A theorem for attractor of endomorphism on k-affinoid spaces

In this section, we copy [Xie25, Appendix A].

Fix an algebraically closed non-Archimedean complete field $\Bbbk$ of characteristic zero, typically $\Bbbk = \mathbb{C}_p$.

Let A be a strictly and reduced $\Bbbk$-affinoid algebra, and $X = \mathscr{M}(A)$ be its Berkovich spectrum. Let $f : X \to X$ be a finite morphism of Berkovich spaces over $\Bbbk$.

Definition 1. Let ρ_A be the spectral radius on A. For any $g \in A$, we define

$$\rho_f(g) := \lim_{n\to\infty} \rho_A\left((f^*)^n(g)\right)$$

where $f^* : A \to A$ is the induced endomorphism on A. Denote by

$$\mathrm{Qnil}_f := \{g \in A \mid \rho_f(g) = 0\}.$$

Proposition 2. There exists a constant $0 < c < 1$ and $m \geq 1$ such that for any $g \in \mathrm{Qnil}_f$, we have

$$\rho_A\left((f^*)^m(g)\right) \leq c\rho_A(g).$$

Assumption We further assume that X is distinguished. And there exists a subvariety $\tilde{Z} \subseteq \tilde{X}$ such that $\tilde{f}(\tilde{X}) = \tilde{Z}$ and $\tilde{f}|_{\tilde{Z}}$ is an automorphism of $\tilde{Z}$.

Recall that A is called distinguished if there exists a surjective morphism $\Bbbk\{x_1, \ldots, x_n\} \to A$ such that the induced norm on A coincides with the spectral norm ρ_A.

Set $Y = V(\mathrm{Qnil}_f) \subseteq X$ be the closed analytic subspace defined by the ideal Qnil_f.

The main result is the following theorem.

Theorem 3 (ref. [Xie25, Theorem 8.3])**.** We have

(a) $\widetilde{\mathrm{Qnil}_f} = I_{\tilde{Z}}$, the ideal of $\tilde{Z}$ in $\tilde{X}$;

(b) Y is distinguished, and $f|_Y : Y \to Y$ is an automorphism;

(c) there exists a unique $\psi : X \to Y$ making $Y \hookrightarrow X$ a section of ψ, such that $f|_Y \circ \psi = \psi \circ f$;

Zariski dense orbit in dimension two Let $X = \mathscr{M}(\Bbbk x, y)$ be the closed unit polydisc of dimension two over $\Bbbk$. Suppose that $f : X \to X$ satisfies

$$\tilde{f} : (x, y) \mapsto (ax + b, 0).$$

In this case, the attractor Y is the line defined by $y = 0$, and $f|_Y$ is an automorphism of Y. Yang: To be revised.

Proposition 4. Suppose that $f|_Y$ is not of finite order and $f^{-1}(Y) \neq X$. Then there exists an affinoid subdomain $U \subseteq X$ such that for every point $x \in U$, the orbit $\{f^n(x)\}_{n\geq 0}$ is Zariski dense in X. Yang:

Date: February 9, 2026, **Author**: Tianle Yang, My Homepage

Appendix

References

[Xie25] Junyi Xie. "The existence of Zariski dense orbits for endomorphisms of projective surfaces". In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).