Lecture 7

Topic 2

First Law of Thermodynamics

Topics

- **2.5 Mass & Energy Analysis of Control Volumes (Open Systems)**

Reading:
Ch 4: 4.1 – 4.4 & 4.6 Borgnakke & Sonntag Ed. 8
Ch 5 Cengel & Boles Ed. 5

2.5 Mass/energy analysis of control volumes

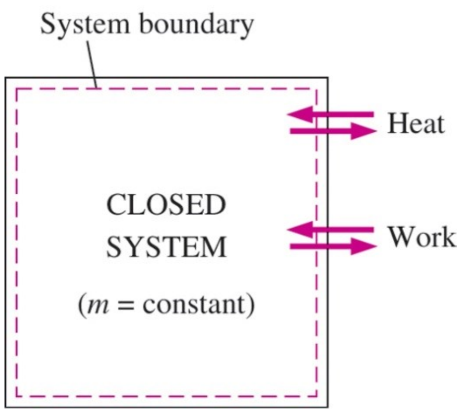
<u>Closed System</u>

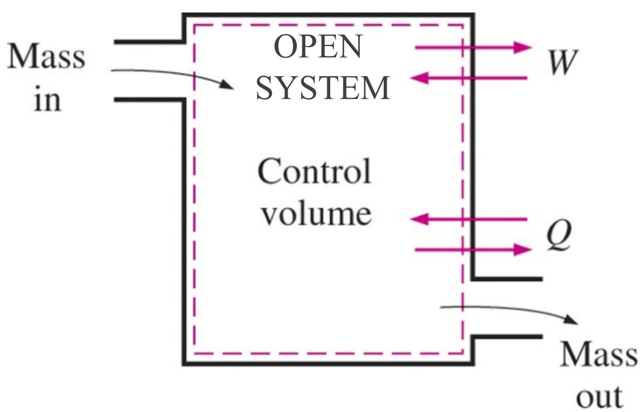
- Constant mass
- Heat & Work cross system boundaries
- Energy equation: $\Delta E_{system} = E_{in} - E_{out}$

$$\Delta U + \Delta KE + \Delta PE = (Q_{in} - Q_{out}) - (W_{out} - W_{in}) + \left(E_{mass,in} - E_{mass,out}\right)$$

<u>Open System</u>

- Mass <u>also</u> crosses system boundaries
- Equations of importance: <u>conservation of mass</u> & <u>conservation of energy</u>
  - Rate of change = +IN – OUT



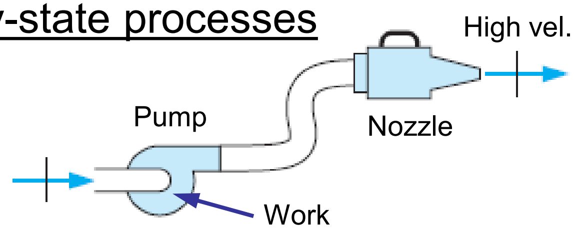


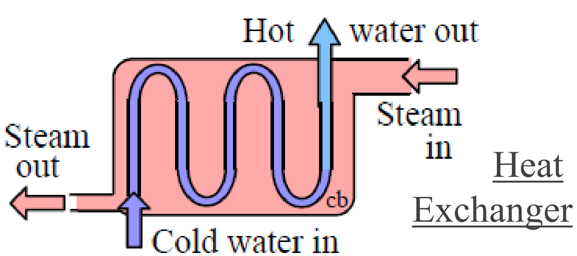
2.5 Mass/energy analysis of control volumes

Types of Open Systems

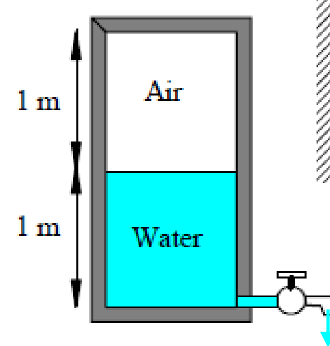
- **Steady-state process**
  - Fluid flow does not change with time (steady flow)
  - Mass IN = Mass OUT; Energy IN = Energy OUT
- **Transient (unsteady) process**
  - Conditions change with time
  - Mass & energy can be stored or depleted
  - Mass IN ≠ Mass OUT; Energy IN ≠ Energy OUT

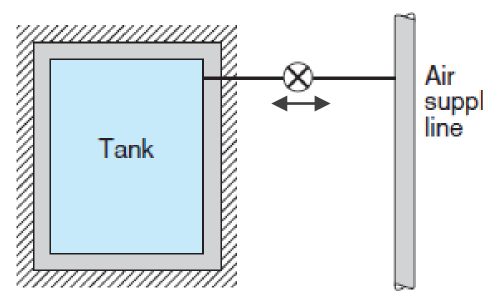
Steady-state processes





Transient processes





2.5.1 Conservation of mass

Conservation of mass:

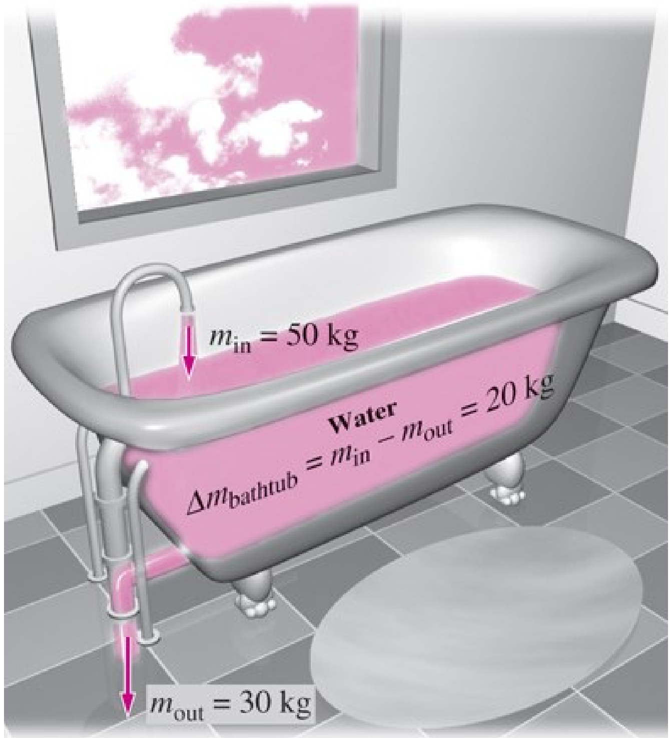
$$\begin{bmatrix}\text{Time rate change}\\ \text{of mass inside}\\ \text{control volume}\end{bmatrix} = \begin{bmatrix}\text{Sum of rate}\\ \text{of mass flowing}\\ \text{INTO control volume}\end{bmatrix} - \begin{bmatrix}\text{Sum of rate}\\ \text{of mass flowing}\\ \text{OUT of control volume}\end{bmatrix}$$

$$\frac{dm_{CV}}{dt} = \Delta\dot{m}_{system} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT} \quad (kg/s)$$

- Closed system: constant mass
  - $\sum \dot{m}_{IN} = \sum \dot{m}_{OUT} = 0$
- Open system: mass crosses boundaries
  - $\Delta\dot{m}_{system} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT}$
  - Steady-state: $\sum \dot{m}_{IN} = \sum \dot{m}_{OUT}$

Water filling in Bathtub

- $\dot{m}_{in} > \dot{m}_{out}; \Delta\dot{m}_{system} > 0$
- $\dot{m}_{out} > \dot{m}_{in}; \Delta\dot{m}_{system} < 0$
- $\dot{m}_{in} = \dot{m}_{out}; \Delta\dot{m}_{system} = 0$

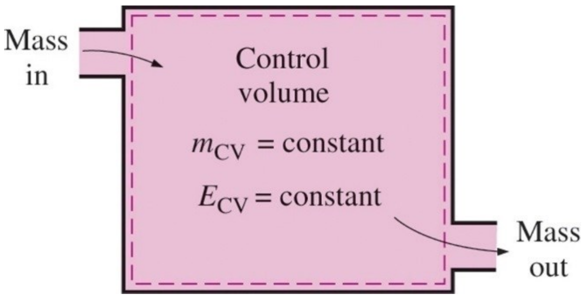


2.5.1 Conservation of mass

Steady-state process

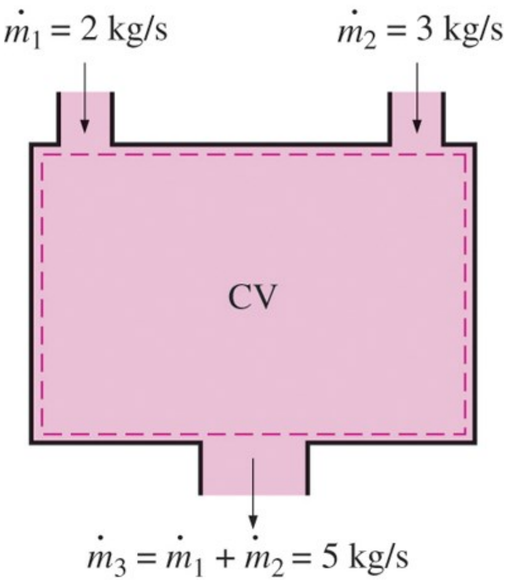
- Constant flow rate through control volume
- Mass remain constant inside the control volume

$$\frac{dm_{CV}}{dt} = \Delta \dot{m}_{CV} = 0$$



- Conservation of mass (steady-state)
  - $\frac{dm_{CV}}{dt} = \Delta \dot{m}_{system} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT} = 0$
  - $\sum \dot{m}_{IN} = \sum \dot{m}_{OUT}$

$$\begin{bmatrix} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{INTO control volume} \end{bmatrix} = \begin{bmatrix} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{OUT of control volume} \end{bmatrix}$$

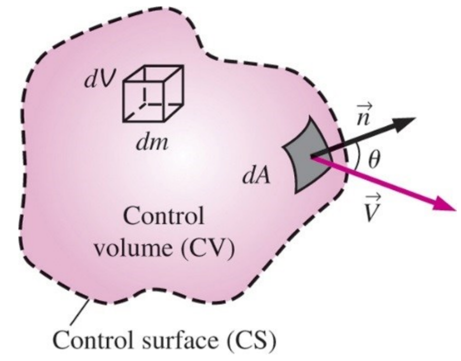


2.5.1 Conservation of mass

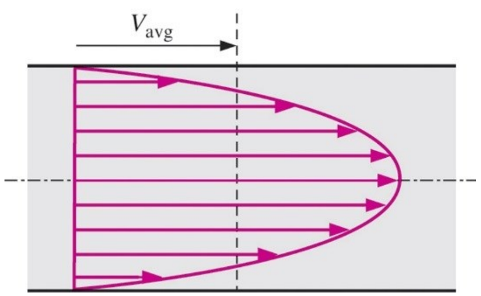
Mass Flow Rate

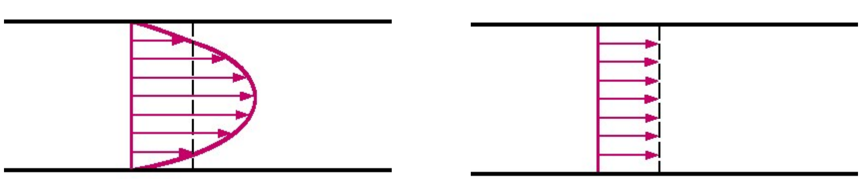
- Mass flow through a cross-sectional area per unit time: $\dot{m}\ \ (kg/s)$

- $\dot{m} = \int_A \rho \vec{V}_n dA = \int_A \frac{1}{v} \vec{V}_n dA$
  - $\rho = 1/v$
  - $\vec{V}_n$: fluid velocity normal to the control surface
  - dA: differential element of flow area



- For constant variables:
  - $\dot{m} = \rho \vec{V}_{ave} A = \frac{\vec{V}_{ave} A}{v}$
    - $\vec{V}_{ave}$: average velocity normal to the control surface



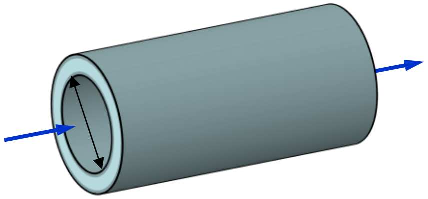


(a) Actual (b) Average

2.5.2 Mass flow rates

Example 2-6

Refrigerant-134a at 200 kPa, 40% quality, flows through a 1.1 cm inside diameter tube with a velocity of 50 m/s. Find the mass flow rate of the refrigerant-134a.



Solution:

- $\dot{m} = \rho \vec{V}_{ave} A$
- $\dot{m} = \frac{\vec{V}_{ave} A}{v}$
- Find v from tables: $v = v_f + x v_{fg} = 0.0007533 + 0.4(0.0984) = 0.0401\ \mathrm{m^3/kg}$
- $\dot{m} = \frac{\vec{V}_{ave} A}{v} = \frac{\vec{V}_{ave}}{v} \cdot \frac{\pi d^2}{4} = \frac{50\ m/s}{0.0401\ m^3/kg} \cdot \frac{\pi (0.011 m)^2}{4}$
- $\dot{m} = 0.1184\ \mathrm{kg/s}$

More exercises at the end of this lecture!

2.5.2 Mass flow rates

Volume flow rate $(\dot{V})$

- Volume of fluid flowing through a cross-section per unit time

$$\dot{V} = \int_A \vec{V}_{ave} dA = \vec{V}_{ave} A \quad (m^3/s)$$

- Relation between mass flow rate $(\dot{m})$ and volume flow rate $(\dot{V})$

$$\dot{m} = \rho \dot{V} = \dot{V}/v \qquad (kg/s)$$

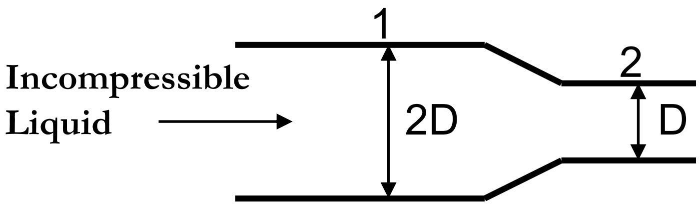
Steady flow of an ***incompressible*** fluid (e.g. *most* liquids)

- Single entrance / exit steady flow
  - $\dot{m}_{in} = \dot{m}_{out} \quad (kg/s)$
  - $\rho_{in}\dot{V}_{in} = \rho_{out}\dot{V}_{out}$
  - $\rho_{in} = \rho_{out} \rightarrow \text{incompressible}$
  - $\dot{V}_{in} = \dot{V}_{out}$
  - $\vec{V}_{in} A_{in} = \vec{V}_{out} A_{out}$

2.5.2 Mass flow rates

Example 2-7: Geometry Effects on Fluid Flow

An incompressible liquid flows through the circular pipe. How is the velocity at location 2 related to the velocity at location 1?



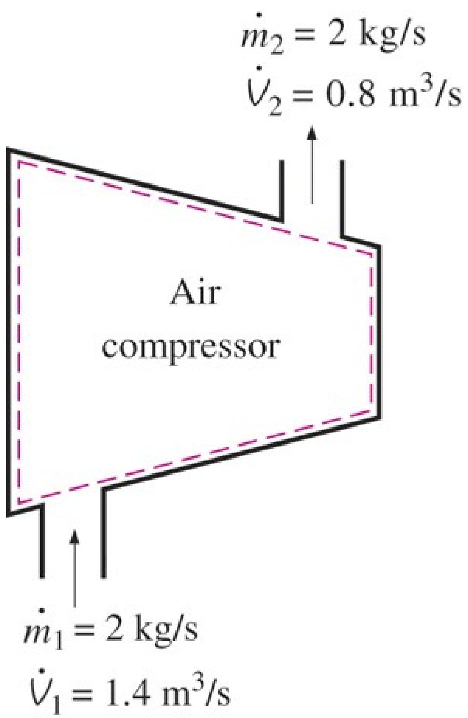
2.5.2 Steady-State, Steady-Flow Processes

Steady flow of a *compressible* fluid (e.g. most gases)

- Mass is always conserved
  - $\rho_1 \vec{V}_1 A_1 = \rho_2 \vec{V}_2 A_2$
- Volume flow rates may not be conserved!
  - Density can increase/decrease

Example 2.8

Air flows into a compressor with $\dot{m}$ = 2 kg/s at 120 kPa, 20^{o}C and exits at 217.5 kPa, 30^{o}C. Assume ideal gas properties. Find the volume flow rates into and out of the compressor.



Solution:

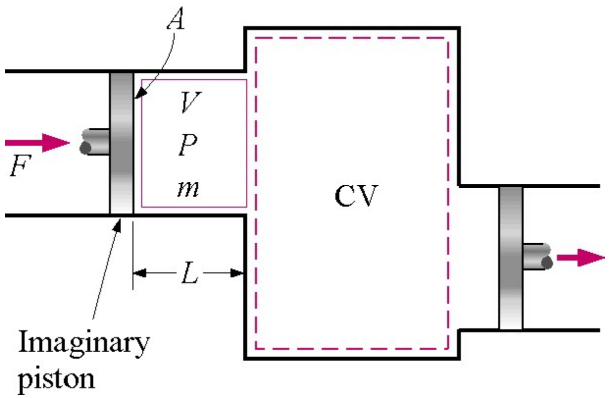
- $\rho_1 = P_1/RT_1 = 120kPa/\left(0.287\frac{kJ}{kgK}293.15K\right) = 1.428\, kg/m^3$
- $\rho_2 = P_2/RT_2 = 217.5kPa/\left(0.287\frac{kJ}{kgK}303.15K\right) = 2.5\, kg/m^3$
- $\dot{V}_1 = \dot{m}_1/\rho_1 = 1.4\, m^3/s$
- $\dot{V}_2 = \dot{m}_2/\rho_2 = 0.8\, m^3/s$

2.5.3 Flow work and energy of a flowing fluid

Introduction to Flow Work

- Energy flows into and out of the control volume with mass.
- Flow Work: the energy required to push the mass into or out of the control volume.

- Theoretical example:
  - Imaginary piston pushes (pulls) a unit of mass into (out of) the control volume.
  - As imaginary piston pushes (pulls) fluid of unit mass into (out of) the control volume, work is done on the fluid.
    - $W_{flow} = \int F ds = \int P dV = PV$
    - $w_{flow} = Pv$



- Flow work (Pv): work done on the unit of mass crossing the control surface

2.5.4 Flow work and energy of a flowing fluid

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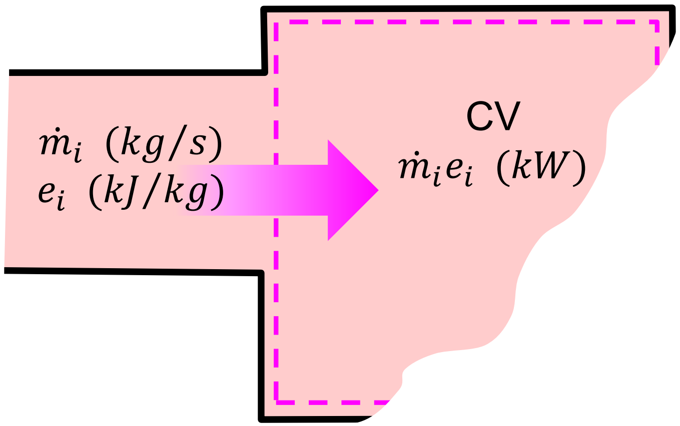
Total energy of flowing fluid

- (Mass) energy crossing a control surface:

$$E_{mass} = internal\ energy + flow\ work + kinetic\ energy + potential\ energy$$

  - $e = u + Pv + \frac{\vec{V}^2}{2} + gz$
  - Recall: $h = u + Pv$
  - $e = h + \frac{\vec{V}^2}{2} + gz$



- Energy transport by mass across a control surface
  - $E_{mass} = me = m\left(h + \frac{\vec{V}^2}{2} + gz\right) \quad (kJ)$

- Rate of energy transport
  - $\dot{E}_{mass} = \dot{m}e = \dot{m}\left(h + \frac{\vec{V}^2}{2} + gz\right) \ (kW)$

2.5.5. Conservation of Energy

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Control Volume Open System

- First law of Thermodynamics (rate form)

$$\begin{bmatrix} \text{Rate of change} \\ \text{of energy inside} \\ \text{control volume} \end{bmatrix} = \begin{bmatrix} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{INTO control volume} \end{bmatrix} - \begin{bmatrix} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{OUT of control volume} \end{bmatrix}$$

$$\underbrace{\Delta \dot{E}_{system}}_{\substack{Rate\ change\ in \\ U,KE,PE}} = \underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{Rate\ of\ net\ energy\ transfer \\ by\ heat,work,mass}} \quad (kW)$$

$$\boxed{\Delta \dot{E}_{system} = \dot{Q}_{net} - \dot{W}_{net} + \sum \underbrace{\dot{m}_i \left(h_i + \frac{\vec{V}_i^{\,2}}{2} + g z_i \right)}_{for\ each\ inlet} - \sum \underbrace{\dot{m}_e \left(h_e + \frac{\vec{V}_e^{\,2}}{2} + g z_e \right)}_{for\ each\ exit} \quad (kW)}$$

- For the above notation:
  - $\dot{Q}_{net} = \dot{Q}_{IN} - \dot{Q}_{OUT}$
  - $W_{net} = \dot{W}_{OUT} - \dot{W}_{IN}$

2.5.5. Conservation of Energy

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Control Volume (CV) Open System

- **Steady-state systems** – constant mass & energy inside a control volume

$$\Delta \dot{E}_{system} = 0 \;(steady\ state)$$

$$0 = \dot{Q}_{net} - \dot{W}_{net} + \sum \underbrace{\dot{m}_i \left(h_i + \frac{\vec{V}_i^{\,2}}{2} + g z_i \right)}_{for\ each\ inlet} - \sum \underbrace{\dot{m}_e \left(h_e + \frac{\vec{V}_e^{\,2}}{2} + g z_e \right)}_{for\ each\ exit} \qquad (kW)$$

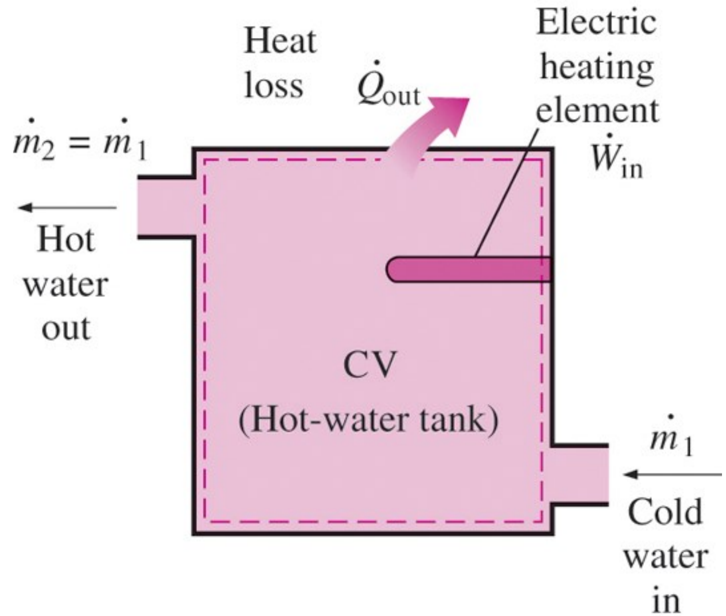
$$\underbrace{\dot{Q}_{net} + \sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^{\,2}}{2} + g z_i \right)}_{\substack{Rate\ of\ energy\ flowing \\ INTO\ the\ CV}} = \underbrace{\dot{W}_{net} + \sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^{\,2}}{2} + g z_e \right)}_{\substack{Rate\ of\ energy\ flowing \\ OUT\ of\ the\ CV}} \qquad (kW)$$

- For the above notation:
  - $\dot{Q}_{net} = \dot{Q}_{IN} - \dot{Q}_{OUT}$
  - $W_{net} = \dot{W}_{OUT} - \dot{W}_{IN}$

2.5.5. Conservation of Energy

SINGLE entrance / exit (steady-state)

- Conservation of mass
  - $\frac{dm_{CV}}{dt} = \Delta\dot{m}_{system} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT}$
  - $\Delta\dot{m}_{system} = 0\ (steady\ state)$
  - $\dot{m}_1 = \dot{m}_2 = \dot{m}$
  - $\frac{1}{v_1}\vec{V}_1 A_1 = \frac{1}{v_2}\vec{V}_2 A_2$



- Conservation of energy ($\dot{E}_{system} = 0, steady\ state$)
  - $\dot{E}_{system} = \dot{Q}_{net} - \dot{W}_{net} + \dot{m}_i\left(h_i + \frac{\vec{V}_i^{\,2}}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{\vec{V}_e^{\,2}}{2} + gz_e\right)$
  - $0 = \dot{Q}_{net} - \dot{W}_{net} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\vec{V}_1^{\,2} - \vec{V}_2^{\,2}}{2}\right) + g(z_1 - z_2)\right]$

2.5.5. Conservation of Energy

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Kinetic and potential energy

- $e = h + \frac{\vec{V}^2}{2} + gz$
- Are KE & PE significant compared to enthalpy?
- $ke = \frac{\vec{V}^2}{2}$ → $\vec{V} = 45\,\frac{m}{s} \rightarrow ke = 1\,kJ/kg$

  $\vec{V} = 140\,\frac{m}{s} \rightarrow ke = 10\,kJ/kg$
- $pe = \mathrm{gz}$ → $z = 100m \rightarrow pe = 0.98\,kJ/kg$

  $z = 1000m \rightarrow pe = 9.8\,kJ/kg$
- Steam ($h \cong$ 2000 to 3000 kJ/kg); Air ($h \cong$ 200 to 6000 kJ/kg).
- KE & PE often neglected if below values above.
- Energy equation → $0 = \dot{Q}_{net} - \dot{W}_{net} + \dot{m}(h_1 - h_2) \quad (kW)$

2.5.6. Transient Processes

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Steady-state vs. Transient

- **Steady-state processes** – constant mass and energy inside CV

$$\Delta\dot{m}_{system} = 0 \rightarrow \sum \dot{m}_{IN} = \sum \dot{m}_{OUT}$$

$$\Delta\dot{E}_{system} = 0 \ \rightarrow \dot{Q}_{net} - \dot{W}_{net} + \dot{m}(h_{in} - h_{out})$$

- **Transient processes** – mass and energy can increase/decrease inside CV

$$\Delta m_{system} = \sum m_{IN} - \sum m_{OUT}$$

$$\Delta U + \Delta KE + \Delta PE \ = Q_{net} - W_{net} + \sum \underbrace{m_i\left(h_i + \frac{\vec{V_i}^2}{2} + gz_i\right)}_{for\ each\ inlet} - \sum \underbrace{m_e\left(h_e + \frac{\vec{V_e}^2}{2} + gz_e\right)}_{for\ each\ exit}$$

Note the drop of the rate term (˙) above the terms

2.5 Example

Example 2-9

A hot-water stream at 80°C enters a mixing chamber with $\dot{m}_1 = 0.5\,kg/s$ and is mixed with a stream of cold water at 20°C. If it is desired that the mixture leaves the chamber at 42°C, determine the mass flow rate of the cold-water stream. Assume that all streams are at a constant pressure of 250 kPa and heat transfer to the surroundings can be neglected.

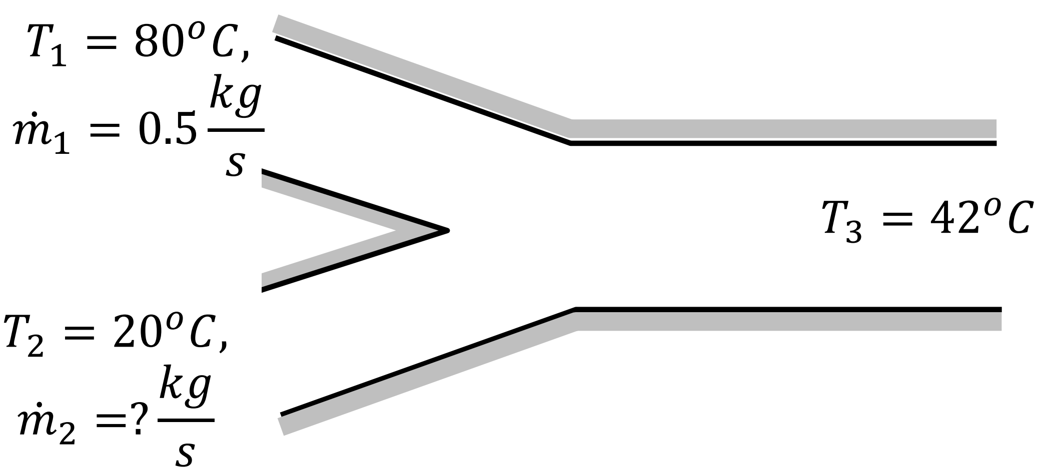


TABLE B.1.4
Compressed Liquid Water

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)
	500 kPa (151.86°C)		
Sat.	0.001093	639.66	640.21
0.01	0.000999	0.01	0.51
20	0.001002	83.91	84.41
40	0.001008	167.47	167.98
60	0.001017	251.00	251.51
80	0.001029	334.73	335.24

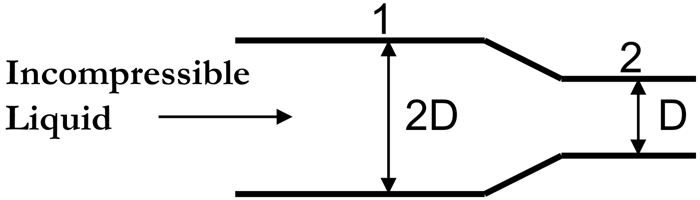
Compressed liquid at 250 kPa is not listed!
How to proceed?

[ans: 0.864 kg/s]

2.5.2 Mass Flow Example Revisited

Example 2-7: Geometry Effects on Fluid Flow

An incompressible liquid flows through the circular pipe. How is the velocity at location 2 related to the velocity at location 1?



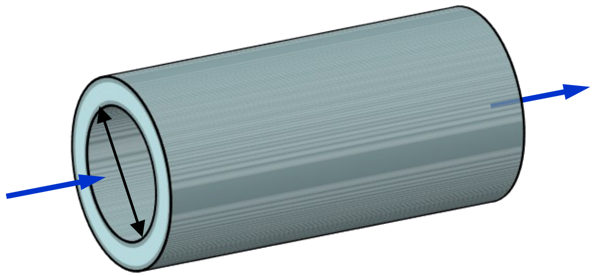
Solution:

- $\sum \dot{m}_{in} = \sum \dot{m}_{out}$
- $\rho_1 \vec{V}_1 A_1 = \rho_2 \vec{V}_2 A_2 \rightarrow \vec{V}_1 A_1 = \vec{V}_2 A_2$
- $\vec{V}_2 = \frac{A_1}{A_2} \vec{V}_1 = \frac{\pi D_1^2/4}{\pi D_2^2/4} * \vec{V}_1$
- $\vec{V}_2 = \left(\frac{D_1}{D_2}\right)^2 \vec{V}_1 = \left(\frac{2D}{D}\right)^2 \vec{V}_1 = 4\vec{V}_1$

2.5.7 Exercises

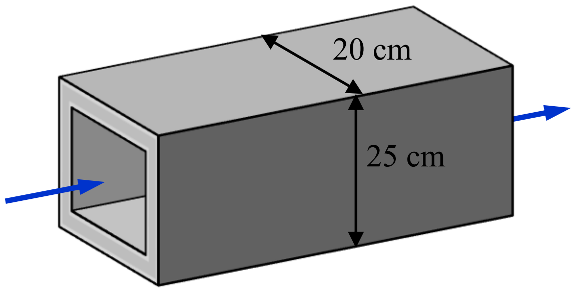
Exercise 2-2: water ($\rho = 1000 \frac{kg}{m^3}$) travels in a pipe with circular cross-sectional area and inner diameter $d = 0.1m$ and average velocity of $2 \frac{m}{s}$. Determine the mass flow rate.

[ans: 15.71 kg/s]



Exercise 2-3: air ($\rho = 1.225 \frac{kg}{m^3}$) travels in a rectangular duct with height 25 cm and width of 20 cm. The average velocity of $5 \frac{m}{s}$. Determine the mass flow rate.

[ans: 0.306 kg/s]



2.5.7 Exercises

Exercise 2-3

Air at 100 kPa, 50°C, flows through a pipe with a volume flow rate of 40 m^3/min. Assuming ideal gas and constant velocity,

a) Determine the mass flow rate through the pipe in kg/s.

b) Determine and the average velocity ($\vec{V}_{avg}$) in m/s if the pipe has a cross sectional area with diameter 30 cm.

[ans: $\dot{m}$ = 0.719 kg/s, $\vec{V}_{avg}$ = 9.43 m/s]