

Lecture 21

Topic 4

Power & Refrigeration Cycles

Topic

- 4.5 Brayton Cycle pt 2

Reading:

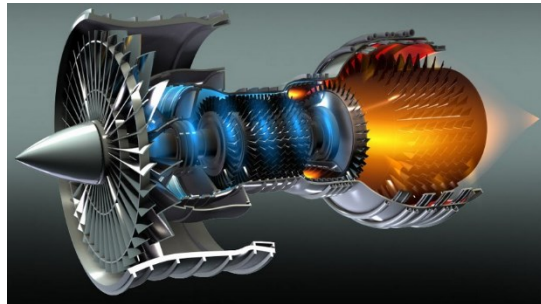
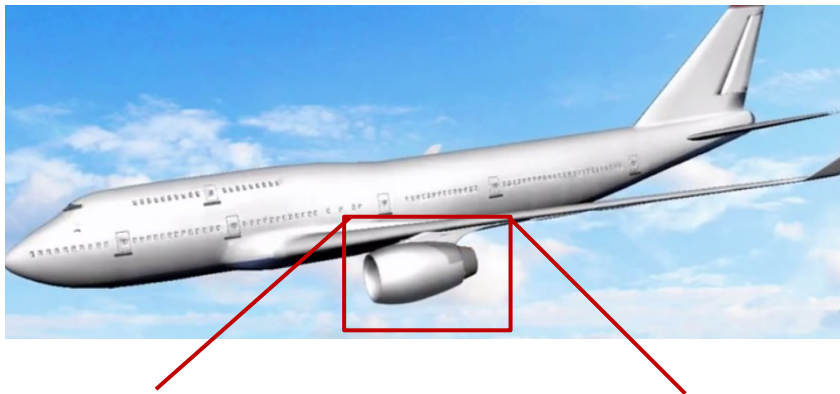
Ch 10: 10.1 – 10.5 Borgnakke & Sonntag Ed. 8

Ch 9: 9-9 – 9-12 Cengel and Boles Ed. 7

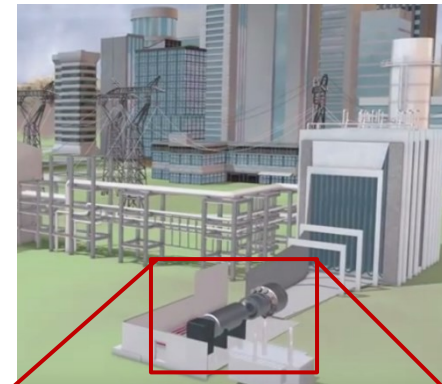
4.5 Brayton Cycle

Brayton Cycle – transfer of heat to useful work out (e.g. electricity to grid or propel an aircraft).

- Air standard ideal cycle approximation for gas-turbine engine



Aircraft: Power → Thrust

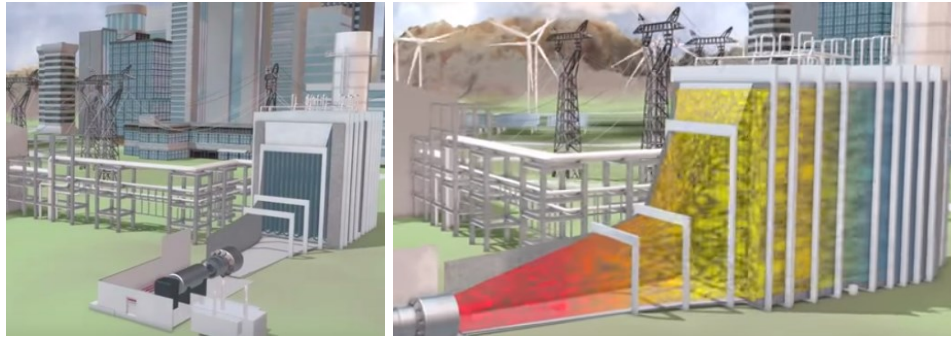


Power Plant: Power → Electricity

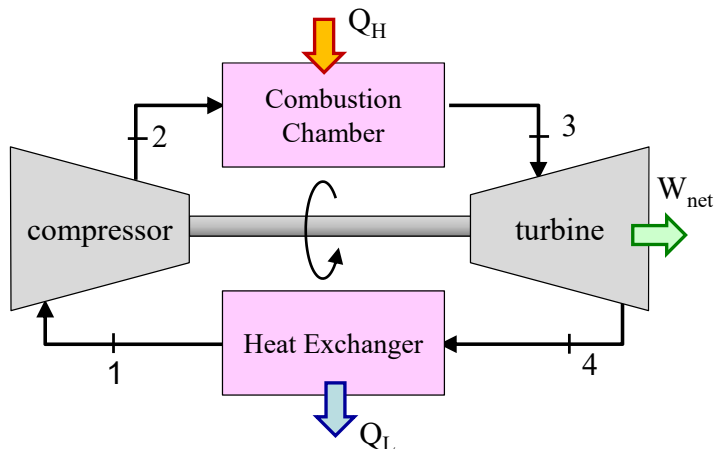
4.5 Brayton Cycle

Brayton Cycle – gas-turbine engine air power plant

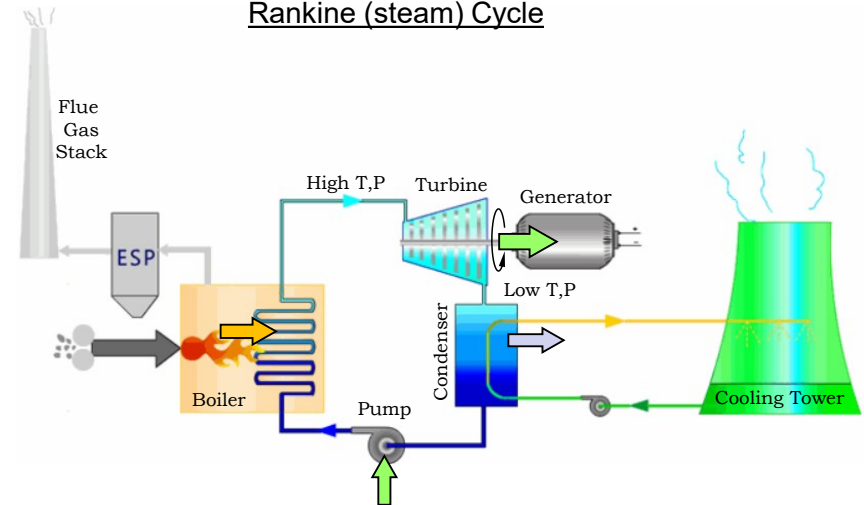
Combined (Dual) Cycle Power Plant: Power → Electricity



Closed Brayton Cycle



Rankine (steam) Cycle



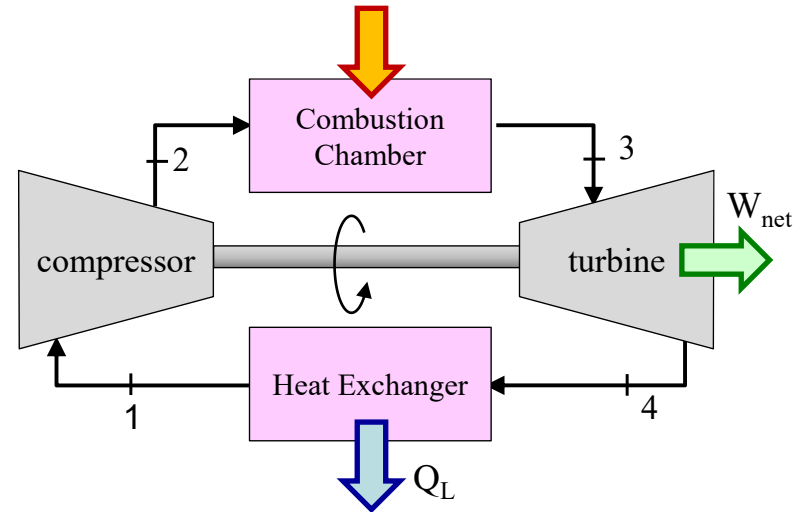
4.5 Brayton Cycle - Basics

Process 1-2: air enters the compressor where; P & T increase.

Process 2-3: high- P air enters combustion chamber. Heat addition at constant pressure.

Process 3-4: high- T gases enter the turbine and expand to atmospheric pressure while producing power.

Process 4-1: Closed: exhaust gases (modelled as air) enter heat exchanger where heat is lost. Air returns back to state 1.



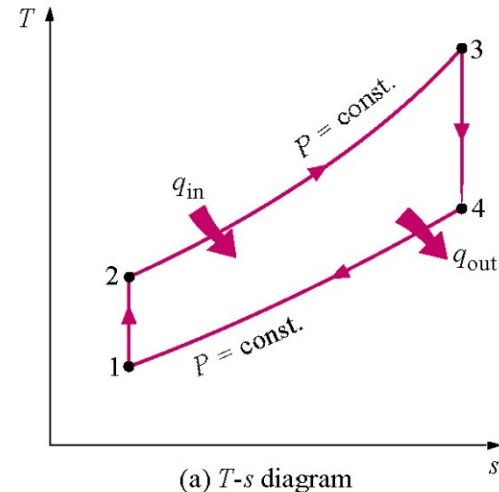
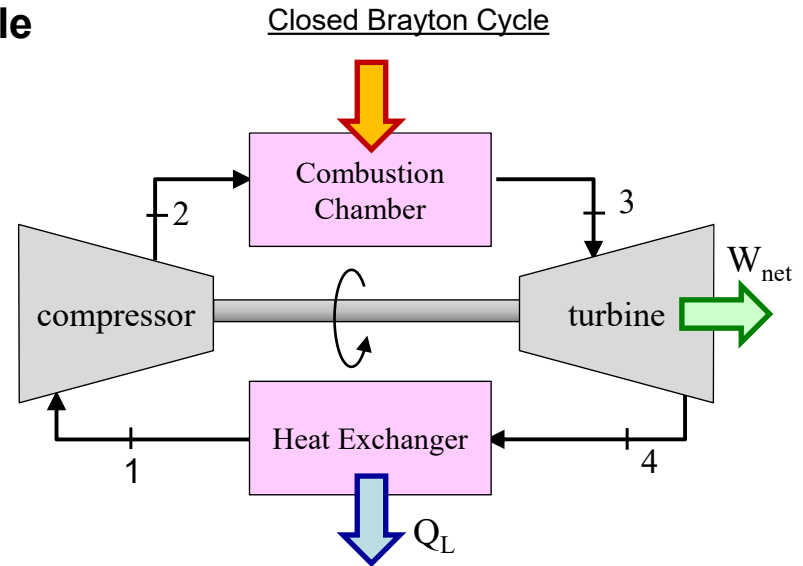
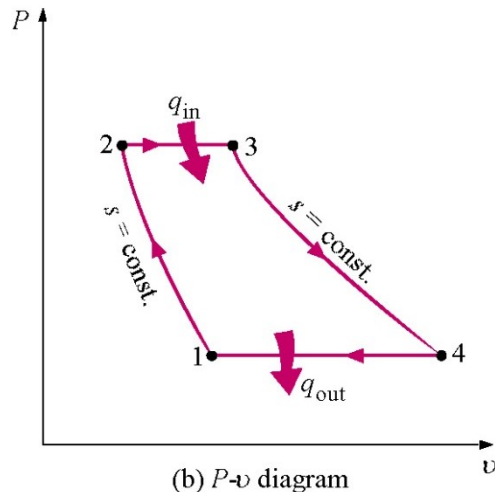
4.5 IDEAL Brayton Cycle

Processes in an air-standard closed Brayton cycle

- Working fluid is air (ideal gas)
- Closed cycle is represented to provide a closed loop in the P-v and T-s diagrams

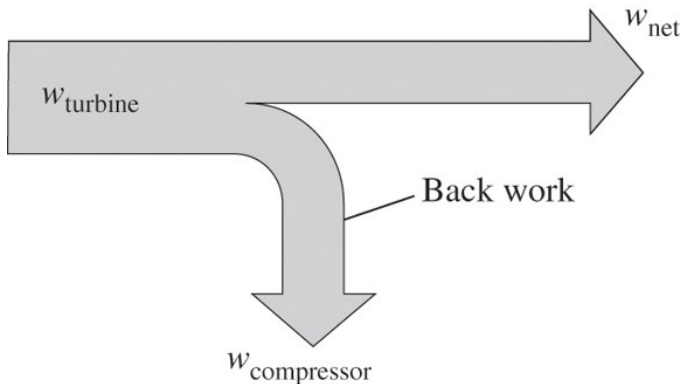
Process Description

1-2	Isentropic compression
2-3	Constant pressure heat addition
3-4	Isentropic expansion
4-1	Constant pressure heat rejection

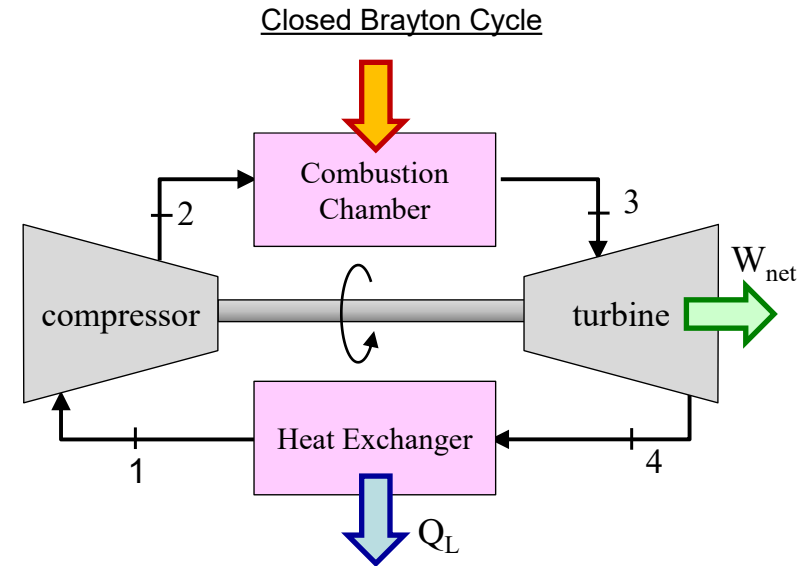


4.5 Brayton Cycle - Basics

- Turbine is connected to compressor via a shaft.
- A fraction of turbine work is used to power the compressor.
- Ratio of compressor to turbine work is called the back work ratio.



$$BWR = \frac{w_{compressor}}{w_{turbine}}$$



- For open Brayton cycle (jet engine):
 $BWR = 1$
 - 100% turbine work powers compressor
- For closed Brayton cycle: $BWR < 1$

4.5 Brayton Cycle – Basic Equations

Ideal Brayton cycle

- Air treated as an ideal gas

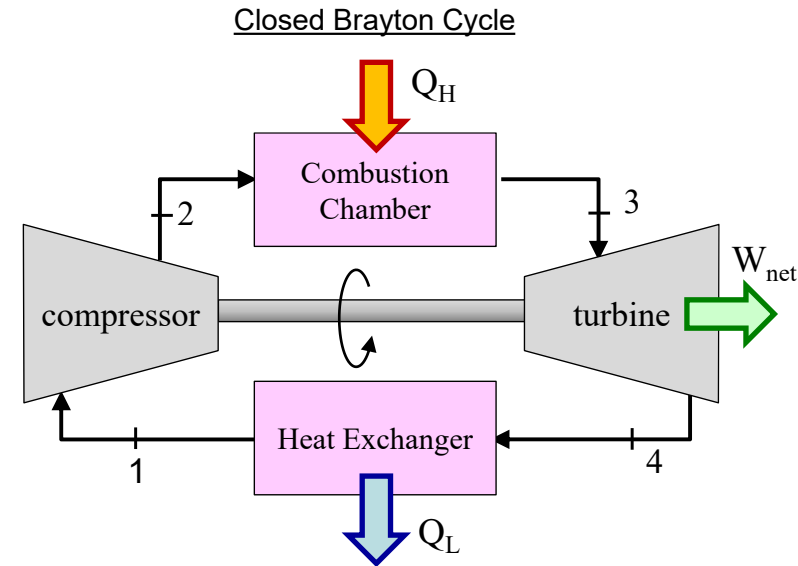
Process 1-2: Isentropic Compression

- 1st Law:

- $0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$
- $\dot{W}_{21,IN} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$

- 2nd Law:

- $s_2 - s_1 = \int \frac{\delta d}{T} + s_{gen} \rightarrow s_2 - s_1 = 0$
- Isentropic relations
- $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \& \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k$



4.5 Brayton Cycle – Basic Equations

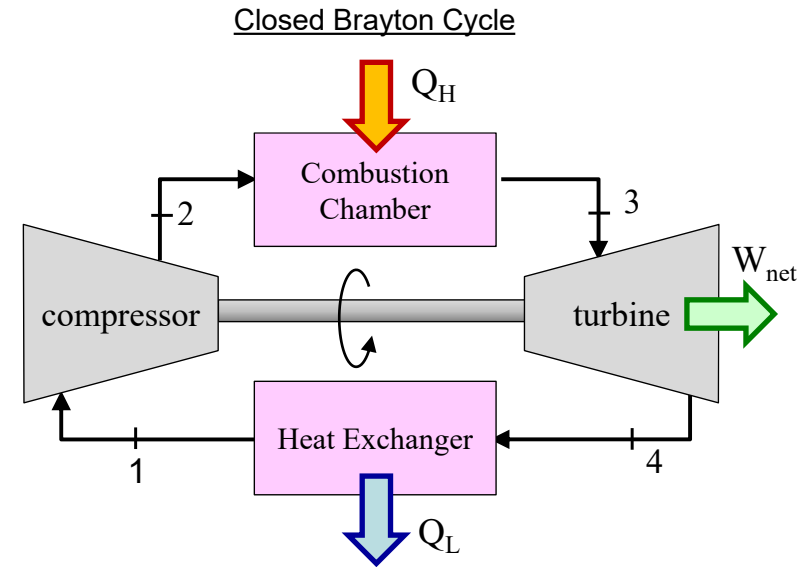
Ideal Brayton cycle

- Air treated as an ideal gas

Process 2-3: Constant pressure heat addition

- 1st Law:

- $$0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$$
- $$\dot{Q}_{32} = \dot{m}(h_3 - h_2) = \dot{m}C_p(T_3 - T_2)$$



4.5 Brayton Cycle – Basic Equations

Ideal Brayton cycle

- Air treated as an ideal gas

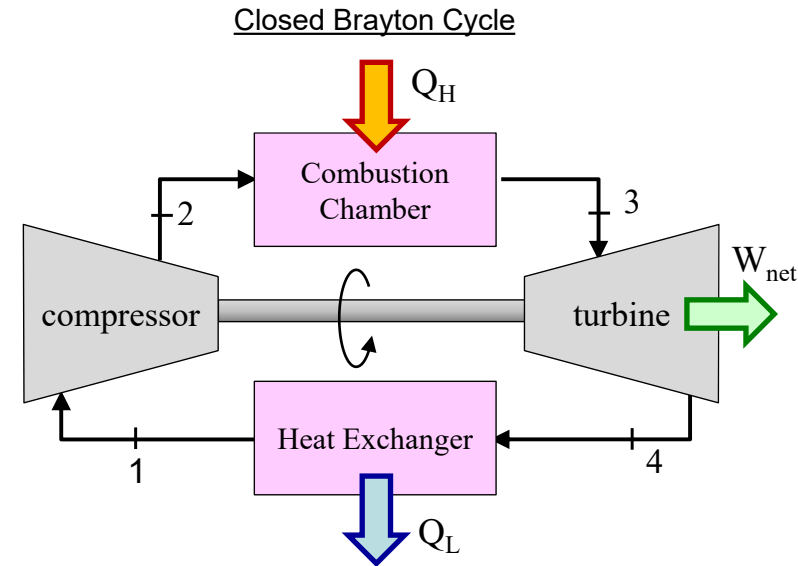
Process 3-4: Isentropic expansion

- 1st Law:

- $$0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$$
- $$\dot{W}_{34} = \dot{m}(h_3 - h_4) = \dot{m}C_p(T_3 - T_4)$$

- 2nd Law:

- $$s_4 - s_3 = \int \frac{\delta d}{T} + s_{gen} \rightarrow s_4 - s_3 = 0$$
- Isentropic relations
- $$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k} = \left(\frac{V_3}{V_4}\right)^{k-1} \quad \& \quad \frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^k$$



4.5 Brayton Cycle – Basic Equations

Ideal Brayton cycle

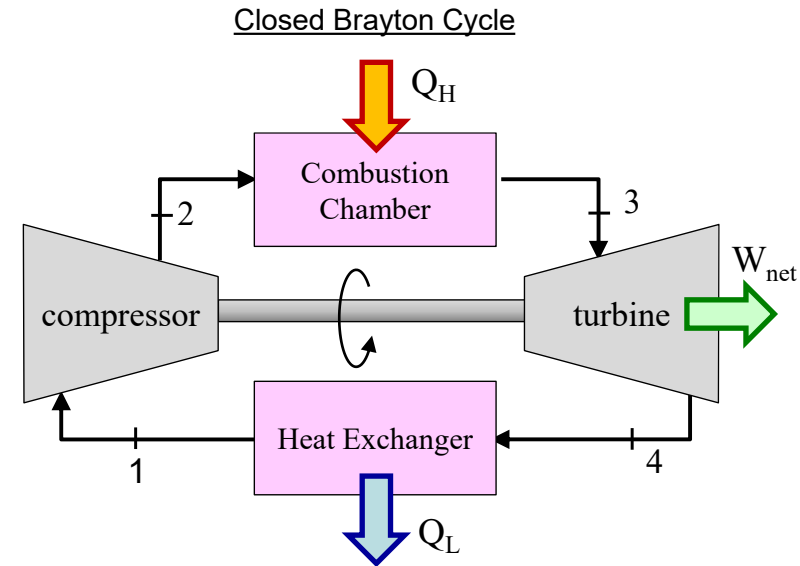
- Air treated as an ideal gas

Process 4-1 CLOSED: constant pressure heat rejection (closed cycle)

- 1st Law:
 - $0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$
 - $\dot{Q}_{41} = \dot{m}(h_4 - h_1) = \dot{m}C_p(T_4 - T_1)$

Efficiency:

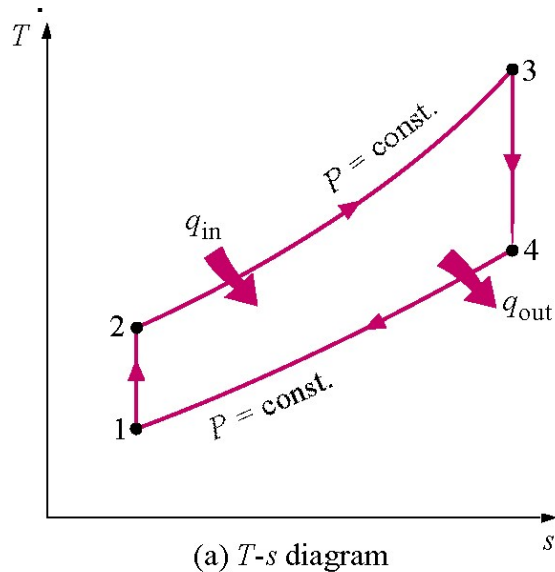
- $\eta_{th,Brayton} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{out} - \dot{W}_{in}}{\dot{Q}_{in}} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$



4.5 Brayton Cycle – Thermal Efficiency



- Net thermal efficiency of an **ideal, closed** Brayton cycle can be expressed as a function of temperatures in the cycle



- $\eta_{th,Brayton} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$
- $\eta_{th,Brayton} = 1 - \frac{mC_p(T_4 - T_1)}{mC_p(T_3 - T_2)}$
- $\eta_{th,Brayton} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$

- Processes 1-2 and 3-4 are isentropic $\rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{k-1/k}$ and $\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{k-1/k}$
- Since $P_3 = P_2$ and $P_4 = P_1 \rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4}$ and $\frac{T_4}{T_1} = \frac{T_3}{T_2}$
- Brayton cycle efficiency: $\eta_{th,Brayton} = 1 - \frac{T_1}{T_2}$

4.5 Brayton Cycle – Thermal Efficiency



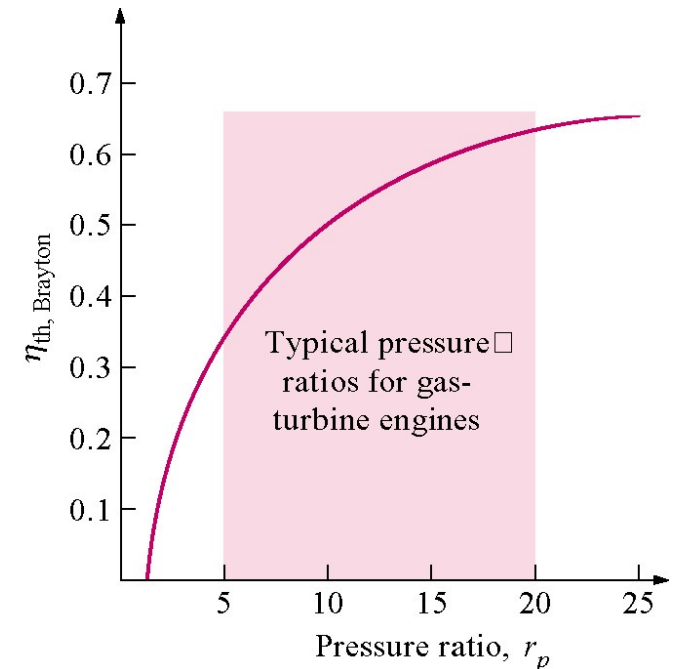
Pressure ratio

- Closed Brayton cycle efficiency is: $\eta_{th,Brayton} = 1 - \frac{T_1}{T_2}$
- Process 1-2 is isentropic, so $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{k-1/k} = (r_p)^{(k-1)/k}$

And, hence

$$\eta_{th, Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

where the pressure ratio
is $r_p = P_2/P_1$



4.5 Compressor and Turbine efficiency

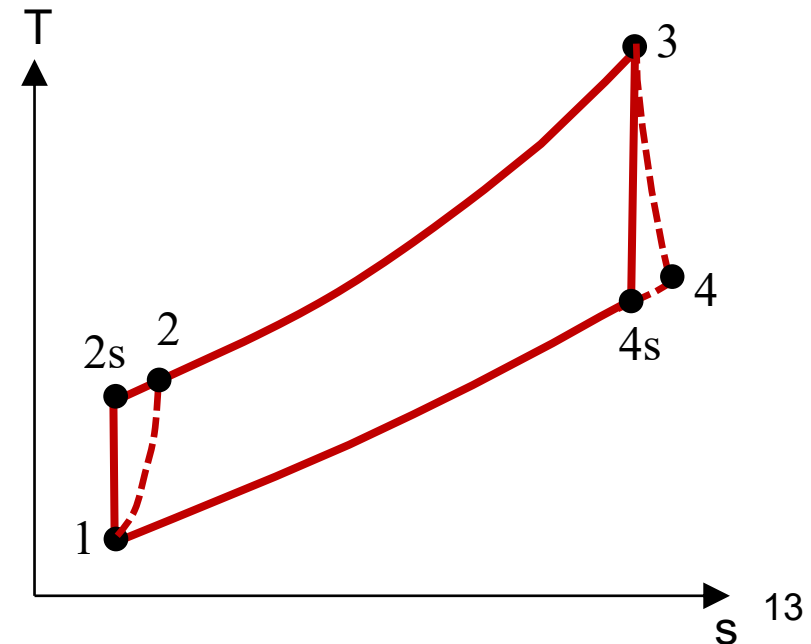
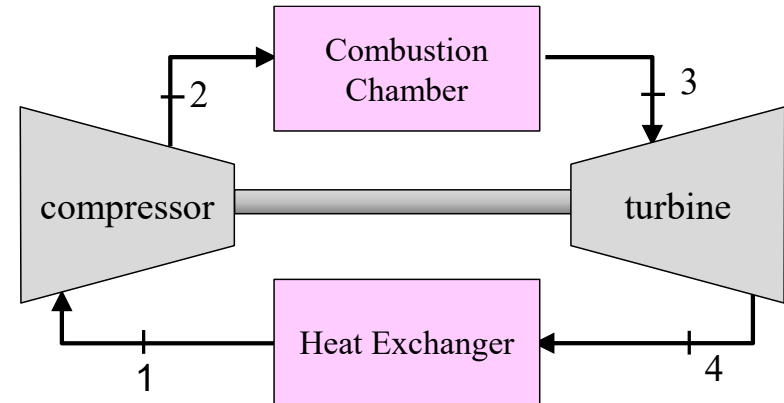


- Compression & expansion processes are not reversible or adiabatic in reality.
- Isentropic efficiencies:

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \quad \& \quad \eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

- Subscript “s” denotes the ideal isentropic case.
- Ideal gas with constant specific heats:

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \quad \& \quad \eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$



4.5 Brayton Cycle – Example

Example 4-8: Consider an ideal closed, air-standard Brayton cycle. Air enters the compressor at 90 kPa, 320K with a flow rate of 50 kg/s. The cycle operates with a pressure ratio of 14 and the maximum temperature reached is 1600 K. The exhaust gas exits the turbine at 90 kPa and sent to a heat exchanger that provides the heat source to operate a Rankine steam power plant. Assume the air exiting the heat exchanger returns to conditions entering the compressor (i.e. state 1).

- Determine the back work ratio of the Brayton cycle.
- What is the Brayton cycle efficiency?
- If the Rankine cycle operates between 10 kPa and 3000 kPa with a highest temperature in the cycle reaching 450°C, determine the mass flow rate of water in the Rankine cycle.

Assume air behaves as an ideal gas with constant specific heats: $C_p = 1.004 \text{ kJ/kgK}$, $R = 0.287 \text{ kJ/kgK}$

