Module 11 self-assessment

Question 1

A ball is drawn at random from a box containing 6 red balls, 4 white and 5 blue ones. Determine the probability it is (a) red, (b) blue, (c) not red and (d) red or white.

> **Solution:**
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> There are a total of 15 balls in the urn, so for (a) $\mathbb{P}(\text{red}) = 6/15$, (b) $\mathbb{P}(\text{blue}) = 1/3$, (c) $\mathbb{P}(\text{not red}) = 1 - \mathbb{P}(\text{red}) = 9/15$, and (d) $\mathbb{P}(\text{red} \cup \text{white}) = \mathbb{P}(\text{red}) \cup \mathbb{P}(\text{white}) = 2/3$ as events are disjoint, that is a ball cannot have more than a single colour, hence the probabilities are added.
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> These answers are found using simple counting taking into consideration that the probability of taking out one ball of any colour is 1,
>
> $$\mathbb{P}(\text{colour}) = \frac{\text{number of balls of that colour}}{\text{total number of balls}}$$

Question 2

For some random events C and D consider that $\mathbb{P}(D) = 0.45$ and $\mathbb{P}(C \cap D) = 0.1$. Find $\mathbb{P}(C^c \cap D)$ where C^c is the complement of event C.

> **Solution:**
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> To make any progress in this question we need to think of a relationship between $D \cap C$ and $D \cap C^c$. For this we will need to make a key observation: that $(D \cap C)$ and $(D \cap C^c)$ are always *disjoint*. As by definition, C and C^c are disjoint, their respective subsets $(C \cap D)$ and $(D \cap C)$ are also disjoint. Effectively we can express D as
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> $$D = (D \cap C) \cup (D \cap C^c)$$
>
> and then taking the probabilities on both sides yields
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> $$\mathbb{P}(D) = \mathbb{P}(D \cap C) + \mathbb{P}(D \cap C^c).$$
>
> Rearranging and introducing the values of the probabilities gives
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> $$\mathbb{P}(D \cap C^c) = \mathbb{P}(D) - \mathbb{P}(D \cap C) = 0.45 - 0.1 = 0.35.$$