

DML conjecture for adelic general points

The main reference of this section is [Xie25, Section 3.2]. We first state the main theorem of this section.

Theorem 1 (ref.[Xie25, Proposition 3.24]). Let \mathbf{k} be a finitely generated field over \mathbb{Q} , X a projective variety defined over \mathbf{k} , and $f : X \dashrightarrow X$ a dominant rational self-map defined over \mathbf{k} .

There exists an iteration $g = f^m$ of f , an embedding $\mathbf{k} \hookrightarrow \mathbb{C}_p$ for some prime $p \geq 3$, an (Yang: analytic) open subset $U \subseteq X(\mathbb{C}_p)$ and an analytic map $\Phi : \mathbb{C}_p^\circ \times U \rightarrow U$ such that

- (1) $U \cong (\mathbb{C}_p^\circ)^d$ analytically, where $d = \dim X$;
- (2) g is well-defined on U , U is invariant under g and $\|g|_U - \text{id}_U\| < 1/p$;
- (3) $\Phi(n, x) = g^n(x)$ for each $n \in \mathbb{Z}_{\geq 0}$ and each $x \in U$;

Lemma 2. Yang: There is a good model for (X, f) .

Example 3. Let $X = E \times E$ with E an elliptic curve without complex multiplication defined over a number field \mathbf{k} , and let $f : X \rightarrow X$ be the endomorphism defined by $(a, b) \mapsto (a + b, b)$. Yang: To be continued.

Appendix