Dynamics 2

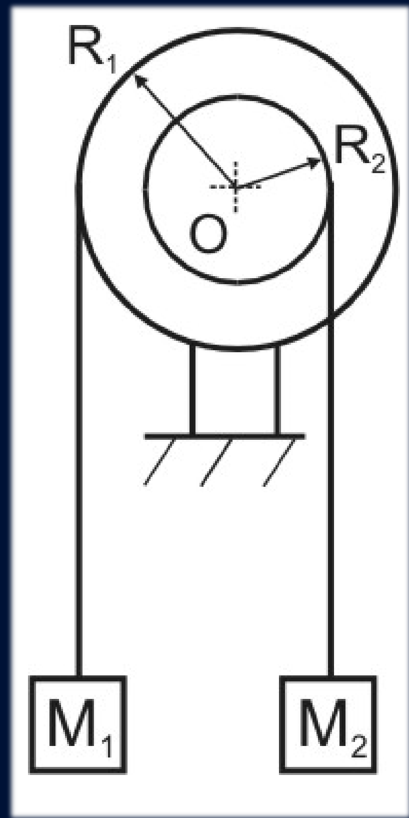
FAR – Kinematics of Rotating Systems
(Dynamics of Systems of Bodies)

Kinematics of Rotating Systems

- can use system Kinematics to solve dynamic problems
  - eg in Example 2.5 (Pulley & Masses)
- several rotating systems of interest
  - friction drives
  - belt drives
  - meshed gears

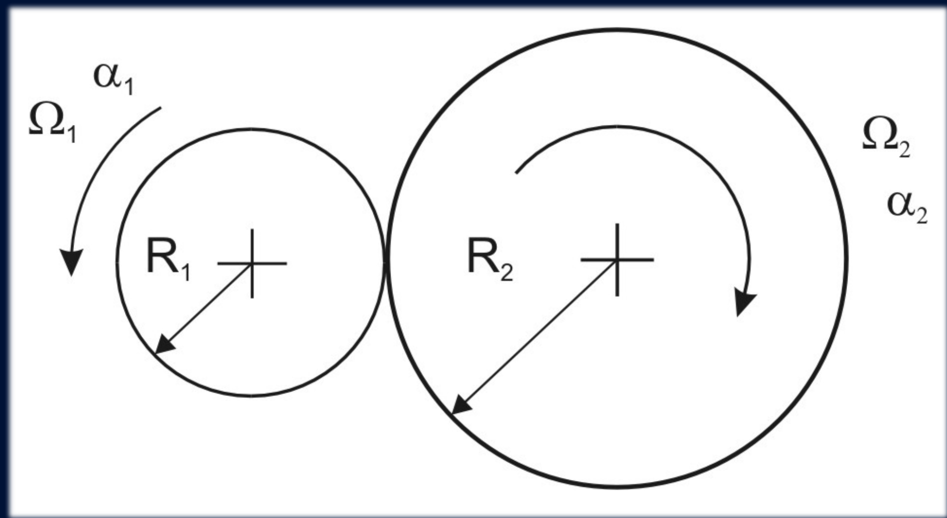
Example 2.5

- Double pulley loaded by 2 masses
- If the masses are released from rest, determine
  - pulley angular acceleration
  - accelerations of the masses
- Data
  - Pulley moment of inertia about its central axis is 22 kgm²
  - M_1 = 24 kg, M_2 = 50 kg
  - R_1 = 0.45 m, R_2 = 0.32 m



Friction Drives

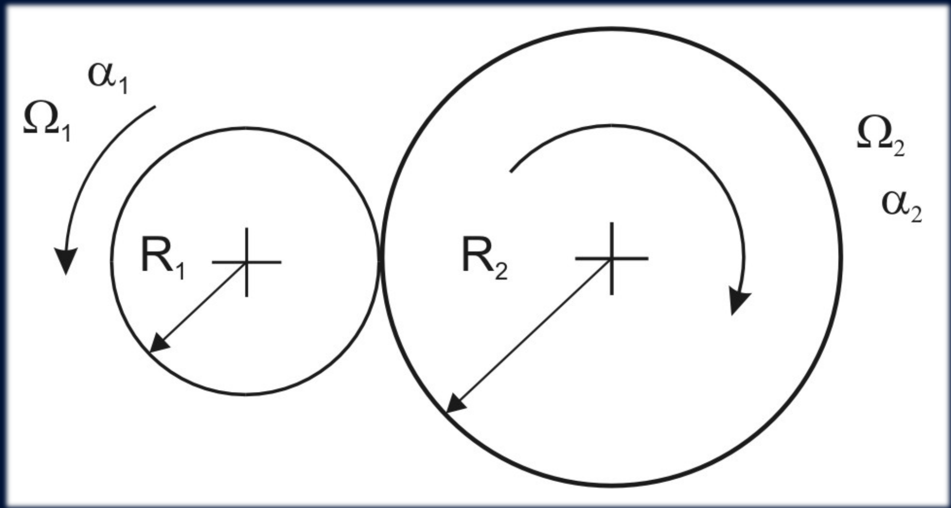
- no slipping $\Rightarrow$ no relative motion at contact point
  - tangential velocities of discs are equal, ie $v_1 = v_2$



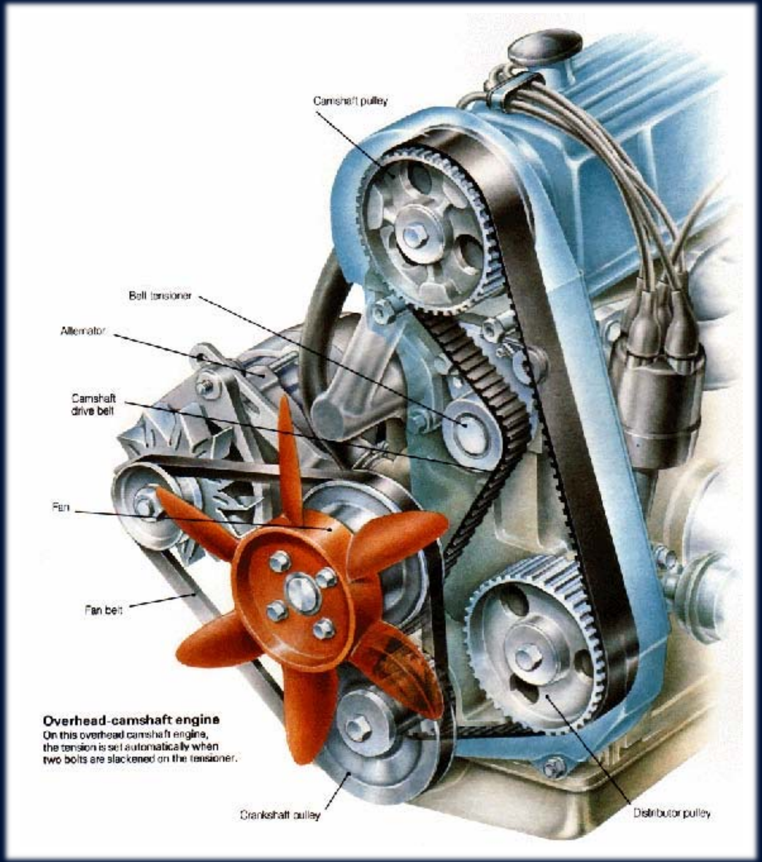
Friction Drives

Friction Drives

- no slipping $\Rightarrow$ no relative motion at contact point
  - tangential velocities of discs are equal, ie $v_1 = v_2$
- also $v = R\Omega$
- giving $R_1\Omega_1 = R_2\Omega_2$
- or $$\Omega_2 = \frac{R_1}{R_2}\Omega_1$$
- And similarly for angular acceleration $$\alpha_2 = \frac{R_1}{R_2}\alpha_1$$

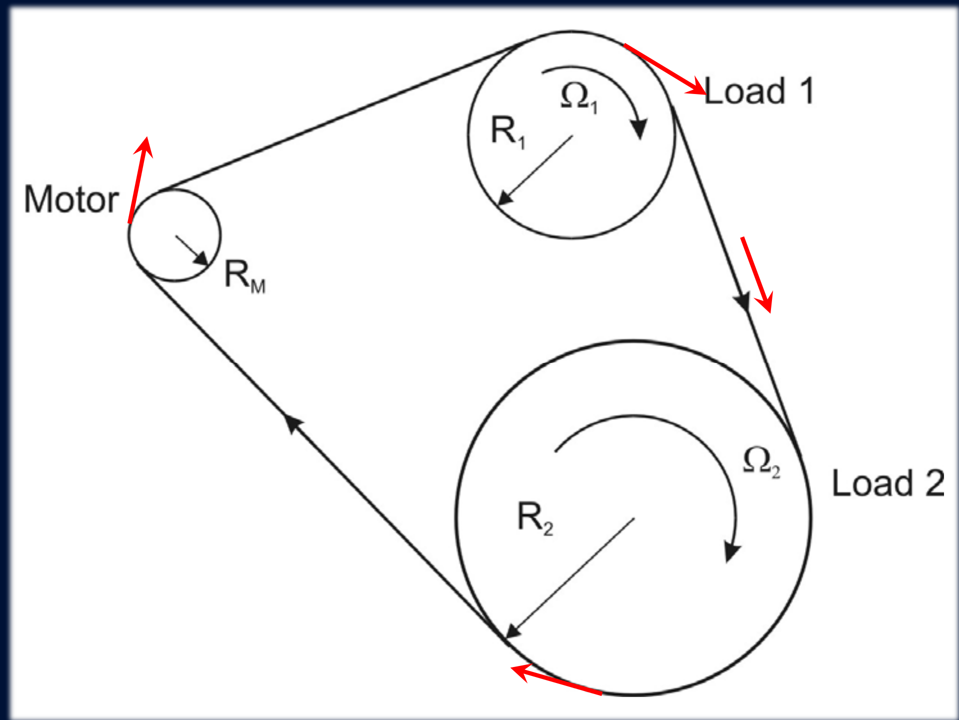


Belt Drives


Camshaft pulley
Belt tensioner
Alternator
Camshaft drive belt
Fan
Fan belt
Overhead-camshaft engine
On this overhead camshaft engine, the tension is set automatically when two bolts are slackened on the tensioner.
Crankshaft pulley
Distributor pulley

Belt Drives

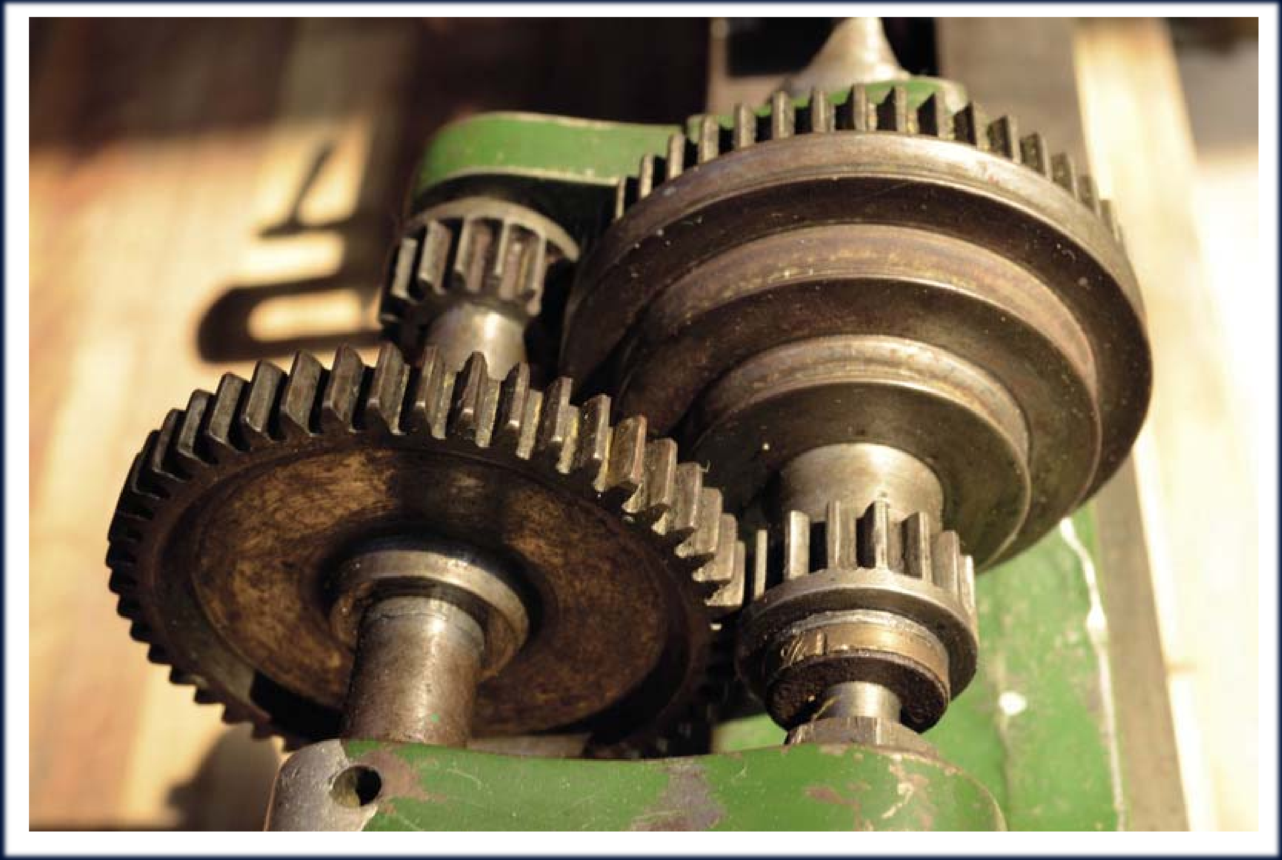
- similar argument
  - to prevent belt stretching/slackening the tangential velocities of the load discs equal that of motor

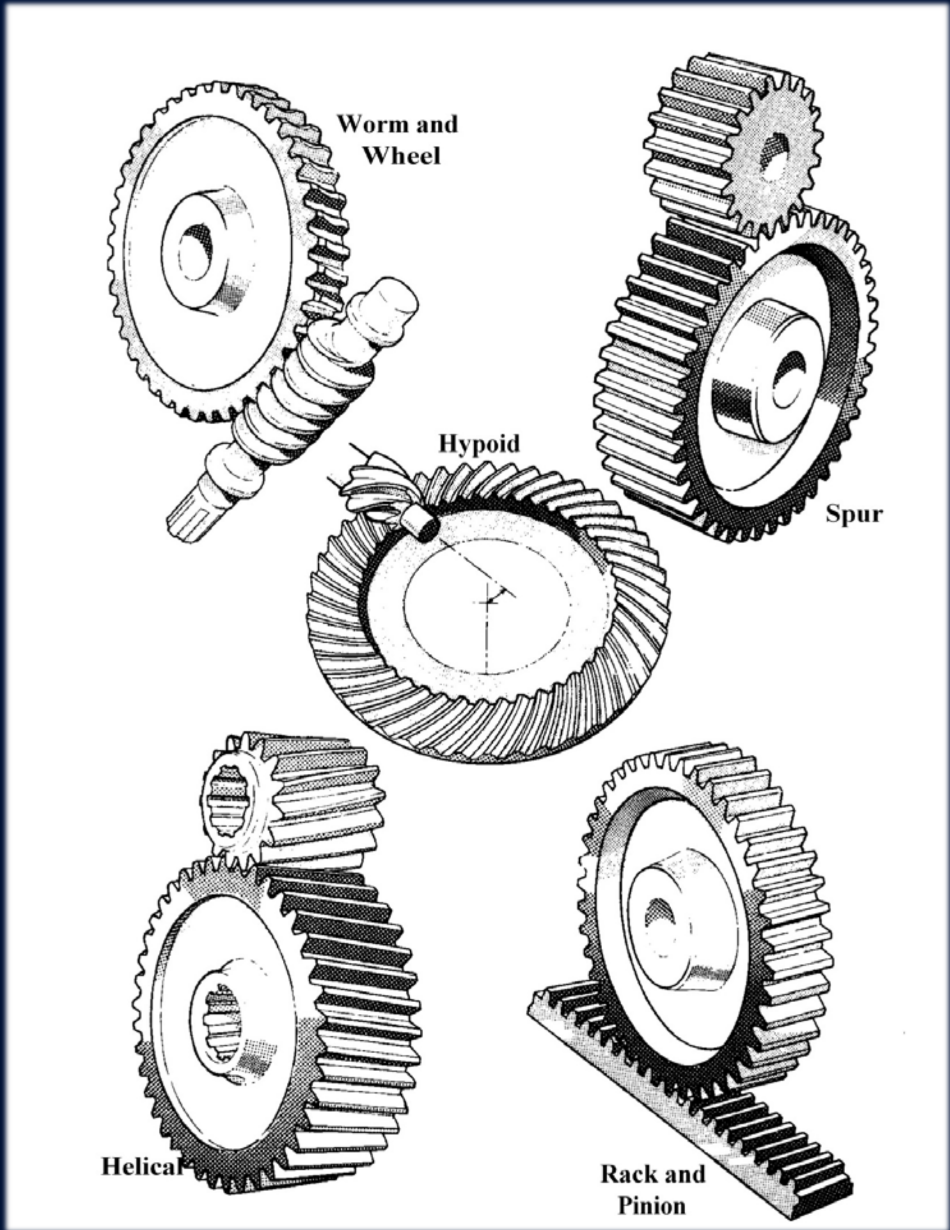


- hence

$$R_M \Omega_M = R_1 \Omega_1 = R_2 \Omega_2$$

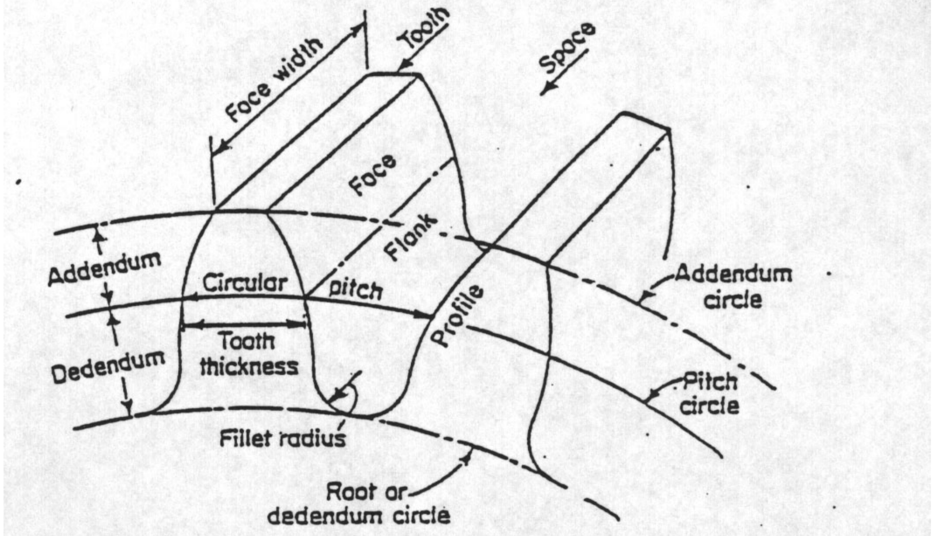
Meshed Gears



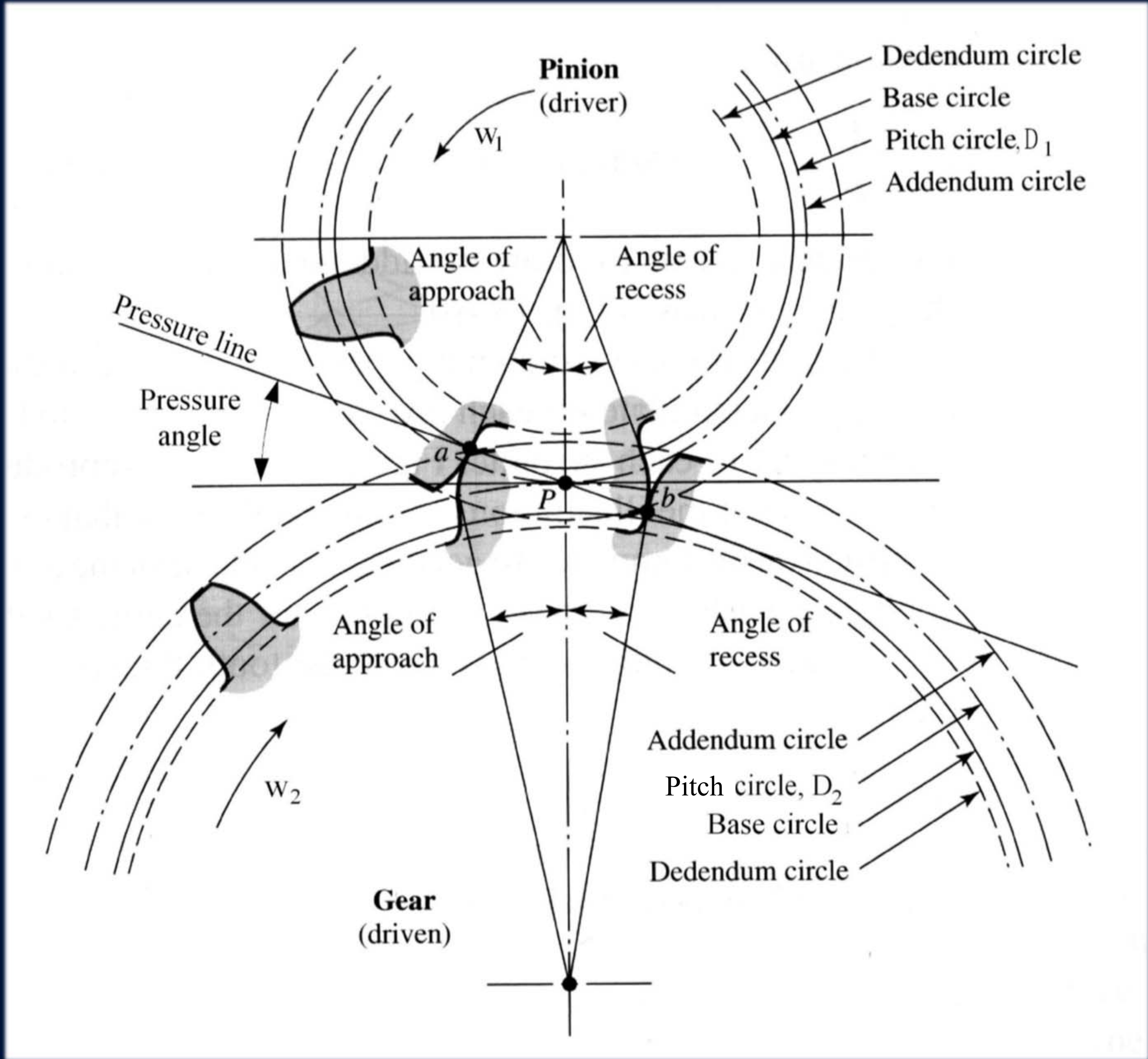

Worm and Wheel
Spur
Hypoid
Helical
Rack and Pinion

Meshed Gears

- gear teeth manufactured to achieve close to uniform transmission of rotary motion between wheels



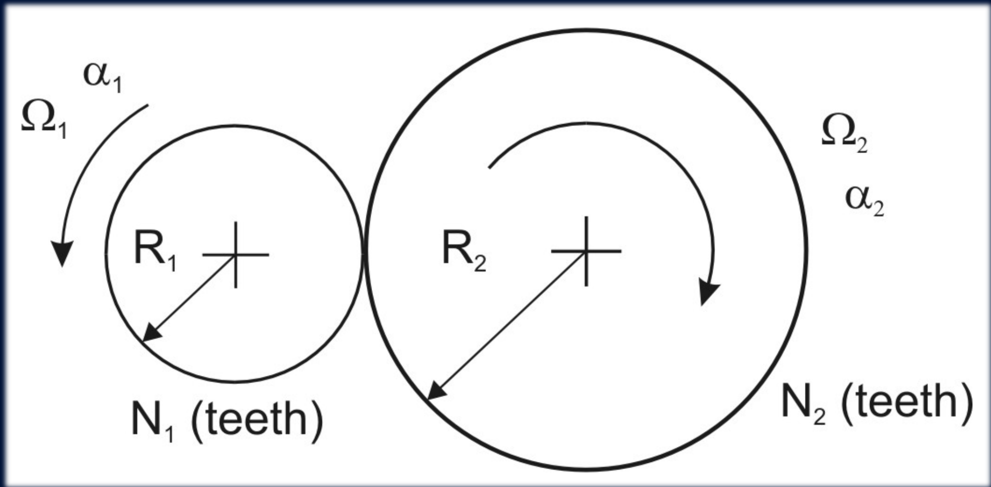
part of Spur gear-wheel showing usual terminology



Pinion
(driver)
w_1
Dedendum circle
Base circle
Pitch circle, D_1
Addendum circle
Angle of approach
Angle of recess
Pressure line
Pressure angle
a
P
b
Angle of approach
Angle of recess
Addendum circle
Pitch circle, D_2
Base circle
Dedendum circle
w_2
Gear
(driven)

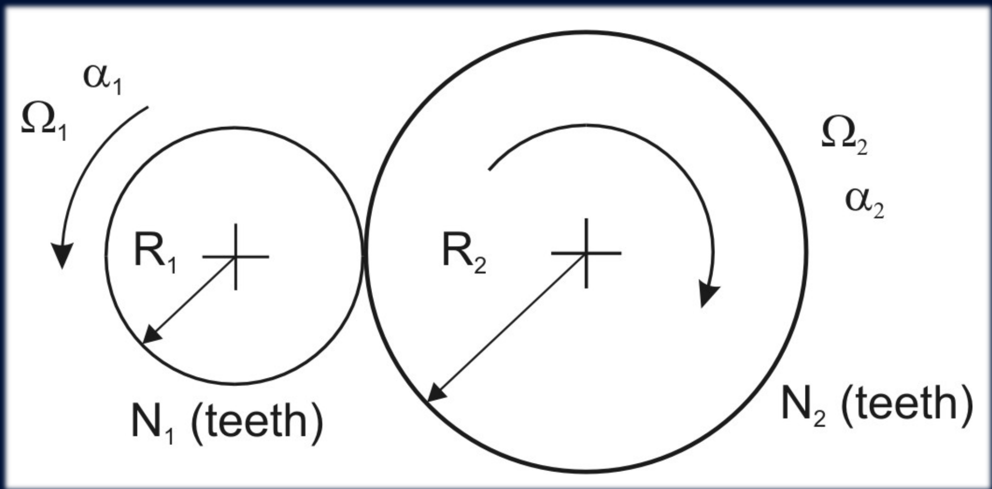
Meshed Gears

- perfectly meshed gears are represented as two discs in perfect rolling contact
  - no slip at contact or 'pitch' point
- effective radii of circles R_1 & R_2
  - pitch circle radii of the gears



Meshed Gears

- for correct meshing
  - circular pitch of the teeth must be identical



Meshed Gears

- for correct meshing
  - circular pitch of the teeth must be identical
- thus $\dfrac{2\pi R_1}{N_1} = \dfrac{2\pi R_2}{N_2}$ or $\dfrac{R_1}{R_2} = \dfrac{N_1}{N_2}$
- the result for friction drive was

$$\Omega_2 = \frac{R_1}{R_2}\Omega_1$$

- hence for meshed gears
- $\Omega_2 = \dfrac{N_1}{N_2}\Omega_1$ and $\alpha_2 = \dfrac{N_1}{N_2}\alpha_1$

Summary

- kinematics of rotating systems
- friction drives
- belt drives
- meshed gears

Example 2.9

- motor operated cable drum (∅ 1.2 m) raises 850 kg load
- motor's gear has 34 teeth
- drum gear G has 164 teeth
- $I_{motor/gear} = 1.8\ kgm^2$
- $I_{drum/gear} = 72\ kgm^2$
- motor armature torque 850 Nm
- calculate the upwards acceleration of the load
  - ignore cable/pulley masses