

Lecture 20

Topic 4

Power & Refrigeration Cycles

Topic

- 4.5 Brayton Cycle

Reading:

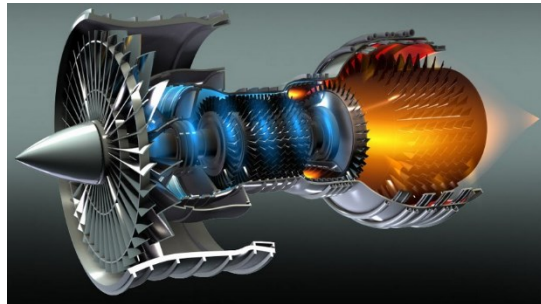
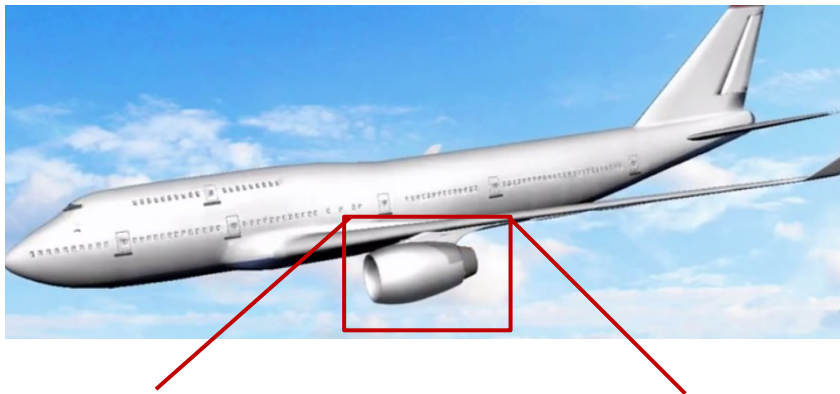
Ch 10: 10.1 – 10.5 Borgnakke & Sonntag Ed. 8

Ch 9: 9-9 – 9-12 Cengel and Boles Ed. 7

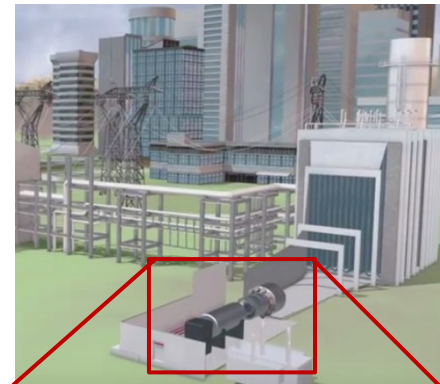
4.5 Brayton Cycle

Brayton Cycle – transfer of heat to useful work out (e.g. electricity to grid or propel an aircraft).

- Air standard ideal cycle approximation for gas-turbine engine



Aircraft: Power → Thrust

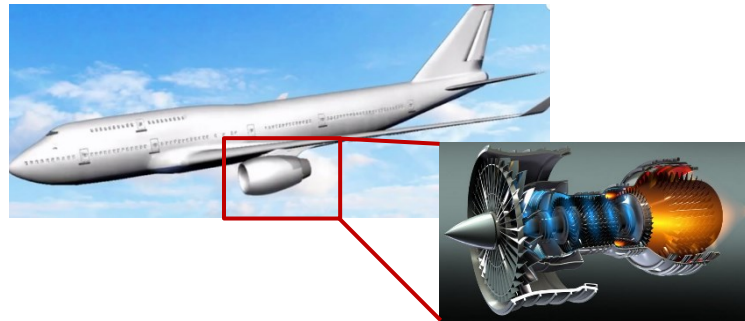


Power Plant: Power → Electricity

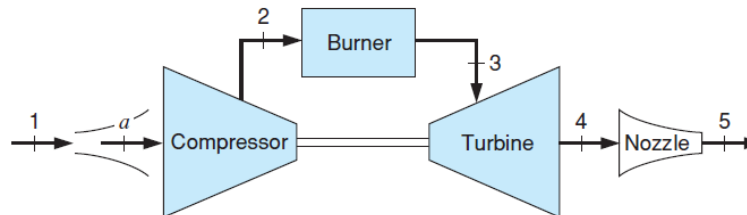
4.5 Brayton Cycle

Brayton Cycle – gas-turbine engine for aircraft

Aircraft: Power \rightarrow Thrust



Open Brayton Cycle



4.5 Ideal Brayton Cycle - Basics



Process 1-2: Ambient air enters the compressor (P & T increases). Ideal: isentropic process.

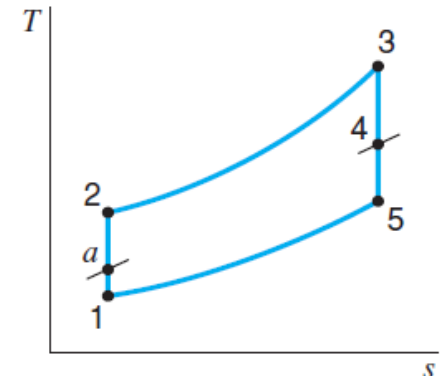
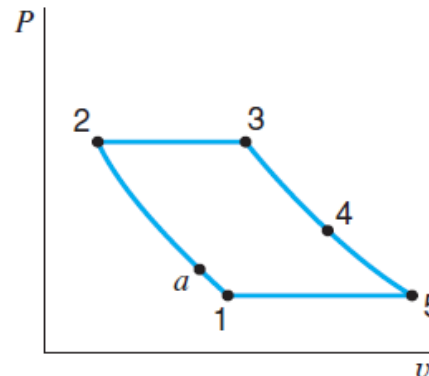
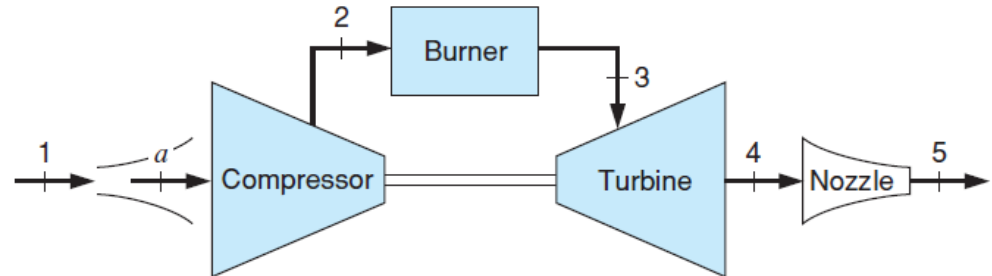
State a: A *diffuser* can exist between states 1 and 2. This increases pressure before the compressor. Ideal: isentropic process

Process 2-3: high- P air enters combustion chamber. Heat addition at constant pressure.

Process 3-4: high- T gases enter the turbine and expand to atmospheric pressure while producing power. Ideal: isentropic process

Process 4-5: Open: exhaust gases enter nozzle where gas expands further. Gases exits the nozzle with high velocity, creating thrust.

Open Brayton Cycle



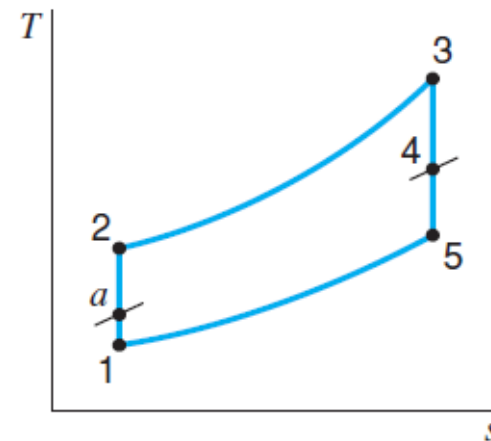
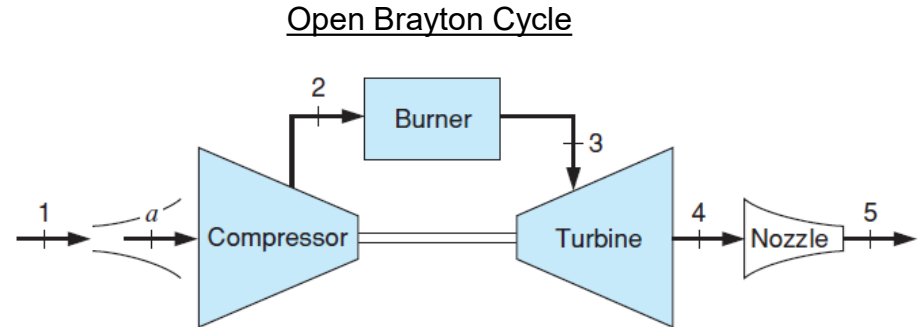
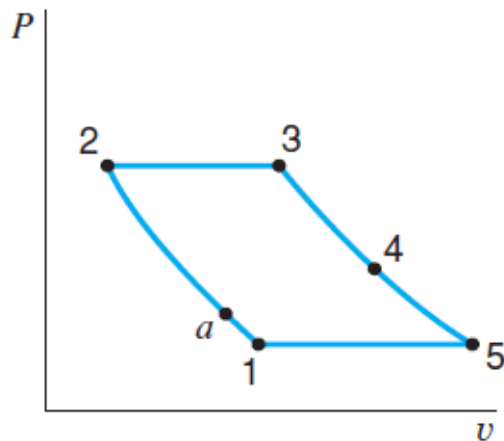
4.5 IDEAL Brayton Cycle

Processes in an air-standard closed Brayton cycle

- Working fluid is modelled as air (ideal gas)
- Representative cycle is a closed loop in the P-v and T-s diagrams

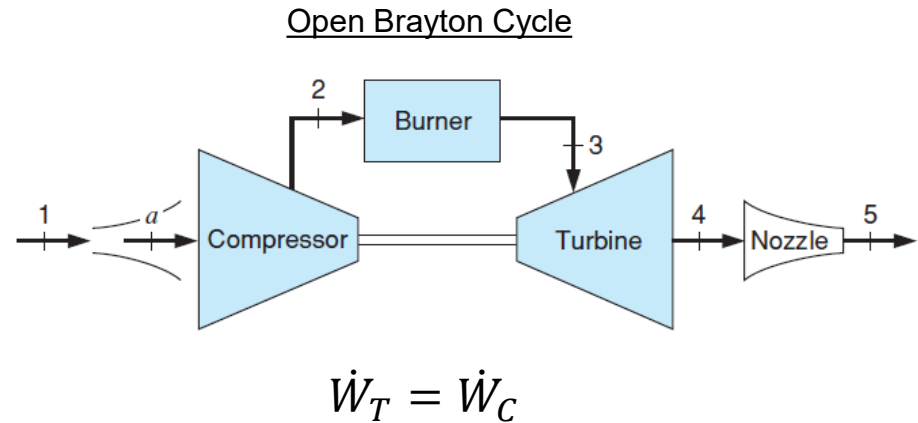
Ideal closed Brayton Cycle:

Process	Description
1-a-2	Isentropic compression
2-3	Constant pressure heat addition
3-4	Isentropic expansion (turbine)
4-5	Isentropic expansion & flow acceleration (nozzle)
5-1	Modelled as constant pressure heat rejection to return to state 1



4.5 Brayton Cycle - Basics

- Turbine is connected to compressor via a shaft.
- For jet engine, ALL of the turbine work is used to directly operate the compressor.
- Theoretically, turbine can also power auxiliary electronics, but this is ignored.



4.5 Brayton Cycle – Basic Equations

Ideal Brayton cycle

- Air treated as an ideal gas

Isentropic Compression Processes

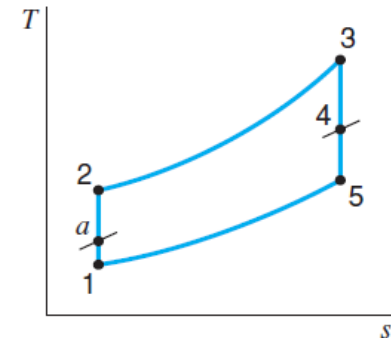
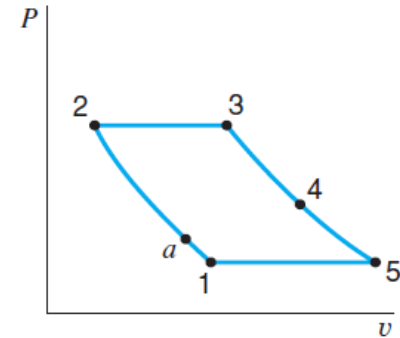
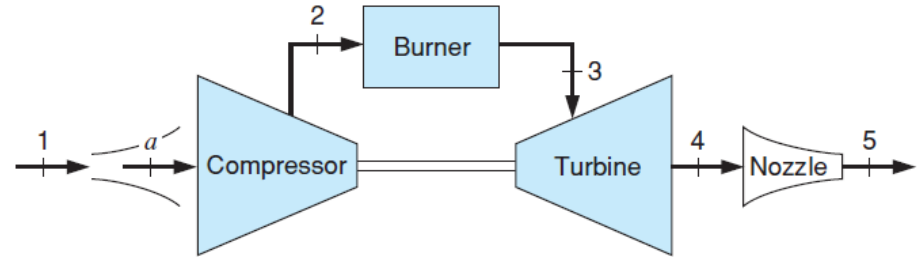
Process 1-a: Diffuser

- 1st Law:

$$\begin{aligned}
 - \quad 0 &= \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe) \\
 - \quad h_1 + \frac{1}{2}\vec{V}_1^2 &= h_a + \frac{1}{2}\vec{V}_a^2 \\
 - \quad h_a - h_1 &= C_p(T_a - T_1) = \frac{1}{2}(\vec{V}_1^2 - \vec{V}_a^2)
 \end{aligned}$$

- 2nd Law:

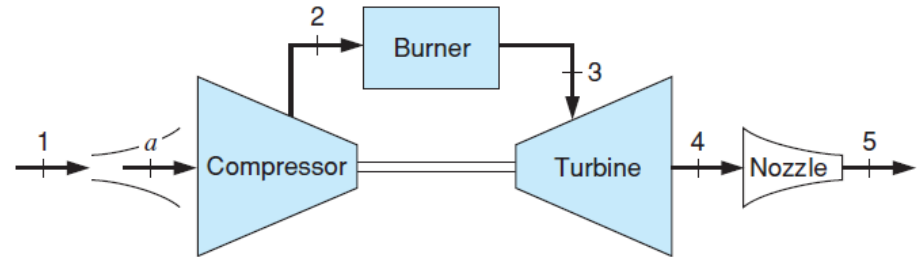
$$\begin{aligned}
 - \quad s_a - s_1 &= \int \frac{\delta d}{T} + s_{gen} \rightarrow s_a - s_1 = 0 \\
 - \quad \text{Isentropic relations} \\
 - \quad \frac{T_a}{T_1} &= \left(\frac{P_a}{P_1}\right)^{(k-1)/k} = \left(\frac{V_1}{V_a}\right)^{k-1} \quad \& \quad \frac{P_a}{P_1} = \left(\frac{V_1}{V_a}\right)^k
 \end{aligned}$$



4.5 Brayton Cycle – Basic Equations

Ideal Brayton cycle

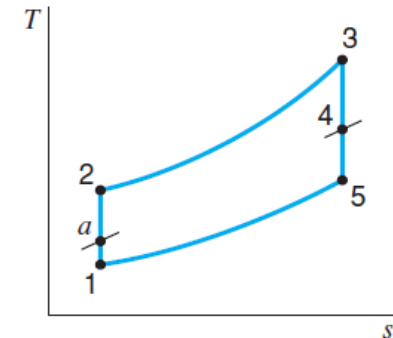
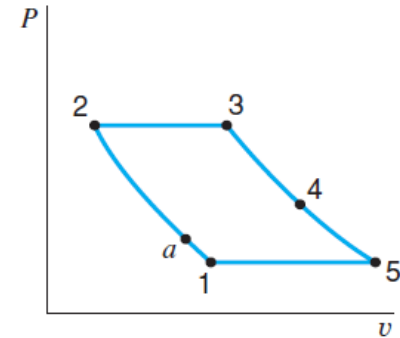
- Air treated as an ideal gas
- No significant change in velocities entering / exiting



Isentropic Compression Processes

Process 1-2: Compressor

- 1st Law:
 - $0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$
 - $\dot{W}_{2a,IN} = \dot{m}(h_2 - h_a) = \dot{m}C_p(T_2 - T_a)$
- 2nd Law:
 - $s_2 - s_a = \int \frac{\delta d}{T} + s_{gen} \rightarrow s_2 - s_a = 0$
 - Isentropic relations
 - $\frac{T_2}{T_a} = \left(\frac{P_2}{P_a}\right)^{(k-1)/k} = \left(\frac{V_a}{V_2}\right)^{k-1} \quad \& \quad \frac{P_2}{P_a} = \left(\frac{V_a}{V_2}\right)^k$



4.5 Brayton Cycle – Basic Equations

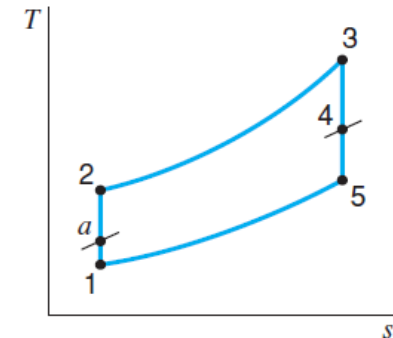
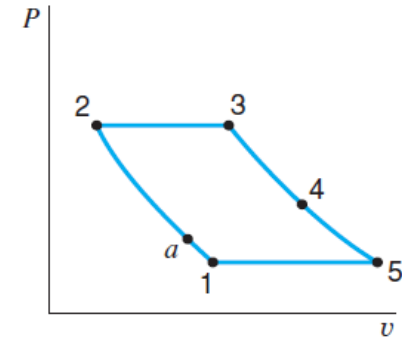
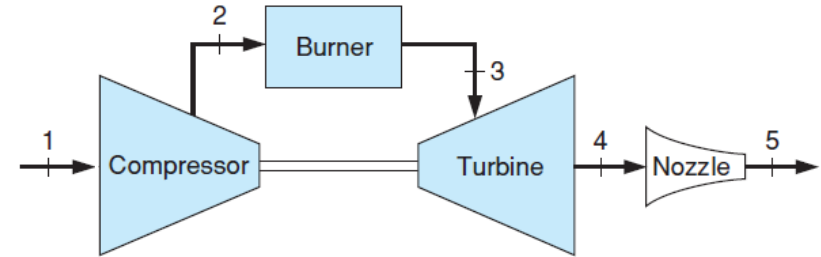
Ideal Brayton cycle

- Air treated as an ideal gas

Isentropic Compression Processes

Process 1-2: No diffuser

- 1st Law:
 - $0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$
 - $\dot{W}_{21,IN} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$
- 2nd Law:
 - $s_2 - s_1 = \int \frac{\delta d}{T} + s_{gen} \rightarrow s_2 - s_1 = 0$
 - Isentropic relations
 - $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \& \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^k$



4.5 Brayton Cycle – Basic Equations

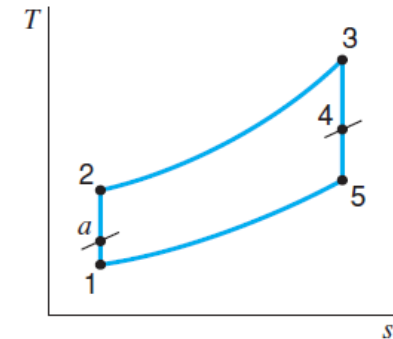
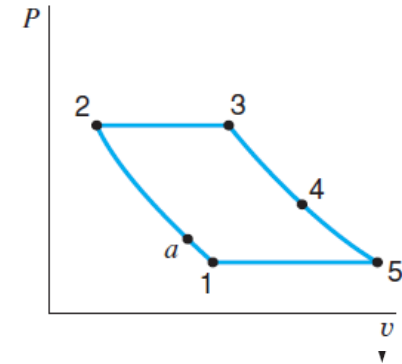
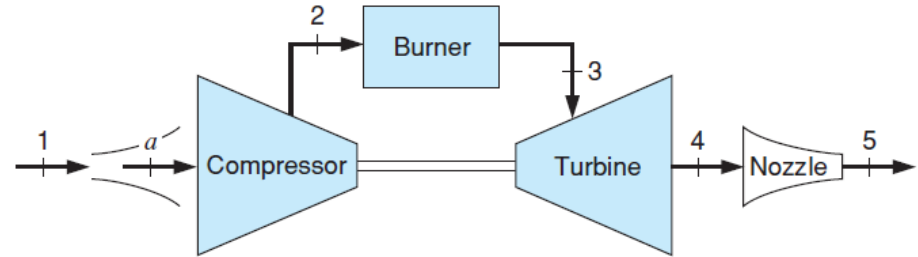
Ideal Brayton cycle

- Air treated as an ideal gas

Process 2-3: Constant pressure heat addition

- 1st Law:

- $0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$
- $\dot{Q}_{32} = \dot{m}(h_3 - h_2) = \dot{m}C_p(T_3 - T_2)$



4.5 Brayton Cycle – Basic Equations

Ideal Brayton cycle

- Air treated as an ideal gas

Process 3-4: Isentropic expansion

- 1st Law:

$$0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$$

$$\dot{W}_{34} = \dot{m}(h_3 - h_4) = \dot{m}C_p(T_3 - T_4)$$

- 2nd Law:

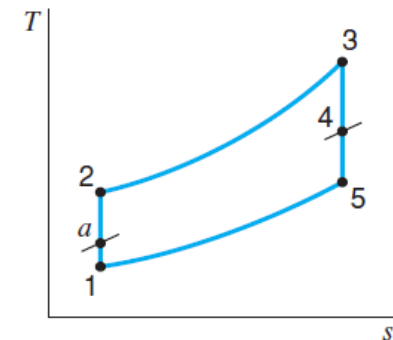
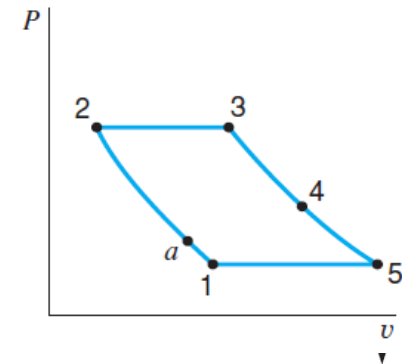
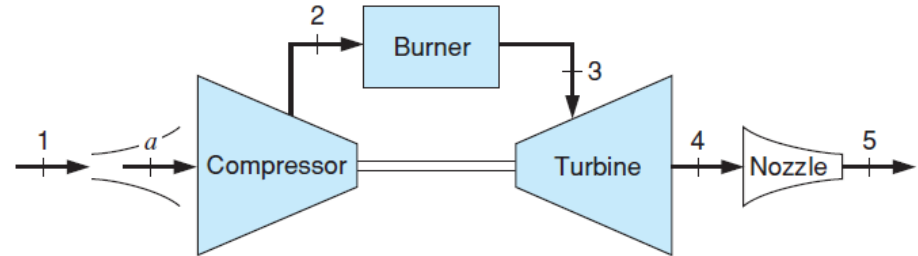
$$s_4 - s_3 = \int \frac{\delta d}{T} + s_{gen} \rightarrow s_4 - s_3 = 0$$

$$\text{Isentropic relations}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k} = \left(\frac{V_3}{V_4}\right)^{k-1} \quad \& \quad \frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^k$$

- For jet engine, we do not need excessive work output from the turbine.

- Turbine work is used to operate the compressor
- $w_{turbine} = w_{compressor}$



4.5 Brayton Cycle – Basic Equations



Ideal Brayton cycle

- Air treated as an ideal gas

Process 4-5 OPEN: additional expansion (adiabatic, reversible) to atmospheric pressure and increased velocity. Can often neglect velocity entering.

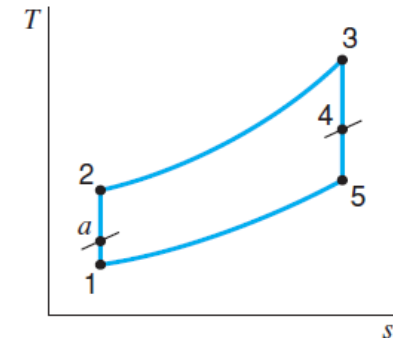
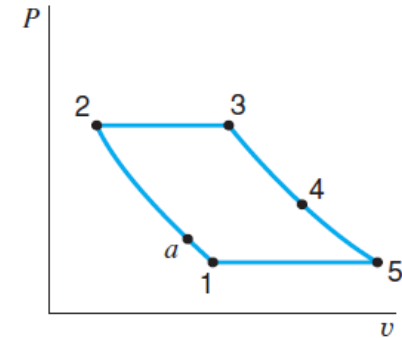
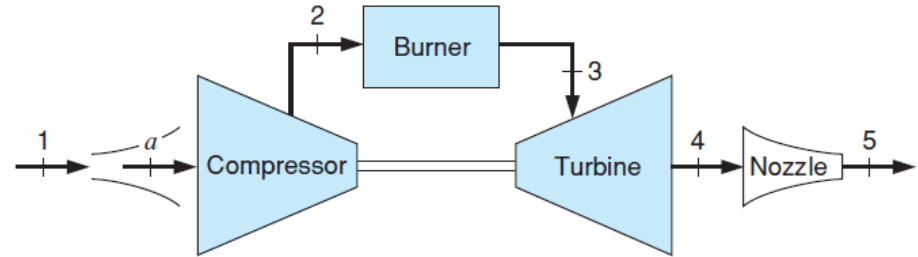
- 1st Law:

- $0 = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke + pe) - \sum \dot{m}_e(h_e + ke + pe)$
- $\vec{V}_5 = \sqrt{2(h_4 - h_5) + \vec{V}_4^2}$
- If neglect velocity entering and using ideal gas:

$$\vec{V}_5 = \sqrt{2C_p(T_4 - T_5)}$$

- 2nd Law:

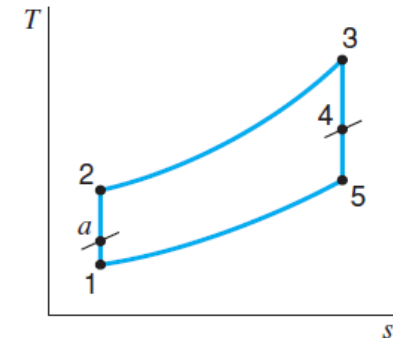
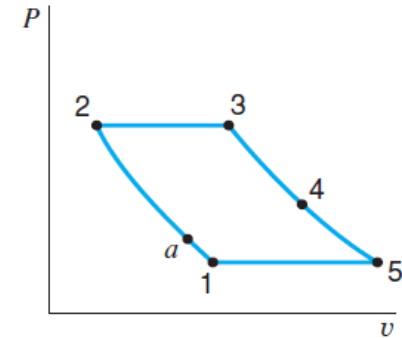
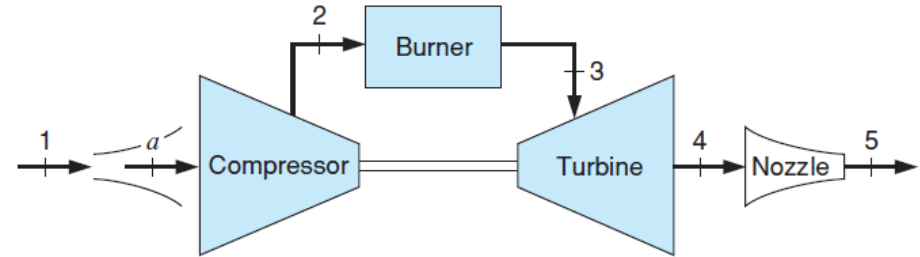
- $s_4 - s_3 = \int \frac{\delta d}{T} + s_{gen} \rightarrow s_4 - s_3 = 0$
- Isentropic relations
- $\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{(n-1)/n} = \left(\frac{V_4}{V_5}\right)^{n-1} \quad \& \quad \frac{P_5}{P_4} = \left(\frac{V_4}{V_5}\right)^n$



4.5 Brayton Cycle – Basic Equations

Jet Engine (Brayton Cycle) Efficiency:

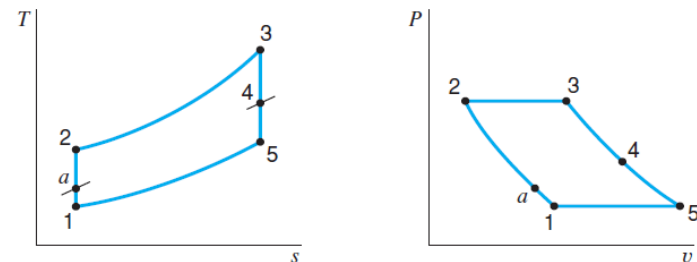
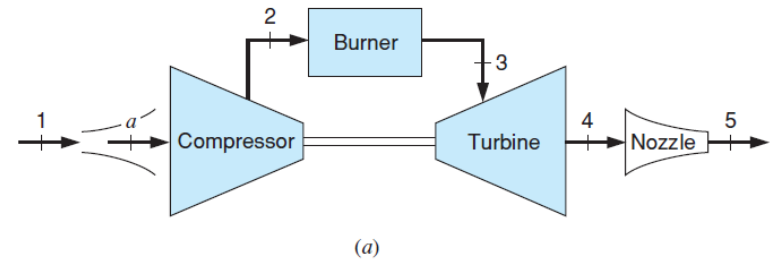
- $\eta_{th,propulsion} = \frac{\dot{W}_{propulsion}}{\dot{Q}_H}$
- Thrust Force: $F_{thrust} = \dot{m}(\vec{V}_{exit} - \vec{V}_{inlet}) = \dot{m}(\vec{V}_5 - \vec{V}_1)$
- $\dot{W}_{propulsion} = F_{thrust} V_{inlet}$
- $\dot{W}_{propulsion} = \dot{m}(\vec{V}_{exit} - \vec{V}_{inlet}) \vec{V}_{aircraft}$
- $\dot{W}_{propulsion} = \dot{m}(\vec{V}_5 - \vec{V}_1) \vec{V}_{aircraft}$
– $\vec{V}_{aircraft} \approx \vec{V}_1$



4.5 Brayton Cycle – Example

Example 4-7: Consider an ideal Brayton cycle for a jet propulsion system (i.e. open Brayton cycle). The pressure and temperature entering the compressor is 90 kPa and 290 K with a mass air flow rate of 10 kg/s (ignore the diffuser). The pressure ratio across the compressor is 14 and the turbine inlet temperature is 1500 K. When the air leaves the turbine, it enters the nozzle and expands to 90 kPa. The work output of the turbine is used to operate the compressor (i.e. $w_{\text{turbine}} = w_{\text{compressor}}$).

- Determine the heat added in the burner.
- Determine the exit temperature and pressure from the turbine
- Determine the air temperature leaving the nozzle.
- Assuming the velocity entering the nozzle is negligible, determine the velocity of the air leaving the nozzle.
- If the velocity enters the cycle (i.e. state 1) at 320 m/s before entering the diffuser, determine the net thrust developed by the engine
- Determine propulsive efficiency if the aircraft is moving at a velocity of 320 m/s.

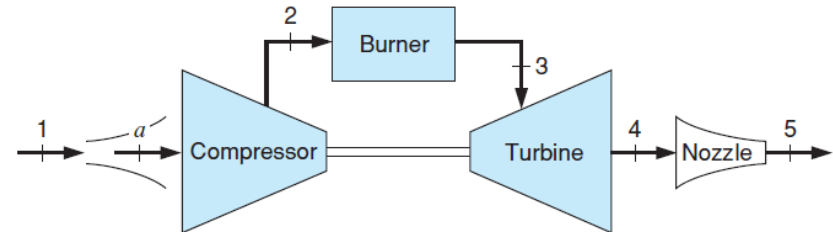


Assume air behaves as an ideal gas with constant specific heats: $C_p = 1.004$ kJ/kgK, $R = 0.287$ kJ/kgK

4.5 Compressor, Turbine & Nozzle Efficiency



- Ideal cycle: isentropic compression, expansion and flow acceleration.
- Such processes are not reversible or adiabatic in realistic operation.
- Subscript “s” denotes the ideal isentropic case.



Compressor efficiency

- $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$ or ideal gas $\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$

Turbine efficiency

- $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$ or ideal gas $\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$

Nozzle efficiency

- $\eta_{nozzle} = \frac{\vec{V}_{5a}^2 - \vec{V}_4^2}{\vec{V}_{5s}^2 - \vec{V}_4^2}$
- If we neglect velocity entering the nozzle
- $\eta_{nozzle} = \frac{\vec{V}_{5a}^2}{\vec{V}_{5s}^2}$

