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1 A theorem for attractor of endomorphism on k -affinoid spaces

In this section, we copy [Xie25ZDOsurfaces].

Fix an algebraically closed non-Archimedean complete field k of characteristic zero, typically $k = \mathbb{C}_p$.

Let A be a strictly and reduced k -affinoid algebra, and $X = \mathcal{M}(A)$ be its Berkovich spectrum. Let $f : X \rightarrow X$ be a finite morphism of Berkovich spaces over k .

Definition 1.1. Let ρ_A be the spectral radius on A . For any $g \in A$, we define

$$\rho_f(g) := \lim_{n \rightarrow \infty} \rho_A((f^*)^n(g))$$

where $f^* : A \rightarrow A$ is the induced endomorphism on A . Denote by

$$\text{Qnil}_f := \{g \in A \mid \rho_f(g) = 0\}.$$

Proposition 1.2. There exists a constant $0 < c < 1$ and $m \geq 1$ such that for any $g \in \text{Qnil}_f$, we have

$$\rho_A((f^*)^m(g)) \leq c\rho_A(g).$$

Assumption We further assume that there exists a subvariety $\tilde{Z} \subseteq \tilde{X}$ such that $\tilde{f}(\tilde{X}) = \tilde{Z}$ and $\tilde{f}|_{\tilde{Z}}$ is an automorphism of \tilde{Z} .

Proposition 1.3. prop of stable and distinguished

Recall that A is called distinguished if there exists a surjective morphism $k\{x_1, \dots, x_n\} \rightarrow A$ such that the induced norm on A coincides with the spectral norm ρ_A .

Set $Y = V(\text{Qnil}_f) \subseteq X$ be the closed analytic subspace defined by the ideal Qnil_f .

Proposition 1.4. We have $\tilde{Y} = \tilde{Z}$. Yang: To be checked

The main result is the following theorem.

Theorem 1.5 (ref. [Xie25ZDOsurfaces]). We have

- (a) Y is distinguished, and $f|_Y : Y \rightarrow Y$ is an automorphism;
- (b) there exists a unique $\psi : X \rightarrow Y$ making $Y \hookrightarrow X$ a section of ψ , such that $f|_Y \circ \psi = \psi \circ f$;

Moreover, there exists $C > 0$, $0 < \lambda < 1$ such that for any $x \in X$ and any $n \geq 0$, we have

$$d(f^n(x), f^n(\psi(x))) \leq C\lambda^n.$$

Yang: To be revised.

Zariski dense orbit in dimension two Let $X = \mathcal{M}(\mathbb{k}x, y)$ be the closed unit polydisc of dimension two over \mathbb{k} . Suppose that $f : X \rightarrow X$ satisfies

$$\tilde{f} : (x, y) \mapsto (ax + b, 0).$$

In this case, the attractor Y is the line defined by $y = 0$, and $f|_Y$ is an automorphism of Y . Yang: To be revised.

Proposition 1.6. Suppose that $f|_Y$ is not of finite order and $f^{-1}(Y) \neq X$. Then there exists an affinoid subdomain $U \subseteq X$ such that for every point $x \in U$, the orbit $\{f^n(x)\}_{n \geq 0}$ is Zariski dense in X . Yang: