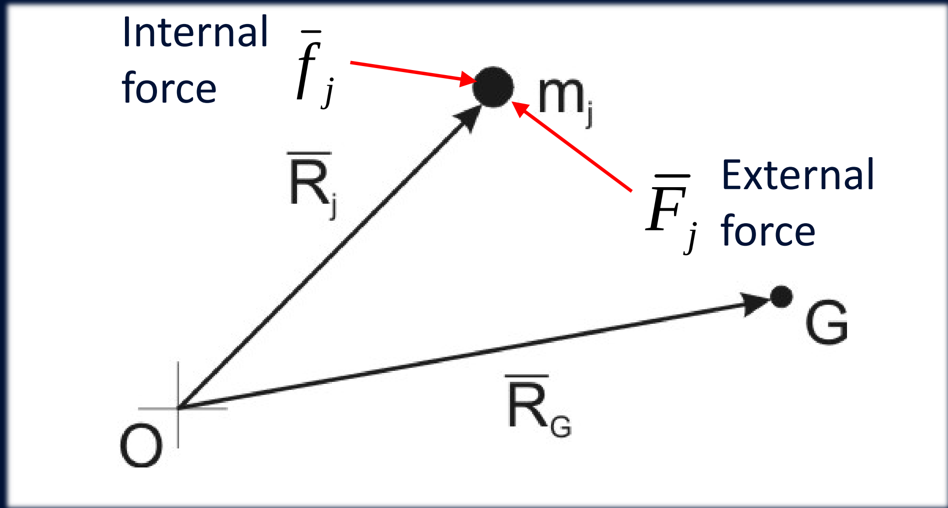
# Dynamics 2

Dynamics of General Systems

(Dynamics of Systems of Bodies)

Law of Motion of Mass Centre

# Law of Motion of Mass Centre




- N2 applies to particle $m_j$

$$m_j \ddot{\bar{R}}_j = \bar{F}_j + \bar{f}_j$$

- N2 for all system particles

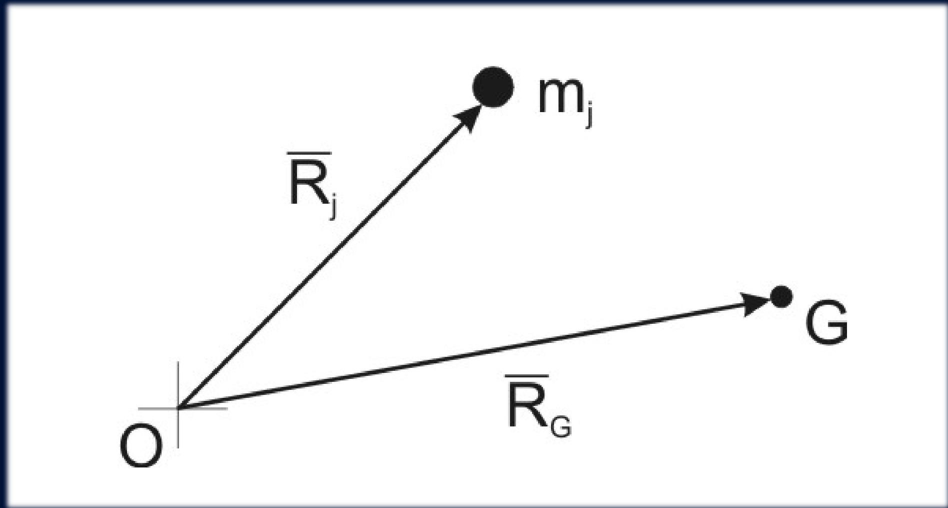
$$\sum^{n} m_j \ddot{\bar{R}}_j = \sum^{n} \bar{F}_j + \cancel{\sum^{n} \bar{f}_j}$$

- RHS: 2nd sum = 0 (sum of internal forces)
- LHS: acceleration of G

$$\sum m_j \ddot{\bar{R}}_j = \frac{d^2}{dt^2}\left(\sum m_j \bar{R}_j\right) = \frac{d^2}{dt^2}\left(M\bar{R}_G\right) = M\ddot{\bar{R}}_G$$

- So $\boxed{M\ddot{\bar{R}}_G = \sum \bar{F}_j}$ ie System Mass × Acceleration of G = Sum of External Forces

# Law of Motion of Mass Centre

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- So $M\ddot{\overline{R}}_G = \sum \overline{F}_j$ **ie System Mass × Acceleration of G = Sum of External Forces**

# Law of Motion of Mass Centre

- we have shown that

**system mass × acceleration of G**

**= (vector) sum of system external forces**

- this parallel result to N2 applies to any collection of particles
  - Generalised Newton 2 or GN2
- the D'Alembert form
  - the sum of external forces and system inertia force = 0
- system inertia force acts as for particles
  - FBD in balance in any direction

# General Moments Theorem

- it can be proven that for any system:

**sum of moments of external forces and particle inertia forces about any point is zero**

- this will be used in study of rigid body motion

# Summary

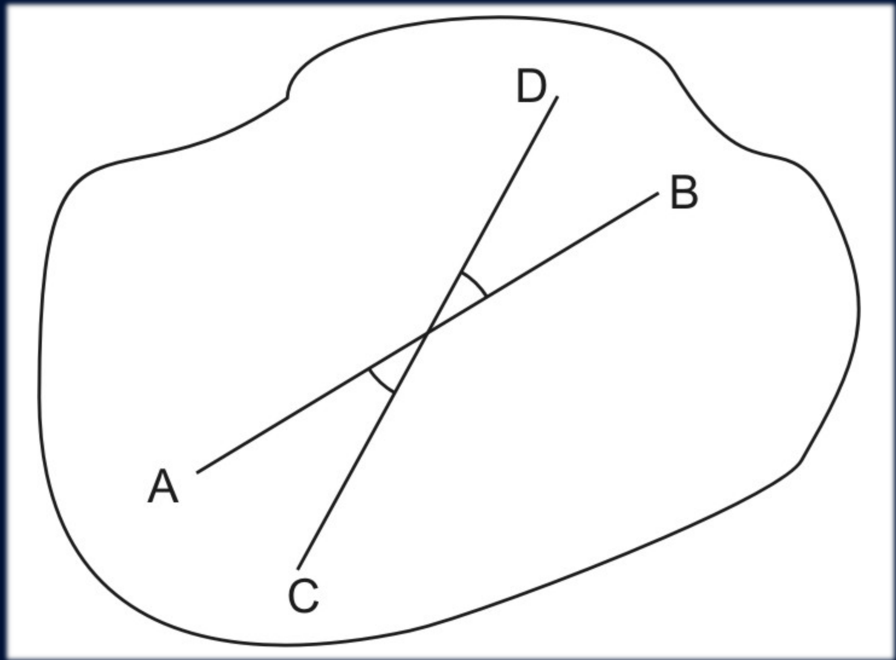
- Generalised Form of Newton’s Second Law
- General Moments Theorem

# Dynamics 2

## Rigid Body Motion & Pure Translation
## (Dynamics of Systems of Bodies)

# Rigid Body Motion in a Plane

- ‘rigid’ implies no deformation during motion
- the length of any inscribed line remains constant
- the angle between any pair of lines remains constant



# Rigid Body Motion in a Plane

- ‘rigid’ implies no deformation during motion
- the length of any inscribed line remains constant
- the angle between any pair of lines remains constant

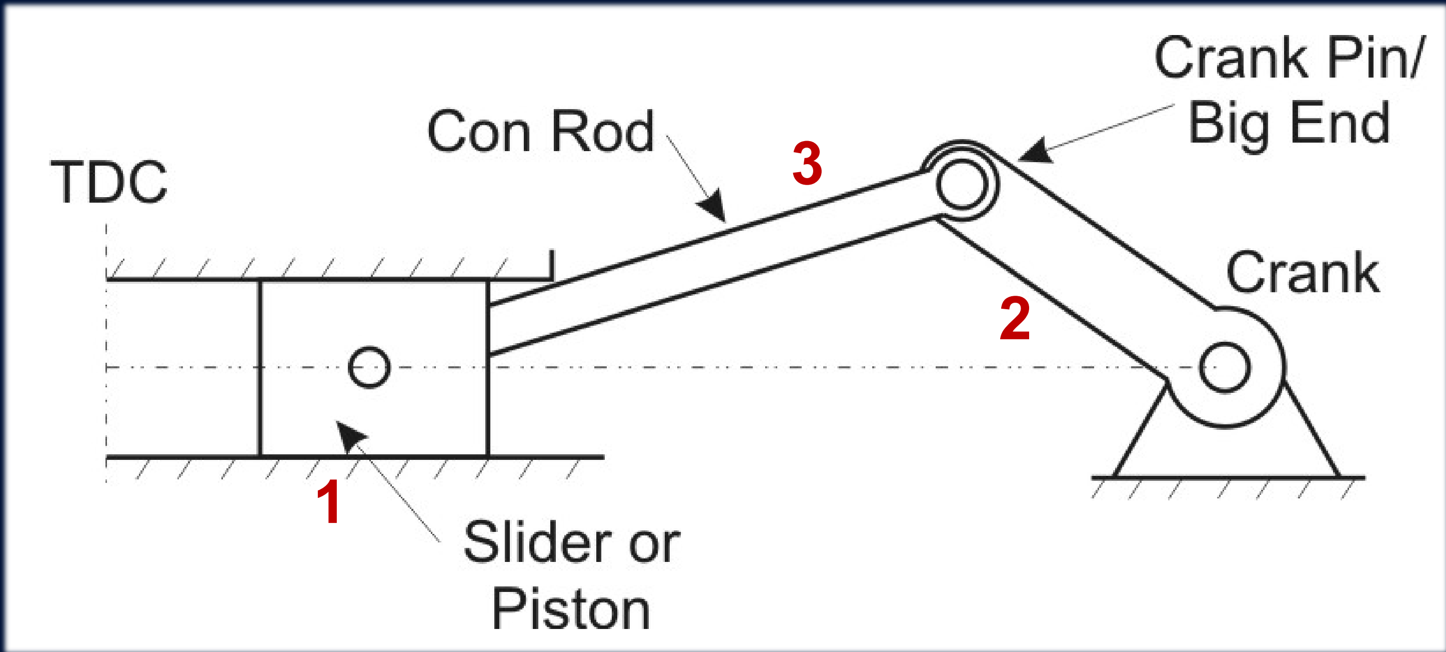
- Kinematics
  - velocities at any two points are not independent
  - same velocity components along line connecting them
  - velocity of one relative to other is at right angles to connecting line
  - i.e. relative to one point the other has circular motion

# Rigid Body Motion & Connected Systems

- we have established General Laws of Dynamics for a System of Mass Particles. We now apply these to:
  - Plane Motion of a Rigid Body
  - Systems of Rigid Connected Bodies

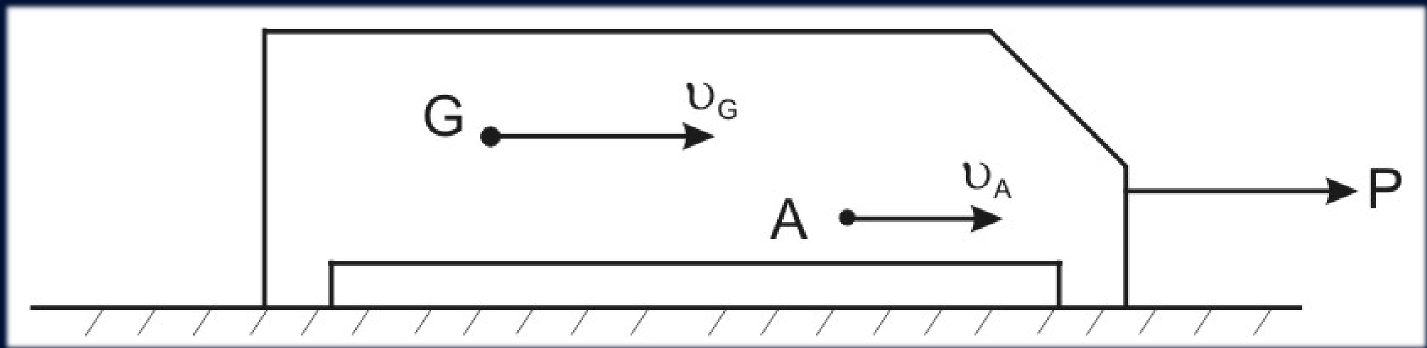
- there are 3 cases of Rigid Body Motion:
  - Pure Translation: body moves without rotation (1)
  - Fixed Axis Rotation: body is restricted to rotational motion about a designated axis (2)
  - General Plane Motion: motion is a combination of translation and rotation (3)

# Rigid Body Motion & Connected Systems



# Pure Translation

- pure translation = no rotation
  - all lines in the body remain in fixed directions during the motion
- kinematics implies that all points have
  - same velocity and same acceleration



## Laws of Motion in PT

- GN2 applies (in d'Alembert form)
  ie. sum of external forces (including weight) plus system inertia forces = 0

- General Moment Theorem Applies
  ie. sum of moments of external forces (including weight) plus moments of particle inertia forces about any point = 0

# Laws of Motion in PT

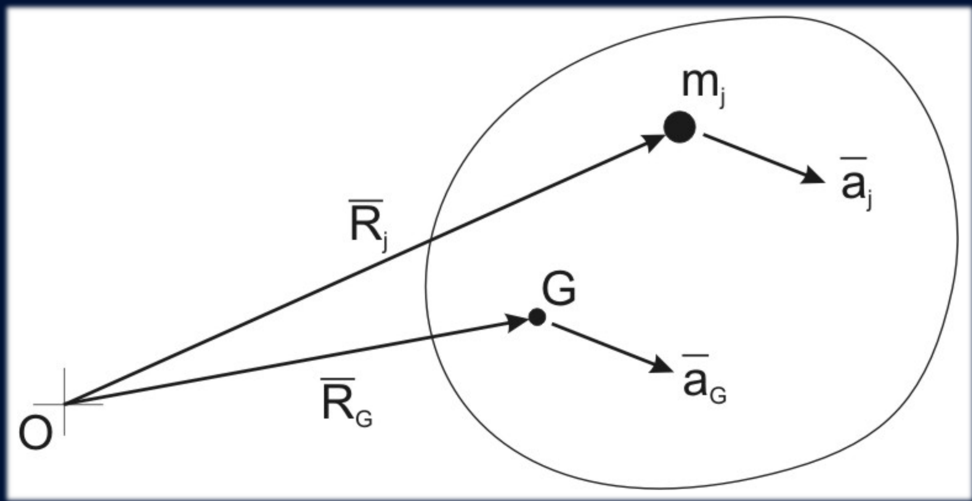
- acceleration is the same for all particles

$$\bar{a}_j = \bar{a}_G = \bar{a}$$

- sum of moments of particle inertia forces about O

$$= \sum \bar{R}_j \times m_j(-\bar{a}_j) = \left(\sum m_j \bar{R}_j\right) \times (-a)$$

$$= M\bar{R}_G \times (-\bar{a}) = \bar{R}_G \times M(-\bar{a})$$



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  - ie moment of system inertia force located at G

- gives the moment law for PT: **sum of moments of external forces plus moment of inertia forces at G = 0**
- FBD must have force and moment balance

# Summary

- Examined rigid body motion
- Defined laws of motion for pure translation

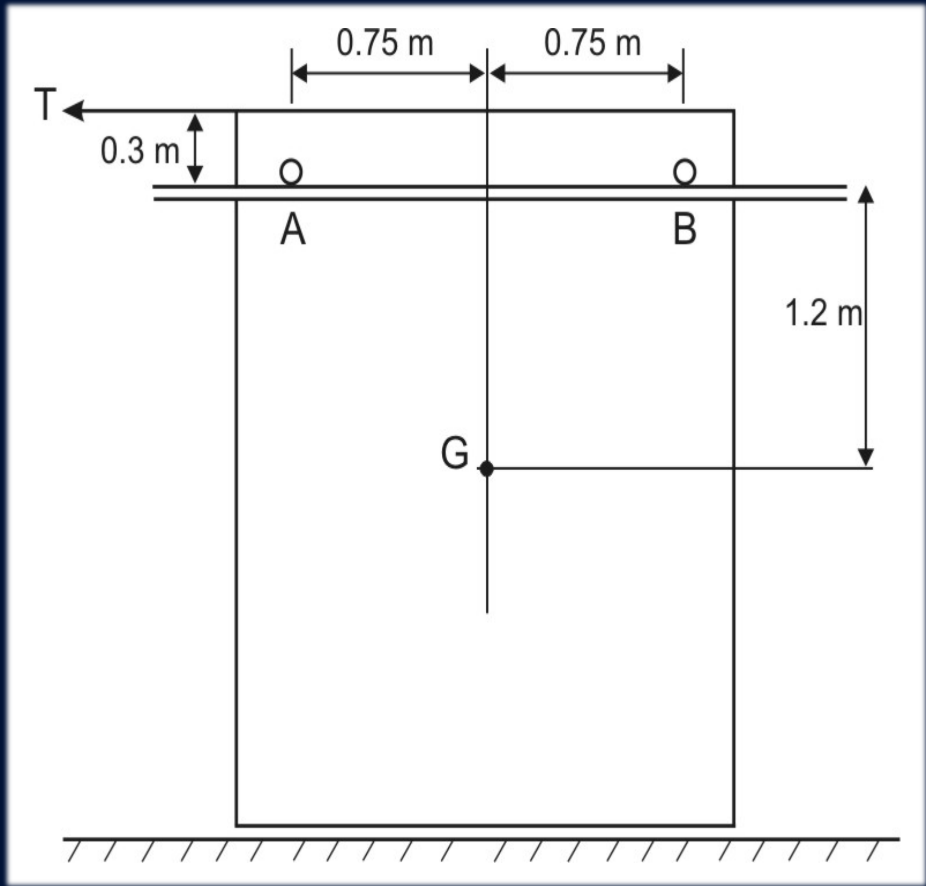
# Dynamics 2

## Rigid Body Motion & Pure Translation
## (Dynamics of Systems of Bodies)

### Worked Example

# Example 2.1

- door on rollers on rail
  - actuated by a cable
  - Mass of door is 200kg
  - rollers are frictionless supports
- what cable tension is required to move the door at 3.6 $m/s^2$ to the left?
- what are the roller loads?
- if acceleration is too high the door lifts off roller and touches floor
  - max accel to avoid this?



# Example 2.3

- Accelerating vehicle - approx case of PT
  - why "approximately"?
- Complete the FBD for a front wheel drive vehicle
  - What is the effect on $V_1$ and $V_2$?
  - Why is this important?
- What is the effect of
  - Rear WD & braking?
- Why are front brakes more powerful than rear brakes?