Dynamics 2 – Tutorial 8

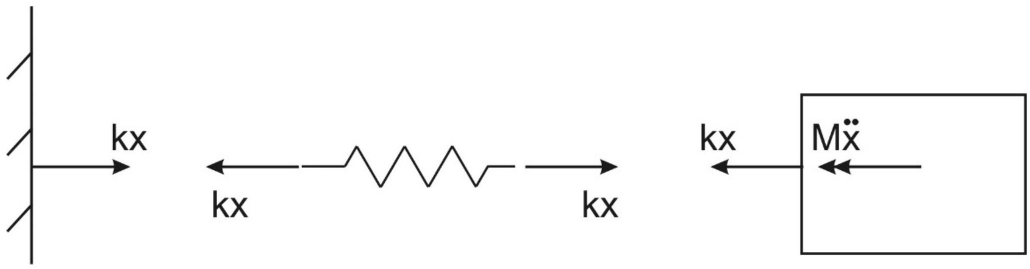
Free Oscillations

Outline Solutions

1.

(a) Frequency and General Solution

FBD (taking x(t) as +ve to the right)



FBD gives

$$M\ddot{x} + Kx = 0$$

$$\ddot{x} + \left(\frac{K}{M}\right)x = 0$$

Compare with

$$\ddot{x} + \omega_O{}^2 x = 0$$

$$\omega_O = \sqrt{\frac{K}{M}}$$

Substituting values

$$\omega_O = \sqrt{\frac{28{,}425}{45}} = 25.133 \text{ rad/s}$$

$$f_O = \frac{\omega_O}{2\pi} = 4 \text{ Hz}$$

General Solution

$$x(t) = C_1 \cos \omega_O t + C_2 \sin \omega_O t$$

(b) Time to reach maximum extension and force on wall with initial velocity v_O = 1.5 m/s

To examine this we need to find the values of the constants in the General Solution using the boundary conditions: x(t) = 0, $\dot{x}(0) = v_O$ = 1.5 m/s

$$x(0) = 0 = C_1 \cdot 1 + C_2 \cdot 0 \qquad \Rightarrow C_1 = 0$$

$$\dot{x}(t) = -\omega_O C_1 \sin \omega_O t + \omega_O C_2 \cos \omega_O t$$
$$\dot{x}(0) = 1.5 = -\omega_O C_1 \cdot 0 + \omega_O C_2 \cdot 1 \qquad \Rightarrow C_2 = 1.5 / \omega_O = 0.0597$$

The solution is therefore

$$x(t) = \frac{v_O}{\omega_O} \sin \omega_O t = 0.0597 \sin 25.13t$$

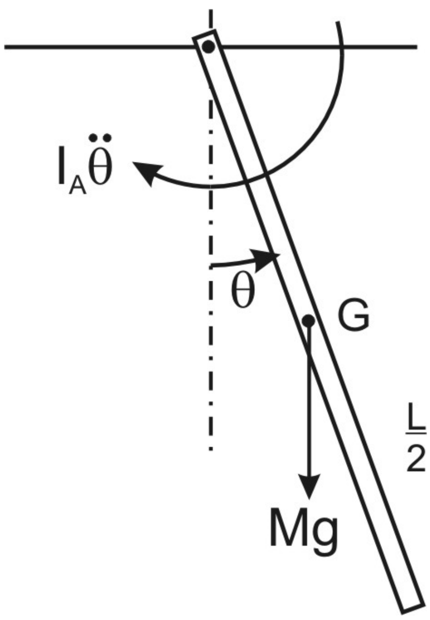
With this solution (sine) the peak displacement (0.0597) is reached at

$$\omega_O t = \frac{\pi}{2} \qquad \Rightarrow t = 0.063 \text{ s} = 63 \text{ ms}$$

The maximum force on the wall also occurs at peak displacement

Force = Kx_{max} = 1697 N

2.



Fixed Axis Rotation

FBD gives

$$I_A\ddot{\theta} + \frac{L}{2}Mg \sin \theta = 0$$

$$\ddot{\theta} + \frac{3g}{2L} \sin \theta = 0$$

Assume small θ and divide by I_A

$$\ddot{\theta} + \frac{MgL}{2I_A}\theta = 0$$

Substituting for $I_A = \frac{1}{3}mL^2$

$$\ddot{\theta} + \frac{3g}{2L}\theta = 0$$

Compare with

$$\ddot{\theta} + \omega_O{}^2\theta = 0$$

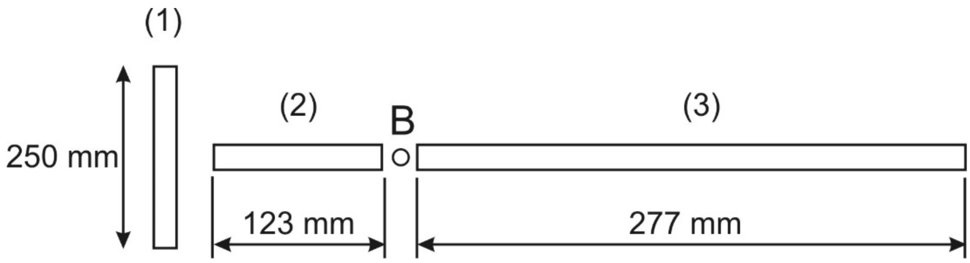
Gives

$$\omega_O = \sqrt{\frac{3g}{2L}} = 4.758 \text{ rad/s or } 0.76 \text{ Hz}$$

3.

(a) Find I_B

Split bar into 3 parts where masses by proportion of length.



m_1 = 1.7308 kg; m_2 = 0.8515 kg; m_3 = 1.9177 kg.

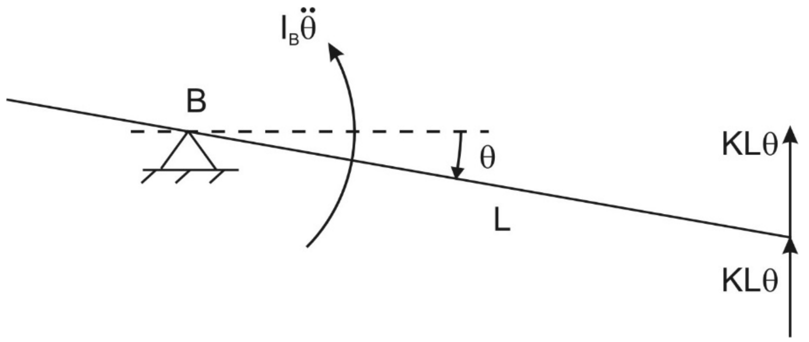
The total I_B

$$I_B = \frac{1}{12} \times 1.7308 \times 0.25^2 + 1.7308 \times 0.123^2 \qquad \text{(part 1)}$$

$$+ \frac{1}{3} \times 0.8515 \times 0.123^2 + \frac{1}{3} \times 1.9177 \times 0.277^2 \qquad \text{(part 2 and 3)}$$

$$I_B = 0.0885 \text{ kgm}^2$$

(b) Value of K for f_O = 6.2 Hz



Let θ be a small angular displacement of bar about pivot B (clockwise +ve).

Spring force is $KL\theta$ and acts upwards

Law of fixed axis rotation about B

$$I_B\ddot{\theta} + (2KL\theta)L = 0$$

$$\ddot{\theta} + \frac{2L^2K}{I_B}\theta = 0$$

$$\Rightarrow \qquad \omega_O = L\sqrt{\frac{2K}{I_B}} \text{ rad/s}$$

Rearranging

$$K = \frac{I_B \omega_O{}^2}{2L^2}$$

Need K for f_O = 6.2 Hz, i.e. $\omega_O = 2\pi \times 6.2 = 38.96$ rad/s

$\Rightarrow \quad K = 875.18$ N/m

(c) Mass required at point C to lower frequency to 6 Hz

New value of $\omega_O = 2\pi \times 6.0 = 37.7$ rad/s.
Adding mass at C will raise I_B by mL^2 and lower ω_O

Let new value for I_B be $I_B{}'$

$$I_B{}' = I_B + mL^2$$

Rearranging natural frequency equation from part (b)

$$I_B{}' = \frac{2L^2K}{\omega_O{}^2}$$

Substituting values gives

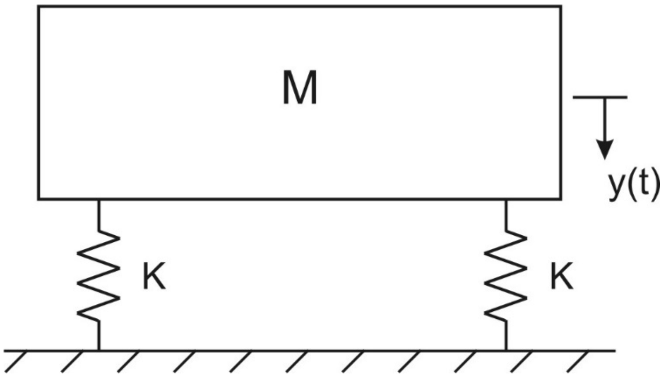
$$I_B{}' = \frac{2 \times 0.277^2 \times 875.18}{37.7^2} = 0.0945\,\text{kgm}^2$$

Hence

$$mL^2 = I_B{}' - I_B$$

$$m = \frac{I_B{}' - I_B}{L^2} = \frac{0.0945 - 0.0885}{0.277^2} = 0.0782 \text{ kg}$$

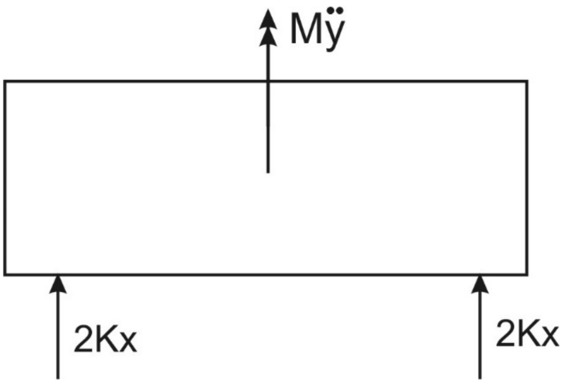
i.e., a 78 gram mass at C will lower frequency to 6 Hz.



M = 240 kg
K = 1000 N/m

(a) Period of oscillation

FBD



From FBD

$$M\ddot{y} + 4Ky = 0$$

$$\ddot{y} + \left(\frac{4K}{M}\right)y = 0$$

$$\Rightarrow \quad \omega_O = \sqrt{\frac{4K}{M}} = 4.0825 \text{ rad/s}$$

Period

$$T_O = \frac{2\pi}{\omega_O} = 1.539 \text{ s}$$

(b) Amplitude of oscillations following impact of mass

Let m = 70 kg be the mass of sand bag, which during motion remains on the platform. By extension from part (a)

$$(M + m)\ddot{y} + 4Ky = 0$$

$$\ddot{y} + \left(\frac{4K}{M + m}\right)y = 0$$

$$\Rightarrow \qquad \omega_O = \sqrt{\frac{4K}{M + m}} = 3.592 \text{ rad/s (note different to part (a))}$$

General Solution

$$y(t) = C_1 \cos \omega_O t + C_2 \sin \omega_O t$$

Requires the initial conditions: Initial displacement x(0) = 0; Initial velocity of combined mass is found as follows

Velocity of sand bag prior to impact following 1.5 m fall

$$v^2 = u^2 + 2gh$$

$$v^2 = 0 + 2 \times 9.81 \times 1.5$$

$\Rightarrow \quad v = 5.42$ m/s

Initial velocity of combined mass from linear momentum equation

$$m \times 5.42 + M \times 0 = (M + m)v_O$$

$\Rightarrow \quad v_O = 1.224$ m/s

As before

$C_1 = 0$

$C_2 = v_O / \omega_O = 0.341$

Solution

$$y(t) = 0.341 \sin \omega_O t = 0.341 \sin 3.592t$$

Hence amplitude of vibration is 0.341 m.