Tutorial 3 – SOLUTIONS

Tutorial 3: First Law of Thermodynamics (closed & open system) including specific heats

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Conceptual Questions:

1. For the classical sign convention of the first law (i.e. $\Delta E_{sys} = Q_{net} - W_{net}$), what is regarded as positive work and positive heat?

Solution: According to $\Delta E_{sys} = Q_{net} - W_{net}$, $Q_{net} = Q_{in} - Q_{out}$ and $W_{net} = W_{out} - W_{in}$. Thus, heat transfer INTO a system and work done BY (out of) a system are positive; heat transfer FROM (out of) a system and work done ON (into) a system are negative.

2. Briefly explain the three modes of heat transfer.

- Conduction: exchange of energy between molecules of a substance with no bulk motion.
- Convection: exchange of energy between a surface and the adjacent liquid or gas that is in motion.
- Radiation: exchange of energy from the surface of one body to the surface of another due to electromagnetic radiation.

3. Using the definition of enthalpy (h = u + Pv) and the definitions du = Cv*dT and dh = Cp*dT, show that $C_P = C_V + R$ for an ideal gas.

Solution:

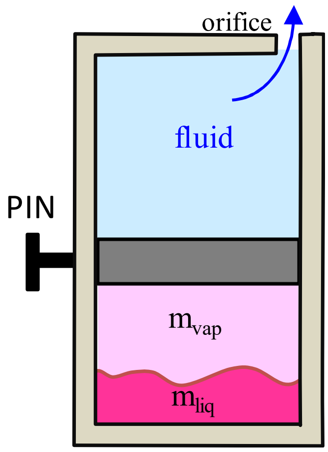
- h = u + Pv; Pv = RT
- dh = du + d(RT)
- dh = C_PdT & du = C_VdT → C_PdT = C_VdT + d(RT) → $C_P = C_V + R$

4. What is the definition of the specific heat ratio (k)?

Solution: k = C_P / C_V

Problem Solving Questions:

5. A piston cylinder device is comprised of two compartments. The bottom compartment is closed and contains refrigerant R-134a. The top compartment is open with a small orifice and contains an unknown fluid. The closed compartment contains 2.5 kg of refrigerant R134a at -20°C. Initially 25% of the refrigerant exists as a vapor, while the remaining exists as a liquid. A pin is used to secure the piston in place.



(a) Heat is transferred to the cylinder via an electric heater, while the pin is still locked. The refrigerant temperature rises to 30°C. Determine the heat transferred (in kJ).

(b) The electric heater is turned off and the pin is removed. The piston moves upward and fluid exits the top compartment such that the pressure in the closed compartment decreases linearly with increasing volume. The cylinder expands until the refrigerant is a saturated vapor at 400 kPa. Determine the <u>final temperature</u>, <u>boundary work</u>, and any <u>heat transfer</u> to the surroundings.

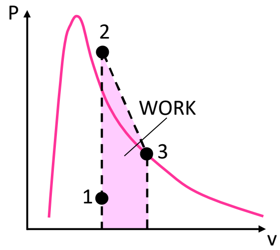
<u>Solution</u>:

- There are several processes occurring, so we will first define them
  - Process 1-2: Refrigerant, initially at a saturated mixture at T_1 = -20°C is heated at constant volume until the refrigerant temperature is 30°C.
  - Process 2-3: Heat addition from the electric heater ceases. Cylinder expands upon pin removal and refrigerant expands as pressure decreasees linearly until the refrigerant reaches a saturated vapor (i.e. x_3 = 1) at P_3=400 kPa. There might be heat transfer to the surroundings during this process.
- <u>Define the system</u>: refrigerant in the cylinder
- Process 1-2: Apply the first law of thermodynamics
  - $\Delta U + \Delta KE + PE = Q_{21} - W_{21} + \Delta E_{mass,21}$
  - System is closed, stationary and fixed volume, thus: $m(u_2 - u_1) = Q_{21}$
  - Determine relevant variables for <u>state 1</u>:
    - $x_1 = 0.25, T_1 = -20C$
    - $v_1 = v_{f@-20C} + x_1 v_{fg@-20C} = 0.000738\frac{m^3}{kg} + 0.25\left(0.14576\frac{m^3}{kg}\right) = 0.03718\frac{m^3}{kg}$
    - $u_1 = u_{f@-20C} + x_1 u_{fg@-20C} = 173.65\frac{kJ}{kg} + 0.25\left(192.85\frac{kJ}{kg}\right) = 221.9\frac{kJ}{kg}$
  - Determine relevant variables for state 2 (T_2 = 30C)
    - Fixed volume & mass: $v_1 = v_2 = 0.03718\frac{m^3}{kg}$
    - $V_1 = V_2 = v_1 m = 0.03718\frac{m^3}{kg} * 2.5kg = 0.0929m^3$
    - $v_2 > v_{g@30C} \rightarrow state\ is\ superheated\ vapor$
    - Need to interpolate in steam tables to find u_2 and P_2

- $\frac{u_2 - u_{500kPa,30C}}{v_2 - v_{500kPa,30C}} = \frac{u_{600kPa,30C} - u_{500kPa,30C}}{v_{600kPa,30C} - v_{500kPa,30C}}$
- $u_2 = u_{500kPa,30C} + \frac{v_2 - v_{500kPa,30C}}{v_{600kPa,30C} - v_{500kPa,30C}}(u_{600kPa,30C} - u_{500kPa,30C})$
- $u_2 = 398.99\frac{kJ}{kg} + \frac{0.03718 - 0.04446}{0.03609 \quad .04446}(397.44 - 398.99)\frac{kJ}{kg} = 397.6\frac{kJ}{kg}$
- $\frac{P_2 - P_{500kPa}}{v_2 - v_{500kPa,30C}} = \frac{P_{600kPa} - P_{500kPa}}{v_{600kPa,30C} - v_{500kPa,30C}}$
- $P_2 = P_{500kPa} + \frac{v_2 - v_{500kPa,30C}}{v_{600kPa,30C} - v_{500kPa,30C}}(P_{600kPa} - P_{500kPa})$
- $P_2 = 500kPa + \frac{0.03718 - 0.04446}{0.03609 - 0.04446}(600 - 500)kPa = 587kPa$

  - Determine heat addition for Process 1-2
    - $Q_{21} = m(u_2 - u_1) = 2.5kg(397.6 - 221.9)\frac{kJ}{kg} = 439.25kJ$

– Process 2-3: Apply First Law of Thermodynamics
  - $\Delta U + \Delta KE + \Delta PE = Q_{32} - W_{32} + \Delta E_{mass,32}$
  - System is closed and stationary, but has moving boundaries
    - $m(u_3 - u_2) = Q_{32} - W_{32}$
  - Determine relevant variables for <u>state 3</u>
    - Saturated vapor at P_3 = 400 kPa
    - $T_3 = T_{sat@400kPa}$. This can be seen as the saturated entry in the superheated R134a tables at 400kPa. T_3 = 8.84°C
    - $u_3 = 383.02\frac{kJ}{kg}$; $v_3 = 0.05136\frac{m^3}{kg}$
    - Find Volume: $V_3 = mv_3 = 2.5kg * 0.05136\frac{m^3}{kg} = 0.1284m^3$
  - Determine the boundary work
    - There is a linear relationship between pressure and volume for process 2-3.
    - Plot process 2-3 on the P-V diagram
    - Pressure will decrease linearly, while volume will increase linearly
    - From lecture (and book): $W_b = \frac{1}{2}(P_3 + P_2)(V_3 - V_2)$
    - $\boldsymbol{W_{b,32} = \frac{1}{2}(400 + 587)kPa(0.1284 - 0.0929)m^3 = 17.5kJ}$
    - This value is also equal to the area under the P-V curve
    - Since the system is expanding, the work is out of the system, so W_{net} is positive



  - Determine the heat transfer in process 2-3
    - From the 1st law of thermodynamics: $m(u_3 - u_2) = Q_{32} - W_{32}$

- $Q_{32} = m(u_3 - u_2) + W_{b,32} = 2.5kg(383.02 - 397.6)\frac{kJ}{kg} + 17.5kJ = -18.95kJ$
- Thus heat is leaving the system towards the surroundings

6. Complete the table below (for substances that can be assumed to be ideal gases, where these are properties at 300K, R_u = 8.314 kJ/(kmolK)).

Substance	**Molar mass kg/kmol**	**R kJ/(kgK)**	**C_p kJ/(kgK)**	**C_v kJ/(kgK)**	**k**
Air	28.97	0.287	1.005	0.718	1.400
Carbon Dioxide	44.01	0.189	0.846	0.657	1.289
Hydrogen	2.016	4.124	14.31	10.18	1.405
Nitrogen	28.01	0.297	1.039	0.743	1.400
Oxygen	32.00	0.260	0.918	0.658	1.395

Solution:

Air

- R = R_u / M -> M = R_u / R = 8.314 / 0.287 = 28.97kg/kmol
- C_v = R / (k-1) = 0.287 / (1.400 - 1) = 0.718kJ/(kgK)
- C_p = C_v + R = 0.718+0.287 = 1.005kJ/(kgK)

Carbon dioxide

- k = C_p / C_v = 0.846 / 0.657 = 1.288
- C_p = C_v + R -> R = C_p - C_v = 0.846 - 0.657 = 0.189kJ/(kgK)
- R = R_u / M -> M = R_u / R = 8.314 / 0.189 = 43.99kg/kmol

Hydrogen

- R = R_u / M = 8.314 / 2.016 = 4.124kJ/(kgK)
- C_p = C_v + R = 10.18+4.124 = 14.30kJ/(kgK)
- k = C_p / C_v = 14.30 / 10.18 = 1.405

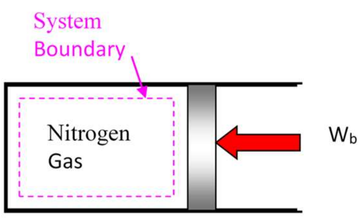
Nitrogen

- R = R_u / M = 8.314 / 28.01 = 0.297kJ/(kgK)
- C_p = C_v + R -> C_v = C_p – R = 1.039 - 0.297 = 0.742kJ/(kgK)
- k = C_p / C_v = 1.039 / 0.742 = 1.400

Oxygen

- k = C_p / C_v -> C_v = C_p / k = 0.918 /1.395 = 0.658kJ/(kgK)
- C_p = C_v + R -> R = C_p - C_v = 0.918 - 0.658 = 0.260kJ/(kgK)
- R = R_u / M -> M = R_u / R = 8.314 / 0.260 = 31.98kg/kmol

7. Three kilograms of nitrogen gas at 27°C, 0.15 MPa are compressed isothermally to 0.3 MPa in a piston-cylinder device. For nitrogen: R = 0.2968 kJ/(kgK) and the critical temperatures and pressures are T_{cr} = 126.2K, P_{cr} = 3.39MPa.



a. Show that nitrogen can be expected to behave as an ideal gas during this process.
b. Determine the minimum work of compression, in kJ.
c. Determine the heat transfer involved in this process?

Solution:

a. Ideal gas is satisfied if T > 2xT_{cr} OR P/P_{cr} << 1.

- Is temperature greater than 2 x T_{cr}? T = (27+273.15) → 300.15K > 2T_{cr}, behaves as ideal gas
- Is P / P_{cr} << 1? 0.15 / 3.39 = 0.0442. Yes, behaves as ideal gas

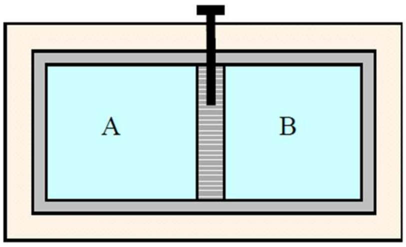
b. Determine the boundary work

- System: Nitrogen in piston cylinder
- Boundary work: constant pressure, ideal gas: $W_b = \int_1^2 PdV = \int_1^2 \frac{mRT}{V} dV$
  - $W_b = mRTln(V_2/V_1)$
  - $m_1 = m_2 \rightarrow P_1V_1/RT_1 = P_2V_2/RT_2 \rightarrow P_1V_1 = P_2V_2 \rightarrow V_2/V_1 = P_1/P_2$
  - $W_b = mRTln(P_1/P_2) = 3kg * 0.2968\ kJ/kgK * 300.15K * ln(0.15/0.30)$
  - $\boldsymbol{W_b = -184.5\ kJ}$**; work is into the system**

c. Determine the heat transfer

- 1st Law: $\Delta U + \Delta KE + \Delta PE = Q_{21} - W_{21} + \Delta E_{mass,21}$
- Closed, stationary system → $\Delta U = Q_{21} - W_{21}$
- Ideal gas: $\Delta U = mC_P(T_2 - T_1)$; temperature is constant so ΔU = 0
- $\boldsymbol{Q_{21} = W_{21} = -184.5kJ}$**; work is into the system, while heat transfer is out of the system**

8. An insulated cylinder is divided into two sections of 0.5 m^3 each by a piston which is locked by a pin. Side A has air at 300 kPa, 360 K, while side B has air at 1.2 MPa, 1000K. The piston is now unlocked so that it is free to move and it conducts heat so that the air comes to a uniform temperature $T_A = T_B$. The piston reaches an equilibrium position. Assume ideal gas with variable specific heats (i.e. use ideal gas AIR TABLES in the BACK of the BOOK (A.7 Borgnakke/Sonntag, or Table A-17 Cengel & Boles) and R = 0.287 kJ/kgK.



(a) Find the mass in section A and B.

(b) Determine the work done of the entire system.

(c) Find the final temperature and pressure

(d) Determine the piston movement in meters if the cross sectional area of the piston
is A = 0.25m^2.

Solution:

(a) P_{1A} = 300 kPa, T_{1A} = 360 K, V_{1A} = 0.5 m^3; P_{1B} = 1200 kPa, T_{1B} = 1000 K; V_{1B} = 0.5m^3

- PV = mRT

- $m_A = P_{1A}V_{1A}/(RT_{1A})$ = 300 kPa * 0.5m^3 / (0.287 kJ/kgK * 360K) = 1.45 kg
- $m_B = P_{1B}V_{1B}/(RT_{1B})$ = 1200 kPa * 0.5m^3 / (0.287 kJ/kgK * 1000K) = 2.09 kg

(b) Take the entire system to be section A + B. There is no movement of any of the boundaries with the surroundings. Boundary work = 0. No other forms of work.
Work = 0 kJ

(c) 1st law of thermodynamics: $\Delta U + \Delta KE + \Delta PE = Q_{21} - W_{21} + \Delta E_{mass,21}$

- System is closed, stationary, and no work. If we also take the system to be A + B, then no heat is transferred to the surroundings (i.e. insulated cylinder). Thus ΔU = 0.
- $\Delta U = 0 = \Delta U_A + \Delta U_B = m_A(u_{2A} - u_{1A}) + m_B(u_{2B} - u_{1B}) = 0$
- Further if $T_{2A} = T_{2B}$, then $u_{2A} = u_{2B} = u_2$ because for an ideal gas, internal energy depends on temperature only.
  - $m_A(u_2 - u_{1A}) = m_B(u_{1B} - u_2)$
  - $Solving\ for\ u_2: u_2 = m_B/(m_A + m_B)\, u_{1B} + m_A/(m_A + m_B)\, u_{1A}$
- From tables:
  - State 1A: 360K → u_{1A} = 257.53 kJ/kg
  - State 1B: 1000K → u_{1B} = 759.19 kJ/kg
- u_2 = 2.09 kg/(1.45+2.09)kg*759.19kJ/kg kJ/kg + 1.45kg/ (1.45+2.09)kg*257.53kJ/kg
- u_2 = 553.59 kJ/kg
- Use the tables to interpolate T_2.
  - $\frac{T_2 - T_{740K}}{u_2 - u_{740K}} = \frac{T_{760K} - T_{740K}}{u_{760K} - u_{740K}}; T_2 = T_{740K} + \frac{u_2 - u_{740K}}{u_{760K} - u_{740K}}(T_{760K} - T_{740K})$
  - $T_2 = 740K + \frac{553.59 - 544.33}{560.32 - 544.33}(760 - 740)K$; **$T_2$ = 751.6K**
- Pressure at state 2
  - If the piston reaches an equilibrium position, then $P_{2A} = P_{2B}$. If this was not the case, then the piston would still be moving and *not* be in equilibrium.
  - Because we do not know the individual volumes, take the entire system at state 2 and solve for P_2
    - $P_2V_{2,A+B} = (m_A + m_B)RT_2$; $P_2 = (m_A + m_B)RT_2 / (V_{1A}+V_{1B})$
    - P_2 = (1.45 + 2.09)kg*0.287kJ/kgK*751.6K / 1m^3; **P_2 = 764.1 kPa**

(d) Find volume displaced in section A or B

- $V_{2A} = m_ART_2 / P_2$; V_{2A} = 1.45kg*0.287kJ/kgK*751.6K / 764.1 kPa = 0.4098 m^2
- Volume displaced = $V_{1A} - V_{2A}$ = 0.5-0.4098 = 0.0902m^3
- Piston displaced = $\Delta V_A / A_{piston}$ = 0.0902m^3 / 0.25m^2; **Distance = 0.36 m**