Dynamics 2 – Tutorial 5

Fixed Axis Rotation Law and Moments of Inertia: Properties and Theorems

Outline Solutions

1.

Consider as 3 joined bars. Want I_A so find the moments of inertia for each bar about point A and then sum.

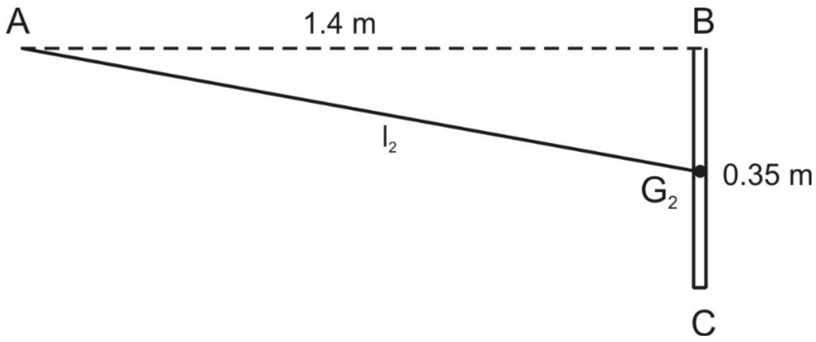
Bar 1 – One end of bar 1 is at point A so use the result for MMI about end of uniform bar

$$I_A = \frac{1}{3} m_1 l_1^{\,2}$$

m_1 = 74.67 kg by proportion

$$\Rightarrow \qquad I_A = \frac{1}{3}(74.67)1.4^2 = 48.78 \text{ kgm}^2$$

Bar 2 – The bar is remote from A. Find the MMI about the centroid of bar 2 (mass said to act about there) and then the parallel axis theorem to refer to point A.



I_{G2} about centroid of bar 2 (mass m_2 = 18.67 kg)

$$I_{G2} = \frac{1}{12} m_2 l_{BC}^{\ 2} = \frac{1}{12}(18.67)0.35^2 = 0.191 \text{ kgm}^2$$

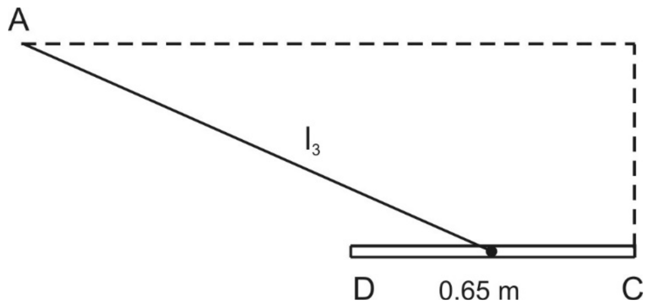
To use parallel axis theorem, find the distance between centroid of G and A. By Pythagoras

$$l_2 = \sqrt{1.4^2 + 0.175^2} = 1.4109 \text{ m}$$

Hence

$$I_A = 0.191 + 18.67 \times 1.4109^2 = 37.35 \text{ kgm}^2$$

Bar 3 – A similar approach for bar 3



$$I_{G3} = I_{G3} = \frac{1}{12} m_3 l_{CD}{}^2 = \frac{1}{12}(34.67)0.65^2 = 1.221$$

$$l_3 = \sqrt{0.35^2 + 1.075^2} = 1.1305 \text{ m}$$

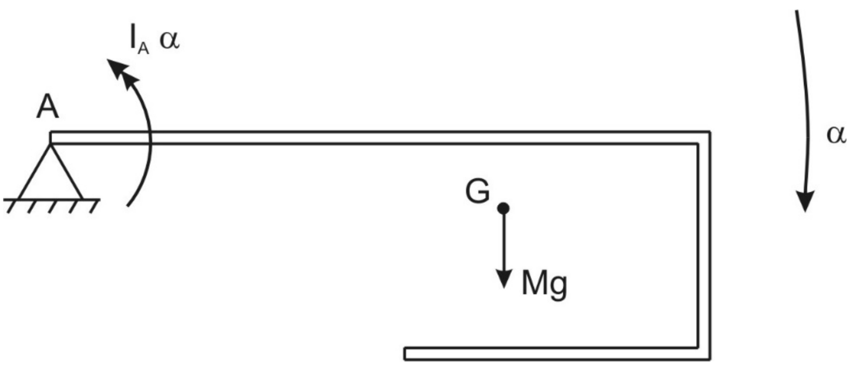
Hence

$$I_A = I_{G3} + m_3\, l_3^2 = 1.221 + 34.67 \times 1.1305^2 = 45.53 \text{ kgm}^2$$

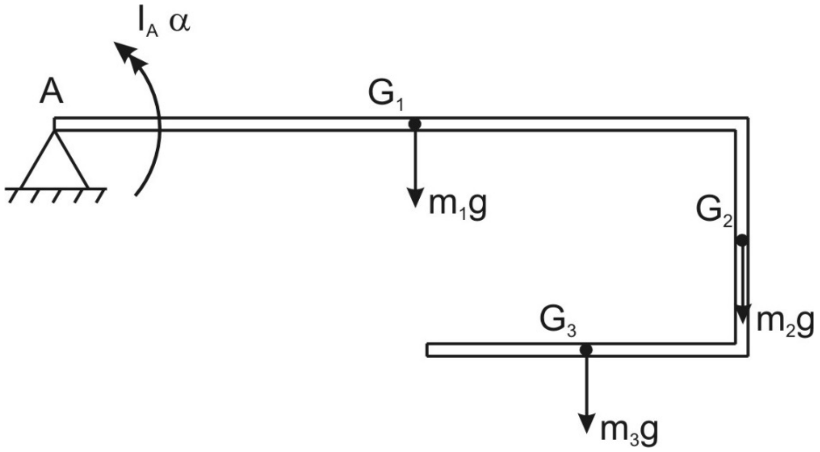
Sum these to give I_A for bar

$$I_A = 131.66 \text{ kgm}^2$$

Second part is to determine the initial Angular Acceleration. Angular acceleration is due to the moment about A of the total bar weight acting through G. But G is not yet located.



Alternatively, take the separate weights of the 3 bar parts (as locations of G for each part are known) which gives the same result.



$$I_A \alpha = (m_1 g)\, 0.7 + (m_2 g)\, 1.4 + m_3 g\, (1.075)$$

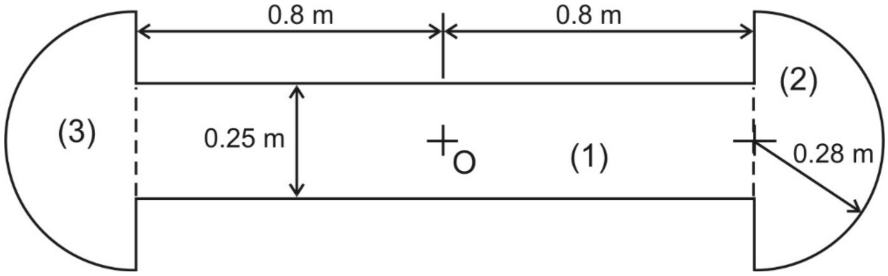
$$131.66\, \alpha = (74.67g)\, 0.7 + (18.67g)\, 1.4 + (34.67)g\, 1.075$$

$$\alpha = \frac{1134.8}{131.7} = 8.62 \text{ rad/s}$$

This is the initial angular acceleration and it will **not** remain constant in this problem. As the bar rotates about A the moment arms of the weights will reduce with G eventually lying below A and with the moment zero.

2.

Consider plate in 3 parts

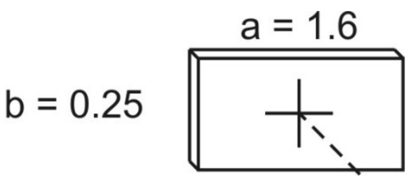


Want I_O for axis perpendicular to page.

Total mass = 170 kg; Total area = (1.6 x 0.25) + $2\left(\frac{\pi \times 0.28^2}{2}\right)$ = 0.6463 m^2

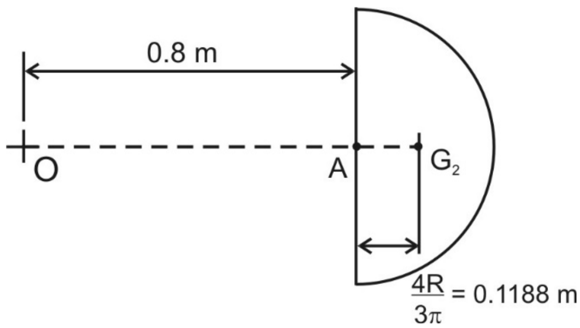
$\Rightarrow$ Mass/Area = 263.04 kg/m^2 M_1 = 105.21 kg, $m_2 = m_3$ = 32.39 kg

Part 1 – I_O can be found using given result for a rectangle.



$$I_O = \frac{1}{12}(105.21)(1.6^2 + 0.25^2) = 22.99 \text{ kgm}^2$$

Part 2 (more difficult). First find I_A for a half-disc.



I_A is half of the answer for a circular disc of mass ($2m_2$).

$$I_A = \frac{1}{2}\left(\frac{1}{2}2m_2R^2\right) = 1.2697 \text{ kgm}^2$$

Then I_{G2} from I_A using parallel axis theorem

$$I_{G2} = I_A - m_2(0.1188^2) = 0.8125 \text{ kgm}^2$$

Then IO from IA using parallel axis theorem (again)

$$I_O = I_{G2} + m_2(0.9188^2) = 28.16 \text{ kgm}^2$$

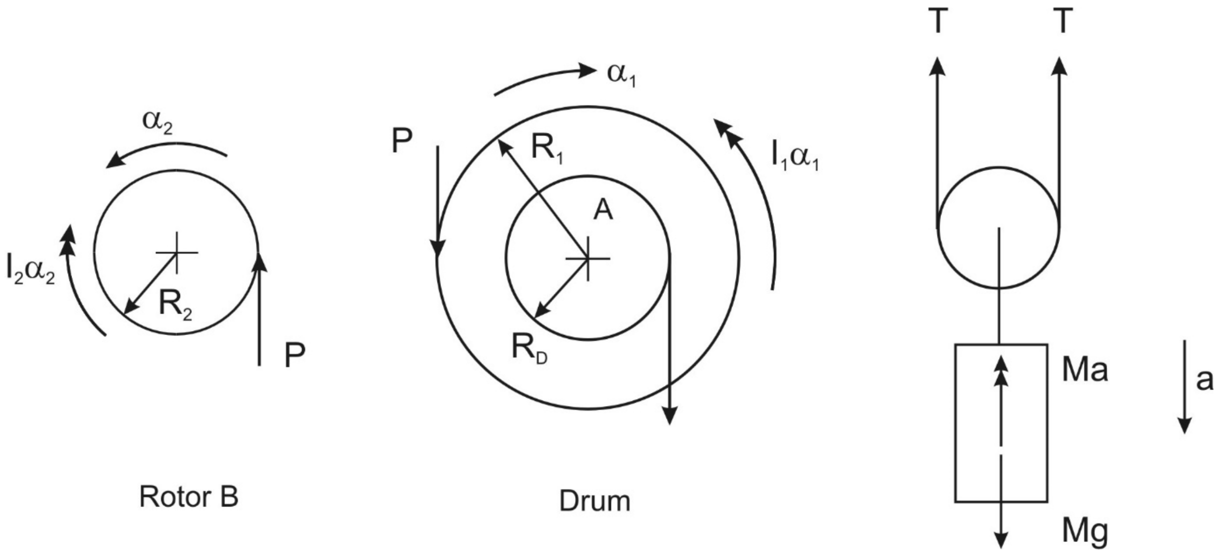
Part 3 gives same I_O as Part 2

Finally total I_O

$$I_O = 22.99 + 2(28.16) = 79.31 \text{ kgm}^2$$

3.

Use 3 FBDs to represent system (note gear tooth force P and inertia couples)



From the FBDs

Drum (assume α_1 clockwise)

$$TR_D - PR_1 = I_1\alpha_1 \qquad (1)$$

Rotor B (assume α_2 anticlockwise)

$$PR_2 = I_2\alpha_2 \qquad (2)$$

Load (assume acceleration a downwards)

$$2T - Mg + Ma = 0 \qquad (3)$$

Also have kinematic links

a) The load pulley makes the wire acceleration twice the load acceleration. Hence

$$R_D\,\alpha_1 = 2a \qquad (4)$$

b) The gears in mesh link α_1 and α_2

$$\alpha_2 = \frac{R_1}{R_2}\,\alpha_1 = \frac{N_1}{N_2}\,\alpha_1 \qquad (5)$$

The question requires a to be found. The best procedure is to put P (2) and T from (3) into (1) to get

$$\frac{m}{2}(g-a)R_D - \left(\frac{R_1}{R_2}\right)I_2\alpha_2 = I_1\alpha_1$$

Now use kinematics (4, 5) to write a and α_2 in terms of α_1. This gives an equation with only one unknown α_1 to solve for:

$$Mg\frac{R_D}{2} - \frac{M}{4}R_D{}^2\alpha_1 - \left(\frac{N_1}{N_2}\right)^2 I_2\alpha_1 = I_1\alpha_1$$

Collect terms to get

$$\left[I_1 + \left(\frac{N_1}{N_2} \right)^2 I_2 + \frac{M}{4} R_D{}^2 \right] \alpha_1 = Mg \frac{R_D}{2}$$

Substituting values ($M = 250$ kg, $I_1 = 60(1.45^2) = 126.15$ kgm^2, $I_2 = 12.5$, $(N_1/N_2) = 4$, $R_D = 0.8$ m) gives

$$366.15\ \alpha_1 = 981$$

$$\Rightarrow \qquad \alpha_1 = 2.769 \text{ rad/s}^2$$

and

$$a = R_D \left(\frac{\alpha_1}{2} \right) = 1.072 \text{ m/s}^2$$

Time for load to fall 4m from $s = \frac{1}{2} at^2$

$$4 = \tfrac{1}{2}(1.072)t^2 \Rightarrow t = 2.73 \text{ secs}$$

Tension T can be found from: $T = \dfrac{M}{2}(g - a) = 1092$ N