Dynamics 2

Resonance and Damping
(Oscillatory Motion)

Applications of Oscillation Theory

- Oscillation theory required for many complex dynamical areas relating to the design and operation of Mechanical Engineering systems
  - e.g. failure due to oscillating stress or 'Fatigue' as a result of prolonged oscillations

- Two categories of oscillations
  - Free Oscillations - response to initial conditions and no external dynamic loads [we've seen these so far]
  - Forced Oscillations - response to external dynamic loads with 'Resonance' as the most significant feature

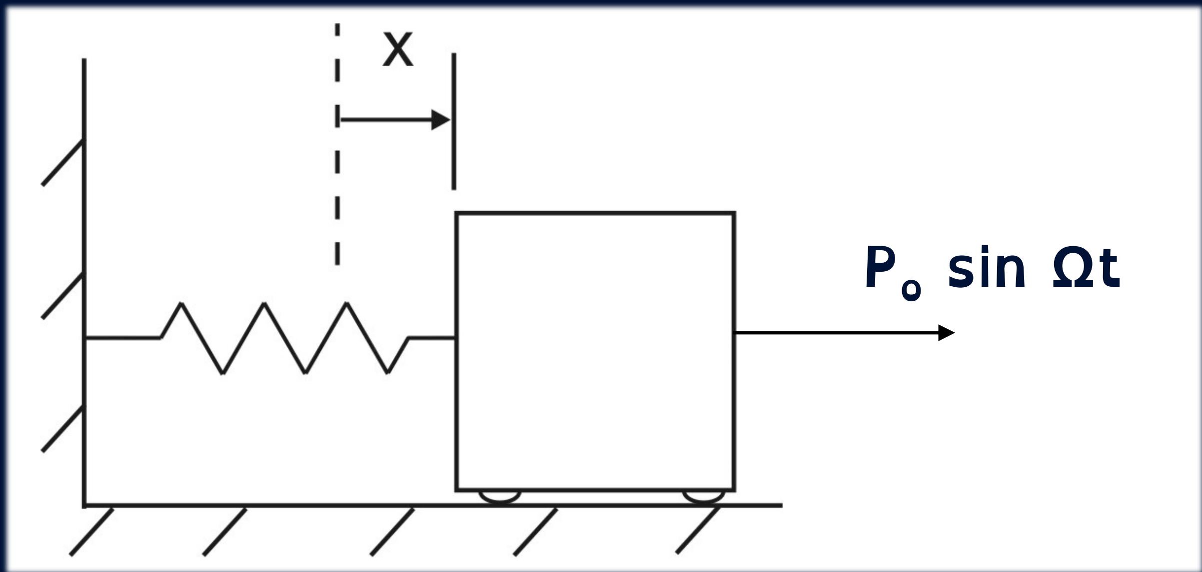
Resonance

- Resonance occurs when external cyclic disturbing forces have a frequency content near to system natural frequency then vibrations are greatly (sometimes dangerously) magnified - **resonance**

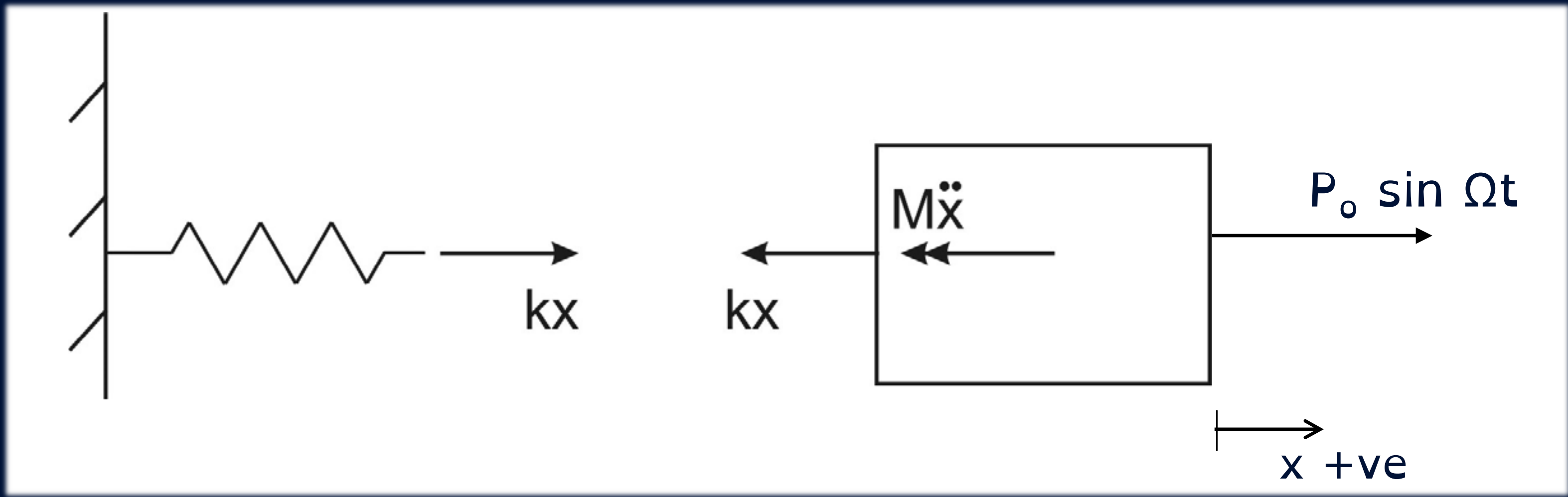
- Common sources of cyclic disturbances
  - rotating out-of-balance forces
  - cyclic power sources such as IC engines
  - prolonged vibratory ground motion as in earthquakes
  - tower shadow effects on rotating turbine blades

Resonance

- Illustrated by considering a sinusoidal (cyclic) load applied to a mass-spring system
  - $P_o \sin \Omega t$ is a force of amplitude P_o oscillating at frequency Ω



Differential Equation of Forced Oscillations



$$M\ddot{x} + Kx = P_0 \sin \Omega t$$

$$\ddot{x} + \frac{K}{M} x = \frac{P_0}{M} \sin \Omega t$$

$$\boxed{\ddot{x} + \omega_0^2 x = \frac{P_0}{M} \sin \Omega t}$$

$$\omega_0 = \sqrt{\frac{K}{M}}$$ (as before)

Differential Equation of Forced Oscillations

$$\ddot{x} + \omega_0^2 x = \frac{P_0}{M} \sin \Omega t$$

Non-homogeneous DE with general solution:

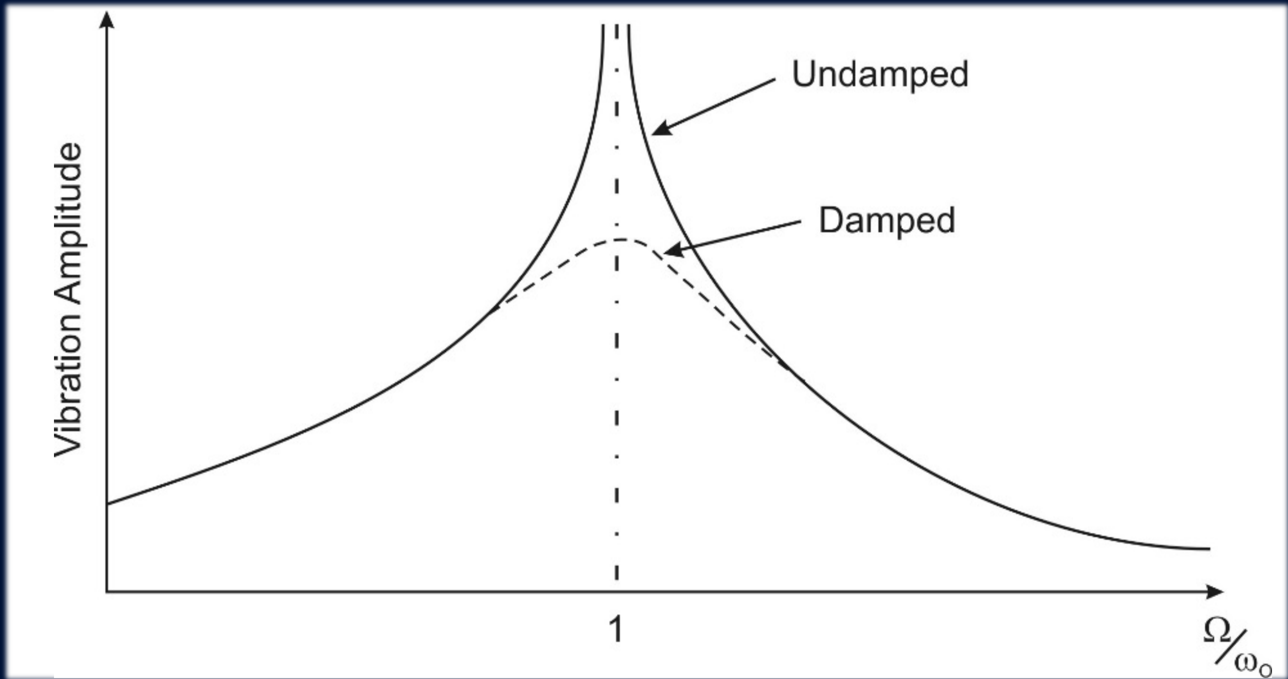
$x(t) =$ complimentary function(CF)
+ particular integral (PI)

solution in this case is:

$$x(t) = \underbrace{\boxed{C_1 \cos \omega_0 t + C_2 \sin \omega_0 t}}_{\text{Free vibration}} + \underbrace{\boxed{\frac{P_0 \sin \Omega t}{M(\omega_0^2 - \Omega_0^2)}}}_{\text{Forced oscillation}}$$

Resonance

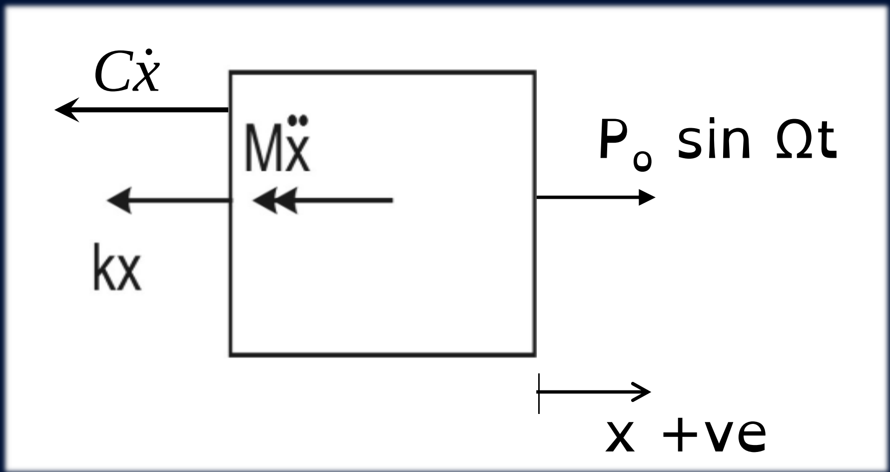
- last term on r.h.s. is large if $\Omega \rightarrow \omega_0$ - this is resonance
- in real systems, energy dissipation or 'damping' prevents infinite vibration amplitude at $\Omega = \omega_0$



$$\frac{P_0 \sin \Omega t}{M(\omega_0^2 - \Omega_0^2)}$$

Damping

- damping can be represented as a viscosity force proportional to the velocity of the mass
- in a direction opposite to the motion of the mass
- damping constant is usually written as C
- FBD for mass, including damping:



Damping

- DE: $$M\ddot{x} + C\dot{x} + Kx = P_0 \sin \Omega t$$

$$\ddot{x} + \frac{C}{M}\dot{x} + \frac{K}{M}x = \frac{P_0}{M}\sin \Omega t$$

written in Canonical form

$$\ddot{x} + 2\delta\omega_0\dot{x} + \omega_0^2 x = \frac{P_0}{M}\sin \Omega t$$

where

$$2\delta\omega_0\dot{x} \equiv \frac{C}{M}$$

δ is the non-dimensional damping ratio

Damping

$$\ddot{x} + 2\delta\omega_0\dot{x} + \omega_0^2 x = \frac{P_0}{M}\sin\Omega t$$

where

$$2\delta\omega_0\dot{x} \equiv \frac{C}{M}$$

δ is the non-dimensional damping ratio and can take the values:

δ = 0 for **undamped** systems

δ < 1 for **under damped** systems

δ > 1 for **over damped** systems

δ = 1 for **critically damped** systems

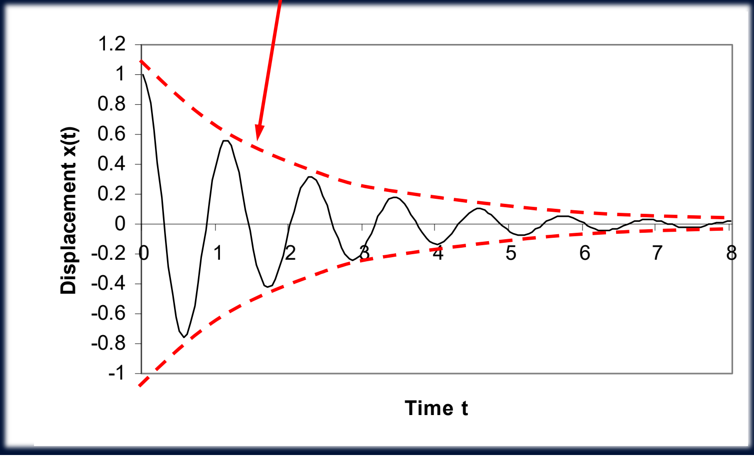
Damping

- Solution x(t) = CF + PI

$$x(t) = \boxed{e^{-\delta\omega_0 t}}[C_1 \cos\omega_0 t + C_2 \sin\omega_0 t]$$

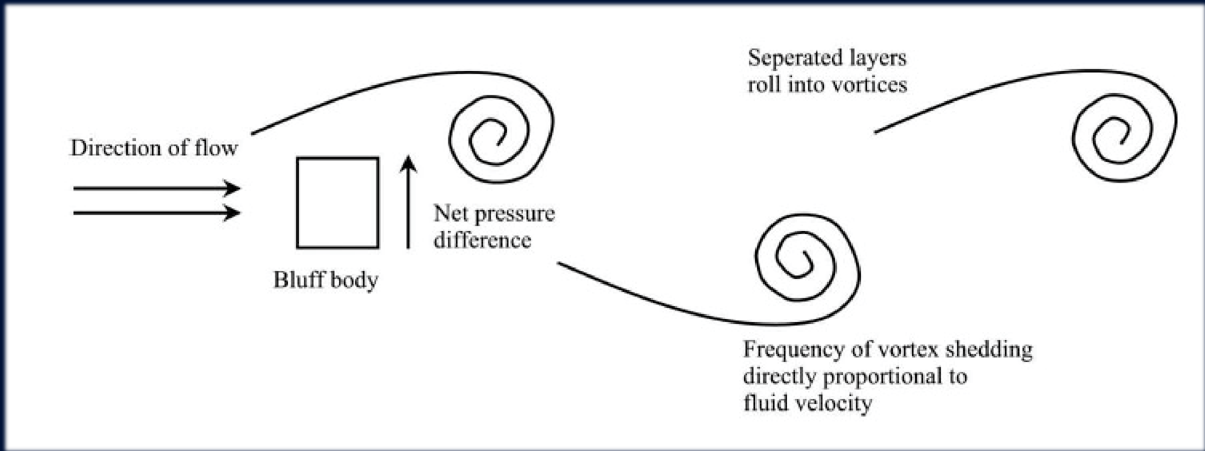
$$+ \frac{P_0}{M[(\omega_0^2 - \Omega_0^2) + \boxed{4\delta^2\omega_0^2\Omega^2}]^{1/2}} \sin\Omega t$$

1st part: 'free vibration' dying away exponentially

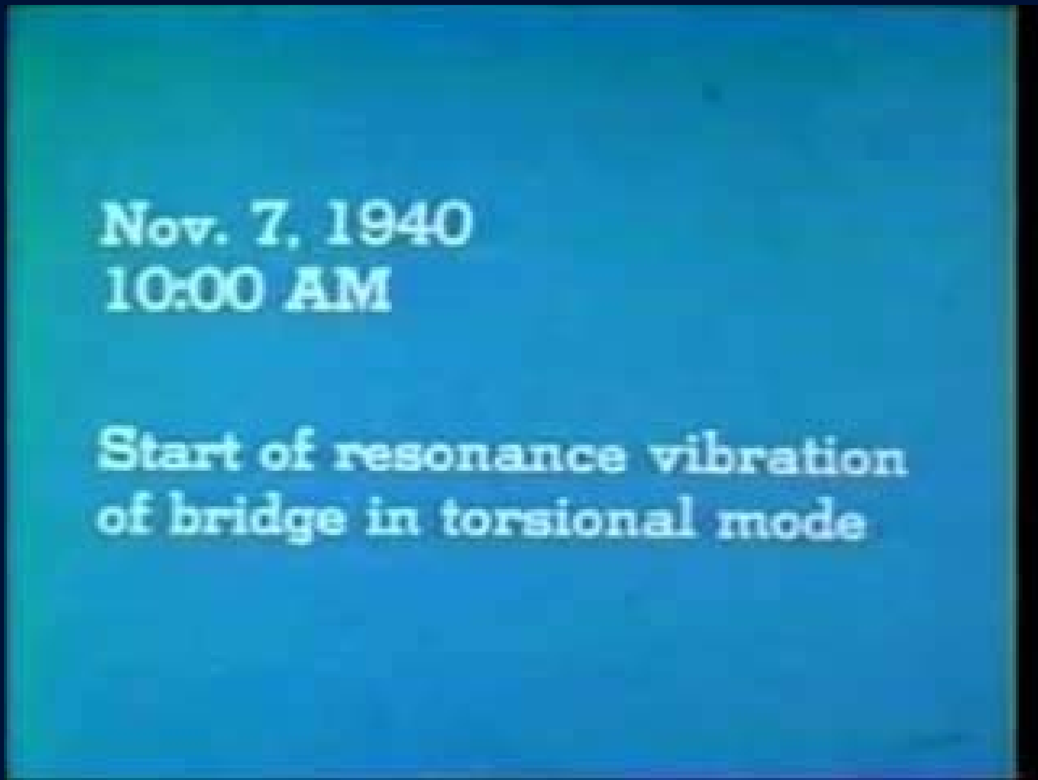


2nd part: damped forced vibration

- 'steady state' solution
- resonance as $\Omega \rightarrow \omega_0$
- amplitude finite as damped (non zero elements in denominator)


Seperated layers
roll into vortices
Direction of flow
Net pressure
difference
Bluff body
Frequency of vortex shedding
directly proportional to
fluid velocity

When, the forcing frequency is similar to the natural frequency



Tacoma Bridge

- Forcing: aerodynamics driven by wind with the forcing frequency determined by the wind speed and bridge geometry
- Natural frequency: can be calculated by bridge structure

See at https://youtu.be/3mclp9QmCGs?t=57s

Vortex shedding – mitigated by the introduction of external corkscrew fins (or strakes)

Summary

- Investigated resonance, for undamped free vibrations through to forced damped vibration (of a single degree of freedom system)
- Looked at causes and effects, including links to fluid mechanics