02450 EXERCISE 9 1 April, 2025

AUC and ensemble methods with PYTHON

Objective: The objective of today's exercise is to understand (i) how the ensemble methods bagging and boosting is used to improve the performance of classifiers, (ii) how the receiver operating characteristic (ROC) is used to evaluate the performance of two-class classification problems, and (iii) how artificial neural network and logistic regression can be generalized to multi-class classification.

Material: Lecture notes "Introduction to Machine Learning and Data Mining" as well as the files provided from DTU Learn.

9.1 Ensemble methods

In this part of the exercise we will consider ensemble methods to improve the classification performance. In particular we will consider bagging and boosting methods.

In bagging we randomly sample with replacement the same number of samples as the size of the training data. This is also denoted bootstrapping and it can be shown that on average each bootstrap sample will contain approximately 63% of the samples in the data.

In boosting we adaptively change the distribution that we sample the training examples from so that the classifiers will focus on examples that are hard to classify. A particularly well known boosting method is "AdaBoost" which is described in section 15.4 of the lecture notes Introduction to Machine Learning and Data Mining.

In this part of the exercise we will work with a synthetic data set which has two classes and two attributes, x_1 and x_2 . This data set cannot be classified correctly with any linear classifier, such as logistic regression or an artificial neural network with one hidden unit, as long as only x_1 and x_2 are used as features.

9.1.1 Load the artificial dataset (Data/synth5 file) with loadmat function. Inspect the data by making a scatterplot. Why can this data set not be classified correctly using logistic regression?

Inspect and run the script $\exp_{1_1.py}$. The script fits an ensemble of logistic regression models to the data, using the bootstrap aggregation (bagging) algorithm. Use L=100 bootstrap samples. This requires creating 100 bootstrapped training sets, fit a logistic regression model to each, and combine the results of their outputs to make the final classification. Explain how the error rate is computed (on the training set) and how the decision boundary is plotted.

Script details:

- \cdot To generate the bootstrap sample, you need to draw N data objects with replacement from the data set.
- You can use the function bootstrap() function from the toolbox to generate random numbers from a discrete distribution. You can write something like
 X_bs, y_bs = bootstrap(X, y, N, weights) to make a bootstrap sample.
- · To make the final classification, take a majority vote amongst the classifiers in the ensemble. You can use simple class BinClassifierEnsemble from the toolbox.
- To plot the decision boundary, you can use the function dbplot() or dbprobplot() from the toolbox. It requires as an input an object that implements predict(X) and predict_proba(X) methods. You can call it e.g. like this: dbprobplot(fitted_classifier, X, y, 'auto', resolution=200)

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Bagging is known to be most effective for non-linear classifiers. Show that bagging only leads to a limited performance improvement for the logistic regression classifier.

Try also the data set in Data/synth6 which is the one used as an example in the lecture.

9.1.2 In the script $\exp_1_2.py$, the script $\exp_1_1.py$ has been modified so that boosting is used instead of bagging. The script fits an ensemble of logistic regression models to the data, using the AdaBoost algorithm. Notice the script uses L=100 rounds of boosting. This requires creating a randomly chosen training set, fit a logistic regression model to it, evaluate its performance and update the weights accordingly, and compute a classifier importance. This process is repeated L=100 times, and ultimately the trained ensemble of classifiers is combined to make a final classification. Compute the error rate (on the training set) and make a plot of the decision boundary.

Script details:

- · Read the hints to the previous exercise.
- · Note that you will need two sets of weights in the algorithm. One is a weight for each of the N data objects, that is adapted when the boosting algorithm proceeds (we could call these weights). The other is the importance weights for the L trained classifiers (we could call these alpha).
- · You can use the function bootstrap() to generate random numbers from a discrete distribution, as before. You will need to update the 'weights' parameter in according to AdaBoost algorithm, in every iteration.
- · To make the final classification, take a weighted majority vote amongst the classifiers in the ensemble (weighted by alpha). Note that alpha needs to be normalized so that it sums to one. ii

Show that, if you use enough rounds of boosting, the data set can be perfectly classified.

Try also the data set in Data/synth6 which is the one used as an example in the lecture.

Let us return to the bagging algorithm, which we found to be of little use for logistic regression. Now, we will try the algorithm on a non-linear classifier: the decision tree. Bagging applied to decision trees is often called "random forests". The Python's package sklearn.ensemble has implemented bagging for random trees, see the class RandomForestClassifier().

9.1.3 The data set Data/synth5 we have used so far can trivially be fitted using a decision tree. Can you explain why? We will consider a different data set, which you can load from the file Data/synth7. Inspect and run the script ex9_1_3.py. Explore the data set and explain in what sense this is more challenging for a decision tree.

Notice the script fits a random forest (an ensemble of bagged decision trees) to the data using L=100 rounds of bagging. Compute the error rate (on the training set) and make a plot of the decision boundary.

Script details:

· Type help(sklearn.ensemble) to learn about the functions for fitting random forests with bagging.

For comparison, you can try also to fit a regular decision tree (without bagging or pruning). Does bagging appear to improve on the classification, and if so, in what sense? Observe how the classification rate and decision boundaries decrease when you reduce number of bootstrap iterations.

Try the script on the other synthetic multi-class data sets you have studied before, (Data/synth5...Data/synth7).

9.2 The receiver operating characteristic (ROC)

The receiver operating characteristic (ROC) is commonly used to evaluate and compare the performance of two-class classifiers, where one class is denoted "positive" and the other is denoted "negative." The ROC is a graphical approach for displaying the tradeoff between the true positive rate (y-axis) and the false positive rate (x-axis). For classifiers such as logistic regression and artificial neural networks that estimate the class labels by thresholding a continuous output variable, an ROC curve can be plotted by varying the threshold value.

9.2.1 Inspect and run the script ex9_2_1.py. The script loads the wine data (Data/wine2) into Python with the loadmat function. Notice how the data is divided into 50% for training and 50% for test using stratification such that training and test have roughly equal class proportions. Fit a logistic regression model to the training set to classifiy the wines as red or white. Consider the red wines as "positive" and white wines as "negative." Notice how the script makes a plot of the ROC curve showing that the AUC is around 0.99.

Script details:

- · You have fitted a logistic regression model to the wine data before in exercise 5.2.6.
- · As before, use cross-validation. Use StratifiedKFold method to ensure that training and test sets have roughly equal class proportions (stratification).
- There is a function in the course toolbox called rocplot() that can make the ROC plot and compute the AUC score for you. Import it and type help(rocplot) to learn how to use it. Notice that the rocplot code assumes the score values are all distinct.
- 9.2.2 Consult the script ex9_2_2.py. The experiment in exercise 9.2.1 is repeated, but this time it is examined how well the type of wine can be classified using only the "Alcohol" attribute. Explain how it can be seen that the alcohol contents of the wine is not very useful for classifying wine as red or white.

Discussion:

♦ You have showed that using only the single attribute "Alcohol" to classify the wine as red or white performs worse than using all attributes. When using logistic regression, is it always best to use as many attributes as possible for the classification?

9.3 Tasks for the report

Continue working on the tasks for the reports as described in the previous exercise note.

References