Dynamics 2 – Tutorial 8

Kinetic Energy and the Power Method for Systems

Outline Solutions

1(a) Disc Mass = 35kg; R = 0.45m; υ = 2.5m/s

For general plane motion

$$\text{KE} = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_O\Omega^2$$

But for rolling without slip

$$\text{R}\Omega = \upsilon$$

Hence

$$\text{KE} = \frac{1}{2}Mv^2 + \frac{1}{2}I_O\frac{v^2}{R^2}$$

Also, for a uniform disc

$$I_O = \frac{1}{2}MR^2 = 3.544$$

Giving

$$\text{KE} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2} = \frac{3}{4}Mv^2 = 164.07\ \text{N}$$

(b) Let M = total mass; m = wheel mass; M_O = body mass

Use result from 1(a) for wheels

$$\text{KE} = \tfrac{1}{2}M_Ov^2 + 4\left(\tfrac{3}{4}mv^2\right) = \frac{1}{2}(M_O + 6m)v^2 = 235v^2$$

(c) M = 250 kg; I_O = 45 kgm^2; R = 0.8 m

Kinetic Energy of System

$$\text{KE} = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_O\Omega^2$$

But

$$\text{R}\Omega = 2v$$

Hence

$$\text{KE} = \tfrac{1}{2}(250)v^2 + \tfrac{1}{2}(45)\frac{4v^2}{0.8^2} = 265.6v^2$$

2. Find downwards acceleration of mass M. Let Q = bearing friction torque on Drum

Net system power is power in from falling mass minus friction power

Net power $= Mgv - Q\Omega$

or

$$= \left(Mg - \frac{2Q}{R} \right) v$$

Kinetic Energy (from 1(c))

KE = $265.6 v^2$

Thus Net power = $\frac{d}{dt}(KE)$ gives

$$\left(Mg - \frac{2Q}{R} \right) v = 531.2\, va$$

Gives

a = 4.05 m/s^2

Velocity of M after 6m fall

$$v_2^2 = v_1^2 + 2as = 0 + 2 \times 4.05 \times 6$$

$$\Rightarrow v_2 = 6.97\,\text{m/s}$$

3. Let v = upwards velocity of load mass
- v = downwards velocity of counterweight
- Ω_1 = angular velocity of drum (clockwise)
- Ω_2 = angular velocity of motor (anti-clockwise)
- M = load mass
- m = counterweight mass
- I_D = Drum Inertia
- I_M = Motor Inertia

Kinetic Energy

$$\text{KE} = \frac{1}{2} M v^2 + \frac{1}{2} m v^2 + \frac{1}{2} I_D \Omega_1^2 + \frac{1}{2} I_M \Omega_2^2$$

Kinematics:

$\Omega_1 R_D = v$ $\qquad$ R_D = 0.6m

$\Omega_2 = 9\Omega_1$

Giving

$$\text{KE} = \frac{1}{2} \left[M + m + \frac{I_D}{R_D{}^2} + 81 \frac{I_M}{R_D{}^2} \right] v^2 = 4048 v^2$$

Power Equation:

Net power = $\frac{d}{dt}(KE)$

Let τ = Motor Torque

$$\tau\Omega_2 - Mgv + mgv = 8096va$$
$$\tau\left(9\frac{v}{R_D}\right) - Mgv + mgv = 8096va$$

v cancels

$$9\frac{\tau}{R_D} = 8096a + Mg - mg$$

Gives

τ = 2574 Nm   (for a = 3.8 m/s^2)

This is "slightly" less than for problem using D'Alembert (must be due to rounding errors!)

4. From 1(a) the KE for disc rolling at velocity v

$$\text{KE} = \frac{3}{4}Mv^2$$

As the wheel is in pure rolling, the friction force acting at the contact point (where velocity is zero) does not dissipate energy.

The power source is due to the weight

Net power = (Mg sinθ)v

Hence

$$Mg\sin\theta v = \frac{d}{dt}(KE) = \frac{6}{4}Mva$$
$$\Rightarrow \quad a = \tfrac{2}{3}g\sin\theta$$

5. Similarly to Question 4, the contact friction forces at the wheels do not dissipate energy.

Power source is due to weight (Mg sinθ)v

Power sink is due to brake torque Q on rear wheels: $-2Q\Omega = -2Q\frac{\upsilon}{R}$

If speed is constant $\frac{d}{dt}$(KE) = 0, hence Net Power = 0 giving

$$\text{Mg}\sin\theta = 2\frac{Q}{R}$$
$$\Rightarrow \quad \text{Q} = 177.7 \text{ Nm}$$

6. Let M = total Mass; m = wheel mass; I_W = wheel inertia; I_E = Engine Inertia; Ω = Wheel Angular Velocity; β = Overall Gear Ratio; $\beta = \frac{\Omega_E}{\Omega}$; R = wheel rolling radius

From Kinematics, $R\Omega = v$ hence

$$\text{KE} = \underbrace{\frac{1}{2}Mv^2}_{\text{Translational KE}} + \underbrace{4\left\{\frac{1}{2}I_W\Omega^2\right\}}_{\text{Wheel KE}} + \underbrace{\frac{1}{2}I_E\Omega_E^2}_{\text{Engine KE}}$$

$$\text{KE} = \frac{1}{2}\left[M + 4\frac{I_W}{R^2} + \beta^2\frac{I_E}{R^2}\right]v^2$$

$$\text{KE} = \frac{1}{2}\left[1278 + 29\beta^2\right]v^2$$

Power supply from Engine

$$\tau\Omega_E = \tau\beta\frac{v}{R}$$

Use Power Law

$$\tau\frac{\beta}{R}v = \left\{1278 + 29\beta^2\right\}va$$

$$\Rightarrow \qquad a = \frac{800\beta}{1278 + 29\beta^2}$$

Third gear	β = 5.29	:-	a = 2.025 m/s^2
Fifth gear	β = 3.1	:-	a = 1.593 m/s^2

Air resistance: Drag force is of the form F_D = Area × C_D × $\frac{1}{2}\rho v^2$, so power loss to Air Resistance is proportional to v^3