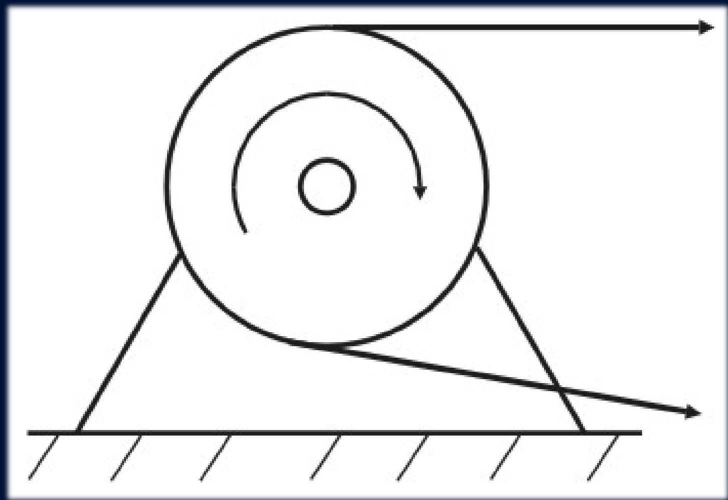
Dynamics 2

Fixed Axis Rotation & Moment of Inertia
(Dynamics of Systems of Bodies)

Fixed Axis Rotation

- examples
  - pulley

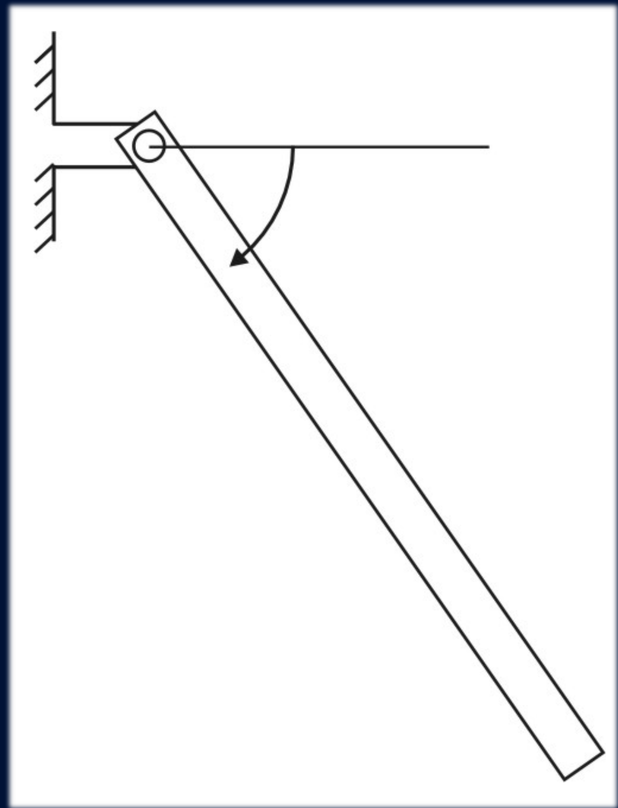


Fixed Axis Rotation

- examples
  - pulley

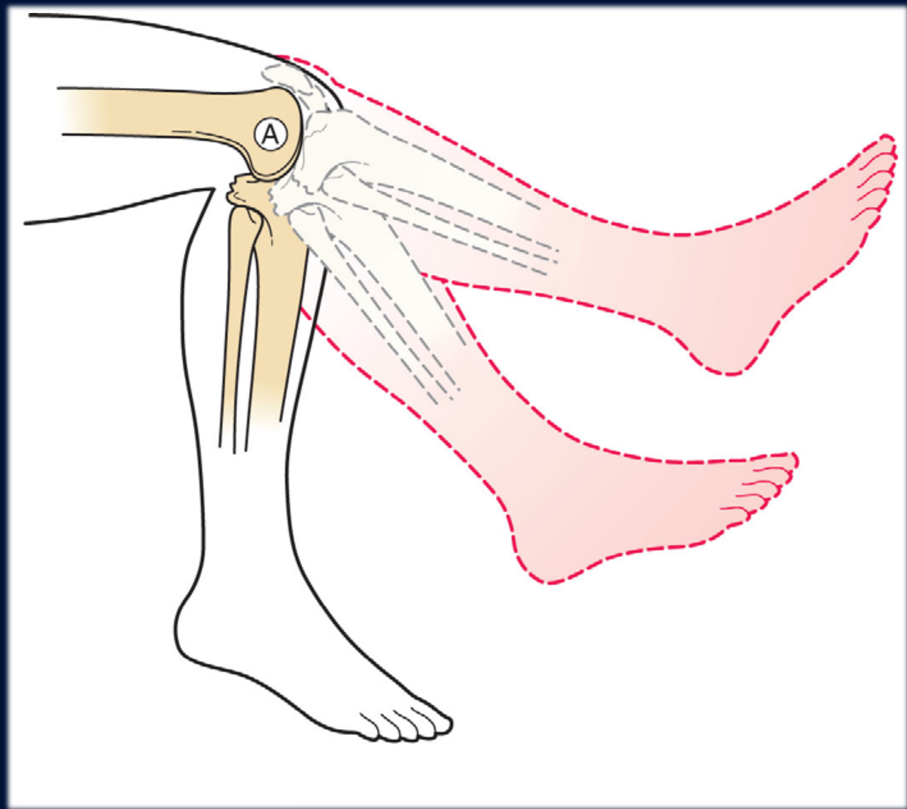
Fixed Axis Rotation

- examples
  - pulley
  - hinged bar



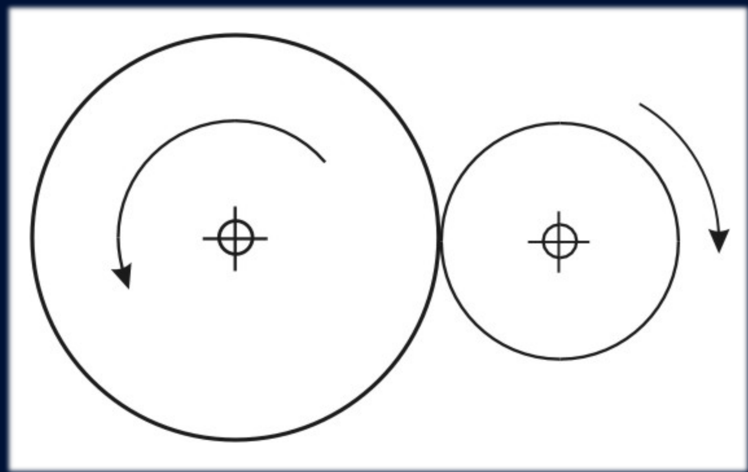
Fixed Axis Rotation

- examples
  - pulley
  - hinged bar



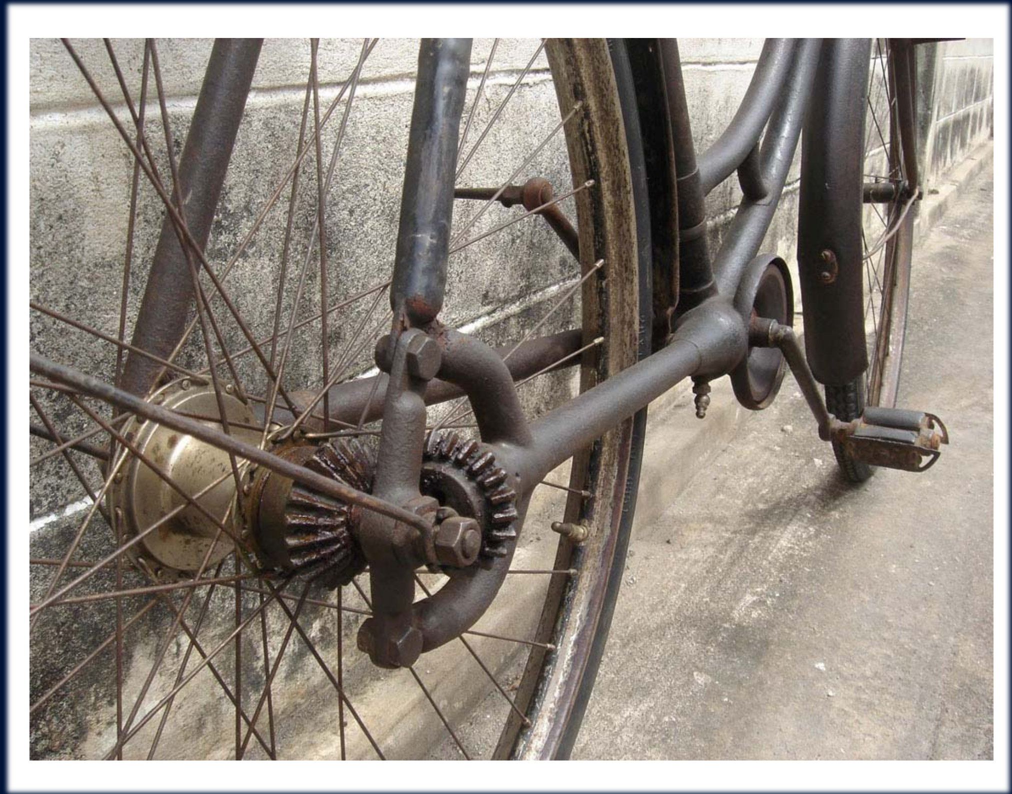
Fixed Axis Rotation

- examples
  - pulley
  - hinged bar
  - meshed gears



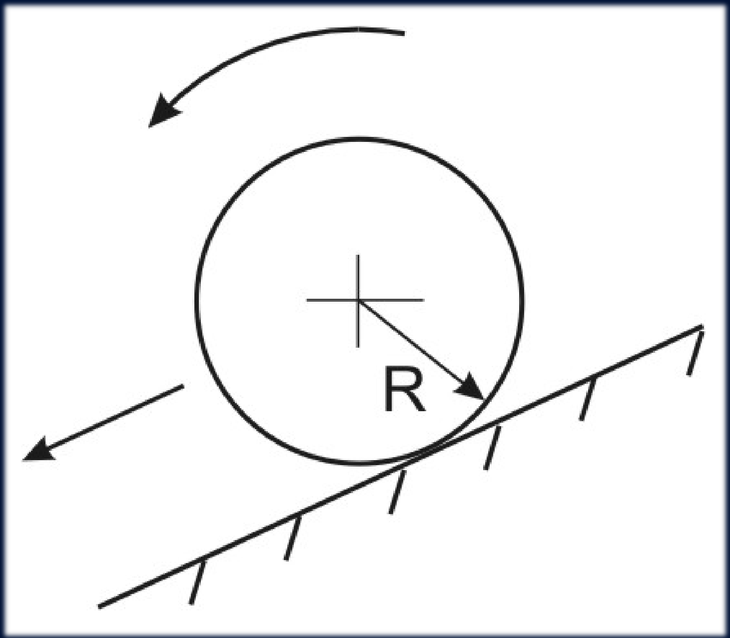
Fixed Axis Rotation

- examples
  - pulley
  - hinged bar
  - meshed gears



Fixed Axis Rotation

- examples
  - pulley
  - hinged bar
  - meshed gears
- **not** fixed axis rotation
  - rolling wheel

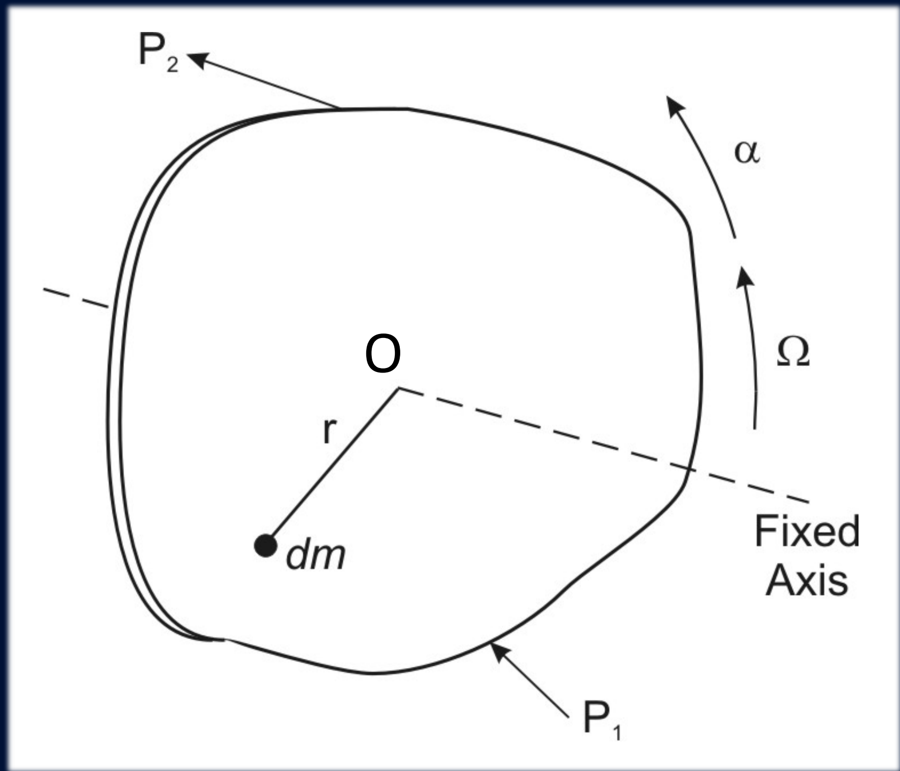


Properties of Bodies in FAR

- all mass particles move in circular paths about the fixed axis (through point O)

- O is not necessarily G

- body motion is angular
  - angular velocity
  - angular acceleration



Laws of Motion for FAR

- Law of Fixed Axis Rotational Motion

  "the sum of the moments of the external forces about the fixed axis in the direction of the angular acceleration = (mass moment of inertia about the axis) × (angular acceleration of the body)"

- or

$$\sum \text{moments about point O} = I_O\,\alpha$$

Laws of Motion for FAR

- we can re-write this law in d'Alembert form

$$\sum \text{moments about point O} + (-\, I_O\, \alpha) = 0$$

- ie the sum of the moments is zero if FBD includes an "inertia couple", $I_O\, \alpha$

  - acting in opposite direction to α

Laws of Motion for FAR

- GN2 applies

  ie "the sum of the external forces and the system inertia force = 0"

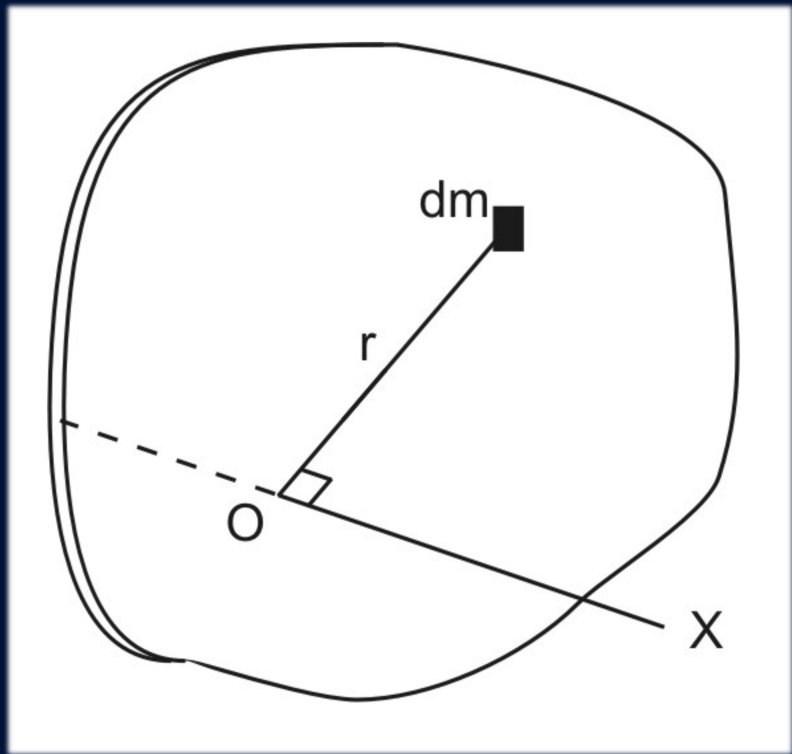
- external forces include forces at the pivot point
- system inertia force is (total mass × acceleration of G) in the opposite direction to a_G
  - and is zero if the fixed axis passes through G
- important for pivot bearing loads and rotating unbalance

Moment of Inertia of Body

- the mass moment of inertia for the body about X axis

$$I_X = \sum r_j^2 dm_j = \int_{BODY} r^2 dm$$

- ie mass moment of inertia of the individual particles summed over the body



Moment of Inertia of Body

- the mass moment of inertia for the body about X axis

$$I_o = \sum r_j^2 dm_j = \int_{BODY} r^2 dm$$

  - i.e. mass moment of inertia of the individual particles summed over the body
- a property of the mass distribution of the body about a specified axis through a specified point [+ve]
- *r* is the perpendicular distance from the particle to the axis
- units are kgm^2
- calculation for simple shapes by integration

MMI Examples

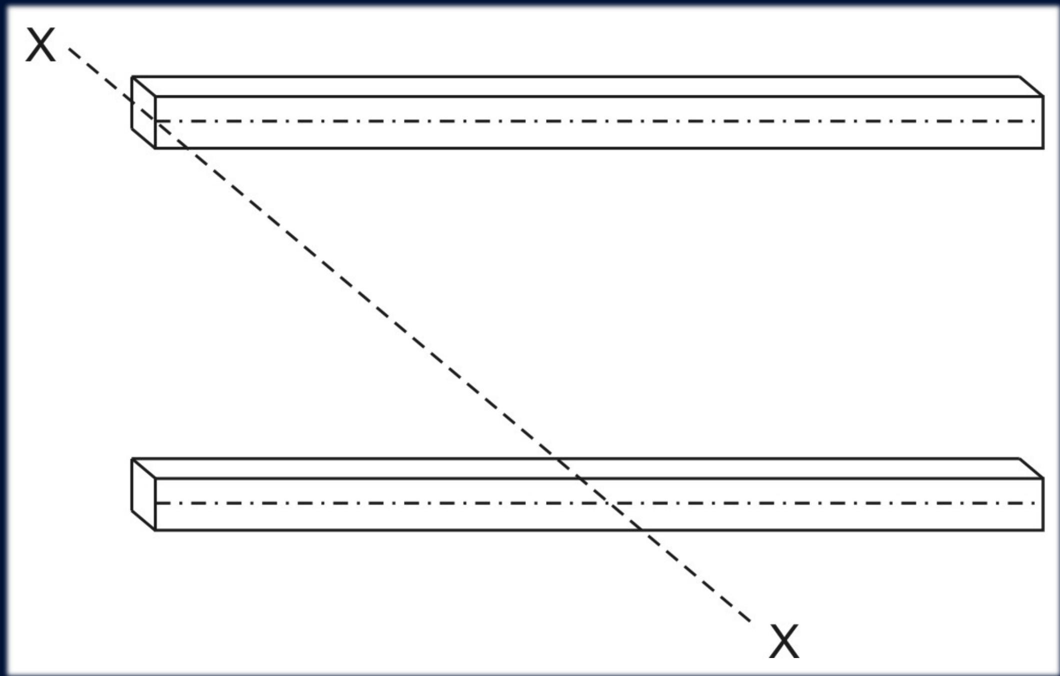
solid bars [mass m, length l]

- axis X through
  - the end

$$I_X = \frac{1}{3}ML^2$$

  - the centre

$$I_X = \frac{1}{12}ML^2$$



MMI Examples

solid bars [mass m, length l]

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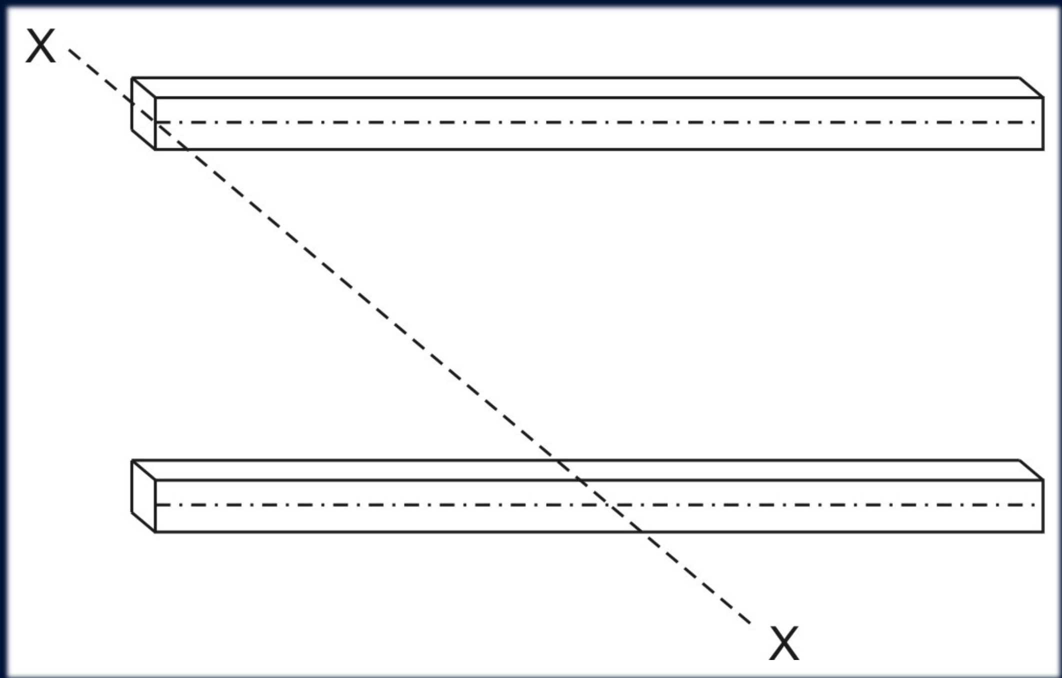

Summary

- Fixed axis rotation of rigid bodies
- Mass Moment of Inertia (MMI)

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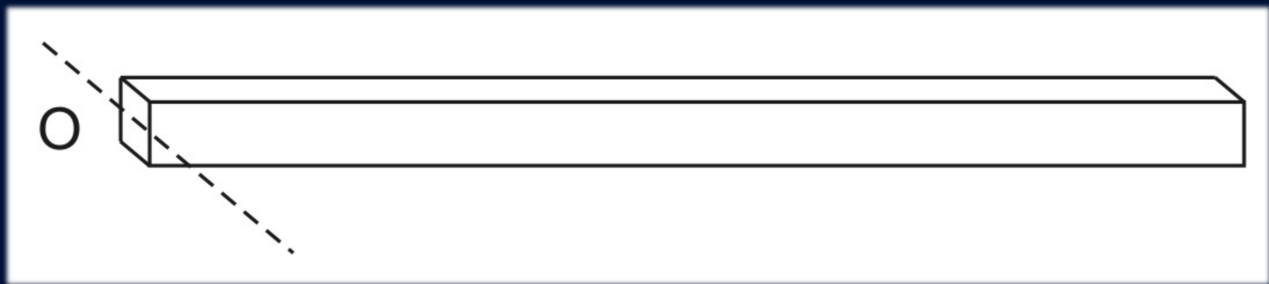
Worked Examples of Moment of Inertia

Example 2.7 – MMI by Integration

Calculate the mass moment of inertia of a uniform bar of length *l*, about an axis through one end.

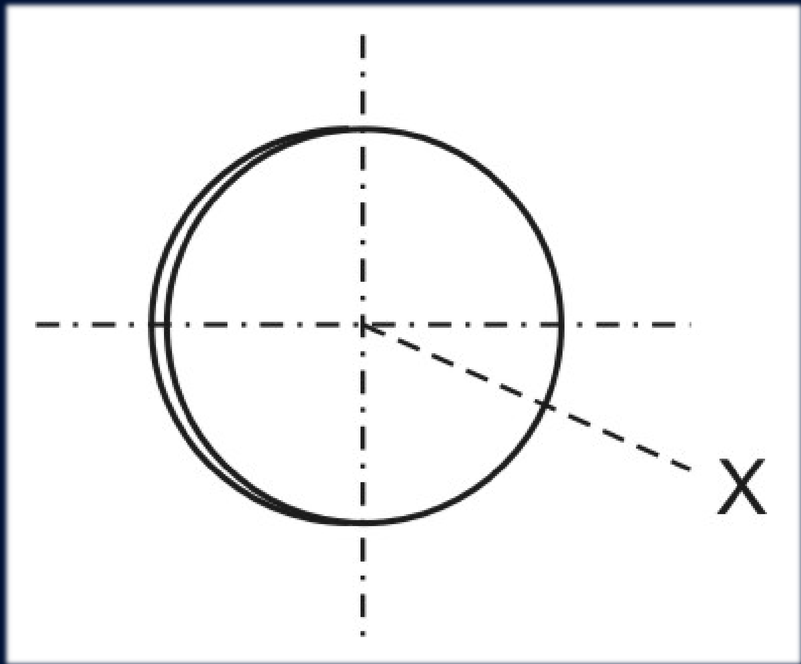


MMI Examples

- uniform disc
  - mass M, radius R

- axis X through centre

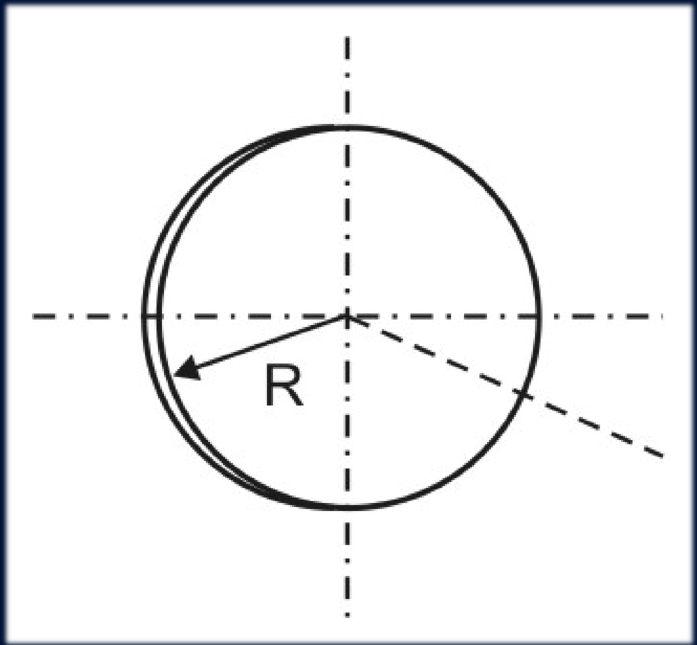
$$I_X = \frac{1}{2} M R^2$$



Example 2.6 – MMI by Integration

Uniform circular disc radius R with axis through centre perpendicular to disc

- Derive disc's moment of inertia from first principles



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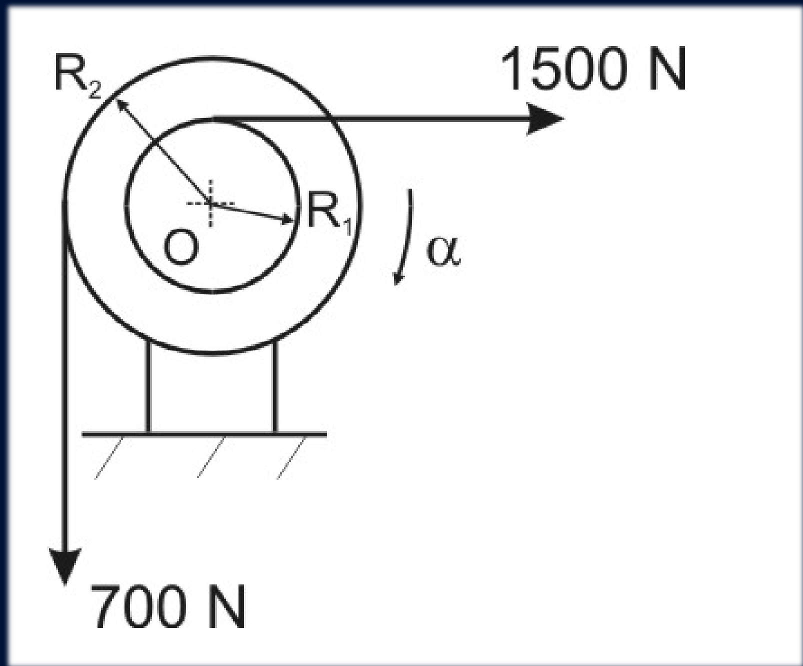
Worked Examples of Fixed Axis Rotation

Example 2.4

- Calculate the angular acceleration of wheel
    - where $I_O = 5\ kgm^2$, $R_1 = 0.3\ m$, $R_2 = 0.6\ m$



Example 2.4

- Calculate the angular acceleration of wheel
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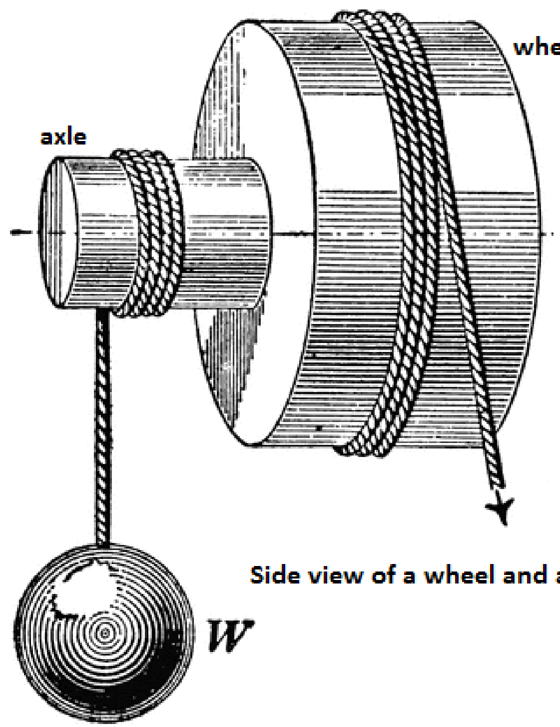

Side view of a wheel and axle.

Example 2.5

- Double pulley loaded by 2 masses
- If the masses are released from rest, determine
  - pulley angular acceleration
  - accelerations of the masses
- Data
  - Pulley moment of inertia about its central axis is 22 kgm^2
  - M_1 = 24 kg, M_2 = 50 kg
  - R_1 = 0.45 m, R_2 = 0.32m