MPC-Based Approach to Active Steering for Autonomous Vehicle Systems

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Abstract: In this paper a novel approach to autonomous steering systems is presented. A model predictive control (MPC) scheme is designed in order to stabilize a vehicle along a desired path while fulfilling its physical constraints. Simulation results show the benefits of the systematic control methodology used. In particular we show how very effective steering maneuvers are obtained as a result of the MPC feedback policy. Moreover, we highlight the trade off between the vehicle speed and the required preview on the desired path in order to stabilize the vehicle. The paper concludes with highlights on future research and on the necessary steps for experimental validation of the approach.

1 Introduction

Recent trends in automotive industry point in the direction of increased content of electronics, computers, and controls with emphasis on the improved functionality and overall system robustness. While this affects all of the vehicle areas, there is a particular interest in active safety, which very nicely and effectively complements the passive safety counterpart. Passive safety is primarily focused on structural integrity of vehicle. Active safety on the other hand is primarily used to avoid accidents and at the same time facilitate better vehicle controllability and stability especially in emergency situations, such as what may occur when suddenly encountering slippery parts of the road (Costlow, 2005).

The progress of active safety related functions can be summarized as follows. At the beginning, the work was primarily focusing on longitudinal dynamics part of motion, in particular, on more effective braking (ABS) and traction control. This was followed by work on different vehicle stability control systems (Tseng et al., 2005) (which are also known under different acronyms such as Electronic Stability Program, ESP, Vehicle Stability Control, VSC, Interactive Vehicle Dynamics, IVD, and Dynamic Stability Control, DSC). Essentially, these systems use brakes on one side to stabilize the vehicle in extreme limit handling situations through controlling the yaw motion. Related to this was also the effort on Four Wheel Steer (4WS) control. However, the early efforts which found limited production applications were primarily focused on improved handling without explicit emphasis on active safety *per se*. Similar can be said about the early introduction of active suspensions, which primarily focused on improved ride and handling. Recent systems include additional intervention with brakes but as a function of excessive vehicle roll, so that in this case the brakes are used to prevent vehicle roll over (Tseng and Xu, 2003).

It is anticipated that the future systems will be able to increase the effectiveness of active safety interventions beyond what is currently available. This will be facilitated not only by additional actuator types such as 4WS, active steering, active suspensions, or active differentials, but also by additional sensor information, such as the increased inclusion of onboard cameras, as well as infrared and other sensor alternatives. All these will be further complemented by GPS information including pre-stored mapping. In this context, it is possible to imagine that future vehicles would be able to identify obstacles on the road such as an animal, a rock, or fallen tree/branch, and assist the driver by following the best possible path, in terms of avoiding the obstacle and at the same time keeping the vehicle on the road at a safe distance from incoming traffic.

At this stage, we assume this “ultimate” obstacle avoidance system will be possible sometime in the future and we propose a double lane change scenario on a slippery road, with a vehicle equipped with a fully autonomous steering system. The control input is the front steering angle and the goal is to follow the desired trajectory or target as close as possible while fulfilling various constraints reflecting vehicle physical limits.

In this scenario, a robot driver (Tseng et al., 2005) can be used as follows. The robot driver can learn the desired path by first going very slowly through this double lane change maneuver. Subsequently, the robot driver tries to negotiate the same trajectory with increased entry speed. This way, we test the vehicle’s overall stability and behavior on slippery, i.e. snowy, road. In addition, this example can also be seen as precursor to fully autonomous vehicles. This approach can be used in the military applications to transfer the load without any associated personnel casualties. A contemporary example of this is the “Grand Challenge” race driving that has been undertaken through autonomous driving (Behringer et al., 2004). The main facilitators for this system are GPS, infrared sensors, cameras and path planning and following control routines.

Additional source of information can also come from surrounding vehicles and environments which may convey the information from the vehicle ahead about road condition, which can give a significant amount of preview to the controller. This is particularly useful if one travels on snow or ice covered surfaces. In this case, it is very easy to reach the limit of vehicle handling capabilities. We reemphasize that in the present paper we assume a given desired trajectory and we will design a controller that can best follow the trajectory on slippery road at the highest possible entry speed.

Anticipating sensor and infrastructure trends toward increased integration of information and control actuation agents, it is then appropriate to ask what is the best and optimum way in controlling the vehicle maneuver for given obstacle avoidance situation. This will be done in the spirit of Model Predictive Control, MPC (Garcia et al., 1989; Mayne et al., 2000) along the lines of our ongoing internal research efforts dating from early 2000 (Asgari and Hrovat, 2002). We use a *nonlinear model* of the plant to *predict* the future evolution of the system (Mayne et al., 2000). Based on this prediction, at each time step t a performance index is optimized under operating constraints with respect to a sequence of future steering moves in order to best follow the given trajectory on a slippery road. The first of such optimal moves is the *control* action applied to the plant at time t. At time $t+1$, a new optimization is solved over a shifted prediction horizon.

In this paper we use nonlinear MPC (NLMPC). This allows us to: *(i)* increase the stability boundary of the controlled system compared to linear controllers, *(ii)* test its computational complexity, *(iii)* create a benchmark controller against which other sub-optimal controllers can be compared. The simulation results presented in this work show the benefits of the systematic control methodology used. In particular we show how complex steering maneuvers are relatively easily obtained as a result of the MPC feedback policy.

The paper is structured as follows. Section 2 describes the vehicle dynamical model used with a brief discussion on tire models. Section 3 formulates the control problem. Section 4 briefly discussed the nonlinear optimization tools used. The double lane change scenario and the simulation results are presented in Section 5. Computational tools and new methodologies which may lead to real-time implementation of the proposed approach are highlighted in Section 6. This is then followed by concluding remarks in Section 7 which highlight future research directions.

2 Modeling

This section describes the vehicle and tire model we used for simulations and control design. We use the following nomenclature.

Nomenclature

F_l, F_c	longitudinal and lateral tire forces
F_x, F_y	forces in car body frame
F_z	normal tire load
H_p	output prediction horizon
H_u	control prediction horizon
I	car inertia
J	cost function
Q, R	MPC weighting matrices
T_s	sampling time
X, Y	absolute car position inertial coordinates
a, b	car geometry (distance of front and rear wheels from center of gravity)

g	gravitational constant
m	car mass
r	wheel radius
s	slip ratio
u	commanded input signals of the prediction model
Δu	commanded input signal changes of the prediction model
v_l, v_c	longitudinal and lateral wheel velocities
x, y	local lateral and longitudinal coordinates in car body frame
$\dot{x}$	vehicle speed
α	slip angle
β	sideslip
δ	wheel steering angle
η	prediction model outputs
ξ	prediction model states
μ	road friction coefficient
ψ	heading angle
$f_l(\cdot)$, $f_c(\cdot)$	functions describing longitudinal and lateral tire models

Subscripts and Superscripts

$(\cdot)_f$	front wheel
$(\cdot)_r$	rear wheel
$(\cdot)_k$	time step
$(\cdot)_{ref}$	reference tracking signals
$\hat{(\cdot)}$	predicted variables
$\overline{(\cdot)}$	upper limit on variables
$\underline{(\cdot)}$	lower limit on variables

2.1 Vehicle Model

We use a "bicycle model" to describe the dynamics of the car and assume constant normal tire load, i.e., F_{z_f}, F_{z_r} = constant. Such model captures the most relevant nonlinearities associated to lateral stabilization of the vehicle. Figure 1 depicts a diagram of the vehicle model, which has the following longitudinal, lateral and turning or yaw degrees of freedom (DOF)

$$m\ddot{x} = m\dot{y}\dot{\psi} + 2F_{x_f} + 2F_{x_r} \tag{1a}$$
$$m\ddot{y} = -m\dot{x}\dot{\psi} + 2F_{y_f} + 2F_{y_r}, \tag{1b}$$
$$I\ddot{\psi} = 2aF_{y_f} - 2bF_{y_r}, \tag{1c}$$

The vehicle's equations of motion in an absolute inertial frame are

$$\dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi, \tag{2a}$$
$$\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi. \tag{2b}$$

Longitudinal and lateral tire forces lead to the following forces acting on the center

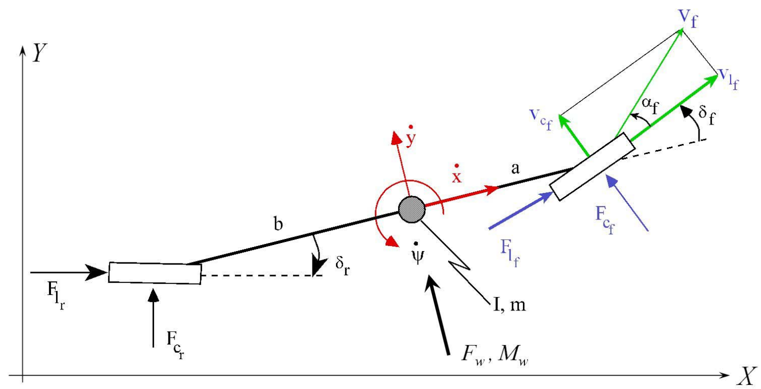


Figure 1 The simplified vehicle dynamical model.

of gravity:

$$F_y = F_l \sin\delta + F_c \cos\delta, \tag{3a}$$
$$F_x = F_l \cos\delta - F_c \sin\delta. \tag{3b}$$

Tire forces for each tire are given by

$$F_l = f_l(\alpha, s, \mu, F_z), \tag{4a}$$
$$F_c = f_c(\alpha, s, \mu, F_z), \tag{4b}$$

where α is the slip angle of the tire and s is the slip ratio defined as

$$s = \begin{cases} s = \frac{r\omega}{v} - 1 \text{ if } v > r\omega, v \neq 0 \text{ for braking} \\ s = 1 - \frac{v}{r\omega} \text{ if } v < r\omega, \omega \neq 0 \text{ for driving} \end{cases} \tag{5}$$

The slip angle represents the angle between the wheel velocity and the direction of the wheel itself:

$$\alpha = \tan^{-1} \frac{v_c}{v_l}. \tag{6}$$

In equation (6), v_c and v_l are the lateral (or cornering) and longitudinal wheel velocities, respectively, which are expressed as

$$v_l = v_y \sin\delta + v_x \cos\delta, \tag{7a}$$
$$v_c = v_y \cos\delta - v_x \sin\delta, \tag{7b}$$

and

$$v_{y_f} = \dot{y} + a\dot{\psi} \quad v_{y_r} = \dot{y} - b\dot{\psi}, \tag{8a}$$
$$v_{x_f} = \dot{x} \qquad\quad v_{x_r} = \dot{x}. \tag{8b}$$

The parameter μ represents the road friction coefficient and F_z is the total vertical load of the vehicle. This is distributed between the front and rear wheels based on the geometry of the car model, described by the parameters a and b:

$$F_{z_f} = \frac{bmg}{2(a+b)}, \tag{9a}$$

$$F_{z_r} = \frac{amg}{2(a+b)}. \tag{9b}$$

The terminology and nomenclature used for describing the tire orientation and forces is illustrated in Figure 2.

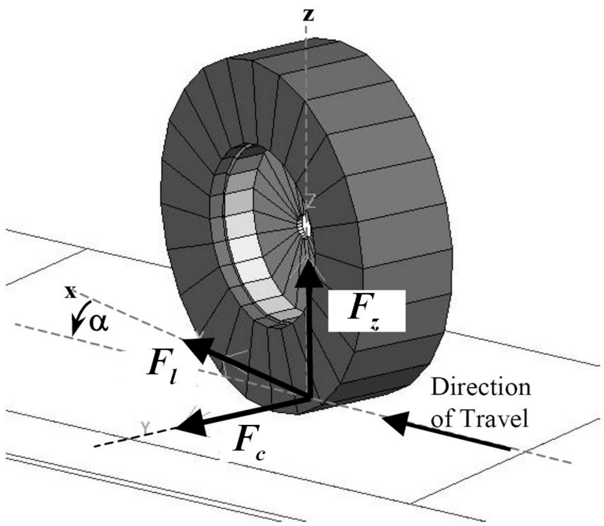


Figure 2 Illustration of tire model nomenclature (Lacombe, 2000)

Using the equations (1)-(9), the nonlinear vehicle dynamics can be described by the following compact differential equation assuming a certain slip ratio s and friction coefficient value μ:

$$\dot{\xi} = f_{s,\mu}(\xi, u), \tag{10a}$$

$$\eta = h(\xi), \tag{10b}$$

where the state and input vectors are $\xi = [y,\ \dot{y},\ \dot{x},\ \psi,\ \dot{\psi},\ Y,\ X]$ and $u = \delta_f$ respectively, and the output map is given as

$$h(\xi) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xi. \tag{11}$$

2.2 Tire Model

Except for aerodynamic forces and gravity, all of the forces which affect vehicle handling are produced by the tires. Tire forces provide the primary external influence and, because of their highly nonlinear behavior, cause the largest variation in vehicle handling properties throughout the longitudinal and lateral maneuvering range. As a result, it is important to use a realistic nonlinear tire model, especially when investigating large control inputs that result in response near the limits of the maneuvering capability of the vehicle. In such situations, the lateral and longitudinal motions of the vehicle are strongly coupled through the tire forces, and large values of longitudinal slip and slip angle can occur simultaneously.

The many existing tire models are predominantly “semi-empirical” in nature, where model structure is determined through analytical considerations, but key parameters still depend on tire data measurements. Those models range from extremely simple (where lateral forces are computed as a function of slip angle, given only a measured slope at $\alpha = 0$ and a measured value of the maximum lateral force) to relatively complex algorithms, which use tire data measured at many different loads and slip angles.

The model for tire tractive and cornering forces (4) used in this paper are described by a Pacejka model (Bakker et al., 1987). This is a complex, semi-empirical model that takes into consideration the interaction between the tractive force and the cornering force in combined braking and steering. The longitudinal and cornering forces are assumed to depend on the normal force, slip angle, surface friction, and longitudinal slip. Sample plots of predicted longitudinal and lateral force versus longitudinal slip and slip angle are shown in Figures 3-4. These plots are shown for the front tire of the “bicycle” model, which represents the two front tires of the actual car.

At this stage the dynamic effects of tires while negotiating sudden changes of road/drive condition (Deur et al., 2005) have been ignored in this first level of investigation. The modeling of tire dynamics may be important from the standpoint of development of high performance ABS, traction control, and IVD systems of future. In addition, the use of dynamic model yields an advantage of avoiding the static tire model numerical difficulties at low vehicle speeds (Deur et al., 2005). Our future work will include consideration of the above tire dynamic effects, as needed. However, for the present study, which serves to establish the main trends and approaches, the above static tire model is appropriate.

3 Problem Formulation

Given (*i*) the tire model described in Section 2.2, (*ii*) the nonlinear dynamics of the model equations (10), and (*iii*) a desired path, we seek to minimize deviations from their references of the heading angle ψ and of the lateral distance Y. We optimize over δ_f when $\delta_r = 0$ (assuming Active/Augmented Front Steer, AFS or 2WS) while external signal evolutions of tire slip and road friction (s,μ) act on the vehicle dynamics. The Four-Wheel Steering (4WS) optimization over δ_f and δ_r is the topic of current ongoing research.

In the sequel we describe how a nonlinear Model Predictive Controller (MPC) can be designed for the posed problem. The main concept of MPC is to use a *model* of the plant to *predict* the future evolution of the system (Garcia et al., 1989; Mayne et al., 2000; Morari and Lee, 1999; Mayne and Michalska, 1993; Hrovat, 1996; Borrelli et al., 2001; Zheng et al., 1994). Based on this prediction, at each time step t a certain performance index is optimized under operating constraints with

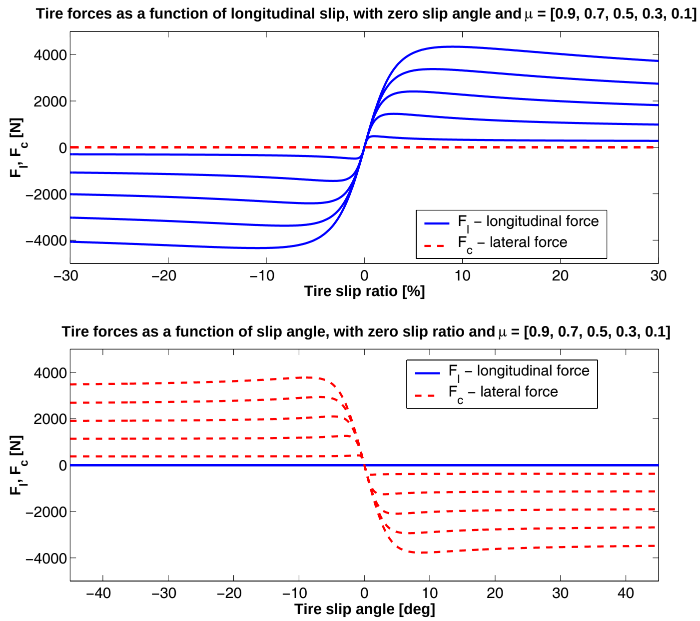


Figure 3 Longitudinal and lateral tire forces with different μ coefficient values.

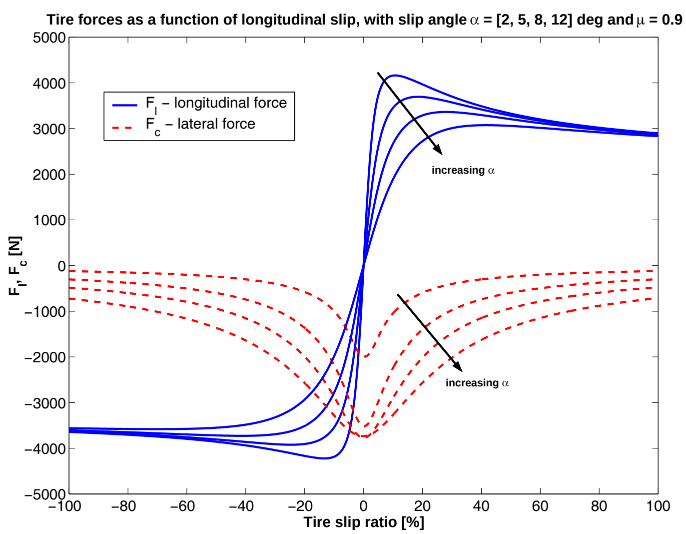


Figure 4 Longitudinal and lateral tire forces with combined braking and cornering.

respect to a sequence of future input moves. The first of such optimal moves is the *control* action applied to the plant at time t. At time $t+1$, a new optimization is solved over a shifted prediction horizon. We regard the slip history as an external input to model we are considering (we assume that there is an independent traction controller yielding the slip signal evolution).

In order to obtain a finite dimensional optimal control problem we discretize the system dynamics (10) with the Euler method, to obtain

$$\xi(k+1) = f^{dt}_{s,\mu}(\xi(k), \Delta u(k)), \tag{12a}$$

$$\eta(k+1) = h(\xi(k)), \tag{12b}$$

where the Δu formulation is used, i.e., $u(k) = u(k-1) + \Delta u(k)$ and $u(k) = \delta_f(k)$, $\Delta u(k) = \Delta\delta_f(k)$.

We consider the following cost function:

$$J(\xi(t), \Delta\mathcal{U}_t) = \sum_{i=1}^{H_p} \left\| \hat{\eta}_{t+i,t} - \eta_{ref\,t+i,t} \right\|^2_Q + \sum_{i=0}^{H_c-1} \|\Delta u_{t+i,t}\|^2_R \tag{13}$$

where, as in standard MPC notation (Mayne et al., 2000; Mayne and Michalska, 1993), $\Delta\mathcal{U}_t = \Delta u_{t,t}, \ldots, \Delta u_{t+H_c-1,t}$ is the optimization vector at time t and $\hat{\eta}_{t+i,t}$ denotes the output vector predicted at time $t+i$ obtained by starting from the state $\xi_{t,t} = \xi(t)$ and applying to system (12) the input sequence $\Delta u_{t,t}, \ldots, \Delta u_{t+i,t}$. H_p and H_c denote the output prediction horizon and the control horizon, respectively. As in standard MPC schemes, $H_p > H_c$ and the control signal is assumed constant for all $H_c \leq t \leq H_p$, i.e., $\Delta u_{t+i,t} = 0\ \forall i \geq H_c$. The reference signal η_{ref} represents the desired outputs, where $\eta = [\psi, Y]'$. Q and R are weighting matrices of appropriate dimensions. In (13) the first summand reflects the desired performance on target tracking, the second summand is a measure of the steering effort.

At each time step t the following finite horizon optimal control problem is solved:

$$
\begin{aligned}
&\text{(14a)} && \min_{\Delta\mathcal{U}_t} && J(\xi_t, \Delta\mathcal{U}_t) \\
&\text{(14b)} && \text{subj. to} && \xi_{k+1,t} = f^{dt}_{s,\mu}(\xi_{k,t}, \Delta u_{k,t}), \\
&\text{(14c)} && && \eta_{k,t} = h(\xi_{k,t}), \\
& && && \qquad k = t, \ldots, t+H_p \\
&\text{(14d)} && && \delta_{f,min} \leq u_{k,t} \leq \delta_{f,max} \\
&\text{(14e)} && && \Delta\delta_{f,min} \leq \Delta u_{k,t} \leq \Delta\delta_{f,max} \\
& && && \qquad k = t, \ldots, t+H_c-1 \\
&\text{(14f)} && && u_{k,t} = u_{k-1,t} + \Delta u_{k,t}
\end{aligned}
$$

We denote by $\Delta\mathcal{U}^*_t \triangleq [\Delta u^*_{t,t}, \ldots, \Delta u^*_{t+H_c-1,t}]'$ the sequence of optimal input increments computed at time t by solving (14) for the current observed states $\xi_{t,t}$, assuming wind, slip and friction coefficient values constant and equal to the estimates/given values at time t over the prediction horizon.

The resulting state feedback control law is

$$\delta_f(t)(\xi(t)) = \delta_f(t-1) + \Delta u^*_{t,t}(\xi(t)) \tag{15}$$

It is well known that stability is not ensured by the MPC law (14)–(15). Usually the problem (14) is augmented with a terminal cost l_N and a terminal constraint set $\mathcal{X}_f$. Those are chosen to ensure closed-loop stability. A treatment of sufficient stability conditions goes beyond the scope of this work and can be found in the surveys (Mayne et al., 2000; Mayne, 2001). We assume that the reader is familiar with the basic concept of MPC and its main issues, we refer to (Mayne et al., 2000) for a comprehensive treatment of the topic.

4 Nonlinear Programming Solvers

Linear and nonlinear MPC has been a success in process industry (Qin and Badgwell, 1997). However MPC implementation requires significant computation infrastructure which might not be available on processes with fast sampling time and limited computational resources. Recently, thanks to the combination of new research results and faster computing units, it has been possible to extend the implementation of MPC design to new areas such as aerospace and automotive (see the recent NLMPC experimental test carried out on an autonomous modified T-33 Aircraft (Keviczky and Balas, 2005; Keviczky et al., 2004), and the experimental test of a hybrid MPC controller on a Ford vehicle (Borrelli et al., 2001)).

We used the commercial NPSOL software package (Gill et al., 1998) for solving the nonlinear programming problem (14). NPSOL is a set of Fortran subroutines for minimizing a smooth function subject to constraints, which may include simple bounds on the variables, linear constraints and smooth nonlinear constraints. The user provides subroutines to define the objective and constraint functions and (optionally) their first derivatives. NPSOL uses a sequential quadratic programming (SQP) algorithm, in which each search direction is the solution of a QP subproblem. Bounds, linear constraints and nonlinear constraints are treated separately. NPSOL requires relatively few evaluations of the problem functions. Hence it is especially effective if the objective or constraint functions are expensive to evaluate.

Other nonlinear software packages are currently under investigation. In particular we are investigating the performance of the *publicly available* solver IPOPT (Biegler et al., 2002), an *interior point* algorithm. The algorithm follows a barrier approach, where the bounds (14d)-(14e) are replaced by a logarithmic barrier term which is added to the objective function (13). The barrier algorithm attempts to solve the nonlinear programming problem (14) by solving a sequence of barrier problems with decreasing barrier parameter. For a given barrier parameter $\mu > 0$ the logarithmic term in the objective function serves to push the iterates into the strict interior of the region defined by the bounds. As the barrier parameter μ, is driven to zero the influence of the logarithmic term is diminished and, under certain assumptions, one is also able to recover solutions that make a bound active.

The interior point approach avoids having to identify the active inequalities at the solution and the combinatorial difficulty associated with it. The interested reader is referred to the work of Wächter and Biegler (Wächter, 2002; Wächter and Biegler, 2001) and the work of Biegler *et. al.* (Biegler et al., 2002) for details on the convergence and implementation. IPOPT has been developed for solving large-scale nonlinear programming problems. Preliminary results seems to show a good performance also on the class of problems presented in this paper.

5 Double Lane Change on Snow Using Active Steering

The nonlinear MPC steering controller described in Section 3 is implemented to perform a sequence of double lane change at increasing entry speed. The desired path is shown in Figure 5. Section 5.1 describes details of the simulation scenario, and Section 5.2 presents our most important results and findings. It should be pointed out that the same controller can be used to control the vehicle during different manoeuvres. In (Borrelli et al., 2005b) the same MPC controller is used in order to evaluate the effect of an external sidewind on the vehicle. The study allowed to estimate the maximum wind speed that a MPC-based active steering system is able to contain so that the vehicle remains stable.

5.1 Scenario Description

In this scenario, the objective is to follow the desired path shown in Figure 5 as close as possible. This test represents an obstacle avoidance emergency maneuver in which the vehicle is entering a double lane change maneuver on snow with a given initial forward speed. The control inputs in a 2WS scenario are the front steering angle and the goal is to follow this trajectory as close as possible by minimizing the vehicle deviation from the target path. The use of front and rear steering angles in a 4WS scenario is the topic of current ongoing research. The experiment is repeated with increasing entry speeds until the vehicle loses control. It should be noticed that the vehicle is coasting during the maneuver, i.e., no braking or traction torque has been applied to the wheels.

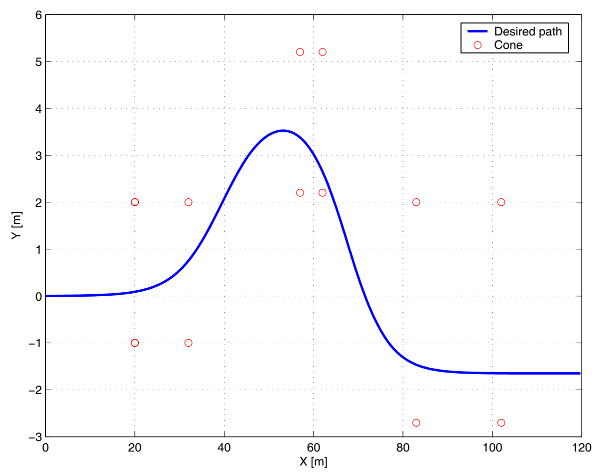


Figure 5 Reference path to be followed.

A NLMPC controller, designed with the procedure described in Section 3, has been tested for different longitudinal speeds, using the front steering angle δ_f as the only control input. The reference signals (desired tracking signals) $\eta_{ref} = \begin{bmatrix} \psi_{ref} \\ Y_{ref} \end{bmatrix}$

are specified by the following set of equations:

$$\psi_{ref} = \frac{d_{y_1}}{2}(1+\tanh(z_1)) - \frac{d_{y_2}}{2}(1+\tanh(z_2)) \tag{16a}$$

$$Y_{ref} = \arctan\left(d_{y_1}\left(\frac{1}{\cosh(z_1)}\right)^2\left(\frac{1.2}{d_{x_1}}\right) - d_{y_2}\left(\frac{1}{\cosh(z_2)}\right)^2\left(\frac{1.2}{d_{x_2}}\right)\right) \tag{16b}$$

$$z_1 = \frac{shape}{d_{x_1}}(X - X_{s_1}) - \frac{shape}{2} \tag{16c}$$

$$z_2 = \frac{shape}{d_{x_2}}(X - X_{s_2}) - \frac{shape}{2} \tag{16d}$$

where $shape = 2.4$, $d_{x_1} = 25$, $d_{x_2} = 21.95$, $d_{y_1} = 4.05$, $d_{y_2} = 5.7$, $X_{s_1} = 27.19$ and $X_{s_2} = 56.46$.

Unless differently specified, the following parameters have been used for the 2WS NLMPC controller:

- sample time: $T = 0.05$ sec;
- constraints on maximum and minimum steering angles $-30 \text{ deg} \leq \delta_f \leq 30 \text{ deg}$
- constraints on maximum and minimum changes in steering angles

  $-20 \text{ deg/s} \leq \Delta\delta_f \leq 20 \text{ deg/s}$

The controller tuning parameters at a given longitudinal vehicle speed are the prediction horizon H_p, control horizon H_u and the weighting matrices Q and R.

5.2 Simulation Results

The NLMPC (14)-(15) has been tested for different vehicle entry speed ranging from 5 m/s to 17 m/s. The minimum length of prediction and control horizons required for vehicle stabilization and acceptable performance has been estimated and reported on the diagonal elements of the Table 1. For the controllers marked as "Unstable", a stabilizing tuning (in term of matrices Q and R) working at the specified vehicle speed has not been found. For instance, with a vehicle speed of 10 m/s, the minimum required prediction and control horizons for stabilizing the vehicle is seven and two steps, respectively.

The empty cells in the table denote controllers that provide better performance than the controllers on the diagonal with additional computational load.

Vehicle speed [m/s]	Prediction horizon (H_p) 2	7	10
5	$H_u = 1$		
10	Unstable $\forall H_u \leq 2$	$H_u = 2$	
15	Unstable $\forall H_u \leq 2$	Unstable $\forall H_u \leq 7$	$H_u = 4$
17	Unstable $\forall H_u \leq 2$	Unstable $\forall H_u \leq 7$	$H_u = 7$

Table 1 Summary of results using different controllers at different vehicle speeds.

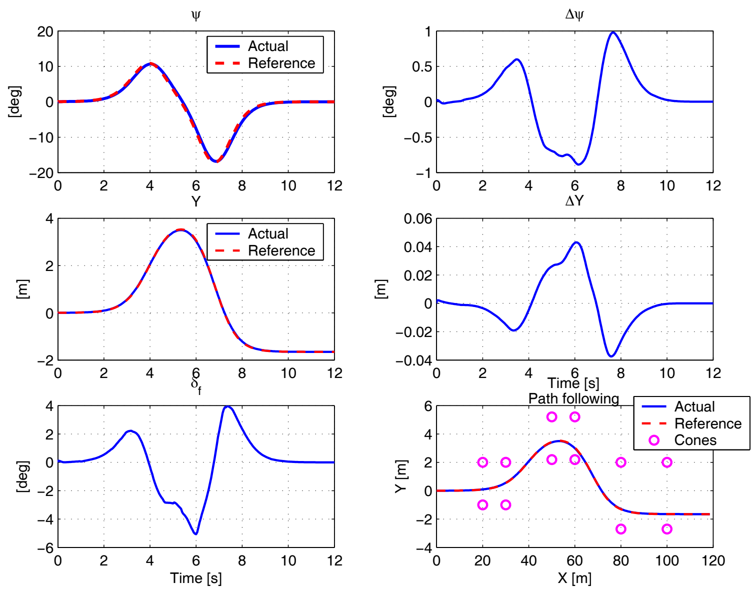


Figure 6 Double lane change maneuver at 10 m/s with $H_p = 7$ and $H_u = 2$.

Figures 6-7 show simulation results obtained at an entry speed of 10 m/s, using a controller with prediction horizon of 7 steps (0.35 sec) and control horizon of 2 steps (0.1 sec). A good reference-tracking with smooth front steering angle and front lateral force is obtained. It can be concluded that 10 m/s is not a critical speed for the controller. This scenario is not critical also from a computational point of view, since the maximum NPSOL computational time was 0.15 seconds.

In Figures 8 and 9 the simulation results with an entry speed of 15 m/s, $H_p = 10$ and $H_u = 4$ are shown. The seventeen meters per second is the maximum vehicle entry speed which the NLMPC steering system is able to handle. The simulation results presented in Figures 10 and 11 show that the vehicle is still stable, despite a large lateral deviation of 4m from the reference which leads to a cone hit. A prediction horizon of 10 steps (0.5 sec) and a control horizon of 7 steps (0.35 sec) were necessary for vehicle stabilization. It can be observed that the vehicle starts to loose control at the beginning of the second lane change where the maneuver exhibits the some elements similar to a counter steering. It should be pointed out that the NLMPC tuning at 15 m/s requires less effort than for the 17 m/s case.

In Figure 12 the lateral and longitudinal vehicle speeds are shown, for entry speeds of 10 m/s, 15 m/s and 17 m/s. It is interesting to note that at 17 m/s the longitudinal vehicle speed first decreases and then increases. This phenomenon is not observed at 15 m/s.

The effects of active constraints on the computational time and on maneuver policy have also been analyzed. To this aim, the double lane change maneuver has been performed at 12 m/s, with $H_p = 10$, $H_u = 4$ and by tightening the limits on the steering angle to ± 2 deg.

Figures 13 and 14 show the unconstrained and constrained case, respectively. The constrained case uses a different maneuver with an unavoidable higher error on

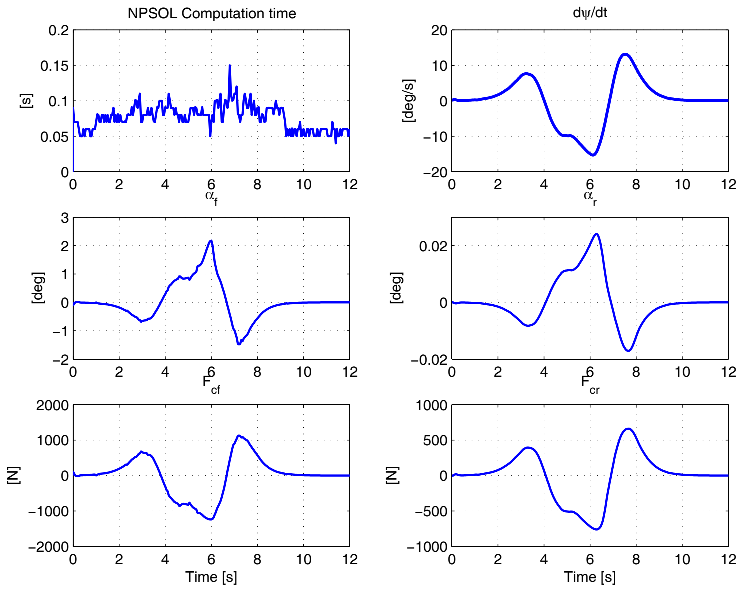


Figure 7 Double lane change maneuver at 10 m/s with $H_p = 7$ and $H_u = 2$. NPSOL computation time, yaw rate, tire forces and slip angles.

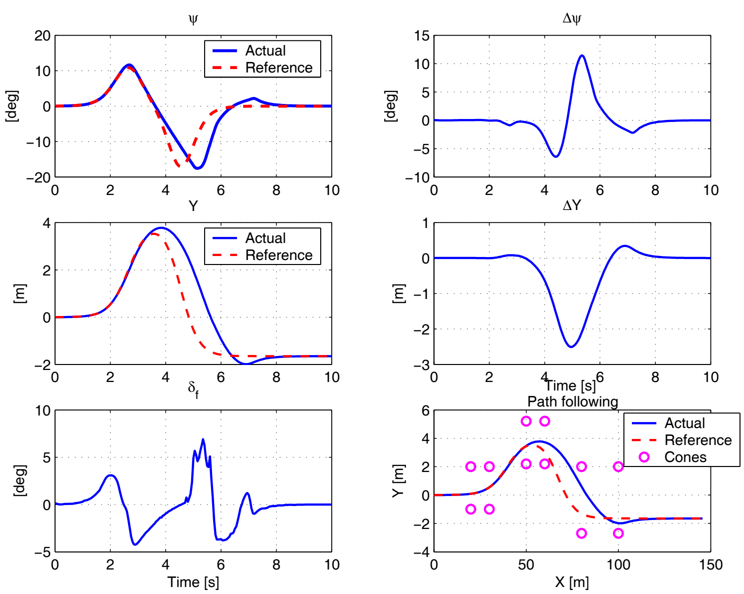


Figure 8 Double lane change maneuver at 15 m/s with $H_p = 10$ and $H_u = 4$.

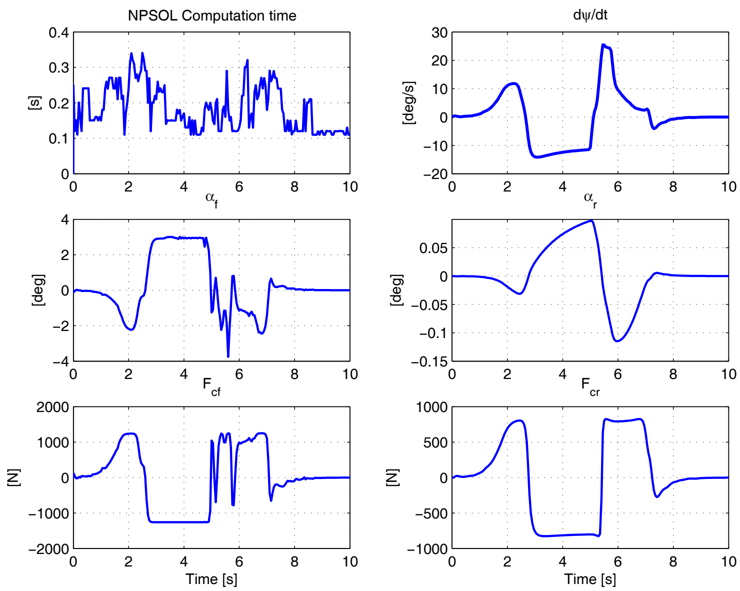


Figure 9 Double lane change maneuver at 15 m/s with $H_p = 10$ and $H_u = 4$.

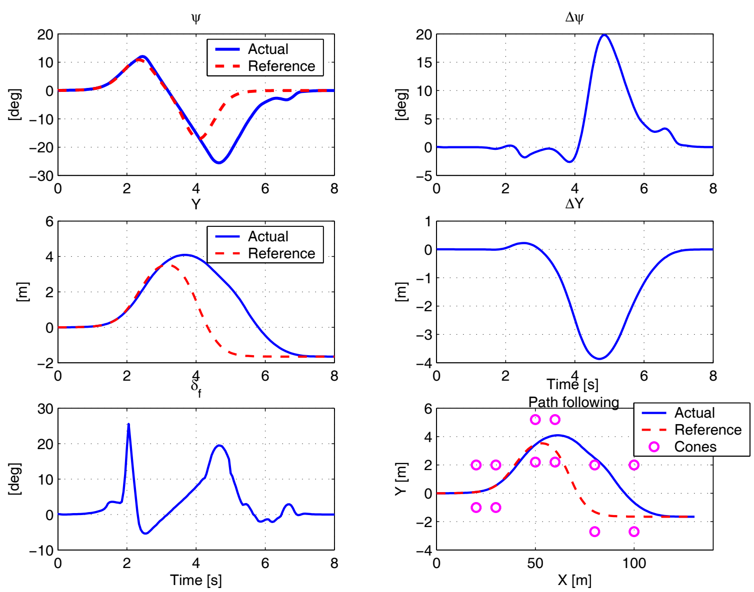


Figure 10 Double lane change maneuver at 17 m/s with $H_p = 10$ and $H_u = 7$.

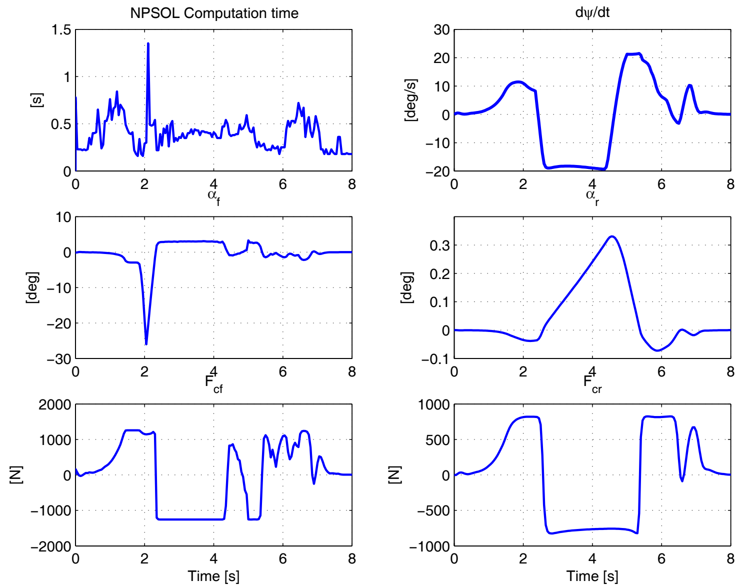


Figure 11 Double lane change maneuver at 17 m/s with $H_p = 10$ and $H_u = 7$. NPSOL computation time, yaw rate, tire forces and slip angles.

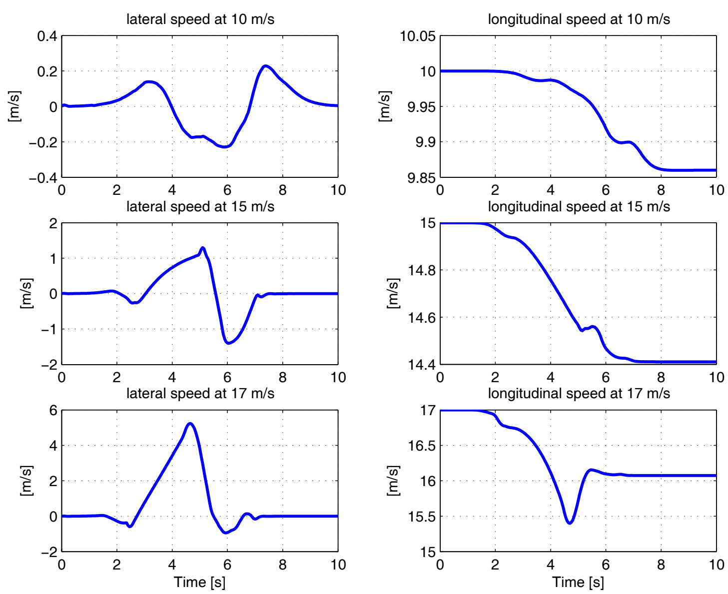


Figure 12 Lateral and longitudinal vehicle speeds at entry speeds of 10 m/s, 15 m/s and 17 m/s.

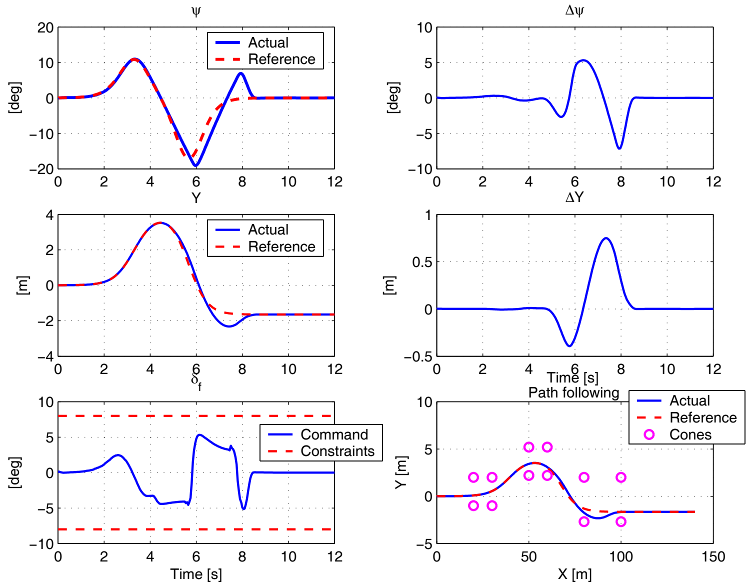


Figure 13 Simulation results with inactive constraints at 12 m/s.

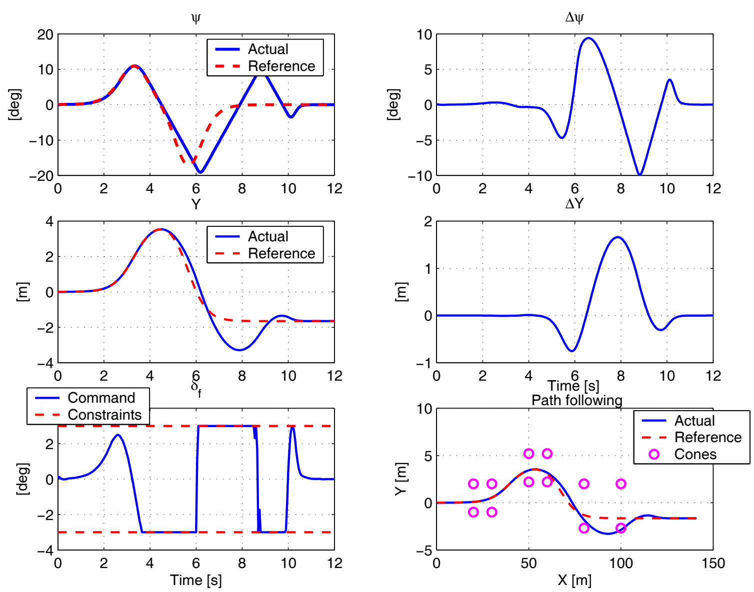


Figure 14 Simulation results with active constraints at 12 m/s.

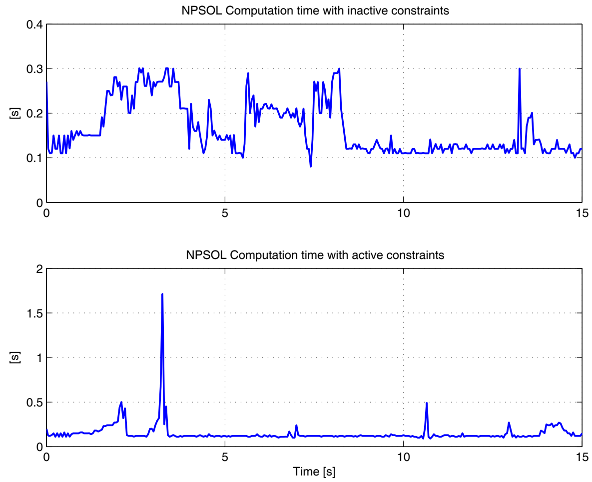


Figure 15 NPSOL computation time in the case of active and inactive constraints on steering angle.

the lateral deviation Y and on the yaw angle ψ. In Figure 15, the computation time required by the nonlinear solver is shown. The computational complexity increases when the constraints on the steering angle become active.

As an independent test of above results, it should be pointed out that they are comparable with the corresponding results obtained through the optimization done at Ford Research Laboratories using SOCS (The Boeing Company, 2004) and iSIGHT (Engineouse Software Inc, 2005) software.

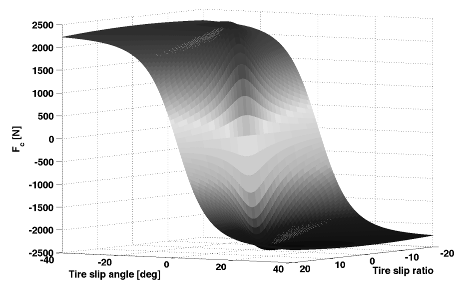
6 Real-Time Implementation

In this section we discuss possible approaches which can provide suboptimal and real-time implementable MPC schemes. This is the topic of current ongoing research directed towards the experimental validation of the results presented in this paper. Also, increasing power of computational infrastructures and improvements of optimization tools will be relevant for the future developments of this work and are not considered here.

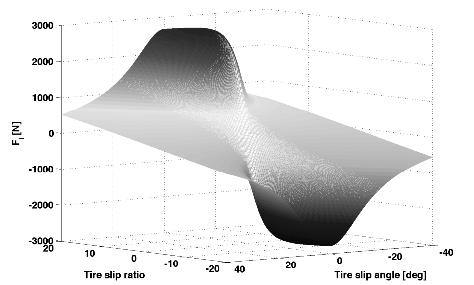
The critical point of the work is the complexity associated to the solution of a constrained nonlinear optimization problem. Linear and piecewise-linear vehicle models might lead to suboptimal control problems with faster solution time and acceptable performance.

6.1 Piecewise-linear Models

By following a similar approach of the works of (Borrelli et al., 2001; Giorgetti et al., 2005a,b) in the automotive field, the model equations (10) can be piecewise-linearized. This would enable the use of Mixed Logical Dynamical (MLD) models (Bemporad and Morari, 1999) to describe the vehicle dynamics. The op-



(a) Lateral tire force as a function of slip angle α and slip ratio s ($\mu = 0.9$).



(b) Longitudinal tire force as a function of slip angle α and slip ratio s ($\mu = 0.9$).

Figure 16 Nonlinear tire model.

timization problem becomes a mixed-integer quadratic programming (MIQP) or a mixed-integer linear programming (MILP) depending on the norm used in the cost function. The MPC controller requires the MILP or MIQP to be solved on line at each sampling step. In a second phase, if the solution of a MILP or MIQP is prohibitive, the explicit piecewise affine form of the MPC law can be computed off line by using the multiparametric mixed integer programming (mp-MILP or mp-MIQP) solvers (Dua and Pistikopoulos, 2000; Bemporad et al., 2002; Borrelli et al., 2005a). The resulting control law has the piecewise affine form and although the resulting piecewise affine control action is *identical* to the MPC designed, the on-line complexity is reduced to the simple evaluation of a piecewise affine function.

The use of piecewise-linear models allows nonlinear predictions in the MPC control design. In particular, for long prediction horizons, it might be critical to predict that the lateral tire forces do not increase linearly with the tire slip angle but they start decreasing beyond a certain peak. On the other hand, this approach requires the generation of piecewise linear approximations to the tire model described in Section 2.2 and shown in Figure 16. Such approximation might require a high number of affine terms in the model, which could lead to an explosion of the mp-MIQP or mp-MILP solution complexity. Nevertheless, it has been possible to obtain piecewise affine explicit solutions in the filed of direct torque engines for highly nonlinear multidimensional systems (T. Geyer, 2005).

6.2 Linear Models

A further reduction can be obtained by linearizing the vehicle dynamics around a certain control input and state trajectory. This would lead to a linear time-varying (LTV) prediction model, where the linear state matrices change at every future time step. The problem complexity is reduced to a quadratic program, which requires less computational resource and could be implemented in real time. However, efficient linearization of the nonlinear vehicle dynamics around the chosen trajectory presents an additional challenge. Analytic expressions could be obtained for the Jacobian matrices using the equations of motion. Another approach is to calculate the Jacobian numerically for all future time steps online, which leads to a more significant computational burden. The main ingredients of this procedure are described briefly next in this section.

Given a nonlinear dynamical system in the following form

$$\dot{\xi} = f(\xi, u), \tag{17a}$$
$$\eta = h(\xi), \tag{17b}$$

we consider small changes in the system states, inputs and outputs around a trajectory given by $\xi_0(t), u_0(t), \eta_0(t)$:

$$\xi(t) = \xi_0(t) + \delta\xi(t), \tag{18a}$$
$$u(t) = u_0(t) + \delta u(t), \tag{18b}$$
$$\eta(t) = \eta_0(t) + \delta\eta(t). \tag{18c}$$

If the $\delta\xi(t), \delta u(t), \delta\eta(t)$ changes are small enough, we can obtain a linearized system around the $\xi_0(t), u_0(t), \eta_0(t)$ trajectory in the following way:

$$\delta\dot{\xi} = \overbrace{\left.\frac{\partial f(\xi,u)}{\partial \xi}\right|_{\xi_0(t),u_0(t)}}^{A(t)} \delta\xi + \overbrace{\left.\frac{\partial f(\xi,u)}{\partial u}\right|_{\xi_0(t),u_0(t)}}^{B(t)} \delta u, \tag{19a}$$
$$\delta\eta = \underbrace{\left.\frac{\partial h(\xi)}{\partial \xi}\right|_{\xi_0(t)}}_{C(t)} \delta\xi. \tag{19b}$$

Using the above linearized system, we can construct a quadratic programming based MPC, which uses LTV prediction model. Let $\mathcal{A}_k$, $\mathcal{B}_k$, $\mathcal{C}_k$, $\mathcal{D}_k$ denote the discrete-time state matrices of the vehicle model linearized at $t = kT_s$.

Based on (20), the prediction for the outputs has the form

$$\mathcal{Y}(k) = \Psi_k \hat{\xi}(k) + \Theta_k \Delta\mathcal{U}(k). \tag{21}$$

Substituting the predicted output (21) into the cost function of (13), we get a quadratic expression in terms of the control changes $\Delta\mathcal{U}(k)$:

$$J(k) = \Delta\mathcal{U}(k)^T \mathcal{H}_k \Delta\mathcal{U}(k) - \Delta\mathcal{U}(k)^T \mathcal{G}_k + const, \tag{22}$$

where

$$\mathcal{H}_k = \Theta_k^T Q_e \Theta_k + R_e, \quad \mathcal{G}_k = 2\Theta_k^T Q_e \mathcal{E}(k),$$
$$const = \mathcal{E}^T(k) Q_e \mathcal{E}(k),$$

and $\mathcal{E}(k)$ is defined as a tracking error between the future target trajectory and the free response of the system, i.e. $\mathcal{E}(k) = \mathcal{Y}_{ref}(k) - \Psi_k \hat{\xi}(k)$. Q_e and R_e are block diagonal matrices of appropriate dimensions with Q and R on the main diagonal, respectively.

This approach leads to a QP problem of minimizing (22) with respect to the vector of control changes $\Delta\mathcal{U}(k)$, where the state matrices describing the prediction model are LTV and change in each implementation cycle.

As an alternative approach, one can locally linearize the optimization problem (14), and compute local explicit solution to multiparametric quadratic programs, as proposed in (Tøndel and Johansen, 2005). In particular, the method

By using successive substitution, it is straightforward to derive that the prediction model of performance outputs (signals of interest) over the prediction horizon is given by equation (20).

$$
\underbrace{\begin{bmatrix} \hat{\eta}(k+1 \mid k) \\ \hat{\eta}(k+2 \mid k) \\ \vdots \\ \hat{\eta}(k+H_c \mid k) \\ \hat{\eta}(k+H_c+1 \mid k) \\ \vdots \\ \hat{\eta}(k+H_p \mid k) \end{bmatrix}}_{\mathcal{Y}(k)} = \underbrace{\begin{bmatrix} \mathcal{C}_{k+1}\mathcal{A}_k \\ \mathcal{C}_{k+2}\mathcal{A}_{k+1}\mathcal{A}_k \\ \vdots \\ \mathcal{C}_{k+H_c}\prod_{i=k}^{k+H_c-1}\mathcal{A}_i \\ \mathcal{C}_{k+H_c+1}\prod_{i=k}^{k+H_c}\mathcal{A}_i \\ \vdots \\ \mathcal{C}_{k+H_p}\prod_{i=k}^{k+H_p-1}\mathcal{A}_i \end{bmatrix}}_{\Psi_k} \hat{\xi}(k)+
$$

$$
\underbrace{\begin{bmatrix} \mathcal{C}_{k+1}\mathcal{B}_k & \mathcal{D}_{k+1} & \cdots & 0 \\ \mathcal{C}_{k+2}\mathcal{A}_{k+1}\mathcal{B}_k & \mathcal{C}_{k+2}\mathcal{B}_{k+1} & \mathcal{D}_{k+2} & \vdots \\ \vdots & \vdots & \ddots & \ddots \\ \mathcal{C}_{k+H_c}\prod_{i=k+1}^{k+H_c-1}\mathcal{A}_i\mathcal{B}_k & \mathcal{C}_{k+H_c}\prod_{i=k+2}^{k+H_c-1}\mathcal{A}_i\mathcal{B}_{k+1} & \cdots & \mathcal{C}_{k+H_c}\mathcal{B}_{k+H_c-1} \\ \mathcal{C}_{k+H_c+1}\prod_{i=k+1}^{k+H_c}\mathcal{A}_i\mathcal{B}_k & \mathcal{C}_{k+H_c+1}\prod_{i=k+2}^{k+H_c}\mathcal{A}_i\mathcal{B}_{k+1} & \cdots & \mathcal{C}_{k+H_c+1}\mathcal{A}_{k+H_c}\mathcal{B}_{k+H_c-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{C}_{k+H_p}\prod_{i=k+1}^{k+H_p-1}\mathcal{A}_i\mathcal{B}_k & \mathcal{C}_{k+H_p}\prod_{i=k+2}^{k+H_p-1}\mathcal{A}_i\mathcal{B}_k & \cdots & \mathcal{C}_{k+H_p}\prod_{i=k+H_c}^{k+H_p-1}\mathcal{A}_i\mathcal{B}_{k+Hc-1} \end{bmatrix}}_{\Theta_k} \times
$$

$$
\underbrace{\begin{bmatrix} \Delta u(k \mid k) \\ \vdots \\ \Delta u(k+H_c-1 \mid k) \end{bmatrix}}_{\Delta\mathcal{U}(k)} \tag{20}
$$

uses quadratic approximations to the nonlinear cost and linear approximations to the constraints equations. This allows to approximate the NLP (14) solution with a set of quadratic programs (QP), for which explicit equivalent solution functions can be calculated by means of of multiparametric quadratic programs (mp-QP). The advantage is that these approximations can be computed off-line as an explicit, piecewise linear function of the state. The online computational burden is reduced to evaluation of a simple piecewise linear look-up table. Possible issues may arise from the interplay between the desired approximation accuracy and the solution complexity.

As last remark, the two major drawbacks deriving from the use of local linear models should be pointed out. First, predictions will be based on linear models, i.e., over a given prediction horizon the MPC controller will not be able to predict a change of slope in the cornering tire curve. Second, the state constraints, if present, might not be fulfilled for the initial nonlinear model, unless robust MPC formulations are used (Campo and Morari, 1987; Allwright and Papavasiliou, 1992; Kothare et al., 1996; Scokaert and Mayne, 1998; Lee and Yu, 1997).

7 Concluding Remarks and Future Work

We have presented a novel MPC-based approach for active steering control design. The approach has been used to develop, in a systematic way, a feedback controller for a double lane change maneuver on slippery surfaces such as snow covered roads. Simulation results showed that complex steering maneuvers are relatively easily obtained as a result of the MPC feedback policy, leading to the capability of stabilizing a vehicle with a speed up to 17 m/s. The results correlates well with the one obtained through Ford internal research activities.

In addition, the trade off between the vehicles speed and the prediction horizon H_p has been highlighted. The minimum prediction horizon which provides acceptable performance and vehicle stability has been quantified at several vehicle speeds. The role of constraints has been discussed as well. We show how the introduction of steering constraints can be handled in a systematic way by the MPC-design. A loss of performance, without loss of vehicle stability, has been observed in the case of vehicle entry speed of 12 m/s when reducing the constraints on the steering angle from ± 30 deg to ± 2 deg.

The computational complexity of the feedback control policy has been highlighted and the simulation results have shown the possibility to experimentally validate the methodology at low vehicle speed, typically below 10 m/s. For high vehicle speeds we are currently investigating new methodologies and computational tools for approximating the nonlinear MPC with a real-time implementable control scheme. Such sub-optimal MPC approaches have been proposed in Section 6.2.

Additional comments on future research issues are summarized as follows:

- The experiment setups rely on the assumption that the steering angles are the only control inputs. A vehicle dynamic control algorithm could be based on other control inputs as well, such as brakes, transmission, longitudinal vehicle speed, electrically controlled differentials and others.

- Actual implementation of the analyzed control schemes will have to incorporate state estimation from available measurements. This is a significant area of research on its own. In this paper we assumed that the states associated to the prediction model can be measured on the actual testbed.

- As indicated in the Introduction, the present work assumes autonomous driving conditions that can be achieved via a steering robot driver. In the future, one could extend the proposed approach to the case where Active Front and Rear Wheel Steer are primarily used to assist a driver, especially in critical, safety-oriented maneuvers. In this context, the driver and vehicle form a combined plant to be controlled, where typical driver models (or in more advanced stage, an on-line adaptive/learning driver) could be used.

- An additional degree of flexibility can be provided in the double lane change scenario by optimizing for the reference trajectory as well, instead of following a pre-specified path. For example, this type of optimization would incorporate the knowledge about potential obstacles (facilitated by cameras, GPS and vehicle to vehicle communications) and the constrained dynamics of the vehicle. This would result in a reference trajectory that would avoid obstacles and optimize drivability within the given road surface conditions and vehicle speed.

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