

A theorem for attractor of endomorphism on k-affinoid spaces

In this section, we copy [Xie25, Appendix A].

Fix an algebraically closed non-Archimedean complete field \mathbb{k} of characteristic zero, typically $\mathbb{k} = \mathbb{C}_p$.

Let A be a strictly and reduced \mathbb{k} -affinoid algebra, and $X = \mathcal{M}(A)$ be its Berkovich spectrum. Let $f : X \rightarrow X$ be a finite morphism of Berkovich spaces over \mathbb{k} .

Definition 1. Let ρ_A be the spectral radius on A . For any $g \in A$, we define

$$\rho_f(g) := \lim_{n \rightarrow \infty} \rho_A((f^*)^n(g))$$

where $f^* : A \rightarrow A$ is the induced endomorphism on A . Denote by

$$\text{Qnil}_f := \{g \in A \mid \rho_f(g) = 0\}.$$

Proposition 2. There exists a constant $0 < c < 1$ and $m \geq 1$ such that for any $g \in \text{Qnil}_f$, we have

$$\rho_A((f^*)^m(g)) \leq c\rho_A(g).$$

Assumption We further assume that X is distinguished. And there exists a subvariety $\tilde{Z} \subseteq \tilde{X}$ such that $\tilde{f}(\tilde{X}) = \tilde{Z}$ and $\tilde{f}|_{\tilde{Z}}$ is an automorphism of \tilde{Z} .

Set $Y = V(\text{Qnil}_f) \subseteq X$ be the closed analytic subspace defined by the ideal Qnil_f .

The main result is the following theorem.

Theorem 3 (ref. [Xie25, Theorem 8.3]). We have

- (a) $\widetilde{\text{Qnil}}_f = I_{\tilde{Z}}$, the ideal of \tilde{Z} in \tilde{X} ;
- (b) Y is distinguished, and $f|_Y : Y \rightarrow Y$ is an automorphism;
- (c) there exists a unique $\psi : X \rightarrow Y$ making $Y \hookrightarrow X$ a section of ψ , such that $f|_Y \circ \psi = \psi \circ f$;

Let $X = \mathcal{M}(\mathbb{k}x, y)$ be the closed unit polydisc of dimension two over \mathbb{k} . Suppose that $f : X \rightarrow X$ satisfies

$$\tilde{f} : (x, y) \mapsto (ax + b, 0).$$

Proposition 4.

Appendix

References

- [Xie25] Junyi Xie. “The existence of Zariski dense orbits for endomorphisms of projective surfaces”. In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).

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