Set Theory

1. Explain how sets and subsets are represented in computer using bit strings.
2. Let $U = \{FORTORN, PASCAL, ADA, COBOL, LISP, BASIC, C^{++}, FORTH\}$
   $B = \{C^{++}, ADA, BASIC\}, C = \{PASCAL, ADA, LISP, C^{++}\},$
   $D = \{FORTORN, PASCAL, ADA, BASIC, FORTH\}, E = \{PASCAL, ADA, COBOL, LISP, C^{++}\}$
   In each of the following, represent the given set by an array of zeros and ones-
   $(a)\ B \cup C$ $(b)\ C \cap D$ $(d)\ B \cap (D \cap E)$ $(e)\ B \cup E^c$ $(f)\ C^c \cap (B \cup C)$
3. How many bit strings of length eight either start with a 1 bit or end with the two bits 00.
4. Find the number of integers from 1 to 1000 that are not divisible by $5, 6$ and 8.
5. Determine integers between 1 and 250 that are divisible by any of the integers $2, 3, 5$ and 7.
6. Of 1000 applicants for a mountain-climbing trip in the Himalayas, 450 get altitude sickness, 622 are not in good enough shape, and 30 have allergies. An applicant qualifies if and only if this applicant does not get altitude sickness, is in good shape, and does not have allergies. If there are 111 applicants who get altitude sickness and are not in good enough shape, 14 who get altitude sickness and have allergies, 18 who are not in good enough shape and have allergies, and 9 who get altitude sickness, are not in good enough shape, and have allergies, find (i) How many applicants qualify? (ii) How many got rejected due to exactly two of the deficiencies? (iii) How many got rejected due to exactly one of the deficiencies? (iv) How many did not qualify?.
7. The college catering service must decide if the mix of food that is supplied for receptions is appropriate. Of 100 people questioned, 37 say they eat fruits, 33 say they eat vegetables, 9 say they eat cheese and fruits, 12 eat cheese and vegetables, 10 eat fruits and vegetables, 12 eat only cheese, and 3 report they eat all three offerings. How many people surveyed eat (i) Cheese (ii) exactly one (iii) exactly two (iv) none of the offerings?
8. Thirty cars were assembled in a factory. The options available were a radio, an air conditioner and white wall tires. It is known that 15 of the cars have radios, 8 of them have air conditioners, and 6 of them have white-wall tires. Moreover, 3 of them have all three options. Determine atleast how many cars do not have any options at all.
9. 75 students went to an amusement park where they can ride on the marry-go-round, roller coaster, and Ferris wheel. It is known that 20 of them have taken all three rides and 55 of them have taken at least two of the three rides. Each ride costs \$0.50 and the total receipt of the amusement park was \$70. Determine the number of children who did not try any of the rides.
10. A survey of 500 television watchers produced the following information- 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games and 50 do not watch any of the three kind of games. How many watch (i) all three kinds of the games? (ii) exactly one of the games? (iii) exactly two of the games?
11. A computer company receives 350 applications from computer graduates for a job planning a line of new web servers. Suppose that 220 of these people majored in computer sciences, 147 majored in business and 51 majored both in computer science and in business. How any of these applicants majored neither in computer science nor in business? [Ans 316]
12. In a survey of 260 college students, the following data were obtained-64 had taken a mathematics course, 58 had taken a business course, 94 had taken a computer science course, 28 had taken both mathematics and a business course, 26 had taken both a mathematics and a computer science course, 22 had taken both a business course and a computer science course, 14 had taken all three types of courses. Determine (a) How many students had taken none of these three courses? (b) Of the student surveyed, how many had taken only a computer science course? [$Ans.\ (a)\ 106\ (b)\ 60$]
13. A survey was conducted among 1000 people. Of these, 595 are democrats, 595 wear glasses and 550 like ice cream; 395 of them are democrats who wear glasses, 350 of them are democrats who like ice cream and 400 of them wear glasses and like ice cream; 250 of them are democrats who wear glasses and like ice cream. How many of them who are not democrats, do not wear glasses and do not like ice cream? How many of them are democrats and who do not wear glasses and do not like ice cream? [Ans. $(a)\ 155\ (b)\ 100$]
14. Out of a total 130 students, 60 were wearing hats to class, 51 are wearing scarves, and 30 are wearing both hats and scarves. Of the 54 students, who are wearing sweaters, 26 are wearing hats, 21 are wearing scarves, and 21 are wearing both hats and scarves. Everyone wearing neither a hat nor a scarf is wearing gloves. (a) How many students are wearing gloves? (b) How many students not wearing a sweater are wearing hats but not scarves? (c) How many students not wearing a sweater are wearing neither a hat nor a scarf?

15. Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study biology and mathematics, 7 study mathematics and physics, 10 study biology and physics, and 30 do not study any of the three subjects. Find the number of students studying exactly one of the three subjects. [*Ans.* 48]

16. (*a*) Among 50 students in a class, 26 got an A in the first examination, and 21 got an A in the second examination. If 17 students didn't get an A in either examination, how many students got an A in both the examination?

    (*b*) If the number of students who got an A in the first examination is equal to that in the second examination, if the total number of students who got an A in exactly one examination is 40 and if 4 students didn't get an A in either examination, determine the number of students who got an A in the first examination only, who got an A in the second examination only and who got an A in both examinations.

17. In a survey of brand preferences for toothpastes, 82% of the population (number of people covered for survey) liked at least one of the brands: I, II and III. 40% of those asked liked brand I, 25% liked brand II, and 35% liked brand III. If 5% of those asked showed liking for all three brands, then what % of those asked like more than one of the brands? [*Ans.* 10]

18. It is known that at a university, 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis, would you believe this claim? Why?

19. The 60,000 fans who attended the homecoming football game bought up all the paraphernalia for their cars. Altogether, 20,000 bumper stickers, 36,000 window decals and 12,000 key rings were sold. We know that 52,000 fans bought at least one item and no one bought more that one of a given item. Also 6,000 fans bought both decals and key rings, 9,000 bought both decals and bumper stickers and 5,000 bought both key rings and bumper stickers. (*a*) How many fans bought all three items? (*b*) How many fans bought exactly one item? (*c*) Some one questioned the accuracy of total number of purchasers: 52,000 (given that all the other numbers have been confirmed to be correct). This person claimed that total number of purchasers to be either 60,000 of 44,000. How do you dispel the claim?

20. The journalism 101 class recently took a survey to determine where the city's people obtained their news from. Unfortunately, some of the reports were damaged. What we know is that 88 people said they obtained news from television, 73 from local news paper, and 46 from news magazines. 34 people reported that they obtained news from television and the local news paper,16 said they obtained their news from television and a news magazine, and 12 obtained their news from local news paper and a news magazine. A total of five reported that they used all three media. If 166 people were surveyed, how many use none of the media to obtain their news? How many obtain their news from a news magazine exclusively?