THE UNIVERSITY *of* EDINBURGH
School of Engineering

Tutorial 4: Mass & Energy Analysis of Steady State Processes and Devices

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Conceptual Questions:

1. Can a steady-state device have boundary work?
No. Any change in size of the control volume would require either a change in mass inside or a change in state inside, neither of which is possible in a steady-state process.

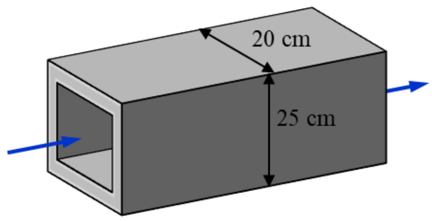
2. Apply the energy balance for throttling process. Does the gas temperature change during the throttling process when the working fluid is an ideal gas?
When throttling an ideal gas, the temperature does not change.

$$h_i = h_e$$
$$h_e - h_i = 0$$
$$\int_i^e C_p(T)dT = 0$$
$$or$$
$$T_e = T_i$$

3. How does a nozzle or spray head generate kinetic energy
By accelerating the fluid from a high pressure towards the lower pressure, which is outside the nozzle. The higher pressure pushes harder than the lower pressure so there is a net force on any mass element to accelerate it.

Problem Solving Questions:

4. Air (ρ=1.225 kg/m^3) travels in a rectangular duct with height 25 cm and width of 20 cm. The average velocity of 5 m/s. Determine the mass flow rate.



$$\dot{m} = \rho \vec{V}_{ave} A$$

$$A = l * w$$

$$\dot{m} = \rho \vec{V}_{ave}(l * w)$$

$$\dot{m} = (1.225\ kg/m^3)(5\ m/s)(0.20m * 0.25m)$$

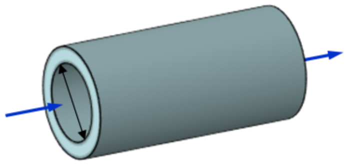
$$\boldsymbol{\dot{m} = 0.306\ kg/s}$$

[ans: 0.306 kg/s]

5. Air at 100 kPa, 50°C, flows through a pipe with a volume flow rate of 40 m^3/min. Assuming ideal gas and constant velocity,

a. Determine the mass flow rate through the pipe in kg/s.

b. Determine the average velocity ($\vec{V}_{avg}$) in m/s if the pipe has a cross sectional area with diameter 30 cm.



$$\dot{m} = \rho \vec{V}_{ave} A = \rho \dot{V}$$

$$\dot{m} = \rho \dot{V}$$

Need to convert $\dot{V}$ from m^3/min to m^3/s and find ρ

$$\dot{V} = \frac{40\ m^3}{min} * \frac{min}{60\ s} = 0.667\ m^3/s$$

Use ideal gas law to calculate ρ (use Table A.5 to find R=0.287 kJ/kg-K)

$$\rho = \frac{P}{RT}$$

$$\rho = \frac{100\ kPa}{(0.287\ kJ/kg \cdot K)(323K)} = 1.079\ m^3/kg$$

$$\dot{m} = \rho \dot{V} = (1.079\ m^3/kg)(0.667\ m^3/s)$$

$$\boldsymbol{\dot{m} = 0.719\ kg/s}$$

$$\dot{m} = \rho \vec{V}_{ave} \frac{\pi d^2}{4}$$

$$\vec{V}_{ave} = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \frac{\pi d^2}{4}}$$

$$\vec{V}_{ave} = \frac{\dot{m}}{\rho \frac{\pi d^2}{4}} = \frac{4 \dot{m}}{\rho \pi d^2} = \frac{4 \cdot (0.719\ kg/s)}{(1.079\ m^3/kg) \cdot \pi \cdot (0.30m)^2}$$

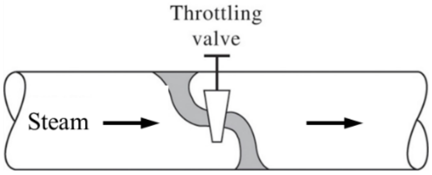
$$\boldsymbol{\vec{V}_{ave} = 9.43\ m/s}$$

[ans: (a) $\dot{m}$ = 0.719 kg/s , (b) $\vec{V}_{avg}$ = 9.43 m/s]

6. One way to determine the quality of saturated steam is to throttle the steam to a low enough pressure that it exists as a superheated vapor. Saturated steam at 400 kPa and quality x_1 is throttled to 100 kPa, 100°C.

a. Determine the initial quality of the steam at 400 kPa.

b. Does the water temperature increase or decrease in this process?



Conservation of mass : $\dot{m}_1 = \dot{m}_2 = \dot{m}$

Conservation of Energy : $\dot{Q}_{net} + \dot{m}_i\left(h_1 + \frac{\vec{V_1}^2}{2} + gz_1\right) = \dot{W}_{net} + \dot{m}_2\left(h_2 + \frac{\vec{V_2}^2}{2} + gz_2\right)$

Assume adiabatic ($\dot{Q}_{net} = 0$)*, no work* ($\dot{W}_{net} = 0$)*, no significant change in velocity* ($\Delta\vec{V} = 0$)*, no height differential* ($\Delta z = 0$)*, which leaves*

$$\dot{m}h_1 = \dot{m}h_2 \rightarrow h_1 = h_2$$

Using T_2 and P_2 we can find the enthalpy at the outlet using table B.1.1 (T_{sat} is approximately 100^0C for 100 kPa so the values for that entry in the table are fine to use)

$$h_2 = 2675.5 = h_1$$

Then use the staturated water values for h_f and h_{fg} at 400kPa from Table B.1.2 to calculate the quality

$$h_1 = h_{f@400kPa} + xh_{fg@400kPa}$$

$$x = \frac{h_1 - h_{f@400kPa}}{h_{fg@400kPa}}$$

$$x = \frac{2675.5 - 604.66}{2133.4}$$

$$\boldsymbol{x = 0.971}$$

We can then find T_1 in Table B.1.2 as $T_1 = T_{sat@400kPa}$

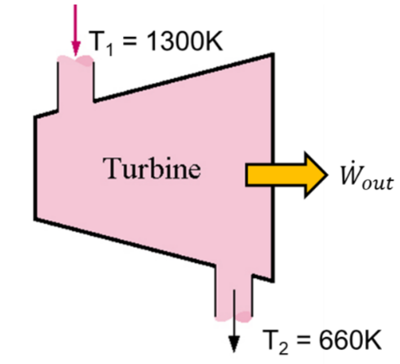
$$\boldsymbol{T_1 = 143.6\text{℃}\ which\ is\ > T_2\ so\ the\ temperature\ decreases}$$

[ans: (a) x = 0.971 , (b) temperature decreases]

7. High pressure air at 1300K flows into an aircraft gas turbine and undergoes a steady-state, steady-flow, adiabatic process to the turbine exit at 660K.

a. Calculate the work done per unit mass of air flowing through the turbine using $C_{p,ave}$ = 1.138 kJ/kgK.

b. Calculate the work done per unit mass when using the ideal gas air tables in the back of the book (i.e. Table A7.1 (Borgnakke & Sonntag).



Conservation of mass : $\dot{m}_1 = \dot{m}_2 = \dot{m}$

Conservation of Energy : $\dot{Q}_{net} + \dot{m}_i\left(h_1 + \frac{\vec{V_1}^2}{2} + gz_1\right) = \dot{W}_{net} + \dot{m}_2\left(h_2 + \frac{\vec{V_2}^2}{2} + gz_2\right)$

For an adiabatic turbine $\dot{W}_{net} = \dot{m}(h_1 - h_2)$

Part a tells us to use the Cp method (so treat air as an ideal gas)

For an ideal gas $\dot{m}(h_1 - h_2) = \dot{m}C_p(T_1 - T_2)$

Want work per kg so $w_{net} = \dot{W}/\dot{m}$

$$w_{out} = C_p(T_1 - T_2)$$

$$w_{out} = (1.138)(1300 - 660)$$

$$\boldsymbol{w_{out} = 728.3\ kJ/kg}$$

Now using Table A.7.1 to calculate the work out per kg

Back to the original turbine equation

$$\dot{W}_{net} = \dot{m}(h_1 - h_2)$$

$$w_{net} = \dot{W}/\dot{m} = h_1 - h_2$$

$h_1 = 1395.89$ kJ/kg

$h_2 = 670.78$ kJ/kg

$$w_{out} = 1395.89 - 670.78$$

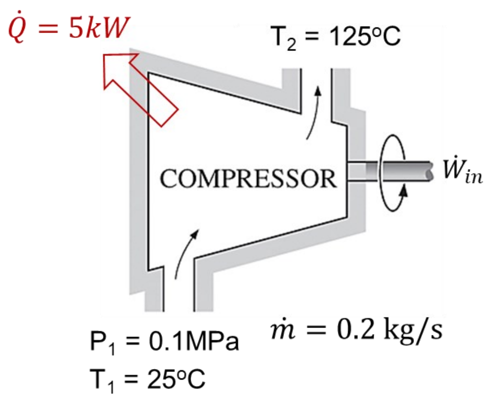
$$\boldsymbol{w_{out} = 725.1\ kJ/kg}$$

[ans: (a) w_{out} = 728.3 kJ/kg, (b) w_{out} = 725.1 kJ/kg]

8. Nitrogen gas is compressed in a steady-state, steady-flow, adiabatic process from 0.1MPa, 25°C. During the compression process the temperature becomes 125°C. The mass flow rate of nitrogen is 0.2 kg/s. Assume ideal gas with constant specific heat C_p = 1.039 kJ/kg·K

a. Determine the work done on the nitrogen, in kW.

b. If heat is lost to the surroundings at 5 kW, determine the work done on the nitrogen, in kW



Conservation of mass : $\dot{m}_1 = \dot{m}_2 = \dot{m}$

Conservation of Energy : $\dot{Q}_{net} + \dot{m}_i\left(h_1 + \frac{\vec{V_1}^2}{2} + gz_1\right) = \dot{W}_{net} + \dot{m}_2\left(h_2 + \frac{\vec{V_2}^2}{2} + gz_2\right)$

For part a) an adiabatic compressor $\dot{W}_{in} = \dot{m}(h_2 - h_1)$

N_2 is treated as an ideal gas so the ideal gas relation for enthalpy can be used

$$\dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$

$$\dot{W}_{in} = \dot{m}C_p(T_2 - T_1)$$

$$\dot{W}_{in} = (0.2)(1.039)(125 - 25)$$

$$\boldsymbol{\dot{W}_{in} = 20.8\ kW}$$

In part b) we can no longer assume adiabatic so the equation becomes

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) + Q_{loss} = \dot{m}C_p(T_2 - T_1) + Q_{loss}$$

$$\dot{W}_{in} = (0.2)(1.039)(125 - 25) + 5$$

$$\boldsymbol{\dot{W}_{in} = 25.8\ kW}$$

[ans: (a) $\dot{W}_{in}$= 20.8 kW ,(b) $\dot{W}_{in}$= 25.8 kW]

9. Apply the conservation of energy equation to a pump and include the change in kinetic and potential energy of the fluid streams entering and leaving the system.

$$\dot{Q}_{net} + \dot{m}_i\left(h_1 + \frac{\vec{V_1}^2}{2} + gz_1\right) = \dot{W}_{net} + \dot{m}_2\left(h_2 + \frac{\vec{V_2}^2}{2} + gz_2\right)$$

Show that if we remove the pumping work, we will arrive at the Bernoulli's equation for frictionless, incompressible fluid flow through a pipe.

Bernoulli's Equation: $\frac{P_2}{\rho} + \frac{\vec{V}_2^2}{2} + gz_2 = \frac{P_1}{\rho} + \frac{\vec{V}_1^2}{2} + gz_1$

So let's start with the conservation of energy equation.

$$\dot{Q}_{net} + \dot{m}_i\left(h_1 + \frac{\vec{V_1}^2}{2} + gz_1\right) = \dot{W}_{net} + \dot{m}_2\left(h_2 + \frac{\vec{V_2}^2}{2} + gz_2\right)$$

From the problem statement $\dot{W}_{net} = 0$

For incompressible liquids $\dot{Q}_{net}$ *is negligible*

this leaves

$$\dot{m}_1\left(h_1+\frac{\vec{V_1}^2}{2}+gz_1\right)=\dot{m}_2\left(h_2+\frac{\vec{V_2}^2}{2}+gz_2\right)$$

we can then use the definition of enthalpy

$$h=u+Pv$$

$$\dot{m}_1\left(u_1+P_1v_1+\frac{\vec{V_1}^2}{2}+gz_1\right)=\dot{m}_2\left(u_2+P_2v_2+\frac{\vec{V_2}^2}{2}+gz_2\right)$$

Conservation of mass : $\dot{m}_1=\dot{m}_2=\dot{m}$

$$u_1+P_1v_1+\frac{\vec{V_1}^2}{2}+gz_1=u_2+P_2v_2+\frac{\vec{V_2}^2}{2}+gz_2$$

For an incompressible liquid, the process is approximately isothermal so $\Delta u\approx 0$

$$P_1v_1+\frac{\vec{V_1}^2}{2}+gz_1=P_2v_2+\frac{\vec{V_2}^2}{2}+gz_2$$

recall $v=\frac{1}{\rho}$

and for an incompressible liquid specific volume does not change so

$$v_1=v_2=v=\frac{1}{\rho}$$

$$\frac{P_1}{\rho}+\frac{\vec{V_1}^2}{2}+gz_1=\frac{P_2}{\rho}+\frac{\vec{V_2}^2}{2}+gz_2$$