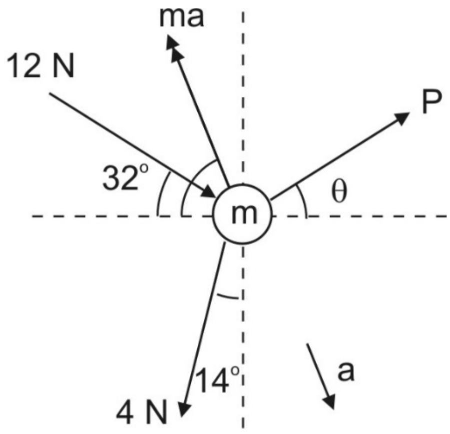
Dynamics 2 – Tutorial 1

Dynamics of a Single Particle and D'Alembert's Method

Outline Solutions

1.



From FBD:

R(→) $P \cos \theta - ma \cos 68 + 12 \cos 32 - 4 \sin 14 = 0$ (1)

R(↑) $P \sin \theta + ma \sin 68 - 12 \sin 32 - 4 \cos 14 = 0$ (2)

$P \cos \theta = ma \cos 68 - 12 \cos 32 + 4 \sin 14$ (1a)
$= -3.365$

$P \sin \theta = - ma \sin 68 + 12 \sin 32 + 4 \cos 14$ (2b)
$= -4.224$

By Pythagoras

$P = \sqrt{[(P \sin \theta)^2 + (P \cos \theta)^2]} = 5.4$ N

$\sin \theta = -4.224 / 5.4 = -0.782$

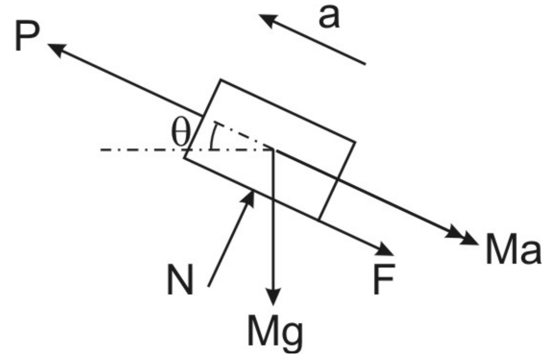
$\theta = \sin^{-1} (-0.782) = -51.5°$

But both Pcosθ and Psinθ are –ve, therefore P acts in the 3rd quadrant, and θ = 180+51.5 = 231.5°

2.

Mass is sliding so $F = \mu N$.

$a = 2.6$ m/s^2; $M = 14$kg ; $\mu = 0.18$; $\theta = 25^{\circ}$



From FBD

R (‖) $F + Ma - P + Mg \sin \theta = 0$ (1)

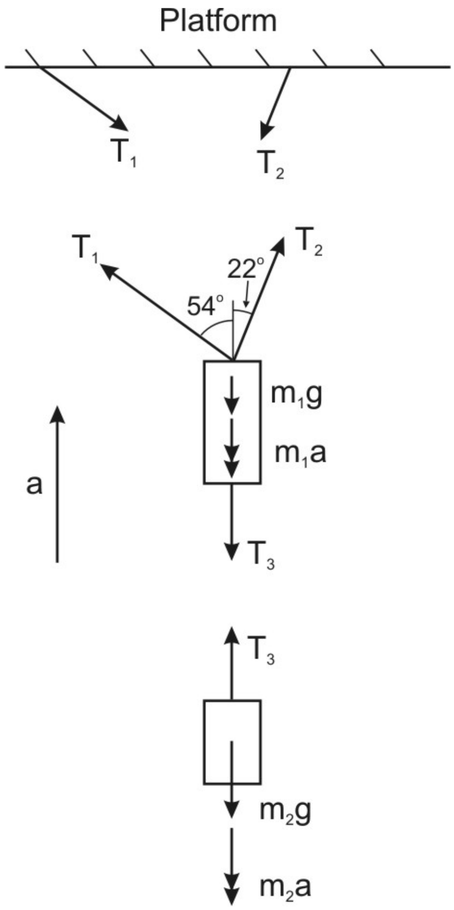
R (⊥) $N - Mg \cos \theta = 0$ (2)

(2) gives $N = 124.47$

$\Rightarrow$ $F = \mu N = 22.41$

(1) gives $P = 116.85$ N

3.



Platform and masses have same acceleration.
$a = 6$; $m_1 = 1.2$; $m_2 = 0.8$

Note use of N3 in FBDs.

For m_1:

$$\uparrow \quad T_1 \cos 54 + T_2 \cos 22 - m_1 g - m_1 a - T_3 = 0$$

$$\rightarrow \quad T_1 \sin 54 = T_2 \sin 22$$

$\Rightarrow$ $T_2 = 2.16T_1$

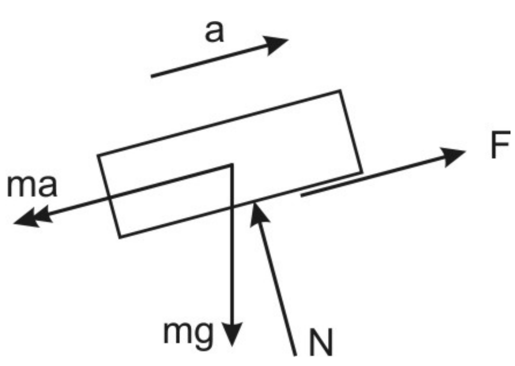
For m2:

$$\uparrow \quad T_3 - m_2 g - m_2 a = 0$$
$\Rightarrow$ $T_3 = 12.65$ N

Use T_3 and T_2 to get T_1 from first equation:

$\Rightarrow$ $T_1 = 12.21$ N
$\Rightarrow$ $T_2 = 26.37$ N

4.



Box mass = 550kg; slope $\theta = 15^{o}$; $\mu = 0.35$

Find accel of truck at point of slip of box.
Assume box is just on point of sliding;
accel of box is same as accel of truck.

From FBD get:
$\|$ $F = ma + mg \sin\theta$
$\perp$ $N = mg \cos\theta$
Also $F = \mu N$

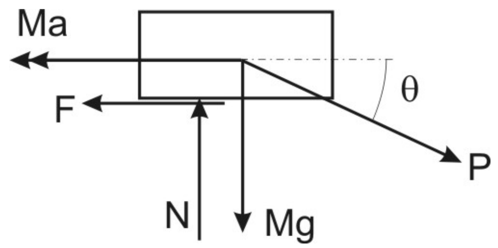
Hence
$a = g[\mu\cos\theta - \sin\theta] = 0.778 \text{m/s}^2$

5.

Mass M = 12kg; $\mu = 0.35$; P = 200N.

P is applied by a wire at angle θ below horizontal. Find acceleration of mass for $\theta = 35°$ and $\theta = 65°$

For $\theta = 35°$: *Assume* mass M is sliding to the right. Find a.



From FBD:

$\rightarrow$ $P \cos\theta = Ma + F$

$\uparrow$ $P \sin\theta + Mg = N$

Also $F = \mu N$

Hence

$$Ma = P(\cos\theta - \mu \sin\theta) - \mu Mg$$

$$12a = 200(\cos\theta - 0.35 \sin\theta) - 41.2$$

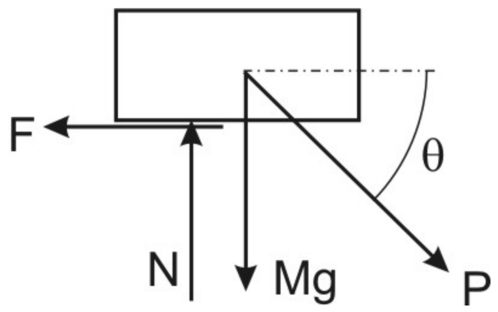
$\Rightarrow$ $a = 6.86 \text{ m/s}^2$

For $\theta = 65°$: get $a = -1.68 \text{ m/s}^2$

However, the solution began with the assumption that the acceleration was to the right and the friction was taken to the left. The negative answer for a is not valid.

We could assume the mass acceleration to the left, but that is clearly not possible as the force P is to the right. The remaining possibility is that the mass does not slide.

Assume the mass does not slide.



$\theta = 65°$; P = 200N

From FBD:

$\rightarrow$ $F = P \cos\theta$

$\uparrow$ $Mg + P \sin\theta = N$

$\Rightarrow$ F = 84.52N

$\Rightarrow$ N = 299 N

But $F_{max} = \mu N = 0.35 \times 299 = 104.6$ N.

Since $F < F_{max}$ assumption is valid.

$\Rightarrow$ $a = 0$