Tutorial 2 – SOLUTIONS

Tutorial 2: Properties of Pure Substances & First Law Analysis

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Conceptual Questions:

1. Water in a kettle is heated at constant pressure. Describe the immediate changes in phase, temperature, and volume if the initial water is:

(a) saturated vapor
- The phase will change to superheated vapor and the temperature and volume will increase rapidly.

(b) compressed liquid
- The phase will remain a compressed liquid and the temperature and volume will increase

(c) saturated liquid
- The liquid will evaporate and produce water vapor giving a saturated liquid-vapor mixture. The temperature will remain constant until all liquid has evaporated, but the volume will increase.

(d) superheated vapor
- The phase will remain a superheated vapor, but the temperature and volume will increase rapidly.

2. Why are temperature and pressure dependent properties in the saturated mixture region?

Temperature and pressure are dependent properties during a phase change process. This is because both temperature and pressure represent the energy of the system. If either pressure or temperature increases, the energy stored in the system increased as well. This means that the latent heat needed to change the phase of matter from liquid to vapour will be smaller.

3. Is it possible to have water vapour at 0°C? If so, how?

Yes, if the pressure is lower than the triple point (e.g. below 0.61 kPa at 0.01°C)

4. Describe the variables in the relationship: $u = u_f + xu_{fg}$. Particularly, what do the subscripts indicate? Where does one find these properties for a given substance?

Solution:
- u = average internal energy (of a saturated mixture of liquid and vapour)
- u_f = internal energy of a saturated liquid; subscript "f" represents the sat. liq.
- u_g = internal energy of a saturated vapour; subscript "g" represents the sat. vap.
- u_{fg} = difference between internal energy of saturated liquid and sat. vap.
- x = quality or dryness fraction (defined as mass of sat. vap. divided by total mass of substance)

– Properties for water can be found in sat. water and stream tables (i.e. tables1 or 2)

Problem Solving Questions:

5. (a) A sealed, rigid container holds 2kg of nitrogen at 101.3 kPa (1.013 bar) and 30°C. What is the volume of the container and how many moles of nitrogen are present? What is the molar mass of nitrogen? (R_{N2} = 0.2968 kJ/kg K and R_u = 8.314 kJ/kmol K).

Solution:

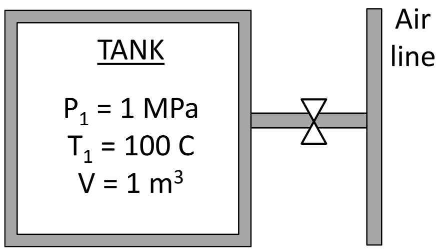
- m_1 = 2kg, P_1 = 101.3 kPa, T_1 = 30°C → (30+273.15)K = 303.15K
- Find Volume:
    - PV = mRT → V = mRT/P = (2kg) * (0.2968 kJ/kgK) * (303.15K) / (101.3 kPa)
    - V = 1.78 m^3
- Find number of moles: different approaches can be used, but one option is:
    - PV = NR_uT
    - N = PV / (R_uT) = (101.3 kPa) * (1.78 m^3) / ((8.314 kJ/kmolK) * (303.15K))
    - N = 0.072 kmol
- Find molar mass
    - m = N*M → M = m / N = (2 kg) / (0.072 kmol); M = 28 kg / kmol

(b) The container is heated so that the temperature of nitrogen increases by 100°C. Assuming ideal gas, what is the change in pressure in the container***?***

Solution:

- Combined ideal gas law: $P_1V_1 = m_1RT_1$ & $P_2V_2 = m_2RT_2$
- Mass & volume are constant, thus V_1 / mR = (T_1 / P_1) = (T_2 / P_2)
- T_2 = T_1 + (100°C) = 403.15 K =
- Solve for P_2:
    - P_2 = (P_1 * (T_2 / T_1) = 101.3 kPa * (403.15K / 303.15K) = 134.7 kPa
    - $P_2 - P_1$ = (134.7 – 101.3) kPa = 33.4 kPa

6. A 1-m^3 rigid tank with air at 1 MPa and 100°C is connected to an air-line. A valve is opened and allows additional air to enter the tank until the pressure reaches 5 MPa. The valve is then closed and the temperature inside the tank is 160°C. Assume air as an ideal gas with R = 0.287 kJ/kgK.



(a) What is the mass of air inside the tank before and after the process?

(b) Heat is transferred from the tank to the surroundings and cools the air in the tank down to room temperature (25°C). What is the pressure inside the tank afterward?

Solution:

(a) P_1 = 1000 kPa, T_1 = 373.15 K, V_1 = 1 m^3 = V_2; P_2 = 5000 kPa, T_2 = 433.15 K

- PV = mRT
- $m_1 = P_1V/(RT_1)$ = 1000 kPa * 1m^3 / (0.287 kJ/kgK * 373.15K) = 9.34 kg
- $m_2 = P_2V/(RT_2)$ = 5000 kPa * 1m^3 / (0.287 kJ/kgK * 433.15K) = 40.2 kg
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(b) Find P_3:

- PV = mRT; T_3 = 298.15 K, V_3 = 1 m^3, m = 40.2 kg
- $P_3 = mRT_3/V$ = 40.2 kg * 0.287 kJ/kgK * 298.15K / 1 m^3 = 3439 kPa or 3.44 MPa

7. Complete the following table for the following property substances. Use tables in back of book for various substances

	Substance	***P [kPa]***	***T [°C]***	***Quality, x***	***v [m³/kg]***	***u [kJ/kg]***	***h [kJ/kg]***	***Phase***
A	R-134a	294	0	0.6	0.0418	306.71	319.01	Sat. liq.-vap.
B	R-134a	572.8	20	1	0.0361	389.19	409.84	Sat. vapour
C	R-134a	172.3	-10	-	0.1200	372.9	393.09	Superheat vapor
D	Ammonia[1]	1000	200	-	0.2267	1683.7	1910.4	Superheat vapor
E	Ammonia[1]	190.2	-20	0.75	0.4679	997	1085.8	Sat. liq.-vap
F	Nitrogen[1]	500	100 K	-	0.0555	67.85	94.36	Superheat Vapor
G	CO_2[1]	1000	-5	-	0.0468	309.6	356.4	Superheat Vapor

[1] Tables can be found in 'Fundamentals of Thermodynamics (8th Ed.)' by Borgnakke and Sonntag

8. A 0.15 m^3 rigid tank contains 2 kg of water at 150 kPa (1.5 bar). Determine the

a) Temperature
b) Enthalpy
c) Mass of each phase of water

Solution:

a) v = V/m = 0.15 m^3/ 2kg = 0.075m^3/kg

- $v_f < v < v_g$, thus the phase is sat. liq.-vap.
- Temperature will be the saturation temperature at 150 kPa; T = 111.37°C

b) Find quality: $x = (v – v_f) / (v_g – v_f)$; v_f = 0.001053 m^3/kg, v_g = 1.15933 m^3/kg;

- x = (0.075 – 0.001053) / (1.15933-0.001053); x = 0.064
- Find enthalpy: $h = h_f + x * (h_g-h_f) = h_f + x*h_{fg}$; h_f = 467.08 kJ/kg, h_{fg} = 2226.46 kJ/kg
- h = 467.08 + 0.064*2226.46 kJ/kg; h = 609.6 kJ/kg

c) $m_f =(1-x)m_{total}$ = (1-0.064) * 2 kg = m_f = 1.872 kg

- $m_g = x*m_{total}$ = 0.064 * 2 kg = m_g = 0.128 kg.

9. Find the internal energy of water at the given states for 7 MPa (70 bar) and plot the states on T-v, P-v diagrams. Show where the states lie with respect to the saturated liquid-vapor dome. (Note: rough estimations of state location on the diagrams are expected if exact numerical values on axis are not provided).

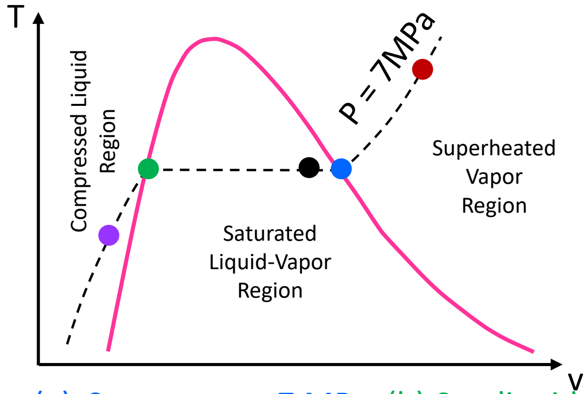
a) Saturated vapor
b) Saturated liquid
c) T = 700°C
d) T = 100°C
e) Quality (x) = 95%

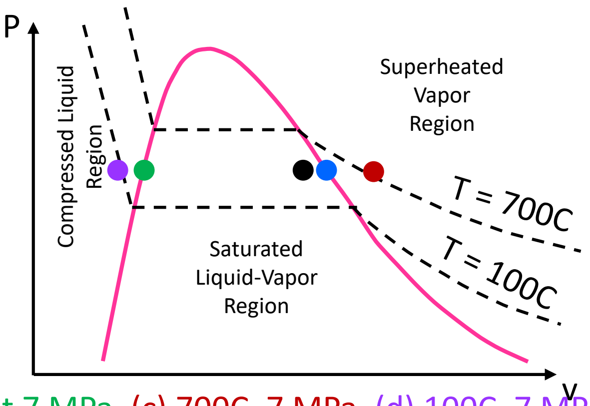
Solution:

a) $u = u_{g@7000\ kPa}$ = 2580.48 kJ/kg
b) $u = u_{f@7000\ kPa}$ = 1257.51 kJ/kg
c) $T_{sat@7000\ kPa}$ = 285.8°C. Since 700°C > T_{sat}, the state is superheated
- From superheated tables, and interpolation: $u_{@6000kPa,\ 700C}$ = 3453.15 kJ/kg, $u_{@8000kPa,\ 700C}$ = 3444.00 kJ/kg
- $\frac{u_{7000kPa}-u_{6000kPa}}{P_{7000kPa}-P_{6000kPa}} = \frac{u_{8000kPa}-u_{6000kPa}}{P_{8000kPa}-P_{6000kPa}}$
- $u_{7000kPa} = u_{6000kPa} + \frac{P_{7000kPa}-P_{6000kPa}}{P_{8000kPa}-P_{6000kPa}} * (u_{8000kPa} - u_{6000kPa})$
- $u_{@7000kPa,\ 700C}$ = 3453.15 + (1/2) * (3444.00-3453.15) kJ/kg = 3448.6 kJ/kg

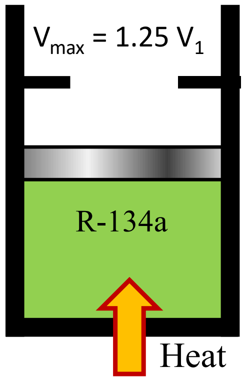
d) $P_{sat@100^oC}$ = 101.3 kPa. Since P > P_{sat}, the state is a compressed liquid.
- Use compressed liquid tables and will need interpolation; $u_{@5000kPa,\ 100C}$ = 417.50 kJ/kg, $u_{@10000kPa,\ 100C}$ = 416.09 kJ/kg
- $\frac{u_{7000kPa}-u_{5000kPa}}{P_{7000kPa}-P_{5000kPa}} = \frac{u_{10000kPa}-u_{5000kPa}}{P_{10000kPa}-P_{5000kPa}}$
- $u_{7000kPa} = u_{5000kPa} + \frac{P_{7000kPa}-P_{5000kPa}}{P_{10000kPa}-P_{5000kPa}} * (u_{10000kPa} - u_{5000kPa})$
- $u_{@7000kPa,\ 700C}$ = 417.5 + 2/5 * (416.09-417.5) kJ/kg = 416.94 kJ/kg

e) Saturated liq.-vap. $u = u_f + x*(u_g-u_f)$.
- $u_{f@7000kPa}$ = 1257.51 kJ/kg, $u_{g@7000kPa}$ = 2580.48 kJ/kg, $u_{fg@7000kPa}$ = 1322.97 kJ/kg
- u = (1257.51 + 0.95 * 1322.97) kJ/kg = 2514.33 kJ/kg





(a) Sat. vapor at 7 MPa, (b) Sat. liquid at 7 MPa, (c) 700C, 7 MPa, (d) 100C, 7 MP
(e) Sat. liquid-vapor mixture at 7 MPa with x = 0.95%



10. A piston/cylinder device with initial volume of 0.5 L contains refrigerant R134a at -10°C, 200 kPa. Stops are placed so that the cylinder can expand an additional 25% of the initial volume. Heat is transferred to the cylinder until the refrigerant temperature reaches 30°C.
 (a) Is the piston at the stops in the final state?
 (b) What is the work done during this process?
 (c) What is the heat transfer during the process?
 (d) If the diameter of the piston is 0.04 m atmospheric pressure of 101.3 kPa is acting on the piston, what is the mass of the piston in kg?

Solution:

- System: refrigerant in cylinder
- Determine State 1 properties
  - T_1 = -10°C, P_1 = 200 kPa → superheated vapor
  - v_1 = 0.10013 m^3/kg, u_1 = 372.31 kg/kg (Table B.5.2)
- Determine mass of refrigerant
  - $m_1 = V_1 / v_1$ = 0.0005 m^3 / (0.10013 m^3/kg) = 0.00499 kg
  - Mass is fixed during process 1-2
- Determine whether piston hits stops before reaching 30°C
  - Remember piston will rise under a constant pressure process (force of piston remains constant) until the piston reaches the stops at which point the volume will no longer increase, but pressure will increase.
  - Determine the maximum allowable volume in the cylinder
    - Specific volume: $v_{max} = (v_1 * 1.25)$ = 0.1252 m^3/kg
    - Volume: $V_{max} = (V_1 * 1.25)$ = 0.00625 m^3
  - Under a constant pressure process (200 kPa), determine if this volume is reached before the temperature reaches 30°C.

        - In superheated table at 200 kPa, determine what scenario occurs first:
            - Specific volume reaches 0.1252 m^3/kg before T = 30^{o}C → piston will reach stops and then pressure & temperature will increase
            - Temperature reaches 30^{o}C before v = 0.1252 m^3/kg → volume will not reach maximum before final temperature is reached and pressure will remain constant
        - Answer: Temperature reaches 30^{o}C before v = 0.1252 m^3/kg
    - Determine the final volume for 30^{o}C, 200 kPa: v_2 = 0.11889 m^3/kg
        - $V_2 = v_2 * m$ = 0.11889 m^3/kg * 0.00499kg = 0.000593 m^3 < V_{max}
    - <u>ANSWER: piston stops before reaching stops</u>

- Determine the boundary work
    - $W_b = \int_1^2 PdV = P(V_2 - V_1)$ $(i.e. constant\ pressure\ process)$
    - $\underline{\boldsymbol{W_b = 200kPa * (0.000593 - 0.0005)m^3 = 0.0186kJ\ \ OR\ \ 18.6J}}$
- Determine heat transfer: apply first law of thermodynamics
    - $\Delta U + \Delta KE + \Delta PE = Q_{21} - W_{21} + \Delta E_{mass}$
    - Stationary, closed system gives: $\Delta U = Q_{21} - W_{21}$
    - $Q_{21} = m(u_2 - u_1) + W_{21}$
    - u_2 = 403.1 kJ/kg (Table B.5.2; R134a: 30C, 200kPa)
    - $\underline{\boldsymbol{Q_{21} = 0.00499kg(403.71 - 372.31)\frac{kJ}{kg} + 0.0186kJ = 0.172kJ}}$
- Determine the mass of the piston
    - FBD gives: $P_{cyl} * A = P_o * A + m_{piston} * g$
    - $m_{piston} = \pi * d^2/4\ (P_{cyl} - P_o) / g$
    - Convert pressure into Pa
    - $m_{piston} = \pi*(0.04\ m)^2/4\ (200{,}000 - 101{,}300) / 9.81\ m/s^2$
    - <u>m_{piston} = 12.6 kg</u>