Tutorial 5 – SOLUTIONS

Tutorial 5: (1) Carnot cycles for heat engines & heat pumps & (2) Entropy Basics

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Problem Solving Questions:

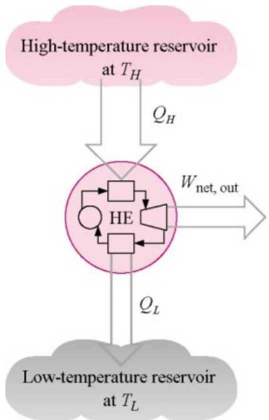
1. Complete the table below for the following property substances.

Substance	Pressure (kPa)	Temperature (C)	State	Specific entropy (kJ/kgK)
water	5000	120	Compressed liquid	1.5232
water	198.5	120	Saturated liquid	1.5275
water	10	50	Superheated vapor	8.1749
R-134a	85.1	-30	Sat. Mixture, x = 0.5	1.3
R-134a	100	-26.54	Saturated Vapor	1.7456
Ammonia	54.5	-45	Sat. Mixture, x = 0.4	2.364
Ammonia	150	80	Superheated vapor	6.4877

2. A heat engine receives 150MW of heat energy. The engine produces 50 MW of net work.

(a) Determine the cycle thermal efficiency

(b) Determine the heat rejected by the cycle to the lower temperature reservoir.



Solution:

a. $n_{TH} = \frac{W_{net,out}}{Q_H} = \frac{50}{150} = 0.333 \text{ or } 33.3\%$

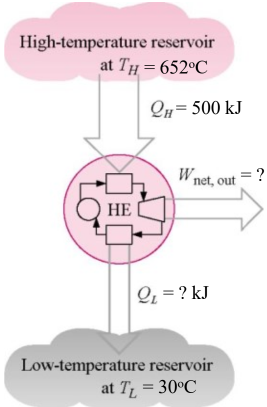
b. 1st law (steady state): $E_{in}=E_{out}$. $Q_H = W_{net,out} + Q_L$

– $Q_L = Q_H - W_{net,out} = 150 - 50 = 100$ **MW**

3. A Carnot heat engine receives 500 kJ of heat per cycle from a high-temperature heat reservoir at 652ºC and rejects heat to a low-temperature heat reservoir at 30ºC. Determine:

(a) The thermal efficiency of this Carnot engine.

(b) The amount of heat rejected to the low-temperature heat reservoir.



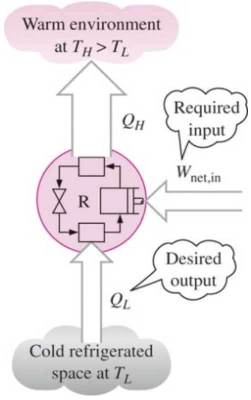
Solution:

a. $\boldsymbol{n_{TH} = \frac{W_{net,out}}{Q_H} \rightarrow CARNOT \rightarrow 1 - \frac{T_L}{T_H} = 1 - \frac{303.15K}{925.15K} = 0.672\ or\ 67.2\%}$

b. $\boldsymbol{n_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{Q_L}{Q_H}} \rightarrow Q_L = (1 - \eta_{TH})Q_H$

- $Q_L = (1 - 0.672)500kJ = \boldsymbol{164kJ}$
- OR one can notice that: $\frac{T_L}{T_H} = \frac{Q_L}{Q_H};\ Q_L = \frac{T_L}{T_H} Q_H$
- $Q_L = (T_L / T_H) * Q_H = (303.15K / 925.15K) * 500kJ =$ **164 kJ**

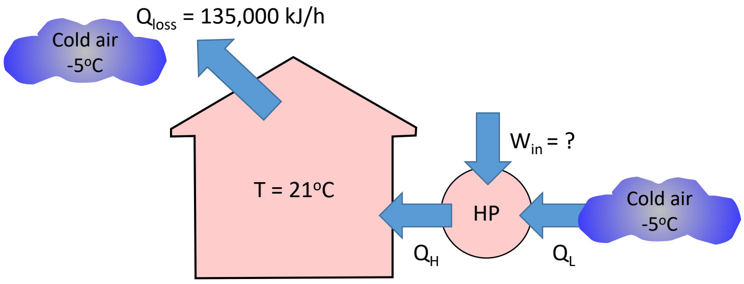
4. An inventor claims to have developed a refrigerator with a COP of 13.5 that maintains the refrigerated space at 2ºC while operating in a room where the temperature is 25ºC. Evaluate the maximum possible (Carnot) COP of a refrigerator operating in these conditions. Is the inventor's claim true?



Solution:

- *The maximum COP can be obtained for a refrigerator operating on a Carnot cycle:* $COP_R = \frac{Q_L}{Q_H - Q_L} \rightarrow CARNOT \rightarrow \frac{T_L}{T_H - T_L} = \frac{275.15K}{(298.15 - 275.15)K} = \boldsymbol{11.96}$
- The maximum COP is less than what the inventor is claiming. The inventor's claim would then be false.

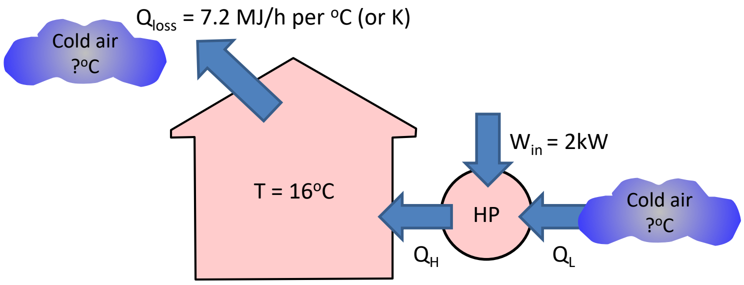
5. A heat pump is to be used to heat a building during the winter. The building is to be maintained at 21°C at all times. The building is estimated to be losing heat at a rate of 135,000 kJ/h when the outside temperature drops to -5°C. Determine the minimum power required to drive the heat pump unit for this outside temperature.



Solution:

- *Take the house to be the control volume*
  - *1st Law (steady state):* *E_{in} = E_{out} →* $\dot{Q}_H = \dot{Q}_{Loss} = 135{,}000\ kJ/h$
- *Take the heat pump as the control volume*
  - *1st law: steady state): E_{in} = E_{out} →* $\dot{Q}_L + \dot{W}_{in} = \dot{Q}_H$
  - $COP = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L}$
  - *The minimum power required to operate the heat pump is when the heat pump is reversible (i.e. operates on the Carnot cycle)*
  - $COP = \frac{T_H}{T_H - T_L} = \frac{294.15K}{(294.15-268.15)K} = 11.31$
  - $\dot{W}_{in} = \frac{\dot{Q}_H}{COP} = \frac{135{,}000 kJ/h}{11.31} = 11{,}936\frac{kJ}{h} \to \mathbf{3.316 kW}$

6. A house loses heat at a rate of 7.2 MJ/h per °C difference between the inside and outside of the house. Calculate the lowest outside temperature for which a heat pump requiring a power input of 2 kW can maintain the house at 16°C.



Solution:

- *Take the house to be the control volume*
  - *1st Law (steady state): E_{in} = E_{out} →* $\dot{Q}_H = \dot{Q}_{Loss} = 7.2\frac{MJ}{h}(T_H - T_L)$
  - $\dot{Q}_H = 7.2\frac{MJ}{h}(T_H - T_L) \to convert\ \frac{MJ}{h}\ to\ \frac{kJ}{s}(kW) \to 7.2\frac{MJ}{h} * 1000\frac{kJ}{MJ} * \frac{hr}{3600sec} = 2kW$

- $\dot{Q}_H = 2kW(T_H - T_L)$

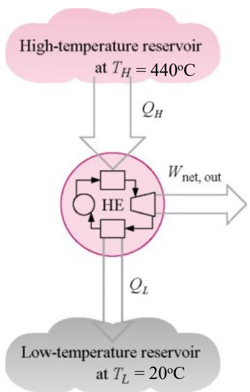
– *Take the heat pump as the control volume*

- *1st law: steady state): $E_{in} = E_{out}$* → $\dot{Q}_L + \dot{W}_{in} = \dot{Q}_H$
- $COP = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L}$
- *To calculate the lowest temperature outside, one needs to consider the best possible heat pump, which is a heat pump that runs on a Carnot cycle.*
- $COP = \frac{T_H}{T_H - T_L} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{2kW(T_H - T_L)}{\dot{W}_{in}} \rightarrow T_H = (16 + 273.15K) = 289.15K$
- $2kW * (T_H - T_L)^2 = T_H\dot{W}_{in} \rightarrow 2kW * (T_H - T_L)^2 = T_H * 2kW$
- $(T_H - T_L)^2 = T_H \rightarrow T_H^2 - T_H - 2T_HT_L + T_L^2 = 0$
- *Plug in value for T_H and solve for quadratic formula*
    - $T_L^2 - 578.3 * T_L + 83318.6 = 0 \rightarrow T_L = 272.15K \ or\ 306.15K$
    - *T_L must be less then T_H, T_L = 272.15K or 1°C.*
- *OR can plug in value for T_H and rearrange:*
- $(T_H - T_L)^2 = T_H \rightarrow (289 - T_L)^2 - 289 = 0$
- $(289 - T_L)^2 - 17^2 = 0 \rightarrow (289 - T_L - 17) * (289 - T_L + 17) = 0$
- $(272 - T_L) * (306 - T_L) = 0 \rightarrow$ *T_L = 272K or 306K, T_L must be lower than T_H. Thus T_L = 272K or -1C.*

7. 3150 kJ of heat is transferred into a cycle at 440°C. 1294.8 kJ of heat are rejected from the same cycle at 20°C. Heat transfer occurs at constant temperature.

(a) Is the Clausius inequality satisfied and is the cycle reversible or irreversible?

(b) Calculate the net work and cycle efficiency for this cycle.



Solution:

(a) $\oint \frac{\delta Q}{T} \leq 0 \rightarrow \int \left(\frac{\delta Q_{NET}}{T}\right)_{IN} + \int \left(\frac{\delta Q_{NET}}{T}\right)_{OUT} = \left(\frac{Q_{IN}}{T_{IN}}\right) + \left(\frac{-Q_{OUT}}{T_{OUT}}\right)$

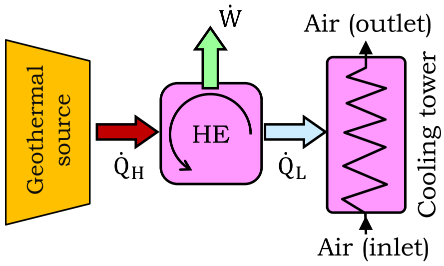
- $\left(\frac{Q_{IN}}{T_{IN}}\right) + \left(\frac{-Q_{OUT}}{T_{OUT}}\right) = \left(\frac{3150kJ}{440+273.15}\right) + \left(\frac{-1294.46kJ}{20+273.15}\right) = (4.417 - 4.417) = 0$
- *Yes, the Clausius inequality is satisfied. Since $\oint \frac{\delta Q}{T} = 0$, that means the cycle is reversible.*

(b) Take the heat engine as the control volume

- *1st law: steady state): $E_{in} = E_{out}$* → $Q_H = Q_L + W_{out}$

- $W_{out} = (3150 - 1294.8)kJ = 1855.2kJ$
- $n_{TH} = \frac{W_{out}}{Q_H} = \frac{1855.2kJ}{3150kJ} 0.589 \; or \; 58.9\%$

8. A power plant generates 150 MW of electrical power. It uses a supply of 1000 MW from a geothermal source and rejects energy to air in a cooling tower.



(a) Find the heat given to the air.

(b) How much air should be flowing to the cooling tower (kg/s) if the air temperature cannot be increased by more than 10°C? Assume air as an ideal gas with constant specific heats

Solution:

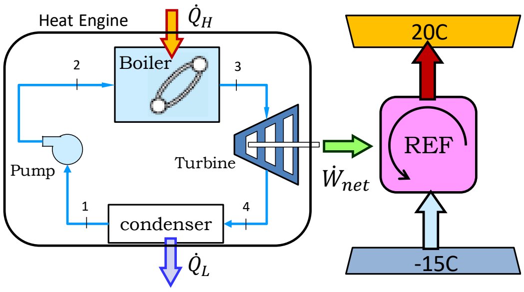
(a) 1st law (steady state): $E_{in} = E_{out} \rightarrow Q_H = W_{out} + Q_L$

- *$Q_L = Q_H - W_{out} = 1000 - 150$ MW $= 850$ MW*

(b) Take cooling tower to be control volume

- *1st law (steady state): $E_{in} = E_{out} \rightarrow \dot{Q}_L = \dot{m}_{air}(h_{in} - h_{out}) = \dot{m}_{air} C_P (T_{in} - T_{out})$*
- $\boldsymbol{\dot{m}_{air} = \dot{Q}_L / (C_P\,(T_{in} - T_{out})) = 850{,}000\ kJ/s\ /\ (1.004\frac{kJ}{kgK} * 10K)}$
- $\boldsymbol{\dot{m}_{air} = 84{,}661\ kg/s}$

9. Water is used as the working fluid in a Carnot cycle heat engine (i.e. power plant). As heat is added to the boiler, the water changes from a saturated liquid to a saturated vapor under constant temperature of 200°C. Heat is rejected from the heat engine in the condenser, where the water remains a saturated mixture ($x_4 > x_1$) at a constant pressure of 20 kPa. The heat engine powers a Carnot cycle refrigerator that operates between -15°C and 20°C.



(a) Find the heat added to the water (q_H) in kJ/kg.

(b) How much heat (i.e. Q_H (in kW)) should be added to the heat engine so that the heat pump (refrigerator) can remove 1 kW from the cold space at -15°C?

(c) What is the mass flow rate of water so that the heat pump (refrigerator) can remove 1 kW from the cold space at -15C?

Solution:

(a) Take the boiler as the control volume

- *1st law (steady state): $E_{in} = E_{out}$; $0 = Q_{in} - W_{out} + m_i h_i - m_e h_e$*
- *$q_H = h_3 - h_2$; the water goes from a saturated liquid (state 2) to a saturated vapor (state 3)*
- $h_2 = h_{f@200C} = 852.43$ kJ/kg; $h_3 = h_{g@200C} = 2793.18$ kJ/kg;
- $q_H = 2793.18 - 852.43$ kJ/kg $= h_{fg@200C} = 1940.75$ kJ/kg
- Note: could also take $q_H = \int Tds = T(s_2 - s_1) = T_{200C} s_{fg@200C} = 473.15K * 4.1014 kJ/kgK = 1940.75\ kJ/kg$

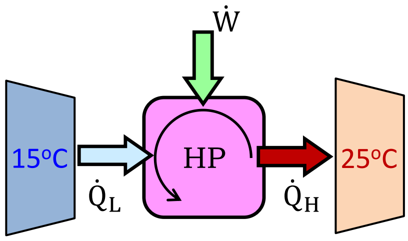
(b) Consider the heat pump (refrigerator) as the control volume

- $\beta = \frac{\dot{Q}_{L,REF}}{\dot{W}_{IN,REF}} \rightarrow CARNOT \rightarrow \frac{T_L}{T_H - T_L}$
  - *note: T's correspond to the low- and high temperature reservoirs acting between the refrigeration cycle.*
- $\beta = \frac{T_L}{T_H - T_L} = \frac{258.15}{293.15 - 258.15} = 7.37$
- $\dot{W}_{IN,REF} = \frac{\dot{Q}_{L,ref}}{\beta} = \frac{1kW}{7.37} = 0.136kW$
- *Take the condenser as the control volume.*
  - *Note: water is condensing as a saturated mixture ($0 < x_4 < 1$) at constant pressure. Temperature will also be constant and will be the saturation pressure at 20kPa (60.06°C).*
  - *Thus: T_L = 60.06°C → 333.2 K*
  - *Efficiency of heat engine:* $n_{TH} = \frac{\dot{W}_{HE}}{\dot{Q}_{H,HE}} \rightarrow CARNOT \rightarrow 1 - \frac{T_L}{T_H}$
  - $n_{TH} = \left(1 - \frac{333.2}{473.15}\right) = 0.296$
  - $\dot{W}_{HE} = 0.296 * \dot{Q}_{H,HE}$
  - *Work output of heat engine is work input to refrigerator:* $\dot{W}_{OUT,HE} = \dot{W}_{IN,REF}$
  - $\dot{W}_{IN,REF} = 0.136kW = 0.296 * \dot{Q}_{H,HE} \rightarrow \boldsymbol{\dot{Q}_{H,HE} = 0.46kW}$

(c) From part (a) we found: $q_{HE} = 1940.75 \frac{kJ}{kg}$

- From part (c), we found: $\dot{Q}_{H,HE} = 0.46kW$
- $\dot{\boldsymbol{m}}_{\boldsymbol{water}} = \dot{Q}_{H,HE} / q_{HE} = 0.46kW / 1940.75 \frac{kJ}{kg} = \boldsymbol{0.000237 \frac{kg}{s} \rightarrow 0.237 \frac{g}{s}}$

10. A reversible heat pump uses 1 kW of power to heat a 25°C room. The heat pump draws in energy from the outside at 15°C.



(a) Determine the heat delivered to the room and the heat removed from the cold temperature reservoir.

(b) Assuming every process is reversible, what are the total rates of entropy into the heat pump from the outside and from the heat pump to the room?

<u>Solution:</u>

(a) Take the heat pump as the control volume

- *1st law (steady state): $E_{in} = E_{out}$; $\dot{Q}_L + \dot{W}_{IN} = \dot{Q}_H$*
- *Inequality of Clausius gives:*
  - $\int\left(\frac{\delta \dot{Q}_{NET}}{T}\right)_{IN} + \int\left(\frac{\delta \dot{Q}_{NET}}{T}\right)_{OUT} = \left(\frac{\dot{Q}_L}{T_L}\right) + \left(\frac{-\dot{Q}_H}{T_H}\right) = 0\ (reversible)$
- $\dot{Q}_L = \dot{Q}_H\left(\frac{T_L}{T_H}\right)$
- *Substitute $\dot{Q}_L = \dot{Q}_H - \dot{W}_{IN}$ into the above equation*
- $\dot{Q}_H - \dot{W}_{IN} = \dot{Q}_H\left(\frac{T_L}{T_H}\right) \rightarrow \dot{Q}_H\left(1 - \frac{T_L}{T_H}\right) = \dot{W}_{IN} \rightarrow \dot{Q}_H = \dot{W}_{IN}/\left(1 - \frac{T_L}{T_H}\right) =$
- $\boldsymbol{\dot{Q}_H = 1kW/\left(1 - \frac{288.15K}{298.15K}\right)}$ ***= 29.8 kW***
- $\boldsymbol{\dot{Q}_L = \dot{Q}_H\left(\frac{T_L}{T_H}\right) = 29.8kW\left(\frac{288.15K}{298.15K}\right)}$ ***= 28.8 kW***

(b) $\frac{\dot{Q}_L}{T_L} = \frac{\dot{Q}_H}{T_H}$ *(inequality of Clausius for reversible cycle);* $\frac{28.8kW}{288.15K} = \frac{29.8kW}{298.15K} = \boldsymbol{0.1kW/k}$