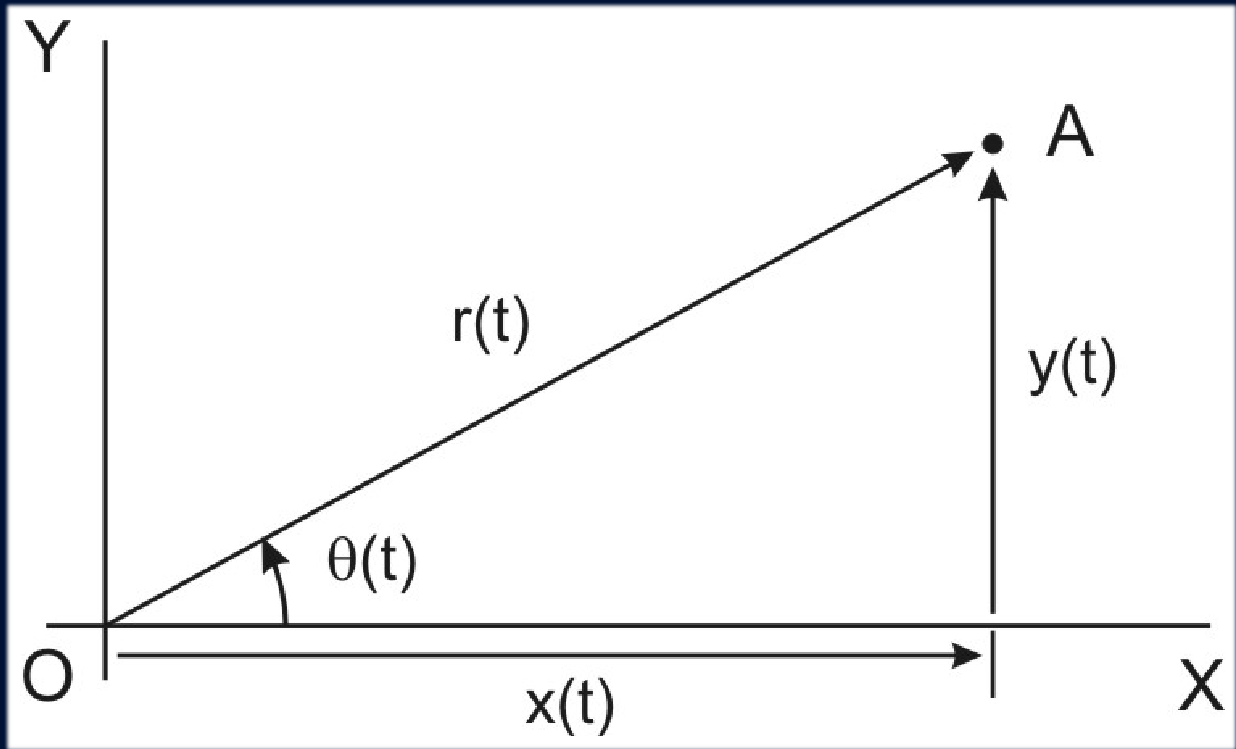
Dynamics 2 (MECE08009)

Particle Motion in Polar Coordinates

(Dynamics of Single Particles)

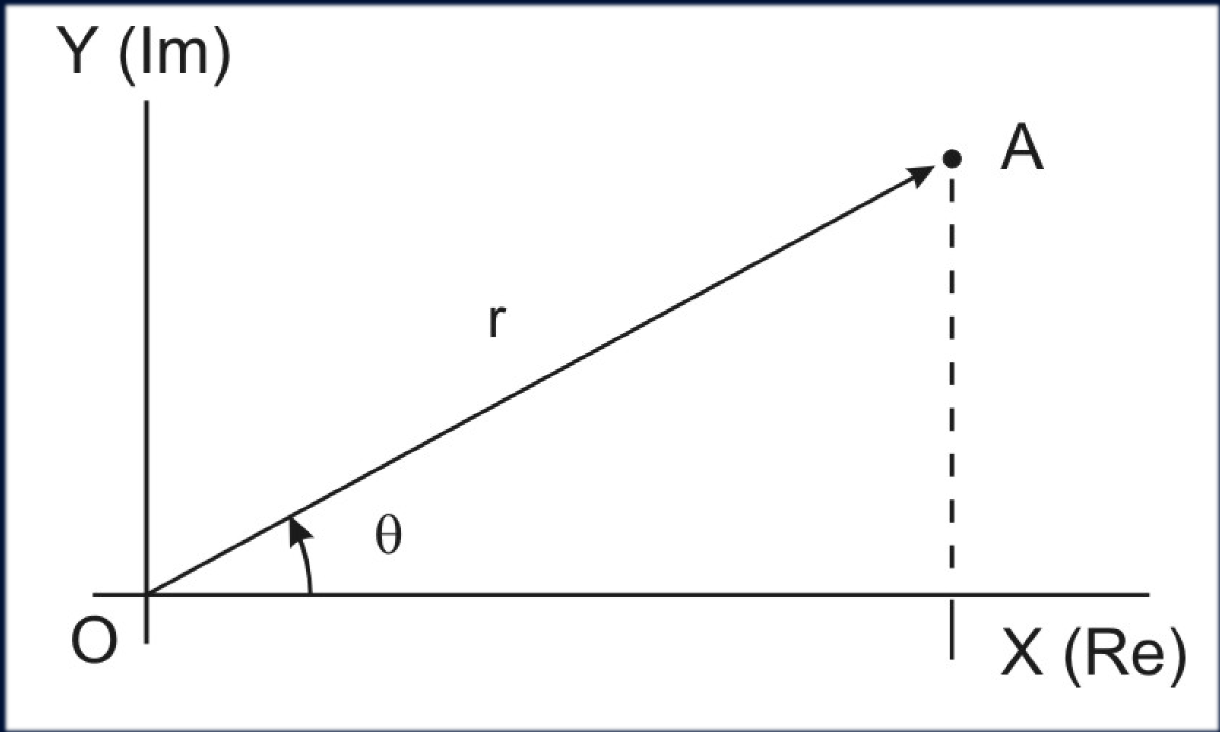
Particle Motion in a Plane in Polar Coordinates

- describe motion with x(t), y(t) [Cartesian Coords]
- may be better to use Polar Coordinates $r(t)$, $\theta(t)$



Velocity & Acceleration in Polar Coordinates

- use complex numbers
  - temporarily call the plane of motion the Complex Plane



Velocity & Acceleration in Polar Coordinates

- instantaneous position of a particle is

  $z = x + \mathrm{i}\, y = \mathrm{r}\, \mathrm{e}^{\mathrm{i}\theta}$

  where (de Moivre's theorem)

  $\mathrm{i} = \sqrt{-1}$

  and r and θ are $f(\mathrm{t})$
- differentiate z with respect to (w.r.t.) time
  - velocity

$$\dot{z} = \dot{r}e^{i\theta} + (r\dot{\theta})ie^{i\theta}$$

- note

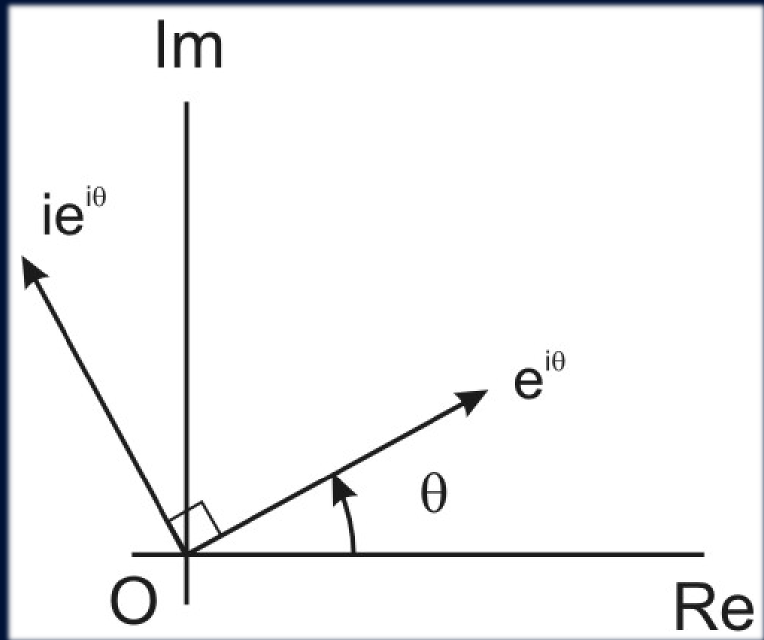
$$\dot{z} = \frac{dz}{dt} \qquad \ddot{z} = \frac{d^2 z}{dt^2}$$

Velocity & Acceleration in Polar Coordinates

- now

$e^{i\theta} = \cos\theta + i\sin\theta$

= a radial unit vector along OA



Velocity & Acceleration in Polar Coordinates

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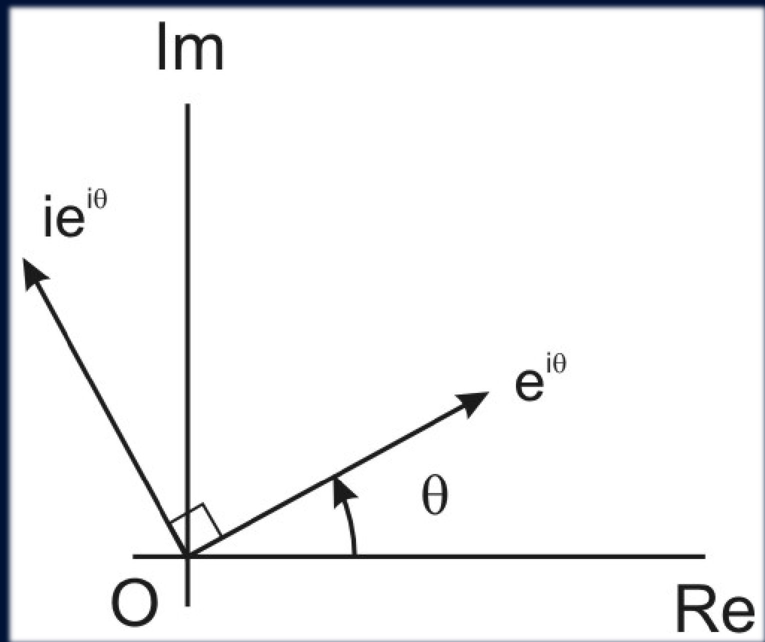
- $\dot{z}$ has two components



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Velocity & Acceleration in Polar Coordinates

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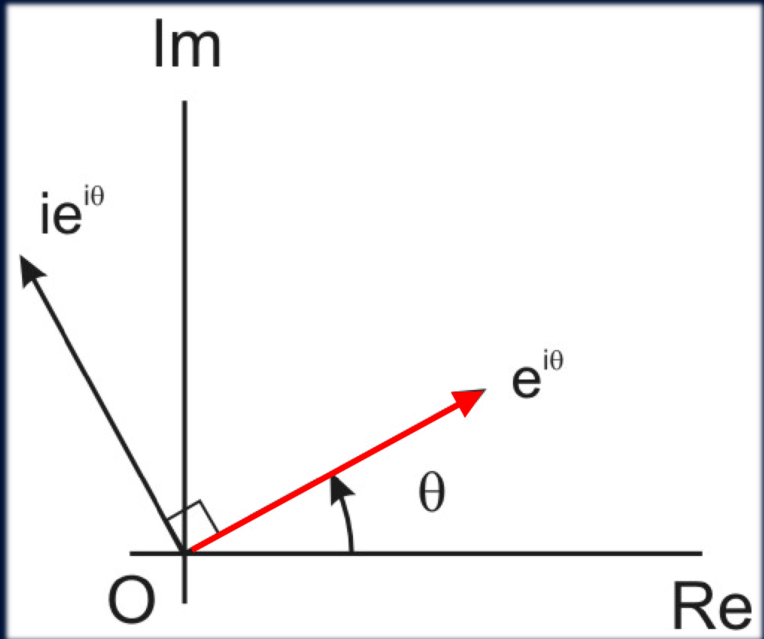
  $e^{i\theta} = \cos\theta + i\sin\theta$

  = a radial unit vector along OA

- $\dot{z}$ has two components
  - radially outwards:

    $\dot{r}$



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Velocity & Acceleration in Polar Coordinates

- now

  $e^{i\theta} = \cos\theta + i\sin\theta$

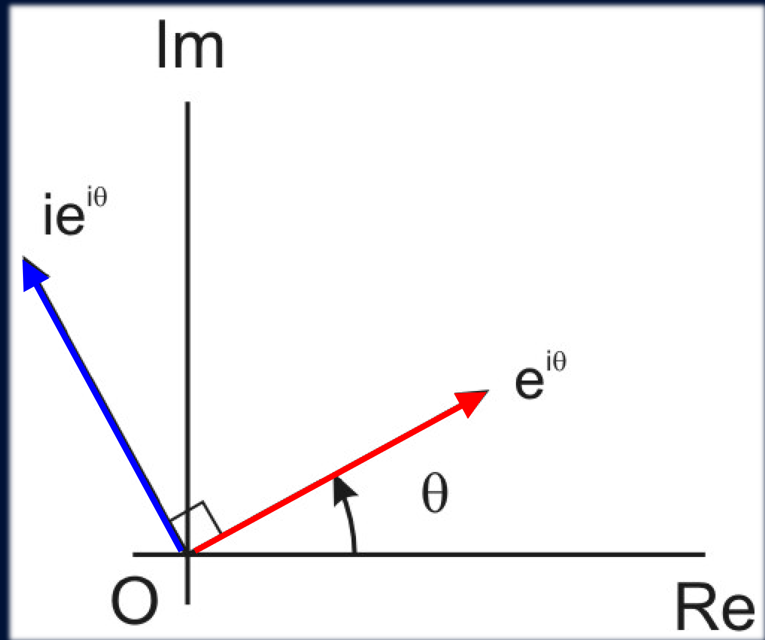
  = a radial unit vector along OA

- $\dot{z}$ has two components
  - radially outwards:

    $\dot{r}$
  - tangentially

    $r\dot{\theta}$

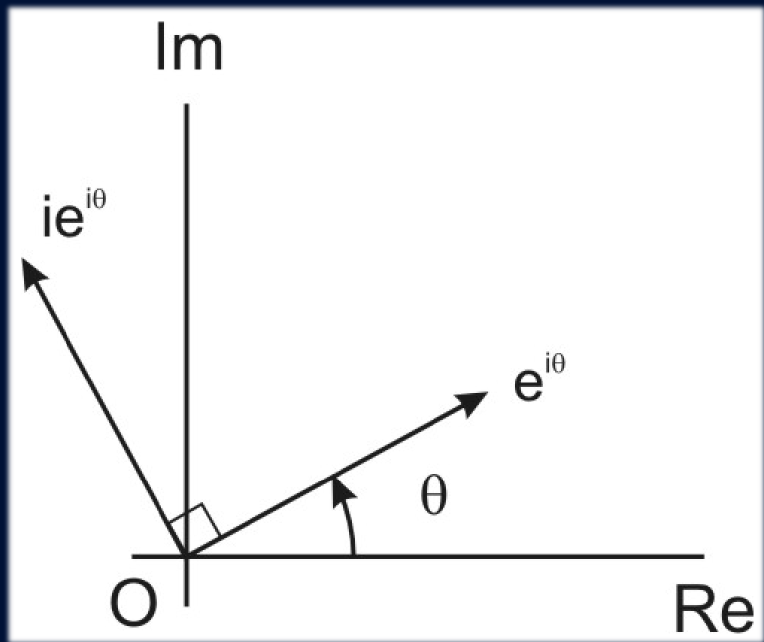
$$\dot{z} = \dot{r}e^{i\theta} + (r\dot{\theta})ie^{i\theta}$$



Velocity & Acceleration in Polar Coordinates

- differentiate $\dot{z}$ w.r.t. time again (acceleration)

$$\ddot{z} = (\ddot{r} - r\dot{\theta}^2)e^{i\theta} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})ie^{i\theta}$$



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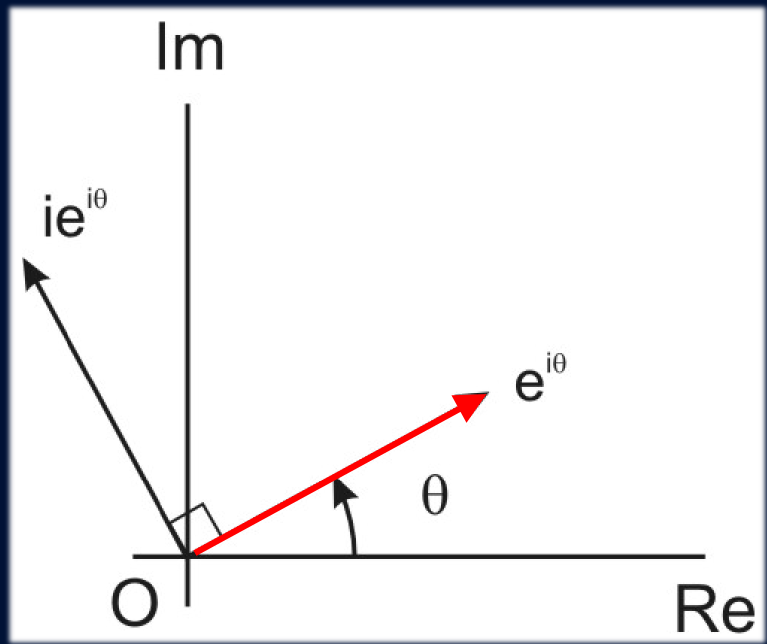

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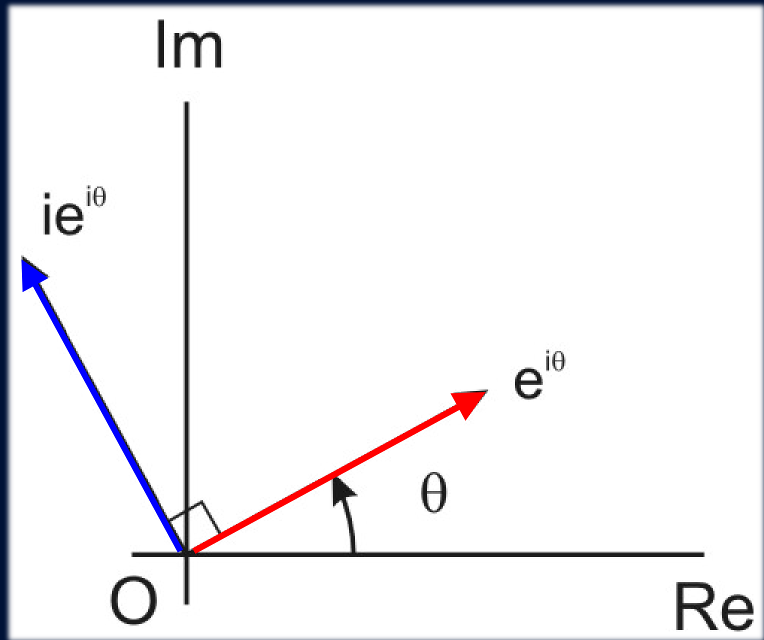
  - radially outwards:

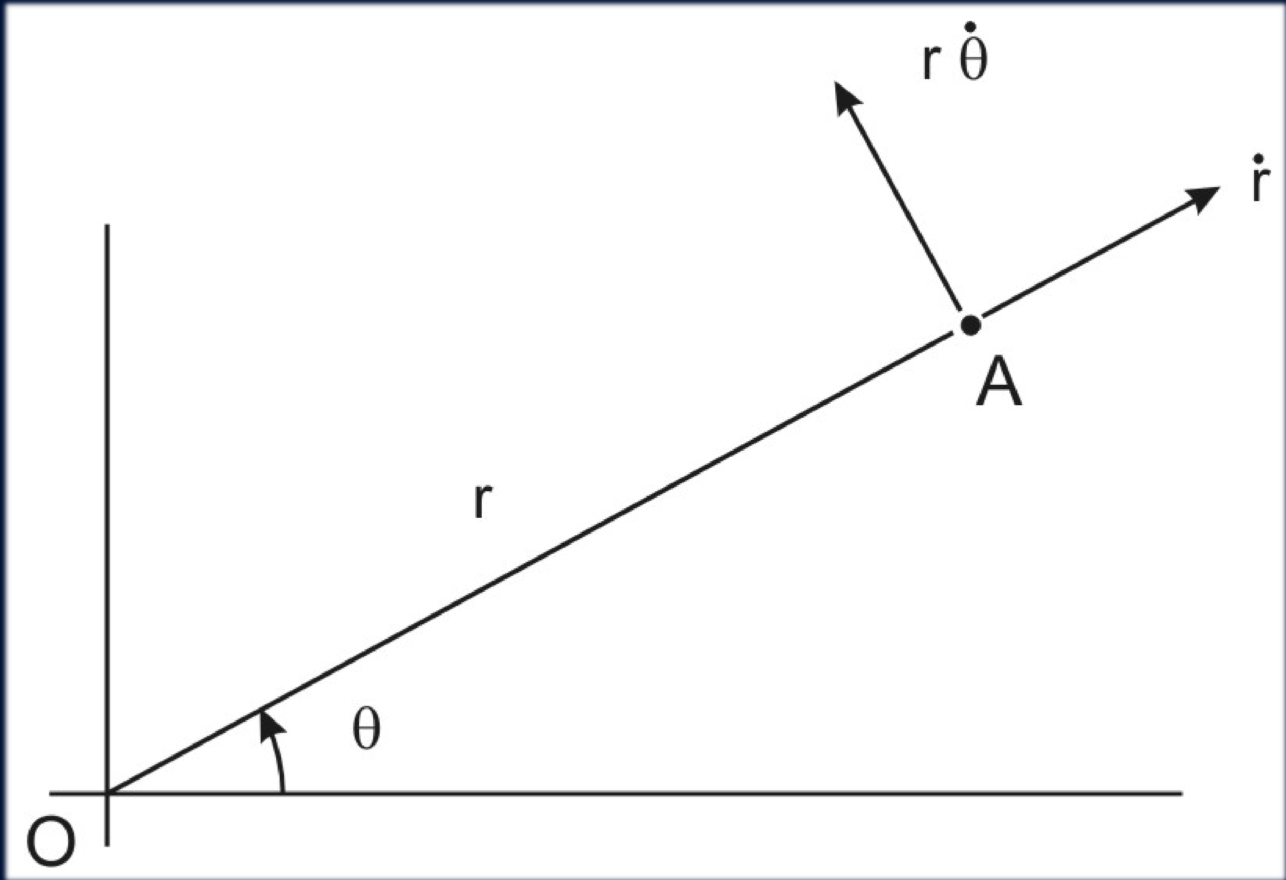
$$\ddot{r} - r\dot{\theta}^2$$

  - tangentially:

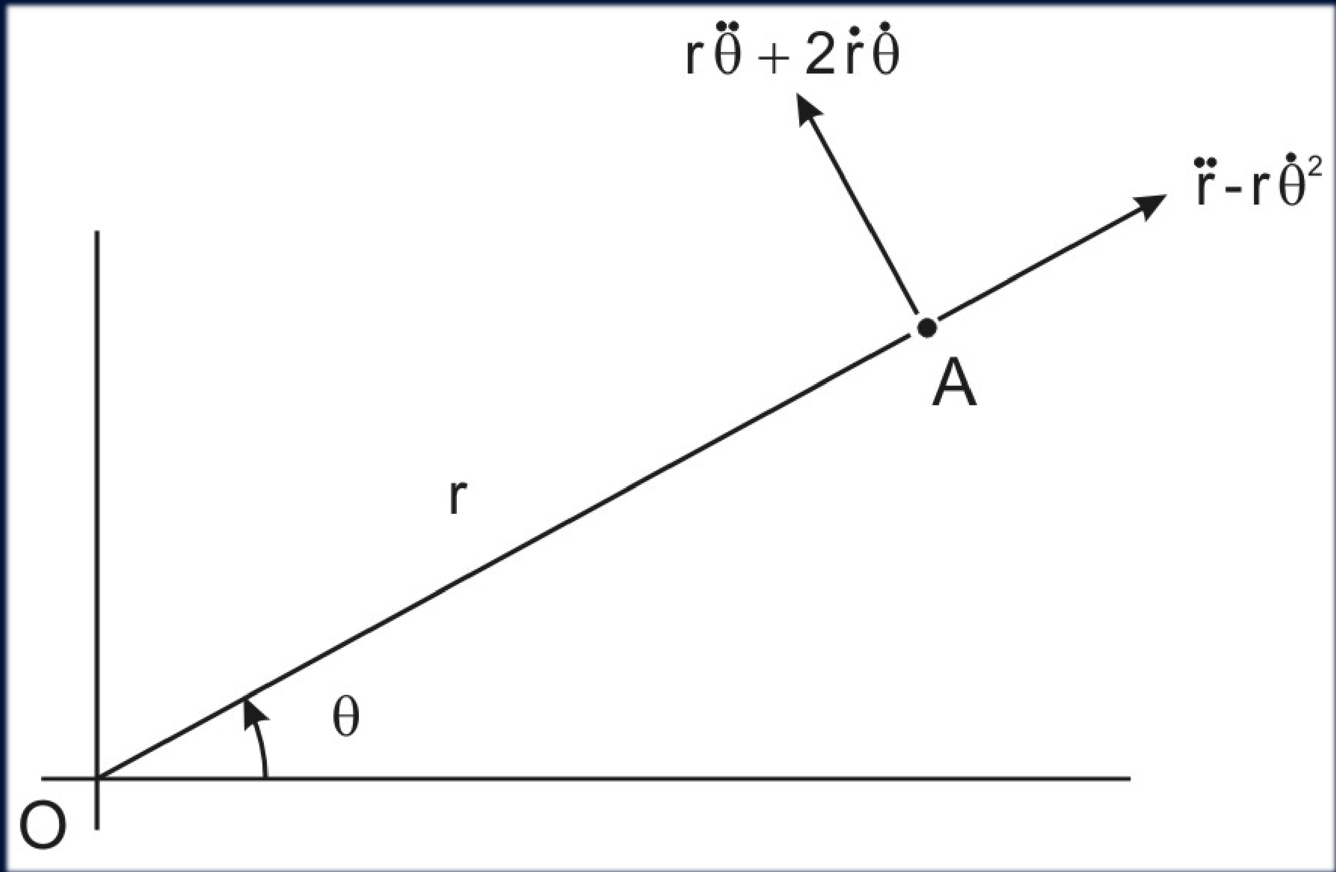
$$r\ddot{\theta} + 2\dot{r}\dot{\theta}$$



Velocity Components



Acceleration Components



Notes on Acceleration Components

$$\ddot{z} = (\ddot{r} - r\dot{\theta}^2)e^{i\theta} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})ie^{i\theta}$$

- in radial component

  $-r\dot{\theta}^2$ is the centripetal acceleration

  - in simpler case of circular motion (r = constant), and the minus sign indicates it is acting inwards

Notes on Acceleration Components

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- in tangential component

  $2\dot{r}\dot{\theta}$ is the Coriolis acceleration

Coriolis Acceleration

Coriolis Acceleration

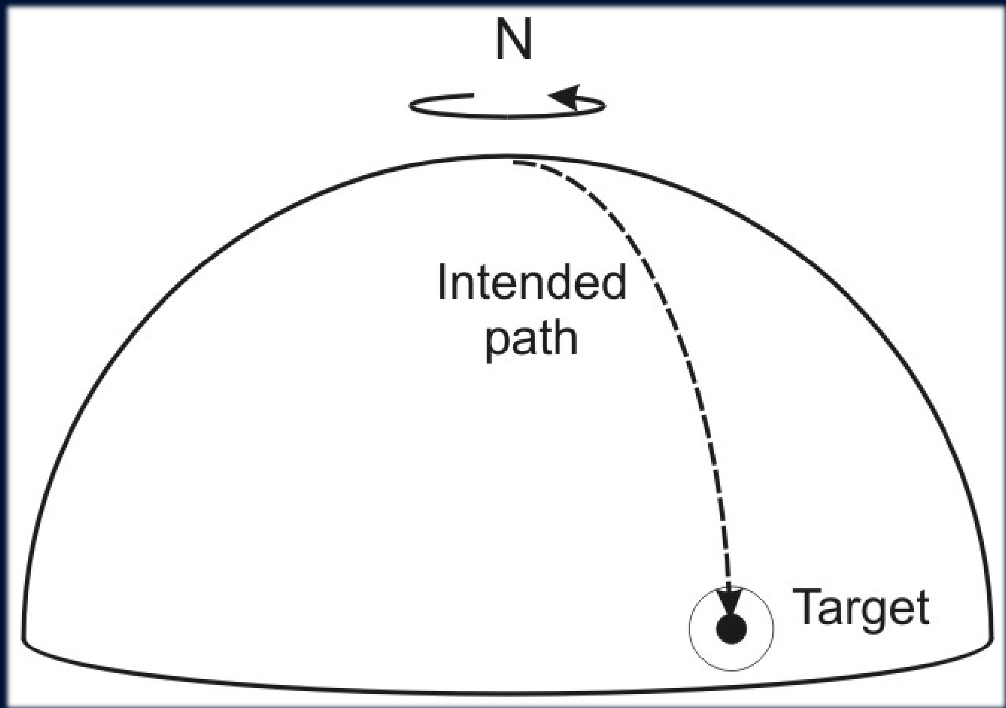
- Gaspard de Coriolis described it in 1835
- inertial force must be included where Newtonian laws are used in a rotating frame of reference
- force acts to the right of the direction of body motion for anti-clockwise rotation of the reference frame or to the left for clockwise rotation
- effect is an apparent deflection of object's path within the rotating system
  - no actual deviation, but apparent
  - arises from the motion of the coordinate system

Coriolis Acceleration

- most obvious in longitudinal paths
- on Earth an object moving North-South will deflect
  - to the right in the Northern Hemisphere
  - to the left in the South
- why?
  - Earth rotates from West to East
  - tangential velocity varies with latitude
    greatest at equator)

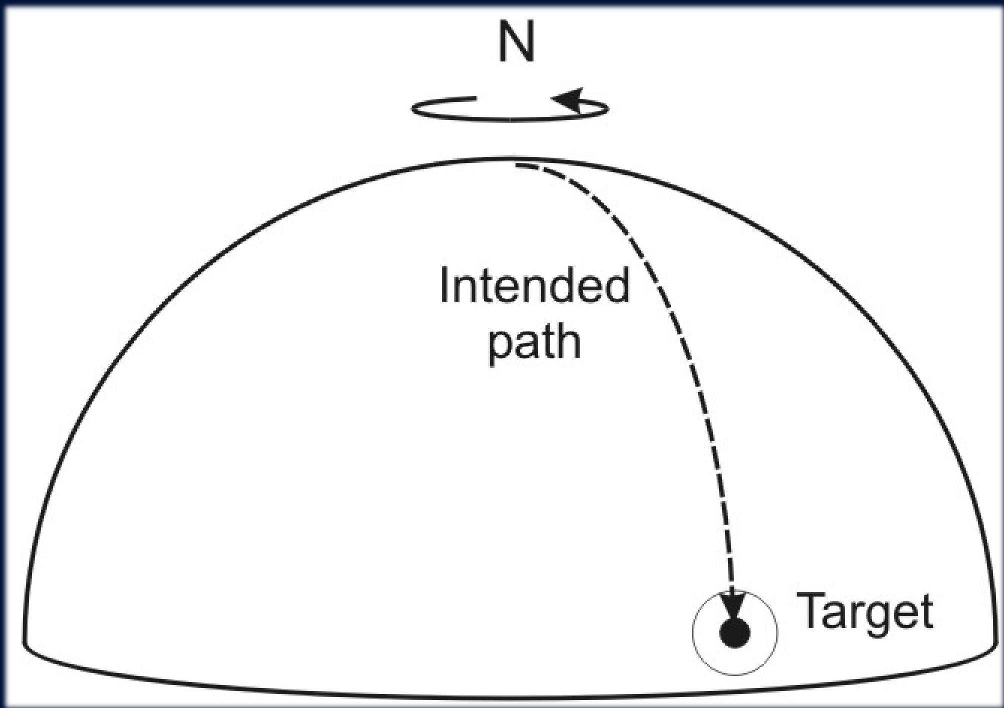
Coriolis Example

- fire a projectile from near to North Pole at target near the equator



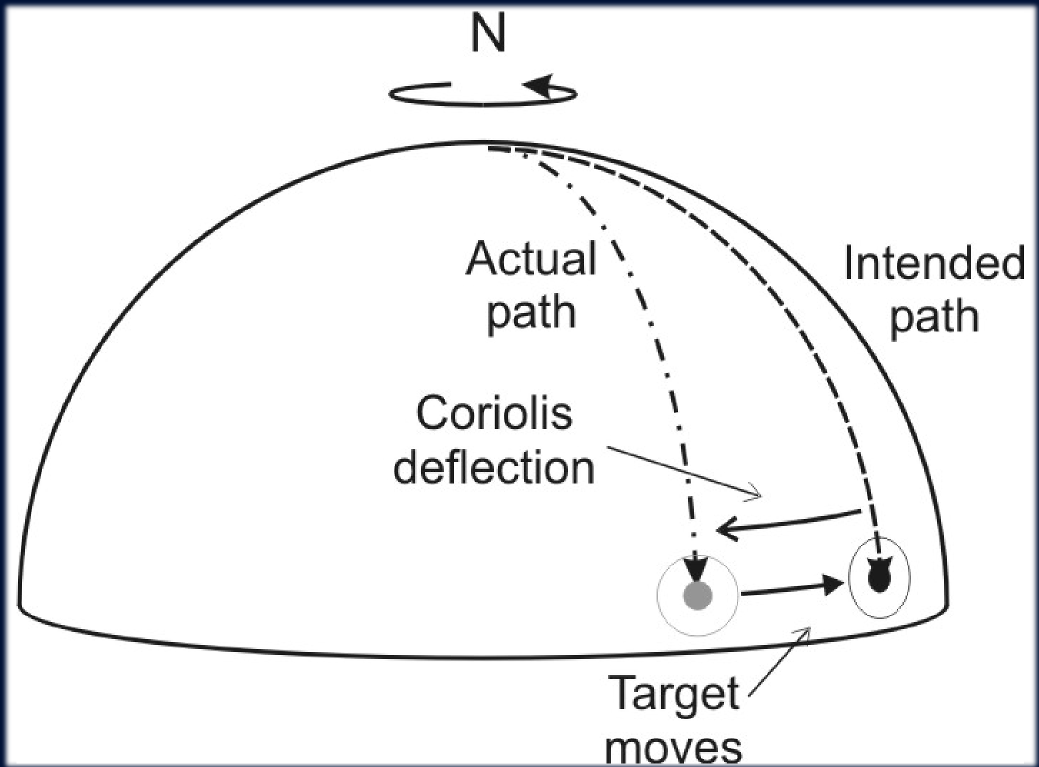
Coriolis Example

- fire a projectile from near to North Pole at target near the equator
  - mostly N-S motion but (some) tangential velocity



Coriolis Example

- target has greater tangential velocity
  - so the projectile lands to the right of the initial target



Coriolis Acceleration

- of major use in
  - astronomy
  - dynamics of the atmosphere (winds)
  - oceanography (currents)

Summary

- Considered the use of polar coordinates to describe general plane motion

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(Dynamics of Single Particles)

Worked Example

Example 1.8

- horizontal disc with radial slot spins at constant 480 rpm. Mass A (0.48 kg) moved inwards by actuator rod B at a constant 18 m/s
- calculate radial & tangential accelerations of mass
- if slot walls have negligible friction, what force is applied to the mass by the actuator?
- show the magnitude and direction of the force applied by the mass to the slot wall