Dynamics 2

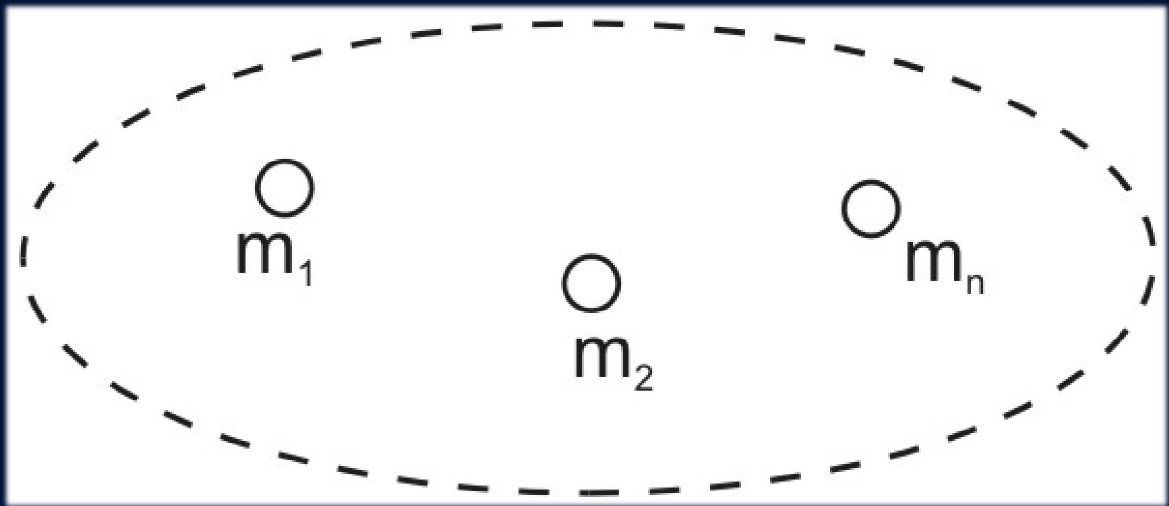
Dynamics of General Systems
(Dynamics of Systems of Bodies)
Introduction

Dynamics of a General System

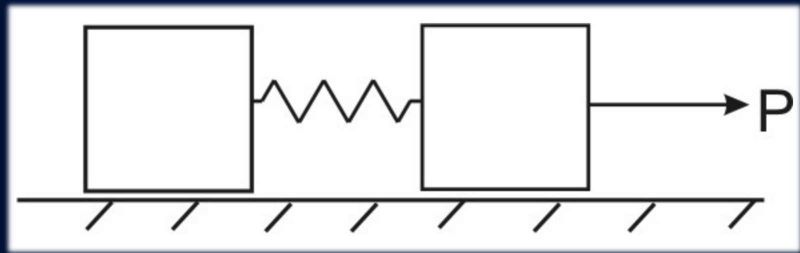
- So far we have investigated
  - the dynamics of a single mass particle
  - used N2 (in d'Alembert's formulation) to examine dynamic behaviour
- Now we extend to wider range of applications
  - bodies
  - assemblies of bodies
- Examine the abstract general case
  - a system of particles

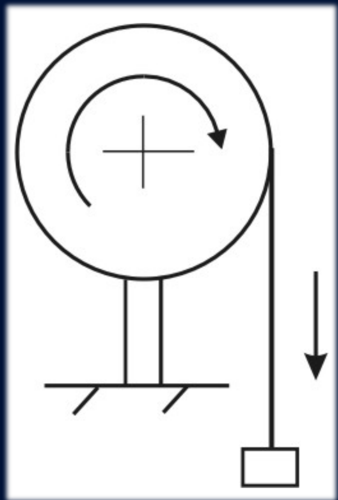
Dynamics of a General System

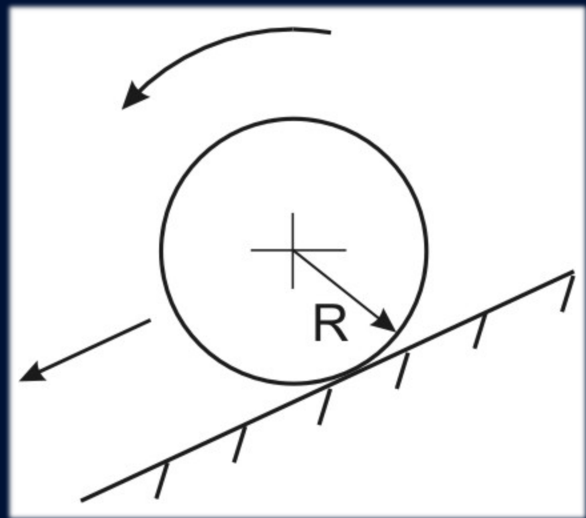
- Systems of Particles have the following properties
  - a boundary (which can be arbitrarily designated)
  - a set of mass particles (joined up/interacting)
  - constant total mass
  - centre of mass G where the system's total weight acts



Examples of Systems







Dynamics of a General System

- In the following sections we will obtain the following important basic results:

  - Definition of internal and external forces in a system
  - Two theorems concerning internal forces
  - Location properties of G
  - Generalised form of Newton’s Second Law for a system
  - Generalised moment theorem for a system

Internal and External Forces

Internal and External Forces

- Distinction between internal or external forces rests on N3
  - internal forces - reactions also act on system
  - external forces - reactions act outside the system

System Internal Force Theorems

- Internal forces occur in equal and opposite collinear pairs, which both act on the system - it can be shown that:
  - the (vector) sum of internal forces for any system is zero
  - the sum, about any point, of the moments of internal forces for any system is zero

Summary

- Defined a body or system of bodies with some examples
- Considered two basic theorems for systems of internal forces

Dynamics 2

Dynamics of General Systems

(Dynamics of Systems of Bodies)

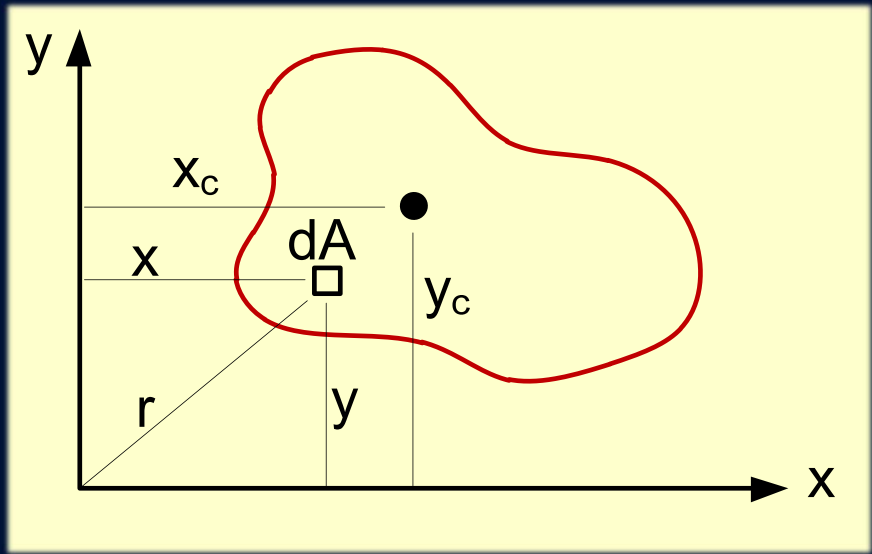
Centre of Gravity

Location of the System Centre of Gravity

- G is where total weight of the system is considered to act

- In a gravity field
  - the total weight acting at G gives the same moment as the sum of the moments of the particle weights
  - It applies about any reference point

Calculation of Centroids

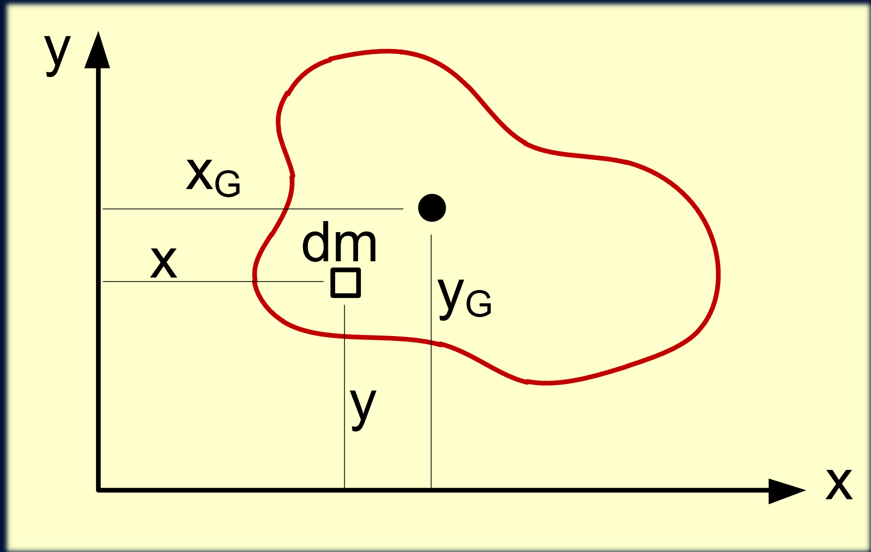


In general this is found from the first moment of area, divided by total area:

$$x_c = \frac{\int x \, \mathrm{d}A}{A} \quad \text{and} \quad y_c = \frac{\int y \, \mathrm{d}A}{A}$$

where x_c and y_c are the coordinates of the centroid and $A = \int \mathrm{d}A$

Calculation of Centre of Gravity



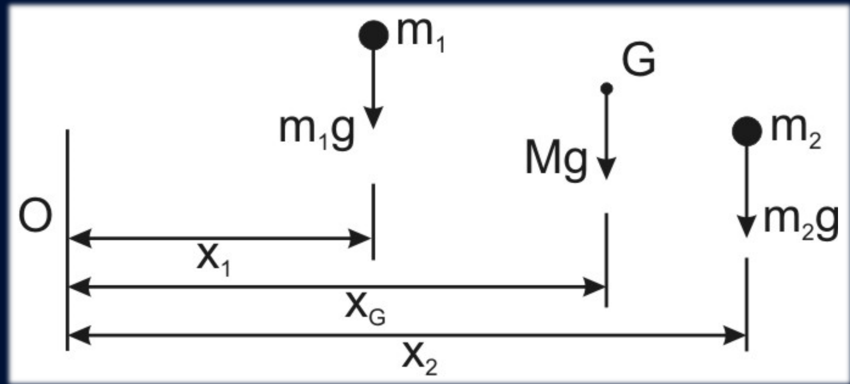
In general this is found from the first moment of mass, divided by total mass:

$$x_G = \frac{\int x \, \mathrm{d}m}{M} \quad \text{and} \quad y_G = \frac{\int y \, \mathrm{d}m}{M}$$

where x_G and y_G are the coordinates of the centroid and $M = \int \mathrm{d}m$

Locating G

- Masses m_1 and m_2
- G is distance x_G from O



- Take moments about O

$$x_G Mg = x_1 m_1 g + x_2 m_2 g$$

$$M x_G = m_1 x_1 + m_2 x_2$$

- In general

$$M x_G = \sum m_j x_j \qquad [\text{also } M y_G = \sum m_j y_j \; ; \; M z_G = \sum m_j z_j]$$

- For continuous bodies can use integrals

Vector form

- $\bar{R}_G$ = the position vector of G from O
  - it has three components x_G, y_G, z_G
- $\bar{R}_j$ = position vector of a typical particle m_j from O
- G for a system: $M\bar{R}_G = \sum^{n} m_j \bar{R}_j$

Vector form

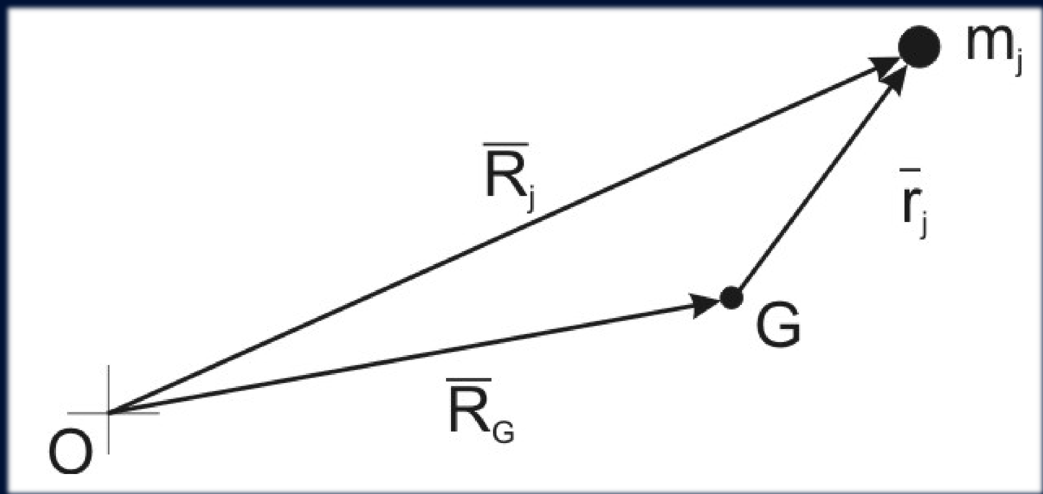
- Extending this, $\bar{r}_j$ = position vector of m_j from G

$$\bar{R}_j = \bar{R}_G + \bar{r}_j \qquad \text{[vector addition]}$$

- giving

$$M\bar{R}_G = \sum m_j(\bar{R}_G + \bar{r}_j)$$

$$\therefore \quad M\bar{R}_G = \sum m_j\bar{R}_G + \sum m_j\bar{r}_j$$



Vector form

- Extending this, $\bar{r}_j$ = position vector of m_j from G

$$\bar{R}_j = \bar{R}_G + \bar{r}_j$$ [vector addition]

- giving

$$M\bar{R}_G = \sum m_j(\bar{R}_G + \bar{r}_j)$$

$$\therefore \quad M\bar{R}_G = \sum m_j\bar{R}_G + \sum m_j\bar{r}_j$$

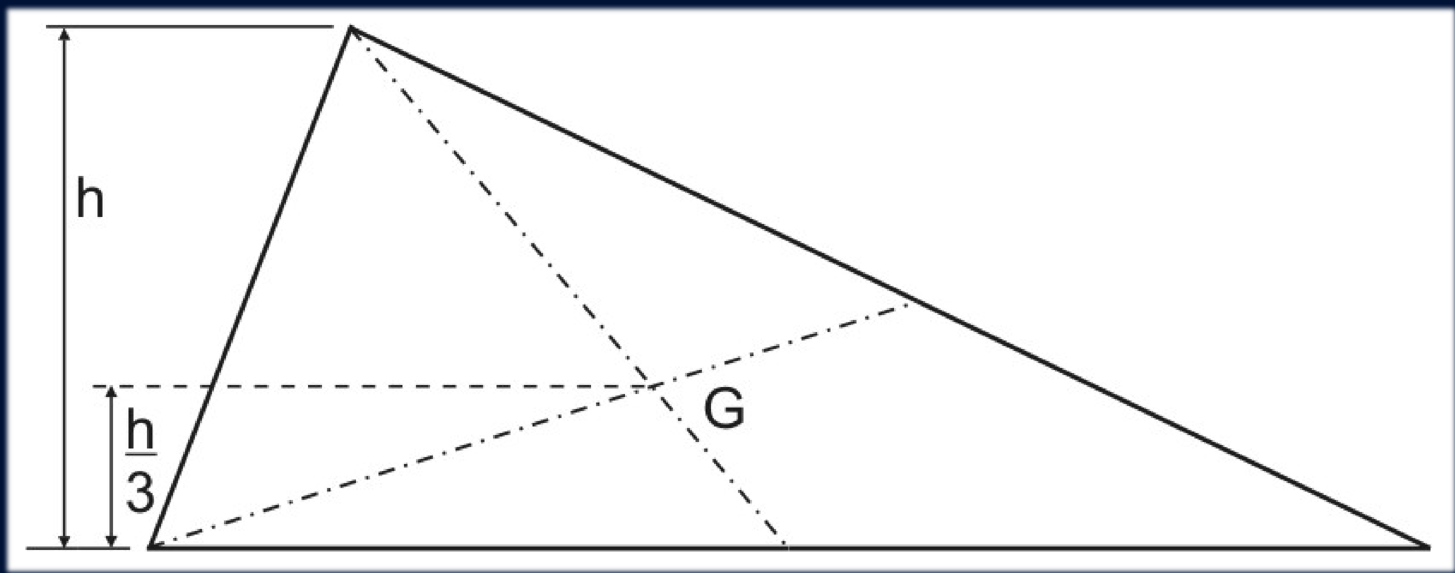
- The 1st term on the RHS is $(\sum m_j)\bar{R}_G = M\bar{R}_G$

$$\therefore \quad \sum m_j\bar{r}_j = 0$$

- Key property of G: the 1st moment of the system mass about G = 0

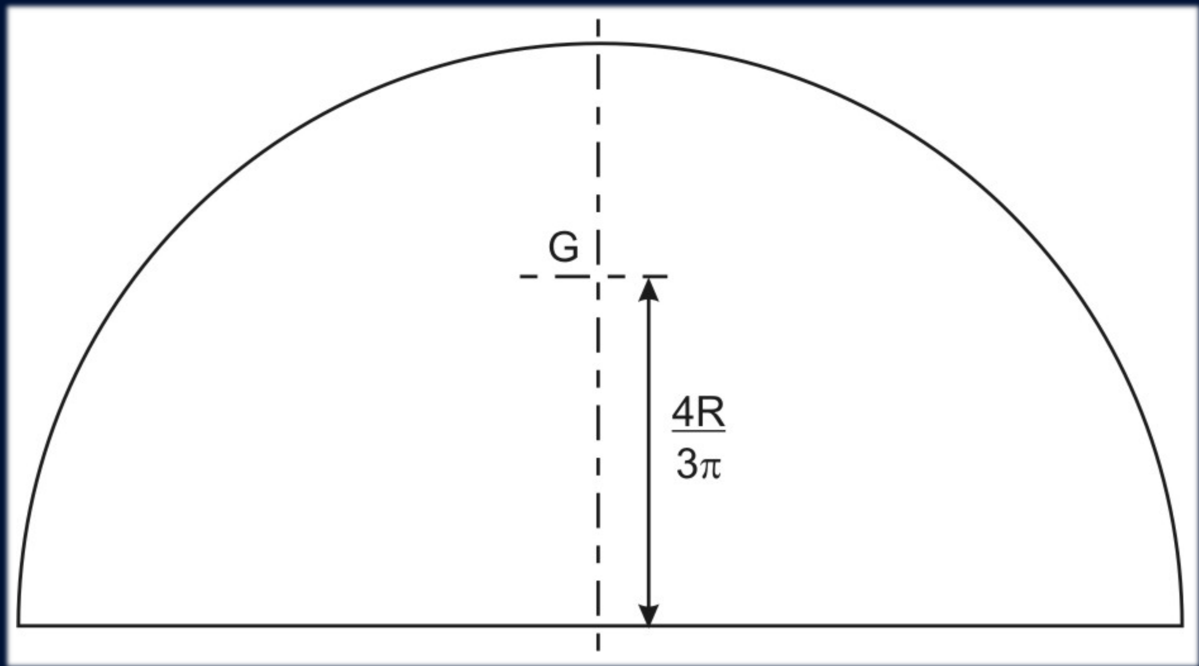
Locating G

- triangular plate



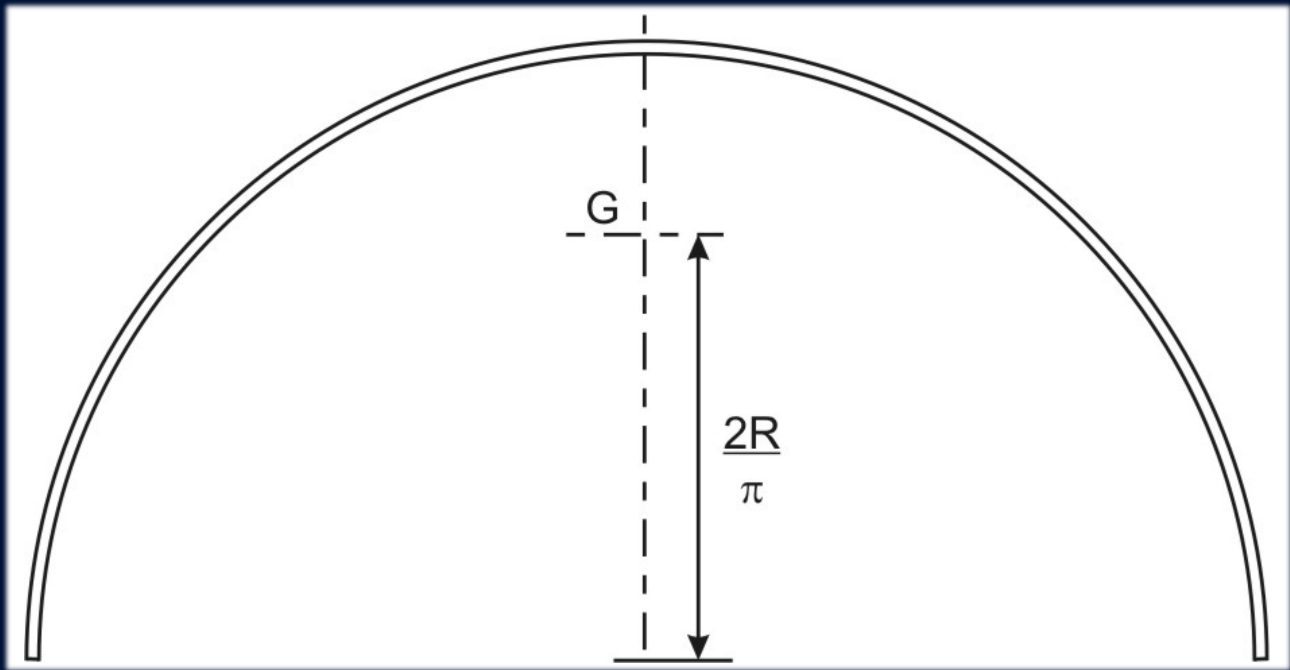
Locating G

- semi-circular plate



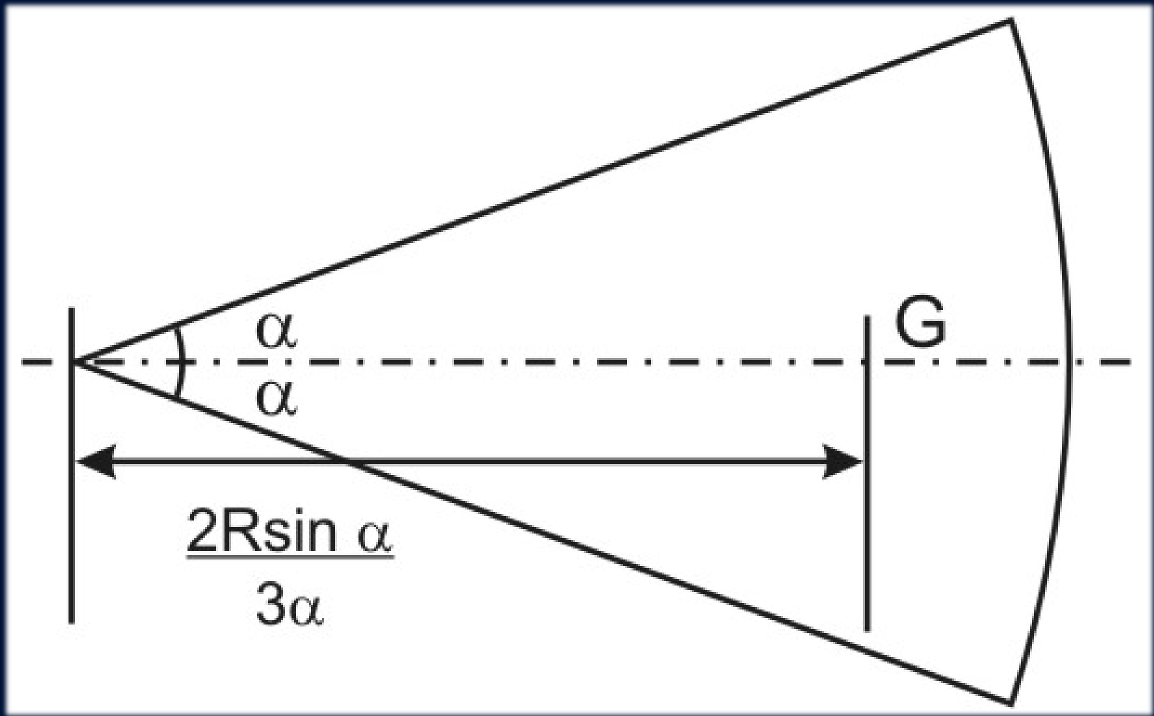
Locating G

- semi-circular ring



Locating G

- sector plate



Summary

- Total weight acting at G gives the same moment as the sum of moments of particle weights
- Demonstrated that the first moment of the system mass about G = 0

Dynamics 2

Dynamics of General Systems

(Dynamics of Systems of Bodies)

Centre of Gravity: Worked Example

Example 2.2

- We have a shaped uniform plate of mass 420 kg
- We need to lift it vertically (with crane) at 2.7 m/s²
  - top edge to remain horizontal
- So attach cable to top edge
  - but where? what is the tension in cable?