Interpolation

The main reference of this section is [Ame11; BGT10; Poo14]. We first state the main theorem of this section.

Theorem 1 (ref.[Poo14, Theorem 1] cf.[BGT10, Theorem 3.3]). Let $\mathbf{k}$ be a complete non-archimedean field of characteristic 0 with $|p|_{\mathbf{k}} = 1/p$. Set $r_p = p^{-1/(p-1)}$. Let $E = E(0,1) = \{x \in \mathbf{k}^d \mid \|x\| \leq 1\}$ be the closed unit ball in $\mathbf{k}^d$. Suppose that $\Phi = (\Phi_1, \cdots, \Phi_d) \in \mathbf{k}^\circ\{\underline{T}\}^d$ satisfies $\|\Phi - \mathrm{id}_E\| \leq r_p$. Here the norm on $\mathbf{k}^d$ or $\mathbf{k}^\circ\{\underline{T}\}^d$ is the supremum norm, i.e., $|\Phi| = \max_{1\leq i\leq d} |\Phi_i|$. Then there exists a function $F \in \mathbf{k}\{\underline{T}, S\}^d$ such that for each $n \in \mathbb{Z}_{\geq 0}$ and each $x \in \mathbf{k}^d$,

$$F(x,n) = \Phi^n(x).$$

Yang: To be checked.

Yang: If f is invertible, can we see that g is unique?

References

[Ame11] E Amerik. "Existence of non-preperiodic algebraic points for a rational self-map of infinite order". In: *Mathematical Research Letters* 18.2 (2011), pp. 251–256 (cit. on p. 1).

[BGT10] Jason P Bell, Dragos Ghioca, and Thomas J Tucker. "The dynamical Mordell-Lang problem for étale maps". In: *American journal of mathematics* 132.6 (2010), pp. 1655–1675 (cit. on p. 1).

[Poo14] Bjorn Poonen. "p-adic interpolation of iterates". In: *Bulletin of the London Mathematical Society* 46.3 (2014), pp. 525–527 (cit. on p. 1).