Tutorial 9 – SOLUTIONS

Tutorial 9: Rankine and Otto Cycles

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Conceptual Questions:

1. How does an Otto cycle deviate from the Carnot cycle?

The absorption and rejection of heat do not occur at a constant temperature which changes the efficiency relations. For Carnot, we can use $\eta_{Carnot}=1-T_C/T_H$, but for the Otto cycle we must account for the temperature variation which ends up resulting in an ideal efficiency of $\eta_{Otto} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$.

2. What simplifications or assumptions are made when treating a gasoline internal combustion engine as an ideal Otto cycle?

- The working fluid is treated as air throughout the entire process. This does not account for the mole number changes due to combustion of the gasoline.
- The heat addition is assumed to occur instantaneously at a constant volume, but the combustion process will take place over a fraction of the crank rotation, meaning the volume will be changing, not staying at top dead center. The combustion is also not instantaneous so there will be some time and environmental change through the process that is not captured.
- The Otto cycle is treated as a closed volume process with no mass transfer, but in a real engine, there is an intake of air/fuel and an exhausting of the combustion gases.

3. Why are there limitations on the allowable compression ratios for internal combustion engines?

- An increase in the compression ratio will lead to an increase in the final temperature which can raise the gas temperature above the fuel autoignition temperature, which can be catastrophic for a spark ignition engine (can lead to a phenomena referred to as engine knock due to the audible disturbance)
- Increasing mechanical stress on components which may reduce component lifetime or result in pre-mature, catastrophic failure

Problem Solving Questions

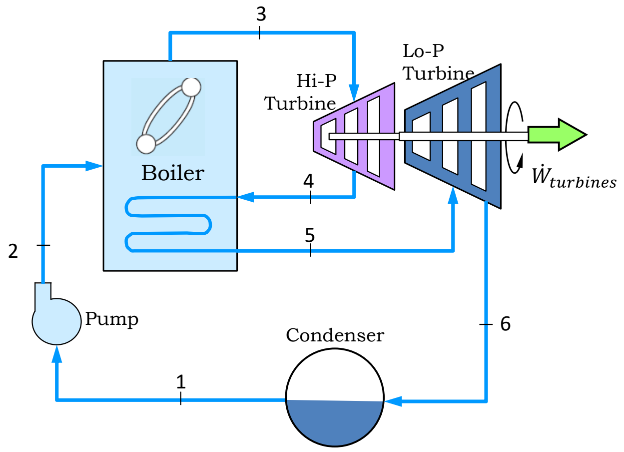
4. A small power plant operates on the ideal Reheat Rankine cycle. Steam flows at 25 kg/s in the boiler to produce steam at 3 MPa, 600°C that enters the high-pressure turbine. The steam exits the high-pressure turbine and re-enters the boiler where it is reheated to 400°C at 500 kPa and then is sent to the low-pressure turbine. The steam/water is sent to the condenser where it exits the condenser at a temperature of 45°C.

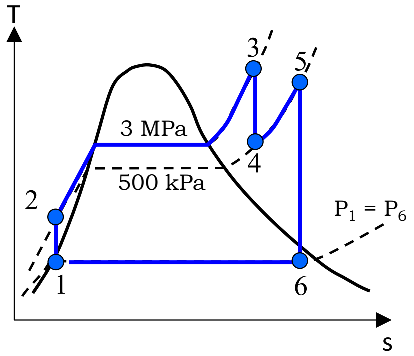
a. Determine the quality of the steam exiting the low-pressure turbine.

b. Determine the cycle efficiency.

c. Now assume that the low-pressure turbine is not reversible and the steam exits as a saturated vapor at 45°C

i. Determine the cycle efficiency.
ii. Determine the rate of entropy generation in the low-pressure turbine.





[ans: a) $x_6 = \mathbf{0.9507}$*, b)* $\eta_{th} = \mathbf{38\%}$ *c) i)* $\eta_{th} = \mathbf{34.7\%}$*, ii)* $\dot{S}_{gen,65} = \mathbf{9.275\,\frac{kW}{K}}$*]*

<u>Solution</u>: ideal Rankine Reheat cycle

Define the states

- State 1: saturated liquid at 45°C; h_1 = 188.42 kJ/kg, v_1 = 0.001010 m^3/kg, P_1 = 9.59 kPa
- State 2: compressed liquid at 3000 kPa
- State 3: superheated vapor at 3000 kPa, 600°C; h_3 = 3682.34 kJ/kg, s_3 = 7.5084 kJ/kgK
- State 4: P_4 = 500 kPa, s_4 = s_3 = 7.5084 kJ/kgK (superheated vapor)
  - Interpolation gives: h_4 = 3093.26 kJ/kg, T_4 = 314°C
- State 5: P_5 = 500 kPa, T_5 = 400°C, h_5 = 3271.83 kJ/kg, s_5 = 7.7937 kJ/kgK
- State 6: s_6 = s_5 = 7.7937, P_6 = $P_{sat@45C}$ = 9.59 kPa (saturated mixture),
  - x_6 = (s_6 – s_f)/s_{fg} = (7.7937 – 0.6386) / 7.5261; **x_6 = 0.9507**
  - h_6 = h_f + $x_6 h_{fg}$ = 188.42+0.9507*2394.77, h_6 = 2465.11 kJ/kg

Apply 1st law to each device

- Pump: $\boldsymbol{w_{21} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101\,\frac{m^3}{kg}(3000 - 9.6)kPa = 3.02\ kJ/kg}$
  - h_2 = w_{in} + h_1 = 3.02 kJ/kg + 188.42 kJ/kg = 191.44 kJ/kg
- Boiler: $q_{32} + q_{54} = (h_3 - h_2) + (h_5 - h_4) = (3682.34 - 191.44) + (3271.83 - 3093.26) = 3669.47\,\frac{kJ}{kg}$
  - $\dot{Q}_H = \dot{m}(q_{32} + q_{54}) = 25\,\frac{kg}{s} * 3669.47\,\frac{kJ}{kg} = 91{,}736.75\ kW$
- Hi-P Turbine: $w_{43} = h_3 - h_4 = 3682.34 - 3093.26 = 589.08\,\frac{kJ}{kg}$
- Lo-P Turbine: $w_{65} = h_5 - h_6 = 3271.83 - 2465.11 = 806.72\,\frac{kJ}{kg}$

- $\dot{W}_{net} = \dot{m}(w_{43} + w_{65} - w_{21}) = 34{,}818.50\ kW$
- Condenser: $q_{16} = h_6 - h_1 = 2465.11 - 188.42 = 2276.69 \frac{kJ}{kg}$
- Thermal efficiency: $\boldsymbol{\eta_{th} = \dot{W}_{net}/\dot{Q}_H = 34{,}818.50/91{,}736.75 = 0.38 = 38\%}$
- If low-pressure turbine is not reversible:
  - State 6: $h_{6'}$ = 2583.19 kJ/kg, $s_{6'} = s_{g@45C}$ = 8.1647kJ/kgK
  - Lo-P turbine: $w_{65} = h_5 - h_{6'} = 3271.83 - 2583.19 = 688.64 \frac{kJ}{kg}$
  - $\dot{W}_{net} = \dot{m}(w_{43} + w_{65} - w_{21}) = 31{,}867.50\ kW$
  - $\boldsymbol{\eta_{th} = \dot{W}_{net}/\dot{Q}_H = 0.347 = 34.7\%}$
  - $\boldsymbol{\dot{S}_{gen,65} = \dot{m}(s_6 - s_5) = 25 \frac{kg}{s}(8.1647 - 7.7937)\frac{kJ}{kgK} = 9.275 \frac{kW}{K} 0}$

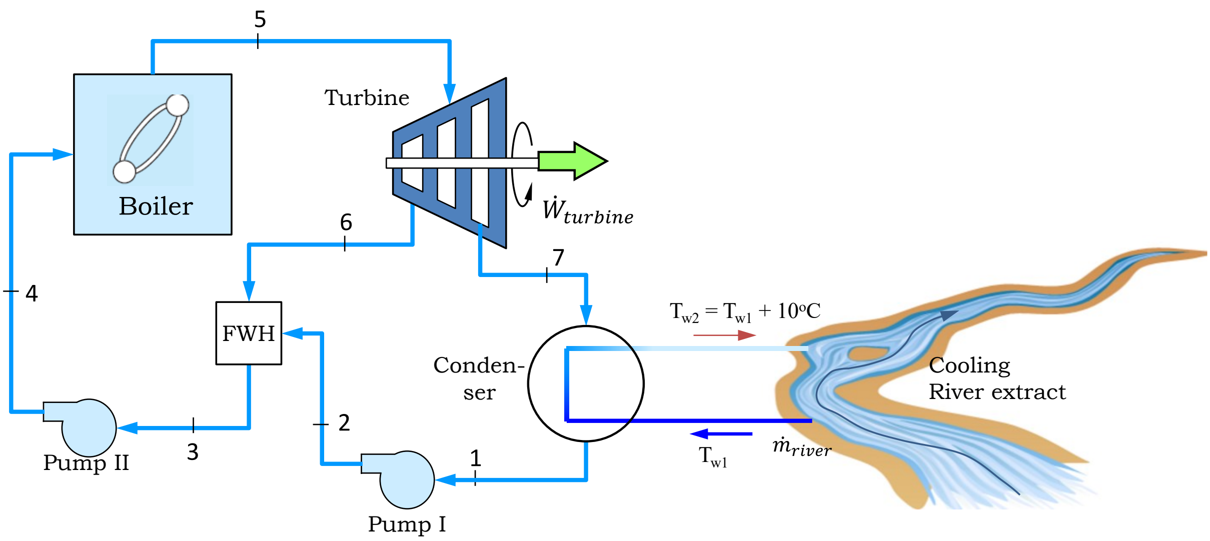
5. An *ideal* steam power plant has high and low working pressures of 20 MPa and 10 kPa. It operates with an open feedwater heater at 1 MPa. The water exiting the FWH is a saturated liquid at 1 MPa. The maximum working temperature is 800°C and the turbine has a total power output of 5 MW.

a. Find the flow rate of steam sent into the FWH.

b. Determine the cycle efficiency.

c. Draw the T-s diagram for this open feedwater heater cycle.

d. To reject the heat in the condenser, a fraction of a river flow is extracted and sent through piping in the condenser in a heat exchanger arrangement. If the government will not allow the heating of the river to exceed 10°C, determine the flow rate of the river allowed to pass through the condenser. Assume the river water to have a constant specific heat of C_P = 4.184 kJ/kgK.



[ans: a) $\dot{m}_6 = 0.60 \frac{kg}{s}$*, b)* $\eta_{th} = 50.3\%$ *d)* $\dot{m}_{river} = 116.16 \frac{kg}{s}$*]*

Solution: ideal Rankine Open Feedwater Heater cycle

Define the states

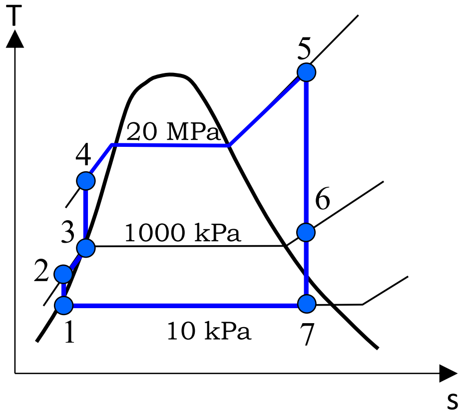
- State 1: saturated liquid at 10 kPa; h_1 = 191.81 kJ/kg, v_1 = 0.001010 m³/kg
- State 2: compressed liquid at 1000 kPa

- State 3: saturated liquid at 1000 kPa; h_3 = 762.79 kJ/kg, s_3 = 2.1386 kJ/kgK
- State 4: compressed liquid at P_4 = 20,000 kPa, $s_4 = s_3$ = 2.1386 kJ/kgK (superheated vapor)
    - Interpolation gives: h_4 = 784.29 kJ/kg, T_4 = 182.55°C
- State 5: P_5 = 20,000 kPa, T_5 = 800°C, h_5 = 4069.80 kJ/kg, s_5 = 7.0544 kJ/kgK
- State 6: $s_6 = s_5$ = 7.0544 kJ/kgK, P_6 = 1000 kPa
    - Interpolation gives: h_6 = 3013.68 kJ/kg, T_6 = 282.74°C
- State 7: P_7 = 10 kPa, $s_7 = s_5$ = 7.0544 kJ/kgK
    - $x_7 = (s_7 - s_f)/s_{fg}$ = (7.0544 – 0.6492) / 7.501; **x_7 = 0.854**
    - $h_7 = h_f + x_7 h_{fg}$ = 191.81+0.854*2392.82, h_7 = 2235.07 kJ/kg

Apply 1st law to each device

- Pump I: $\boldsymbol{w_{21} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101\frac{m^3}{kg}(1000 - 10)kPa = 1.0\ kJ/kg}$
    - $h_2 = w_{in} + h_1$ = 1.0 kJ/kg + 191.81 kJ/kg = 192.81 kJ/kg
- Feed water heater: $\dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3$
    - Conservation of mass: $\dot{m}_6 + \dot{m}_2 = \dot{m}_3$
    - $\dot{m}_2 = \dot{m}_7; \dot{m}_3 = \dot{m}_5;\ \dot{m}_7 = \dot{m}_5 - \dot{m}_6$
    - $\dot{m}_6 h_6 + \dot{m}_7 h_2 = \dot{m}_5 h_3 \rightarrow \dot{m}_6 h_6 + (\dot{m}_5 - \dot{m}_6)h_2 = \dot{m}_5 h_3$
    - $\dot{m}_6(h_6 - h_2) = \dot{m}_5(h_3 - h_2)$
    - $\dot{m}_6 = \frac{\dot{m}_5(h_3 - h_2)}{(h_6 - h_2)} = \dot{m}_5 \frac{762.79 - 192.81}{3013.15 - 192.81} \rightarrow \dot{m}_6 = 0.202\dot{m}_5$
- Turbine: $\dot{W}_{Turbine} = \dot{m}_5 h_5 - \dot{m}_6 h_6 - \dot{m}_7 h_7 = 5000kW$
    - Conservation of mass: $\dot{m}_5 = \dot{m}_6 + \dot{m}_7$
    - $\dot{m}_7 = \dot{m}_5 - \dot{m}_6 = 0.798\dot{m}_5$
    - $5000kW = \dot{m}_5\left(4069.80\frac{kJ}{kg} - 0.202 * 3013.68\frac{kJ}{kg} - 0.798 * 2235.07\frac{kJ}{kg}\right)$
    - $5000kW = \dot{m}_5(1677.45)\frac{kJ}{kg} \rightarrow \dot{m}_5 = 2.98\frac{kg}{s}$
    - $\boldsymbol{\dot{m}_6 = 0.60\frac{kg}{s}}$
    - $\dot{m}_7 = 2.38\ \frac{kg}{s}$
- Boiler: $\dot{Q}_{54} = \dot{m}_5(h_5 - h_4) = (4069.80 - 784.29) = 9793.15\ kW$
- Pump I: $\dot{W}_{21} = \dot{m}_7 w_{21} = 2.38\ kW$
- Pump II: $\dot{W}_{43} = \dot{m}_2(h_4 - h_3) = 2.98\frac{kg}{s}(784.29 - 762.79)\frac{kJ}{kg} = 64.1\ kW$
- Thermal efficiency: $\eta_{th} = \dot{W}_{net}/\dot{Q}_H = (5000 - 2.38 - 64.1)/9793.15$
    - $\eta_{th} = 0.503 = 50.3\%$

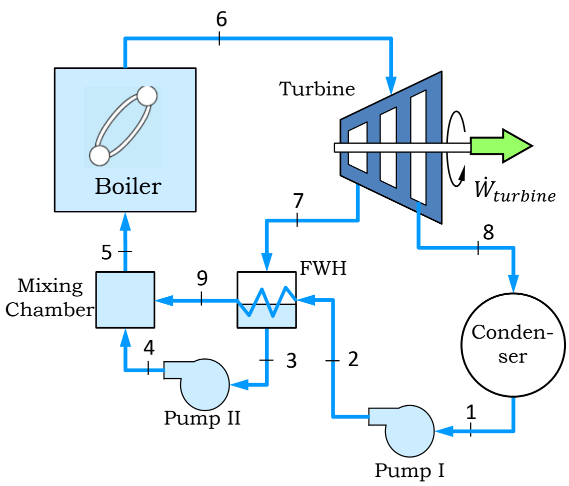
- Condenser: $\dot{Q}_{17} = \dot{m}_7(h_7 - h_1) = 2.38\frac{kg}{s}(2235.07 - 191.81)\frac{kJ}{kg} = 4860.12\ kW$
  - T-s Diagram:

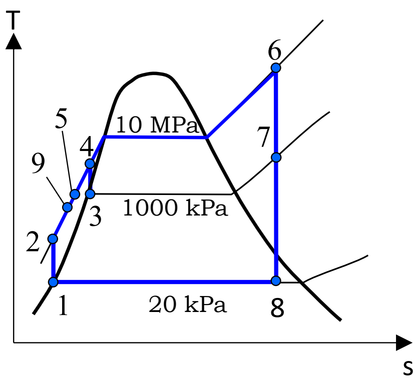


- Condenser heat exchanger with river water
  - $\dot{Q}_{17} = \dot{m}_{river} C_P \Delta T_{water}$
  - $\dot{m}_{river} = \dot{Q}_{17}/(C_P \Delta T_{water}) = 4860.12kW/\left(4.184\frac{kJ}{kgK}10K\right) = 116.16\frac{kg}{s}$

6. A steam power plant operates on an ***ideal*** regenerative Rankine cycle with closed feedwater heater. Steam enters the turbine at 10 MPa and 550°C. Steam is condensed in the condenser at 20 kPa. Steam is extracted from the turbine at 1000 kPa and sent through the closed feedwater heater. This extracted steam leaves the FWH as a saturated liquid at 1000 kPa. The turbine produces 14 MW of power, while the condenser removes 19.5 MW of heat in the condenser.

a. Determine the total mass flow rate of steam through the cycle.
b. Determine the quality of the steam exiting the turbine.
c. Determine the flow rate of steam extracted into the FWH.
d. Determine the cycle efficiency.





[ans: a) $x_8 = 0.837$*, b)* $\dot{m}_{total} = 12.06\frac{kg}{s}$ *c)* $\dot{m}_7 = 2.18\frac{kg}{s}$*, d)* $\eta_{th} = 41.6\%$*]*

Solution: ideal Rankine Closed Feedwater Heater Cycle

Define the states

- State 1: saturated liquid at 20 kPa; h_1 = 251.38 kJ/kg, s_1 = 0.8319 kJ/kgK
- State 2: compressed liquid at 10,000 kPa, $s_2 = s_1$ = 0.8319 kJ/kgK
  - Interpolation gives: h_2 = 261.56 kJ/kg, T_2 = 60.5°C
  - We did not use the expression $w_{21} = h_2 - h_1 = v_1(P_2 - P_1)$ because at such high pressures the specific volume is not constant (remember the above equation is typically suitable for pressures lower than 5 MPa).
- State 3: saturated liquid at 1000 kPa; h_3 = 762.79 kJ/kg, s_3 = 2.1386 kJ/kgK, T_3 = 179.91°C
- State 4: compressed liquid at P_4 = 10,000 kPa, $s_4 = s_3$ = 2.1386 kJ/kgK (superheated vapor)
  - Interpolation gives: h_4 = 861.25 kJ/kg, T_4 = 201.18°C
- State 5: P_5 = 10,000 kPa
  - Additional information requires a formal 1st law analysis of the mixing chamber (described below)
- State 6: P_6 =10,000 kPa, T_6 = 550°C
  - s_6 = 6.7561 kJ/kgK, h_6 = 3500.92 kJ/kg
- State 7: P_7 = 1000 kPa, $s_7 = s_6$ = 6.7561 kJ/kgK
  - Interpolation gives h_7 = 2858.79 kJ/kg, T_7 = 213.48°C
- State 8: P_8 = 20 kPa, $s_8 = s_6$ = 6.7561 kJ/kgK
  - $x_8 = (s_8 - s_f)/s_{fg}$ = (6.7561 – 0.8319) / 7.0766; $\mathbf{x_8 = 0.837}$
  - $h_8 = h_f + x_8 h_{fg}$ = 251.38+0.837*2358.33, h_8 = 2225.66 kJ/kg
- State 9: P_9 = 10,000 kPa
  - Additional information requires a formal 1st law analysis of the feedwater heater (described below)

Apply 1st law to each device

- Condenser: $\dot{Q}_{18} = \dot{m}_8(h_8 - h_1)$
  - $\dot{m}_8 = \dot{Q}_{18}/(h_8 - h_1) = \frac{19{,}500kW}{(2225.66-251.38)kJ/kg}; \dot{m}_8 = 9.88\frac{kg}{s}$
- Turbine: $\dot{W}_{Turbine} = \dot{m}_5 h_6 - \dot{m}_8 h_8 - \dot{m}_7 h_7$
  - Conservation of mass: $\dot{m}_5 = \dot{m}_7 + \dot{m}_8$
  - $\dot{m}_7 = \dot{m}_5 - \dot{m}_8$ (*)
  - Substitute (*) into energy equation:
    - $\dot{W}_{Turbine} = \dot{m}_5 h_6 - \dot{m}_8 h_8 - (\dot{m}_5 - \dot{m}_8)h_7$
  - Solving for $\dot{m}_5$
    - $\boldsymbol{\dot{m}_5 = \left(\dot{W}_{Turbine} + \dot{m}_8(h_8 - h_7)\right)/(h_6 - h_7) = 12.06\frac{kg}{s}}$
  - Solving for mass flow rate through feedwater heater
    - $\boldsymbol{\dot{m}_7 = \dot{m}_5 - \dot{m}_8 = 2.18\frac{kg}{s}}$

- Closed feedwater heater: $\dot{E}_{in} = \dot{E}_{out}$
  - $\dot{m}_7 h_7 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{m}_9 h_9$
  - $\dot{m}_9 = \dot{m}_2 = \dot{m}_8; \dot{m}_3 = \dot{m}_7$
  - $\dot{m}_7(h_7 - h_3) = \dot{m}_8(h_9 - h_2)$
  - Solving for h9: $h_9 = h_2 + \left(\dot{m}_7(h_7 - h_3)\right)/\dot{m}_8 = 725.65 \frac{kJ}{kg}$
  - Interpolating gives: $T_9 = 170.3^oC$
- Mixing chamber: $\dot{E}_{in} = \dot{E}_{out}$
  - $\dot{m}_4 h_4 + \dot{m}_9 h_9 = \dot{m}_5 h_5$
  - $\dot{m}_9 = \dot{m}_8; \dot{m}_4 = \dot{m}_7 \rightarrow \dot{m}_7 h_4 + \dot{m}_8 h_9 = \dot{m}_5 h_5$
  - Solve for h5: $h_5 = \frac{\dot{m}_7}{\dot{m}_5} h_4 + \frac{\dot{m}_8}{\dot{m}_5} h_9 = 734.25 \frac{kJ}{kg}$
  - Interpolating gives: $T_5 = 172.26^oC$
- Boiler: $\dot{Q}_{65} = \dot{m}_5(h_6 - h_5) = 12.06 \frac{kg}{s}(3500.92 - 734.25)\frac{kJ}{kg} = 33{,}377.06\ kW$
- Thermal efficiency: $\eta_{th} = \dot{W}_{net}/\dot{Q}_H = 1 - \dot{Q}_L/\dot{Q}_H$
  - $\eta_{th} = 1 - 19{,}500/33{,}377.06 = 0.416 = 41.6\%$

7. Consider an ideal, air-standard Otto cycle with a compression ratio of 9. Before compression, the air exists at P_1 = 95 kPa, T_1 = 17^{o}C and occupies V_1 = 3.8 Litres. During the constant volume heat addition, 7.5 kJ of heat is transferred to the air. Assume constant specific heats with R_{air} = 0.287 kJ/kgK, C_v = 0.717 kJ/kgK.

a) Determine the maximum temperature and pressure in the cycle.

b) Calculate the thermal efficiency.

c) Calculate the mean effective pressure.

[ans: a) T_3 = 3111.7 K, P_3 = 9169 kPa , b) $\boldsymbol{\eta_{th} = 58.5\%}$*, c) MEP = 1298.4 kPa]*

Solution: ideal Otto cycle

Define each process and find the state variables

- Process 1-2 (isentropic compression): 1st Law: $W_{21} = m(u_2 - u_1) = mC_v(T_2 - T_1)$
  - State 1: P_1 = 95 kPa, T_1 = 290.15K, $v_1 = (RT_1 / P_1)$ = 0.8766 m^3/kg
  - Mass: m = V_1 / v_1 = 0.0038 m^3 / 0.8766 m^3/kg = 0.004335 kg.
  - State 2: Isentropic relations: $T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = T_1 * r^{(k-1)} = 698.7K$, $v_2 = (1/9)v_1$ = 0.0974 m^3/kg; $P_2 = RT_2/v_2$ = 2058.9 kPa
  - $W_{21} = mC_v(T_2 - T_1) = 1.27\ kJ$
- Process 2-3 (constant volume heat addition): $Q_{32} = m(u_3 - u_2) = mC_v(T_3 - T_2)$
  - State 3: $T_3 = \frac{Q_{32}}{mC_v} + T_2$ = 3111.7 K, $P_3 = RT_3/v_3$ = 9169 kPa
- Process 3-4 (isentropic expansion): $W_{43} = m(u_3 - u_4) = mC_v(T_3 - T_4)$

   - State 4: isentropic relations: $T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3 * \left(\frac{1}{r}\right)^{(k-1)} = 1292.11K,$
     $P_4 = RT_4/v_4 = 423.04$ kPa
   - $W_{43} = mC_v(T_3 - T_4) = 5.66\ kJ/kg$
- Process 4-1 (constant volume heat rejection): $Q_{14} = m(u_4 - u_1) = mC_v(T_4 - T_1)$
   - $Q_{14} = 3.11\ kJ/kg$
- Thermal efficiency: $\eta_{th,Otto} = \frac{W_{net}}{Q_H} = \frac{W_{43} - W_{21}}{Q_{32}} = 0.585 = 58.5\%$
- $MEP = \frac{W_{net}}{V_2 - V_1} = 1299.65kPa$

8. A gasoline engine operates on the ideal Otto cycle. Before compression, the air exists at P_1 = 90 kPa and T_1 = 290K. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050K. Assume constant specific heats with R_{air} = 0.287 kJ/kgK and C_v = 0.717 kJ/kgK.

a) Determine the compression ratio.

b) Determine the highest pressure in the cycle.

c) Determine the exhaust temperature (T_4).

d) Determine the specific net work output in kJ/kg.

[ans: a) r = 7.67, b) P_3 = 4883 kPa, c) T_4 = 907.2K, d) w_{net} = 557.5 kJ/kg]

Solution: ideal Otto cycle

Define each process and find the state variables

- State 1: P_1 = 90 kPa, T_1 = 290K, $v_1 = (RT_1 / P_1) = 0.9248$ m³/kg
- Process 2-3 (constant volume heat addition): $q_{32} = u_3 - u_2 = C_v(T_3 - T_2)$
   - $T_2 = T_3 - \frac{q_{32}}{C_v} = 655.3K$
   - Isentropic relations (process 1-2): $P_2 = P_1\left(\frac{T_2}{T_1}\right)^{k/k-1} = 1560.95\ kPa$
   - Ideal Gas: $v_2 = RT_2 / P_2 = 0.1205$ m³/kg
   - Compression ratio: $r = v_1 / v_2 = 7.67$
- State 3: $v_3 = v_2$, $P_3 = RT_3/v_3 = 4883.2$ kPa
- State 4: isentropic relations (process 3-4)
   - $T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = 907.2K$
- Process 1-2: $w_{21} = C_v(T_2 - T_1) = 261.9\ kJ/kg$
- Process 3-4: $w_{34} = C_v(T_3 - T_4) = 819.4\ kJ/kg$
- $w_{net} = w_{34} - w_{21} = 557.5\ kJ/kg$