Module 5 self assessment

Question 1

Find the flux of $\mathbf{f}(x,y) = xy^2\mathbf{\hat{i}} + x\mathbf{\hat{j}}$ through the semicircle

$$c: x^2 + y^2 = 1, \quad y < 0$$

with clockwise direction.

Question 2

Find the flux of $\mathbf{f}(x,y) = y \sin x \hat{\mathbf{i}} + \sin y \hat{\mathbf{j}}$ through a square with vertices $(0,0), (\frac{\pi}{2},0), (\frac{\pi}{2},\frac{\pi}{2})$ and $(0,\frac{\pi}{2})$ in anticlockwise direction.

Question 3

Solve the integral

$$\int_{c} x(x+y) \mathrm{d}s,$$

on a c given parametrically in terms of the position vector $\mathbf{r}(t) = (1+t)\hat{\mathbf{i}} + (2+3t)\hat{\mathbf{j}}$ for $0 \le t \le 1$.

Hint: Recall that the definition of the position vector in Cartesian coordinates is $\mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}$.