Lecture 14

Topic 3
Second Law of Thermodynamics

Topics

− 3.5: 2nd law of thermodynamics & principle of increase of entropy

Reading:
Ch 6&7: 6.11 & 7.1 – 7.6 Borgnakke & Sonntag Ed. 8
Ch 7: 7-10 – 7-13 Cengel and Boles Ed. 7

3.5 2nd Law Analysis

Relating 2nd law of thermodynamics with 1st law of thermodynamics

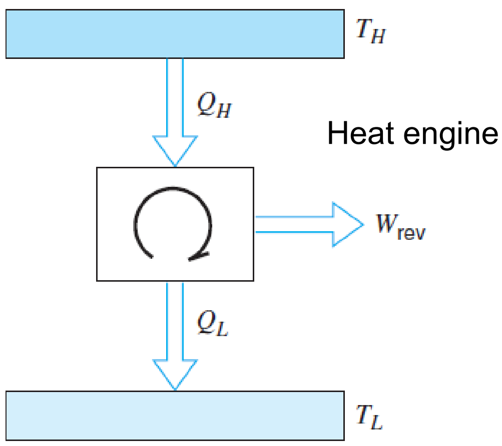
ΔEnergy = + in – out ΔEntropy = + in – out + generation

- Energy Equation: $\underbrace{\Delta E = U_2 - U_1}_{} = \underbrace{\sum m_i e_i - \sum m_e e_e}_{} + \underbrace{Q_{in}}_{} - \underbrace{W_{out}}_{}$
- Entropy Equation: $\underbrace{\Delta S = S_2 - S_1}_{\Delta\ within\ the\ system} = \underbrace{\sum m_i s_i - \sum m_e s_e}_{mass\ entering\ or\ leaving} + \underbrace{\int \frac{dQ}{T}}_{heat\ transfer\ to/from\ surroundings} + \underbrace{S_{gen}}_{work\ related\ terms}$

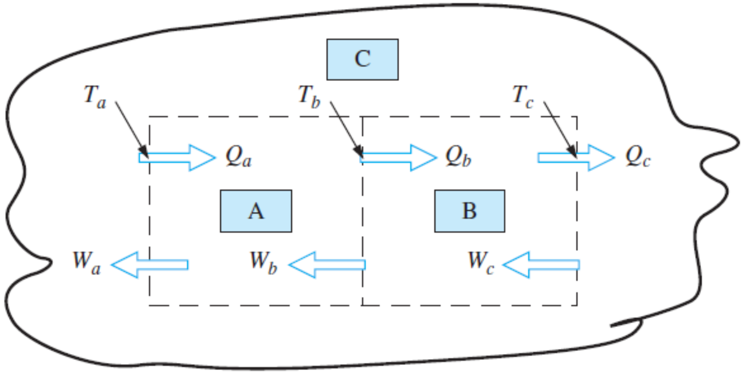
3.5 2nd Law Analysis

Heat term in 2nd law: $\int \frac{dQ}{T}$

- Heat transfer occurring across a boundary at $T_{boundary}$
- If $T_{boundary}$ is not constant: $\int \frac{dQ}{T}$
- If $T_{boundary}$ constant: $\frac{Q}{T_{boundary}}$
- Simplicity: extend boundaries to $T_{boundary}$
  - Heat engine: $\int \frac{dQ}{T} = \frac{Q_H}{T_H} + \frac{-Q_L}{T_L}$
  - Sub-system A: $\int \frac{dQ}{T} = \frac{Q_a}{T_a} + \frac{-Q_b}{T_b}$



Set of sub-systems

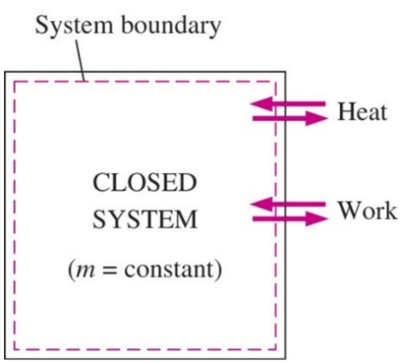


3.5 2nd Law Analysis

- ΔEntropy = + in – out + generation:
  - $dS_{sys} = S_2 - S_1 = \sum m_i s_i - \sum m_e s_e + \int \frac{dQ}{T} + S_{gen}$
  - $\frac{dS}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \int \frac{d\dot{Q}}{T} + \dot{S}_{gen}$

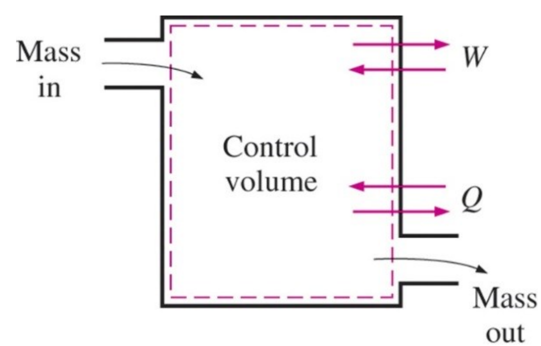
<u>Control Mass (closed system)</u>

- $S_2 - S_1 = \sum \cancelto{0}{m_i} s_i - \sum \cancelto{0}{m_e} s_e + \int \frac{dQ}{T} + S_{gen}$



<u>Control Volume (open system)</u>

- $\frac{dS_{sys}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \int \frac{d\dot{Q}}{T} + \dot{S}_{gen}$



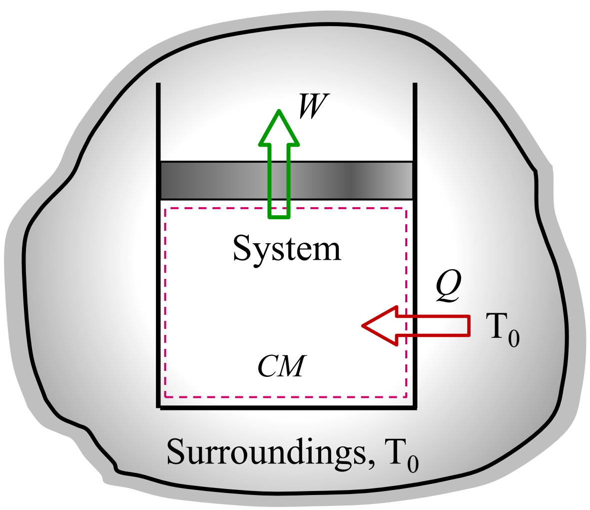
Principle increase of entropy

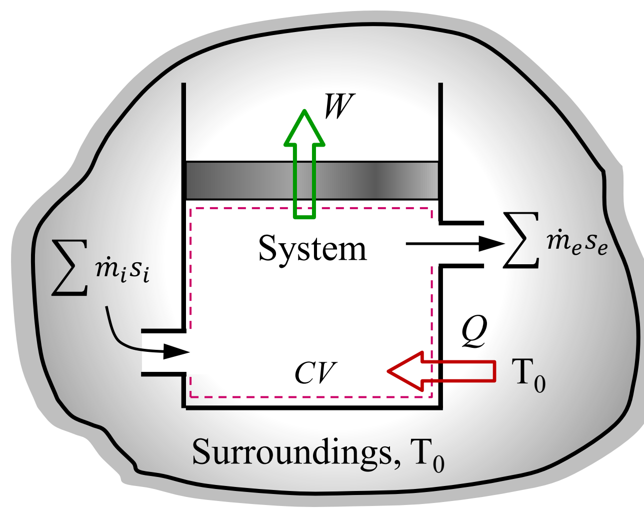
Control Mass (closed system)

- $S_{gen} = dS_{sys} + dS_{surr} \geq 0$
- $dS_{sys} = \int \frac{dQ}{T} + S_{gen}$
- $dS_{sys} = m(s_2 - s_1)$
- $dS_{surr} = -\frac{dQ}{T_0}$

Control Volume (open system)

- $\dot{S}_{gen} = \frac{dS_{sys}}{dt} + \frac{dS_{surr}}{dt} \geq 0$
- $\frac{dS_{sys}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \int \frac{d\dot{Q}}{T} + \dot{S}_{gen}$
- $\frac{dS_{sys}}{dt} = \frac{s_2 - s_1}{dt}$
- $\frac{dS_{surr}}{dt} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \frac{d\dot{Q}}{T_o}$

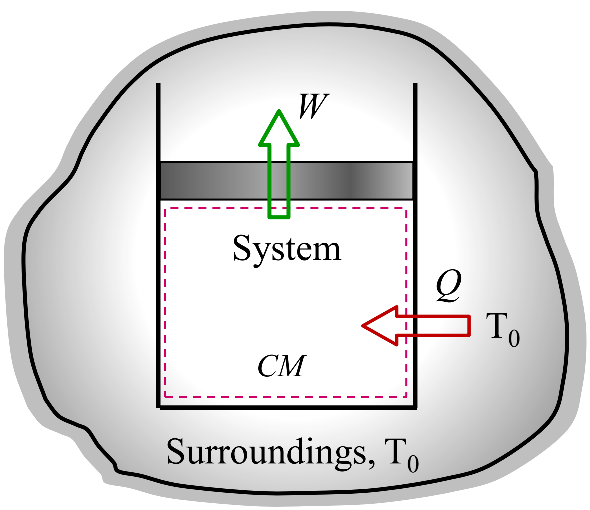


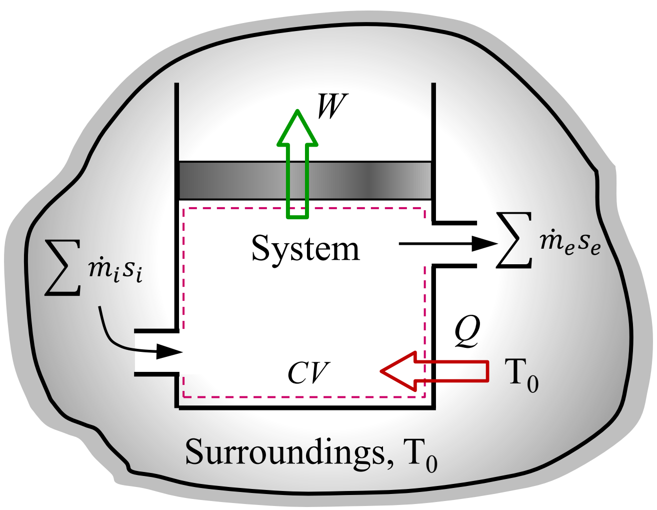


Principle increase of entropy

- Net entropy generated: $S_{gen} = dS_{sys} + dS_{surr}$
  - $S_{gen} = dS_{sys} + dS_{surr} \geq 0$
  - $\dot{S}_{gen} = \frac{dS_{sys}}{dt} + \frac{dS_{surr}}{dt} \geq 0$

$$S_{gen} = \Delta S_{Total} \begin{cases} > 0 & \text{Irreversible processes} \\ = 0 & \text{Reversible processes} \\ < 0 & \text{Impossible processes} \end{cases}$$





3.5.1 Second Law for Control Volumes

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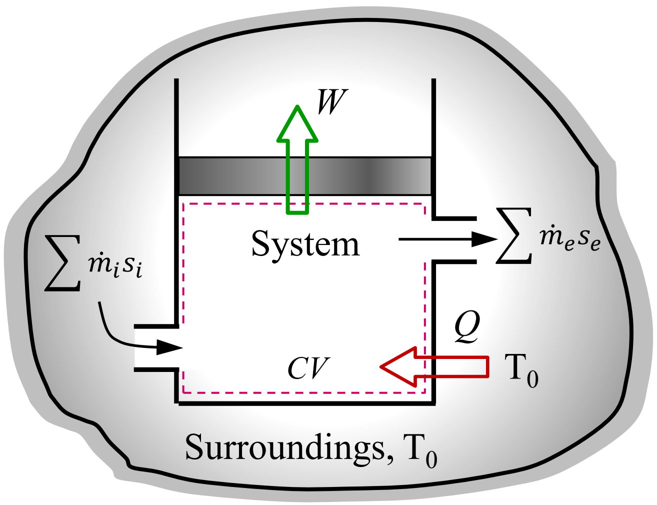
Steady State Systems

- $\frac{dS_{sys}}{dt} = 0 = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \int \frac{d\dot{Q}}{T} + \dot{S}_{gen}$
- $\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \int \frac{d\dot{Q}}{T} + \dot{S}_{gen}$
- Single entrance, single exit flow
  - $\dot{m}(s_e - s_i) = \sum(\dot{Q}/T) + \dot{S}_{gen}$
  - $s_e = s_i + \sum(q/T) + s_{gen}$



3.5.2 Example

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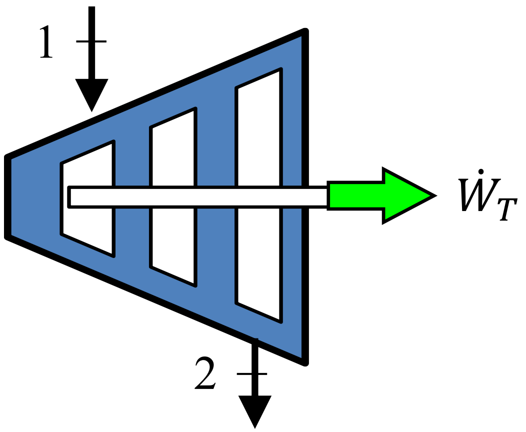
Example 3.6:

Steam enters a turbine at 300°C, 600 kPa at a flow rate of 0.5 kg/s. The steam exits the turbine at 20 kPa. Assume the turbine to be reversible and adiabatic



(a) Determine the quality steam/water exiting the turbine

(b) Determine the work power output of the turbine in kW

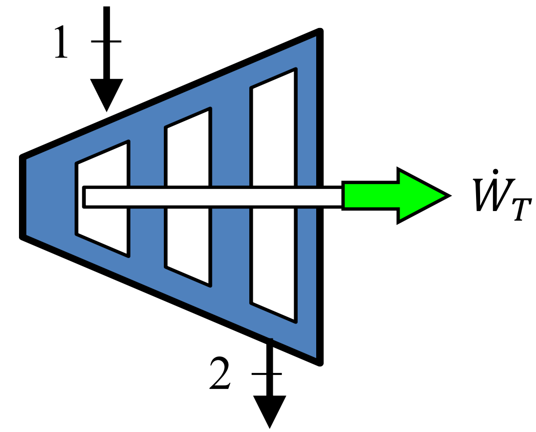
3.5.2 Example

Example 3.7:

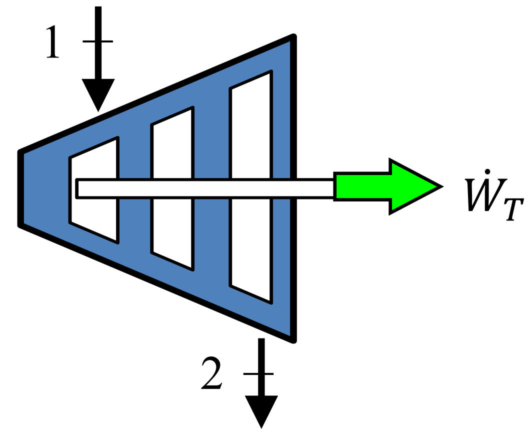
Take example 3.6, but now the turbine is *not* reversible and the steam exits the turbine as a saturated vapor at 20 kPa.

(a) Determine the new power output of the turbine in kW

(b) Determine the rate of entropy generated in the turbine



3.5.3 Isentropic Device Efficiency



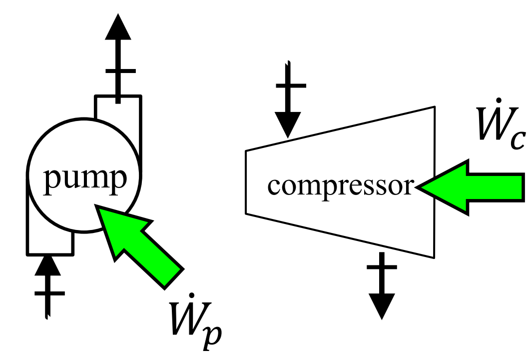
Isentropic Efficiency

- Entropy generation reduces turbine output (less efficient).

- Isentropic efficiency of turbine
  - Efficiency compared to reversible (perfect) process
  - $\eta_{turbine} = \frac{irreversible\ work}{reverible\ work} = \frac{actual\ work}{reverible\ work} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$
  - Example 3.7: $\eta_{turbine} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3061.63 - 2609.7}{3061.63 - 2430.48} = 0.716$

Same applies to **pumps** and **compressors**

- Work is into the system
- Irreversible: more W_{in} required for same ΔP
- $\eta_{compressor} = \frac{reversible\ work\ IN}{irreverible\ work\ IN} = \frac{reverible\ work}{actual\ work} = \frac{h_1 - h_{2s}}{h_1 - h_{2a}}$



3.5.3 Reversible Steady State Process

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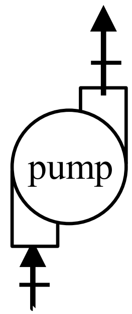
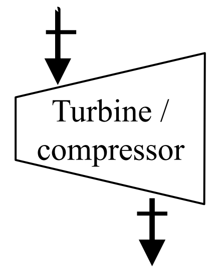
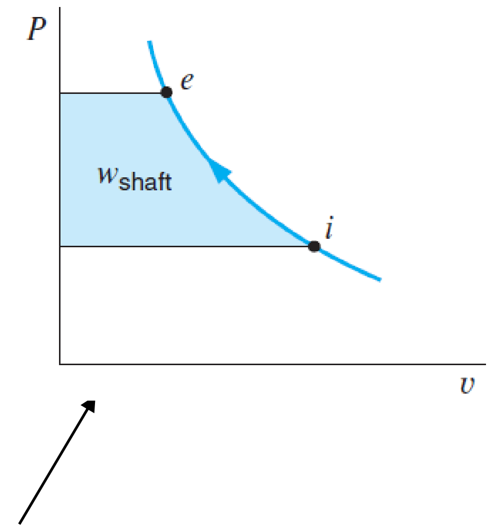
- Consider a steady-flow, work producing/consuming device (e.g. turbine, pump, compressor)



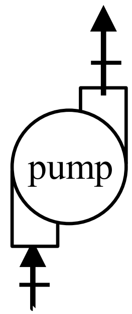


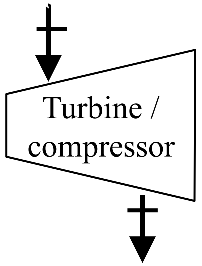
- First law for a steady-flow control volume
  - $\delta w_{rev} = \delta q_{rev} - dh - dke - dpe$
  - Recall: $\delta q_{rev} = Tds \rightarrow \delta w_{rev} = Tds - dh - dke - dpe$
  - Recall (lecture 13): $dh = Tds + vdP$
  - $\delta w_{rev} = Tds - Tds - vdP - dke - dpe$
  - $w_{rev} = -\int_1^2 vdP - \Delta ke - \Delta pe \; (kJ/kg)$
  - If ignore kinetic and potential energies: $w_{rev} = -\int_1^2 vdP \;\; (kJ/kg)$
  - Shaft work (reversible) is *not* area under the curve



3.5.3 Reversible Steady State Process

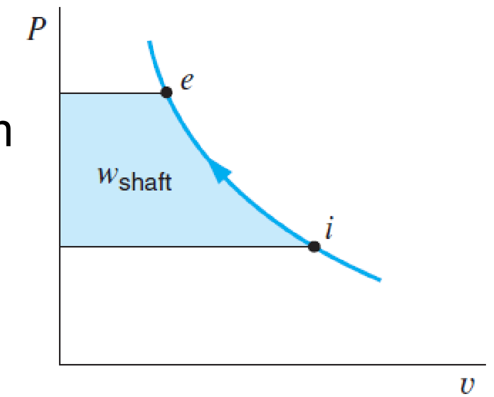
- Consider a steady-flow, work producing/consuming device (e.g. turbine, pump, compressor)





$$w_{rev,out} = -\int_1^2 v dP \ (kJ/kg)$$

- Turbines
  - Fluid pressure drops (dP < 0), work is out of the system
- Pump / Compressor
  - Fluid pressure rises (dP > 0), work is into the system

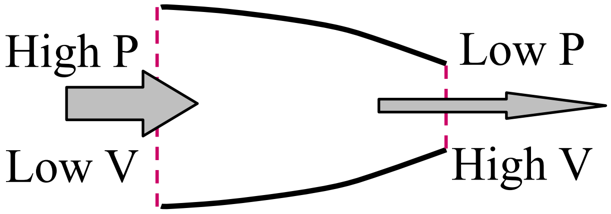


3.5.3 Reversible Steady State Process

Pumping an incompressible liquid

- Incompressible liquid: the specific volume $v \sim$ constant
- $w_{rev} = \int_1^2 v dP + \Delta ke + \Delta pe \rightarrow v\Delta P + \Delta ke + \Delta pe$

- Device involving no work (e.g. nozzles or a pipe section)
  - $0 = v(P_2 - P_1) + \Delta ke + \Delta pe \rightarrow Bernoulli\ equation$



nozzle

Pipe Flow

3.5.4 Example

Example 3.8

Air flows into an insulated nozzle at 1 MPa, 1200K with 15 m/s. The air exits the nozzle at 650 kPa. Assume air as an ideal gas with variable specific heats.

(a) Assuming an adiabatic, reversible process, determine the exit temperature and velocity.

(b) If the process is not reversible, the air expands to 650 kPa, 1100K.

i. Determine the exit velocity for this irreversible process

ii. Determine the specific entropy generated (in kW/kgK)

iii. Determine the nozzle efficiency defined as: $\eta_{nozzle} = \frac{V_{e,irr}^2 - V_i^2}{V_{e,rev}^2 - V_i^2}$

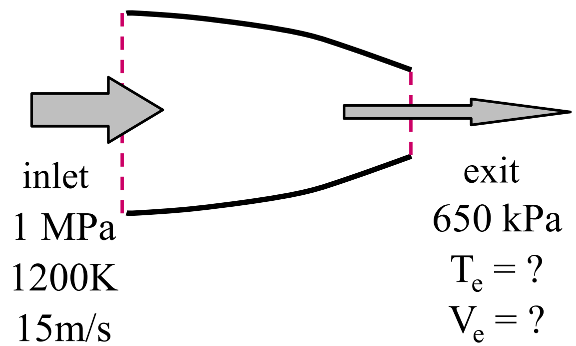


TABLE A7.1

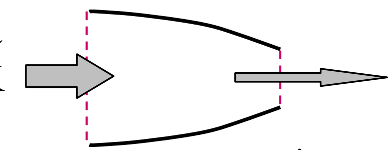
Ideal Gas Properties of Air, Standard Entropy at 0.1-MPa (1-Bar) Pressure

T (K)	u (kJ/kg)	h (kJ/kg)	s_T^0 (kJ/kg-K)	T (K)	u (kJ/kg)	h (kJ/kg)	s_T^0 (kJ/kg-K)
1000	759.19	1046.22	8.13493	1100	845.45	1161.18	8.24449
1050	802.10	1103.48	8.19081	1150	889.21	1219.30	8.29616
1100	845.45	1161.18	8.24449	1200	933.37	1277.81	8.34596

3.5.4 Example

Example 3.8

Air flows into an insulated nozzle at 1 MPa, 1200K with 15 m/s. The air exits the nozzle at 650 kPa.



Solution:

(b)

- State 2 (exit) irreversible (actual):
  - $T_{2,irreversible}$ = 1100K, $s_{T2}{}^0$ = 8.24449 kJ/kgK, $h_{2,irrev}$ = 1161.18 kJ/kg
- 1st law: $\boldsymbol{V_2 = \sqrt{2(h_1 - h_2) + V_1^2} = 483.2\,\frac{m}{s}}$
- 2nd Law: $s_2 - s_1 = s^0_{T2} - s^0_{T1} - R\ln\left(\frac{P_2}{P_1}\right) = s_{gen}$
- $\boldsymbol{s_{gen} = (8.24449 - 8.34596)\frac{kJ}{kgK} - 0.287\frac{kJ}{kgK}\ln\left(\frac{650kPa}{1000kPa}\right) = 0.02216\,\frac{\mathrm{kJ}}{\mathrm{kgK}}}$
- $\boldsymbol{\eta_{nozzle} = \frac{V_{e,irr}^2 - V_i^2}{V_{e,rev}^2 - V_i^2} = \frac{483.2^2 - 15^2}{530^2 - 15^2} = 0.83}$

3.5.5 Exercises

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Exercise 3-3

A Carnot engine as a piston cylinder device has 1 kg of air as the working fluid. Heat is supplied to the air at 800K and rejected by the air at 300K. At the beginning of the heat addition process, the pressure is 0.8 MPa and during heat transfer the volume triples.

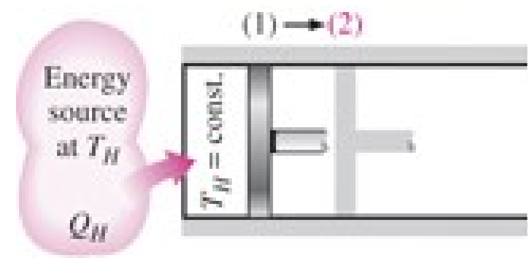
a) Calculate the net cycle work.
b) Calculate the amount of work done in the isentropic expansion process.
c) Calculate the entropy change during the heat rejection process.

Assume that air behaves an ideal gas with constant specific heats (C_v = 0.718kJ/kgK, R=0.287kJ/kgK and k=1.4).

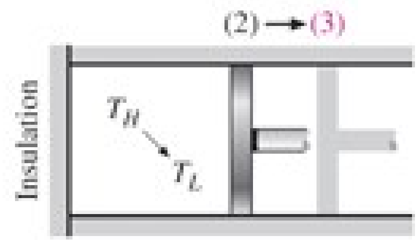
Solutions are worked out in the next couple of slides

3.5.5 Exercises

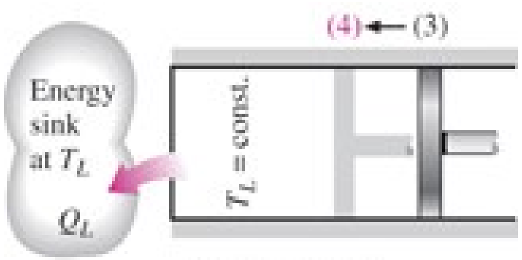
Heat Engine as piston/cylinder device
(closed system)



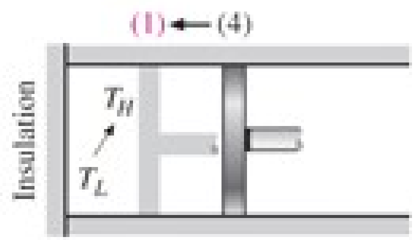
(a) Process 1-2
Constant temp. heat addition



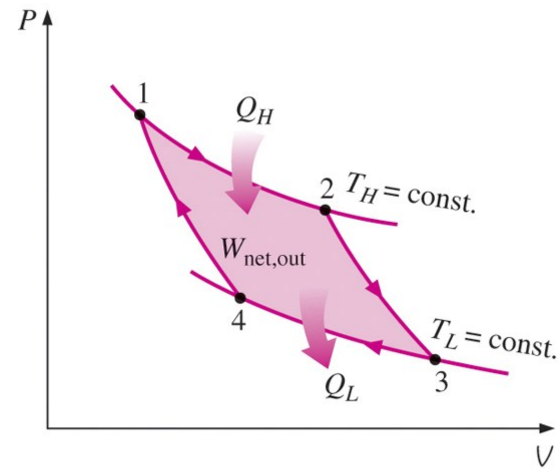
(b) Process 2-3
Reversible, adiabatic expansion

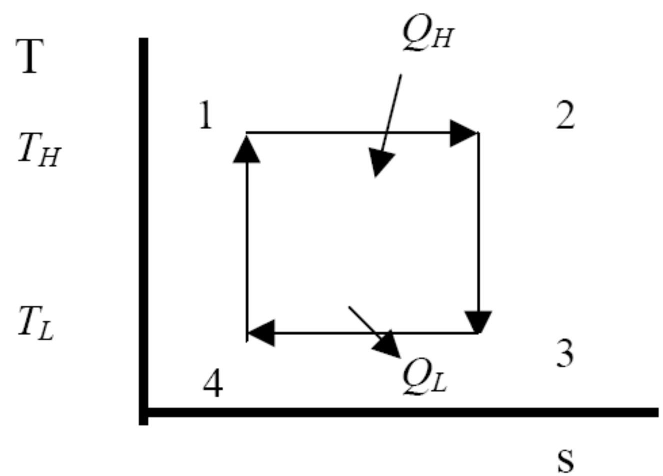


(c) Process 3-4
Constant temp. heat rejection



(d) Process 4-1
Reversible, adiabatic compression

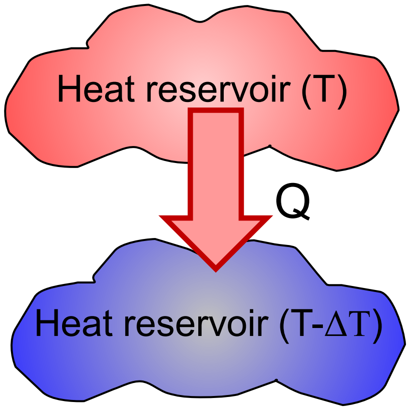




3.5.5 Exercises

Exercise 3-4

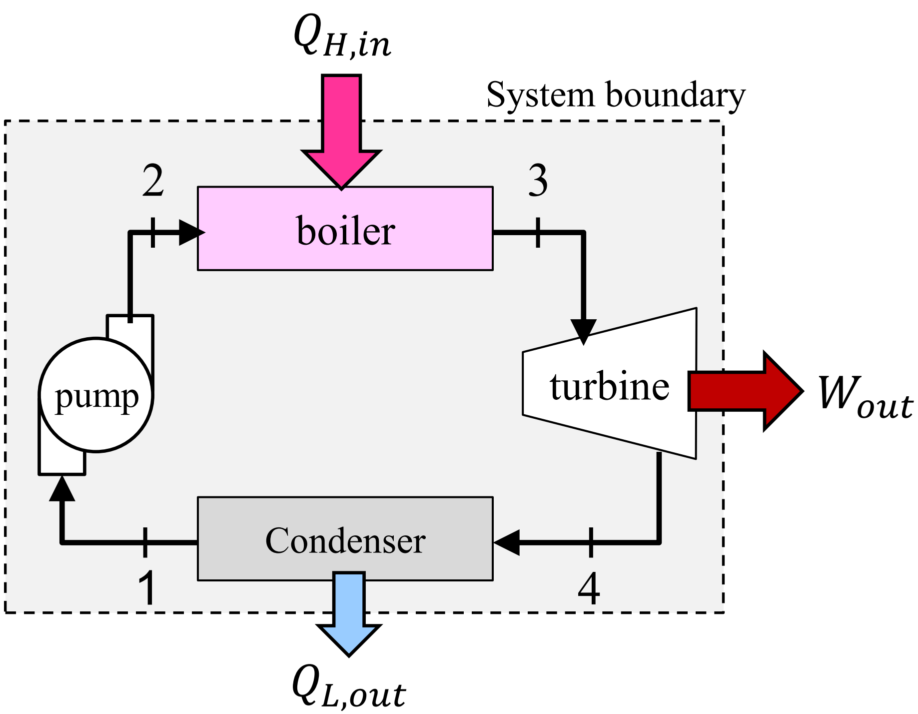
Find the total entropy change, or entropy generation, for the transfer of 1000 kJ of heat energy from a heat reservoir at 1000 K to a heat reservoir at 500 K. Assume the heat reservoirs are internally reversible.

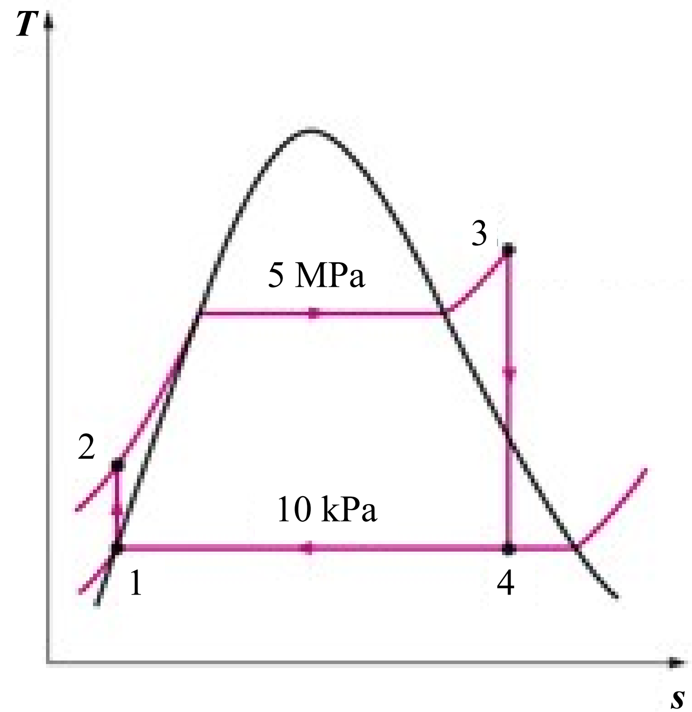


3.5.5 Exercises

Exercise 3-5

Saturated liquid water at 10 kPa leaves the condenser of a steam power plant and is pumped to the boiler pressure of 5 MPa. Calculate the work for an isentropic pumping process.





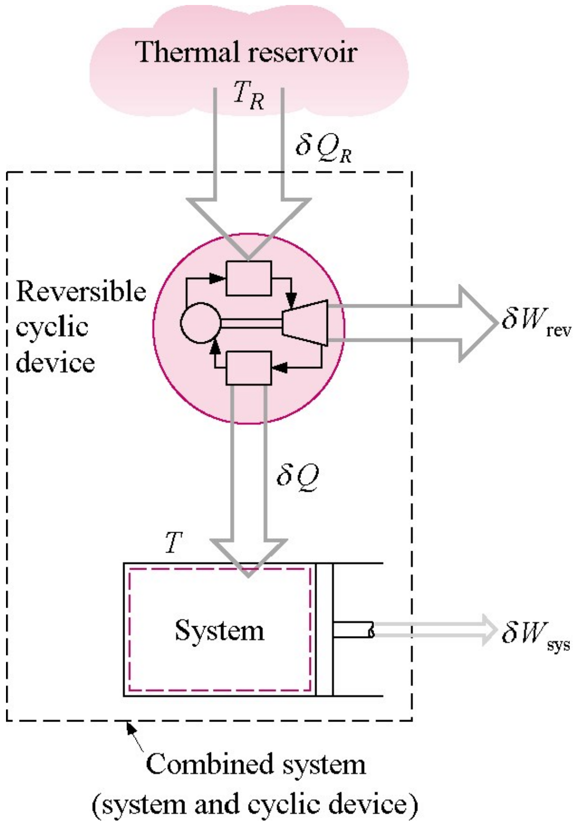
Appendix: Further Detail on Entropy and the Clausius Inequality

Consider a heat reservoir giving up heat to a reversible heat engine, which in turn gives up heat to a piston-cylinder device as shown below.

We apply the first law on an incremental basis to the combined system composed of the heat engine and the system. The change in the total energy of the combined system:

$$E_{in} - E_{out} = \Delta E_c$$

$$\delta Q_R - (\delta W_{rev} + \delta W_{sys}) = dE_c$$



where E_c is the energy of the combined system. Let W_c be the work done by the combined system.

Then the first law becomes

$$\delta W_c = \delta W_{rev} + \delta W_{sys}$$

$$\delta Q_R - \delta W_c = dE_c$$

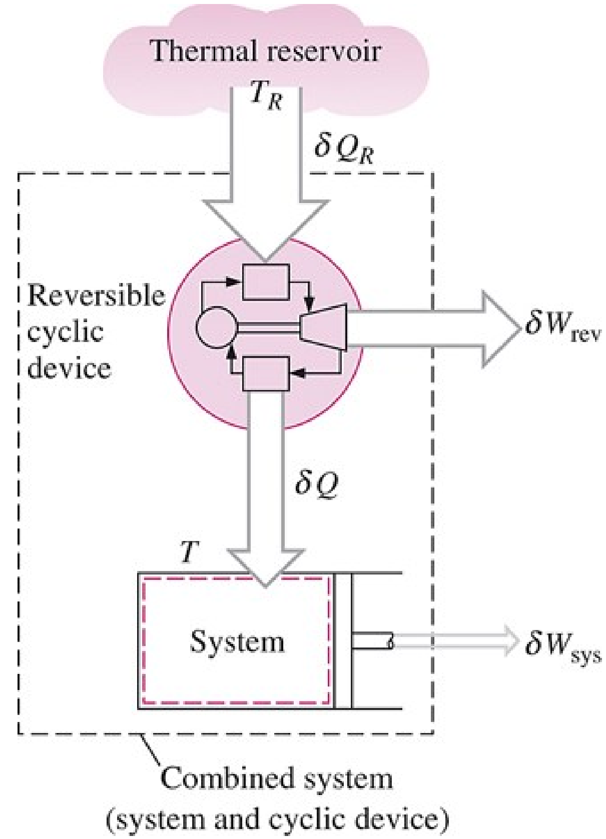
If we assume that the engine is totally reversible, then…

If we assume that the engine is totally reversible, then

$$\boxed{\frac{Q_L}{Q_H} = \frac{T_L}{T_H}}$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

$$\delta Q_R = T_R \frac{\delta Q}{T}$$



where T_R is the constant temperature at which the cyclic device operates.

The total net work done by the combined system becomes

$$\delta W_c = T_R \frac{\delta Q}{T} - dE_c$$

The total work done is found by taking the cyclic integral of the incremental work.

$$W_c = T_R \oint \frac{\delta Q}{T} - \oint dE_c$$

If the system, as well as the heat engine, is required to undergo a cycle, then

$$\oint dE_c = 0$$

and the total net work becomes

$$W_c = T_R \oint \frac{\delta Q}{T}$$

If W_c is positive, we would have a cyclic device exchanging energy with a single heat reservoir and producing an equivalent amount of work; thus, the Kelvin-Planck statement of the second law is violated (it is impossible for any device that operates on a cycle to receive heat from a single reservoir <u>and</u> produce a net amount of work)

But W_c can be zero (no work done) or negative (work is done on the combined system) and not violate the Kelvin-Planck statement of the second law. Therefore, since $T_R > 0$ (absolute temperature), we conclude:

$$W_c = T_R \oint \frac{\delta Q}{T} \leq 0$$

or

$$\oint \frac{\delta Q}{T} \leq 0$$

Here Q is the net heat added *to* the system, Q_{net}.

$$\boxed{\oint \frac{\delta Q_{net}}{T} \leq 0}$$

This equation is called the Clausius inequality.
The inequality holds for the irreversible process.
The equality holds for the reversible process.