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## 1 A theorem for attractor on $k$ -affinoid spaces

In this section, we copy [Xie25, Appendix A].

Fix an algebraically closed non-Archimedean complete field  $\mathbb{k}$  of characteristic zero and with non-trivial valuation, typically  $\mathbb{k} = \mathbb{C}_p$ .

**Setup** Let  $A$  be a strictly and reduced  $\mathbb{k}$ -affinoid algebra, and  $X = \mathcal{M}(A)$  be its Berkovich spectrum. Let  $f : X \rightarrow X$  be a finite morphism of Berkovich spaces over  $\mathbb{k}$ . We further assume that there exists a subvariety  $\tilde{Z} \subseteq \tilde{X}$  such that  $\tilde{f}(\tilde{X}) = \tilde{Z}$  and  $\tilde{f}|_{\tilde{Z}}$  is an automorphism of  $\tilde{Z}$ .

**Definition 1.1.** Let  $\rho_A$  be the spectral radius on  $A$ . For any  $g \in A$ , we define

$$\rho_f(g) := \lim_{n \rightarrow \infty} \rho_A((f^*)^n(g))$$

where  $f^* : A \rightarrow A$  is the induced endomorphism on  $A$ . Denote by

$$\mathrm{Qnil}_f := \{g \in A \mid \rho_f(g) = 0\}.$$

**Proposition 1.2.** There exists a constant  $0 < c < 1$  and  $m \geq 1$  such that for any  $g \in \mathrm{Qnil}_f$ , we have

$$\rho_A((f^*)^m(g)) \leq c\rho_A(g).$$

**Proposition 1.3.** Set  $Y = V(\mathrm{Qnil}_f) \subseteq X$  be the closed analytic subspace defined by the ideal  $\mathrm{Qnil}_f$ . We have  $\tilde{Y} = \tilde{Z}$ . **Yang:** To be checked

The main result is the following theorem.

**Theorem 1.4** (ref. [Xie25, Theorem 8.3]). We have

- (a)  $f|_Y : Y \rightarrow Y$  is an automorphism;
- (b) there exists a unique  $\psi : X \rightarrow Y$  making  $Y \hookrightarrow X$  a section of  $\psi$ , such that  $f|_Y \circ \psi = \psi \circ f$ ;

Moreover, there exists  $C > 0$ ,  $0 < \beta < 1$  such that for any  $x \in X$  and any  $h \in A$ , we have

$$|h(f^n(\psi(x))) - h(f^n(x))| \leq C\beta^n \rho_A(h), \quad \forall n \geq 0.$$

**Yang:** To be revised.

## 1.1 Zariski dense orbit in dimension two

**Zariski dense orbit in dimension two** Let  $X = \mathcal{M}(\mathbb{k}x, y)$  be the closed unit polydisc of dimension two over  $\mathbb{k}$ . Suppose that  $f : X \rightarrow X$  satisfies

$$\tilde{f} : (x, y) \mapsto (ax + b, 0).$$

In this case, the attractor  $Y$  is the line defined by  $y = 0$ , and  $f|_Y$  is an automorphism of  $Y$ . Yang: To be revised.

**Proposition 1.5.** Suppose that  $f|_Y$  is not of finite order and  $f^{-1}(Y) \neq X$ . Then there exists an affinoid subdomain  $U \subseteq X$  such that for every point  $x \in U$ , the orbit  $\{f^n(x)\}_{n \geq 0}$  is Zariski dense in  $X$ . Yang:

## References

- [Xie25] Junyi Xie. “The existence of Zariski dense orbits for endomorphisms of projective surfaces”. In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).