Module 2 self assessment

Question 1

Find the potential of the conservative field

$$\mathbf{C}(x, y) = (2y - \mathrm{e}^{y})\hat{\mathbf{i}} + (2x - x\mathrm{e}^{y})\hat{\mathbf{j}}$$

Question 2

A point mass m positioned at coordinates (x, y, z) is attracted towards the origin $(0, 0, 0)$ with a force whose magnitude is gmr^{-2} where g is an acceleration-related constant and r is the distance between the point mass and the origin.

(i) Assuming that both m and g are known, derive an expression for the force field $\mathbf{f}$ at any point in the 3D frame apart from the origin, justifying your reasoning.

(ii) Show that, aside the origin, the force field is conservative and derive its potential function ϕ, such that $\mathbf{f} = \nabla\phi$. Hint, you may find the vector calculus identity

$$\nabla \times (\phi\mathbf{a}) = \phi\nabla \times \mathbf{a} + \nabla\phi \times \mathbf{a}$$

useful. Here ϕ is a scalar field, and $\mathbf{a}$ a vector field.

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