Dynamics 2

Introduction to Oscillations
(Oscillatory Motion)

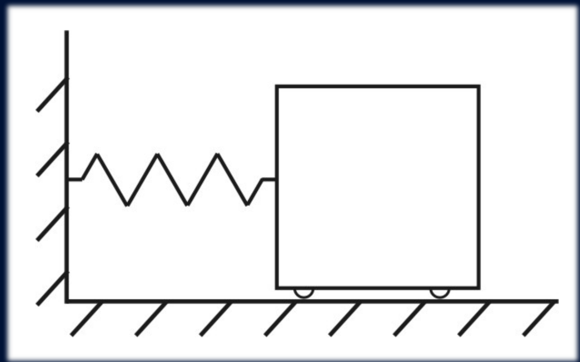
Oscillatory Motion

- Dynamics problems have involved steadily accelerating particles and bodies.
- Large problem areas in Dynamics involve non-steady acceleration

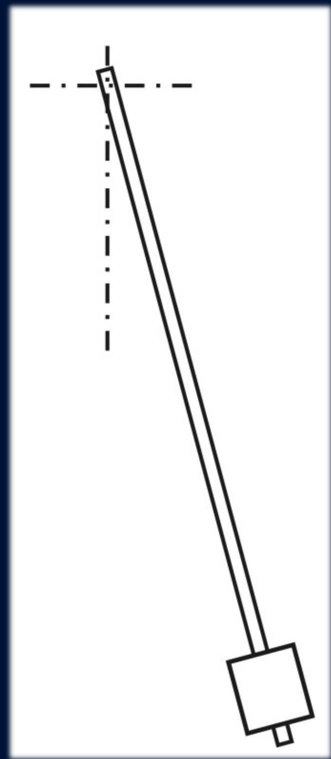
  -ie oscillatory motion
- A consequence of the interaction of two basic properties of material bodies

  Mass + Elasticity → Oscillations

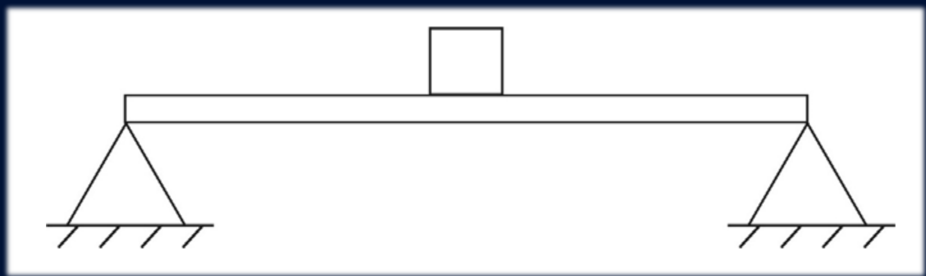
Basic Mechanical Oscillators



mass + spring



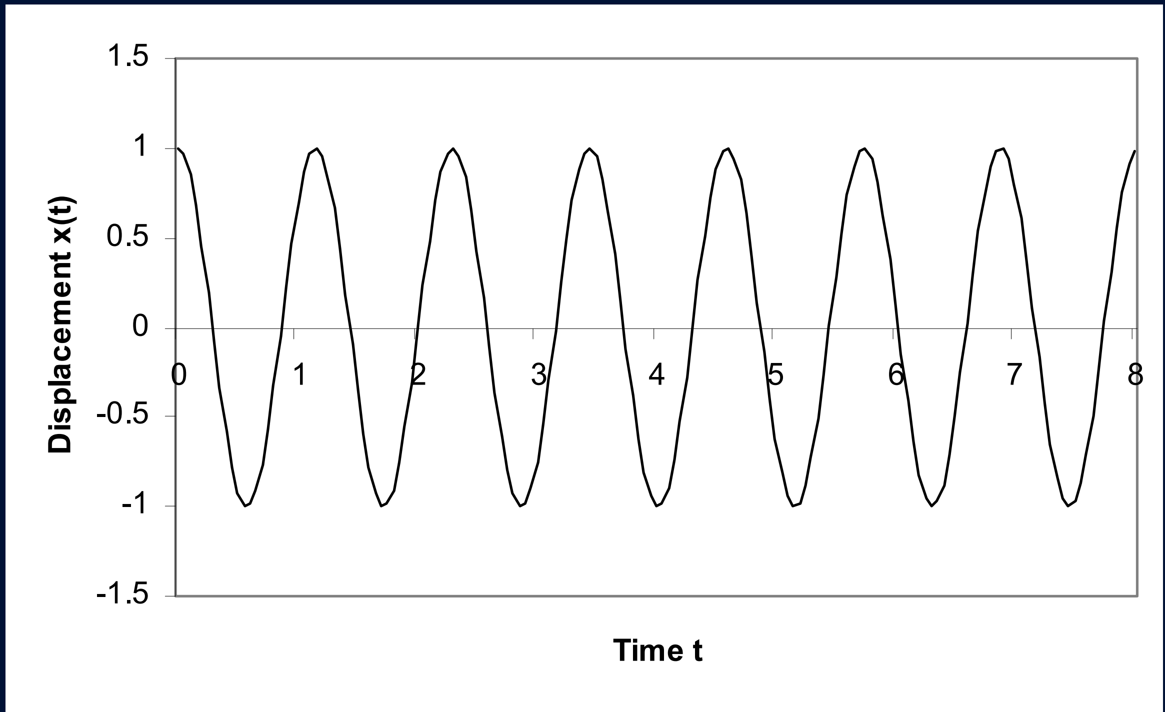
pendulum

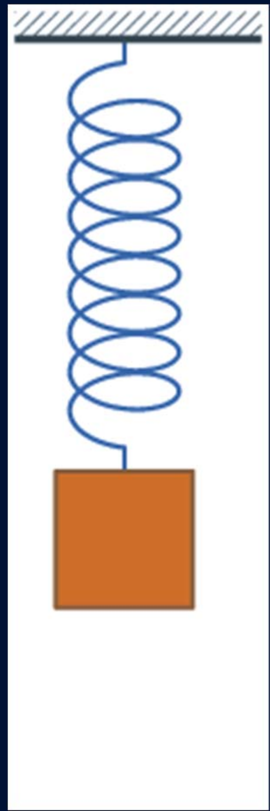


mass on beam

Basic Mechanical Oscillators

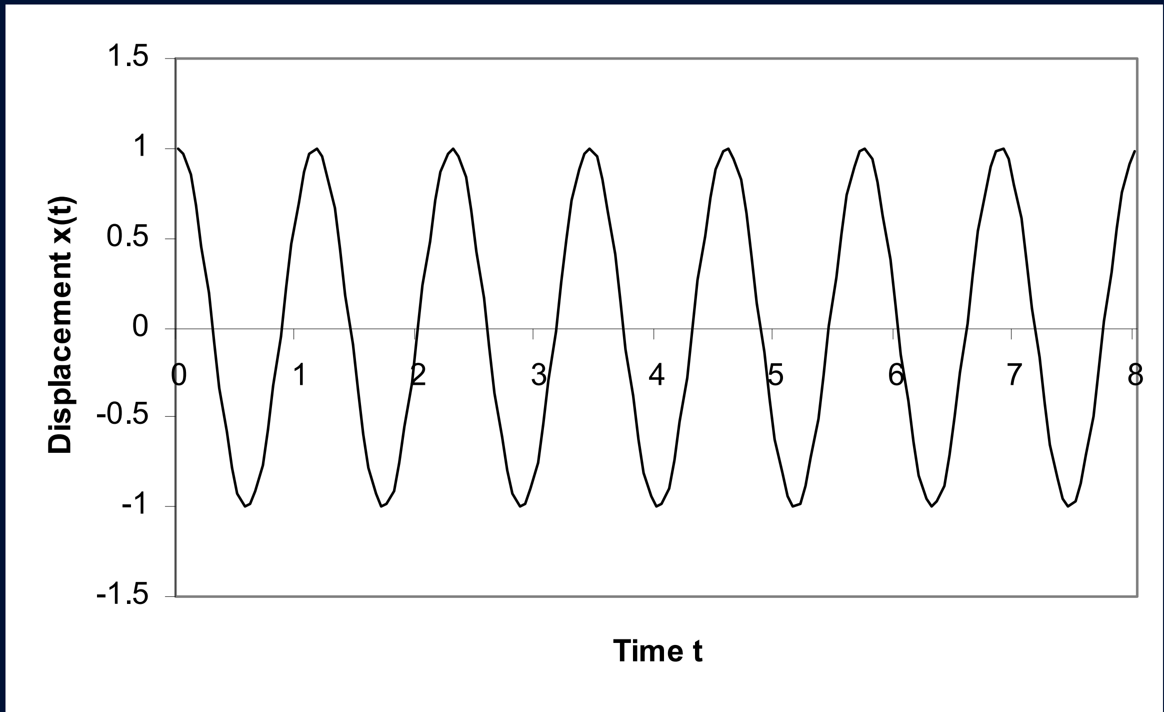
- Disturbances cause oscillations
  - Simple Harmonic Motion (SHM)

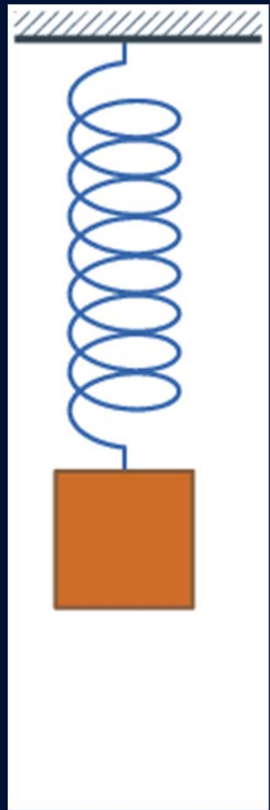




Basic Mechanical Oscillators

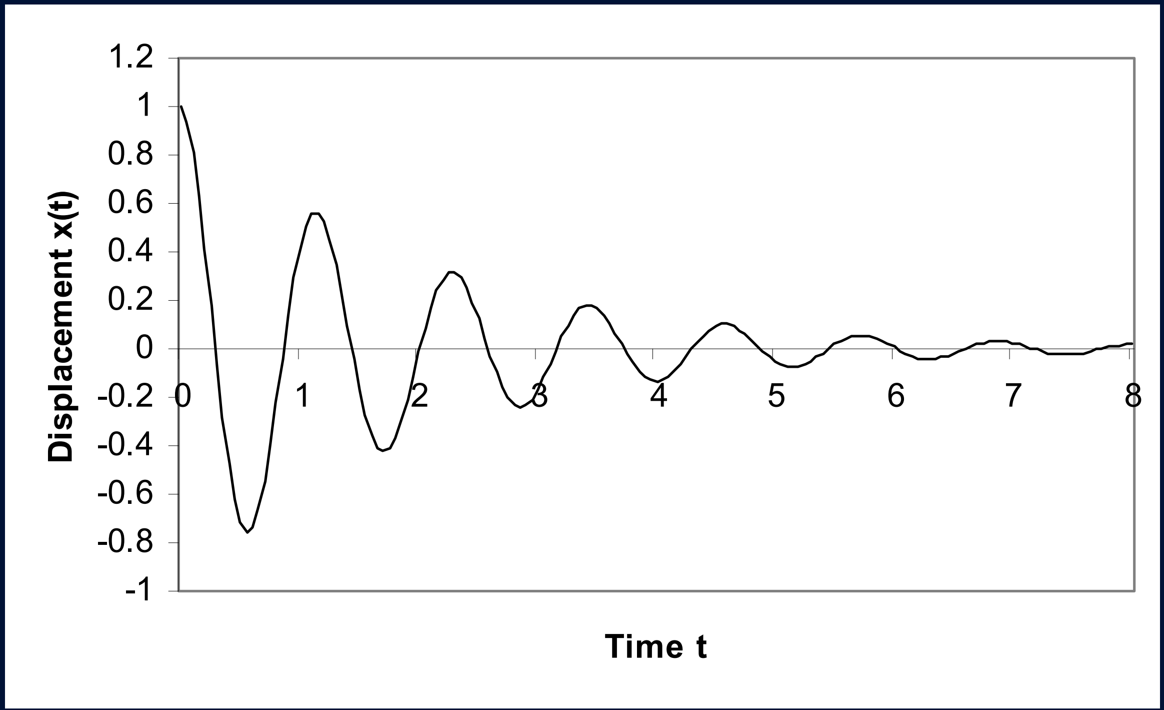
- 'Free vibration' in response to initial disturbance
  - In ideal case, oscillations last indefinitely





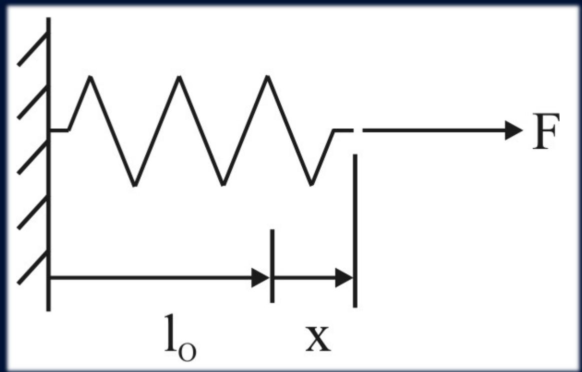
Basic Mechanical Oscillators

- In reality, energy lost by friction
  - oscillations decay due to damping!

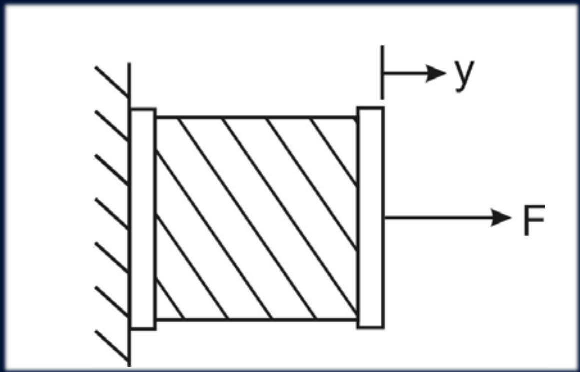


Elasticity and Spring Elements

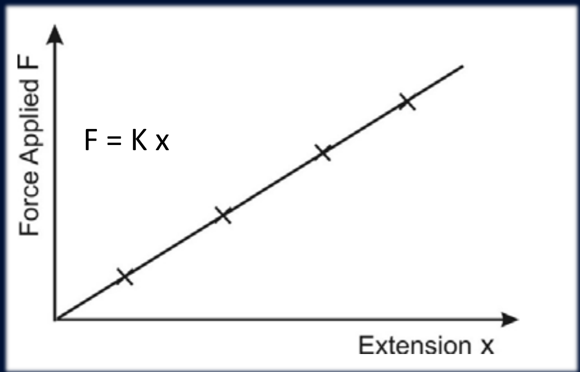
- Coil spring

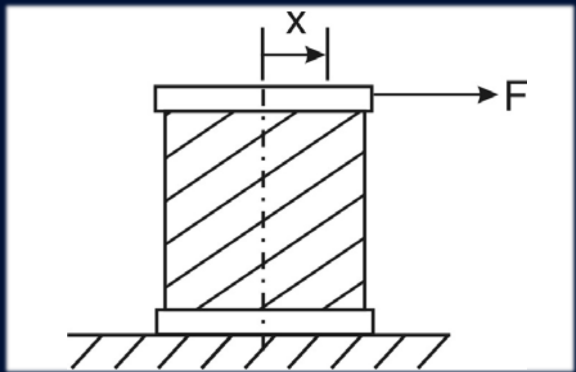


- Rubber bush



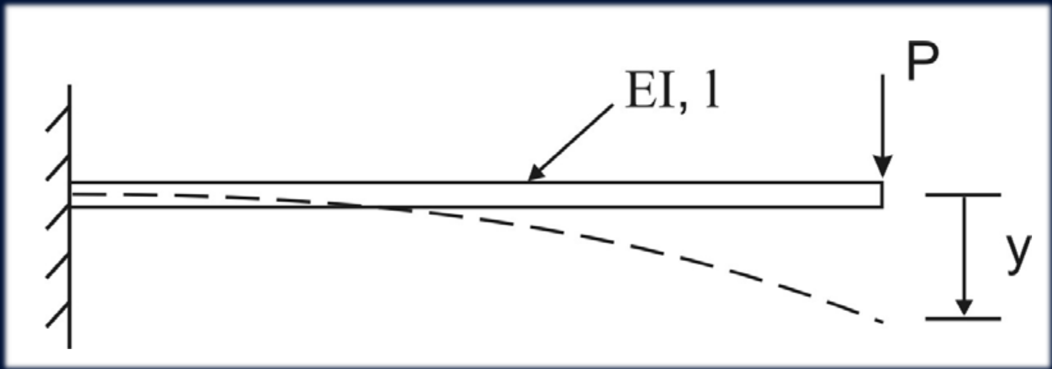
- Hookes Law





Elasticity and Spring Elements

- Beams – Cantilever



$$y = \frac{Pl^3}{3EI}$$

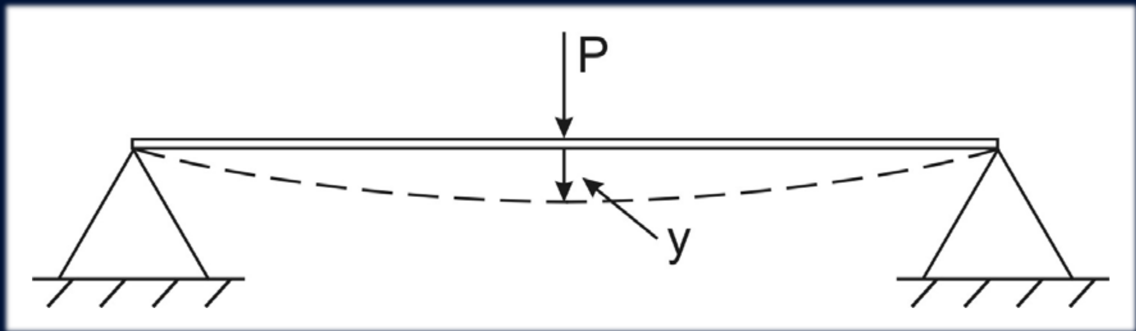
$$P = \left(\frac{3EI}{l^3}\right)y$$

$$\boxed{\text{Flexural Rigidity} = \frac{3EI}{l^3}}$$

E = Young's modulus (N/m^2)
I = Second moment of area (m^4)

Elasticity and Spring Elements

- Beams – Simply supported



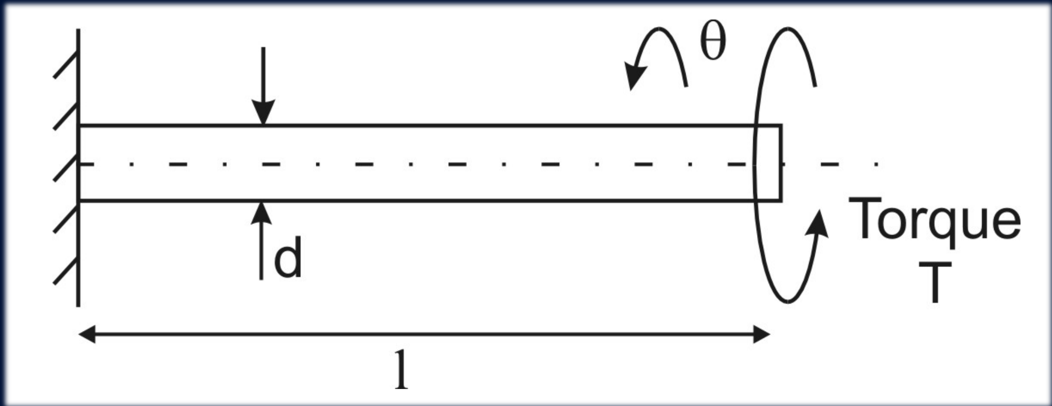
$$y = \frac{Pl^3}{48EI}$$

$$P = \left(\frac{48EI}{l^3}\right) y$$

$$\boxed{\text{Flexural Rigidity} = \frac{48EI}{l^3}}$$

Elasticity and Spring Elements

- Beams – Torsion Bar



$$T = \frac{GI_p}{l}\theta$$

$$T = \left(\frac{G\pi d^4}{32l}\right)\theta$$

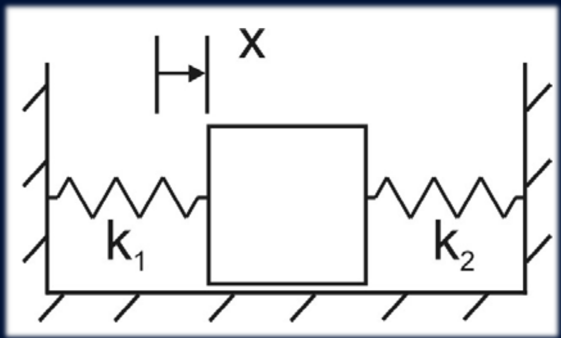
$$\boxed{\text{Torsional Stiffness} = \frac{G\pi d^4}{32l}}$$

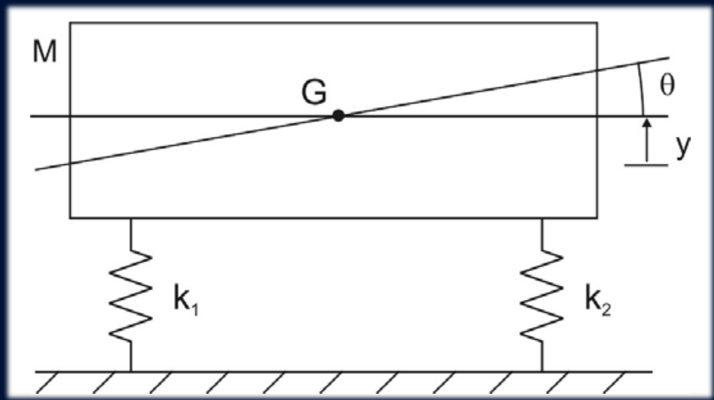
G = modulus of rigidity (N/m^2)

I_p = polar second moment of area (m^4)

Degrees of Freedom (DoF)

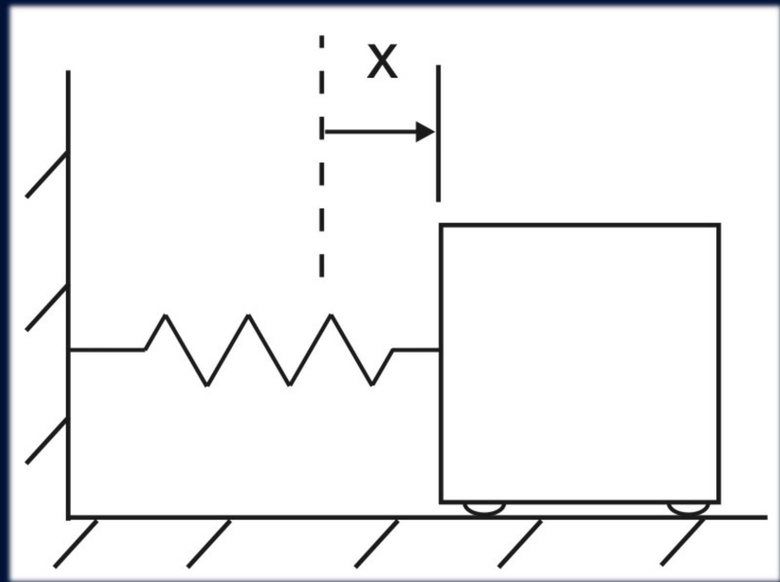
- DoF is the number of coordinates needed to fully specify the instantaneous position of body in motion
- and number of natural frequencies system possesses
  - corresponding 'mode' of oscillation
- Only single DoF systems used in this course
- Examples of 1 (left) and 2 DoF (right) systems





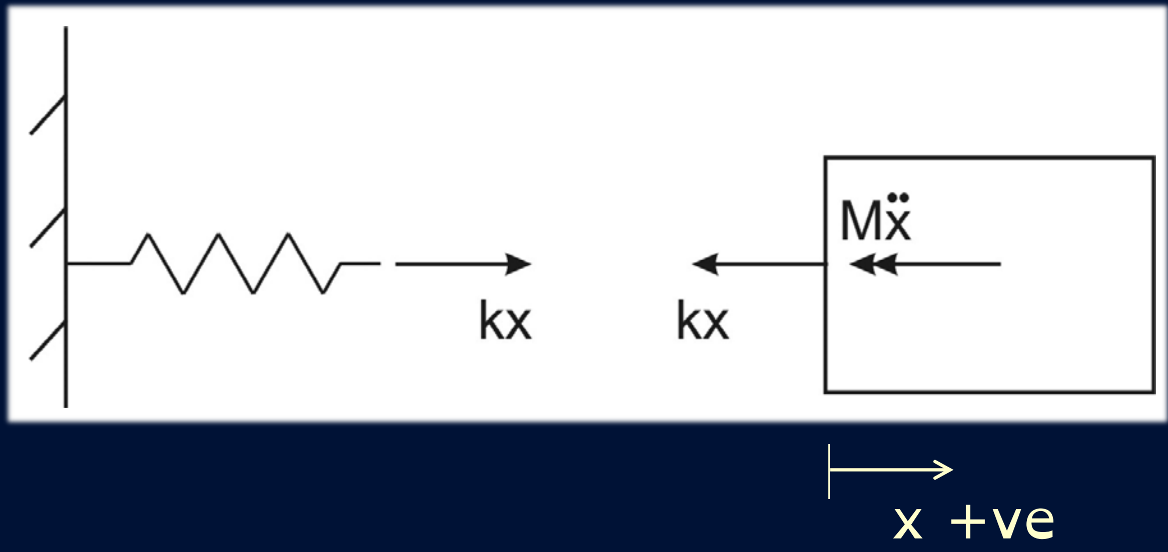
Differential Equation of Free Oscillations

- Simplest mechanical oscillator
- $x(t)$ is instantaneous position of the mass from its equilibrium point (i.e. with the spring un-stretched)



Differential Equation of Free Oscillations

- FBD



$$M\ddot{x} + Kx = 0$$

$$\ddot{x} + \frac{K}{M}x = 0$$

$$\ddot{x} + \omega_0^2 x = 0$$

$$\boxed{\omega_0 = \sqrt{\frac{K}{M}}}$$

Differential Equation of Free Oscillations

- Second order DE

$$\ddot{x} + \omega_0^2 x = 0$$

- General Solution

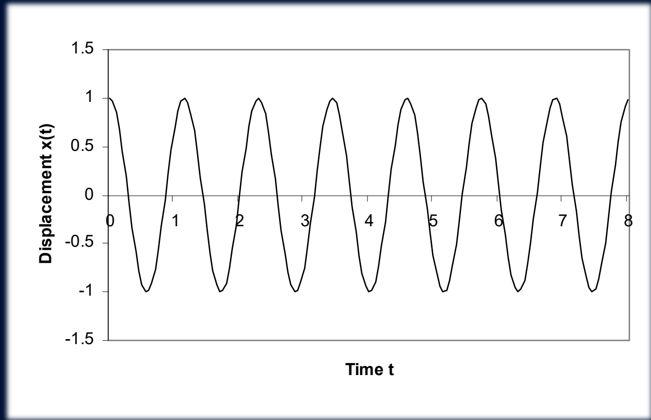
$$x(t) = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

- Simple Harmonic Motion
  - natural frequency (in Hz)

$$f_0 = \frac{\omega_0}{2\pi}$$

  - Period (in seconds)

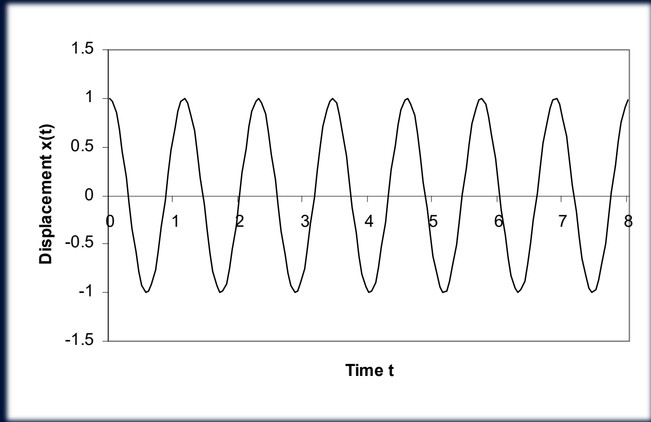
$$T_0 = \frac{2\pi}{\omega_0}$$



Differential Equation of Free Oscillations

- Second order DE

$$\ddot{x} + \omega_0^2 x = 0$$



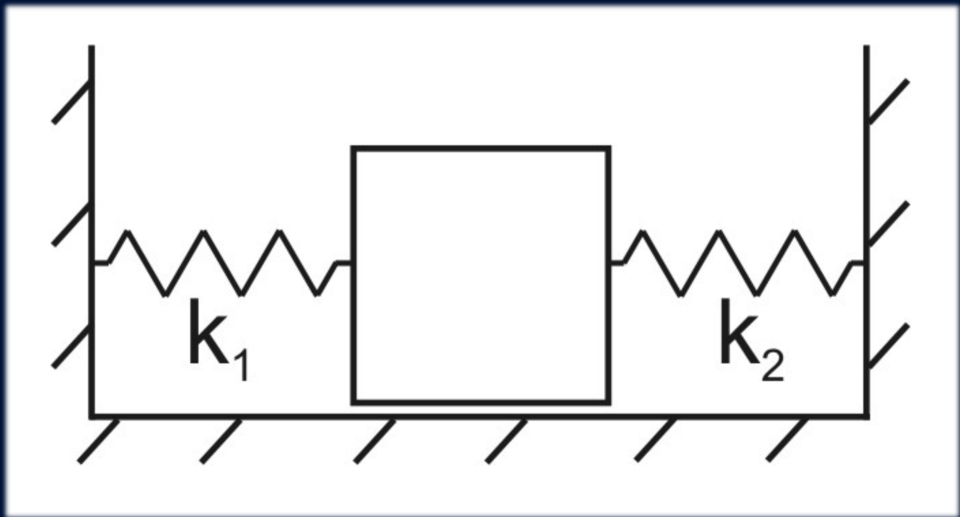
- General Solution

$$x(t) = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

- Magnitude of oscillation determined by the constants C_1 and C_2 which are fixed by the boundary conditions
  - eg, the initial displacement and/or velocity of the mass at $t = 0$.

Example 4.1

- Find
  - system DE
  - natural frequency of oscillation in Hz, if the system is disturbed
- Data
  - $M = 0.5$ kg
  - $k_1 = 80$ N/m
  - $k_2 = 140$ N/m



Dynamics 2

Introduction to Oscillations
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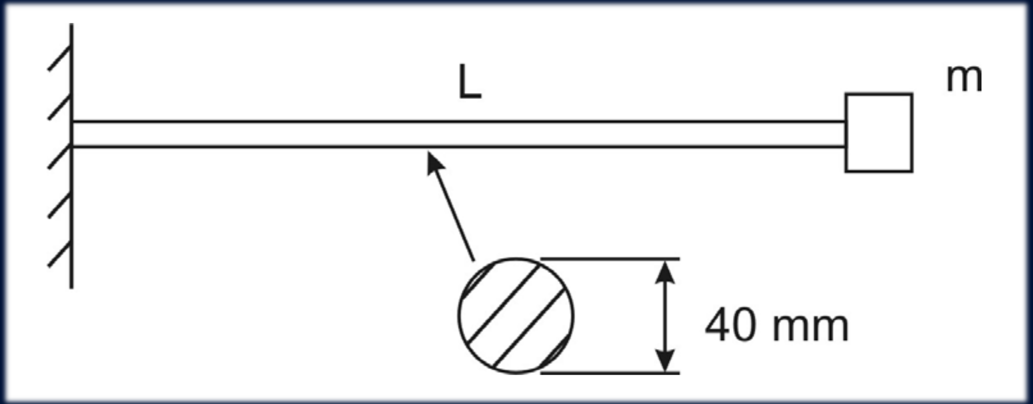
Worked Example: Cantilever

Example 4.2

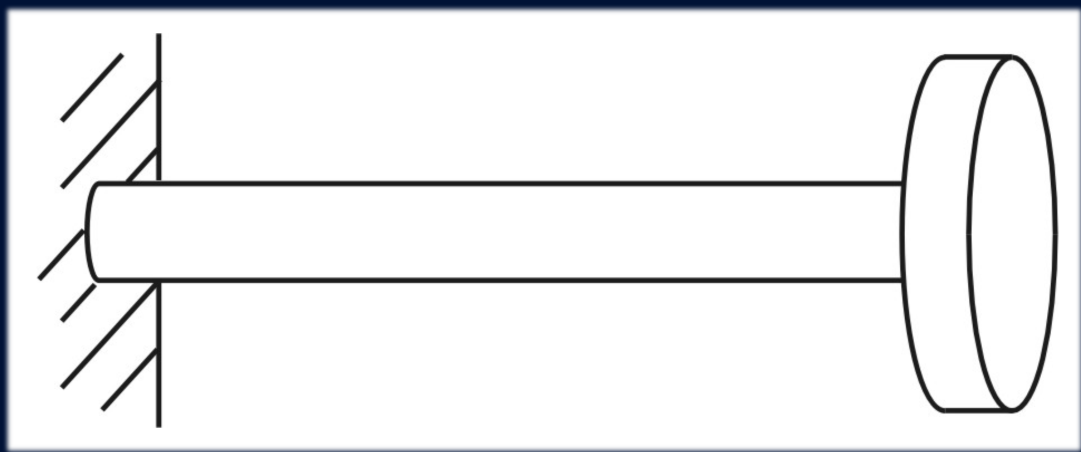
- Mass m (12 kg) is attached to the end of a cantilevered beam.
- Find the system natural frequency of free vibration if disturbed.
- Steel beam
  - $E = 20.7 \times 10^{10}$ N/m^2
  - L = 1.2 m
  - cross section shown.



- What approximations are used?
- What is the effect of gravity?

Example 4.6

- Determine the frequency of torsional oscillations of a uniform disc connected to a solid circular steel bar
- Disc details: R = 0.4 *m*; *m*ass = 20 kg
- Steel bar details: 1 *m*; ∅ = 0.1; G = 80 GPa



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Introduction to Oscillations

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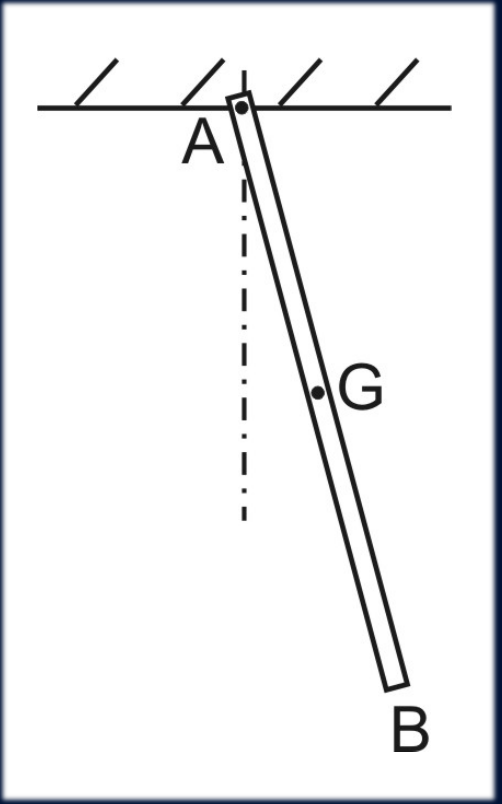
Worked Example: Hinged Pendulum

Example 4.5

- Find an expression for the frequency of small oscillations of the uniform pendulum bar AB
- What would be the effect if the pivot was attached to a lift accelerating upwards?

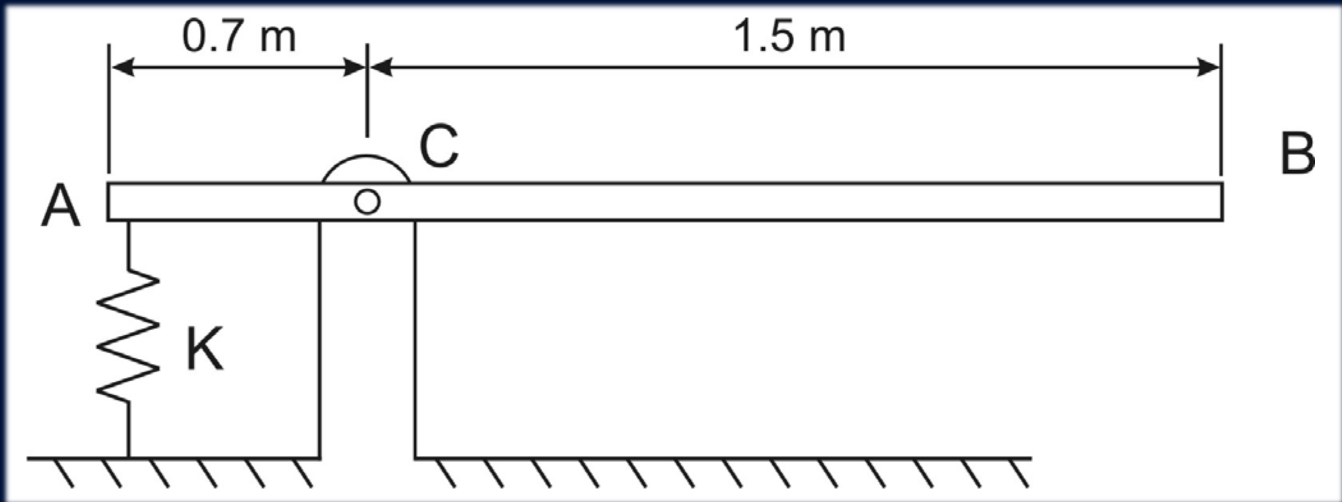


Summary

- Worked example of oscillatory motion in a hinged pendulum with small amplitude of oscillation.

Example 4.4

- Equipment shown is for a childrens' playground
- AB is a 65 kg pivoted steel beam
- Find the stiffness of spring K to give a system frequency of oscillation of 0.8 Hz when a 30 kg child is sitting at B



Example 4.3

- A is a 25 kg crate moving along roller track at 3.5 m/s
- It is stopped by B, a spring loaded buffer
  - B is rigid 32 kg mass restrained by four 90 N/m stiffness springs
- Find
  - maximum displacement of the buffer
  - maximum load transmitted to the wall