End Sem Examination

Name :- Tanishq Chauhan

Roll No :- 21C3184

Subject :- Applied Mathematics - III (ACR3C1)

Semester :- 3rd Semester

Branch :- Computer Science + Engineering

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Applied Mathematics III (ACR3C1)

Answer of Q. No 3 (a)

	Mean	σ	r_{12}	r_{13}	r_{23}
x_1 (inches)	5.8	2.8	0.75		
x_2 (lbs)	42	12		0.43	
x_3 (years)	17	1.5			0.54

X_3 as a function of X_1 & X_2 is given by :-

$$X_3 - \overline{X_3} = b_{31.2}\,(x - \overline{x_1}) + b_{32.1}\,(x_2 - \overline{x_2})$$

Where $\quad b_{31.2} = -\frac{\sigma_3}{\sigma_2} \frac{W_{32}}{W_{33}}$

$$b_{32.1} = -\frac{\sigma_3}{\sigma_2} \frac{W_{32}}{W_{33}}$$

$$W = \begin{vmatrix} 1 & r_{12} & r_{13} \\ r_{12} & 1 & r_{23} \\ r_{31} & r_{32} & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0.75 & 0.43 \\ 0.75 & 1 & 0.54 \\ 0.43 & 0.54 & 1 \end{vmatrix}$$

Therefore,

$$W_{31} = \begin{vmatrix} 0.75 & 0.43 \\ 1 & 0.54 \end{vmatrix}$$

$$= -0.025$$

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$$W_{32} = \begin{vmatrix} 1 & 0.43 \\ 0.75 & 0.54 \end{vmatrix}$$

$$= -0.2175$$

$$W_{33} = \begin{vmatrix} 1 & 0.75 \\ 0.75 & 1 \end{vmatrix}$$

$$= 0.4375$$

Here

$$b_{31.2} = -\frac{\sigma_3}{\sigma_1}\frac{W_{31}}{W_{33}} = \frac{-1.5}{2.8} \times \frac{-0.025}{0.4375}$$

$$= 0.0306$$

$$b_{32.1} = \frac{-\sigma_3}{\sigma_2}\frac{W_{32}}{W_{33}} = \frac{-1.5}{12} \times \frac{-0.2175}{0.4375}$$

$$= 0.0621$$

X_3 on X_1 & X_2 is -

$$X_3 - 17 = 0.0306\,(X_1 - 5.8) + 0.0621\,(X_2 - 42)$$

$$X_3 = 0.0306\,X_1 + 0.0621\,X_2 + 14.21432$$

When $X_1 = 5.6$ & $X_2 = 55$

$$X_3 = 0.0306(5.6) + 0.0621(55) + 14.21432$$

$$\boxed{X_3 = 17.80118 \text{ years}}$$

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Answer of Q. No. 3 (b)

$$f(x) \begin{cases} kx, & 0 \le x < 1, \\ k, & 1 \le x < 2, \\ -kx + 3k, & 2 \le x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) $\sum f(x) = 1$

$$kx + k + (-kx + 3k) = 1$$
$$4k = 1$$

$$\boxed{k = \frac{1}{4}}$$

(ii) for cdf

$$F(x) = P(X \le x)$$

for $x < 0$

$$Fx = 0$$

for $0 \le n < 1$

$$F(n) = \int_{-\infty}^{0} f(x) + \int_{0}^{x} f(n)$$

$$= 0 + \frac{x^2}{2} \times \frac{1}{4} = \frac{x^2}{8}$$

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for $1 \le x < 2$

$$F(x) = f(0) + f(1) + \int_{1}^{x} \frac{1}{4} \, dx$$

$$F(x) = 0 + \frac{1}{8} + \left(\frac{x}{4} - \frac{1}{4} \right)$$

$$F(x) = \left(\frac{x - 1}{4} \right)$$

for $2 \le x < 3$

$$F(x) = F(0) + F(1) + f(2) + \int_{2}^{x} -\frac{1}{4} x + \frac{3}{4} \, dx$$

$$F(x) = 0 + \frac{1}{8} + \frac{1}{4} + \left(\frac{-x^2}{2 \times 4} + \frac{3x}{4} + \frac{4}{8} - \frac{6}{4} \right)$$

$$F(x) = \left(\frac{3}{8} + - \frac{x^2}{8} + \frac{6x}{8} - \frac{8}{8} \right)$$

$$F(x) = \frac{-x^2 + 6x - 5}{8}$$

& for $x > 3$

$$\boxed{F(x) = 1}$$

Now
$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{8}, & 0 \le x < 1 \\ \frac{x-1}{4}, & 1 \le x < 2 \\ \frac{-x^2 + 6x - 5}{8} & 2 \le x < 3 \\ 1 & x > 3 \end{cases}$$

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Answer of Q. No 1 (a)

$f(x) = (x+5)^2 \sin x + \ln(x^2+3)$

$f(3) = (3+5)^2 \sin 3 + \ln(9+3)$

$= 11.5166$

$f(4) = (4+5)^2 \sin 4 + \ln(16+3$

$= -58.3566$

$f(3) \cdot f(4) < 0$

So atleast one real root lies in the interval $(3, 4)$

Let $a_0 = 3$, $b_0 = 4$

The next approximation to the root is given by Regula - Falsi Method

$$x_{k+1} = \frac{a_k f(b_k) - b_k f(a_k)}{f(b_k) - f(a_k)}$$

$K = 0, 1, 2, 3 \ldots\ldots$

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K	a_k	b_k	$f(a_k)$	$f(b_k)$	x_{k+1}	$f(x_{k+1})$
0	3	4	11.5166	-58.3566	3.1648	1.0192
1	3.1648	4	1.0192	-58.3566	3.1791	0.0645
2	3.1791	4	0.0645	-58.3566	3.1800	0.0043
3	3.1800	4	0.0043	-58.3566	3.1801	-0.0024
4	3.1800	3.1801	0.0043	-0.0024	3.1801	-0.0024

x_{k+1} assumes the same value over further iterations $\therefore |x_4 - x_3| < 10^{-4}$

$\therefore$ The value of $x = 3.1801$
Which is smallest positive (+ve) real root.

$\boxed{x = 3.1801}$ is the smallest positive (+ve) real root with four places of decimals using Regula - Falsi Method.

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Answer of Q. No 1. (c)

Taking; $h \Rightarrow \frac{1.8 - 1.2}{2} = \frac{0.6}{2} = 0.3$

$x_0 = 1.2, \; y_0 = 2.6, \; x_n = 1.8$

$y' = \frac{y + x^2 \sin x}{x}$

$\therefore \; f(x,y) = \frac{y + x^2 \sin x}{x}$

Modified Euler Method;

$y_{m+1} = y_m + h f\left(x_m + \frac{1}{2}h, \; y_m + \frac{1}{2} f(x_m, y_m)\right)$

$f(x_0, y_0) = f(1.2, 2.6)$

$= \frac{2.6 + (1.2)^2 \sin(1.2)}{1.2}$

$= 3.2857$

$x_0 + \frac{1}{2}h = \frac{1.2 + 0.3}{2} = 1.35$

$y_0 + \frac{1}{2}h \; f(x_0, y_0) = 2.6 + \frac{0.3}{2} \times 3.2857$

$= 3.0928$

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$$f\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2} h f(x_0, y_0)\right) = f(1.35, 3.0928)$$

$$= 3.6082$$

$$f\left(x_1 + \frac{1}{2} h, y_1 + \frac{1}{2} h f(x_1, y_1)\right) = f(1.65, 4.2757$$

$$= 4.2358$$

$$y_2 = y_1 + hf\left(x_1 + \frac{h_1}{2}, y_1 + \frac{1}{2} h f(x_1, y_1)\right)$$

$$= 3.6824 + 0.3 \times (4.2358)$$

$$= 4.9532$$

$$\therefore y(1.8 = 4.9532$$

So, The answer is :- $\boxed{y(1.8) = 4.9532}$

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Answer of Q. No. 4 ~~(a)~~ (c)

Let H_0 : The family structure doesn't matters in school performance of the students.

H_1 : The family structure matters in school performance of the students.

The expected frequencies written in boxes are obtained by,

$$E_{ij} = \frac{R_i * C_j}{Total}$$

The contingency table is given below :-

		Academic Status			
		School	Graduate	Post Graduate	Total
Family	No Parent	15 $\boxed{34}$	32 $\boxed{24}$	35 $\boxed{24}$	82
	One Parent	92 $\boxed{81}$	52 $\boxed{55}$	48 $\boxed{56}$	192
	Two Parent	48 $\boxed{40}$	22 $\boxed{27}$	25 $\boxed{28}$	95
	Total	155	106	108	369

The test statistics is :-

$$\chi^2 = \sum_i \sum_j \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

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$$x^2 = \frac{(15-34)^2}{34} + \frac{(32-24)^2}{24} + \frac{(35-24)^2}{24} + \frac{(92-81)^2}{81}$$

$$+ \frac{(52-55)^2}{55} + \frac{(48-56)^2}{56} + \frac{(48\ 40)^2}{40} + \frac{(22-27)^2}{27}$$

$$+ \frac{(25-28)^2}{28}$$

$$x^2 = 23.972$$

Here degree of freedom $v = (3-1)(3-1) = 4$

and $X^2_{0.05}(4) = 9.488$.

Since $x^2 = 23.972 > x^2_{0.05}(4) = 9.488$. Therefore family structure matters in school performance of the students

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Answer of Q. No. 4(b)

Car	1	2	3	4	5	6	7	8	9	10	11
Production Material	5.1	6.5	3.6	3.5	5.7	5.0	6.4	4.7	3.2	3.5	6.4
Experimental Material	5	6.5	3.1	3.7	4.5	4.1	5.3	2.6	3	3.5	5.1

$\bar{X}$ = Mean of production material tyres

$$= \frac{5.1 + 6.5 + 3.6 + 3.5 + 5.7 + 5.0 + 6.4 + 4.7 + 3.2 + 3.5 + 6.4}{11}$$

$= 4.87$

$\bar{Y}$ = Mean of experimental material tyre

$$= \frac{5 + 6.5 + 3.1 + 3.7 + 4.5 + 4.1 + 5.3 + 2.6 + 3 + 3.5 + 5.1}{11}$$

$= 4.22$

$n_1 = 11$, $n_2 = 11$ $\Rightarrow$ $n = 11$

v = Degree of freedom $\quad \bar{d} = \bar{x} - \bar{y} = 0.65$

$= 11 - 1$

$= 10$

$d_i = x_i - y_i \quad 0.1 \quad 0 \quad 0.5 \quad 0.2 \quad 1.2 \quad 0.9 \quad 1.1 \quad 2.1 \quad 0.2 \quad 0 \quad 1.3$

$$s^2 = \frac{1}{n-1} \sum (d_i - \bar{d})^2$$

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$$s^2 = \frac{1}{10}\left[(0.1-0.65)^2+(0-0.65)^2+(0.5-0.65)^2+(0.2-0.65)^2+(1.2-0.65)^2 + (0.5-0.65)^2+(1.1-0.65)^2+(2.1-0.65)^2+ (0.2-0.65)^2+(1.3-0.65)^2+(0-0.65)^2\right]$$

$$s^2 = \frac{1}{10}\left[0.3025+0.4225+0.0225+0.7225+0.3025+0.0625 + 0.2025+2.1025+0.2025+0.4225+0.4225\right]$$

$$s^2 = 0.51875$$
$$s = 0.7202$$

Using Student - test

$\because \nu = 10$

$\therefore t_{10}(0.01) = 2.76$

$$\text{Confidence Interval} = \bar{d} \pm \frac{s}{\sqrt{n}} \times t_{10}(0.01)$$

$$= 0.65 \pm \frac{0.72}{\sqrt{11}} \times 2.76$$

$$= [0.65 - 0.59,\ 0.65 + 0.59]$$

$$= [0.06,\ 1.24]$$

Let $H_0 : \bar{d} = 0$, Mean wear of both equipmental wear & permanent wear is equal

$H_1 : \bar{d} > 0$, Mean wear of exp. wear is less than permanent wear.

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$v = 10, \quad t_{10}(0.01) = 2.76$

$$t = \frac{\bar{d}}{S/\sqrt{n-1}} = \frac{0.65}{0.72/\sqrt{10}} = \frac{0.65}{0.23}$$

$t = 2.83$

Since $t = 2.83 > t_{10}(0.01) = 2.76$

Therefore Ho is rejected

∴ ~~Expapmental~~

∴ Experimental material wears less than production wear

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Answer of Q. No - 1(c)

Taking; $h \Rightarrow \frac{1.8 - 1.2}{2} = \frac{\cancel{0.6}}{\cancel{2}} = 0.3$

$x_0 = 1.2, \; y_0 = 2.6, \; x_n = 1.8$

$y' = \frac{y + x^2 \sin x}{x}$

$\therefore f(x, y) = \frac{y + x^2 \sin x}{x}$

Modified Euler Method :

$y_{m+1} = y_m + h f\left(x_m + \frac{1}{2}h, \; y_m + \frac{1}{2} f(x_m, y_m)\right)$

$f(x_0, y_0) = f(1.2, 2.6)$

$= \frac{2.6 + (1.2)^2 \sin(1.2)}{1.2}$

$= 3.2857$

$x_0 + \frac{1}{2}h = 1.2 + \frac{0.3}{2} = 1.35$

$y_0 + \frac{1}{2} h f(x_0, y_0) = 2.6 + \frac{0.3}{2} \times 3.2857$

$= 3.0928$

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$$f\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2} h f(x_0, y_0)\right) = f(1.35, 30928)$$

$$= 3.6082$$

Now,

$$y_1 = y_0 + hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2} h f(x_0, y_0)\right)$$

$$= 2.6 + 0.3 \times 3.6082$$

$$= 3.6824$$

$$\therefore y_1 = y(1.5) = 3.6824$$

Now, Again taking (x_1, y_1) in place of (x_0, y_0) and repeat the process

$$f(x_1, y_1) = f(1.5, 3.6824)$$

$$= \frac{3.6824 + (1.5)^2 \sin(1.5)}{1.5}$$

$$= 3.9512$$

$$x_1 + \frac{1}{2}h = 1.5 + \frac{0.3}{2} = 1.65$$

$$y_1 + \frac{1}{2} hf(x_1, y_1) = 3.6824 + \frac{0.3}{2} \times 3.9512$$

$$= 4.2757$$

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$$f\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}h\, f(x_1, y_1)\right) = f(1.65, 4.2757)$$

$$= 4.2358$$

$$y_2 = y_1 + hf\left(x_1 + \frac{h}{2}, y_1 + \frac{1}{2}h\, f(x_1, y_1)\right)$$

$$= 3.6824 + 0.3 \times (4.2358)$$

$$= 4.9532$$

$$\therefore y(1.8) = 4.9532$$

So, The answer is :- $\boxed{y(1.8) = 4.9532}$

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Answer of Q. No. 2 (b)(i)

Width of river = 40ft

Here, h = 5 and n = 8

By using Simpson's 1/3 rd rule.

Area of cross section = $\int_0^{40} y\,dx$

$$\text{Area} = \frac{h}{3}\left[(y_0 + y_3) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{5}{3}\left[(0+3) + 4(4+9+15+8) + 2(7+12+41)\right]$$

$$= \frac{5}{3}\left[3 + 4(36) + 2(60)\right]$$

$$= \frac{5}{3}\left[3+144+120\right] = \frac{5}{3}\left[267\right] = 5\left[89\right]$$

$$= 445 \text{ Sq feet}$$

$\therefore$ Area of cross section is 445 Sq. feet.

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Answer of Q. No. 2 (b) (ii)

$$I = \int_1^6 \frac{\ln x + e^x \cos x}{\ln(1+x^3)}\, dx$$

Here $y = f(x) = \dfrac{\ln x + e^x \cos x}{\ln(1+x^5)}$

$$h = \frac{6-1}{12} = \frac{5}{12}$$

Simpson's 3/8 Rule

$$I = \frac{3h}{8}\left[(y_0+y_{12}) + 3(y_1+y_2+y_4+y_5+y_7+y_8+y_{10}+y_{11}) + 2(y_3+y_6+y_9)\right]$$

$$I = 3\left(\frac{5}{12}\right)\left(\frac{1}{8}\right)\left[(74.4532 + 3(31.3961) + 2(-8.6548)\right]$$

$$\boxed{I = 23.6456}$$

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Answer of Q No. 2 (b) (ii)

$$I = \int_1^6 \frac{\ln x + e^x \cos x}{\ln(1+x^3)}\, dx$$

Here $y = f(x) = \dfrac{\ln x + e^x \cos x}{\ln(1+x^3)}$

$$h = \frac{6-1}{12} = \frac{5}{12}$$

Now 13 ordinates :-

x	y
1	2.1188
17/12	0.7290
22/12	-0.5165
27/12	-2.0457
32/12	-3.9471
37/12	-6.0565
42/12	-7.8697
47/12	-8.3944
52/12	-6.0599
57/12	1.2606
62/12	15.9233
67/12	39.7182
72/12 (6)	72.3344

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Using Simpson's 3/8 Rule

$$I = \frac{3h}{8}\left[(y_0+y_{12}) + 3(y_1+y_2+y_4+y_5+y_7+y_8+y_{10}+y_{11}) + 2(y_3+y_6+y_9)\right]$$

$$I = 3\left(\frac{5}{12}\right)\left(\frac{1}{8}\right)\left[(74.4532 + 3(31.3961) + 2(-8.6548)\right]$$

$$I = 3\times\left(\frac{5}{12}\right)\left(\frac{1}{8}\right)\left[151.3318\right]$$

$$I = 23.6456$$

So, The answer is $\boxed{I = 23.6456}$

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Answer of Q. No. 2 (c)

T	i
1.2	1.36
2	0.58
2.4	0.34
2.8	0.2
3	0.75
3.3	0.54

Newton's Dividend Difference Table

T	i	Δi	$\Delta^2 i$	$\Delta^3 i$
1.2	1.36	-0.075		
2	0.58	-0.6	0.3125	0
2.4	0.34	-0.35	0.3125	4.8542
2.8	0.2	2.75	5.1667	-13.4074
3	0.75	-0.7	-69	
3.3	0.54			

$\Delta^4 i$	$\Delta^5 i$
2.6968	-7.9734
-14.0474	

$\because$ Time difference is not constant

$f(x) = y_0 \, f(x - x_0) \, f(x_0, x_1) + (x - x_0)(x - x_1) \times$
$f(x_0, x_1, x_2) + [(x - x_0)(x - x_1)(x - x_2)$
$f(x_0, x_1, x_2, x_3)$

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$f(x-x)(x-x_1)(x-x_2)(x-x_4) f(x_0, x_1, x_2, x_3)$

$f(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

$f(x_0, x_1, x_2, x_3, x_4)$

Putting values & Solving

$y(2.52) = 1.36 + (2.52-1.2)(-0.975) + (2.52-1.2)$
$(02.52-2)\ \cancel{(2.52-2.4}\ (0.3125 + (2.52-1.2)$
$(2.52-2) \cdot (2.52-2.4)$
$+ (2.52-1.2)(2.52-2)(2.52-2.4)(2.52-2.8)(2.52-2.6968)$
$+ (2.52-1.2)(2.52-2)(2.52-2.4)(2.52-2.8)(2.52-3)$
(-7.9734)

$= 1.36 - 1.287 + 0.2145 + 0 - 0.622 - 0.0883$

$= 0.137$

$$\boxed{I = y(2.52) = 0.137}$$

So, The answer is :- $I = y(2.52) = 0.137$.

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Answer of Q. No 5 (a)

(i) Stationary Process :-

If a certain probability distribution or average do not depend on time, the process can be dependent on time but statistical measures are independent of time, then this type of random process is called stationary process.

Mathematically,

A continuous - time random process $\{X(t), t \in R\}$ is strict - sence stationary or simply stationary if for all $t_1, t_2 \ldots t_n \in R$ and all $\Delta \in R$, the joint CDF of

$$X(t_1), X(t_2) \ldots, X(t_n)$$

is same as the joint CDF of

$$X(t_1 + \Delta), X(t_2 + \Delta) \ldots, X(t_n + \Delta)$$

That is,

for all real numbers, $x_1, x_2 \ldots x_n$, all have

$$F_{X(t_1)X(t_2)(t_3) \ldots (t_n)}(x_1, x_2 \ldots x_n) =$$

$$F_{X(t_1+\Delta)(t_2+\Delta) \ldots (t_n+\Delta)}(x_1, x_2 \ldots x_n)$$

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(ii) Wide Sense Stationary Process

Covariance stationary or wide sense stationary process (WSS) is called to a process if its mean is constant and the autocorrelation depends using only on the time difference

that is if

$$E\{X(t)\} = \mu$$

$$\text{and } E\{X(t)X(t-\tau)\} = R(\tau).$$
$$= (R(t_1 - t_2)$$