Tutorial 8 – SOLUTIONS

Tutorial 8: Refrigeration Cycles & Rankine Cycles

Note: numerical solution are based on one approach to solving the tutorial questions. Other approaches can also be correct and could lead to slightly different numerical answers.

Conceptual Questions:

1. Why can the Carnot refrigeration cycle not be used in reality?

Solution:

- The Carnot refrigeration cycle is suggested to take place entirely underneath the saturated liquid-vapor dome. The compression process cannot take place when a vapor and liquid are being introduced into the compressor. The compressor does not operate with two phase mixtures; it can only operate with a vapor. The Carnot cycle also replaces the throttle with an isentropic turbine. However, the turbine in this case would be handling a fluid with low quality (more liquid than vapor). The turbine operates best when the working fluid has low moisture content.
- If the Carnot cycle can be executed outside of the saturated region, then it would be difficult to maintain isothermal conditions during the heat rejection as temperature and pressure would not remain constant.

2. What is the ideal Rankine cycle and how does it address practical difficulties associated with implementing the Carnot Rankine Cycle?

Solution:

- The Carnot Rankine cycle is suggested to take place entirely underneath the saturated liquid-vapor dome. First this would suggest that the pump would operate with a saturated liquid-vapor mixture and the pump can only operate with a liquid. Two-phase flows in compressors and pumps should be avoided. Limiting the heat transfer in the boiler to a 2-phase system severely limits the maximum temperature (remember the maximum temperature would have to remain below the critical point of water: 374°C). Lastly, the quality of the steam exiting the turbine would be quite low if all processes are to remain below the saturated dome. Liquid water would cause erosion of the turbine blades and reduce the lifetime of the turbine.
- If the Carnot Rankine cycle operated partly above the saturated liquid-vapor dome (see slide 21 of Lecture 16), then the compression process would require significantly greater work input (not to mention that the pump would continue to operate with a two-phase mixture. The pumping process would go to extremely high pressures and temperatures. The boiler also would not be able to undergo a constant pressure heat addition since the temperature will remain constant, yet above the saturated dome (pressure will vary)

3. What approaches can be used to improve the ideal Rankine cycle efficiency?

Solution:

- Increasing the maximum working temperature (although it should not go much higher than 600°C to avoid surface damage in the turbine)
- Greater expansion by lowering the working condenser pressure

- Increasing the higher working pressure (possibly using a two stage pumping process).
- Reheating the vapor exiting the turbine and sending the working fluid to a second lower temperature turbine to have an additional expansion process
- These methods can be implemented by using a reheat cycle or feed water heater cycle.

Problem Solving Questions

4. Consider a steam power plant operating on an ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed at a pressure of 75 kPa.

a) Determine the quality of the steam exiting the turbine

b) Determine the thermal efficiency of the cycle

[ans: a) x_4 = 0.886, b) $\boldsymbol{\eta_{th}} = \mathbf{26}\%$*]*

Solution: ideal Rankine cycle

Define the states

- State 1: saturated liquid at 75 kPa; h_1 = 384.36 kJ/kg, s_1 = 1.2129 kJ/kgK, v_1 = 0.001037 m^3/kg
- State 2: compressed liquid at 3000 kPa, $s_2 = s_1$ = 1.2129 kJ/kgK.
- State 3: superheated vapor at 3000 kPa, 350°C; h_3 = 3115.25 kJ/kg, s_3 = 6.7427 kJ/kgK
- State 4: $s_4 = s_3$ = 6.7427 kJ/kgK, P_4 = 75 kPa (saturated mixture),
    - $x_4 = (s_4 - s_f)/s_{fg}$ = (6.7427 – 1.2129) / 6.2434; $\mathbf{x_4 = 0.886}$
    - $h_4 = h_f + x_4 h_{fg}$ = 384.36 + 0.886*2278.59; h_4 = 2403.19 kJ/kg

Apply 1st law to each device

- Pump: $\boldsymbol{w_{21} = h_2 - h_1 = v_1(P_2 - P_1) = 0.001037\frac{m^3}{kg}(3000 - 75)kPa = 3.03\ kJ/kg}$
    - $h_2 = w_{in} + h_1$ = 3.03 kJ/kg + 384.36 kJ/kg = 387.39 kJ/kg
- Boiler: $q_{32} = h_3 - h_2 = 3115.25 - 387.39 = 2727.86\frac{kJ}{kg}$
- Turbine: $w_{43} = h_3 - h_4 = 3115.25 - 2403.19 = 712.06\frac{kJ}{kg}$
- Condenser: $q_{14} = h_4 - h_1 = 2403.19 - 384.36 = 2018.83\frac{kJ}{kg}$
- Thermal efficiency: $\eta_{th} = (w_{43} - w_{21})/q_{32} = (712.06 - 3.03)/2727.86$
    - $\boldsymbol{\eta_{th} = 0.26 = 26\%}$

5. A small power plant operating on the Rankine cycle with a water boiler at 3.0 MPa. The cycle has the highest and lowest temperatures of 450°C and 45°C, respectively.

a) Determine the power plant thermal efficiency.

b) Compare this efficiency with that of a Carnot cycle operating between the same high and low temperatures.

[ans: a) $\boldsymbol{\eta_{th}} = \mathbf{35}\%$*, b)* $\boldsymbol{\eta_{Carnot}} = \mathbf{56}\%$*]*

Solution: ideal Rankine cycle

Define the states

- State 1: saturated liquid at 45°C; h_1 = 188.42 kJ/kg, v_1 = 0.001010 m^3/kg, P_1 = 9.6 kPa
- State 2: compressed liquid at 3000 kPa
- State 3: superheated vapor at 3000 kPa, 450°C; h_3 = 3344 kJ/kg, s_3 = 7.0833 kJ/kgK
- State 4: $s_4 = s_3$ = 7.0833 kJ/kgK; P_4 = 9.6 kPa (saturated mixture),
    - $x_4 = (s_4 - s_f)/s_{fg}$ = (7.0833 – 0.6386) / 7.5261; x_4 = 0.856
    - $h_4 = h_f + x_4 h_{fg}$ = 188.42+0.856*2394.77, h_4 = 2238.34kJ/kg

Apply 1st law to each device

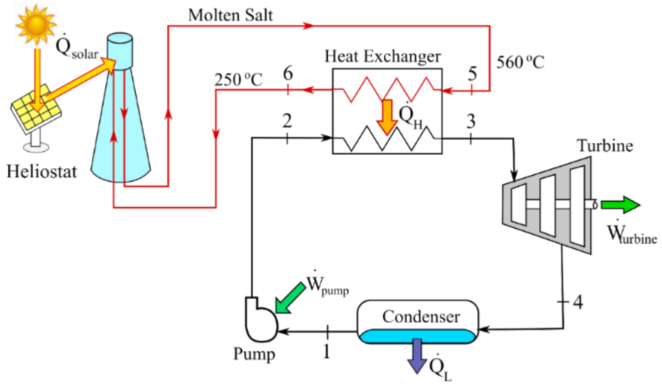
- Pump: $\boldsymbol{w_{21} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101\frac{m^3}{kg}(3000 - 9.6)kPa = 3.02\ kJ/kg}$
    - $h_2 = w_{in} + h_1$ = 3.02 kJ/kg + 188.42 kJ/kg = 191.44 kJ/kg
- Boiler: $q_{32} = h_3 - h_2 = 3344 - 191.44 = 3152.56\frac{kJ}{kg}$
- Turbine: $w_{43} = h_3 - h_4 = 3344 - 2238.34 = 1105.66\frac{kJ}{kg}$
- Condenser: $q_{14} = h_4 - h_1 = 2238.34 - 188.42 = 2049.92\frac{kJ}{kg}$
- Thermal efficiency: $\eta_{th} = (w_{43} - w_{21})/q_{32} = (1105.66 - 3.02)/3152.56$
    - $\boldsymbol{\eta_{th} = 0.35 = 35\%}$
- Carnot efficiency: $\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{45+273.15}{450+273.15} = 0.56 = 56\%$

6. Consider an ideal Rankine cycle where a solar heat is used to heat molten salt to 560oC in a collector. Molten salt delivers heat to the water in a heat exchanger. The salt leaves the heat exchanger at 250oC and has a mass flow rate of m ̇_salt = 20 kg/s. The water has a mass flow rate of 5 kg/s. The lower working pressure of the Rankine cycle is 75 kPa, and the higher working pressure is 5 MPa. Molten salt can be treated with constant specific heats Cp,salt = 1.5 kJ/kgK.

a) What is the heat source delivered to the water (i.e. $\dot{\boldsymbol{Q}}_{\boldsymbol{H}}$, in kW)?

b) If a nearby city consumes 5 MW of electricity, can the solar power plant provide enough power to meet this demand?

c) What is the thermal efficiency?



[ans: a) $\dot{\boldsymbol{Q}}_{\boldsymbol{H}}$ =9300 kW ; b) $\dot{W}_{net}$ =2.37 MW< 5 MW; c) $\boldsymbol{\eta_{th}} = \mathbf{25.5}\%$]

<u>Solution</u>: Determine the properties at each state

- State 1: P_1 = 75 kPa, saturated liquid T_1 = 91.77°C, h_1 = 384.36 kJ/kg, s_1 = 1.2129 kJ/kgK
- State 2: P_2 = 5000 kPa, $s_2 = s_1$ = 1.2129 kJ/kgK
  Can determine h_2 from s_2 (interpolation; h_2 = 390.00 kJ/kg)… or…
  $w_{pump} = h_2 - h_1 = v_1(P_2 - P_1)$; h_2 = 389.47 kJ/kg
- State 3 P_3 = 5000 kPa, Need additional information about solar heater to determine remaining properties
  Solar heater: $\dot{Q}_{solar} = \dot{m}_{salt}C_{salt}(T_5 - T_6) = 20\frac{kg}{s} * 1.5\frac{kJ}{kgK}(560 - 250)C =$ 9300 kW; $\dot{Q}_{solar} = \dot{m}_w(h_3 - h_2) \rightarrow h_3 = \dot{Q}_{solar}/\dot{m}_w + h_2$
  $h_3 = 9300kW/(5\ kg/s) + 389.47\frac{kJ}{kg} = 2249.47\frac{kJ}{kg}$
  $x_3 = (h_3 - h_{f@5MPa}) / h_{fg@5MPa}$ = (2249.47 – 1154.21) / 1640.12 = 0.668
  $s_3 = s_{f@5MPa} + x_3 * s_{fg@5MPa}$ = 2.9201 + 0.668*3.0532 = 5.0104 kJ/kgK
- State 4: $P_4 = P_1$ = 75 kPa, $s_4 = s_3$ = 5.0104 kJ/kgk → saturated mixture
  $x_4 = (s_4 - s_{f@75kPa}) / s_{fg@75kPa}$ = (5.0104 – 1.2129) / 6.2434 = 0.608
  $h_4 = h_{f@75kPa} + x_4 * h_{fg@75kPa}$ = 384.36 + 0.608*2662.96 = 1170.29 kJ/kgK

a) $\dot{Q}_H = \dot{Q}_{solar} = \mathbf{9300\ kW}$

b) $\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pump}$

$\dot{W}_{turbine} = \dot{m}_w(h_3 - h_4)$= $(2249.47 - 1770.29)\frac{kJ}{kg} = 2395.9\ kW$

$\dot{W}_{pump} = \dot{m}_w(h_2 - h_1) = 5\frac{kg}{s}(389.47 - 384.36)\frac{kJ}{kg} = 25.5\ kW$

$\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pump} = 2395.9 - 25.5 = \mathbf{2370.36\ kW}$

$\dot{W}_{net}$ < 5 MW; cannot meet electricity demand

c) $\eta_{th} = \dot{W}_{net}/\dot{Q}_H = 2370.36/9300 = 0.255 = \mathbf{25.5}\%$

7. Consider the system in problem (6), but now a boiler is used in combination with the solar heating system. The boiler supplies enough heat such that the quality of the mixture leaving the turbine is x4 = 0.95.
Assuming the turbine to operate reversibly & adiabatically, determine

a) The maximum temperature in the cycle.

b) The power output of the turbine (in kW).

c) The new thermal efficiency.

[ans: a)T_3 = 558.2°C ; b) $\dot{W}_{net}$ =5.08 MW > 5 MW; c) $\boldsymbol{\eta_{th}} = \mathbf{31.9}\%$]

Solution: States 1 and 2 will remain the same, but 3 and 4 will change

- State 3': superheated steam atP_3 = 5 MPa
  interpolate based off of $s_{3'} = s_4 = s_{f@75kPa} + x_4 * s_{fg@75kPa}$ = 7.14413 kJ/kgK ;
  $T_{3'}$ = 558.2°C; $h_{3'}$ = 3569.25 kJ/kg
- State 4: P_4 = 75 kPa, $s_4 = s_{f@75kPa} + x_4 * s_{fg@75kPa}$ = 7.14413 kJ/kgK
  $h_4 = h_{f@75kPa} + x_4 * h_{fg@75kPa}$ = 2549.02 kJ/kg

a) $\mathbf{T_{max} = T_{3'}}$ **= 558.2°C**

b) $\dot{W}_{turbine} = \dot{m}_w(h_{3'} - h_4)$

$$\dot{W}_{turbine} = 5\frac{kg}{s}(3569.24 - 2549.02)\frac{kJ}{kg} = 5101.1\ kW$$

$$\dot{W}_{pump} = \dot{m}_w(h_2 - h_1) = 5\frac{kg}{s}(389.47 - 384.36)\frac{kJ}{kg} = 25.5\ kW$$

$$\boldsymbol{\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{pump} = 5101.1\ - 25.5 = 5075.6\ kW}$$

net power now meets energy demand

c) $\eta_{th} = \dot{W}_{net}/\dot{Q}_{total}$

$$\dot{Q}_{total} = \dot{m}_w(h_{3'} - h_2) = 5\frac{kg}{s}(3569.25 - 389.47)\frac{kJ}{kg} = 15{,}898.9\ kW$$

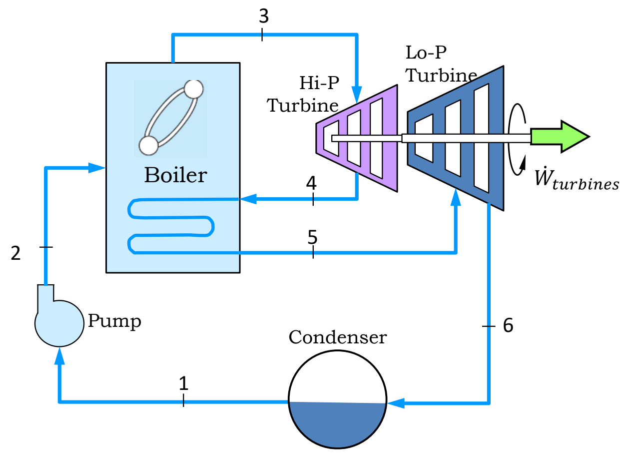
$$\boldsymbol{\eta_{th} = 5075.6/15{,}898.9 = 0.319 = 31.9\%}$$

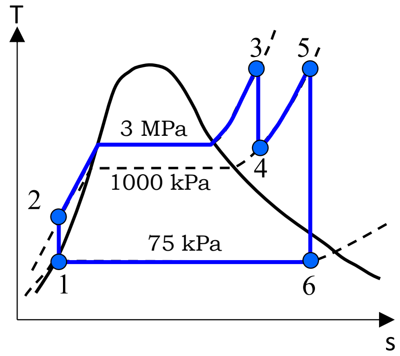
8. Consider the same steam power plant in problem (4). You are not satisfied with the semi-low quality of the mixture exiting the turbine. Therefore, you propose to fix this problem by operating this power plant under an ideal reheat Rankine cycle. The steam exits the original turbine (now termed, high-pressure turbine) at 1000 kPa and is reheated in the boiler to a temperature of 350°C. The steam is then sent into a low-pressure turbine and enters the condenser at 75 kPa.

a) Determine the quality of the steam exiting the low-pressure turbine.

  i) Are you satisfied with this quality (i.e. is $x_6 \geq 0.9$)?

b) Determine the thermal efficiency of this reheat Rankine cycle.





[ans: a) x6 = 0.988, b) $\boldsymbol{\eta_{th}} = \mathbf{38\%}$*]*

Solution: ideal Rankine cycle

States 1-3 are similar to those of problem 4

- State 1: saturated liquid at 75 kPa; h_1 = 384.36 kJ/kg, s_1 = 1.2129 kJ/kgK, v_1 = 0.001037 m^3/kg
- State 2: compressed liquid at 3000 kPa, $s_2 = s_1$ = 1.2129 kJ/kgK, h_2 = 387.39 kJ/kg; $w_{21} = v_1(P_2 - P_1) = 3.03\ kJ/kg$
- State 3: superheated vapor at 3000 kPa, 350°C; h_3 = 3115.25 kJ/kg, s_3 = 6.7427 kJ/kgK
- State 4: P_4 = 1000 kPa, $s_4 = s_3$ = 6.7427 kJ/kgK; superheated vapor
  - T_4 = 210.58°C, h_4 = 2852.13 kJ/kg
- State 5: P_5 = 1000 kPa, T_5 = 350°C, h_5 = 3157.65 kJ/kg, s_5 = 7.301 kJ/kgK
- State 6: $s_6 = s_5$ = 7.301 kJ/kgK, P_6 = 75 kPa (saturated mixture),
  - $x_6 = (s_6 - s_f)/s_{fg}$ = (7.301 – 1.2129) / 6.2434; $\mathbf{x_6 = 0.988}$
  - $h_6 = h_f + x_6 h_{fg}$ = 384.36 + 0.988*2278.59, h_6 = 2635.61 kJ/kg

Apply 1st law to each device

- Boiler: $q_H = q_{32} + q_{54} = (h_3 - h_2) + (h_5 - h_4) = (3115.25 - 387.39) + (3157.65 - 2852.13) = 3033.38\,\frac{kJ}{kg}$
- Hi-P Turbine: $w_{43} = h_3 - h_4 = 3115.25 - 2852.13 = 263.12\,\frac{kJ}{kg}$
- Lo-P Turbine: $w_{65} = h_5 - h_6 = 3157.65 - 2635.61 = 522.04\,\frac{kJ}{kg}$
- Thermal efficiency: $\boldsymbol{\eta_{th} = (w_{43} + w_{65} - w_{21})/q_H = 26\%}$
  - In this circumstance, the efficiency remains the same as it was without the reheat. Often times the reheat cycle is used to merely increase the quality of the steam exiting the turbine. A method to increase the efficiency in this circumstance is the increase the maximum working temperature, both in the original heating process and during the reheat.

9. A steam power plant operates on a Rankine cycle. The power plant operates with a high pressure of 5000 kPa and has a boiler exit temperature of 600°C. The working temperature inside the condenser is 50°C. There is no pressure loss in the boiler and condenser, and the pump operates adiabatically and reversibly. The turbine operates adiabatically, but the steam exits the turbine as a saturated vapour.

a. Determine the lower working pressure in kPa.

b. Determine the thermal efficiency.

c. Determine the isentropic efficiency.

d. Determine the entropy generation (in kJ/kgK) in the condenser if the heat, q_L, is rejected to a river which has a constant temperature of 20°C.

[ans: a) P = 12.350 kPa; b) $\eta_{th} = 30.9\%$ *;c)* $\boldsymbol{\eta = 80.2\%}$ *;* $s_{gen} = 0.756\ kJ/\mathrm{kgK}$*]*

Solution: For completeness, find h & s at all states

- State 1: saturated liquid: T_1 = 50C; v_1 = 0.001012 m^3/kg, h_1 = 209.31 kJ/kg, s_1 = 0.7037 kJ/kgK
- State 2: $h_2 = h_1 + v_1(P_2\text{-}P_1)$ = 209.31 kJ/kg + 0.001012 m^3/kg*(5000-12.350kPa) h_2 = 214.36 kJ/kg, $s_2 = s_1$
- State 3: superheated vapour: h_3 = 3666.47 kJ/kg, s_3 =7.2588 kJ/kgK
- State 4: saturated vapour: $h_4 = h_{g@50C}$ = 2592.06 kJ/kg, $s_4 = s_{g@50C}$ = 8.0762 kJ/kgK

a) Lower working pressure is saturated pressure at 50°C. P = 12.350 kPa

b) $\eta_{TH} = w_{net} / q_H = q_{net} / q_H = 1 - q_L / q_H$

- $w_{net} = w_{43} - w_{21} = (h_3 - h_4) - (h_2 - h_1)$
- $w_{net} = (3666.47 - 2592.06) - (214.36 - 209.31) = 1069.36\ kJ/kg$
- $q_L = h_4 - h_1 = 2592.06 - 209.31 = 2382.75\ kJ/kg$
- $q_H = h_3 - h_2 = 3666.47 - 214.36 = 3452.11\ kJ/kg$
- $\boldsymbol{\eta_{TH}} = w_{net} / q_H = \mathbf{1069.36/3452.11 = 0.309\ or\ 30.9\%}$
- $\boldsymbol{\eta_{TH}} = 1 - q_L / q_H = 1 - 2382.75 / 3452.11 = \mathbf{0.309\ or\ 30.9\%}$

c) Isentropic efficiency: $\eta = w_{actual}/w_{isentropic}$

- $w_{isentropic} = h_3 - h_{4s}$; h_{4s} occurs when $s_4 = s_3$.
- $x_{4s} = (s_4\text{-}s_f)/s_{fg}$ = (7.2588-0.7037)/7.3725 = 0.889
- $h_{4s} = h_f + x_{4s}{*}h_{fg}$ = 209.31 kJ/kg + 0.889*2382.75 kJ/kg; h_{4s} = 2327.57 kJ/kg
- $w_{isentropic}$ = 3666.47 – 2327.57 kJ/kg = 1338.9 kJ/kg
- $w_{actual} = h_3 - h_{g@50C}$ = 3666.47 – 2592.06 = 1074.41 kJ/kg
- $\boldsymbol{\eta = 1074.41/1338.9 = 0.802\ or\ 80.2\%}$

d) $s_{gen} = (s_1 - s_4) + q_L/T_L = (0.7037 - 8.0762)\ kJ/kgK + (2382.75 kJ/kg / 293.15\ K)$
$\boldsymbol{s_{gen} = 0.756\ kJ/\mathrm{kgK}}$