Dynamics 2

Power Method
(Work – Energy Approach)

‘Power’ Method for Systems

- The ‘power’ method provides an alternative to the classical Newtonian approach for linked systems
- Can give quick answers in many cases
- Views system as a whole
  - Gives acceleration but FBD needed for internal forces
- Based on concepts of work, energy and power
- Requires the identification of power sources and sinks for the system

Required Understanding

- Kinetic energy
- Potential energy
- Work done by forces and torques / moments
- Power of forces and torques / moments

Revision

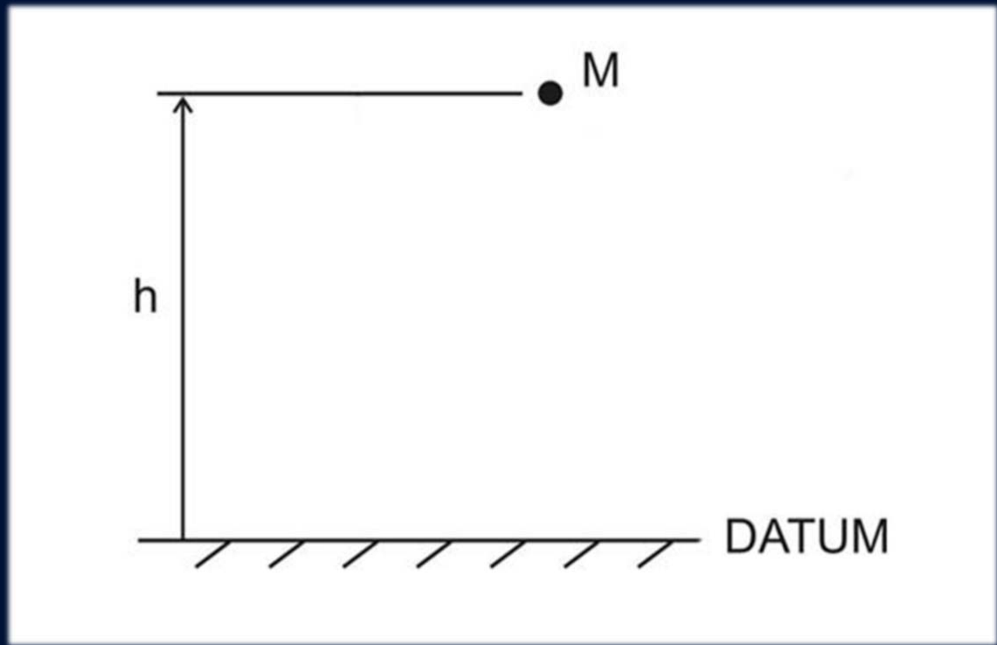
- Energy
  - Scalar quantity (not a vector)
  - The capacity of a system for doing work
  - J, Nm or Ws
- Kinetic energy
  - Particle $$\mathrm{KE} = \frac{1}{2} m v^2$$
  - System $\sum$ particles

Revision

- Potential energy

  - Gravitational PE

    $PE = Mgh$

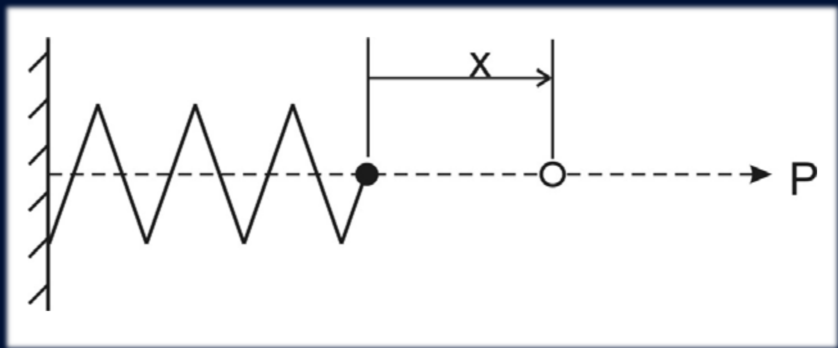


Revision

- Potential energy

  - Spring (Elastic Strain energy)

$$\mathrm{PE} = \frac{1}{2} k\, x^2$$



Revision

- Work
  - done by forces (& torques) when the mass particles on which they act move some distance

- Work done by a force

  Force × distance moved by particle *in direction of force*

$$W = F \times d$$

- Work done by a torque

  Torque × angle of rotation of body *in direction of torque*

$$W = \tau \times \theta$$

- Can be +ve or –ve

Revision

- Power
  - Work rate or work done/sec by force/torque (W,J/s,Nm/s)

- Power of a force

  Force $\times$ velocity of mass at point of application *in direction of force*

$$P = F \times v$$

- Power of a Torque

  Torque $\times$ angular velocity of body *in direction of torque*

$$P = \tau \times \Omega$$

Revision

- Power can be +ve or −ve
  - Positive means energy is flowing into the system
  - Negative means energy is flowing out of the system
- Electric motor
  - Armature torque in same direction as rotor motion
  - Power +ve
- Disc brake
  - Frictional torque from disc pads is in opposite direction to motion
  - Power −ve

Kinetic Energy of Rigid Bodies

- Pure translation $$\mathrm{KE}_{\mathrm{BODY}} = \frac{1}{2} M\, v^2$$
- Fixed axis rotation $$\mathrm{KE}_{\mathrm{BODY}} = \frac{1}{2} I_O\, \Omega^2$$
- General plane motion $$\mathrm{KE}_{\mathrm{BODY}} = \frac{1}{2} M v^2 + \frac{1}{2} I_O\, \Omega^2$$

Summary

- revision of concepts of work and energy
- derivation of Work-Energy Principle

Power Theorems

- Two shown here
  - KE only
  - KE + PE
- Conservative forces

Dynamics 2

Power Method: The Power Theorems
(Work – Energy Approach)

Power Theorem (ex PE)

- Define states 1 and 2 occurring a short time apart
- Energy Balance

$$KE_1$$

$$+$$

$$(\text{Work done } 1 \rightarrow 2 \text{ on system by all system forces})$$

$$= KE_2$$

- Work done includes weight forces

This is the **Work-Energy Principle**

Power Theorem (ex PE)

- Re-write as:

  Work done on system by
  all system forces/torques in a time interval
  = Increase in system KE

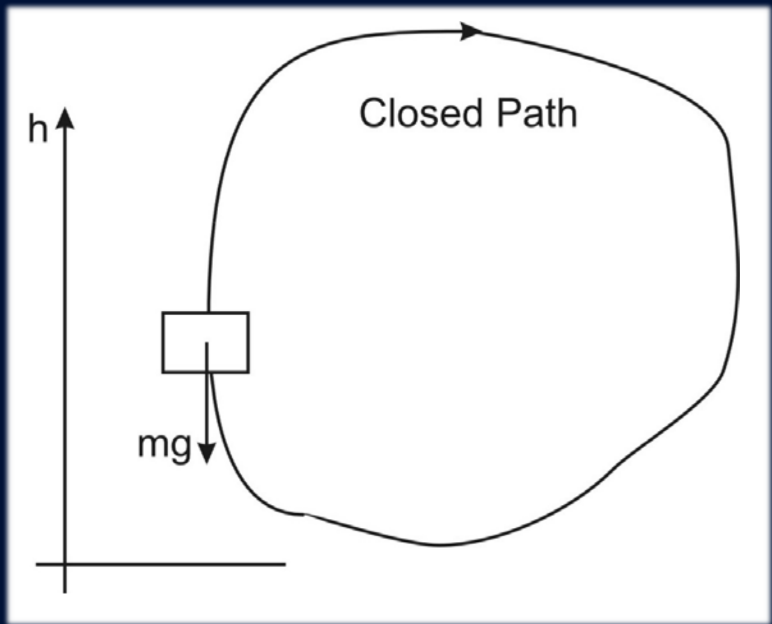
- Let 1 and 2 be close and separated by Δt
  - Dividing by Δt and letting $\Delta t \rightarrow 0$ gives
- Basic System Power Theorem

  work done/sec by all system forces/torques

$$= \frac{d}{dt}\left(\text{System KE}\right)$$

Conservative Forces

- Weights are main example of conservative forces

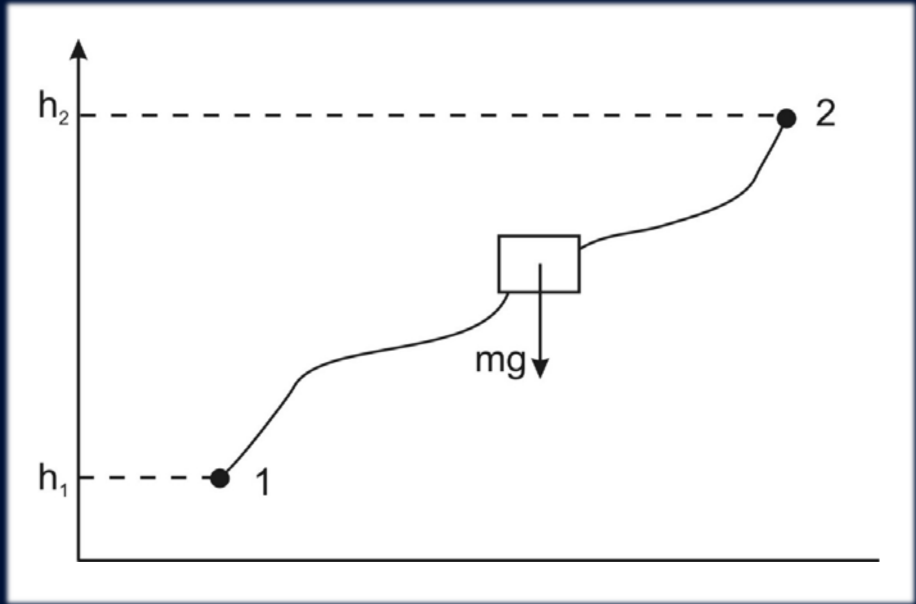


- Work done by the weight force of a mass in any closed path of movement is zero

Conservative Forces

- Work done by weight during a change of position

  Work done ($1 \rightarrow 2$)
  $$= -mg(h_2 - h_1)$$
  $$= mgh_1 - mgh_2$$
  $$= PE_1 - PE_2$$



- i.e. Work done = $PE_1 - PE_2 = -$ (Increase in PE)
- Weight (& 'field' forces) are Conservative Forces
  - Not dependent on path followed

Power Theorem (inc PE)

- Earlier result Basic System Power Theorem

$$\text{Net power of all system forces/torques} = \frac{d}{dt}\left(\text{System KE}\right)$$

- implicitly accounted for PE through system weight forces
  - can modify to explicitly include system PE

Power Theorem (inc PE)

- It was shown that

  Work done by system weights ($1 \rightarrow 2$) = $PE_1 - PE_2$

- Let 1 and 2 be close & separated by Δt

  Work done by weights in $\Delta t = -\Delta(PE)$

- Dividing by Δt and $\Delta t \rightarrow 0$ gives the net power supply from weight forces as:

$$\text{Power of system weight forces} = -\frac{d}{dt}\left(\text{System PE}\right)$$

Power Theorem (inc PE)

- The LHS of original power theorem is split into

Net power of all system forces/torques
plus power of system weight forces $= \dfrac{d}{dt}\left(\text{System KE}\right)$

- or

Net power of system forces/torques (excl. weight)
plus $-\dfrac{d}{dt}\left(\text{System PE}\right) = \dfrac{d}{dt}\left(\text{System KE}\right)$

- Which finally becomes:

Net power of system forces/torques $= \dfrac{d}{dt}\left(\text{System KE} + \text{PE}\right)$

Power Theorems

- Implicitly including PE

$$\text{Net power of system forces/torques} = \frac{d}{dt}\left(\text{System KE}\right)$$

- Explicitly including PE

$$\text{Net power of system forces/torques} = \frac{d}{dt}\left(\text{System KE} + \text{PE}\right)$$

- 2nd includes PE and follows from the simpler 1st
  - often simpler to use the 1st in problems
- 2nd applies to all types of conservative forces in systems - not just weight

Worked Power Examples

- Assume system velocities (v, Ω, etc.)
- Write down the KE for each component
- Sum these and use kinematics to get system KE in terms of v or Ω, e.g.

$$\text{System KE} = \tfrac{1}{2}\,(B)\,\Omega^2$$

- Identify what is putting energy in and out of the system and the work rates (power) to calculate net power in
- Work out d/dt (system KE)
  - Quick way is to use $d/dt(KE) = (B)\,\Omega\,\alpha$
- Use power theorem to calculate variables of interest
  - $d/dt(KE) = \text{net power}$

Summary

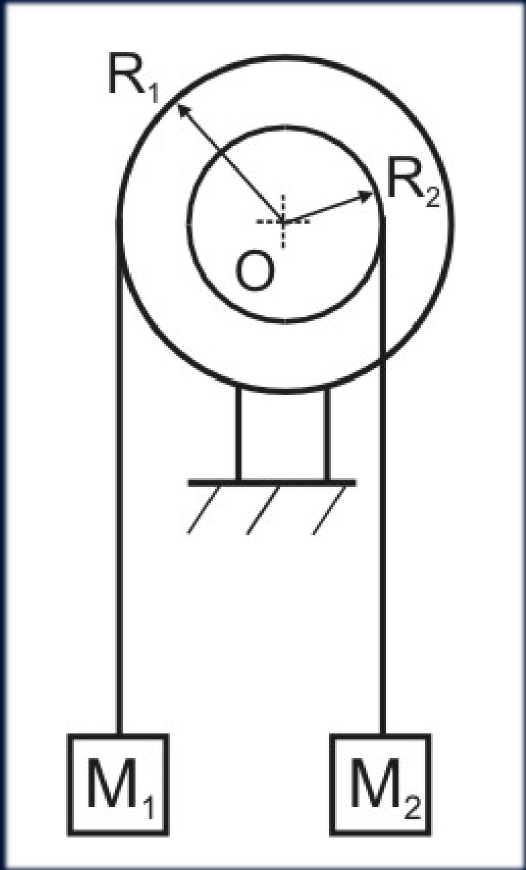
- developed the Power Theorems
- Identified a procedure for applying these

Dynamics 2

Power Method: Worked Example
(Work – Energy Approach)

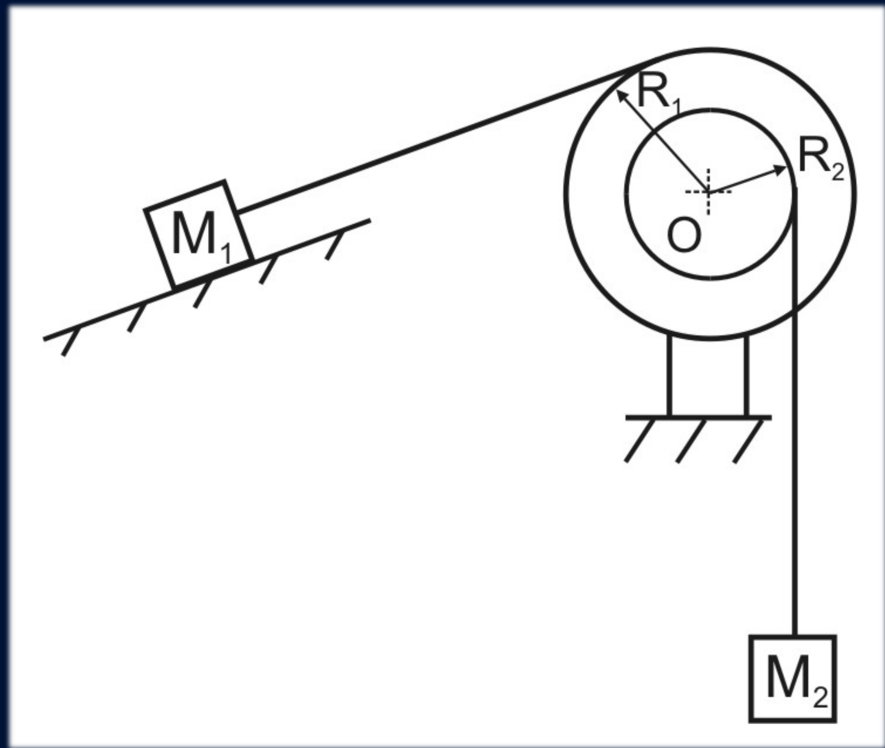
Example 3.1 (same as 2.5?)

- Double pulley loaded by 2 masses
- If the masses are released from rest, determine
  - pulley angular acceleration
  - accelerations of the masses
- Data
  - Pulley $I_O = 22$ kgm^2
  - $M_1 = 24$ kg, $M_2 = 50$ kg
  - $R_1 = 0.45$ m, $R_2 = 0.32$m
- Use power method



Example 3.2

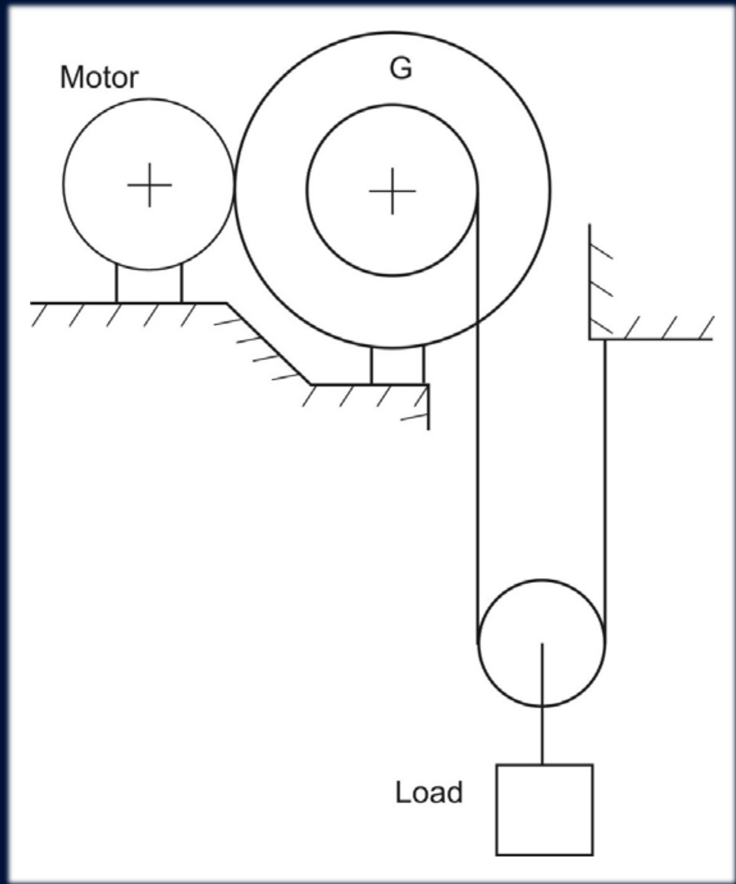
- M_1 is on a 20° incline
  - coefficient of friction = 0.28
- If the masses are released determine
  - Mass accelerations
  - Cable tensions
- Data (as 3.1)
  - Pulley I_O = 22 kgm²
  - M_1 = 24 kg, M_2 = 50 kg
  - R_1 = 0.45 m, R_2 = 0.32m
- Use power method



Example 3.3 (rpt 2.9)

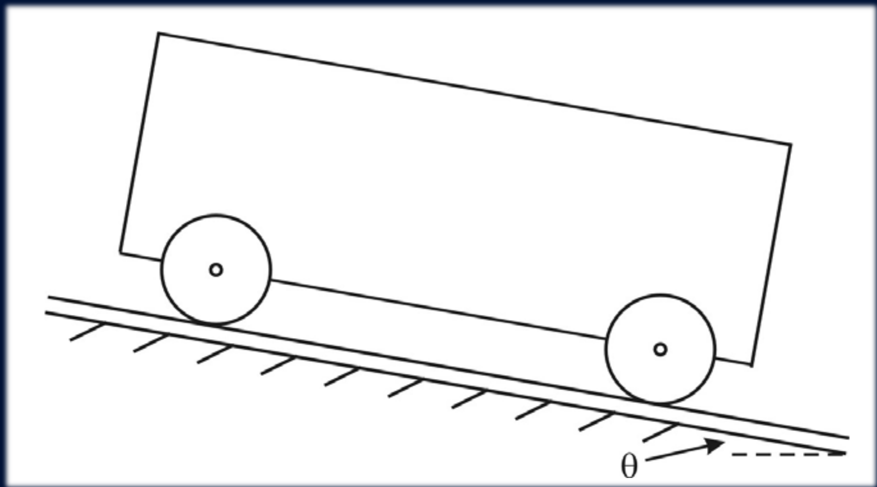
- Motor operated cable drum (∅ 1.2 m) raises 850 kg load
- Motor's gear has 34 teeth
- Drum gear G has 164 teeth
- $I_{Motor/Gear} = 1.8\ kgm^2$
- $I_{Drum/Gear} = 72\ kgm^2$
- Motor armature torque 850 Nm
- Calculate the upwards acceleration of the load

  - Ignore cable/pulley masses
- Use power method



Example 3.4

- Loaded 4 wheel trolley runs on rails down incline ($\theta°$)
  - Total mass 400 kg
  - Wheels as uniform discs of R = 350 mm and mass 35 kg
- If released on a 15° incline what is acceleration?
- If wheel bearings rusty and resist rotation with friction torque of 8.5 Nm what is the acceleration?
  - What is minimum incline for trolley to roll?



Example 3.5

- Electric motor driven via chain drive with a 5/1 reduction.
  - Neglect system masses and MMI's not listed
- Acceleration up a 6° incline (neglect air resistance)?
- Max speed on the level (including air resistance)