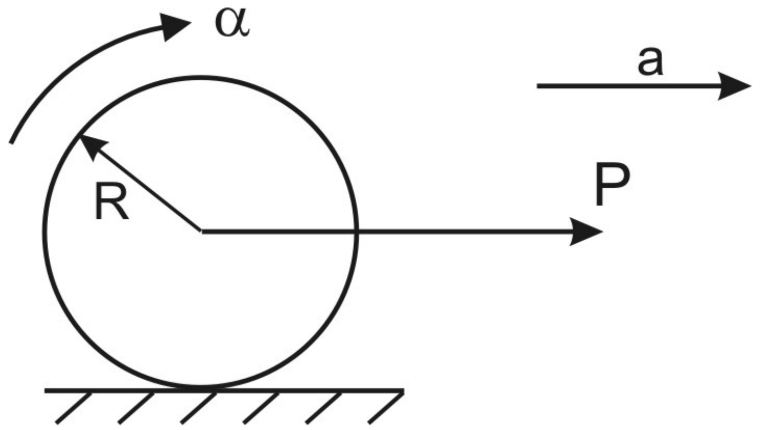
Dynamics 2 – Tutorial 7

Bodies and Systems with General Plane Motion

Outline Solutions

1.



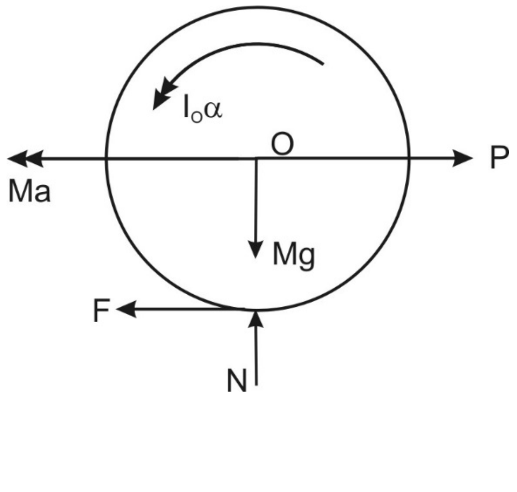
Data:
M = 35kg
R = 0.45
a = 6m/s^2
P = ?

α = Angular acceleration

$\alpha = \frac{a}{R}$ for pure rolling

Disc is in General Plane Motion

FBD of Disc (Note Inertia Force and Inertia Couple)



From FBD get:

$$P - Ma - F = 0$$
$$N - Mg = 0$$

Taking moments about contact point

$$PR - MaR - I_O\alpha = 0$$

Rearrange and use kinematic relationships to get

$$P = \frac{1}{R}\left[MaR + \frac{I_O a}{R}\right] = \left[M + \frac{I_O}{R^2}\right]a$$

And noting that $I_o = 0.5*MR^2$, gives

$$P = \left[35 + \frac{1}{2}\frac{(35)0.45^2}{0.45^2}\right]6 = 315 \text{ N}$$

Then get

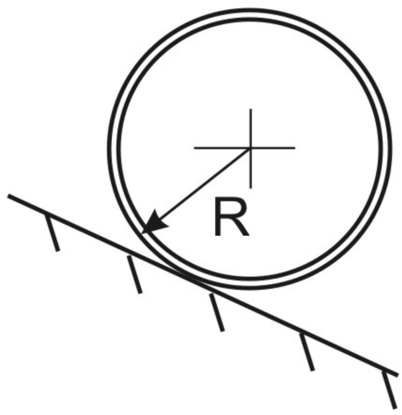
$$F = P - Ma = 105 \text{ N}$$

Is the assumption of pure rolling correct?

It is if $F < F_{MAX}$

$$F_{MAX} = \mu N = \mu Mg = 0.35(35)9.81 = 120.2 \text{ N}$$

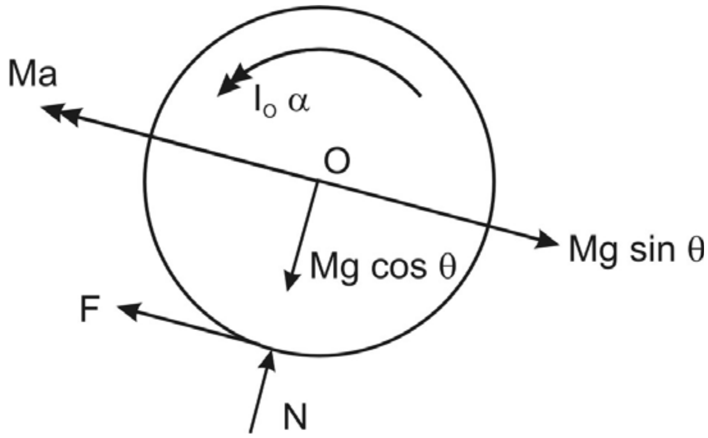
i.e., pure rolling is possible.



Let *a* be the acceleration down slope.

$\alpha = \dfrac{a}{R}$ for rolling

FBD



From FBD

$$Mg\sin\theta - Ma - F = 0$$
$$Mg\cos\theta = N$$

Moments about contact point

$$I_O\alpha + MaR - MgR\sin\theta = 0$$

Also

$$\alpha = \frac{a}{R}$$

For this ring,

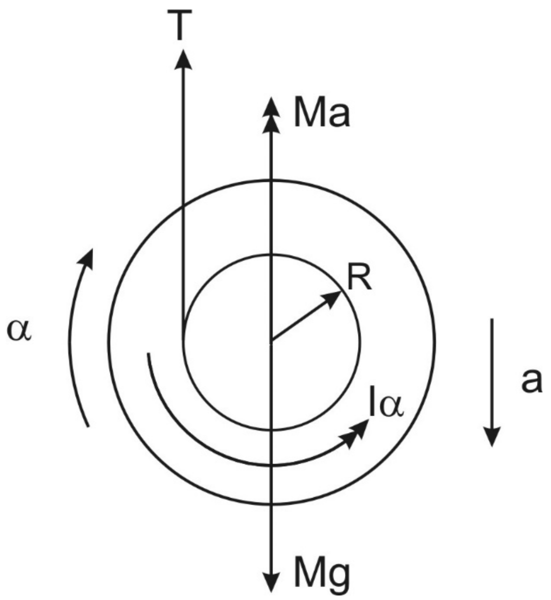
$$I_O = MR^2$$

Moment equation then gives:

$$M_O R^2 \frac{a}{R} + MaR = MgR\sin\theta$$

Gives

$$a = \frac{g}{2}\sin\theta$$



Assume downwards acceleration = a

$$\alpha = \frac{a}{R}$$

where
R is spindle radius and
T is string tension

From FBD

$$T + Ma - Mg = 0 \qquad (1)$$

Taking moments about centre

$$TR - I\,\alpha = 0 \qquad (2)$$

Gives

$$TR = I\frac{a}{R}$$

$$\Rightarrow \quad T = I\frac{a}{R^2} \qquad (3)$$

Put (3) in (1)

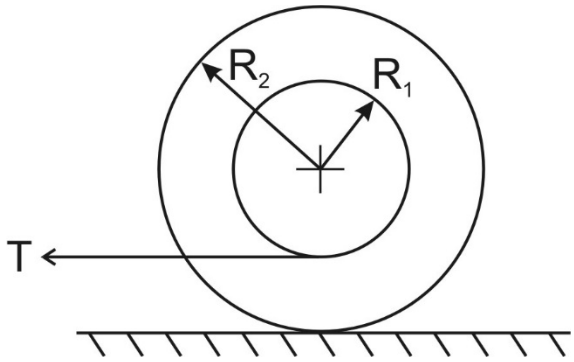
$$I\frac{a}{R^2} + Ma - Mg = 0$$

Gives

$$a = \frac{Mg}{\left[M + \dfrac{I}{R^2}\right]}$$

Substituting values (M = 0.1 kg; R = 0.012 mm; $I = MK^2 = 0.1 \times 0.06^2 = 0.00036$ kgm^2)

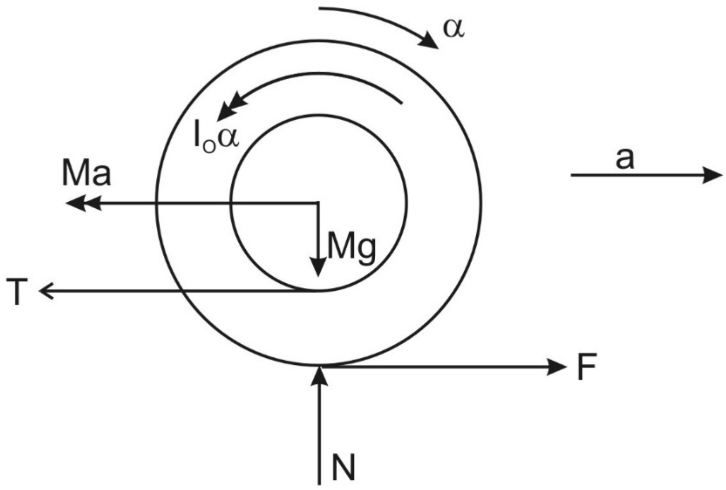
$$\Rightarrow \quad a = 0.377 \text{ m/s}^2$$



Mass = M

Moment of Inertia = I_O

Assume cable drum rolls **to the right** with acceleration a. The FBD is



Kinematics

$$\alpha = \frac{a}{R_2}$$

From the FBD

$$F - T - Ma = 0 \qquad \text{(F unknown)}$$

Take moments about contact point to avoid F (As all moments are anticlockwise the choice of direction for rolling is looking suspect!)

$$MaR_2 + I_O\alpha + T(R_2 - R_1) = 0$$

$$\Rightarrow \quad MaR_2 + I_O\frac{a}{R_2} + T(R_2 - R_1) = 0$$

This is going to give a negative result for a

$$a = \frac{-T(R_2 - R_1)}{\left[MR_2 + \dfrac{I_O}{R_2}\right]}$$

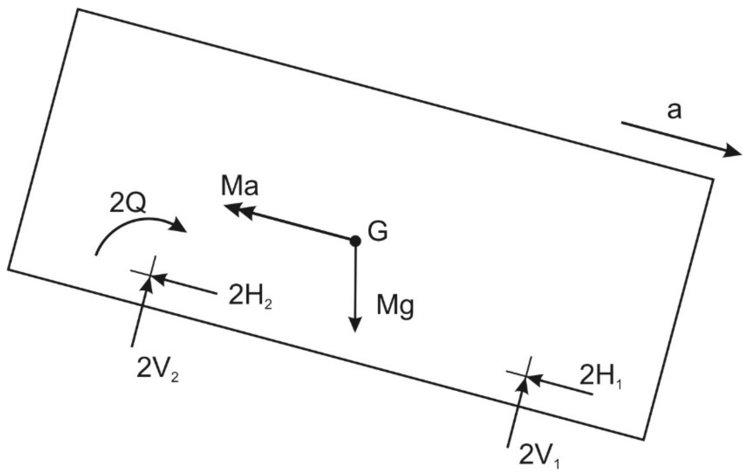
It is definitely negative since $R_2 > R_1$ and the denominator is positive. Hence the assumption was not valid. Starting again and assuming that a is **to the left** gives

$$a = \frac{T(R_2 - R_1)}{\left[MR_2 + \dfrac{I_O}{R_2}\right]}$$

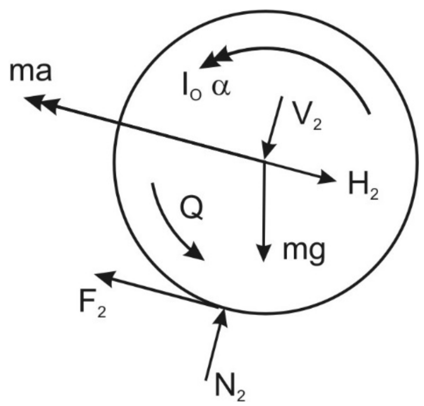
As *a* is positive the assumption is valid.

Assume rolling down the slope with acceleration a. Brake torque Q acts on each rear wheel, opposing the angular acceleration. M = body mass = 260 kg, m = wheel mass = 35 kg. R = 0.35; $I_O = \frac{1}{2}(35)(0.35^2) = 2.144$ kgm^2

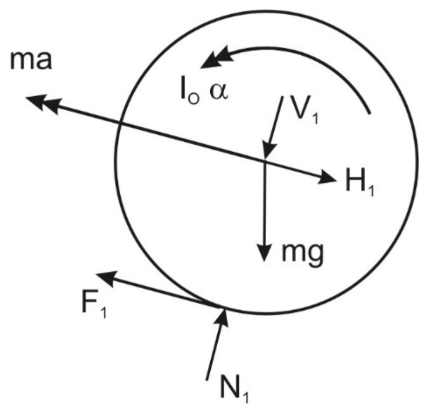
Body FBD



Rear Wheel FBD



Front Wheel FBD



From sum of forces on body parallel to slope

$$Mg\sin\theta - Ma - 2H_1 - 2H_2 = 0 \tag{1}$$

From rear wheel by moments about contact point

$$(H_2 + mg\sin\theta - ma)R - I_O\frac{a}{R} - Q = 0 \tag{2}$$

Front wheels

$$(H_1 + mg\sin\theta - ma)R - I_O\frac{a}{R} = 0 \tag{3}$$

Rearranging 2 and 3 and inserting in 1

$$Mg\sin\theta - Ma - 2\left[ma - mg\sin\theta + I_O\frac{a}{R^2}\right] - 2\left[ma - mg\sin\theta + I_O\frac{a}{R^2} + \frac{Q}{R}\right] = 0 \tag{4}$$

Substituting $I = \frac{1}{2}mR^2$ gives

$$(M + 6m)a = (M + 4m)g \sin\theta - 2\frac{Q}{R} \qquad (5)$$

For part (a) Q = 0

$$a = \frac{(M + 4m)g \sin\theta}{(M + 6m)} = \frac{400 \times 9.81 \sin 15}{470} = 2.16 \text{ m/s}^2$$

For part (b) need to find the deceleration that stops the trolley from 12 m/s in 40 m.

$$v^2 = u^2 + 2as$$

$\Rightarrow$ a = –1.8 m/s^2

Rearranging 5 gives

$$Q = \frac{R}{2}\left[(M + 4m)g \sin\theta - (M + 6m)a\right] = \frac{2}{0.35}\left[400 \times 9.81 \sin 15 - 470 \times -1.8\right]$$

$\Rightarrow$ Q = 325.8 Nm