise or oscillations in the output. These effects are called finite word length effects.

PART B
UNIT-1 - SIGNALS AND SYSTEMS

1.Determine whether the following signals are Linear ,Time Variant, causal and stable

(1) Y(n)=cos[x(n)] Nov/Dec 2008 CSE
(2) Y(n)=x(-n+2)
(3) Y(n)=x(2n)
(4) Y(n)=x(n)+nx(n+1)

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.79)

2. Determine the causal signal x(n) having the Z transform Nov/Dec 2008 CSE

X(z)=$\frac{1}{(1+2z^{-1})(1-z^{-1})^2}$

Refer book : Digital signal processing by Ramesh Babu .(Pg no 2.66)

3. Use convolution to find x(n) if X(z) is given by Nov/Dec 2008 CSE

$$\frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$ **for ROC** $|z| > \frac{1}{2}$

Refer book : Digital signal processing by Ramesh Babu .(Pg no 2.62)

4.Find the response of the system if the input is {1,4,6,2} and impulse response of the system is {1,2,3,1}

April/May2008CSE

Refer book: Digital signal processing by A.Nagoor kani .(Pg no 23-24)

5.find r_{xy} and r_{yx} for x={1,0,2,3} and y={4,0,1,2}. **April/May2008 CSE**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.79)

6. (i) Check whether the system y(n)=ay(n-1)+x(n) is linear ,casual, shift variant, and stable

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.51-1.57)

(ii) Find convolution of $\{5,4,\underset{\uparrow}{3},2\}$ and $\{1,0,3,\underset{\uparrow}{2}\}$ **April/May2008 CSE**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.79)

7. (i) Compute the convolution y(n) of the signals

$$x(n)=\begin{cases} a^n, & -3\le n\le 5 \\ 0, & \text{elsewhere}\end{cases}$$

and

$$h(n)=\begin{cases} 1, & 0\le n\le 4 \\ 0, & \text{elsewhere}\end{cases}$$

Nov/Dec 2007 CSE

8.A discrete-time system can be static or dynamic, linear or nonlinear, Time invariant or time varying, causal or non causal, stable or unstable. Examine the following system with respect to the properties also.

(1) y(n) = cos [x(n)]
(2) y(n)=x(-n+2)
(3) y(n)=x(2n)
(4) y(n)=x(n).cosWo(n)

Nov/Dec 2007 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.185-1.197)

9.(i) Determine the response of the casual system.
y(n)-y(n-1)=x(n)+x(n-1) to inputs x(n)=u(n) and x(n)=2-n u(n). Test its stability.

(ii) Determine the IZT of X(z)=1 / [(1-z-1)(1-z-1)2] **Nov/Dec 2007 CSE**

Refer book : Digital signal processing by A.Nagoor kani . (Pg no 463)

10.(i)Determine the Z-transform of the signal $x(n)=a^n u(n)-b^n u(-n-1)$, b>a and plot the ROC.

Refer John G Proakis and Dimtris G Manolakis, “Digital Signal Processing Principles, Algorithms and Application”, PHI/Pearson Education, 2000, 3rd Edition. Page number (157)

(ii) Find the steady state value given $Y(z)=\{0.5/[(1-0.75z^{-1})(1-z^{-1})]\}$

Refer John G Proakis and Dimtris G Manolakis, “Digital Signal Processing Principles, Algorithms and Application”, PHI/Pearson Education, 2000, 3rd Edition. Page number (207)

(iii) Find the system function of the system described by

y(n)-0.75y(n-1)+0.125y(n-2)=x(n)-x(n-1) and plot the poles and zeroes of

11.(i) find the convolution and correlation for x(n)={0,1,-2,3,-4} and h(n)={0.5,1,2,1,0.5}.

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.79)

(ii)Determine the Impulse response for the difference equation

Y(n) + 3 y(n-1)+2y(n-2)=2x(n)-x(n-1) April/May2008 IT

Refer book : Digital signal processing by Ramesh Babu .(Pg no 2.57)

12. (i) Compute the z-transform and hence determine ROC of x(n) where

$$X(n) = \begin{cases} (1/3)^n \quad u(n).n \geq 0 \\ (1/2)^{-n} \quad u(n).n<0 \end{cases}$$

Refer book : Digital signal processing by Ramesh Babu .(Pg no 2.20)

(iii) prove the property that convolution in Z-domains multiplication in time domain April/May2008 IT

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.77)

13.Find the response of the system if the input is {1,4,6,2} and impulse response of the system is {1,2,3,1} April/May2008CSE

Refer book: Digital signal processing by A.Nagoor kani .(Pg no 23-24)

14.find r_{xy} and r_{yx} for x={1,0,2,3} and y={4,0,1,2}. April/May2008 CSE

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Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.51-1.57)

(ii) Find convolution of {5,4,3,2} and {1,0,3,2} April/May2008 CSE

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$$x(n)=\begin{cases} a^n, & -3\le n\le 5 \\ 0, & \text{elsewhere} \end{cases}$$

and

$$h(n)=\begin{cases} 1, & 0\le n\le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Nov/Dec 2007 CSE

17.A discrete-time system can be static or dynamic, linear or nonlinear, Time invariant or time varying, causal or non causal, stable or unstable. Examine the following system with respect to the properties also.

(1) $y(n) = \cos[x(n)]$

(2) $y(n)=x(-n+2)$

(3) $y(n)=x(2n)$

(4) $y(n)=x(n).\cos W_o(n)$

Nov/Dec 2007 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no 1.185-1.197)

18.(i) Determine the response of the casual system.
$y(n)-y(n-1)=x(n)+x(n-1)$ to inputs $x(n)=u(n)$ and $x(n)=2-n\ u(n)$. Test its stability.

(ii) Determine the IZT of X(z)=1 / [(1-z-1)(1-z-1)2] **Nov/Dec 2007 CSE**

Refer book : Digital signal processing by A.Nagoor kani .
(Pg no 463)

19.(i)Determine the Z-transform of the signal $x(n)=a^n u(n)-b^n u(-n-1)$, b>a and plot the ROC.

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (157)

(ii) Find the steady state value given $Y(z)=\{0.5/[(1-0.75z^{-1})(1-z^{-1})]\}$

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (207)

(iii) Find the system function of the system described by
$y(n)-0.75y(n-1)+0.125y(n-2)=x(n)-x(n-1)$ and plot the poles and zeroes of H(z). **(MAY 2006 ITDSP)**

Refer signals and systems by P. Ramesh babu , page no:10.65
(To find the impulse response h(n) and take z-transform.)

20.(i)Using Z-transform, compute the response of the system
y(n)=0.7y(n-1)-0.12y(n-2+x9n-1)+x(n-2) to the input x(n)=nu(n). Is the system stable?

Refer signals and systems by chitode, page no:4.99

(ii)State and prove the properties of convolution sum. **(MAY 2006 ECESS)**

Refer signals and systems by chitode, page no:4.43 to 4.45

21.State and prove the sampling theorem. Also explain how reconstruction of original signal is done from the sampled signal. (NOV 2006 ECESS)

Refer signals and systems by chitode, page no:3-2 to 3-7

22.Explain the properties of an LTI system. (NOV 2006 ECESS)

Refer signals and systems by chitode, page no:4.47 to 4.49

23.a. Find the convolution sum for the $x(n) = (1/3)^{-n} u(-n-1)$ and $h(n)=u(n-1)$

Refer signals and systems by P. Ramesh babu , page no:3.76,3.77

b. Convolve the following two sequences linearly x(n) and h(n) to get y(n).

x(n)= {1,1,1,1} and h(n) ={2,2}.Also give the illustration

Refer signals and systems by chitode, page no:67

c. Explain the properties of convolution. (NOV2006 ECESS)

Refer signals and systems by chitode, page no:4.43 to 4.45

24. Check whether the following systems are linear or not

1. **$y(n) = x^2(n)$**
2. **$y(n) = nx(n)$ (APRIL 2005 ITDSP)**

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (67)

25.(i)Determine the response of the system described by,

$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$ when the input sequence is $x(n)=4^n u(n)$.

Refer signals and systems by P. Ramesh babu , page no:3.23

(ii)Write the importance of ROC in Z transform and state the relationship between Z transforms to Fourier transform. (APRIL 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (153)

Refer S Poornachandra & B Sasikala, "Digital Signal Processing",
Page number (6.10)

UNIT-II - FAST FOURIER TRANSFORMS

1.By means of DFT and IDFT ,Determine the sequence x3(n) corresponding to the circular convolution of the sequence x1(n)={2,1,2,1}.x2(n)={1,2,3,4}. Nov/Dec 2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no 3.46)

2. State the difference between overlap save method and overlap Add method Nov/Dec 2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no 3.88)

3. Derive the key equation of radix 2 DIF FFT algorithm and draw the relevant flow graph taking the computation of an 8 point DFT for your illustration Nov/Dec 2008 CSE

Refer book : Digital signal processing by Nagoor Kani .(Pg no 215)

4. **Compute the FFT of the sequence x(n)=n+1 where N=8 using the in place radix 2 decimation in frequency algorithm. Nov/Dec 2008 CSE**

Refer book : Digital signal processing by Nagoor Kani .(Pg no 226)

5. Find DFT for {1,1,2,0,1,2,0,1} using FFT DIT butterfly algorithm and

plot the spectrum **April/May2008 CSE**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 4.17)

6. (i)Find IDFT for {1,4,3,1} using FFT-DIF method **April/May2008 CSE**

(ii)Find DFT for {1,2,3,4,1} **(MAY 2006 ITDSP)**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 4.29)

7.Compute the eight point DFT of the sequence x(n)={ ½,½,½,½,0,0,0,0} using radix2 decimation in time and radix2 decimation in frequency algorithm. Follow exactly the corresponding signal flow graph and keep track of all the intermediate quantities by putting them on the diagram. **Nov/Dec 2007 CSE**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 4.30)

8.(i) Discuss the properties of DFT.

Refer book : Digital signal processing by S.Poornachandra.,B.sasikala. (Pg no 749)

(ii)Discuss the use of FFT algorithm in linear filtering. **Nov/Dec 2007 CSE**

Refer book : Digital signal processing by John G.Proakis .(Pg no 447)

9.(i) if $x(n) \xleftrightarrow{\text{N pt DFT}} X(k)$ **then, prove that**

X1(n)x2(n)=1/N [Xt(k) X2 N]. **April/May2008 IT**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 3.34)

(ii) Find 8 Point DFT of $x(n)=\begin{cases}0.5, & 0\le n\le 3\\ 0, & 4\le n\le 7\end{cases}$ **Using DIT FFT** **April/May2008 IT**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 4.32)

10.Derive the equation for radix 4 FFT for N=4 and Draw the butterfly Diagram. **April/May2008 IT**

11. (i) Compute the 8 pt DFT of the sequence x(n)={0.5,0.5,0.5,0.5,0,0,0,0} using radix-2 DIT FFT

Refer P. Ramesh babu, "Signals and Systems".Page number (8.89)

(ii) Determine the number of complex multiplication and additions involved in a N-point Radix-2 and Radix-4 FFT algorithm. **(MAY 2006 ITDSP)**

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (456 & 465)

12.Find the 8-pt DFT of the sequence x(n)={1,1,0,0} **(APRIL 2005 ITDSP)**

Refer P. Ramesh babu, "Signals and Systems". Page number (8.58)

13.Find the 8-pt DFT of the sequence

$x(n)=\begin{cases}1, & 0\le n\le 7\\ 0, & \text{otherwise}\end{cases}$

using Decimation-in-time FFT algorithm **(APRIL 2005 ITDSP)**

Refer P. Ramesh babu, "Signals and Systems".Page number (8.87)

14.Compute the 8 pt DFT of the sequence x(n)={0.5,0.5,0.5,0.5,0,0,0,0} using DIT FFT **(NOV 2005 ITDSP)**

Refer P. Ramesh babu, "Signals and Systems".Page number (8.89)

15.By means of DFT and IDFT , determine the response of an FIR filter with impulse response h(n)={1,2,3},n=0,1,2 to the input sequence x(n) ={1,2,2,1}.

(NOV 2005 ITDSP)

Refer P. Ramesh babu, "Signals and Systems".Page number (8.87)

16.(i)Determine the 8 point DFT of the sequence

x(n)= {0,0,1,1,1,0,0,0}

Refer P. Ramesh babu, "Signals and Systems".Page number (8.58)

(ii)Find the output sequence y(n) if h(n)={1,1,1} and x(n)={1,2,3,4} using circular convolution (APR 2004 ITDSP)

Refer P. Ramesh babu, "Signals and Systems".Page number (8.65)

17. (i)What is decimation in frequency algorithm? Write the similarities and differences between DIT and DIF algorithms. (APR 2004 ITDSP) & (MAY 2006 ECEDSP)

Refer P. Ramesh babu, "Signals and Systems". Page number (8.70-8.80)

18.Determine 8 pt DFT of x (n)=1for -3≤n≤3 using DIT-FFT algorithm (APR 2004 ITDSP)

Refer P. Ramesh babu, "Signals and Systems". Page number (8.58)

19.Let X(k) denote the N-point DFT of an N-point sequence x(n).If the DFT of X(k)is computed to obtain a sequence $x_1(n)$. Determine $x_1(n)$ in terms of x(n) (NOV 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (456 & 465)

UNIT-III - IIR FILTER DESIGN

1.Design a digital filter corresponding to an analog filter H(s)= $\frac{0.5(s+4)}{(s+1)(s+4)}$ **using the impulse invariant method to work at a sampling frequency of 100 samples/sec**

Nov/Dec 2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no5.40)

2.Determine the direct form I ,direct form II ,Cascade and parallel structure for the system Y(n)=-0.1y(n-1)+0.72y(n-2)+0.7x(n)-0.25x(n-2) Nov/Dec 2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no5.61)

3.What is the main drawback of impulse invariant method ?how is this overcome by bilinear transformation? Nov/Dec 2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no5.46)

4.Design a digital butter worth filter satisfying the constraints Nov/Dec 2008 CSE

0.707≤$\left|H(e^{jw}\right|$**≤1 for 0 ≤w≤**$\frac{\pi}{2}$

$\left|H(e^{jw}\right|$**≤0.20 for** $3\frac{\pi}{4}$ **≤w≤**π

With T=1 sec using bilinear transformation .realize the same in Direct form II

Refer book : Digital signal processing by Ramesh Babu .(Pg no5.79)

5. (i)Design digital filter with H(s) =$1/(s2+7s+12)$ **using T=1sec.**

(ii)Design a digital filter using bilinear transform for H(s)=2/(s+1)(s+2)with cutoff frequency as 100 rad/sec and sampling time =1.2 ms April/May2008 CSE

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 341)

6. (i) Realize the following filter using cascade and parallel form with

direct form –I structure

$$\frac{1+z^{-1}+z^{-2+}5z^{-3}}{(1+Z^{-1})(1+2Z^{-1}+4Z^{-2})}$$

(ii) Find H(s) for a 3rd order low pass butter worth filter April/May2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no 5.8)

7.(i) Derive bilinear transformation for an analog filter with system function H(s) =b / (s+a)

Refer book: Digital signal processing by John G.Proakis .(Pg no 676-679)

(ii)Design a single pole low pass digital IIR filter with -3 db bandwidth of 0.2π by use of bilinear transformation. Nov/Dec 2007 CSE

8.(i) Obtain the Direct Form I, Direct Form II, cascade and parallel realization for the following system Y(n)= -0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2)

Refer book : Digital signal processing by Ramesh Babu .(Pg no 5.68)

(ii) Discuss the limitation of designing an IIR filter using impulse invariant method. Nov/Dec 2007 CSE

Refer book : Digital signal processing by A.Nagoor kani . (Pg no 330)

9. Design a low pass Butterworth filter that has a 3 dB cut off frequency of 1.5 KHz and an attenuation of 40 dB at 3.0 kHz April/May2008 IT

Refer book : Digital signal processing by Ramesh Babu .(Pg no 5.14)

10. (i) Use the Impulse invariance method to design a digital filter from an analog prototype that has a system function

April/May2008 IT

$Ha(s)=s+a/((s+a)^2+b^2)$

Refer book : Digital signal processing by Ramesh Babu .(Pg no 5.42)

(ii) Determine the order of Cheybshev filter that meets the following specifications

(1) 1 dB ripple in the pass band 0≤|w| ≤ 0.3 b

(2) Atleast 60 dB attrnuation in the stop band 0.35∏ ≤|w| ≤∏ Use Bilinear Transformation

Refer book : Digital signal processing by Ramesh Babu .(Pg no 5.27)

11.(i) Convert the analog filter system function$H_a(s)=\{(s+0.1)/[(s+0.1)^2+9]\}$ into a digital IIR filter using impulse invariance method.(Assume T=0.1sec) (APR 2006 ECEDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (675)

12.Determine the Direct form II realization for the following system: y(n)=-0.1y(n-1+0.72y(n-2)+0.7x(n)-0.252x(n-2). (APRIL 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (601-7.9b)

13.Explain the method of design of IIR filters using bilinear transform method.

(APRIL 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3^{rd} Edition. Page number (676-8.3.3)

14.Explain the following terms briefly:

(i)Frequency sampling structures

(ii)Lattice structure for IIR filter (NOV 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3^{rd} Edition. Page number (506 &531)

15.Consider the system described by

y(n)-0.75y(n-1)+0.125y(n-2)=x(n)+0.33x(n-1).

Determine its system function (NOV 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3^{rd} Edition. Page number (601-7.37)

16.Find the output of an LTI system if the input is x(n)=(n+2) for 0≤n≤3 and h(n)=a^nu(n) for all n (APR 2004 ITDSP)

Refer signals and systems by P. Ramesh babu , page no:3.38

17.Obtain cascade form structure of the following system:

y(x)=-0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2) (APR 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3^{rd} Edition. Page number (601-7.9c)

18.Verify the Stability and causality of a system with

$H(z)=(3-4Z^{-1})/(1+3.5Z^{-1}+1.5Z^{-2})$ (APR 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3^{rd} Edition. Page number (209)

UNIT-IV - FIR FILTER DESIGN

1.Design a FIR linear phase digital filter approximating the ideal frequency response Nov/Dec 2008 CSE

$$Hd(w) = \begin{Bmatrix} 1, for\ |w| \le \frac{\pi}{6} \\ 0, for\ \frac{\pi}{6} < |w| \le \pi \end{Bmatrix}$$

With T=1 Sec using bilinear transformation .Realize the same in Direct form II

Refer book : Digital signal processing by Nagoor Kani .(Pg no 367)

2.Obtain direct form and cascade form realizations for the transfer function of the system given by

$$H(z) = \left(1 + \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$$ **Nov/Dec 2008 CSE**

Refer book : Digital signal processing by Nagoor Kani .(Pg no 78)

3.Explain the type I frequency sampling method of designing an FIR filter.

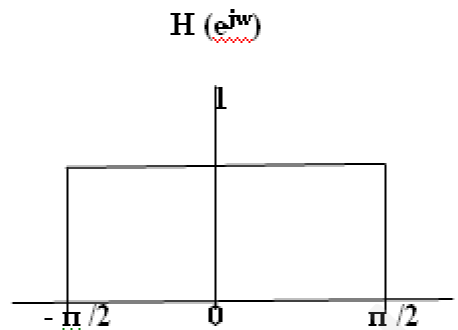
Nov/Dec 2008
CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no6.82)

4.Compare the frequency domain characteristics of various window functions .Explain how a linear phase FIR filter can be used using window method. Nov/Dec 2008 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no6.28)

5. Design a LPF for the following response .using hamming window with N=7



April/May2008 CSE

6. (i) Prove that an FIR filter has linear phase if the unit sample response satisfies the condition h(n)= ±h(M-1-n), n=0,1,….M-1. Also discuss symmetric and antisymmetric cases of FIR filter. Nov/Dec 2007

Refer book: Digital signal processing by John G.Proakis .
(Pg no 630-632)

(ii) Explain the need for the use of window sequences in the design of FIR filter. Describe the window sequences generally used and compare their properties.
Nov/Dec 2007 CSE

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 292-295)

7.(I) Explain the type 1 design of FIR filter using frequency sampling technique. Nov/Dec 2007 CSE

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 630-632)

(ii)A low pass filter has the desired response as given below

$$\mathbf{Hd(e^{jw})=}\begin{cases} e^{-i3w}, & 0\le w<\prod/2 \\ 0, & \prod/2\le<\prod \end{cases}$$

Determine the filter coefficients h(n) for M=7 using frequency sampling method.

Nov/Dec 2007
CSE

8.(i) For FIR linear phase Digital filter approximating the ideal frequency response

$$\mathbf{Hd(w) = 1 \quad \le|w| \le\prod /6}$$

$$\mathbf{0 \quad \prod /6\le |w| \le\prod}$$

Determine the coefficients of a 5 tap filter using rectangular Window

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 415

(ii) Determine the unit sample response h(n) of a linear phase FIR filter of Length M=4 for which the frequency response at w=0 and w= ∏/2 is given as Hr(0) ,Hr(∏/2) =1/2

April/May2008 IT

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 310)

9.(i) Determine the coefficient h(n) of a linear phase FIR filter of length M=5 which has symmetric unit sample response and frequency response

$$H_r(k) = \begin{cases} 1 & \text{for } k=0,1,2,3 \\ 0.4 & \text{for } k=4 \\ 0 & \text{for } k=5,6,7 \end{cases}$$

April/May2008 IT(NOV 2005 ITDSP)

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 308)

(ii) Show that the equation $\sum_{n=0}^{m-1} h(n)=\sin(wj-wn)=0$,is satisfied for a linear phase FIR filter of length 9

April/May2008 IT

10. Design linear HPF using Hanning Window with N=9

$$H(w) = \begin{cases} 1 & -\pi \text{ to } W_c \text{ and } W_c \text{ to } \pi \\ 0 & \text{otherwise} \end{cases}$$

April/May2008 IT

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 301)

11.Explain in detail about frequency sampling method of designing an FIR filter.

(NOV 2004 ITDSP) & (NOV 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (630)

12.Explain the steps involved in the design of FIR Linear phase filter using window method.

(APR 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (8.2.2 & 8.2.3)

13.(i)What are the issues in designing FIR filter using window method?

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (8.2)

(ii)An FIR filter is given by

y(n)=2x(n)+(4/5)x(n-1)+(3/2)x(n-2)+(2/3)x(n-3) find the lattice structure coefficients **(APR 2004 ITDSP)**

Refer S Poornachandra & B Sasikala, "Digital Signal Processing", Page number (FIR-118)

UNIT-V - FINITE WORD LENGTH EFFECTS

1.Draw the circuit diagram of sample and hold circuit and explain its operation

Nov/Dec 2008 CSE/ Nov/Dec 2007 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no1.172)

2. The input of the system y(n)=0.99y(n-1)+x(n) is applied to an ADC .what is the power produced by the quantization noise at the output of the filter if the input is quantized to 8 bits

Nov/Dec 2008 CSE

Refer book : Digital signal processing by Nagoor Kani .(Pg no 423)

3.Discuss the limit cycle in Digital filters **Nov/Dec 2008 CSE**

Refer book : Digital signal processing by Nagoor Kani .(Pg no 420)

4.What is vocoder? Explain with a block diagram Nov/Dec 2008 CSE/ Nov/Dec 2007 CSE

Refer book : Digital signal processing by Ramesh Babu .(Pg no10.7)

(ii) Discuss about multirate Signal processing **April/May 2008 CSE**

Refer book : Digital signal processing by Ramesh Babu .(Pg no 8.1)

5. (i) Explain how the speech compression is achieved .

(ii) Discuss about quantization noise and derive the equation for finding quantization noise power. **April/May2008CSE**

Refer book : Digital signal processing by Ramesh Babu.(Pg no 7.9-7.14)

6. Two first order low pass filter whose system functions are given below are connected in cascade. Determine the overall output noise power. H1(z) = 1/ (1-0.9z-1) and H2(z) = 1/ (1-0.8z-1) **Nov/Dec 2007 CSE**

Refer book: Digital signal processing by Ramesh Babu. (Pg no 7.24)

7. Describe the quantization errors that occur in rounding and truncation in two's complement. **Nov/Dec 2007 CSE**

Refer book : Digital signal processing by John G.Proakis .(Pg no 564)

8. Explain product quantization and prove $\sigma_{err}^2 = \sum_{i=1}^{m} \sigma^2_{oi}$ **April/May2008 IT**

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 412)

9.A cascade Realization of the first order digital filter is shown below ,the system function of the individual section are $H1(z)=1/(1-0.9z^{-1})$ and $H2(z) =1/(1-0.8z^{-1})$ .Draw the product quantization noise model of the system and determine the overall output noise power

April/May2008 IT

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 415)

9. (i) Show dead band effect on y(n) = .95 y(n-1)+x(n) system restricted to 4 bits .Assume x(0) =0.75 and y(-1)=0

Refer book : Digital signal processing by A.Nagoor kani .(Pg no 423-426)

11. Explain the following terms briefly:

(i)Perturbation error

(ii)Limit cycles **(NOV 2005 ITDSP)**

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number(7.7.1 &7.7.2)

12.(i) Explain clearly the downsampling and up sampling in multirate signal processing. **(APRIL 2005 ITDSP)**

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing

Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (784-790)

(ii)Explain subband coding of speech signal

(NOV 2003 ITDSP) & (NOV 2004 ITDSP) & (NOV 2005 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number(831-833)

13.(i) Derive the spectrum of the output signal for a decimator

(ii) Find and sketch a two fold expanded signal y(n) for the input

(APR 2004 ITDSP) &(NOV 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (788)

14.(i)Propose a scheme for sampling rate conversion by a rational factor I/D.

(NOV 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (790)

15. Write applications of multirate signal processing in Musical sound processing

(NOV 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (952)

16. With examples illustrate (i) Fixed point addition (ii) Floating point multiplication (iii) Truncation (iv) Rounding.(APR 2005 ITDSP) & (NOV 2003 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (7.5)

17. Describe a single echo filter using in musical sound processing.

(APRIL 2004 ITDSP)

Refer John G Proakis and Dimtris G Manolakis, "Digital Signal Processing Principles, Algorithms and Application", PHI/Pearson Education, 2000, 3rd Edition. Page number (12.5.3)

Reg. No. :

M 2459

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Fifth Semester

Electronics and Communication Engineering

EC 333 — DIGITAL SIGNAL PROCESSING

(Common to Bio-Medical Engineering)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Check for linearity and causality of the system $y(n) = Cos\ \omega nT$.

2. State two properties of Z-transform.

3. Prove that convolution in the time domain is multiplication in the frequency domain.

4. Draw the basic butterfly of DIT – FFT structure.

5. What is limit cycle oscillation?

6. State the advantages and disadvantages of FIR filters.

7. Write the expression for Hanning window.

8. When do you decimate a signal?

9. What is interpolation?

10. What is quantization noise?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Represent the signal $y(n) = x(2n) + x(n-1)$ where $x(n)$ is the input and y(n) is the output. (8)

(ii) Explain the procedure to perform linear and circular convolution. (8)

Or

(b) Explain in detail the steps in the computation of FFT using DIF algorithm. (16)

12. (a) Design a FIR filter with the following characteristics using rectangular window with $M = 7$ and determine $h(n)$ (16)

$$H_d(e^{j\omega}) = 1,\ 0 \le |\omega| \le \pi/2 = 0,\ \pi/2 \le |\omega| < \pi$$

Or

(b) Discuss the various window functions available for constructing linear phase FIR filters. (16)

13. (a) Design a Butterworth filter with the following characteristics using bilinear transformation method using $T = 1$ s. (16)

$$0.8 \le \left|H(e^{j\omega})\right| \le 1 \text{ for } 0 \le \omega \le 0.2\pi$$

$$\left|H(e^{j\omega})\right| \le 0.2 \text{ for } 0.6\pi \le \omega \le \pi$$

Or

(b) Explain briefly how cascade and parallel realization of filters are done. (16)

14. (a) Explain fixed and floating point representation in detail. (16)

Or

(b) Explain the various errors that occur in a DSP system. (16)

15. (a) With a neat block diagram explain decimation and interpolation. (16)

Or

(b) Discuss mean, variance, co-variance of a Discrete Random signal. (16)

DEPARTMENT OF ECE

2 MARKS & QUESTION- ANSWERS

EC 1302- Digital Signal Processing

Class: S5 ECE (A&B)

Electronics & Communication Engineering.

Digital Signal Processing
S5ECE A&B

1. What is a continuous and discrete time signal?
Ans:
Continuous time signal: A signal x(t) is said to be continuous if it is defined for all time t. Continuous time signal arise naturally when a physical waveform such as acoustics wave or light wave is converted into a electrical signal. This is effected by means of transducer.(e.g.) microphone, photocell.
Discrete time signal: A discrete time signal is defined only at discrete instants of time. The independent variable has discrete values only, which are uniformly spaced. A discrete time signal is often derived from the continuous time signal by sampling it at a uniform rate.

2. Give the classification of signals?
Ans:
Continuous-time and discrete time signals
Even and odd signals
Periodic signals and non-periodic signals
Deterministic signal and Random signal
Energy and Power signal

3. What are the types of systems?
Ans:
Continuous time and discrete time systems
Linear and Non-linear systems
Causal and Non-causal systems
Static and Dynamic systems
Time varying and time in-varying systems
Distributive parameters and Lumped parameters systems
Stable and Un-stable systems.

4. What are even and odd signals?
Ans:
Even signal: continuous time signal x(t) is said to be even if it satisfies the condition x(t)=x(-t) for all values of t.
Odd signal: he signal x(t) is said to be odd if it satisfies the condition x(-t)=-x(t) for all t. In other words even signal is symmetric about the time origin or the vertical axis, but odd signals are anti-symmetric about the vertical axis.

5. What are deterministic and random signals?

Ans:

Deterministic Signal: deterministic signal is a signal about which there is no certainty with respect to its value at any time. Accordingly we find that deterministic signals may be modeled as completely specified functions of time.

Random signal: random signal is a signal about which there is uncertainty before its actual occurrence. Such signal may be viewed as group of signals with each signal in the ensemble having different wave forms

.(e.g.) The noise developed in a television or radio amplifier is an example for random signal.

6. What are energy and power signal?

Ans:

Energy signal: signal is referred as an energy signal, if and only if the total energy of the signal satisfies the condition $0<E<\infty$. The total energy of the continuous time signal x(t) is given as

$E=\lim_{T\to\infty}\int x^2(t)dt$, integration limit from –T/2 to +T/2

Power signal: signal is said to be powered signal if it satisfies the condition $0<P<\infty$. The average power of a continuous time signal is given by

$P=\lim_{T\to\infty}1/T\int x2(t)dt$, integration limit is from-T/2 to +T/2.

7. What are the operations performed on a signal?

Ans:

Operations performed on dependent variables:

Amplitude scaling: y (t) =cx (t), where c is the scaling factor, x(t) is the continuous time signal.

Addition: $y(t)=x_1(t)+x_2(t)$

Multiplication y (t)=x1(t)x2(t)

Differentiation: y (t)=d/dt x(t)

Integration (t) =$\int x(t)dt$

Operations performed on independent variables

Time shifting

Amplitude scaling

Time reversal

8. What are elementary signals and name them?

Ans:

The elementary signals serve as a building block for the construction of more complex signals. They are also important in their own right, in that they may be used to model many physical signals that occur in nature.

There are five elementary signals. They are as follows

Unit step function

Unit impulse function

Ramp function
Exponential function
Sinusoidal function

9. What are the properties of a system?

Ans:

Stability: A system is said to be stable if the input x(t) satisfies the condition(t) $|\leq M_x<\infty$ and the out put satisfies the condition $|y(t)|\leq M_y<\infty$ for all t.

Memory: A system is said to be memory if the output signal depends on the present and the past inputs.

Invertibility: A system is said to be invertible if the input of the system con be recovered from the system output.

Time invariance: A system is said to be time invariant if a time delay or advance of the input signal leads to an identical time shift in the output signal.

Linearity: A system is said to be linear if it satisfies the super position principle

$$i.e.)\ R(ax_1(t)+bx_2(t))=ax_1(t)+bx_2(t)$$

10. What is memory system and memory less system?

Ans:

A system is said to be *memory system* if its output signal at any time depends on the past values of the input signal. circuit with inductors capacitors are examples of memory system..

A system is said to be *memory less system* if the output at any time depends on the present values of the input signal. An electronic circuit with resistors is an example for memory less system.

11. What is an invertible system?

Ans:

A system is said to be *invertible system* if the input of the system can be recovered from the system output. The set of operations needed to recover the input as the second system connected in cascade with the given system such that the output signal of the second system is equal to the input signal applied to the system.

$$H^{-1}\{y(t)\}=H^{-1}\{H\{x(t)\}\}.$$

12. What are time invariant systems?

Ans:

A system is said to be time invariant system if a time delay or advance of the input signal leads to an idenditical shift in the output signal. This implies that a time invariant system responds idenditically no matter when the input signal is applied. It also satisfies the condition

$$R\{x(n-k)\}=y(n-k).$$

13. Is a discrete time signal described by the input output relation $y[n]= r^n x[n]$ time invariant.

Ans:

A signal is said to be time invariant if R{x[n-k]}= y[n-k]

R{x[n-k]}=R(x[n]) / x[n]→x[n-k]

$=r^n x[n-k]$ ---------------- (1)

y[n-k]=y[n] / n→n-k

$=r^{n-k} x[n-k]$ -------------------(2)

Equations (1)≠Equation(2)

Hence the signal is time variant.

14. Show that the discrete time system described by the input-output relationship y[n] =nx[n] is linear?

Ans:

For a sys to be linear $R\{a_1x_1[n]+b_1x_2[n]\}=a_1y_1[n]+b_1y_2[n]$

L.H.S:$R\{ a_1x_1[n]+b_1x_2[n] \}=R\{x[n]\}$ /$x[n] \rightarrow a_1x_1[n]+b_1x_2[n]$

$= a_1 nx_1[n]+b_1 nx_2[n]$ -------------------(1)

R.H.S: $a_1y_1[n]+b_1y_2[n]= a_1 nx_1[n]+b_1 nx_2[n]$ --------------------(2)

Equation(1)=Equation(2)

Hence the system is linear

15. What is SISO system and MIMO system?

Ans:

A control system with single input and single output is referred to as single input single output system. When the number of plant inputs or the number of plant outputs is more than one the system is referred to as multiple input output system. In both the case, the controller may be in the form of a digital computer or microprocessor in which we can speak of the digital control systems.

16. What is the output of the system with system function H_1 and H_2 when connected in cascade and parallel?

Ans:

When the system with input x(t) is connected in cascade with the system H1 and H2 the output of the system is

$$y(t)=H_2\{H_1\{x(t)\}\}$$

When the system is connected in parallel the output of the system is given by

$$y(t)=H_1x_1(t)+H_2x_2(t).$$

17. What do you mean by periodic and non-periodic signals?

A signal is said to be periodic if

x(n+N)=x(n)

Where N is the time period.

A signal is said to be non-periodic if

x(n+N)≠x(n) .

18.Determine the convolution sum of two sequences x(n) = {3, 2, 1, 2} and

h(n) = {1, 2, 1, 2}

Ans: y(n) = {3,8,8,12,9,4,4}

19.Find the convolution of the signals

x(n) = 1 n=-2,0,1
= 2 n=-1
= 0 elsewhere.

Ans: y(n) = {1,1,0,1,-2,0,-1}

20.Detemine the solution of the difference equation

y(n) = 5/6 y(n-1) – 1/6 y(n-2) + x(n) for $x(n) = 2^n u(n)$

Ans: $y(n) = -(1/2)^n u(n) + 2/3(1/3)^n u(n)+8/5\ 2^n u(n)$

21.Determine the response y(n), n>=0 of the system described by the second order difference equation

y(n) – 4y(n-1) + 4y(n-2) = x(n) – x(n-1) when the input is $x(n) = (-1)^n u(n)$ and the initial condition are y(-1) = y(-2)=1.

Ans:$y(n) = (7/9-5/3n)2^n u(n) +2/9(-1)^n u(n)$

22. Differentiate DTFT and DFT

DTFT output is continuous in time where as DFT output is Discrete in time.

23.Differentiate between DIT and DIF algorithm

DIT – Time is decimated and input is bi reversed format output in natural order
DIF – Frequency is decimated and input is natural order output is bit reversed format.

24. How many stages are there for 8 point DFT

8

25 How many multiplication terms are required for doing DFT by expressional method and FFT method

expression –n2 FFT N /2 log N

26. Distinguish IIR and FIR filters

FIR	IIR
Impulse response is finite They have perfect linear phase	Impulse Response is infinite They do not have perfect linear phase
Non recursive	Recursive
Greater flexibility to control the shape of magnitude response	Less flexibility

27. Distinguish analog and digital filters

Analog	digital
Constructed using active or passive components and it is described by a differential equation	Consists of elements like adder, subtractor and delay units and it is described by a difference equation
Frequency response can be changed by changing the components	Frequency response can be changed by changing the filter coefficients
It processes and generates analog output	Processes and generates digital output
Output varies due to external conditions	Not influenced by external conditions

28. Write the expression for order of Butterworth filter?

The expression is $N=\log (\lambda /€)^{1/2}/\log (1/k)^{½}$

29. Write the expression for the order of chebyshev filter?

$N=\cosh^{-1}(\lambda /e)/\cosh^{-1}(1/k)$

30. Write the various frequency transformations in analog domain?

LPF to LPF:s=s/Ωc
LPF to HPF:s=Ωc/s
LPF to BPF:s=s2xlxu/s(xu-xl)
LPF to BSF:s=s(xu-xl)?s2=xlxu. X=Ω

31. Write the steps in designing chebyshev filter?

1. Find the order of the filter.
2. Find the value of major and minor axis. λ
3. Calculate the poles.
4. Find the denominator function using the above poles.
5. The numerator polynomial value depends on the value of n.

If n is odd: put s=0 in the denominator polynomial.
If n is even put s=0 and divide it by $(1+e2)^{1/2}$

32. Write down the steps for designing a Butterworth filter?

1. From the given specifications find the order of the filter
2 find the transfer function from the value of N

3. Find Ωc
4 find the transfer function ha(s) for the above value of Ωc by su s by that value.

33. State the equation for finding the poles in chebyshev filter

sk=acos¢k+jbsin¢k,where ¢k=∏/2+(2k-1)/2n)∏

34. State the steps to design digital IIR filter using bilinear method

Substitute s by 2/T (z-1/z+1), where T=2/Ω (tan (w/2) in h(s) to get h (z)

35. What is warping effect?

For smaller values of w there exist linear relationship between w and .but for larger values of w the relationship is nonlinear. This introduces distortion in the frequency axis. This effect compresses the magnitude and phase response. This effect is called warping effect

36. Write a note on pre warping.

The effect of the non linear compression at high frequencies can be compensated. When the desired magnitude response is piecewise constant over frequency, this compression can be compensated by introducing a suitable rescaling or prewar ping the critical frequencies.

37. Give the bilinear transform equation between s plane and z plane

s=2/T (z-1/z+1)

38. Why impulse invariant method is not preferred in the design of IIR filters other than low pass filter?

In this method the mapping from s plane to z plane is many to one. Thus there ire an infinite number of poles that map to the same location in the z plane, producing an aliasing effect. It is inappropriate in designing high pass filters. Therefore this method is not much preferred.

39. By impulse invariant method obtain the digital filter transfer function and the differential equation of the analog filter h(s) =1/s+1

H (z) =1/1-$e^{-T}z^{-1}$

Y/x(s) =1/s+1
Cross multiplying and taking inverse lap lace we get,
D/dt(y(t)+y(t)=x(t)

40. What is meant by impulse invariant method?

In this method of digitizing an analog filter, the impulse response of the resulting digital filter is a sampled version of the impulse response of the analog filter. For e.g. if the transfer function is of the form, 1/s-p, then

$H(z) = 1/1-e^{-pT}z^{-1}$

41. What do you understand by backward difference?

One of the simplest methods of converting analog to digital filter is to approximate the differential equation by an equivalent difference equation.

$d/dt(y(t)/t=nT=(y(nT)-y(nT-T))/T$

42. What are the properties of chebyshev filter?

1. The magnitude response of the chebyshev filter exhibits ripple either in the stop band or the pass band.
2. The poles of this filter lies on the ellipse

43. Give the Butterworth filter transfer function and its magnitude characteristics for different orders of filter.

The transfer function of the Butterworth filter is given by

$H(j\Omega) = 1/1+j(\Omega/\Omega c)\ N$

44. Give the magnitude function of Butterworth filter.

The magnitude function of Butterworth filter is

$|h(j\Omega)=1/[1+(\Omega/\Omega c)^{2N}]^{1/2}$,N=1,2,3,4,….

45. Give the equation for the order N, major, minor axis of an ellipse in case of chebyshev filter?

The order is given by $N=\cosh^{-1}(((10^{.1\alpha p})-1/10^{.1\alpha s}-1)1/2))/\cosh^{-1}\Omega s/\Omega p$

$A = (\mu^{1/N}-\mu^{-1/N})/2\Omega p$

$B=\Omega p\ (\mu^{1/N}+ \mu^{-1/N})/2$

46. Give the expression for poles and zeroes of a chebyshev type 2 filters

The zeroes of chebyshev type 2 filter $SK=j\Omega s/\sin k\Phi k$, k=1….N

The poles of this filter xk+jyk

$xk= \Omega s\sigma k/ \Omega s^2+\sigma k^2$

$yk=\Omega s\Omega k/ \Omega s^2+\sigma k^2 \quad \sigma k=a\cos\Phi k$

47. How can you design a digital filter from analog filter?

Digital filter can de designed from analog filter using the following methods

1. Approximation of derivatives
2. Impulse invariant method
3. Bilinear transformation

48. write down bilinear transformation.

s=2/T (z-1/z+1)

49. List the Butterworth polynomial for various orders.

N	Denominator polynomial
1	S+1
2	S^2+.707s+1
3	(s+1)(s^2+s+1)
4	(s^2+.7653s+1)(s^2+1.84s+1)
5	(s+1)(s^2+.6183s+1)(s^2+1.618s+1)
6	(s^2+1.93s+1)(s^2+.707s+1)(s^2+.5s+1)
7	(s+1)(s^2+1.809s+1)(s^2+1.24s+1)(s^2+.48s+1)

50. Differentiate Butterworth and Chebyshev filter.

Butterworth dampimg factor 1.44 chebyshev 1.06
Butterworth flat response damped response.

51. What is filter?

Filter is a frequency selective device ,which amplify particular range of frequencies and attenuate particular range of frequencies.

52. What are the types of digital filter according to their impulse response?

IIR(Infinite impulse response)filter
FIR(Finite Impulse Response)filter.

53. How phase distortion and delay distortion are introduced?

The phase distortion is introduced when the phase characteristics of a filter is nonlinear with in the desired frequency band.
The delay distortion is introduced when the delay is not constant with in the desired frequency band.

54. what is mean by FIR filter?

The filter designed by selecting finite number of samples of impulse response (h(n) obtained from inverse fourier transform of desired frequency response H(w)) are called FIR filters

55. Write the steps involved in FIR filter design

Choose the desired frequency response $H_d(w)$
Take the inverse fourier transform and obtain $H_d(n)$
Convert the infinite duration sequence $H_d(n)$ to h(n)
Take Z transform of h(n) to get H(Z)

56. What are advantages of FIR filter?

Linear phase FIR filter can be easily designed .

Efficient realization of FIR filter exists as both recursive and non-recursive structures.

FIR filter realized non-recursively stable.

The round off noise can be made small in non recursive realization of FIR filter.

57. What are the disadvantages of FIR FILTER

The duration of impulse response should be large to realize sharp cutoff filters.

The non integral delay can lead to problems in some signal processing applications.

58. What is the necessary and sufficient condition for the linear phase characteristic of a FIR filter?

The phase function should be a linear function of w, which inturn requires constant group delay and phase delay.

59. List the well known design technique for linear phase FIR filter design?

Fourier series method and window method

Frequency sampling method.

Optimal filter design method.

60. Define IIR filter?

The filter designed by considering all the infinite samples of impulse response are called IIR filter.

61. For what kind of application , the antisymmetrical impulse response can be used?

The ant symmetrical impulse response can be used to design Hilbert transforms and differentiators.

62. For what kind of application , the symmetrical impulse response can be used?

The impulse response ,which is symmetric having odd number of samples can be used to design all types of filters ,i.e , lowpass,highpass,bandpass and band reject.

The symmetric impulse response having even number of samples can be used to design lowpass and bandpass filter.

63.What is the reason that FIR filter is always stable?

FIR filter is always stable because all its poles are at the origin.

64.What condition on the FIR sequence h(n) are to be imposed n order that this filter can be called a liner phase filter?

The conditions are

(i) Symmetric condition h(n)=h(N-1-n)

(ii) Antisymmetric condition h(n)=-h(N-1-n)

65. Under what conditions a finite duration sequence h(n) will yield constant group delay in its frequency response characteristics and not the phase delay?

If the impulse response is anti symmetrical ,satisfying the condition

$$H(n)=-h(N-1-n)$$

The frequency response of FIR filter will have constant group delay and not the phase delay .

66. State the condition for a digital filter to be causal and stable?

A digital filter is causal if its impluse response $h(n)=0$ for $n<0$.

A digital filter is stable if its impulse response is absolutely summable ,i.e,

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

67. What are the properties of FIR filter?

1.FIR filter is always stable.
2.A realizable filter can always be obtained.
3.FIR filter has a linear phase response.

68. When cascade from realization is preferred in FIR filters?

The cascade from realization is preferred when complex zeros with absolute magnitude less than one.

69. What are the disadvantage of Fourier series method ?

In designing FIR filter using Fourier series method the infinite duration impulse response is truncated at $n=\pm(N-1/2)$.Direct truncation of the series will lead to fixed percentage overshoots and undershoots before and after an approximated discontinuity in the frequency response .

70. What is Gibbs phenomenon?
OR
What are Gibbs oscillations?

One possible way of finding an FIR filter that approximates $H(ej\omega)$would be to truncate the infinite Fourier series at $n=\pm(N-1/2)$.Abrupt truncation of the series will lead to oscillation both in pass band and is stop band .This phenomenon is known as Gibbs phenomenon.

71. What are the desirable characteristics of the windows?

The desirable charaterstics of the window are

1.The central lobe of the frequency response of the window should contain most of the energy and should be narrow.
2.The highest side lobe level of the frequency response should be small.
3.The sides lobes of the frequency response should decrease in energy rapidly as ω tends to π .

72. Compare Hamming window with Kaiser window.

Hamming window	Kaiser window

1.The main lobe width is equal to8π/N and the peak side lobe level is –41dB. 2.The low pass FIR filter designed will have first side lobe peak of –53 dB	The main lobe width ,the peak side lobe level can be varied by varying the parameter α and N. The side lobe peak can be varied by varying the parameter α.

73.What is the necessary and sufficient condition for linear phase characteristics in FIR filter?

The necessary and sufficient condition for linear phase characteristics in FIR filter is the impulse response h(n) of the system should have the symmetry property,i.e,

$$H(n) = h(N-1-n)$$

Where N is the duration of the sequence .

74.What are the advantage of Kaiser widow?

1.It provides flexibility for the designer to select the side lobe level and N .

2. It has the attractive property that the side lobe level can be varied continuously from the low value in the Blackman window to the high value in the rectangle window .

75.What is the principle of designing FIR filter using frequency sampling method?

In frequency sampling method the desired magnitude response is sampled and a linear phase response is specified .The samples of desired frequency response are defined as DFT coefficients. The filter coefficients are then determined as the IDFT of this set of samples.

76.For what type of filters frequency sampling method is suitable?

Frequency sampling method is attractive for narrow band frequency selective filters where only a few of the samples of the frequency response are non-zero.

77.What is meant by autocorrelation?

The autocorrelation of a sequence is the correlation of a sequence with its shifted version, and this indicates how fast the signal changes.

78.Define white noise?

A stationary random process is said to be white noise if its power density spectrum is constant. Hence the white noise has flat frequency response spectrum.

$S_X(w) = \sigma_x^2, -\pi \leq w\pi$

79.what do you understand by a fixed-point number?

In fixed point arithmetic the position of the binary point is fixed. The bit to the right represent the fractional part of the number & those to the left represent the integer part.

For example, the binary number 01.1100 has the value 1.75 in decimal.

80.What is the objective of spectrum estimation?

The main objective of spectrum estimation is the determination of the power spectral density of a random process. The estimated PSD provides information about the structure of the random process which can be used for modeling, prediction or filtering of the deserved process.

81.List out the addressing modes supported by C5X processors?

1. Direct addressing
2. Indirect addressing
3. Immediate addressing
4. Dedicated-register addressing
5. Memory-mapped register addressing
6. Circular addressing

82.what is meant by block floating point representation? What are its advantages?

In block point arithmetic the set of signals to be handled is divided into blocks. Each block have the same value for the exponent. The arithmetic operations with in the block uses fixed point arithmetic & only one exponent per block is stored thus saving memory. This representation of numbers is more suitable in certain FFT flow graph & in digital audio applications.

83.what are the advantages of floating point arithmetic?

1. Large dynamic range
2. Over flow in floating point representation is unlike.

84.what are the three-quantization errors to finite word length registers in digital filters?

1. Input quantization error 2. Coefficient quantization error 3. Product quantization error

85.How the multiplication & addition are carried out in floating point arithmetic?

In floating point arithmetic, multiplication are carried out as follows,

Let f1 = M1*2c1 and f2 = M2*2c2. Then f3 = f1*f2 = (M1*M2) 2(c1+c2)

That is, mantissa is multiplied using fixed-point arithmetic and the exponents are added.

The sum of two floating-point number is carried out by shifting the bits of the mantissa of the smaller number to the right until the exponents of the two numbers are equal and then adding the mantissas.

86.What do you understand by input quantization error?

In digital signal processing, the continuous time input signals are converted into digital using a b-bit ACD. The representation of continuous signal amplitude by a fixed digit produce an error, which is known as input quantization error.

87.List the on-chip peripherals in 5X.

The C5X DSP on-chip peripherals available are as follows:

1. Clock Generator
2. Hardware Timer
3. Software-Programmable Wait-State Generators
4. Parallel I/O Ports
5. Host Port Interface (HPI)
6. Serial Port
7. Buffered Serial Port (BSP)
8. Time-Division Multiplexed (TDM) Serial Port
9. User-Maskable Interrupts

88.what is the relationship between truncation error e and the bits b for representing a decimal into binary?

For a 2's complement representation, the error due to truncation for both positive and negative values of x is 0>=xt-x>-2-b

Where b is the number of bits and xt is the truncated value of x.

The equation holds good for both sign magnitude, 1's complement if x>0

If x<0, then for sign magnitude and for 1's complement the truncation error satisfies.

89.what is meant rounding? Discuss its effect on all types of number representation?

Rounding a number to b bits is accomplished by choosing the rounded result as the b bit number closest to the original number unrounded.

For fixed point arithmetic, the error made by rounding a number to b bits satisfy the inequality

$$\frac{-2-b}{2} \leq xt-x \leq \frac{2-b}{2}$$

for all three types of number systems, i.e., 2's complement, 1's complement & sign magnitude.

For floating point number the error made by rounding a number to b bits satisfy the inequality

$$-2-b \leq E \leq 2-b \qquad \text{where } E=\frac{xt-x}{x}$$

90.what is meant by A/D conversion noise?

A DSP contains a device, A/D converter that operates on the analog input x(t) to produce xq(t) which is binary sequence of 0s and 1s.

At first the signal x(t) is sampled at regular intervals to produce a sequence x(n) is of infinite precision. Each sample x(n) is expressed in terms of a finite number of bits given the sequence xq(n). The difference signal e(n)=xq(n)-x(n) is called A/D conversion noise.

91.what is the effect of quantization on pole location?

Quantization of coefficients in digital filters lead to slight changes in their value. This change in value of filter coefficients modify the pole-zero locations. Some times the pole locations will be changed in such a way that the system may drive into instability.

92.which realization is less sensitive to the process of quantization?

Cascade form.

93.what is meant by quantization step size?

Let us assume a sinusoidal signal varying between +1 and -1 having a dynamic range 2. If the ADC used to convert the sinusoidal signal employs b+1 bits including sign bit, the number of levels available for quantizing x(n) is 2b+1. Thus the interval between successive levels

$$q = \frac{2}{2b+1} = 2-b$$

Where q is known as quantization step size.

94.How would you relate the steady-state noise power due to quantization and the b bits representing the binary sequence?

Steady state noise power

Where b is the number of bits excluding sign bit.

95.what is overflow oscillation?

The addition of two fixed-point arithmetic numbers cause over flow the sum exceeds the word size available to store the sum. This overflow caused by adder make the filter output to oscillate between maximum amplitude limits. Such limit cycles have been referred to as over flow oscillations.

96.what are the methods used to prevent overflow?

There are two methods used to prevent overflow

1. Saturation arithmetic 2. Scaling

97.what are the two kinds of limit cycle behavior in DSP?

1.zero input limit cycle oscillations

2.Overflow limit cycle oscillations

98.What is meant by "dead band" of the filter

The limit cycle occur as a result of quantization effect in multiplication. The amplitudes of the output during a limit cycle are confined to a range of values called the dead band of the filter.

99.Explain briefly the need for scaling in the digital filter implementation.

To prevent overflow, the signal level at certain points in the digital filter must be scaled so that no overflow occurs in the adder.

100.What are the different buses of TMS320C5X and their functions?

The C5X architecture has four buses and their functions are as follows:

Program bus (PB):

It carries the instruction code and immediate operands from program memory space to the CPU.

Program address bus (PAB):

It provides addresses to program memory space for both reads and writes.

Data read bus (DB):

It interconnects various elements of the CPU to data memory space.

Data read address bus (DAB):

It provides the address to access the data memory space.

Part B

1. Determine the DFT of the sequence

$x(n) = 1/4$, for $0 <= n <= 2$

0, otherwise

Ans: The N point DFT of the sequence x(n) is defined as

$$x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \qquad K=0,1,2,3,\ldots N-1$$

$x(n) = (1/4, 1/4, 1/4)$

$X(k) = \frac{1}{4} e^{-j2\pi k/3}[1+2\cos(2\pi k/3)]$ where $k = 0,1,\ldots\ldots\ldots,N-1$

2. Derive the DFT of the sample data sequence $x(n) = \{1,1,2,2,3,3\}$ and compute the corresponding amplitude and phase spectrum.

Ans: The N point DFT of the sequence x(n) is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \qquad K=0,1,2,3,\ldots N-1$$

$X(0) = 12$

$X(1) = -1.5 + j2.598$

$X(2) = -1.5 + j0.866$

$X(3) = 0$

$X(4) = -1.5 - j0.866$

$X(5) = -1.5 - j2.598$

$X(k) = \{12, -1.5 + j2.598, -1.5 + j0.866, 0, -1.5 - j0.866, -1.5 - j2.598\}$

$|X(k)| = \{12, 2.999, 1.732, 0, 1.732, 2.999\}$

$\angle X(k)=\{0,-\pi/3,-\pi/6,0,\pi/6,\pi/3\}$

3.Given x(n) = {0,1,2,3,4,5,6,7} find X(k) using DIT FFT algorithm.

Ans: Given N = 8

$W_N^k = e^{-j(2\pi/N)k}$
$W_8^0 = 1$
$W_8^1 = 0.707-j0.707$
$W_8^2 = -j$
$W_8^3 = -0.707-j0.707$

Using butterfly diagram

X(k) = {28,-4+j9.656,-4+j4,-4+j1.656,-4,-4-j1.656,-4-j4,-4-j9.656}

4.Given X(k) = {28,-4+j9.656,-4+j4,-4+j1.656,-4,-4-j1.656,-4-j4,-4-j9.656} ,find x(n) using inverse DIT FFT algorithm.

$W_N^k = e^{j(2\pi/N)k}$
$W_8^0 = 1$
$W_8^1 = 0.707+j0.707$
$W_8^2 = j$
$W_8^3 = -0.707+j0.707$

x(n) = {0,1,2,3,4,5,6,7}

5.Find the inverse DFT of X(k) = {1,2,3,4}

Ans: The inverse DFT is defined as

$$x(n)=(1/N)\sum_{k=0}^{N-1} x(k)e^{j2\pi nk/N} \qquad n=0,1,2,3,\ldots N-1$$

x(0) = 5/2
x(1) = -1/2-j1/2
x(2) = -1/2
x(3) = -1/2+j1/2
x(n) = {5/2, -1/2-j1/2, -1/2, -1/2+j1/2}

6. Design an ideal low pass filter with a frequency response $H_d(e^{jw}) =1$ for $-\prod/2 <= w <= \prod/2$

0 otherwise

find the value of h(n) for N=11 find H(Z) plot magnitude response

a. Find h(n) by IDTFT
b. Convert h(n) in to a fine length by truncation
c. H(0)=1/2,

$h(1)=h(-1)=0.3183$
$h(2)=h(-2)=0$
$h(3)=h(-3)= -0.106$
$h(4)=h(-4)=0$
$h(5)=h(-5)=0.06366$

d. Find the transfer function H(Z) which is not realizable conver in to realizable by multiplying by $z^{-(N-1/2)}$
e. H'(Z) obtained is $0.06366-0.106z^{-2}+.3183Z^{-4}+.5Z^{-5}+.3183Z^{-6}-.106Z^{-8}+0.06366Z^{-10}$
f. Find H (e^{jw}) and plot amplitude response curve.

7. Design an ideal low pass filter with a frequency response $H_d(e^{jw}) =1$ for $-\prod/4<=|w|<=\prod$
0 otherwise
find the value of h(n) for N=11 find H(Z) plot magnitude response

g. Find h(n) by IDTFT
h. Convert h(n) in to a fine length by truncation
i. $H(0)=0.75$
$h(1)=h(-1)=-.22$
$h(2)=h(-2)=-.159$
$h(3)=h(-3)= -0.075$
$h(4)=h(-4)=0$
$h(5)=h(-5)=0.045$
j. Find the transfer function H(Z) which is not realizable conver in to realizable by multiplying by $z^{-(N-1/2)}$
k. H'(Z) obtained is $0.045-0.075z^{-2}$ -.159 Z-3-$0.22Z^{-4}+0.75Z^{-5}-.22Z^{-6}$ $-0.159Z^{-7}$ -. $075Z^{-8}+0.045Z^{-10}$
l. Find H (e^{jw}) and plot amplitude response curve.

8. Design band pass filter with a frequency response $H_d(e^{jw}) =1$ for $-\prod/3<=|w|<=2\prod/3$
0 otherwise
find the value of h(n) for N=11 find H(Z) plot magnitude response

m. Find h(n) by IDTFT
n. Convert h(n) in to a fine length by truncation
o. Find the transfer function H(Z) which is not realizable conver in to realizable by multiplying by $z^{-(N-1/2)}$
p. H'(Z) obtained Find H (e^{jw}) and plot amplitude response curve.

9. Design band reject filter with a frequency response $H_d(e^{jw}) = 1$ for $\prod/4 <= |w| <= 3\prod/4$

0 otherwise

find the value of h(n) for N=11 find H(Z) plot magnitude response

q. Find h(n) by IDTFT
r. Convert h(n) in to a fine length by truncation
s. Find the transfer function H(Z) which is not realizable conver in to realizable by multiplying by $z^{-(N-1/2)}$
t. H'(Z) obtained Find H (e^{jw}) and plot amplitude response curve.

10. Derive the condition of FIR filter to be linear in phase.

Conditions are
Group delay and Phase delay should be constant
And
And show the condition is satisfied

11Derive the expression for steady state I/P Noise Power and Steady state O/P Noise Power.

Write the derivation.

12 Draw the product quantatization model for first order and second order filter

Write the difference equation and draw the noise model.

13 For the second order filter Draw the direct form II realization and find the scaling factor S0 to avoid over flow

Find the scaling factor from the formula

$$I = \frac{1+r2}{(1-r2)(1-2r2\cos2\text{ø} = r4)}$$

14 Explain Briefly about various number representation in digital computer.

1 Fixed point
2 Floating point
3 Block floating point

Signed magnitude represenation
1's Complement
2's Complement
etc

15 Consider the transfer function H(Z)=H1(Z)H2(Z) where H1(Z) =1/1-a1Z-1
H2(z) =1/ 1-a2Z-1
Find the o/p Round of noise power Assume a1=0.5 and a2= 0.6 and find o.p round off noise power.

Draw the round of Noise Model.
By using residue method find σ01
By using residue method find σ 02
$= \sigma_{01}\ 2 + \sigma\ 02\ \ 2$

$$\text{Ans:}\ \frac{2^{-2b}\,(5.43)}{12}$$

16.Explain the architecture of DSP processor .

Diagram. & explanation.

17.Describe briefly the different methods of power spectral estimation?

1. Bartlett method
2. Welch method
3. Blackman-Tukey method

and its derivation.

18.what is meant by A/D conversion noise. Explain in detail?

A DSP contains a device, A/D converter that operates on the analog input x(t) to produce xq(t) which is binary sequence of 0s and 1s.

At first the signal x(t) is sampled at regular intervals to produce a sequence x(n) is of infinite precision. Each sample x(n) is expressed in terms of a finite number of bits given the sequence xq(n). The difference signal e(n)=xq(n)-x(n) is called A/D conversion noise.

+ derivation.

19 onsider the transfer function H(Z)=H1(Z)H2(Z) where H1(Z) =1/1-a1Z-1
H2(z) =1/ 1-a2Z-1
Find the o/p Round of noise power Assume a1=0.7 and a2= 0.8and find o.p round off noise power.

Draw the round of Noise Model.
By using residue method find σ01
By using residue method find σ 02
$= \sigma_{01}\ 2 + \sigma\ 02\ \ 2$

20.Given X(k) = {1,1,1,1,1,1,1,1,} ,find x(n) using inverse DIT FFT algorithm.

$$W_N^k = e^{j(2\pi/N)k}$$

Find x(n)

21.Find the inverse DFT of $X(k) = \{3,4,5,6\}$

Ans: The inverse DFT is defined as

$$x(n)=(1/N)\sum_{k=0}^{N-1} x(k)e^{j2\pi nk/N} \qquad n=0,1,2,3,\ldots N-1$$

22. Explain various addressing modes of TMS processor.

Immediate.
Register
Register indirect
Indexed

& its detail explanation.

23 Derive the expression for steady state I/P Noise Variance and Steady state O/P Noise Variance

Write the derivation.

24. Explain briefly the periodogram method of power spectral estimation?

Write the derivation with explanation.

25. Explain various arithmetic instruction of TMS processor.

All arithmetic instruction with explanation.

Reg. No. : | | | | | | | | | | | |

V 4558

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Fifth Semester

(Regulation 2004)

Electronics and Communication Engineering

EC 1302 — DIGITAL SIGNAL PROCESSING

(Common to B.E. (Part-Time) Fourth Semester Regulation 2005)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Define the properties of convolution.

2. Draw the basic butterfly diagram of radix – 2 FFT.

3. What are the merits and demerits of FIR filters?

4. What is the relationship between analog and digital frequency in impulse invariant transformation?

5. What are the three types of quantization error occurred in digital systems?

6. What is meant by limit cycle oscillations?

7. What is a periodogram?

8. Determine the frequency resolution of the Bartlett method of power spectrum estimates for a quality factor Q = 15. Assume that the length of the sample sequence is 1500.

9. What is meant by pipelining?

10. What is the principal feature of the Harvard architecture?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Discuss in detail the important properties of the Discrete Fourier Transform. (8)

(ii) Find the 4 point DFT of the sequence (8)

$x(n) = Cos\ n\pi/4$.

Or

(b) (i) Using decimation-in-time draw the butterfly line diagram for 8 point FFT calculation and explain. (8)

(ii) Compute an 8 point DFT using DIF FFT radix 2 algorithm. (8)

$X(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

12. (a) (i) Determine the magnitude response of an FIR filter (M = 11) and show that the phase and group delays are constant. (8)

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}$$

(ii) If the desired response of a low-pass filter is

$$H_d(e^{iw}) = e^{-j3w}, \quad -3\pi/4 \le w \le 3\pi/4$$
$$0, \quad 3\pi/4 < |w| \le \pi$$ (8)

Determine $H(e^{iw})$ for $M = 7$ using a Hamming window.

Or

(b) (i) For the analog transfer function $H(s) = \dfrac{1}{(s+1)(s+2)}$ determine $H(z)$ using impulse invariant technique. Assume $T = 1s$. (6)

(ii) Design a digital Butterworth filter that satisfies the following constraint using bilinear transformation $(T = 1s)$ (10)

$$0.9 \le |H(e^{iw})| \le 1 \quad \text{for } 0 \le w \le \pi/2$$
$$|H(e^{iw})| \le 0.2 \quad \text{for } 3\pi/4 \le w \le \pi$$

13. (a) (i) Discuss in detail the Truncation error and Round-off error for sign magnitude and two's complement representation. (8)

(ii) Explain the quantization effects in converting analog signal into digital signal. (8)

Or

(b) (i) A digital system is characterized by the difference equation

$$y(n) = 0.9y\ (n-1) + x(n)$$

With $x(n) = 0$ and initial condition $y(-1) = 12$. Determine the dead band of the system. (4)

(ii) What is meant by the co-efficient quantization? Explain. (12)

14\. (a) (i) Explain the Barlett method of averaging periodograms. (8)

(ii) What is the relationship between autocorrelation and power spectrum? Prove it. (8)

Or

(b) (i) Derive the mean and variance of the power spectral estimate of the Blackman and Tukey method. (8)

(ii) Obtain the expression for mean and variance of the auto correlation function of random signals. (8)

15\. (a) (i) Describe the multiplier and accumulator unit in DSP processors. (6)

(ii) Explain the architecture of TMS 320 C5X DSP processor. (10)

Or

(b) (i) Discuss in detail the four phases of the pipeline techniques. (8)

(ii) Write short notes on :

(1) Parallel logic unit. (4)

(2) Circular registers. (4)

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C 3189

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2007.

Fifth Semester

(Regulation 2004)

Electronics and Communication Engineering

EC 1302 — DIGITAL SIGNAL PROCESSING

(Common to B.E. (Part-Time) Fourth Semester - Regulation 2005)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The first five DFT coefficients of a sequence $x[n]$ are $X(0) = 20$, $X(1) = 5 + j2$, $X(2) = 0$, $X(3) = 0.2 + j0.4$, $X(4) = 0$. Determine the remaining DFT coefficients.

2. What are the advantages of FFT algorithm over direct computation of DFT?

3. Show that the filter with $h[n] = \{-1, 0, 1\}$ is a linear phase filter.

4. Find the digital transfer function $H(z)$ by using impulse invariant method for the analog transfer function $H(s) = \dfrac{1}{s+2}$. Assume T = 0.5 sec.

5. Identify the various factors which degrade the performance of the digital filter implementation when finite word length is used.

6. What is meant by limit cycle oscillation in digital filters?

7. Define power spectral density and cross power spectral density.

8. What are the disadvantages of non-parametric methods of power spectral estimation?

9. Differentiate between Von Neumann and Harvard architectures.

10. State the merit and demerit of multiported memories.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Prove the following properties of DFT when $H[k]$ is the DFT of an N-point sequence $h[n]$.

(1) $H[k]$ is real and even when $h[n]$ is real and even.

(2) $H[k]$ is imaginary and odd when $h[n]$ is real and odd. (8)

(ii) Compute the DFT of $x[n] = e^{-0.5n}$, $0 \le n \le 5$. (8)

Or

(b) (i) From first principles obtain the signal flow graph for computing 8-point DFT using radix-2 decimation-in-frequency FFT algorithm. (8)

(ii) Using the above signal flow graph compute DFT of $x[n] = \cos(n\pi/4)$, $0 \le n \le 7$. (8)

12. (a) A bandpass FIR filter of length 7 is required. It is to have lower and upper cut-off frequencies of 3 kHz and 6 kHz respectively and is intended to be used with a sampling frequency of 24 kHz. Determine the filter coefficients using Hanning window. Consider the filter to be causal. (16)

Or

(b) Determine $H(z)$ for a Butterworth filter satisfying the following specifications :

$$0.8 \le |H(e^{j\omega})| \le 1, \; 0 \le \omega \le \pi/4$$
$$|H(e^{j\omega})| \le 0.2, \; \pi/2 \le \omega \le \pi$$

Assume T = 0.1 sec. Apply bilinear transformation method.

13. (a) (i) Consider the truncation of negative fraction numbers represented in $(\beta+1)$-bit fixed point binary form including sign bit. Let $(\beta-b)$ bits be truncated. Obtain the range of truncation errors for signed magnitude, 2's complement and 1's complement representations of the negative numbers. (8)

(ii) The coefficients of a system defined by

$$H(z)=\frac{1}{(1-0.4z^{-1})(1-0.55z^{-1})}$$ are represented in a number with a sign bit and 3 data bits. Determine the new pole locations for (1) direct realization and (2) cascade realization of first order systems. Compare the movements of the new poles away from the original ones in both the cases. (8)

Or

(b) (i) Consider $(b+1)$-bit (including sign bit) bipolar A/D converter. Obtain an expression for signal to quantization noise ratio. State the assumptions made. (8)

(ii) A causal IIR filter is defined by the difference equation $y[n]=x[n]-0.9y[n-1]$. The unit sample response $h[n]$ is computed such that the computed values are rounded to one decimal place. Show that the filter exhibits dead band effect. Determine the dead band range. (8)

14. (a) (i) Compute the autocorrelation and power spectral density for the signal $X(t)=A\cos(2\pi f_0 t+\Phi)$ when A and f_0 are constants. Φ is a random variable which is uniformly distributed over the interval $(-\pi, \pi)$. (8)

(ii) Explain briefly the periodogram method of power spectral estimation. (8)

Or

(b) (i) Describe briefly Bartlett, Welch and Blackman-Tukey methods of power spectral estimation. (10)

(ii) Determine the frequency resolution of the Bartlett, Welch and Blackmann-Tukey methods of power spectral estimation for a quality factor $Q=20$. Assume that overlap in Welch's method is 50% and the length of the sample is 2000. (6)

15. (a) (i) Explain what is meant by instruction pipelining. Explain with an example, how pipelining increases the through put efficiency. (8)

(ii) Explain the operation of TDM serial ports in P-DSPs. (8)

Or

(b) (i) With a suitable diagram describe the functions of multiplier/adder unit of TMS 320C54X. (8)

(ii) Explain the operation of CSSU of TMS320C 54X and explain its use considering the Viteri operator. (8)

T 8164

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fifth Semester

Electronics and Communication Engineering

EC 1302 — DIGITAL SIGNAL PROCESSING

(Common to B.E. (Part–Time) Fourth Semester Regulation – 2005)

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Determine the DTFT of a sequence $x(n) = a^n u(n)$.

2. What is FFT?

3. Obtain the block diagram representation of a FIR system.

4. Give any two properties of Butterworth filter and Chebyshev filter.

5. Express the fraction 7/8 and −7/8 in sign magnitude, 2's complement and 1's complement.

6. (a) What are the quantization errors due to finite word length registers in digital filters?

   (b) What are the different quantization methods?

7. What is zero padding? Does zero padding improve the frequency resolution in the spectral estimate?

8. Explain deterministic and random signals with examples.

9. Give the digital signal processing application with the TMS 320 family.

10. What is the advantage of Harvard architecture of TMS 320 series?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Calculate the DFT of the sequence $x(n) = \{1, 1, -2, -2\}$. (6)

(ii) Determine the response of LTI system by radix –2 DIT FFT. (10)

Or

(b) (i) Derive the equation for Decimation–in–time algorithm for FFT. (8)

(ii) How do you linear filtering by FFT using save–add method? (8)

12. (a) Design a high pass filter using hamming window, with a cut–off frequency of 1.2 radians/sec and N = 9.

Or

(b) Design a digital Butterworth filter satisfying the constraints

$$0.707 \le \left|H(e^{jw})\right| \le 1 \text{ for } 0 \le w \le \pi/2$$

$$\left|H(e^{jw})\right| \le 0.2 \text{ for } 3\pi/4 \le w \le \pi$$

with T = 1 sec using bilinear transformation. Realize the filter in each case using the most convenient realisation form.

13. (a) Find the output roundoff noise power for the following transfer function $H(z) = H_1(z) H_2(z)$ where $H_1(z) = \dfrac{1}{1 - a_1 z^{-1}}$ and $H_2(z) = \dfrac{1}{1 - a_2 z^{-1}}$ and $a_1 = 0.5$ and $a_2 = 0.6$.

Or

(b) (i) Explain the characteristics of a limit cycle oscillation with respect to the system described by the difference equation $y(n) = 0.95\, y(n-1) + x(n)$. Determine the dead band of the filter. (10)

(ii) Draw the product quantisation noise model of second order IIR system. (6)

14. (a) Determine the performance characteristics of non–parametric power spectrum estimators Welch, Bartlett and Blackman and Tukey.

Or

(b) (i) Give the key features of the digital signal processor. (7)

(ii) Write short notes on : (9)

(1) 32–bit accumulator

(2) 16 × 16 bit parallel multiplier

(3) Shifter.

15. (a) (i) Explain the function of auxiliary registers in the indirect addressing mode to point the data memory location. (8)

(ii) Write a program to use the auxiliary register in memory pointing and looping. (8)

Or

(b) (i) Write a program to compute the following equation :

$Y = A * X_1 + B * X_2 + C * X_3$. (8)

(ii) Write a program to perform addition of two 64 bit numbers. (8)

R 3301

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Fifth Semester

(Regulation 2004)

Electronics and Communication Engineering

EC 1302 — DIGITAL SIGNAL PROCESSING

(Common to B.E. (Part-Time) Fourth Semester Regulation 2005)

Time : Three hours | Maximum : 100 marks

Answer **ALL** questions.

PART A — (10 × 2 = 20 marks)

1. State and prove Parseval's relation for DFT.

2. What do you mean by the term "bit reversal" as applied to FFT?

3. In the design of FIR digital filters, how is Kaiser window different from other windows?

4. Find the digital transfer function $H(z)$ by using impulse invariant method for the analog transfer function $H(s) = \frac{1}{s+2}$. Assume $T = 0.1$ sec.

5. Express the fraction (-7/32) in signed magnitude and two's complement notations using 6 bits.

6. What do you mean by limit cycle oscillations in digital filters?

7. Define unbiased estimate and consistent estimate.

8. Define the terms : autocorrelation sequence and power spectral density.

9. What are the factors that may be considered when selecting a DSP processor for an application?

10. What is pipelining?

PART B — ($5 \times 16 = 80$ marks)

11. (a) Two finite duration sequences are given by

$x[n] = \sin(n\pi/2)$ for $n = 0, 1, 2, 3$

$h[n] = 2^n$ for $n = 0, 1, 2, 3$

(i) Calculate the 4 point DFT $X[k]$. (5)

(ii) Calculate the 4 point DFT $H[k]$. (5)

(iii) If $Y[k] = X[k]H[k]$, determine the inverse DFT $y[n]$ of $Y[k]$ and sketch it. (6)

Or

(b) (i) Obtain an 8-point decimation-in-frequency FFT flow graph from first principles. (8)

(ii) Using the above flow graph compute DFT of

$x[n] = \cos(n\pi/4), 0 \le n \le 7$. (8)

12. (a) (i) Describe the design of FIR filters using frequency sampling technique. (8)

(ii) The desired frequency response of a low pass filter is given by

$$H_D(e^{j\omega}) = \begin{cases} e^{(-j2\omega)}, & -\pi/4 \le \omega \le \pi/4 \\ 0, & \pi/4 \le |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h_D[n]$. Obtain the coefficients $h[n]$ of FIR filter using a rectangular window defined by

$$w[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}.$$ (8)

Or

(b) Design a digital Butterworth filter satisfying the following specifications

$0.7 \le |H(e^{j\omega})| \le 1, \quad 0 \le \omega \le 0.2\pi$

$|H(e^{j\omega})| \le 0.004, \quad 0.6\pi \le \omega \le \pi$

Assume $T = 1$ sec. Apply impulse-invariant transformation. (16)

13. (a) (i) Consider the truncation of negative fraction numbers represented in $(\beta+1)$- bit fixed point binary form including sign bit. Let $(\beta - b)$ bits be truncated. Obtain the range of truncation errors for signed magnitude, 2's complement and 1's complement representations of the negative numbers. (8)

(ii) An 8-bit ADC feeds a DSP system characterized by the following transfer function

$$H(z) = \frac{1}{z+0.5}$$

Estimate the steady state quantization noise power at the output of the system. (8)

Or

(b) (i) The coefficients of a system defined by

$H(z) = \dfrac{1}{(1-0.4z^{-1})(1-0.55z^{-1})}$ are represented in a number system with a sign bit and 3 data bits using signed magnitude representation and truncation. Determine the new pole locations for direct realization and for cascade realization of first order systems. (8)

(ii) An IIR causal filter has the system function $H(z) = \dfrac{z}{z-0.97}$. Assume that the input signal is zero-valued and the computed output signal values are rounded to one decimal place. Show that under these stated conditions, the filter output exhibits dead band effect. What is the dead band range? (8)

14. (a) (i) With suitable relations, explain briefly the periodogram method of power spectral estimation. Examine the consistency and bias of periodogram. (10)

(ii) Explain power spectrum estimation using the Bartlett method. (6)

Or

(b) (i) Explain how the Blackman and Tukey method is used in smoothing the periodogram? Derive the mean and variance of the power spectral estimate of the Blackman and Tukey method. (10)

(ii) Determine the frequency resolution of the Bartlett, Welch and Blackman-Tukey methods of power spectral estimation for a quality factor Q = 15. Assume that overlap in Welch's method is 50% and the length of the sample is 1500. (6)

15\. (a) (i) Explain how Harvard architecture as used by the TMS 320 family differs from the strict Harvard architecture. Compare this with the architecture of a standard von Neumann processor. (8)

(ii) A multiplier - accumulator, with three pipe stages, is required for a digital signal processor. Sketch a block diagram of a suitable configuration for the MAC. With the aid of a timing diagram, explain how the MAC works. (8)

Or

(b) (i) In relation to DSP processor, explain the following techniques :

SIMD, VLIW.

In each case, clearly point out the advantages and disadvantages of the technique in signal processing. (8)

(ii) Explain the operation of CSSU of TMS32OC 54X and explain its use considering the Viterbi operator. (8)

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