Dynamics 2

General Plane Motion

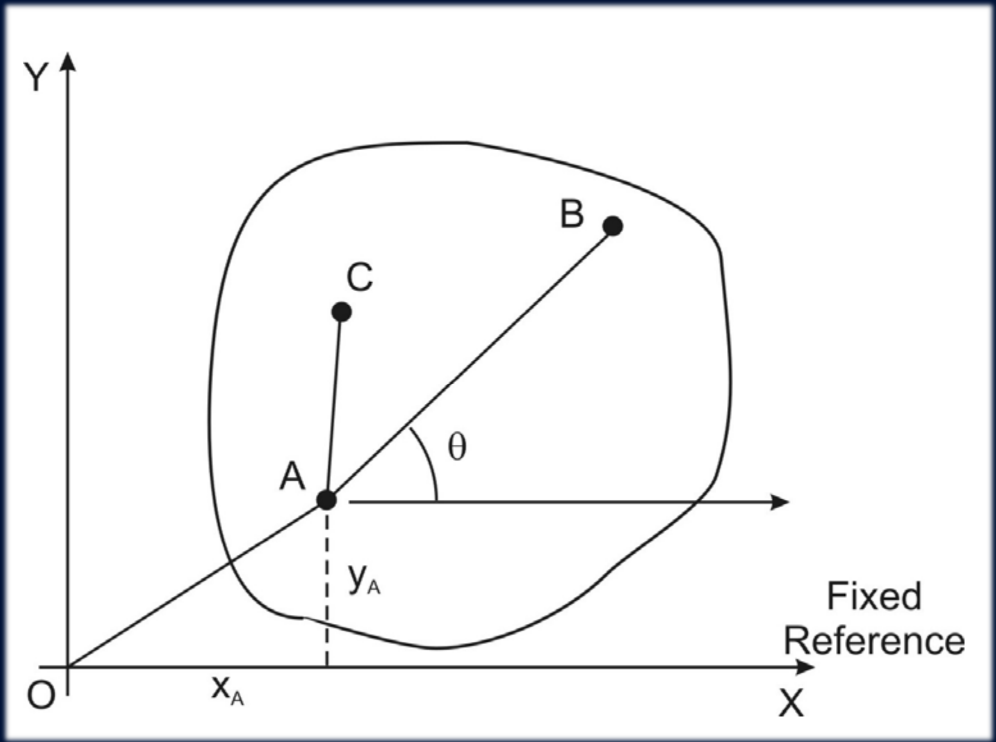
(Dynamics of Systems of Bodies)

General Plane Motion

- Combination of
  - Pure Translation
  - Fixed Axis Rotation
- GPM for short

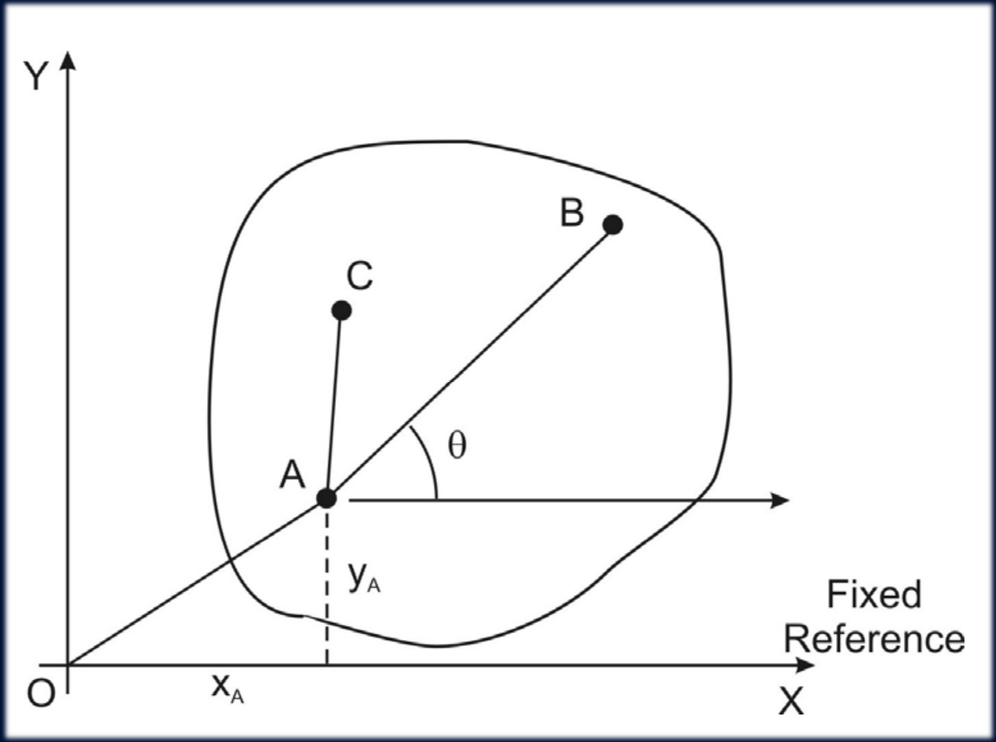
Properties of Bodies in GPM

- Position defined by 3 coordinates
  - A = ref point
  - position ($\mathbf{x_A}$, $\mathbf{y_A}$)
  - AB is fixed line
  - position of B fixed by θ
  - position of C fixed by $\angle CAB$



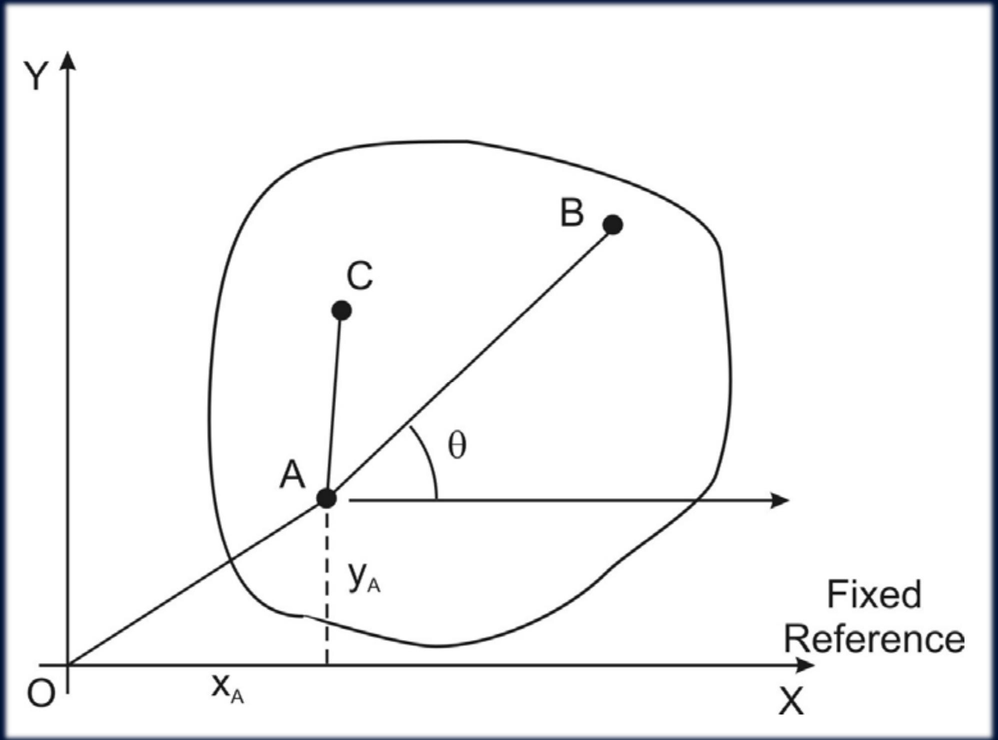
Properties of Bodies in GPM

- Position defined by 3 coordinates
- Has 3 degrees of freedom
  - X, Y, θ



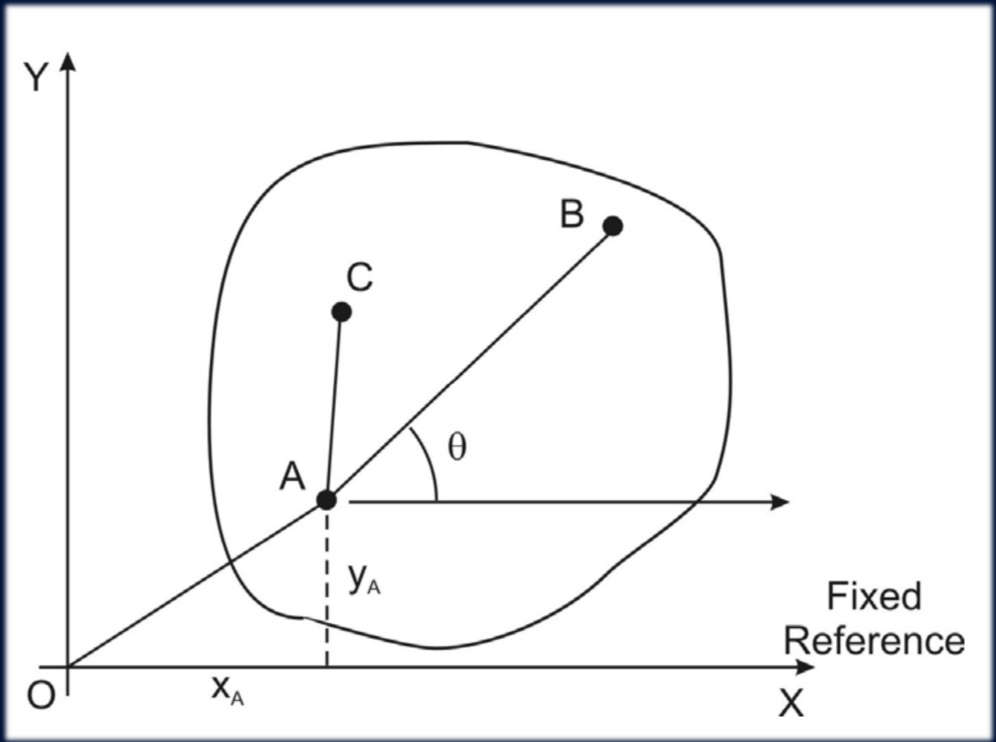
Properties of Bodies in GPM

- Position defined by 3 coordinates
- Has 3 degrees of freedom
- G as reference
  - valuable



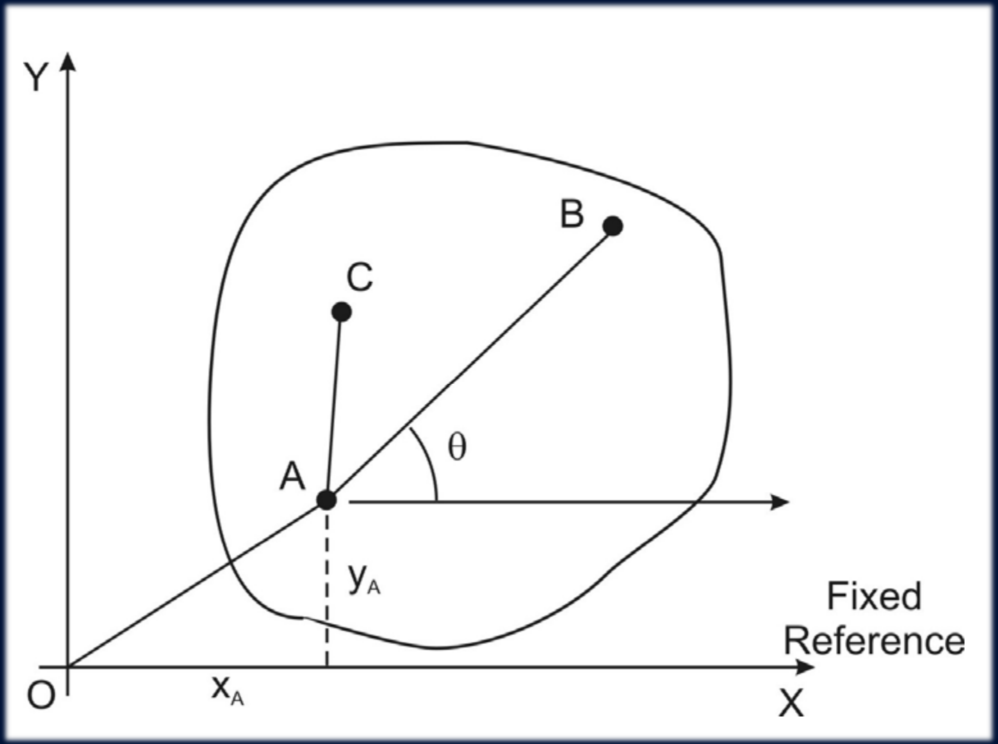
Properties of Bodies in GPM

- Position defined by 3 coordinates
- Has 3 degrees of freedom
- G as reference
- Particle velocity
  - ref point vel.
  - Ω



Properties of Bodies in GPM

- Position defined by 3 coordinates
- Has 3 degrees of freedom
- G as reference
- Particle velocity
- Particle accel
  - ref point accel.
  - Ω and α

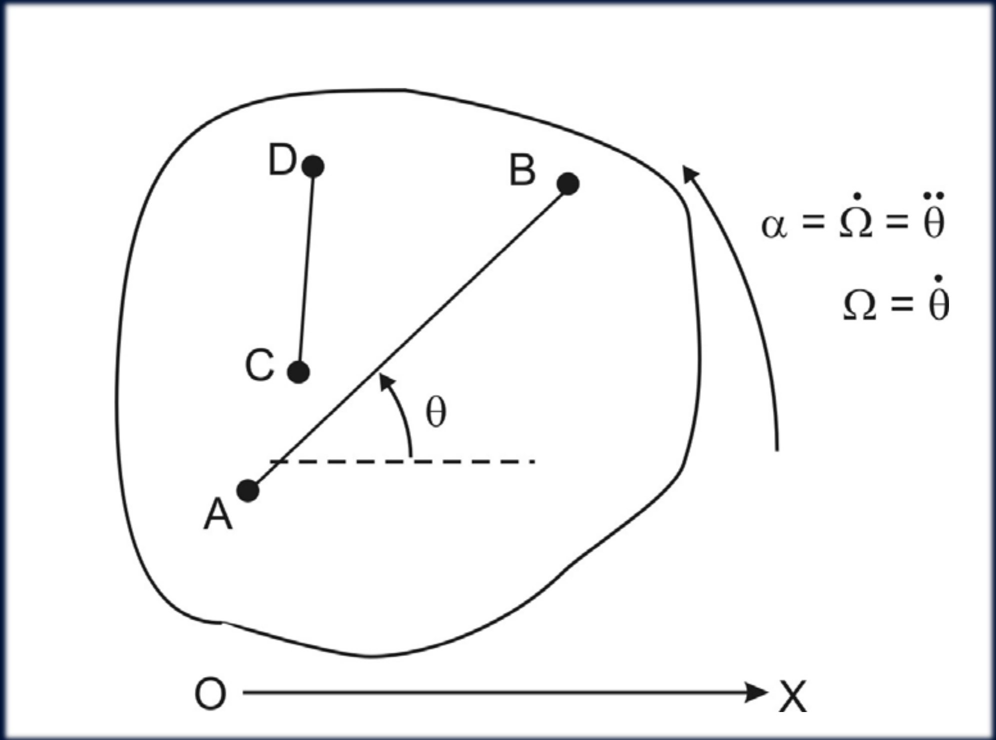


Angular Velocity in GPM

- Defintiion: time derivative of angle between any straight line in body & fixed reference direction
- AB is line
- OX = fixed ref
- θ = angle between AB and OX
- Angular velocity Ω

$$\frac{d\theta}{dt} \quad \text{or} \quad \dot{\theta}$$

- Similar for α

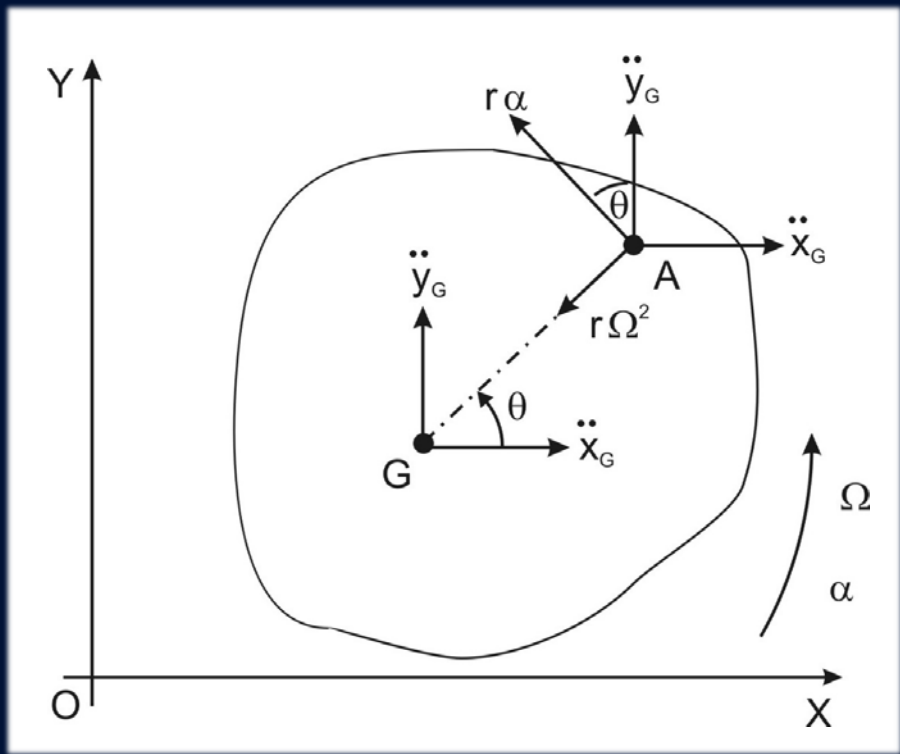


Angular Velocity in GPM

- Do NOT need to refer to rotating about a point
  - Rotating about a fixed point is special case (FAR)
- Can locate the instantaneous centre (IC)
  - Meet in Design
- If velocities for any two points on body are known
  - project lines at 90° - where they meet is the IC
  - ie at instant body is rotating about IC
- IC is itself a moving point in GPM

Acceleration Components in GPM

- Choose G as reference point for motion
  - A is another point
  - GA at angle θ to OX
- G has accel components
  - $\ddot{x}_G$ and $\ddot{y}_G$
- Ω angular velocity
- α angular accel
- Since $\overline{a}_A = \overline{a}_G + \overline{a}_{GA}$
  - Acc for A as shown



Dynamic Laws in GPM

- Force Law (GN2)

  sum of External Forces + body Inertia Force = 0

- Moment Law

  sum about any point of moments of the External Forces

  + the moment of the body Inertia Force acting at G

  + the Inertia Couple

  = 0

- Inertia force = ma, inertia couple = Iα as before

Summary

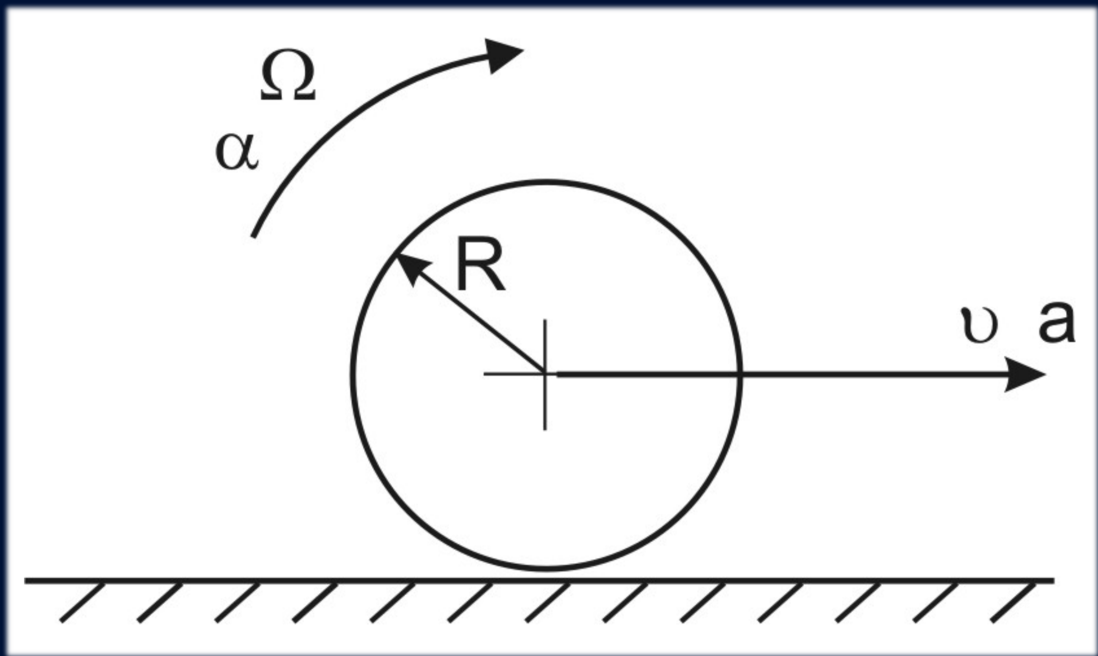
- Properties of General Plane Motion
- Velocity and acceleration components
- Dynamics Laws
- Rolling Wheel

Rolling Wheels

- 3 worked examples on GPM
  - All use rolling wheels

Rolling Wheels

- Pure rolling – no slip at contact point



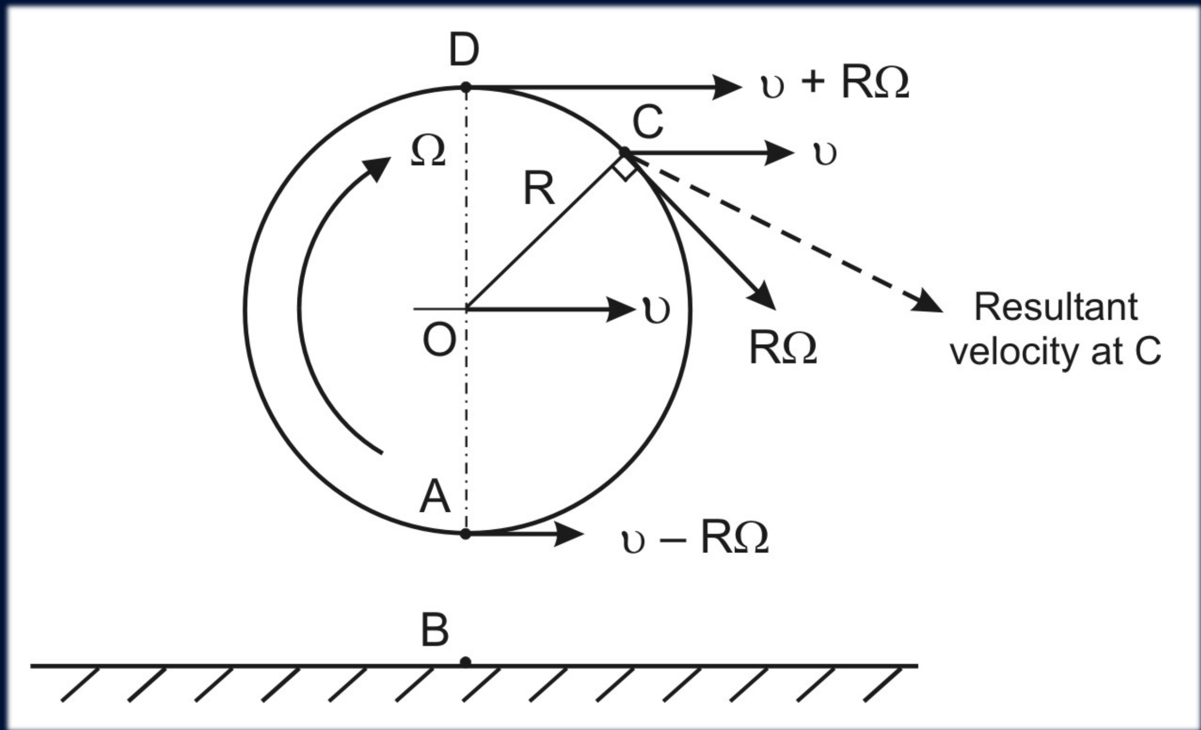
Velocity

- For no slip at A:

$$v_A = v_B$$

$$v - R\Omega = 0$$

$$v = R\Omega$$

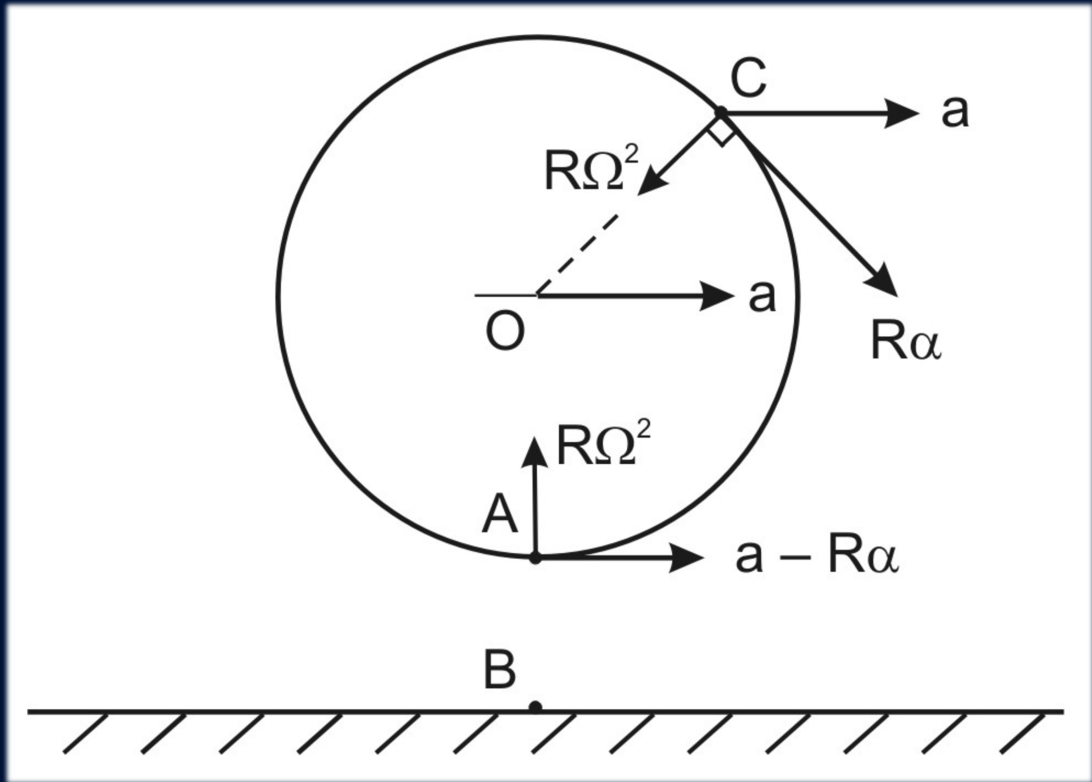


Acceleration

- For no slip at A:

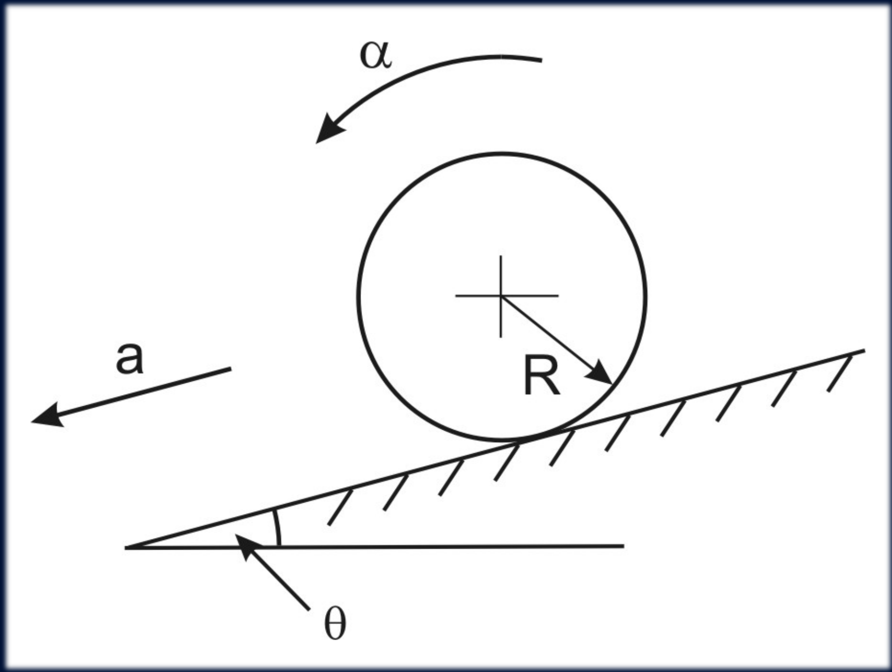
$$a - R\alpha = 0$$

$$a = R\alpha$$



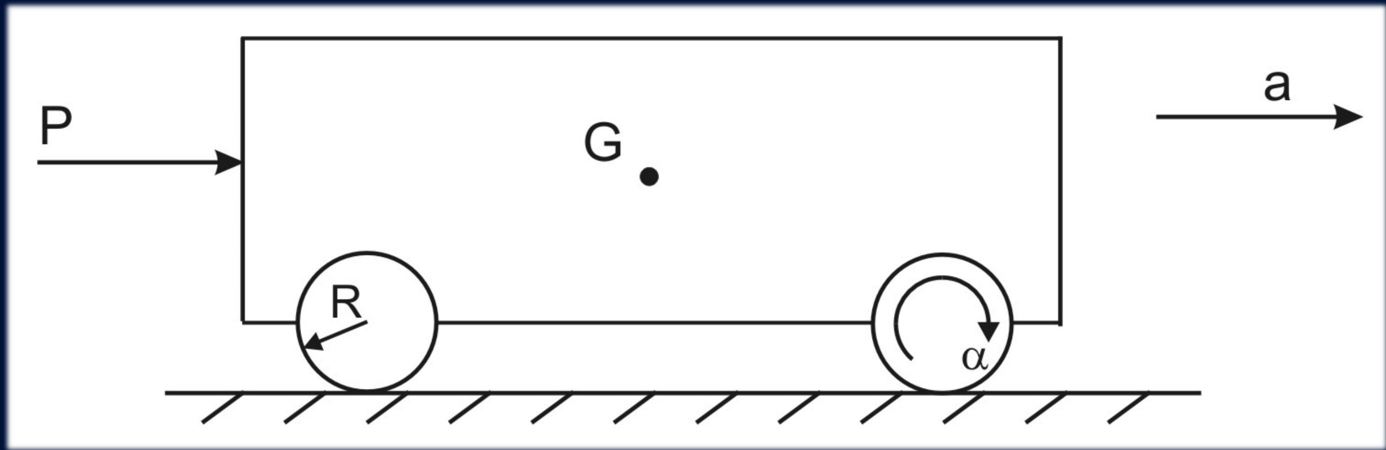
Example 2.10

- Uniform circular disc (mass 30 kg, ∅ 1.6 m) released from rest on a 12° slope
  - calculate acceleration down the slope assuming pure rolling
  - magnitude of friction force?
- If μ for the surface is 0.25
  - maximum incline for pure rolling?



Example 2.11

- 4 wheel un-powered trolley has total mass 200 kg
  - Wheels are uniform discs m = 25 kg, radius = 350 mm
  - What force P is required to give $a = 1.4$ m/s^2?
- When the trolley reaches $v = 15$ m/s, P removed and brakes activated on rear wheels.
  - What brake torque per wheel is required to stop trolley in 18 m?



Example 2.12

- Truck accelerates from rest at 2.5 m/s^2
- Unrestrained cylindrical load midway on 10m bed
  - mass of 300 kg (uniform cylinder R = 0.8 m)
  - μ for truck surface is 0.2
- Will the cylinder roll or slide, or stay put?
- Will it roll off the back of the truck
  - if so how long would it take?