Module 13 self-assessment

Question 1

A civil engineer is studying an "exit-right-only" lane ending on a set of traffic lights. The lane is long enough to fit a queue of 7 cars. Let X be the number of cars in the lane at the end of of a randomly chosen red light. From some observations she did, she thinks that the the probability $\mathbb{P}(X = x) \propto (x + 1)(8 - x)$ for $x \in \{0, 1, 2, 3, 4, 5, 6, 7\}$. Find the PMF of X and the probability of $X \geq 5$.

Question 2

Consider a continuous random variable X with a probability density function

$$p_X(x) = \frac{2}{a^2}x, \quad 0 \leq x \leq a$$

Find its cumulative $F_X(x)$, the expectations $\mathbb{E}[X]$, and $\mathbb{E}[X^2 - a^2]$, and the variance $\mathrm{Var}(X)$.