Tutorial 7 – SOLUTIONS

Tutorial 7: More 2nd Law Analysis and Refrigeration Cycles

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

1. A piston/cylinder device contains 2 kg of water at 5 MPa and 100°C. Heat is added from a reservoir at 600°C to the water until it reaches 600°C. The piston/cylinder device expands during this process with a constant force acting on the piston.

(a) Determine the work done.

(b) Determine the heat transfer to the water.

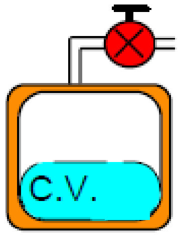
(c) Determine the total entropy production (in kJ/K) for the system and the surroundings

[ans: a)776.6 kJ, b) 6488 kJ, c) 4.48 kJ/K]

Solution:

- Process: constant pressure expansion of steam/water as working fluid
- 1st Law: $\boldsymbol{U_2 - U_1 = Q_{21} - W_{21}}$
- 2nd Law: $\boldsymbol{m(s_2 - s_1) = \int \frac{Q_{21}}{T} + S_{gen}}$
- Find properties at state 1:
    - v_1 = 0.001041 m^3/kg, u_1 = 417.58 kJ/kg, h_1 = 422.78 kJ/kg, s_1 = 1.303 kJ/kgK
- Find properties at state 2:
    - v_2 = 0.07870 m^3/kg, u_2 = 3273.3 kJ/kg, h_2 = 3666.8 kJ/kg, s_2 = 7.260 kJ/kgK
- Find work from expansion:
    - $W_{21} = mP(v_2 - v_1) = 2kg * 5000kPa * (0.07870 - 0.001041)\frac{m^3}{kg} = 776.6kJ$
    - This is the work out of the system and to the surroundings
- Find the heat transfer to the system:
    - $\boldsymbol{Q_{21} = m(u_2 - u_1) + W_{21} = 2kg(3273.3 - 417.58)\frac{kJ}{kgK} + 776.6kJ = 6488.0kJ}$
- Find the total entropy production
    - $\boldsymbol{S_{gen} = \Delta S_{system} + \Delta S_{surroundings}}$
    - $\boldsymbol{S_{gen} = m(s_2 - s_1) - \frac{Q_{21}}{T} = 2kg * (7.260 - 1.303)\frac{kJ}{kgK} - \frac{6488kJ}{873.15K} = 4.48kJ/K}$

2. A rigid, insulated vessel contains superheated vapour steam at 3 MPa, 600ºC. A valve on the vessel is opened, allowing steam to escape. The overall process is irreversible, but the steam remaining inside the vessel goes through a *reversible, adiabatic* expansion. Determine the fraction of steam that has escaped when the final state inside is a saturated vapour.

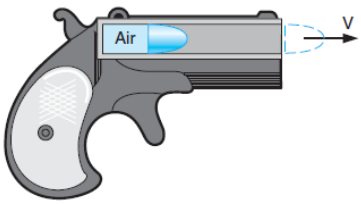


[ans: 0.949]

<u>Solution</u>: Control volume: remaining steam inside the tank

- Conservation of mass: $m_{stream,exit} = m_1 - m_2$
- 1st Law: $\boldsymbol{u_2 - u_1 = q_{21} - w_{21} - \dot{m}_{exit} h_{exit}}$
- 2nd Law: $\boldsymbol{s_2 - s_1 = \int \frac{Q_{21}}{T} + S_{gen} = 0}$ **(reversible, adiabatic)**
- Find the entropy of the steam
  - $s_1 = 7.510$ kJ/kgK (at T_1, P_1) = s_2
- <u>Find T_2</u>:
  - Temperature at state 2 will be determined when $s_2 = s_g$ at T_2. Use the tables to find where $s_g = 7.510$ kJ/kgK. Use table 1 for steam tables.
  - Through interpolation: $T_2 = 87.54$ºC or 360.7K.
- Find v_2: $v_2 = v_g$ at T_2 → interpolation $v_2 = 2.578\ m^3/kg$
- If the control volume is ONLY the steam that remains, then the mass can be replaced with the specific volume (i.e. only the volume changes):
- Find the fraction of the mass that has escaped: recall $m = V / v$
  - $\frac{m_{exit}}{m_1} = \frac{m_1 - m_2}{m_1} = 1 - \frac{m_2}{m_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{0.1324}{2.578} = 0.949$

3. Consider a small air piston with a cylinder volume of 1 cm^3 at 250 kPa and 27ºC. The bullet acts as a piston initially held by a trigger. The bullet is released so that the air expands in an *adiabatic, reversible* process. If the pressure should be 100 kPa as the bullet leaves the cylinder, find the final volume and the work done by the air. Assume ideal gas with constant specific heats (Cv = 0.717 kJ/kgK, Cp = 1.004 kJ/kgK, R = 0.287 kJ/kgK)



[ans: V_2 = 1.92 cm^3, Work = 0.145 J]

<u>Solution</u>: Assume a reversible, adiabatic process

- 1st Law: $\boldsymbol{u_2 - u_1 = q_{21} - w_{21} \rightarrow u_2 - u_1 = -w_{21}}$

- 2nd Law: $\boldsymbol{s_2 - s_1 = \int \frac{Q_{21}}{T} + S_{gen} \rightarrow s_2 - s_1 = S_{gen} = 0}$ (reversible, adiabatic)
- Find temperature of state 2:
    - $T_2 = T_1(P_2/P_1)^{(k-1)/k} = 300 * (100/250)^{0.4/1.4} = 230.9K$
- Find volume of state 2:
    - $V_2 = V_1 P_1 T_2/(P_2 T_1) = 1cm^3 * 250\ kPa * 230.9K/(100kPa * 300K) = 1.92cm^3$
- Find the work: Isentropic expansion
    - $W_{21} = \frac{1}{1-k}(P_2V_2 - P_1V_1) = \frac{1}{1-1.4}(100 * 1.92e^{-6} - 250 * 1e^{-6}) = 0.000145\ kJ$

4. A spring-loaded piston/cylinder setup contains 1.5 kg of air at 27°C and 160 kPa. It is now heated in a process where pressure is linear in volume (i.e. P = *A* + *B**V) to twice the initial volume where it reaches 900K. Assuming an ideal gas with constant specific heats (Cv = 0.717 kJ/kgK, Cp = 1.004 kJ/kgK, R = 0.287 kJ/kgK), find

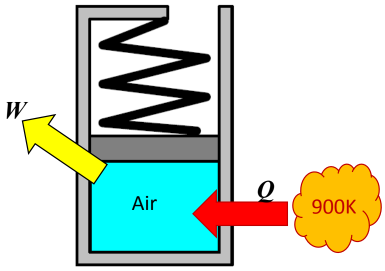


Fig. 4

a) the work performed
b) the heat transfer
c) the total entropy generated assuming a source (i.e. surroundings) at 900K.

[Ans: a) 161.4 kJ, b) 806.7 kJ, c) 0.584 kJ/K]

Solution:

- 1st Law: Conservation of energy: $\boldsymbol{m(u_2 - u_1) = Q_{21} - W_{21}}$
- 2nd Law: $\boldsymbol{m(s_2 - s_1) = \int \frac{Q_{21}}{T} + S_{gen}}$
- Process: expansion: P = *A* + *B**V, fluid (air)

State 1: T_1 = 300K, P_1 = 160 kPa

- $V_1 = mRT_1/P_1$ = (1.5kg * 0.287kJ/kg/K * 300K) / 160 kPa = 0.8072 m^3

State 2: T_2 = 900K, V_2 = 2* V_1 = 1.614 m^3

- $P_2 = mRT_2/V_2$ = (1.5kg * 0.287kJ/kg/K * 900K) / 1.614m^3 = 240 kPa
- Work: $\boldsymbol{W_{21} = \int PdV = \frac{1}{2}(P_1 + P_2) * (V_2 - V_1) = 161.4kJ}$
    - Work is out of the system
- Heat Transfer: $\boldsymbol{Q_{21} = m(u_2 - u_1) + W_{21} = mC_v(T_2 - T_1) + W_{21}}$
- $\boldsymbol{Q_{21} = 1.5\ kg * 0.717 \frac{kg}{kgK}\ (900 - 300)K + 161.4kJ = 806.7\ kJ}$
- Sgen: $\boldsymbol{S_{gen} = m(s_2 - s_1) - \int \frac{Q_{21}}{T} = m\left(C_p ln\frac{T_2}{T_1} - Rln\frac{P_2}{P_1}\right) - \int \frac{Q_{21}}{T}}$
- $\boldsymbol{S_{gen} = 1.5kg * \left(1.004\frac{kJ}{kgK} ln\frac{900K}{300K} - 0.287\frac{kJ}{kgK} ln\frac{240kPa}{160kPa}\right) - \frac{806.7kJ}{900K} = 0.584\frac{kJ}{K}}$

5) Steam enters a turbine at 300°C, 600 kPa and exhausts as a saturated vapor at 20 kPa. Assume the turbine to be adiabatic.

a) Determine isentropic efficiency of the turbine.

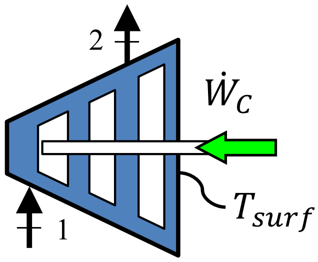
b) Determine the amount of entropy generated during this process.

[ans: (a) $\eta_{turbine} = 0.716$*, (b)* $s_{gen} = 0.5362\frac{kJ}{kgK}$*]*

Solution:

- State 1: P_1 = 600 kPa, T_1 = 300C → superheated vapor
  - h_1 = 3061.63 kJ/kg, s_1 = 7.3723 kJ/kgK
- State 2: P_2 = 20 kPa, saturated vapor
  - $h_2 = 2609.70\frac{kJ}{kg};\ s_2 = 7.9085\frac{kJ}{kgK}$
- State 2 (isentropic): $s_2 = s_1$ = 7.3723 kJ/kgK, P_2 = 20 kPa
  - $x_2 = (s_2 - s_f)/s_{fg} = (7.3723 - 0.8319)/7.0766 = 0.924$
  - $h_2 = h_f + x_2 h_{fg} = 251.38 + 0.924 * 2358.33 = 2430.48\frac{kJ}{kg}$
- 1st Law: Conservation of energy:
  - Actual: $w_{21,out,actual} = h_1 - h_2 = 3061.63 - 2609.70$
  - $w_{21,out,actual} = 451.93\frac{kJ}{kg}$
  - Isentropic: $w_{21,out,isentropic} = h_1 - h_{2,s} = 3061.63 - 2430.48$
  - $w_{21,out,isentropic} = 631.15\frac{kJ}{kg}$
- Isentropic efficiency: $\eta_{turbine} = w_{actual}/w_{isentropic} = \frac{451.93}{631.15} = 0.716$
- 2nd Law: $s_2 - s_1 = \int \frac{q_{21}}{T} + s_{gen} \rightarrow s_{gen} = s_2 - s_1$
  - $s_{gen} = 7.9085 - 7.3723 = 0.5362\frac{kJ}{kgK}$

6) Air is compressed in an axial flow compressor operating at steady state from 300K, 100 kPa to a pressure of 400 kPa. Heat loss from the compressed air occurs at the rate of 34.5 kJ/kg on the compressor's surface where the temperatures is constant at 50°C. Assuming variable specific heats for air, determine the minimum compressor work (in kJ/kg air) in order to accomplish this pressure increase. Take R_{air} = 0.287 kJ/kgK.

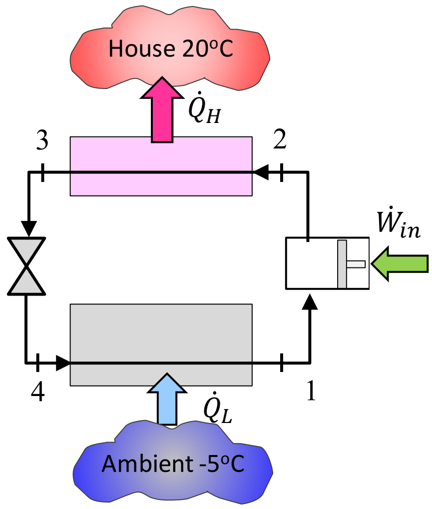


Solution:

- ***1st law (steady state): $E_{in} = E_{out}$; $0 = W_{in} - Q_{loss} + mh_1 - mh_2$***
- ***$w_c = h_2 - h_1 + q_{loss}$***
- **State 1 (Thermodynamic air tables): P_1 = 100 kPa, T_1 = 300K, h_1 = 300.47 kJ/kg, s_{T1}^0 = 6.86926 kJ/kgK.**
- **State 2: P_2 = 400 kPa; Need more information about state 2**
- **Entropy equation**
- $s_2 - s_1 = \int_1^2 \frac{-dq}{T} + S_{gen}$; **in order to achieve the minimum work input, the process must be reversible. If reversible, then S_{gen} = 0**
- $s_2 - s_1 = \int_1^2 \frac{dq}{T}$
- **For ideal gas with variable specific heats:** $s_2 - s_1 = s_{T2}^0 - s_{T1}^0 - R\ln\left(\frac{P_2}{P_1}\right)$
- $s_{T2}^0 - s_{T1}^0 - R\ln\left(\frac{P_2}{P_1}\right) = \int_1^2 \frac{-dq}{T}$; **solve for s_{T2}^0.**
- $s_{T2}^0 = s_{T1}^0 + R\ln\left(\frac{P_2}{P_1}\right) + \int_1^2 \frac{-dq}{T} \to 6.86926\frac{kJ}{kgK} + 0.287\frac{kJ}{kgK}\ln(4) - \frac{34.5\frac{kJ}{kg}}{323.15K} = 7.160\frac{kJ}{kgK}$
- **Use thermodynamic air tables to interpolate to find T_2 and h_2:**
- **T_2 = 400.3K, h_2 = 401.60 kJ/kg**

$w_c = h_2 - h_1 + q_{loss}$ = 401.60 kJ/kg – 300.47 kJ/kg + 34.5 kJ/kg = 135.63 kJ/kg

7) A refrigeration system is used as a heating device (i.e. heat pump). The refrigeration system uses R-410A. The cycle is used to warm a house and maintain a constant house temperature of 20°C. The electric power required to operate the heat pump is 2 kW and it exchanges heat with the ambient at -5°C. The high and low operating pressures of the refrigeration cycle are 2000 kPa and 400 kPa, respectively. Assume the cycle to operate on the ideal refrigeration cycle



a) Determine the COP of the heat pump.

b) Determine the heating rate in kW.

c) Determine the change of entropy for the surroundings in kW/K.

Solution:

- Heat pump cycle: Determine the states
- State 1: (sat. vapor at 400 kPa); h_1 = 271.90 kJ/kg, s_1 = 1.0779 kJ/kgK
- State 2: P_2 = 2000 kPa, $s_2 = s_1 > s_{g@2000kPa}$ (superheated vapor); Interpolation
  - $\boldsymbol{h_2 = \frac{(1.0779-1.0099)kJ/kgK}{(1.0878-1.0099)kJ/kgK}(320.62-295.49)\frac{kJ}{kg} + 295.49\frac{kJ}{kg} = 317.43\frac{kJ}{kg}}$
- Compressor: 1st law (steady state)
  - $w_{in} = h_2 - h_1$ = (317.43 - 271.90) kJ/kg = 45.53 kJ/kg
- Mass flow rate of refrigerant: $\boldsymbol{\dot{m} = \dot{W}_{IN}/w_{IN} = 2kW/45.53\frac{kJ}{kg} = 0.0439\frac{kg}{s}}$
- State 3: Saturated liquid at 2000 kPa; interpolate to find h_3
  - $\boldsymbol{h_3 = \frac{(2000-1885.1)kPa}{(2140.2-1885.1)kPa}(114.95-106.14)\frac{kJ}{kg} + 106.14\frac{kJ}{kg} = 110.11\frac{kJ}{kg}}$
- Condenser: 1st Law (steady state)
  - $\boldsymbol{\dot{Q}_{32} = \dot{m}(h_2 - h_3) = 0.0439\frac{kg}{s}(317.43-110.11)\frac{kJ}{kg} = 9.1kW}$
- Find COP: $\boldsymbol{COP = \frac{\dot{Q}_{32}}{\dot{W}_{IN}} = \frac{9.1kW}{2kW} = 4.55}$
- Rate of entropy for the surroundings: $\boldsymbol{\Delta S_{surr} = \frac{\dot{Q}_{32}}{T_{house}} - \frac{\dot{Q}_{14}}{T_{ambient}}}$
  - Evaporator (1st Law): $\boldsymbol{\dot{Q}_{14} = \dot{m}(h_1 - h_4);\ h_4 = h_3}$
  - $\boldsymbol{\dot{Q}_{14} = \dot{m}(h_1 - h_4) = 0.0439\frac{kg}{s}(271.90-110.11)\frac{kJ}{kg} = 7.10kW}$
  - We can also compute from: $\boldsymbol{\dot{Q}_{14} = \dot{Q}_{32} - \dot{W}_{IN} = 9.1 - 2.0 = 7.1kW}$
  - $\boldsymbol{\Delta S_{surr} = \frac{9.1kW}{293.15K} - \frac{7.1kW}{268.15K} = 0.00456\ kW/K}$

8) Consider the heat pump in problem (7), however, the compressor is now irreversible (but still adiabatic) and the R-410a refrigerant exits the compressor at 2000 kPa, 65°C.

a. Determine the increase in compressor work.
b. Determine the heating rate (Q_{32})
c. Determine the COP given the new conditions of the compressor.
d. Determine the entropy generated during the compression process.

Solution:

- Heat pump cycle: Determine the states
- State 1: (sat. vapor at 400 kPa); h_1 = 271.90 kJ/kg, s_1 = 1.0779 kJ/kgK
- State 2: P_2 = 2000 kPa, T_2 = 65°C (superheated vapor); Interpolation
  - $\boldsymbol{h_2 = \frac{(65-60)C}{(80-60)C}(343.22-320.62)\frac{kJ}{kg} + 320.62\frac{kJ}{kg} = 326.3\frac{kJ}{kg}}$
  - $\boldsymbol{s_2 = \frac{(65-60)C}{(80-60)C}(1.1537-1.0878)\frac{kJ}{kgK} + 1.0878\frac{kJ}{kgK} = 1.104\frac{kJ}{kg}}$
- Compressor: 1st law (steady state, irreversible, but still adiabatic)
  - $\boldsymbol{\dot{W}_{21} = \dot{m}(h_2-h_1) = 0.0439\frac{kg}{s}(326.3-271.9)\frac{kJ}{kg} = 2.39\ kW}$
  - **Increase in compressor work is 0.39 kW.**
- Condenser: 1st Law (steady state)
  - $\boldsymbol{\dot{Q}_{32} = \dot{m}(h_2-h_3) = 0.0439\frac{kg}{s}(326.3-110.11)\frac{kJ}{kg} = 9.49kW}$
- Find COP: $\boldsymbol{COP = \frac{\dot{Q}_{32}}{\dot{W}_{IN}} = \frac{9.49kW}{2.39kW} = 3.97}$
- Compressor: 2nd Law Analysis
  - $\boldsymbol{\dot{m}(s_2-s_1) = \int\frac{Q_{21}}{T} + S_{gen}}$
  - $\boldsymbol{S_{gen} = \dot{m}(s_2-s_1) = 0.0439\frac{kg}{s}(1.104-1.0779)\frac{kJ}{kgK} = 0.001146\frac{kW}{K}}$

9) A refrigerator uses R-134a as the working fluid and operates on an IDEAL vapor-compression refrigeration cycle between 100 kPa and 800 kPa. The mass flow rate of the refrigerant is 0.05 kg/s.

a) Determine the quality of the refrigerant entering the evaporator.

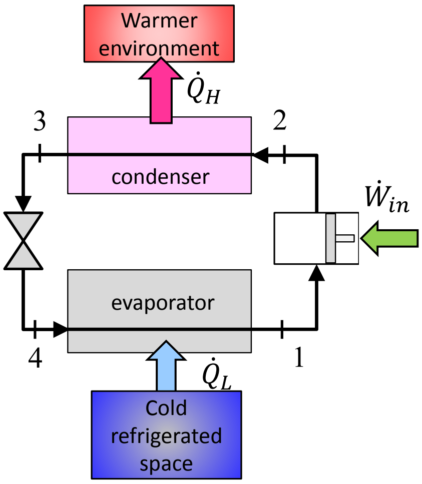
b) Determine the rate of heat removal from the refrigerated space.

c) Determine the power input to the compressor.

d) Determine the COP.

e) Determine the heat rejected to the warmer environment.

f) Show the processes on the T-s diagram.



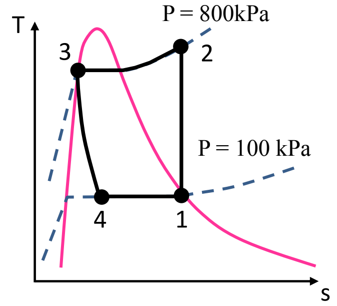
[ans: (a) $\boldsymbol{x_4 = 0.36}$*, (b)* $\boldsymbol{\dot{Q}_{14} = 6.92kW}$*, (c)* $\boldsymbol{\dot{W}_{21} = 2.16kW}$ *(d) COP = 3.2, (e)* $\boldsymbol{\dot{Q}_{32} = 9.08kW}$*]*

Solution:

- State 1: (sat. vapor at 100 kPa); h_1 = 381.98 kJ/kg, s_1 = 1.7456 kJ/kgK
- State 2: P_2 = 800 kPa, $s_2 = s_1 > s_{g@800kPa}$ (superheated vapor); Interpolation
  - $h_2 = \frac{(1.7456-1.7446)kJ/kgK}{(1.7768-1.7446)kJ/kgK}(435.11 - 424.86)\frac{kJ}{kg} + 424.86\frac{kJ}{kg} = 425.20\frac{kJ}{kg}$
  - $T_2 = \frac{(1.7456-1.7446)kJ/kgK}{(1.7768-1.7446)kJ/kgK}(50 - 40)C + 40C = 40.33C$
- State 3: (sat. liquid at 800 kPa);
  - $h_3 = \frac{(800-771)kPa}{(887.6-771)kPa}(249.1 - 241.79)\frac{kJ}{kg} + 241.79\frac{kJ}{kg} = 243.61\frac{kJ}{kg}$
  - $T_3 = \frac{(800-771)kPa}{(887.6-771)kPa}(35 - 30)C + 30C = 31.2C$
  - $s_3 = \frac{(800-771)kPa}{(887.6-771)kPa}(1.1673 - 1.1437)\frac{kJ}{kgK} + 1.1437\frac{kJ}{kgK} = 1.1496\frac{kJ}{kgK}$
- State 4: $h_4 = h_3$ = 243.61; interpolation needed at 100 kPa
  - $h_{f@100kPa} = \frac{(100-85.1)kPa}{(101.3-85.1)kPa}(165.8 - 161.12)\frac{kJ}{kg} + 161.12\frac{kJ}{kg} = 165.42\frac{kJ}{kg}$
  - $h_{fg@100kPa} = \frac{(100-85.1)kPa}{(101.3-85.1)kPa}(216.36 - 218.68)\frac{kJ}{kg} + 218.68\frac{kJ}{kg} = 216.55\frac{kJ}{kg}$
  - $s_{f@100kPa} = \frac{(100-85.1)kPa}{(101.3-85.1)kPa}(0.8690 - 0.8499)\frac{kJ}{kgK} + 0.8499\frac{kJ}{kgK} = 0.8675\frac{kJ}{kgK}$
  - $s_{fg@100kPa} = \frac{(100-85.1)kPa}{(101.3-85.1)kPa}(0.8763 - 0.8994)\frac{kJ}{kgK} + 0.8499\frac{kJ}{kgK} = 0.8994\frac{kJ}{kgK}$

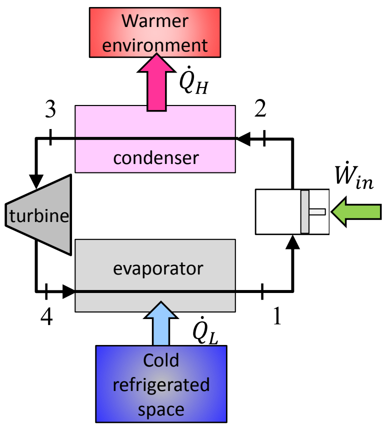
- Quality: $x_4 = (h_4 - h_{f@100kPa})/h_{fg@100kPa} = (243.61 - 165.42)/216.55$
  - $x_4 = 0.36$
- $T_3 = T_{sat@100kPa} = -26.54C$

- Evaporator: 1st law (steady state)
  - $\dot{Q}_{14} = \dot{m}(h_1 - h_4) = 0.05\frac{kg}{s}(381.98 - 243.61)\frac{kJ}{kg} = 6.92kW$
- Compressor: 1st law (steady state)
  - $\dot{W}_{21} = \dot{m}(h_2 - h_1) = 0.05\frac{kg}{s}(425.20 - 381.98)\frac{kJ}{kg} = 2.16kW$
- COP: $COP = \frac{\dot{Q}_{14}}{\dot{W}_{21}} = \frac{6.92kW}{2.16kW} = 3.2$
- Condenser: 1st law (steady state)
  - $\dot{Q}_{32} = \dot{m}(h_2 - h_3) = 0.05\frac{kg}{s}(425.20 - 243.61)\frac{kJ}{kg} = 9.08kW$



10) Take the same refrigeration system as in problem 9, but now the throttle is replaced by an isentropic turbine.

a) Determine the quality of the refrigerant entering the evaporator.
b) Draw the processes on the T-s diagram
c) Determine the new cooling capacity.
d) Determine the percentage increase of COP.
e) Why are turbines not placed in refrigeration cycles?



[ans: (a) $x_4 = 0.32$, (c) $\dot{Q}_{14} = 7.36\ kW$, (d) COP = 3.41 (6.56% increase)]

Solution:

- States 1-3 are the same as in problem 9.
- State 4: now $h_4 \neq h_3$, rather $s_4 = s_3 = 1.1496$ kJ/kgK
- At 100 kPa $s_f < s_2 < s_g$. Interpolation needed to get s_f and s_g at 100 kPa.
  - $s_{f@100kpa}$ = 0.8675 kJ/kgK, $s_{fg@100kPa}$ = 0.87815 kJ/kgK
  - $x_4 = (s_4 - s_{f@100kPa})/s_{fg@100kPa} = (1.1496 - 0.8675)/0.87815$

- $x_4 = 0.32$
- $h_4 = h_f + x_4 h_{fg} = 165.42\frac{kJ}{kg} + 0.32 * 216.55\frac{kJ}{kg} = 234.72\frac{kJ}{kg}$
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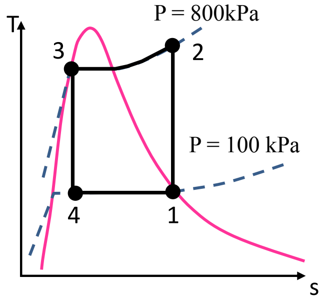

- Evaporator: 1st law (steady state)
  - $\dot{Q}_{14} = \dot{m}(h_1 - h_4) = 0.05\frac{kg}{s}(381.98 - 234.72)\frac{kJ}{kg} = 7.36kW$
- COP: $COP = \frac{\dot{Q}_{14}}{\dot{W}_{21}} = \frac{7.36kW}{2.16kW} = 3.41$
  - COP percentage increase: (3.41-3.2) / 3.2 x 100% = 6.56%
- Although the isentropic turbine allows for greater cooling capacity, the turbine would be quite large to fit in a refrigerator unit. Also in practicality, the turbine is not adiabatic or reversible, thus entropy will not be constant across the turbine. Any work out of the turbine could ideally be used to run the compressor. However, even in the best case scenario, the work output of an isentropic turbine is: $\dot{W}_{43} = \dot{m}(h_3 - h_4) = 0.05\frac{kg}{s}(243.61 - 234.72)\frac{kJ}{kg} = 0.45\ kW$. This amount of work output is not worth placing a very large turbine within the refrigeration unit.