Tutorial 10 – SOLUTIONS

Tutorial 10: Diesel & Brayton Cycles

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Conceptual Questions:

1. How does an ideal Otto cycle differ from an ideal Diesel cycle?

Solution:

- The difference between the ideal Otto and ideal Diesel cycle is in the heat addition process. For Otto cycles, heat is added at constant volume, while for Diesel cycles heat is added at constant pressure.

2. Is the thermal efficiency of an Otto cycle expected to be higher or lower than that of a Diesel cycle? Why

Solution:

- Thermal efficiency is a function of compression ratio. Diesel engines have a higher compression ratio than petrol cycles operating on the Otto cycle, thus under conventional operation, diesel engines have higher thermal efficiencies than spark-ignition petrol engines that operate on the Otto cycle.

3. Why does the Otto cycle have a lower compression ratio than the Diesel cycle?

Solution:

- The Otto cycle operates with a petrol fuel, which is resistive to auto-ignition, while the diesel cycle operates with a diesel fuel, which is easier to react spontaneously and cause heat release via compression ignition. In the diesel cycle, fuel is injected late during the injection stroke and can ignite upon injection. The fuel self-ignites because the air is at a high temperature/pressure due to the higher compression. The petrol fuel cannot operate on such a high compression ratio because the fuel is often introduced into the engine with the air. Thus the air and fuel are both compressed. If the compression ratio is raised for the Otto cycle, all of the fuel's energy can be released at once and cause high P/T which can damage the engine.

4. An ideal Otto cycle with specified compression ratio is executed using (a) air, (b) argon, and (c) ethane as the working fluid. For which working fluid will the thermal efficiency be the highest? Why?

Solution:

- The efficiency of the Otto cycle can be expressed as: $\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$. At a fixed compression ratio, the only parameter that will change is *k*. For each working fluid: $k_{air} = 1.4$, $k_{argon} = 1.667$, and $k_{ethane} = 1.186$. The thermal efficiency increases when *k* is larger. Thus the efficiency will be the highest for argon. If we presume a compression ratio of 9: $\eta_{th,Argon} = 77\%, \eta_{th,air} = 58\%, \eta_{th,ethane} = 33\%$

Problem Solving Questions

5. Consider an air-standard, ideal Diesel cycle has a compression ratio of 14. Air exists at T_1 = 30°C and 100 kPa at the beginning of the compression process and the temperature increases to T_3 = 1450K at the end of the heat addition process. Assume air behaves as an ideal gas with constant specific heats with R = 0.287 kJ/kgK and C_v = 0.717 kJ.kgK.

a) Determine the cutoff ratio

b) Determine the heat rejected per unit mass

c) Determine the thermal efficiency.

[ans: a) v_3/v_2 = 1.66, b) $\boldsymbol{q_{14} = 226.19\frac{kJ}{kg}}$*, c)* $\boldsymbol{\eta_{th,Diesel} = 61\%}$ *]*

Solution: ideal Diesel cycle

- State 1: P_1 = 100 kPa; T_1 = 303.15K, $v_1 = RT_1/P_1$ = 0.870 m^3/kg
- State 2: (isentropic relations): $T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = T_1 * r^{(k-1)} = 871.2K$
    - $v_2 = \frac{v_1}{r} = 0.06215\, m^3/kg,\ \ P_2 = \frac{RT_2}{v_2} = 4023.3\ kPa$
- State 3: $P_3 = P_2$ = 4023.3 kPa, T_3 = 1450K, $v_3 = RT_3/P_3$ = 0.1034 m^3/kg
- Cutoff ratio: v_3/v_2 = 1.66
- Process 2-3 (constant P heat addition): $q_{32} = h_3 - h_2 = C_p(T_3 - T_2)$
    - $C_P = R + C_v$ = 1.004 kJ/kgK
    - $q_{32} = C_p(T_3 - T_2) = 581.13kJ/kg$
- State 4: (isentropic relations): $T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = 618.6K$, $P_4 = RT_4/v_4$ = 204 kPa
- Process 4-1 (constant V heat rejection): $q_{14} = u_4 - u_1 = C_v(T_4 - T_1) = 226.19\frac{kJ}{kg}$
- $\eta_{th,Diesel} = 1 - \frac{q_{14}}{q_{32}} = 0.61 = 61\%$

6. A Diesel engine has 6 cylinders each with a bore of 0.1m and a stroke of 0.11m. The engine has a compression ratio of 19 and is operating at 2000 RPM (revolutions per minute - each cycle takes two revolutions to complete the engine cycle). The mean effective pressure is 1400 kPa. Assume air behaves as an ideal gas with constant specific heats with R = 0.287 kJ/kgK and C_v = 0.717 kJ.kgK.

a. With the total of 6 cylinders, find the engine power in kW and in horsepower (hp).

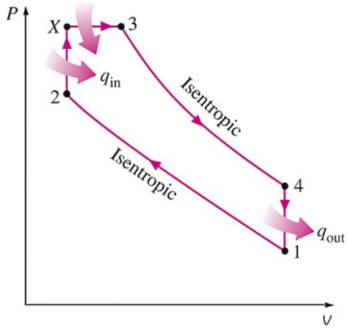
[ans: a) $\dot{W} = 121kW \rightarrow 162\ hp$***]***

Solution: ideal Diesel cycle

- $\Delta V = \frac{\pi}{4}bore^2 stroke = 0.000864m^3$
- $MEP = W_{net}/\Delta V \rightarrow W_{net} = MEP * \Delta V = 1.2096\frac{kJ}{cycle}$
- Every 2nd revolution provides this power.

- Thus: $\dot{W} = WxN_{cylinder}xRPM$. Here we need to keep track of units
  - $\dot{W} = 1.2096\frac{kJ}{cycle}x\,(6cylinders)\,x\,\left(2000\frac{rev}{min}\right)\,x\left(\frac{1cycle}{2rev}\right)\,x\,\left(\frac{1min}{60sec}\right) = 121kW$
  - $\dot{W} = 121kW \rightarrow 162\ hp$

7. An air-standard dual cycle has a compression ratio of 14 and cutoff ratio of 1.2. The pressure ratio during the constant-volume heat addition process is 1.5. The initial pressure and temperature at the start of the isentropic compression process are 80 kPa and 20°C. Assume air behaves as an ideal gas with constant specific heats with R = 0.287 kJ/kgK and C_v = 0.717 kJ.kgK.



a. Determine the maximum pressure and temperature reached in the cycle.

b. Determine the overall heat added to the system.

c. Determine the thermal efficiency and MEP.

[ans: a) $P_X = 4827.9\ kPa,\ T_3 = 1516.4\ K$, *b)* $q_{total} = 555.76\frac{kJ}{kg}$ *c)* $\eta_{th,dual} = 69.7\%;$ $MEP = 396.8\ kPa]$

Solution: ideal Dual cycle with air as an ideal gas

Define the states

- State 1: P_1 = 80 kPa; T_1 = 293.15K, $v_1 = RT_1/P_1$ = 1.052 m^3/kg
- State 2: (isentropic relations): $T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = T_1 * r^{(k-1)} = 842.4K$
  - $v_2 = \frac{v_1}{r} = 0.07512\,m^3/kg,\ \ P_2 = \frac{RT_2}{v_2} = 3218.62\ kPa$
- Process 1-2
  - $w_{21} = C_v(T_2 - T_1) = 393.8\frac{kJ}{kg}$
- Process 2-X (constant V heat addition): $q_{X2} = u_X - u_2 = C_v(T_X - T_2)$
  - State X: $v_x = v_2$ = 0.07512 m^3/kg, $P_x = 1.5(P_2)$ = 4827.93 kPa, $T_X = P_xv_x/R$ = 1263.67 K
  - $q_{X2} = C_v(T_X - T_2) = 302.01\ kJ/kg$
- State 3: $P_3 = P_X$ = 4827.93 kPa, v_3 / v_2 = 1.2 → v_3 = 0.09014 m^3/kg
  - $T_3 = P_3v_3/R$ = 1516.4K
- Process X-3 (constant P heat addition & expansion): $u_3 - u_X = q_{3X} - w_{3X}$
  - $w_{3X} = P_{3,X}(v_3 - v_X) = 72.53\frac{kJ}{kg}$
  - $q_{3X} = u_3 - u_X + w_{3X} = C_P(T_3 - T_X) = 253.74\frac{kJ}{kg}$
  - $q_{total} = q_{X2} + q_{3X} = 555.76\frac{kJ}{kg}$
- Process 3-4 (isentropic expansion): $w_{43} = u_3 - u_4 = C_v(T_3 - T_4)$

   - State 4: (isentropic relations): $T_4 = T_3 \left(\frac{v_3}{v_4}\right)^{k-1} = 527.7K$, $v_4 = v_1$;
     $P_4 = RT_4/v_4 = 204$ kPa
   - $w_{43} = C_v(T_3 - T_4) = 708.9 \frac{kJ}{kg}$
- Process 4-1 (constant V heat rejection): $q_{14} = u_4 - u_1 = C_v(T_4 - T_1) = 168.15 \frac{kJ}{kg}$
- $\eta_{th,Diesel} = \frac{w_{43}+w_{3X}-w_{21}}{q_{X2}+q_{3X}} = 1 - \frac{q_{14}}{(q_{3X}+q_{X2})} = 0.697 = 69.7\%$
- MEP = w_{net} / ($v_1 - v_2$) = 396.8 kPa

8. A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 9. Air enters the compressor at 300 K, 100 kPa and enters the turbine at 1300 K.

a) Determine the maximum pressure in the cycle.

b) Determine the back work ratio.

c) Calculate the thermal efficiency.

[ans: a) P_{max} = 900 kPa , b) BWR = 0.43, c) $\boldsymbol{\eta_{th}} = \mathbf{46.6}\%$]

Solution: ideal Brayton cycle

Define each process and find the state variables

- Process 1-2 (isentropic compression): 1st Law: $w_{21} = (h_2 - h_1) = C_P(T_2 - T_1)$
   - State 1: P_1 = 100 kPa, T_1 = 300 K
   - State 2: Isentropic relations: $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = T_1 * r_p^{\frac{(k-1)}{k}} = 562\ K$
     $P_2 = r_p * P_1$ = 900 kPa
   - $w_{21} = C_P(T_2 - T_1) = 263.1\ kJ/kg$
- Process 2-3 (constant pressure heat addition): $q_{32} = h_3 - h_2 = C_P(T_3 - T_2)$
   - $P_3 = P_2 = 900$ kPa = P_{max}
   - $q_{32} = h_3 - h_2 = C_P(T_3 - T_2) = 740.9 \frac{kJ}{kg}$
- Process 3-4 (isentropic expansion): $w_{43} = h_3 - h_4 = C_P(T_3 - T_4)$
   - State 4: isentropic relations: $T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 693.9\ K$
   - $w_{43} = C_P(T_3 - T_4) = 608.5\ kJ/kg$
- Back work ratio: w_c / w_t = 0.43
- Thermal efficiency: $\eta_{th,Brayton} = \frac{w_{net}}{q_H} = \frac{w_{43} - w_{21}}{q_{32}} = 0.466 = 46.6\%$

9. Consider an ideal closed Brayton cycle in which air enters the compressor at 300K, 100 kPa. Combustion adds 860 kJ/kg of heat in the burner. The maximum temperature is 1500 K. Assume air as an ideal gas with constant specific heats, C_P = 1.004 kJ/kgK, R = 0.287 kJ/kgK.

a) Determine the pressure ratio.

b) Determine the highest pressure in the cycle.

c) Determine the exhaust temperature (T_4) exiting the turbine.

d) Determine the cycle efficiency.

[ans: a) r_p = 14.45, b) P_{max} = 1445 kPa, c) T_4 = 699.4K, d) $\boldsymbol{\eta_{th}} = \mathbf{53.3}\%$]

<u>Solution</u>: ideal closed Brayton cycle

Define each process and find the state variables

- Process 1-2 (isentropic compression): 1st Law: $w_{21} = h_2 - h_1 = C_P(T_2 - T_1)$
  - State 1: P_1 = 100 kPa, T_1 = 300K
  - State 2: P_2 = ?, T_2 = ?
- Process 2-3 (constant volume heat addition): $q_{32} = h_3 - h_2 = C_P(T_3 - T_2)$
  - $T_2 = T_3 - \frac{q_{32}}{C_P} = 643.4\ K$
  - Isentropic relations (process 1-2): $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} = r_p = 14.45$
  - $P_2 = r_P P_1 = 1445\ kPa$
- State 3: $P_3 = P_2 = P_{max}$ = 1445 kPa
- State 4: isentropic relations (process 3-4)
  - $T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 699.4\ K$
- Process 1-2: $w_{21} = C_P(T_2 - T_1) = 344.8\ kJ/kg$
- Process 3-4: $w_{34} = C_P(T_3 - T_4) = 803.8\ kJ/kg$
- Efficiency: $\eta_{net} = \frac{w_{34} - w_{21}}{q_{32}} = 0.534 = 53.4\%$

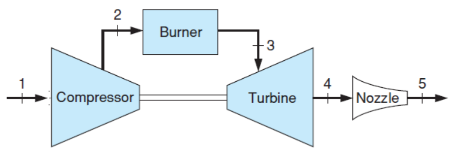
10. A jet aircraft operates on the open Brayton cycle. The aircraft is flying at an altitude of 4900 m where the ambient air pressure is 55 kPa and ambient air temperature is 260 K. The ambient air enters the compressor at these conditions with a mass flow rate of 10 kg/s. The pressure ratio across the compressor is 14. The maximum temperature of the cycle is 1450 K. The air exhausts from the turbine at 250 kPa and enters a nozzle where the air further exhausts to the atmospheric pressure of 55 kPa. Assume air behaves as an ideal gas with *variable* specific heats (i.e. use Table A7.1). Assume the turbine to operate isentropically, while compressor and nozzle operate adiabatically, but *irreversibly*.

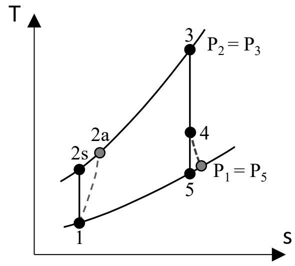
a. Determine energy required to operate the compressor in kW.

b. Determine heat delivered in the combustor in kW.

c. If the nozzle has an isentropic efficiency of 90%, determine the air velocity exiting the nozzle. Assume the velocity entering the nozzle is negligible to that leaving.

d. Determine the thrust force and propulsion efficiency if the aircraft is moving at a speed of 360 m/s.





[ans: a) $\dot{W}_c = 4099.8$*, b)* $\dot{Q}_{32} = 9050.9\ kW$ *c)* $V_{5,a} = 846.9\frac{m}{s}$*; d)* $F_{thrust} = 4869\ N$, $\eta_{propulsion} = 19.4\%$*]*

Solution: Open Brayton Cycle with variable specific heats

Define the states using table A7.1

- State 1: P_1 = 55 kPa; T_1 = 260 K, h_1 = 260.32 kJ/kg, s_{T1}^o = 6.72562 kJ/kgK
- State 2: $P_2 = 14*P_1$ = 770 kPa
  - Further information needed to define other state 2 variables (see analysis for process 3-4)
- State 3: P_3 = 770 kPa, T_3 = 1450 K, h_3 = 1575.40 kJ/kg, s_{T3}^o = 8.57111 kJ/kgK
- State 4: P_4 = 250 kPa
  - $s_4 - s_3 = s_{T4}^o - s_{T3}^o - Rln\left(\frac{P_4}{P_3}\right) = 0$
  - $s_{T4}^o = s_{T3}^o + Rln\left(\frac{P_4}{P_3}\right) = 8.24825\ kJ/kgK$
  - $Interpolation\ gives: T_4 = 1103.6\ K, h_4 = 1165.42\ kJ/kg$
- State 5: P_5 = 55 kPa
  - Find isentropic values first:
    - $s_{5,s} - s_4 = s_{T5}^o - s_{T4}^o - Rln\left(\frac{P_5}{P_4}\right) = 0$
    - $s_{T5}^o = s_{T4}^o + Rln\left(\frac{P_5}{P_4}\right) = 7.8137\ kJ/kgK$
    - $Interpolation\ gives: T_{5,s} = 749.4\ K, h_{5,s} = 766.95\ kJ/kg$

- Process 2-3: constant pressure heat addition
    - $\dot{Q}_{32} = \dot{m}(h_3 - h_2); need\ h_2\ (see\ process\ 3-4)$
- Process 3-4: isentropic expansion
    - $\dot{W}_{43} = \dot{m}(h_3 - h_4) = 4099.8\ kW$
    - $\dot{W}_{43} = \dot{W}_{compressor}$
    - $\dot{W}_{compressor} = \dot{m}(h_2 - h_1) \rightarrow h_2 = \frac{\dot{W}_{compressor}}{\dot{m}} + h_1 = 670.3\frac{kJ}{kg}$
    - $\dot{Q}_{32} = \dot{m}(h_3 - h_2) = 9050.9\ kW$
- Process 4-5: Expansion through nozzle with increase in velocity
    - Find isentropic velocity first:
        - 1st Law: $h_4 = h_{5,s} + \frac{1}{2}V_{5,s}^2$
        - $V_{5,s} = \sqrt{2\left(h_4 - h_{5,s}\right)} = 892.7\ m/s$
    - Find actual velocity based on nozzle isentropic efficiency
        - $\eta_{nozzle} = \frac{V_{5,a}^2 - V_4^2}{V_{5,s}^2 - V_4^2}$
        - $V_4 \sim 0 \rightarrow V_{5,a}^2 = \eta_{nozzle} * V_{5,s}^2 \rightarrow V_{5,a} = 846.9\ m/s$
        - $1st\ Law: h_4 = h_{5,a} + \frac{1}{2}V_{5,a}^2 \rightarrow h_{5,a} = 806.8\ kJ/kg$
        - $Interpolation\ gives: T_{5,a} = 785.9\ K$
- Find thrust force: $F_{thrust} = \dot{m}\left(V_5 - V_{aircraft}\right)$
    - $F_{thrust} = 10\frac{kg}{s}\left(846.9\frac{m}{s} - 360\frac{m}{s}\right) = 4869N$
- Propulsion power: $\dot{W}_{propulsion} = F_{thrust}V_{aircraft}$
    - $\dot{W}_{propulsion} = 4869N * 360\frac{m}{s} = 1752.8\ kW$
- Propulsion efficiency: $\eta_{propulsion} = \frac{\dot{W}_{propulsion}}{\dot{Q}_{32}} = \frac{1752.8kW}{9050.9kW} = 19.4\%$