Roll No.....

BE-II Examination June 2021
Computer Science Engineering
CER4C1 Discrete Structures

Duration: 3 Hours **Max. Marks [60]**

Note: Attempt all the 5 questions with any two of (a), (b) and (c) from each.

Q.1 (a) The college catering service must decide if the mix of food that is supplied for receptions is appropriate. Of 100 people questioned, 37 say they eat fruits, 33 say they eat vegetables, 9 say they eat cheese and fruits, 12 eat cheese and vegetables, 10 eat fruits and vegetables, 12 eat only cheese, and 3 report they eat all three offerings. How many people surveyed eat (i) Cheese (ii) exactly one (iii) exactly two (iv) none of the offerings? **[6]**

(b) (i) In an asynchronous transfer mode ($\boldsymbol{ATM}$) (a communications protocol used on backbone networks), data are organized into cells of **53** bytes. How many $\boldsymbol{ATM}$ cells can be transmitted in **30** seconds over a link operating at- **[3]**

(I) **256** kilobits per second

(II) **200** kilobits per second

(III) **1** megabit per second?

[**1** kilobits= **1000** bits, **1** byte= **8** bits]

(ii) Define characteristic function of a set. Show that- **[3]**

(I) $f_{A\cup B} = f_A + f_B - f_A.f_B$ (II) $f_{A\oplus B} = f_A + f_B - 2f_A.f_B$.

(c) Define equivalence relation and equivalence class. Show that if $\boldsymbol{R} \subseteq \boldsymbol{A}\times\boldsymbol{A}$ is an equivalence relation and let $a, b \in A$ be arbitrary, then- **[6]**

(i) $[a] \neq \phi$

(ii) $b \in [a] \Longrightarrow [b] = [a]$

(iii) $[a] = [b] \Longleftrightarrow aRb$

(iv) if $[a] \cap [b] \neq \phi$, then $[a] = [b]$.

Q.2 (a) Answer the following- **[6]**

(i) Explain which rules of inference are used for each step-

"Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

(ii) Determine whether the argument is valid. If valid, what rule of inference is being used?

"If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$."

(iii) Express the negation of the statement $\exists y(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$ so that all negation symbols immediately precede predicates.

(b) What relevant conclusion (or conclusions) can be derived from the following premises-

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural program will go on ", "If the sport day is held, the trophy will be awarded ", "the trophy was not awarded ". **[6]**

(c) What relevant conclusion or conclusions can be drawn?

"All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu." "Cheeseburgers are not healthy to eat." **[6]**

Q.3 **(a)** Write principle of strong induction and prove that $\sqrt{5}$ is irrational. **[6]**

(b) **(i)** Show that there must be at least **306** ways to choose **4** integers from **25** to **40** (both inclusive) so that all the choices have the same sum. **[3]**

(ii) For the generating function provide a closed formula for the sequence it determines- **[3]**

(c) Use generating functions to find the number of ways to make change for ***Rs.* 120** using **(i)** $Rs.5, Rs.20$ and $Rs.30$ if at least one of each denomination is used **(ii)** $Rs.5, Rs.10, Rs.20$ and $Rs.30$ if at least one and no more than four of each denomination is used. **[6]**

Q.4 **(a)** Apply Dijkstra's algorithm to find the shortest path from the vertex P to Q in graph shown in figure **1**. **[6]**

(b) Use Huffman coding to encode these symbols with given frequencies- **[6]**

$$A : 0.30 \; B : 0.30 \; C : 0.13 \; D : 0.12 \; E : 0.10 \; F : 0.05$$

(c) **(i)** Using Kruskal's algorithm to find minimal spanning tree for the weighted graph given figure **2**. Also find the cost for it. **[3]**

(ii) Represent the expression $((x+2) \uparrow 3 * y - (3+x)) - 5$ using a binary tree. Also write Prefix, Infix, Postfix notation. **[3]**

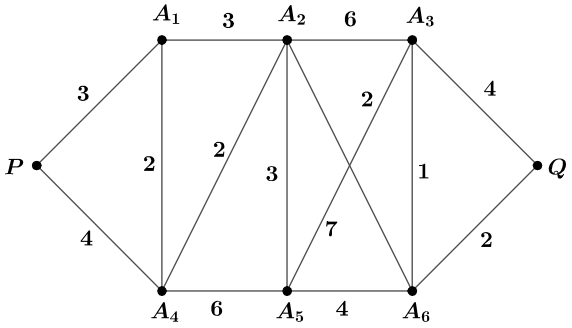

Figure 1

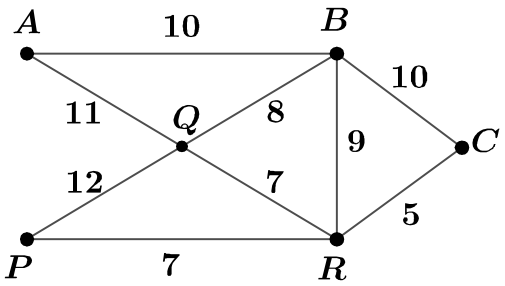

Figure 2

Q.5 **(a)** Show that the set $\{1, -1, i, -i\}$ of four fourth roots of unity is an Abelian group with respect to multiplication. **[6]**

(b) Show that $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to addition modulo **6** and multiplication modulo **6.** **[6]**

(c) **(i)** Change the following Boolean function to disjunctive normal form- **[3]**

$$f(x, y, z, t) = \left[x'.y + x.y.z' + x.y'.z + x'.y'.z'.t + t'\right]'$$

(ii) Draw a circuit for the following Boolean function and replace it by a simpler one- **[3]**

$$F(x, y, z) = x.y.z + (z + y).x'$$

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