

Lecture 3

Topic 1.2

Properties of Pure Substances Part 2

- Phase change in pure substances
 - Thermodynamic tables
 - Ideal gas equation of state

Reading:

Ch 2 Borgnakke & Sonntag Ed. 8

Ch3. Cengel & Boles Ed. 7

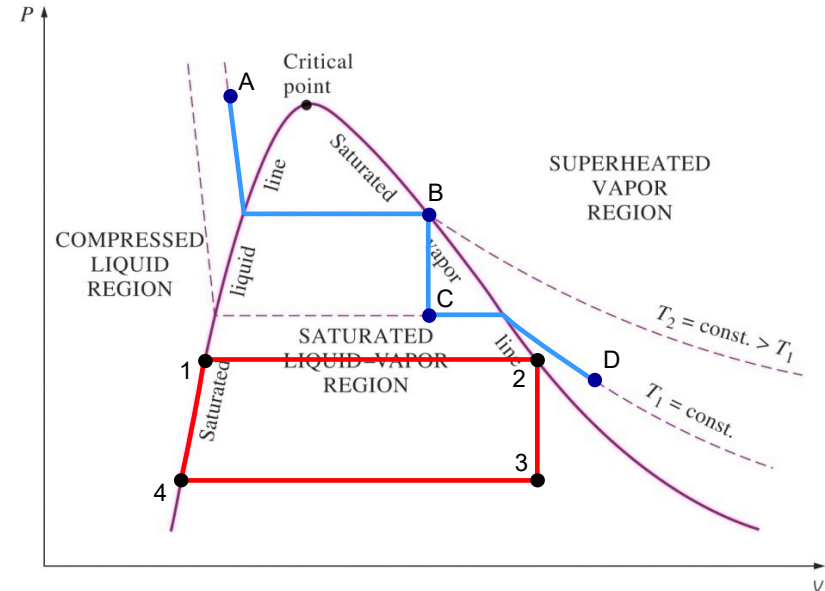
**please ignore compressibility factor & equations of state*

Overview



Last Time (Lecture 2)

- Phase change
- P - v - T phase diagrams
 - State is defined by two independent variables – *state postulate*
 - P is unique function of T and v (vice versa)

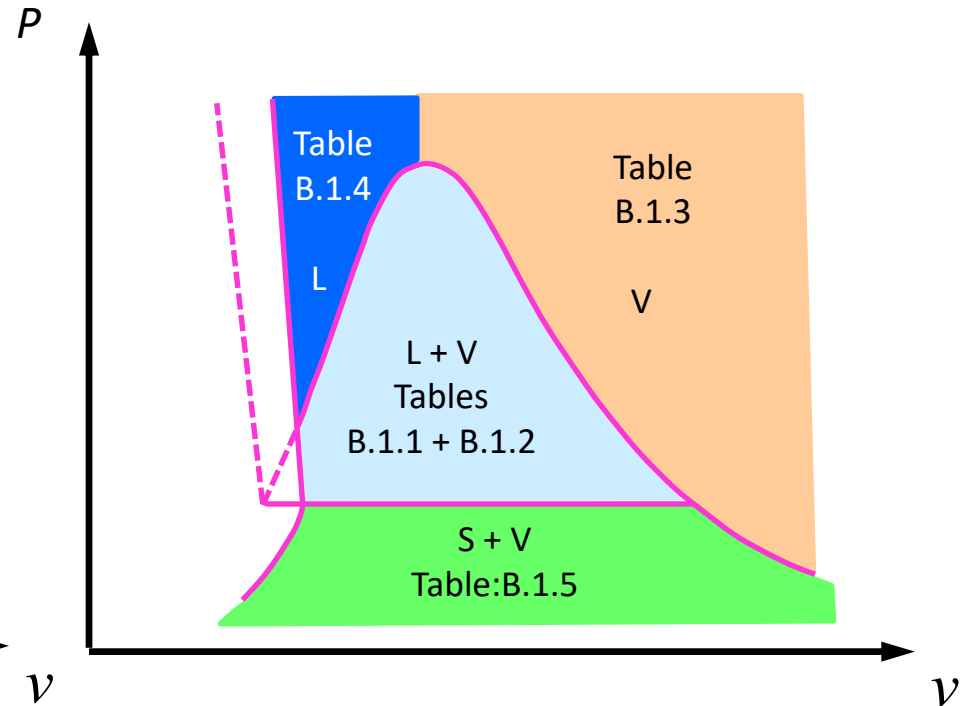
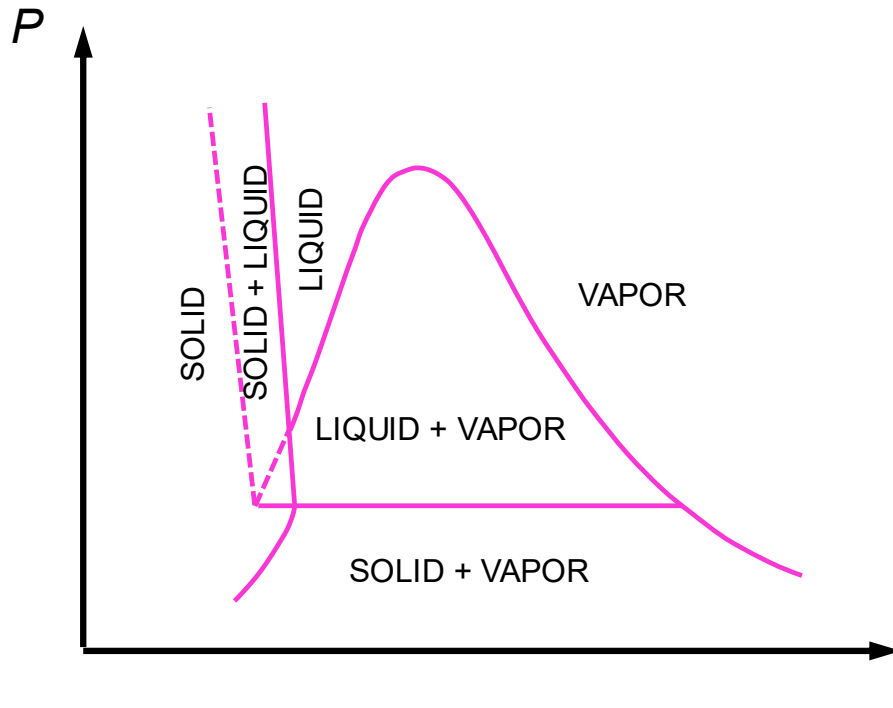


This Lecture

- Identifying properties of thermodynamic state
 - Thermodynamic Tables
 - Steam tables (i.e. water) will be presented in this lecture
 - Tables for other substances also exist in the book
 - Refrigerants, air, gases, etc.

1.2.5 Using property tables

- Property tables will be used extensively in this course
- Below is a PV diagram to identify relevant tables for given phases of WATER
 - Water is a substance chosen to be a prime example since we experience it daily
 - Note similar tables exists for other substances



1.2.5.0 Properties of interest

- Pressure
 - Normal component of force per unit area: $P = F/A$
 - Units in tables: kPa, MPa, bar
- Temperature
 - Comparative measurement of hot and cold
 - Units in tables: °C, K
- Specific volume (v)
 - Volume per unit mass: $v = V/m$
 - Reciprocal of density: $v = \rho^{-1}$
 - Units in tables: $[m^3/kg]$
- Internal energy (U or u)
 - Energy contained within a substance, excluding kinetic energy and potential energy (introduced in Lecture 4+5)
 - Units in tables are per unit mass: $u = U/m [kJ/kg]$
- Enthalpy (H or h)
 - Convenient grouping of internal energy, pressure & volume: $H = U+PV$ (introduced in Lecture: 7)
 - Units in tables are per unit mass: $h = H/m [kJ/kg]$
- Entropy (S or s)
 - Measure of molecular disorder of a system (introduced in Lecture 9)
 - Units in tables are per unit mass: $s = S/m [kJ/kg * K]$

1.2.5.1 Saturated Tables

Saturated Water Tables: B.1.1 & B.1.2

- Temperature and pressure are dependent properties
 - B.1.1 – temperature is independent property
 - B.1.2 – pressure is the independent property

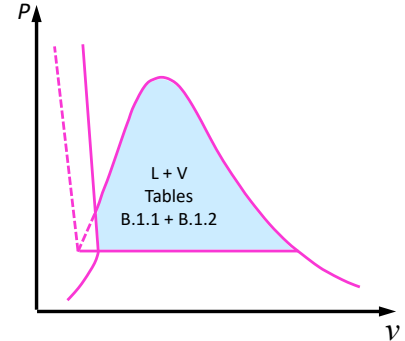


TABLE B.1.1
Saturated Water

Temp. (°C)	Press. (kPa)	Specific Volume, m ³ /kg		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
0.01	0.6113	0.001000	206.131	206.132
5	0.8721	0.001000	147.117	147.118
10	1.2276	0.001000	106.376	106.377
15	1.705	0.001001	77.924	77.925
20	2.339	0.001002	57.7887	57.7897

Specific Temp. Corresponding saturation Press. Specific volume properties

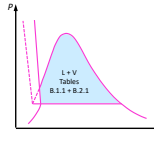
TABLE B.1.2
Saturated Water Pressure Entry

Press. (kPa)	Temp. (°C)	Specific Volume, m ³ /kg		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
0.6113	0.01	0.001000	206.131	206.132
1	6.98	0.001000	129.20702	129.20802
1.5	13.03	0.001001	87.97913	87.98013
2	17.50	0.001001	67.00285	67.00385
2.5	21.08	0.001002	54.25285	54.25385

Specific Press. Corresponding saturation Temp. Specific volume properties

- Saturation pressure: pressure at which the liquid and vapor phases are in equilibrium at a given temperature.
- Saturation temperature: temperature at which the liquid and vapor phases are in equilibrium at a given pressure.

1.2.5.1 Saturated Tables



Saturated Water Tables: B.1.1 & B.1.2

- Properties with subscript f
 - Saturated Liquid
- Properties with subscript g
 - Saturated vapor
- Properties with subscript fg
 - Difference between saturated vapor and saturated liquid values
 - E.g.: $v_{fg} = v_g - v_f$
- Saturated mixture
 - States in between Sat. Liquid & Sat. Vapor
 - Values in between f and g

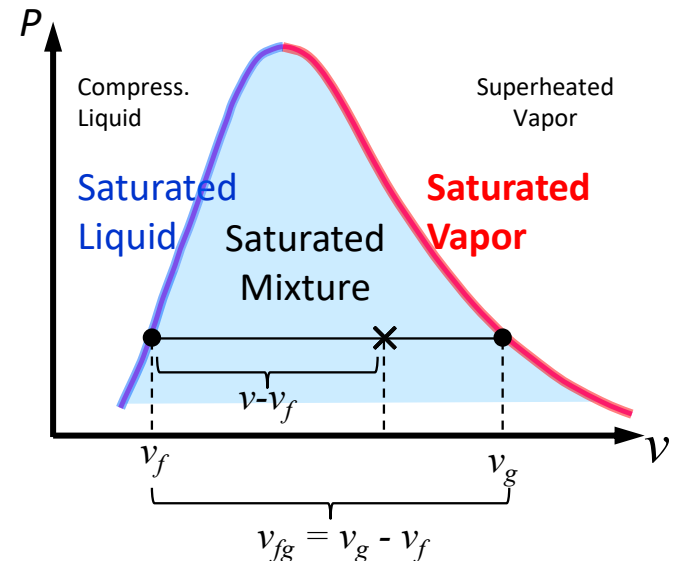
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20	2.339	0.001002	57.7887	57.7897

Specific
Temp.

Corresponding
saturation Press.

Specific volume
properties



1.2.5.1 Saturated Tables



Example 1-5

Two vessels each contain 50 kg of water. The first vessel contains **saturated liquid water** at 90°C, while the second vessel contains **saturated vapour** at 90°C. Determine the pressure in each vessel and the required volume of each vessel.

Sat. Liq.
90°C

Sat. Vap.
90°C

Solution:

Vessel 1: Saturated liquid

Vessel 2: Saturated vapour

TABLE B.1.1
Saturated Water

Temp. (°C)	Press. (kPa)	Specific Volume, m ³ /kg		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
75	38.58	0.001026	4.13021	4.13123
80	47.39	0.001029	3.40612	3.40715
85	57.83	0.001032	2.82654	2.82757
90	70.14	0.001036	2.35953	2.36056
95	84.55	0.001040	1.98082	1.98186
100	101.3	0.001044	1.67185	1.67290

1.2.5.1 Saturated Tables



Exercise 1-4: Performed in Tophat

A vessel contains 50 kg of saturated water vapour at atmospheric pressure (i.e. 100 kPa). Determine the temperature and volume of the tank.

(a) 99.62°C , 0.05215 m^3

(b) 105.99°C , 84.7 m^3

(c) 99.62°C , 84.7 m^3

(d) 105.99°C , 0.0524 m^3

TABLE B.1.2

Saturated Water Pressure Entry

Press. (kPa)	Temp. ($^{\circ}\text{C}$)	Specific Volume, m^3/kg		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
...
50	81.33	0.001030	3.23931	3.24034
75	91.77	0.001037	2.21607	2.21711
100	99.62	0.001043	1.69296	1.69400
125	105.99	0.001048	1.37385	1.37490
150	111.37	0.001053	1.15828	1.15933
175	116.06	0.001057	1.00257	1.00363

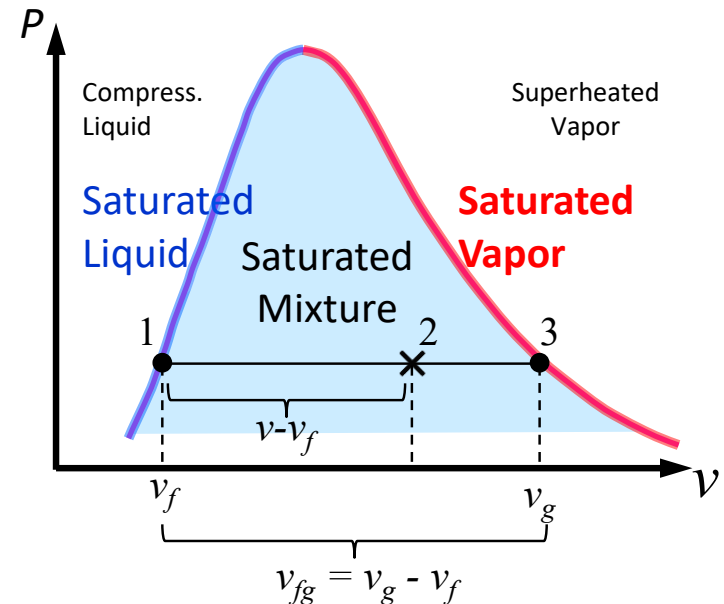
1.2.5.1 Saturated Tables

Quality of Saturated liquid-vapour MIXTURE

- Parameter values (e.g. v , u , h , s) depend on the quantity of vapour and liquid in the mixture
- Quality**: ratio of vapour mass to the total mass of mixture

$$x = \frac{\text{mass}_{\text{saturated vapor}}}{\text{mass}_{\text{total}}} = \frac{m_g}{m_f + m_g}$$

- Saturated liquid: $x = 0$
 - All mass is liquid; No vapor (i.e. state 1)
- Saturated vapor: $x = 1$
 - All mass is vapor; No liquid (i.e. state 3)
- Saturated Mixture
 - $0 \leq x \leq 1$ (i.e. state 2)



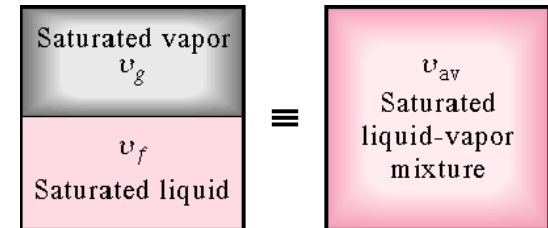
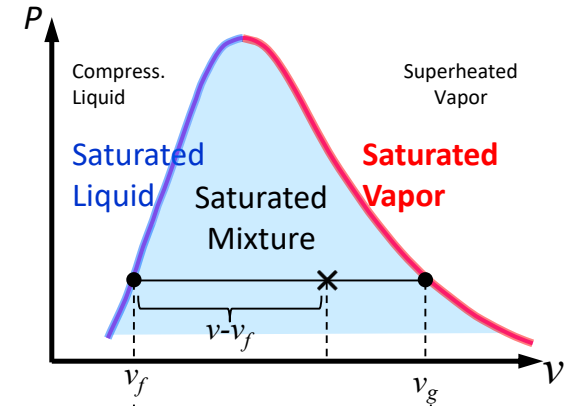
1.2.5.1 Saturated Tables

Property Values of Saturated liquid-vapour MIXTURE

- Average value given in terms of quality

Example: Container of Saturated MIXTURE, find v_{mix}

- Liquid: mass m_f and occupies volume V_f
- Vapour: mass m_g and occupies volume V_g

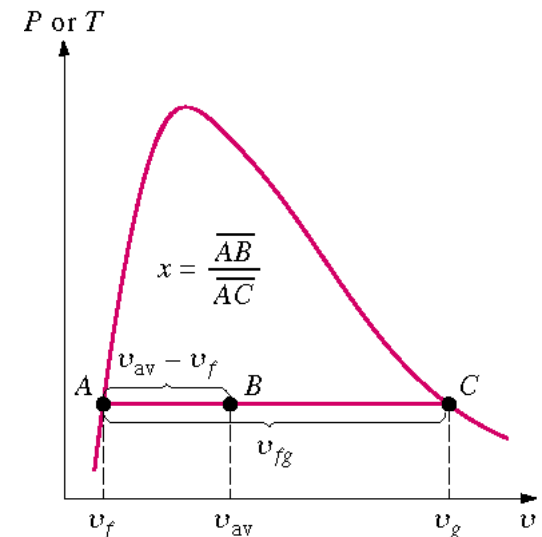
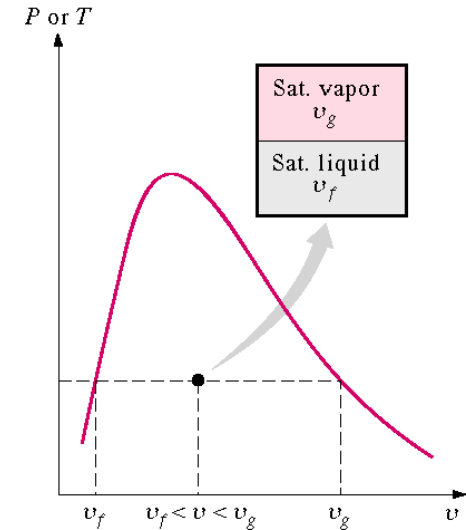


1.2.5.1 Saturated Tables

Lever Rule: linear interpolation (more info in tutorial session)

- Extensive properties (e.g. u , h , s) of a saturated mixture can be determined by the level rule
- Example: let y be any extensive property
 - The y value of a saturated liquid-vapor mixture lies between y_f and y_g values
 - $y_B = y_f + x(y_g - y_f)$
 - $y_g - y_f = y_{fg}$
 - $y_B = y_f + xy_{fg}$
 - Quality is related to horizontal distances between sat. liquid & sat. vapor

$$x = \frac{y - y_f}{y_{fg}}$$



1.2.5.1 Saturated Tables

Exercise 1-5

Consider a closed container of water at a pressure of 700 kPa that contains 1.78 kg of saturated liquid and 0.22 kg of saturated vapor. What is the specific volume of the mixture?

Liquid- vapour mixture

$$m_f = 1.78 \text{ kg}$$

$$m_g = 0.22 \text{ kg}$$

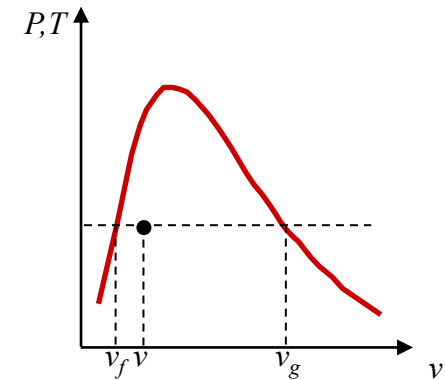
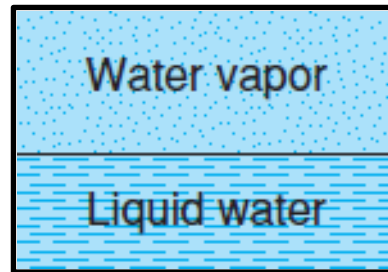


TABLE B.1.2

Saturated Water Pressure Entry

Press. (kPa)	Temp. (°C)	Specific Volume, m ³ /kg			Internal Energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg-K		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
600	158.85	0.001101	0.31457	0.31567	669.88	1897.52	2567.40	670.54	2086.26	2756.80	1.9311	4.8289	6.7600
650	162.01	0.001104	0.29158	0.29268	683.55	1886.51	2570.06	684.26	2076.04	2760.30	1.9627	4.7704	6.7330
700	164.97	0.001108	0.27176	0.27286	696.43	1876.07	2572.49	697.20	2066.30	2763.50	1.9922	4.7158	6.7080
750	167.77	0.001111	0.25449	0.25560	708.62	1866.11	2574.73	709.45	2056.98	2766.43	2.0199	4.6647	6.6846
800	170.43	0.001115	0.23931	0.24043	720.20	1856.58	2576.79	721.10	2048.04	2769.13	2.0461	4.6166	6.6627

1.2.5.2 Compressed Liquid

- Compressed liquid: substance's pressure is greater than saturation pressure.
 - E.g. water at 60C, 500 kPa
 - Also called subcooled liquid: temp. is lower than saturation temp. at given pressure
 - E.g. water at 30C, 19.941 kPa
- Table B.1.4
- Property values: vary little with pressure
 - x100 press. inc. often yields changes less than 1%.
- Water: at $P < 5$ Mpa, values are approx. equal to the saturated liquid values at the same temperature.
 - Approximate intensive property: $y \cong y_f@T$

TABLE B.1.1

Saturated Water

Temp. (°C)	Press. (kPa)	Specific Volume, m ³ /kg		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
60	19.941	0.001017	7.66969	7.67071
65	25.03	0.001020	6.19554	6.19656
70	31.19	0.001023	5.04114	5.04217
75	38.58	0.001026	4.13021	4.13123
80	47.39	0.001029	3.40612	3.40715
85	57.83	0.001032	2.82654	2.82757

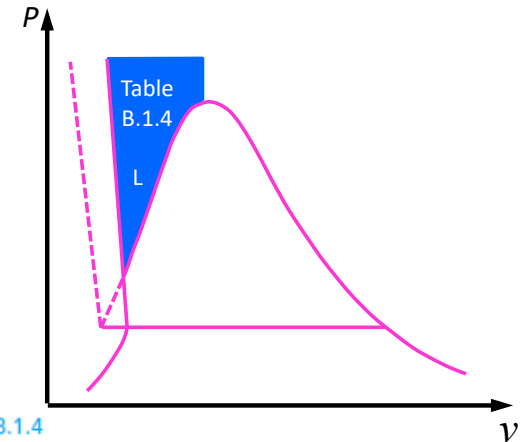


TABLE B.1.4

Compressed Liquid Water

Temp. (°C)	v (m ³ /kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg·K)
500 kPa (151.86°C)				
Sat.	0.001093	639.66	640.21	1.8606
0.01	0.000999	0.01	0.51	0.0000
20	0.001002	83.91	84.41	0.2965
40	0.001008	167.47	167.98	0.5722
60	0.001017	251.00	251.51	0.8308

1.2.5.3 Superheated Vapor

- Temperature is higher than saturation temperature
- Pressure is lower than saturated pressure
- E.g. water at 100 kPa, 200C
 - $T > T_{sat} @ 100kPa \rightarrow$ superheated vapor
- Table B.1.3
 - Function of T for given P

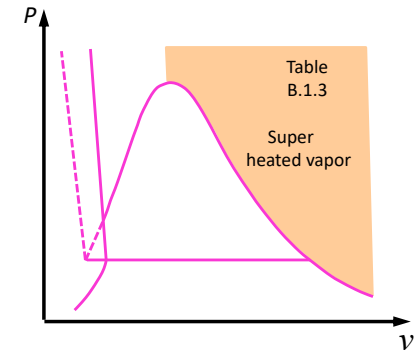


TABLE B.1.3
Superheated Vapor Water

Temp. (°C)	v (m ³ /kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
100 kPa (99.62°C)				
Sat.	1.69400	2506.06	2675.46	7.3593
150	1.93636	2582.75	2776.38	7.6133
200	2.17226	2658.05	2875.27	7.8342
250	2.40604	2733.73	2974.33	8.0332
300	2.63876	2810.41	3074.28	8.2157
400	3.10263	2967.85	3278.11	8.5434
500	3.56547	3131.54	3488.09	8.8341
600	4.02781	3301.94	3704.72	9.0975
700	4.48986	3479.24	3928.23	9.3398
800	4.95174	3663.53	4158.71	9.5652
900	5.41353	3854.77	4396.12	9.7767
1000	5.87526	4052.78	4640.31	9.9764

TABLE B.1.2
Saturated Water Pressure Entry

Press. (kPa)	Temp. (°C)	Specific Volume, m ³ /kg		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
50	81.33	0.001030	3.23931	3.24034
75	91.77	0.001037	2.21607	2.21711
100	99.62	0.001043	1.69296	1.69400

1.2.5.4 Saturated Solid-Vapour

- At low temperatures, P-v-T surface is steep and flat
- Solid: Incompressible substance where v is a weak function of P & T
- $v = v(T) = v_i$
- Same concept as saturated liquid-vapor
 - Subscript “i” refers to solid
 - Subscript “g” refers to vapor
 - Lever rule

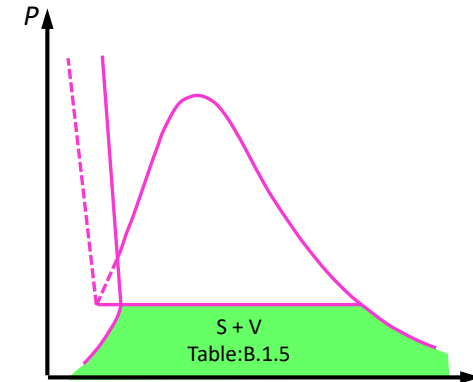


TABLE B.1.5

Saturated Solid-Saturated Vapor, Water

Temp. (°C)	Press. (kPa)	Specific Volume, m ³ /kg		
		Sat. Solid v_i	Evap. v_{ig}	Sat. Vapor v_g
0.01	0.6113	0.0010908	206.152	206.153
0	0.6108	0.0010908	206.314	206.315
−2	0.5177	0.0010905	241.662	241.663
−4	0.4376	0.0010901	283.798	283.799
−6	0.3689	0.0010898	334.138	334.139
−8	0.3102	0.0010894	394.413	394.414

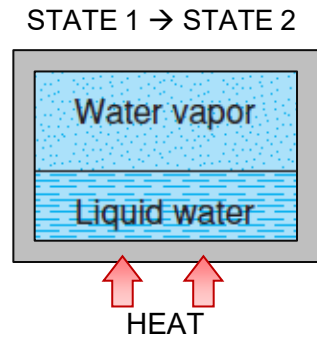
1.2.5 Using property tables



Example 1-6

Consider the *closed, rigid* container of water shown. The pressure is 700 kPa, the mass of the saturated liquid is 1.78 kg, and the mass of the saturated vapor is 0.22 kg. Heat is added to the water until the pressure increases to 8 MPa.

Does the liquid level rise or fall? Find the final temperature of the water.



1.2.6 Ideal gas equation of state



Ideal gas equation of state

- Relationship between temperature, pressure & volume
- Robert Boyle (1662)
 - Pressure inversely proportional to volume: $P \propto 1/V$
- Jacques Charles and Joseph Gay-Lussac (1802)
 - At low pressures, volume is proportional to temperature
- Variants of ideal gas equation

$$P = R \left(\frac{T}{v} \right) \quad Pv = RT \quad PV = mRT \quad P = \rho RT$$

Remember: Absolute temperature (K) and absolute pressure

1.2.6 Ideal gas equation of state



Gas constant (R): $Pv = RT$

- Each gas has specific value of R (units $\text{kJ/kg}\cdot\text{K}$)
- Relation to **universal gas constant** ($R_u = 8.314 \text{ kJ/kmol}\cdot\text{K}$)

$$R = \frac{R_u}{M} \qquad PV = nR_u T$$

Molecular weight (M)

- Ratio of mass (m) to moles (N) of a substance.

$$M = m/N$$

- Molar mass: ratio of mass to moles. Same value regardless of units.
 - Example: air:

$$M_{air} = 28.97 \frac{\text{g}}{\text{gmol}} = 28.97 \frac{\text{kg}}{\text{kmol}} = 28.97 \frac{\text{lbm}}{\text{lbmol}}$$

1.2.6 Ideal gas equation of state



The combined gas law

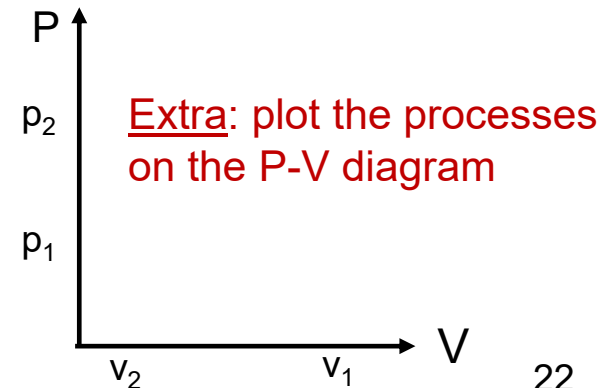
- Fixed mass: properties of an ideal gas at two different states can be related. E.g. $m_1 = m_2$

$$\begin{array}{l} P_1 V_1 = m_1 R T_1 \\ P_2 V_2 = m_2 R T_2 \end{array} \left. \vphantom{\begin{array}{l} P_1 V_1 = m_1 R T_1 \\ P_2 V_2 = m_2 R T_2 \end{array}} \right\} \rightarrow m_1 = m_2 \longrightarrow \frac{P_1 V_1}{R T_1} = \frac{P_2 V_2}{R T_2} \longrightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Example 1-8

An ideal gas with fixed mass & initial temperature of 25°C undergoes two processes described below. Determine the final temperature of the gas.

- Process 1-2: The volume is held constant while the pressure doubles.
- Process 2-3: The pressure is held constant while the volume is reduced to 1/3 of the original volume.



1.2.6 Ideal gas equation of state

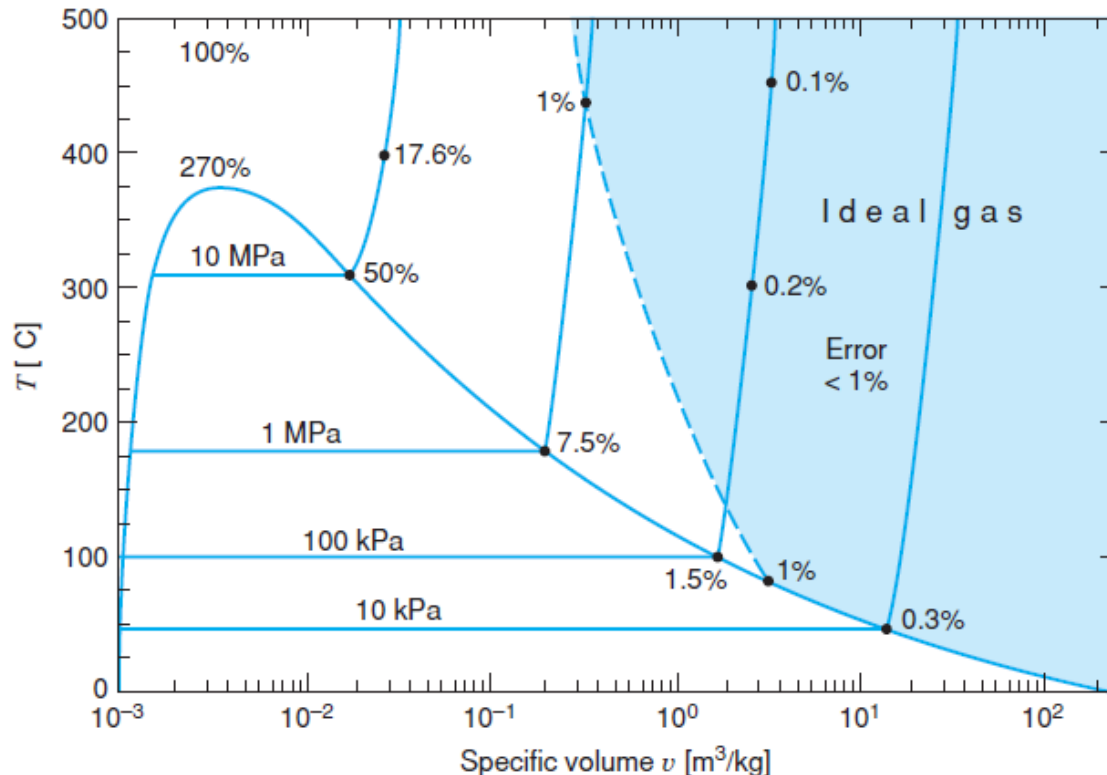


- IDEAL GAS: good *approximation* for the behavior of many gases encountered in real engineering problems.
- When can Ideal Gas assumption be used? → low density gases

(1) gas pressure is very small compared to the critical pressure (e.g. $10xP \leq P_{crit}$).

OR

(2) gas temperature is twice the critical temperature

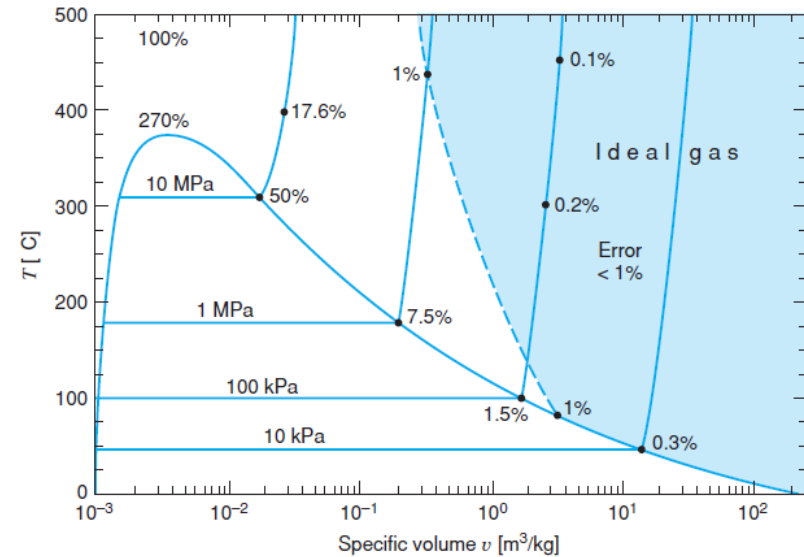


- Steam often mistaken as an ideal gas.
- Only sometimes can steam be approximated as an ideal gas
- Graph shows % error when assuming ideal gas for water
- Common gases that can be treated as ideal gas
 - Air, N_2 , O_2 , H_2 , CO_2 , CO, Ar
 - Often valid for temperatures $< 1000K$

1.2.6 Ideal gas equation of state



- IDEAL GAS: good *approximation* for the behavior of many gases and vapors encountered in real engineering problems.



Example 1-7:

When can Ideal Gas assumption be used?

a) Water: Behaves as an ideal gas in the superheated region and depends on both temperature and pressure. Often when water is encountered in engineering problems, the ideal gas assumption may not be used to solve problems. We must use the real gas relations (e.g. data from the property tables).

b) Oxygen/Nitrogen: The critical pressure and temperature for oxygen/nitrogen is 5.08 MPa and 154.8 K, respectively. For temperatures 300-1000 K and pressures less than 50 MPa, oxygen/nitrogen is considered to be an ideal gas and the ideal gas equation can be used.

1.2.7 Exercises



Exercise 1-6

Show that at 2000 kPa and 200°C water is a compressed liquid and find the percent difference associated with approximating internal energy as internal energy for a saturated liquid at 200°C.

Solution:

Hint: Percent difference: $\frac{|u_{approx.} - u_{real}|}{u_{approx.}} \times 100\%$

Table B.1.1

Temp. (°C)	Press. (kPa)	Internal Energy, kJ/kg		
		Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g
195	1397.8	828.36	1764.43	2592.79
200	1553.8	850.64	1744.66	2595.29
205	1723.0	873.02	1724.49	2597.52

Table B.1.4

Temp. (°C)	v (m ³ /kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
2000 kPa (212.42°C)				
Sat.	0.001177	906.42	908.77	2.4473
0.01	0.000999	0.03	2.03	0.0001
20	0.001001	83.82	85.82	.2962
40	0.001007	167.29	169.30	.5716
60	0.001016	250.73	252.77	.8300
80	0.001028	334.38	336.44	1.0739
100	0.001043	418.36	420.45	1.3053
120	0.001059	502.84	504.96	1.5259
140	0.001079	588.02	590.18	1.7373
160	0.001101	674.14	676.34	1.9410
180	0.001127	761.46	763.71	2.1382
200	0.001156	850.30	852.61	2.3301

Ans: Sat. Tables (approx. soln) : $u_f @ 200^\circ\text{C} = 850.64 \text{ kJ/kg}$; Compressed Liquid Tables: $u = 850.3 \text{ kJ/kg}$
Percent difference = 0.04%

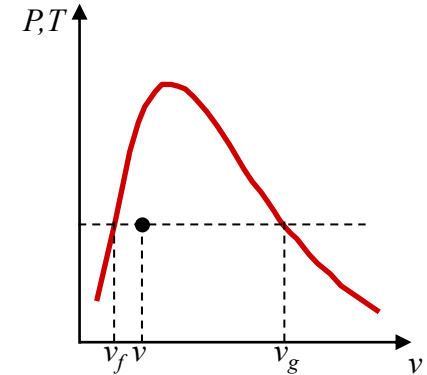
1.2.7 Exercises



Exercise 1-7:

Fill out the following table for substance water:

Remember: quality only exists if it is underneath the 'dome'



	P [kPa]	T [°C]	Quality, x	v [m ³ /kg]	u [kJ/kg]	h [kJ/kg]	s [kJ/kgK]
A	500	20					
B	500			0.20			
C	1400	275					
D		300	0.8				
E		500				3450	

1.2.7 Exercises



Exercise 1-8

Calculate the specific volume of nitrogen at 300 K and 8.0 MPa and compare the result with the value given in a nitrogen table as $v = 0.01484 \text{ m}^3/\text{kg}$. The critical temperature of nitrogen is 126.2 K, its critical pressure is 3.39 MPa and its gas constant (R) is 0.2968 kJ/kgK. Why are values using the ideal gas equation different than the tables?

Ans: 0.01113 m³/kg

Exercise 1-9

Determine the particular gas constant for air ($M=28.97 \text{ kJ}/(\text{kmol}\cdot\text{K})$) and hydrogen ($M=2.016 \text{ kJ}/(\text{kmol}\cdot\text{K})$).

Ans: 0.2869 kJ/(kmol K), 4.124 kJ / (kmol K)