Dynamics 2

Fixed Axis Rotation & Moment of Inertia Theorems
(Dynamics of Systems of Bodies)

Transformation Theorems for MMI

- Parallel Axis Theorem
  - applies to all rigid bodies

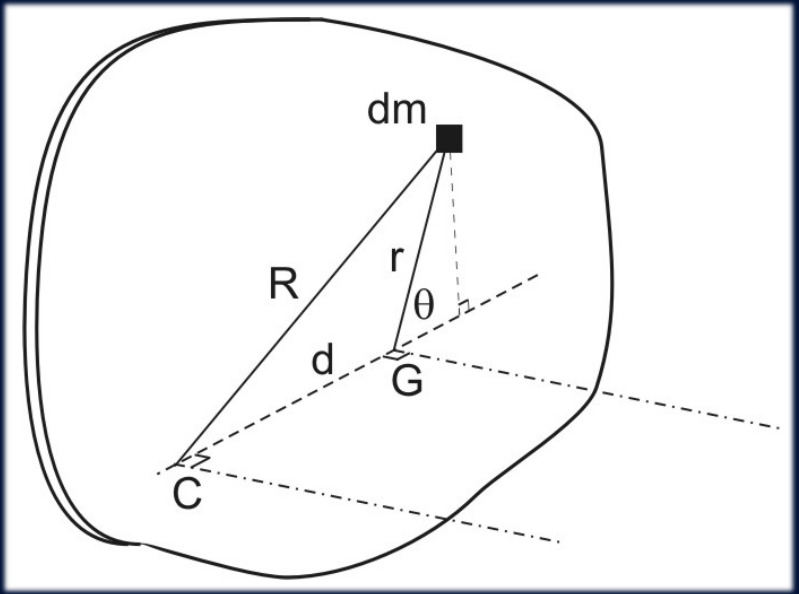
- Perpendicular Axis Theorem
  - applies only to thin bodies (plates/laminae)

Parallel Axis Theorem

- I_G is the moment of inertia of a body about an axis through G
- I_C is the moment of inertia about a *parallel axis* through some other point C of the body

$$I_C = I_G + Md^2$$

where *M* is the body mass and *d* is perpendicular distance between the parallel axes



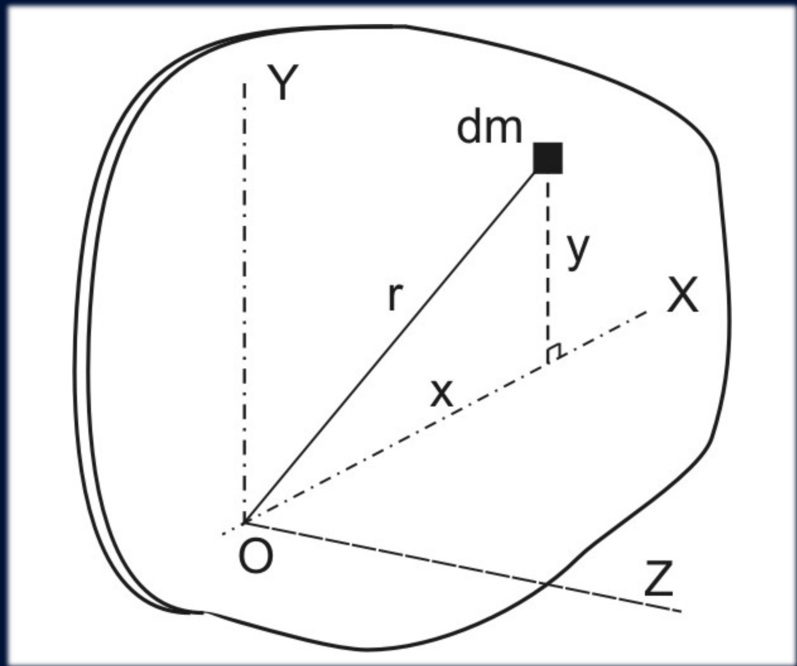
- note that $I_C \geq I_G$ ie smallest moment of inertia is about G

Perpendicular Axis Theorem

- if O is any point
  - OX and OY are a pair of rectangular axes in the plane of the body
  - OZ is an axis perpendicular to the body

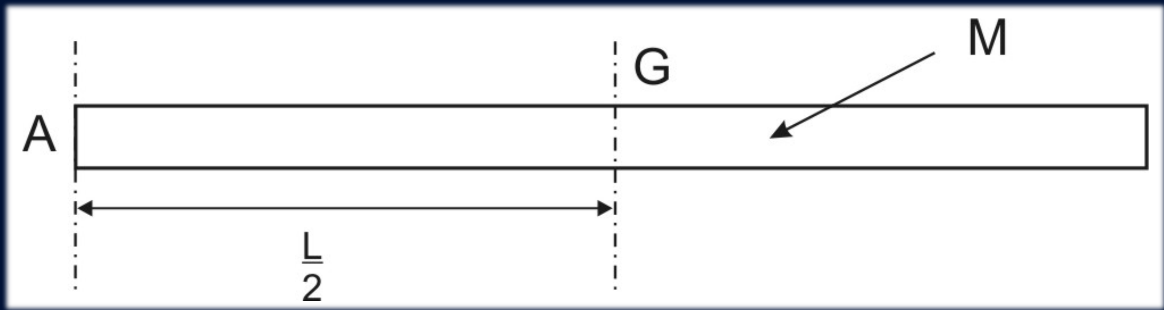
- then for O

$$I_Z = I_X + I_Y$$



Using Theorems

- what is the MMI of this bar about the axis through G



$$I_A = \frac{ML^2}{3}$$ [about end A - derived previously]

$$I_A = I_G + M\left(\frac{L}{2}\right)^2$$

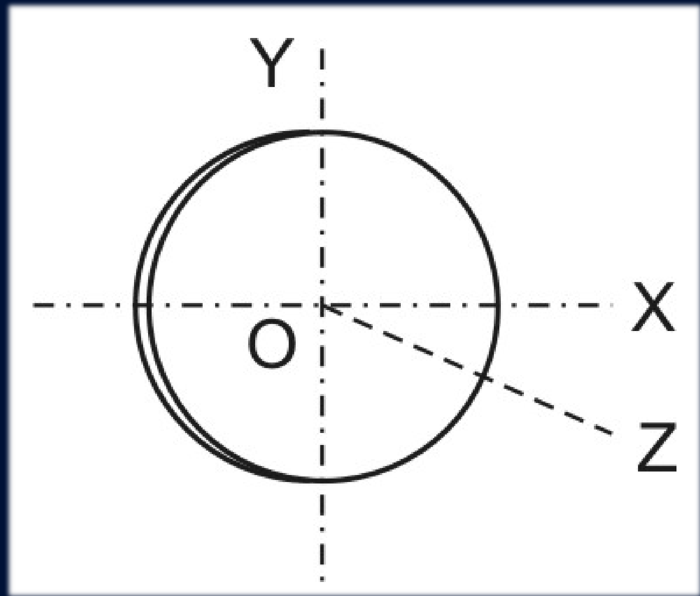
$$I_G = I_A - \frac{ML^2}{4} = \frac{ML^2}{12}$$

Using Theorems

- MMI of disc about a diameter

$$I_Z = \frac{1}{2}MR^2$$ (about centre of disc – as proved)

$$I_Z = I_X + I_Y$$ (⊥ axes theorem)



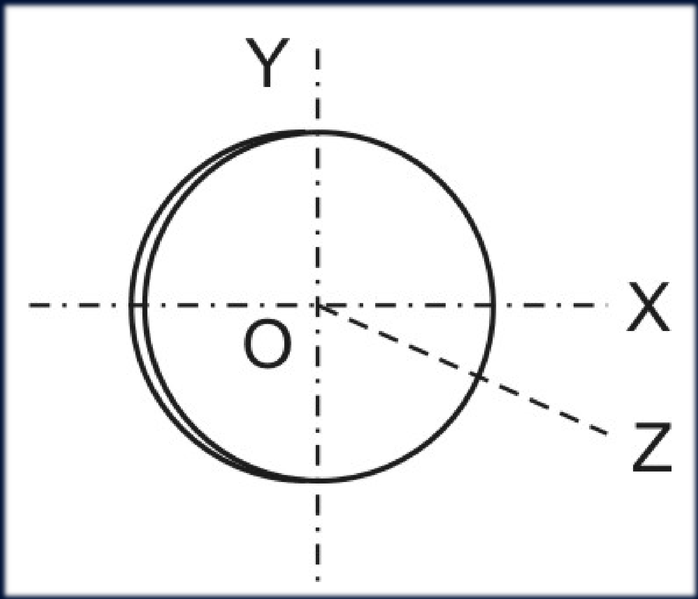
Using Theorems

- MMI of disc about a diameter

$$I_Z = \frac{1}{2} MR^2$$ (about centre of disc – as proved)

$$I_Z = I_X + I_Y$$ (⊥ axes theorem)

- then $I_X = I_Y$



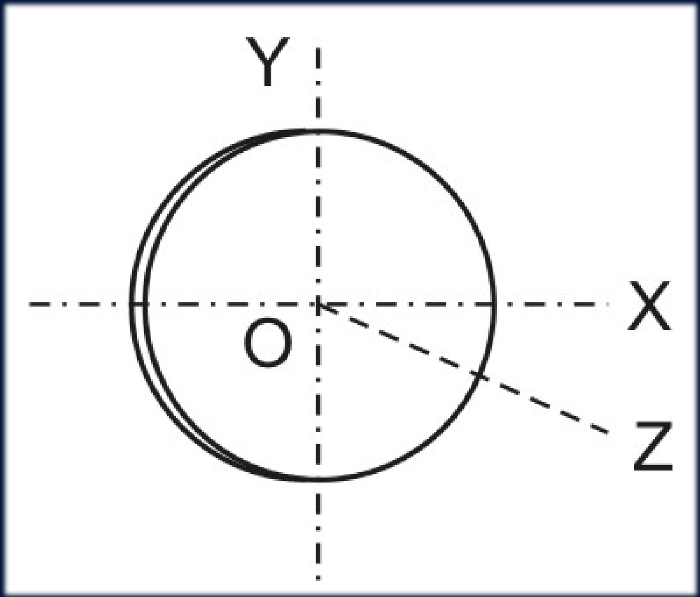
Using Theorems

- MMI of disc about a diameter

$$I_Z = \frac{1}{2}MR^2$$ (about centre of disc – as proved)

$$I_Z = I_X + I_Y$$ (⊥ axes theorem)

- then $I_X = I_Y$
- and hence $I_X = \frac{I_Z}{2} = \frac{1}{4}MR^2$



Radius of Gyration

- a frequently used concept
  - related to Moment of Inertia
- any Moment of Inertia can be written
  - (Mass of the Body) × K^2
  - $I_O = MK^2$
- K is an equivalent length quantity
  - Radius of Gyration

Radius of Gyration - Disc

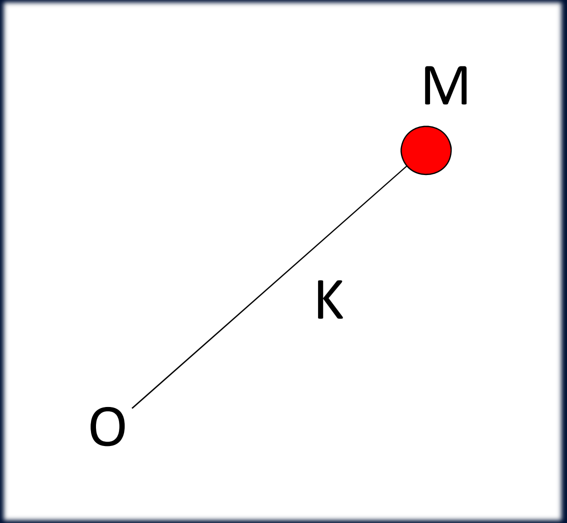
- uniform disc of radius R about centre:

$$I_O = \frac{1}{2} M R^2$$

- The equivalent Radius of Gyration is then:

$$K = \frac{R}{\sqrt{2}}$$

- can be thought of as a concentrated mass M located distance K from axis
- K may often be given in data



Summary

- Parallel Axis Theorem
- Perpendicular Axis Theorem
- Radius of Gyration

Dynamics 2

Fixed Axis Rotation & Moment of Inertia Theorems: Worked Examples

(Dynamics of Systems of Bodies)

Example 2.8

- find I_B and I_G for axes perpendicular to the plane for the fabricated steel bar shown

  - Total mass = 45 kg

  - Total length = 0.65 m ; Mass/Length = 69.23 kg/m