

Module 10 self-assessment

Question 1

Elastic energy is the potential mechanical energy stored in the volume of a material when that volume is compressed or stretched. The elastic energy W corresponding to a volume R of a certain material is $q^2 R / (2EI)$, where q is its stress and E and I are constants. Find the elastic energy of a cylindrical volume of radius a and length l in which the stress varies directly as the distance from its axis, being zero at the axis and q_0 at the outer surface. Justify your answer by commenting on the influence of a and l on this energy.

Question 2

Compute the integral

$$\iiint_R z \, dV,$$

for a region R at the intersection of two unit spheres centred at $(0, 0, 0)$ and $(0, 0, 1)$ respectively.

Question 3

Let R be the solid enclosed by the paraboloids $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. Setup an iterated integral in cylindrical coordinates for the volume of R . You are not required to solve it.