Lecture 12

Topic 3

Second Law of Thermodynamics

Topics

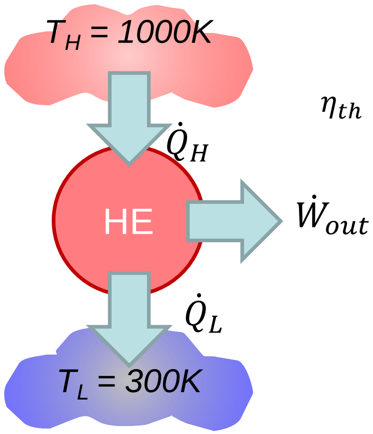
- 3.3 Definition of entropy

Reading:
Ch 6: 6.1 – 6.4 Borgnakke & Sonntag Ed. 8
Ch 7: 7-1 – 7-6 Cengel and Boles Ed. 7

3.3 Revisiting Second Law thus far

Lecture 11: Reversible Processes & Carnot Cycle

- Define maximum possible thermal efficiency
- Heat engine: $\eta_{TH} = 1 - \frac{Q_L}{Q_H} \underbrace{=}_{Carnot} 1 - \frac{T_L}{T_H}$
- Refrigerator: $\beta = \frac{Q_L}{Q_H - Q_L} \underbrace{=}_{Carnot} \frac{T_L}{T_H - T_L}$



$$\eta_{th} = 1 - \frac{300\ K}{1000\ K} = 0.7$$

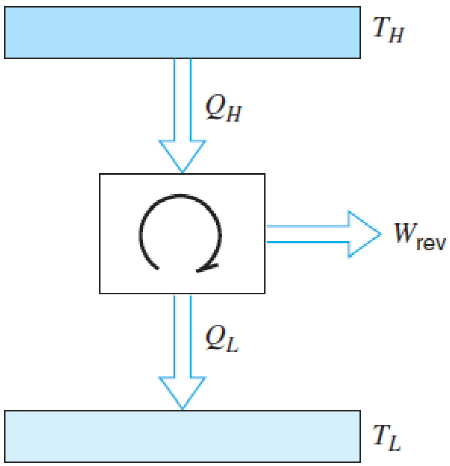
Lectures 12 – 14

- 2nd law analysis
- Quantitative information about irreversibilities in processes
- Introduction to entropy (s) and entropy balance

3.3.1 Inequality of Clausius

- The second law, is based on the Clausius inequality

$$\oint \frac{\delta Q_{net}}{T} \leq 0$$



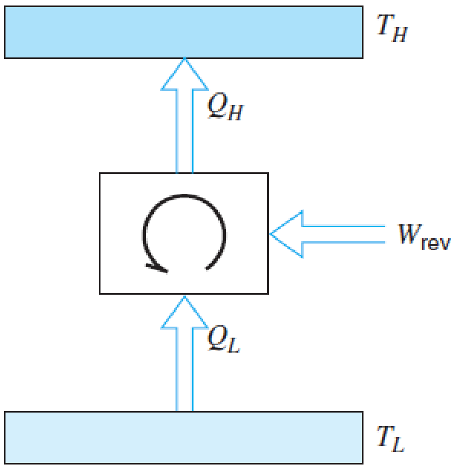
Consider a heat engine operating between T_H and T_L

- $\oint \delta Q = Q_H - Q_L > 0$
- For constant T_H and T_L
  - $\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$
- **Carnot:** Q's become T's
  - $\oint \frac{\delta Q}{T} = \frac{T_H}{T_H} - \frac{T_L}{T_L} = 0$
- **Irreversible**: $W_{irr} < W_{rev}$ (less work out)
- 1st law: $W = Q_H - Q_L$ (keep Q_H constant)
  - $Q_H - Q_{L,\,irr} < Q_H - Q_{L,\,rev}$
  - $Q_{L,\,irr} > Q_{L,\,rev}$ (more heat to T_L)
  - $\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_{L,irr}}{T_L} < 0$
- **Reversible** heat engines
  - $\oint \frac{\delta Q}{T} = 0$
- **Irreversible** heat engines
  - $\oint \frac{\delta Q}{T} < 0$

3.3.1 Inequality of Clausius

- The second law, is based on the Clausius inequality

$$\oint \frac{\delta Q_{net}}{T} \leq 0$$



Consider a refrigerator operating between T_H and T_L

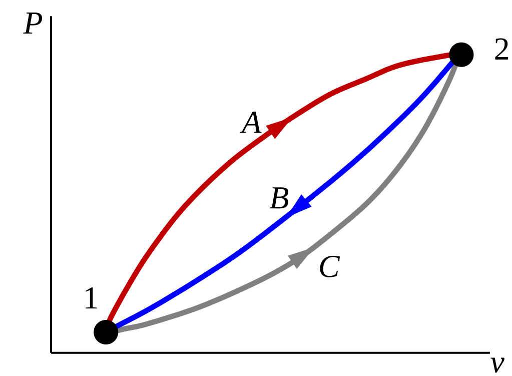
- $\oint \delta Q = -Q_H + Q_L < 0$
- For constant T_H and T_L
  - $\oint \frac{\delta Q}{T} = \frac{Q_L}{T_L} - \frac{Q_H}{T_H}$
- **Carnot**: Q's become T's
  - $\oint \frac{\delta Q}{T} = \frac{T_L}{T_L} - \frac{T_H}{T_H} = 0$
- **Reversible** refrigerators
  - $\oint \frac{\delta Q}{T} = 0$

- **Irreversible**: $W_{irr} > W_{rev}$ (more work required)
- 1st law: $W = Q_H - Q_L$ (keep Q_L constant)
  - $Q_{H,irr} - Q_L > Q_{H,rev} - Q_L$
  - $Q_{H,\,irr} > Q_{H,\,rev}$ (more heat to T_H)
  - $\oint \frac{\delta Q}{T} = \frac{Q_L}{T_L} - \frac{Q_{H,\,irr}}{T_H} < 0$
- **Irreversible** refrigerators
  - $\oint \frac{\delta Q}{T} < 0$

3.3.2 Entropy, S

- Introduction to **entropy** (S)

- Consider reversible processes along path A-B
- Inequality of Clausius
  - $\oint \left(\frac{\delta Q_{net}}{T}\right)_{rev} = 0$
  - $\oint \left(\frac{\delta Q_{net}}{T}\right)_{rev} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A - \int_2^1 \left(\frac{\delta Q}{T}\right)_B = 0$



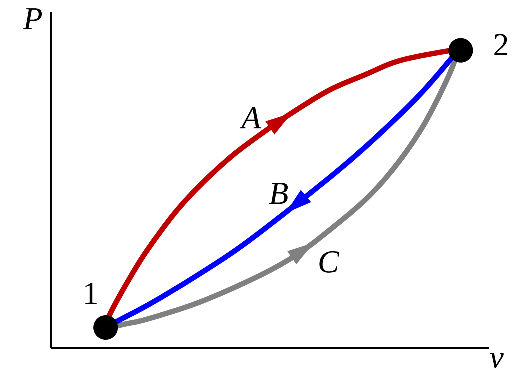
- Consider reversible processes along path C-B
- Inequality of Clausius
  - $\oint \left(\frac{\delta Q_{net}}{T}\right)_{rev} = 0$
  - $\oint \left(\frac{\delta Q_{net}}{T}\right)_{rev} = \int_1^2 \left(\frac{\delta Q}{T}\right)_C - \int_2^1 \left(\frac{\delta Q}{T}\right)_B = 0$

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A = \int_1^2 \left(\frac{\delta Q}{T}\right)_C$$

- $\oint \left(\frac{\delta Q_{net}}{T}\right)$ is same for reversible processes between states 1 & 2

3.3.2 Entropy, S

- $\dfrac{\delta Q}{T}$ is independent of path and is function of end states
- $\dfrac{\delta Q}{T}$ is a property



- **Entropy**: $dS \equiv \left(\dfrac{\delta Q}{T}\right)_{rev}$ $[kJ/K]$
  - Measure of energy no longer available to perform useful work
  - Measure of disorder of the system

- Entropy change during an internally reversible process:

$$dS = \left.\frac{\delta Q_{net}}{T}\right|_{\text{int rev}} \qquad S_2 - S_1 = \int_1^2 \left.\frac{\delta Q_{net}}{T}\right|_{\text{int rev}}$$

3.3.2 Entropy, S

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- Intensive property *s*, (kJ/kgK)
  - Similar to *u, v, h*
  - Can be used to define the state

Entropy of a pure substance (e.g. water, refrigerant)

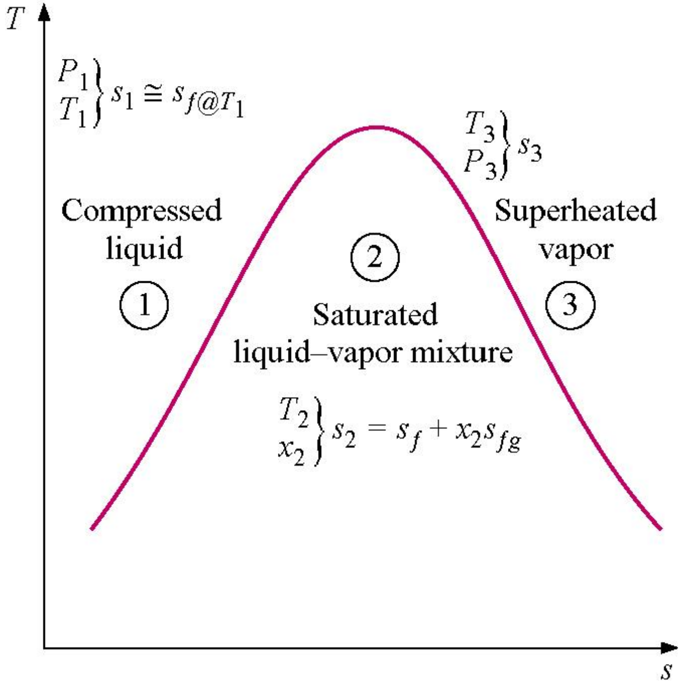
- Thermodynamic tables

Reference value for water:

$$T = 0.01^{o}C,\ s_f = 0\ \text{kJ/kgK}$$

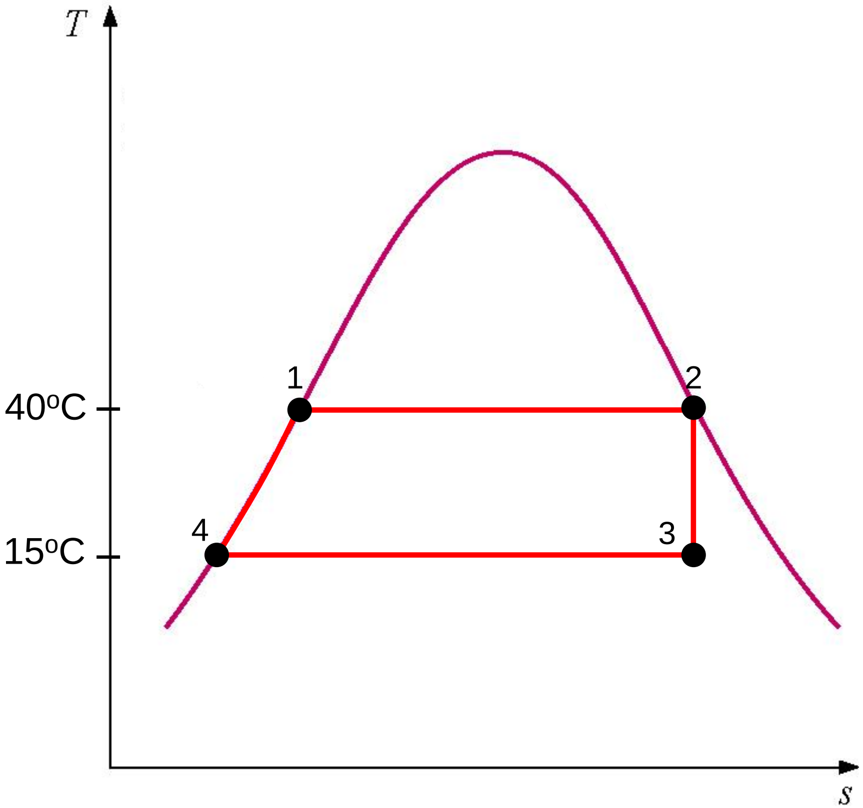
TABLE B.1.1 (*continued*)
Saturated Water

Temp. (°C)	Press. (kPa)	Entropy, kJ/kg-K Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
0.01	0.6113	0	9.1562	9.1562
5	0.8721	0.0761	8.9496	9.0257
10	1.2276	0.1510	8.7498	8.9007
15	1.705	0.2245	8.5569	8.7813
20	2.339	0.2966	8.3706	8.6671
25	3.169	0.3673	8.1905	8.5579



3.3.2 Entropy, S

Property diagrams – T vs. s diagram



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30	4.246	0.4369	8.0164	8.4533
35	5.628	0.5052	7.8478	8.3530
40	7.384	0.5724	7.6845	8.2569

<u>Draw Processes on T-s diagram</u>

1→2 Entropy increase from sat. liquid to sat. vapor at 40ºC

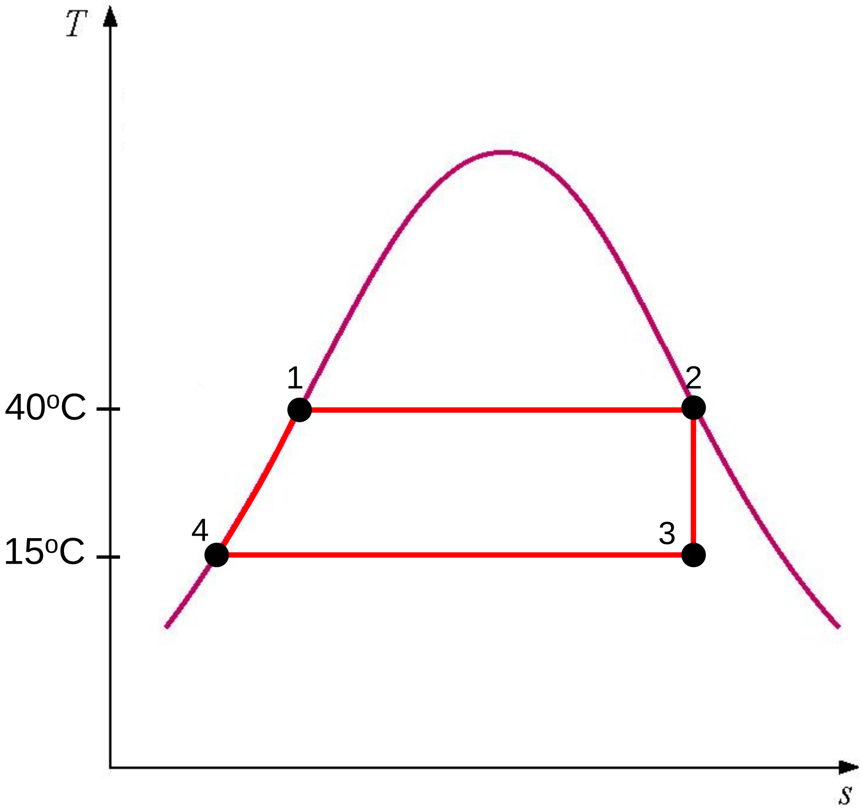
2→3 Temp decrease to T=15ºC under constant s

3→4 Entropy decrease to sat. liquid at T = 15ºC

4→1 T & s increase as sat. liquid to return to state 1 (40C)

3.3.2 Entropy, S

Property diagrams – T vs. s diagram



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<u>Find Δs for each process:</u>

1→2 $s_2 - s_1 = s_g - s_f = s_{fg@40C}$

2→3 $s_3 - s_2 = 0$, isentropic process

3→4 $s_4 - s_3 = s_{f@15C} - s_3 \rightarrow s_{f@15C} - s_{g@40C}$

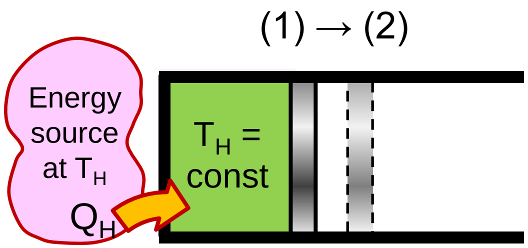
4→1 $s_1 - s_4 = s_{f@40C} - s_{f@15C}$

3.3.3 ΔS for Reversible Processes

- Evaluate the change of entropy for reversible processes

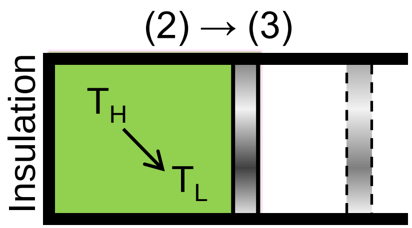
$$dS = \int \left(\frac{\delta Q}{T}\right)_{rev}$$

Carnot Cycle



Process 1-2: Reversible isothermal heat addition.

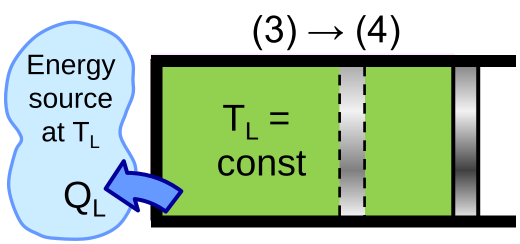
$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev} \rightarrow T_H = Const. \rightarrow \boldsymbol{S_2 - S_1 = \frac{Q_H}{T_H}}$$



Process 2-3: Reversible, adiabatic expansion. Temp. T_H to T_L.

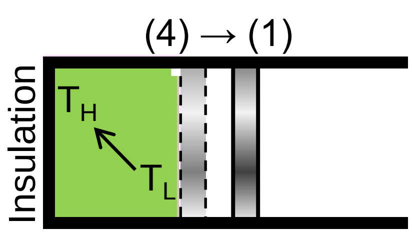
$$S_3 - S_2 = \int_2^3 \left(\frac{\delta Q}{T}\right)_{rev} \rightarrow Q = 0 \rightarrow \boldsymbol{S_3 - S_2 = 0}$$

Adiabatic + reversible = isentropic



Process 3-4: Reversible isothermal heat rejection.

$$S_4 - S_3 = \int_3^4 \left(\frac{\delta Q}{T}\right)_{rev} \rightarrow T_L = Const. \rightarrow \boldsymbol{S_4 - S_3 = \frac{-Q_L}{T_L}}$$



Process 4-1: Reversible adiabatic compression. Temp. T_L to T_H.

$$S_1 - S_4 = \int_4^1 \left(\frac{\delta Q}{T}\right)_{rev} \rightarrow Q = 0 \rightarrow \boldsymbol{S_1 - S_4 = 0}$$

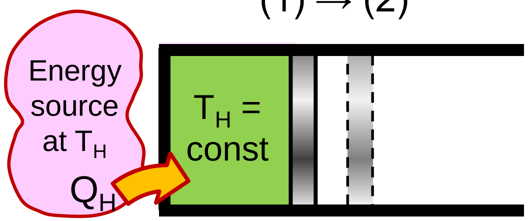
Adiabatic + reversible = isentropic

3.3.3 ΔS for Reversible Processes

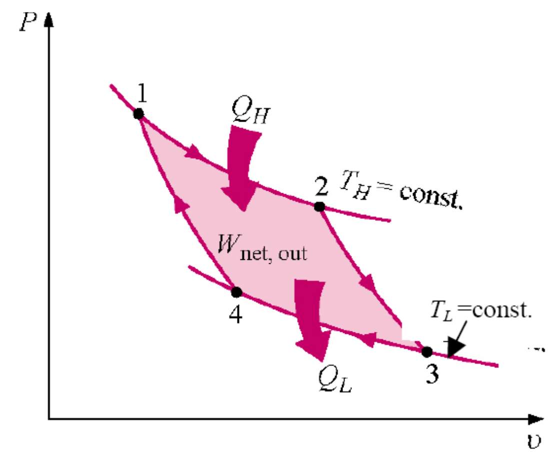
- Evaluate the change of entropy for reversible processes

Carnot Cycle

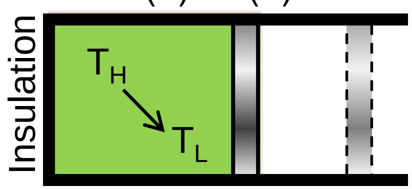
(1) → (2)



$$S_2 - S_1 = \frac{Q_{21}}{T_H} = \frac{Q_H}{T_H}$$

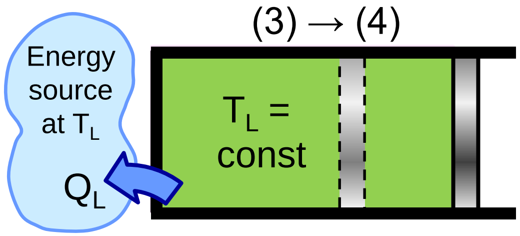


(2) → (3)



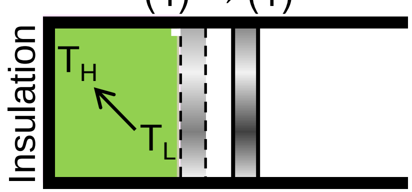
$$S_3 - S_2 = 0$$

(3) → (4)



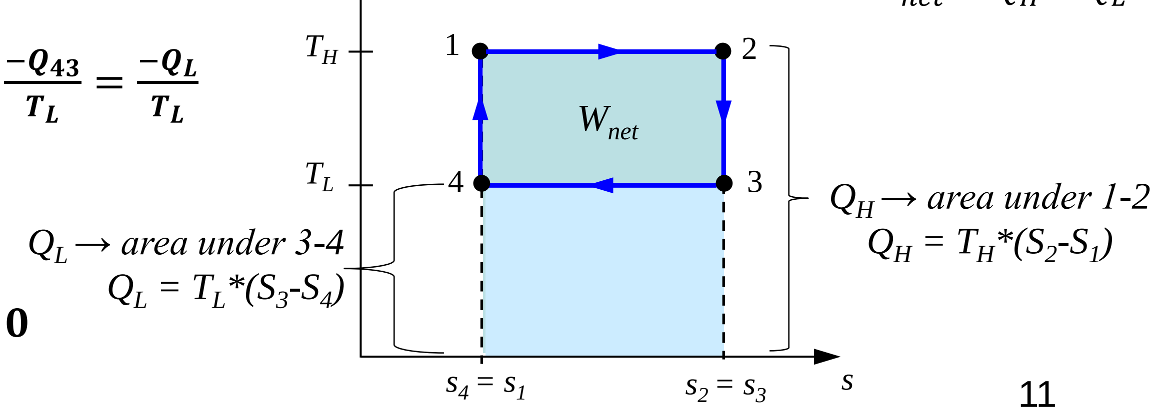
$$S_4 - S_3 = \frac{-Q_{43}}{T_L} = \frac{-Q_L}{T_L}$$

(4) → (1)



$$S_1 - S_4 = 0$$

1st Law: $W_{net} = Q_H - Q_L$



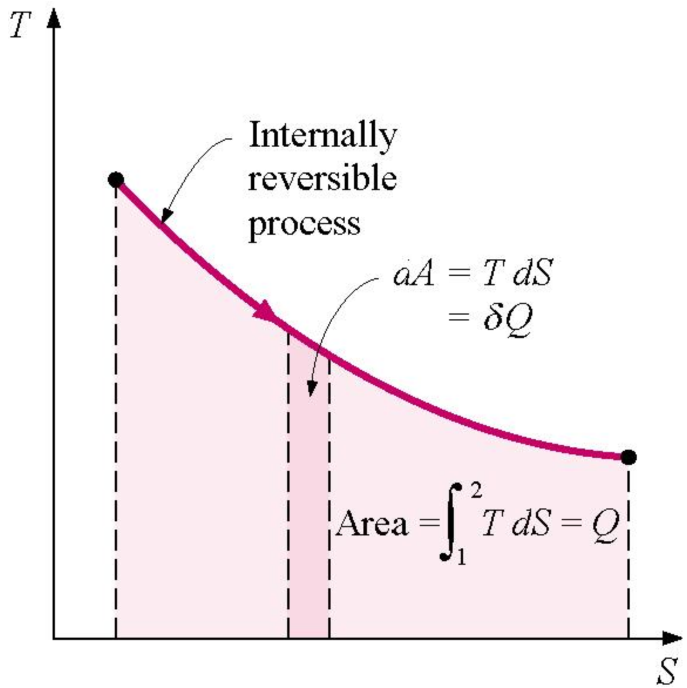
3.3.3 ΔS for Reversible Processes

Heat Transfer - Area under a *T-S* Curve

- For reversible processes

$$dS = \frac{\delta Q_{net}}{T} \qquad \delta Q_{net} = TdS$$

- Incremental heat transfer in a reversible process
  - The differential area under the *T-S* diagram.



$$Q_{net} = \int_1^2 TdS$$

3.3.3 ΔS for Reversible Processes

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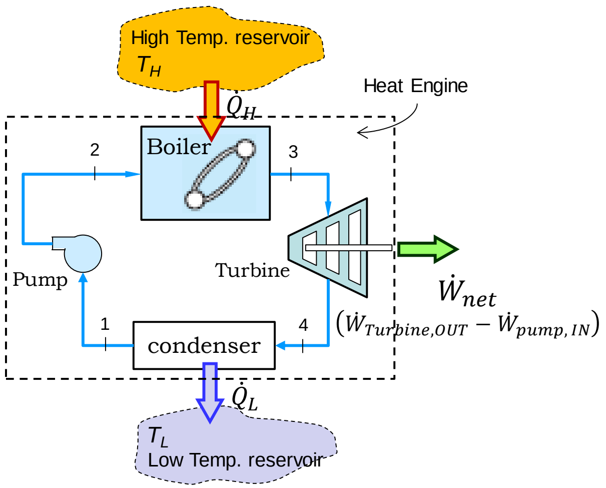
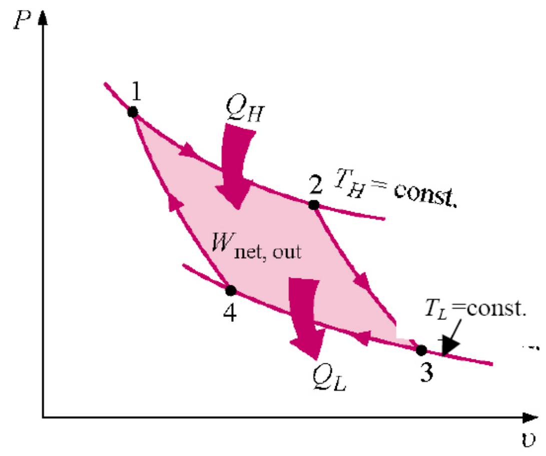
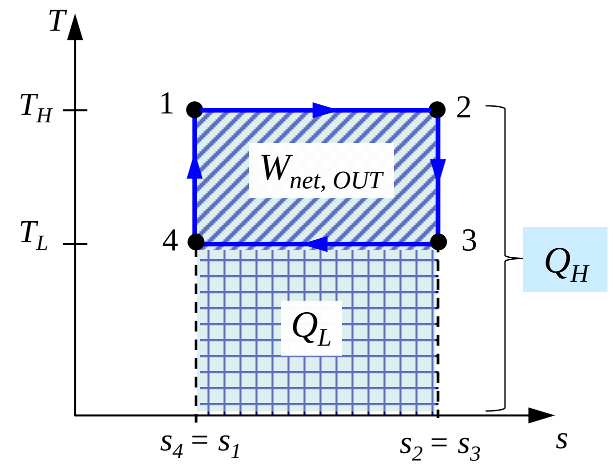
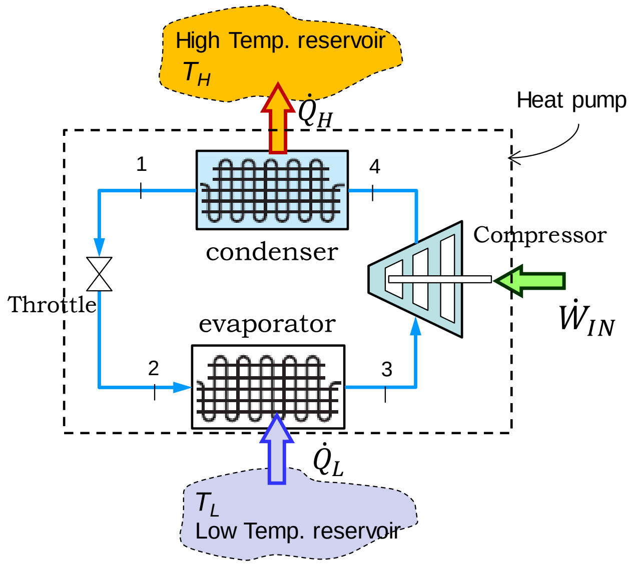
Carnot Cycle as steam power plant

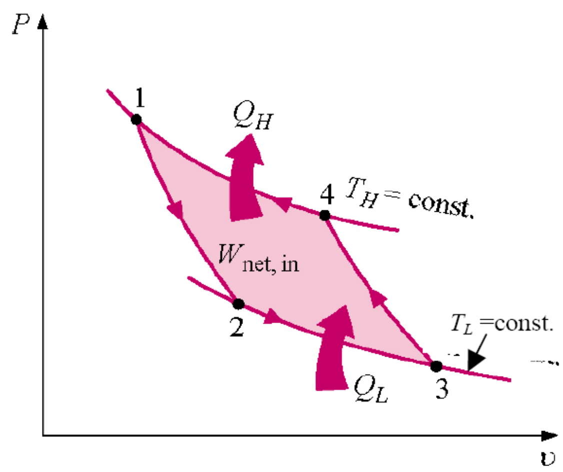


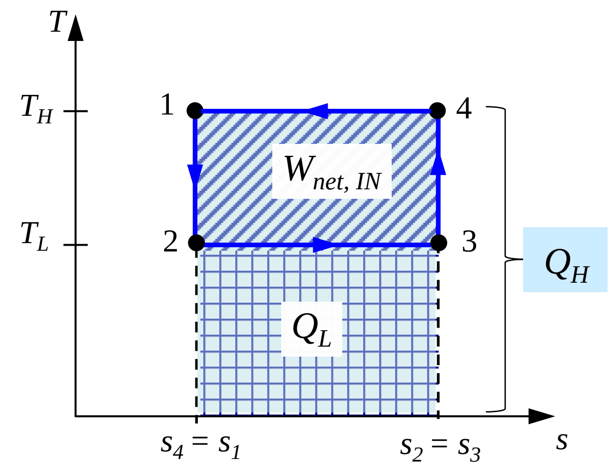




Carnot Cycle as refrigerator



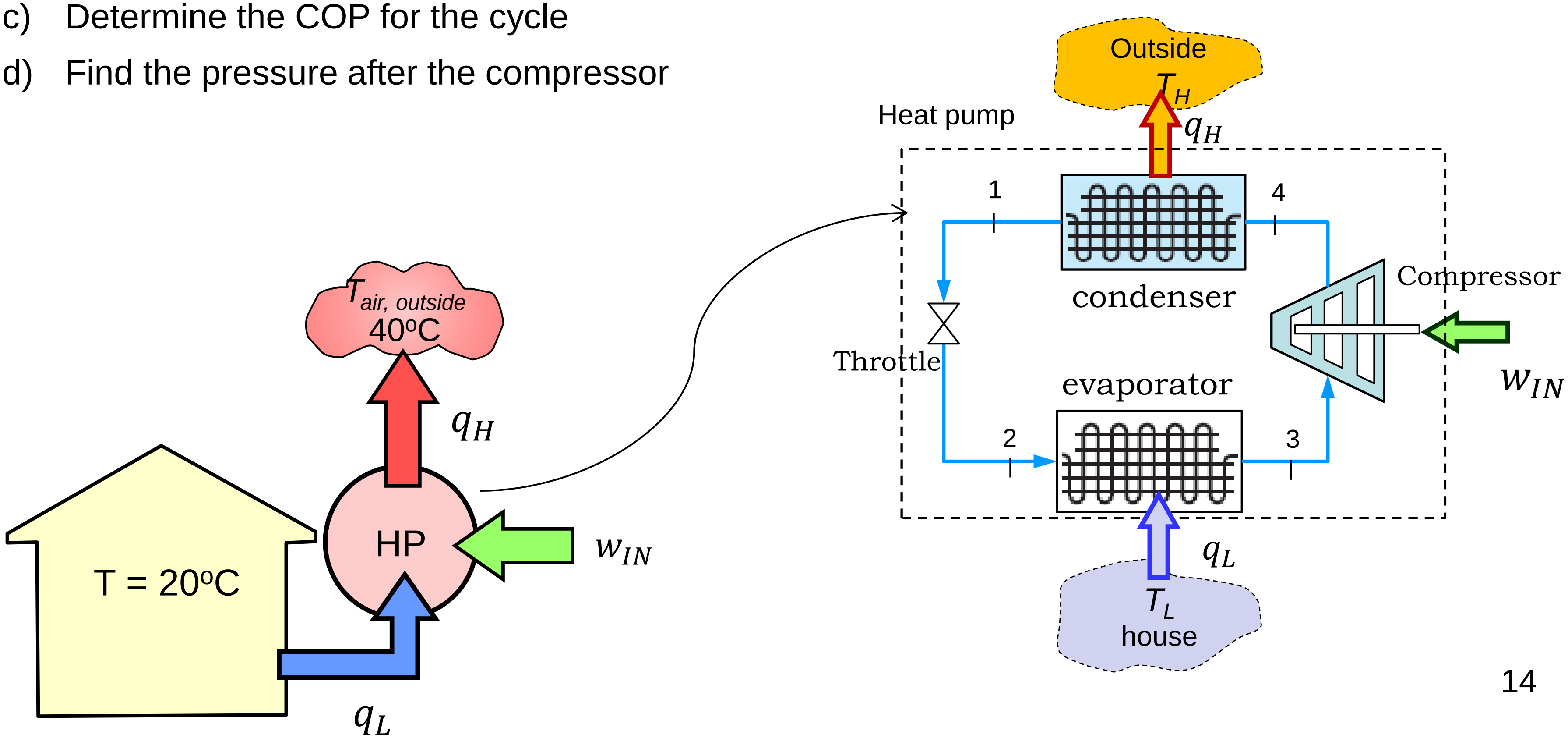




3.3.4 Example

Example 3.3: A <u>Carnot Heat Pump</u> is used to cool a house. The heat pump operates with ammonia as the working fluid. Heat is rejected in the condenser as the ammonia changes from a saturated vapour to a saturated liquid at 40ºC. Heat is added in the evaporator as the ammonia has a constant temperature of 20ºC. Assume the compressor and throttle to be adiabatic and reversible.

a) Draw this cycle on the T-s diagram
b) Determine q_H and q_L
c) Determine the COP for the cycle
d) Find the pressure after the compressor



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a) Draw this cycle on the T-s diagram

b) Determine q_H and q_L

c) Determine the COP for the cycle

d) Determine the quality of state 2

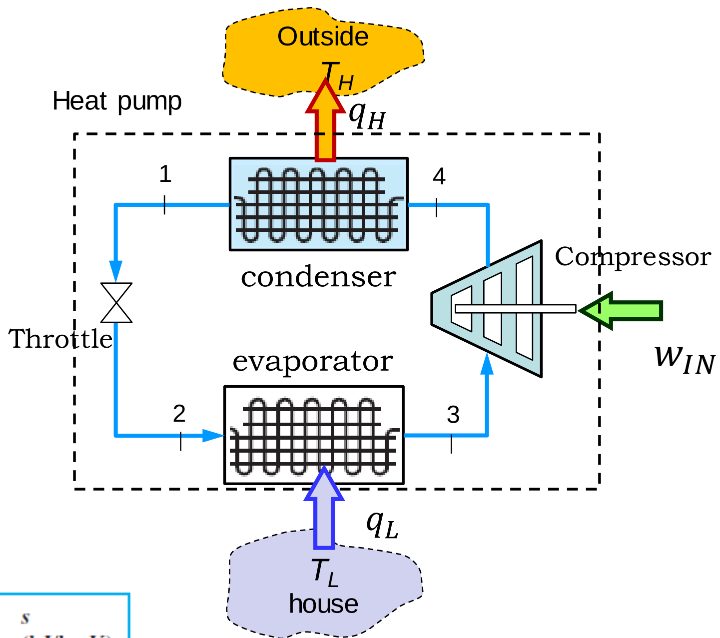

TABLE B.2.1 (*continued*)
Saturated Ammonia

Temp. (°C)	Press. (kPa)	Enthalpy, kJ/kg Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Entropy, kJ/kg-K Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
20	857.5	274.30	1185.9	1460.2	1.0408	4.0452	5.0860
25	1003.2	298.25	1165.2	1463.5	1.1210	3.9083	5.0293
30	1167.0	322.42	1143.9	1466.3	1.2005	3.7734	4.9738
35	1350.4	346.80	1121.8	1468.6	1.2792	3.6403	4.9196
40	1554.9	371.43	1098.8	1470.2	1.3574	3.5088	4.8662

TABLE B.2.2 (*continued*)
Superheated Ammonia

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	1000 kPa (24.90°C)			
Sat.	0.12852	1334.9	1463.4	5.0304
20	—	—	—	—
30	0.13206	1347.1	1479.1	5.0826
40	0.13868	1369.8	1508.5	5.1778

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	1200 kPa (30.94°C)			
Sat.	0.10751	1337.8	1466.8	4.9635
40	0.11287	1360.0	1495.4	5.0564
50	0.11846	1383.0	1525.1	5.1497
60	0.12378	1404.8	1553.3	5.2357

3.3.4 Exercises

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Exercise 3-1

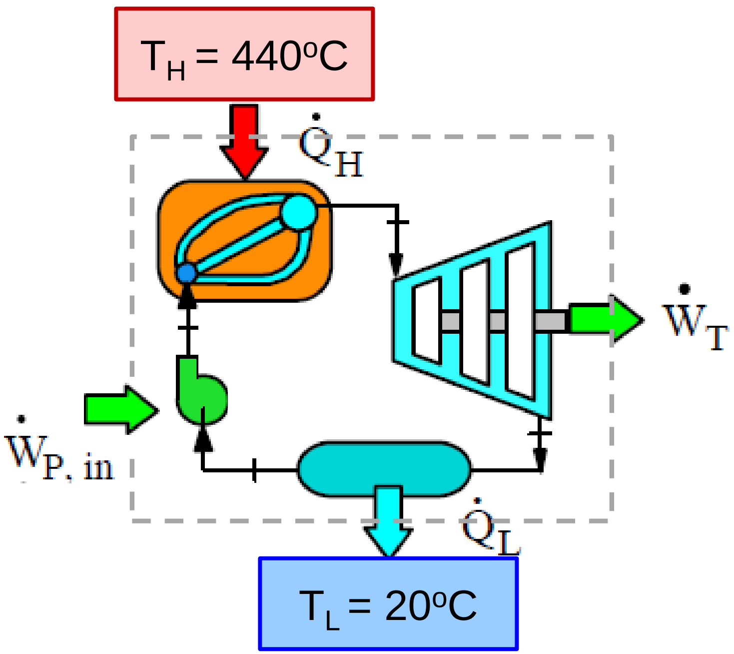
Consider a particular power plant where the heat added and rejected both occur at constant temperature. No other processes experience any heat transfer. The heat is added in the amount of Q_H = 3150 kJ at 440^{o}C and is rejected in the amount of Q_L = 1950 kJ at 20^{o}C.

a) Is the Clausius inequality satisfied?

b) Is the cycle reversible or irreversible?

c) Find the thermal efficiency of the power plant

d) How does the thermal efficiency differ if the power plant operated on the Carnot Cycle?



ans:

a) The Clausius inequality is satisfied

b) The cycle is irreversible since $\oint \frac{\delta Q}{T} < 0$.

c) $\eta_{TH} = 38.1\%$

d) $\eta_{TH} = 58.9\%$