

# Adelic topology

The main reference of this section is [Xie25].

## 1 Basic adelic subset

Let  $\mathbf{k}$  be a finitely generated field over  $\mathbb{Q}$  and  $\mathbb{k}$  be its algebraic closure.

Let  $X$  be a variety over  $\mathbf{k}$  with  $\mathbf{k}$  a finitely generated field over  $\mathbb{Q}$ . Set  $M_{\mathbf{k}}$  be the set of places of  $\mathbf{k}$ . For each  $v \in M_{\mathbf{k}}$ .

Let  $\mathcal{J}_{\mathbf{k}}$  be the set of all embeddings  $\sigma : K \rightarrow \mathbb{C}_{\sigma}$  over  $\mathbf{k}$ , where  $\mathbb{C}_{\sigma} = \mathbb{C}_p$  or  $\mathbb{C}$  according to whether  $v$  is non-archimedean or archimedean. For each  $\sigma \in \mathcal{J}_{\mathbf{k}}$ , set  $\mathcal{I}_{\sigma} = \{\tau : \mathbb{k} \rightarrow \mathbb{C}_v \mid \tau|_{\mathbf{k}} = \sigma\}$ . For every  $\tau \in \mathcal{J}_{\mathbf{k}}$ , we have an induced map  $\phi_{\tau} : X(\mathbb{k}) \rightarrow X(\mathbb{C}_{\tau})$ .

On  $X(\mathbb{C}_{\tau})$ , we have the analytic topology induced from  $\mathbb{C}_{\tau}$ .

**Definition 1.**

$$X_{\mathbf{k}}(\sigma, U) := \bigcup_{\tau \in \mathcal{I}_{\sigma}} \phi_{\tau}^{-1}(U) \subseteq X(\mathbb{k})$$

is called a *basic adelic open subset* of  $X(\mathbb{k})$ , where  $U \subseteq X(\mathbb{C}_{\sigma})$  is an open subset in the analytic topology.

**Theorem 2.** Suppose that  $X$  is irreducible. Then for any finite collection of places  $\sigma_1, \dots, \sigma_n \in M_{\mathbf{k}}$  and any basic adelic open subsets  $X_{\mathbf{k}}(\sigma_i, U_i)$  for  $i = 1, \dots, n$ , the intersection

$$\bigcap_{i=1}^n X_{\mathbf{k}}(\sigma_i, U_i)$$

is non-empty.

## 2 General adelic subset

## 3 Adelic topology

Yang: Is  $A^1(\mathbb{Q})$  adelic closed in  $A^1(\overline{\mathbb{Q}})$  with adelic topology? If so, why?

Yang: Describe all adelic closed subset in  $A^1(\overline{\mathbb{Q}})$ .

## Appendix

## References

- [Xie25] Junyi Xie. “The existence of Zariski dense orbits for endomorphisms of projective surfaces”. In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).