

Adelic topology

The main reference of this section is [Xie25].

1 Basic adelic subset

Let \mathbf{k} be a finitely generated field over \mathbb{Q} and \mathbb{k} be its algebraic closure.

Let X be a variety over \mathbf{k} with \mathbf{k} a finitely generated field over \mathbb{Q} . Set $M_{\mathbf{k}}$ be the set of places of \mathbf{k} . For each $v \in M_{\mathbf{k}}$.

Let $\mathcal{I}_{\mathbf{k}}$ be the set of all embeddings $\sigma : K \rightarrow \mathbb{C}_{\sigma}$ over \mathbf{k} , where $\mathbb{C}_{\sigma} = \mathbb{C}_p$ or \mathbb{C} according to whether v is non-archimedean or archimedean. For each $\sigma \in \mathcal{I}_{\mathbf{k}}$, set $\mathcal{I}_{\sigma} = \{\tau : \mathbb{k} \rightarrow \mathbb{C}_{\tau} \mid \tau|_{\mathbf{k}} = \sigma\}$. For every $\tau \in \mathcal{I}_{\mathbf{k}}$, we have an induced map $\phi_{\tau} : X(\mathbb{k}) \rightarrow X(\mathbb{C}_{\tau})$.

On $X(\mathbb{C}_{\tau})$, we have the analytic topology induced from \mathbb{C}_{τ} .

Definition 1.

$$X_{\mathbf{k}}(\sigma, U) := \bigcup_{\tau \in \mathcal{I}_{\sigma}} \phi_{\tau}^{-1}(U) \subseteq X(\mathbb{k})$$

is called a *basic adelic open subset* of $X(\mathbb{k})$, where $U \subseteq X(\mathbb{C}_{\sigma})$ is an open subset in the analytic topology.

Theorem 2. Suppose that X is irreducible. Then for any finite collection of places $\sigma_1, \dots, \sigma_n \in M_{\mathbf{k}}$ and any basic adelic open subsets $X_{\mathbf{k}}(\sigma_i, U_i)$ for $i = 1, \dots, n$, the intersection

$$\bigcap_{i=1}^n X_{\mathbf{k}}(\sigma_i, U_i)$$

is non-empty.

2 General adelic subset

3 Adelic topology

Yang: Is $\mathbb{A}^1(\mathbb{Q})$ adelic closed in $\mathbb{A}^1(\overline{\mathbb{Q}})$ with adelic topology? If so, why?

Yang: Describe all adelic closed subset in $\mathbb{A}^1(\overline{\mathbb{Q}})$.

Appendix

References

- [Xie25] Junyi Xie. “The existence of Zariski dense orbits for endomorphisms of projective surfaces”. In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).