A theorem for attractor on k-affinoid spaces

In this section, we copy [Xie25, Appendix A].

Fix an algebraically closed non-Archimedean complete field $\Bbbk$ of characteristic zero and with non-trivial valuation, typically $\Bbbk = \mathbb{C}_p$.

Setup Let A be a strictly and reduced $\Bbbk$-affinoid algebra, and $X = \mathscr{M}(A)$ be its Berkovich spectrum. Let $f : X \to X$ be a finite morphism of Berkovich spaces over $\Bbbk$. We further assume that there exists a subvariety $\tilde{Z} \subseteq \tilde{X}$ such that $\tilde{f}(\tilde{X}) = \tilde{Z}$ and $\tilde{f}|_{\tilde{Z}}$ is an automorphism of $\tilde{Z}$.

Definition 1. Let ρ_A be the spectral radius on A. For any $g \in A$, we define

$$\rho_f(g) := \lim_{n\to\infty} \rho_A\left((f^*)^n(g)\right)$$

where $f^* : A \to A$ is the induced endomorphism on A. Denote by

$$\mathrm{Qnil}_f := \{g \in A \mid \rho_f(g) = 0\}.$$

Proposition 2. There exists a constant $0 < c < 1$ and $m \geq 1$ such that for any $g \in \mathrm{Qnil}_f$, we have

$$\rho_A\left((f^*)^m(g)\right) \leq c\rho_A(g).$$

Proposition 3. Set $Y = V(\mathrm{Qnil}_f) \subseteq X$ be the closed analytic subspace defined by the ideal Qnil_f. We have $\tilde{Y} = \tilde{Z}$. Yang: To be checked

The main result is the following theorem.

Theorem 4 (ref. [Xie25, Theorem 8.3])**.** We have

(a) $f|_Y : Y \to Y$ is an automorphism;

(b) there exists a unique $\psi : X \to Y$ making $Y \hookrightarrow X$ a section of ψ, such that $f|_Y \circ \psi = \psi \circ f$;

Moreover, there exists $C > 0$, $0 < \beta < 1$ such that for any $x \in X$ and any $h \in A$, we have

$$|h(f^n(\psi(x))) - h(f^n(x))| \leq C\beta^n \rho_A(h), \quad \forall n \geq 0.$$

Yang: To be revised.

1 Zariski dense orbit in dimension two

Zariski dense orbit in dimension two Let $X = \mathscr{M}(\Bbbk x, y)$ be the closed unit polydisc of dimension two over $\Bbbk$. Suppose that $f : X \to X$ satisfies

$$\tilde{f} : (x, y) \mapsto (ax + b, 0).$$

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In this case, the attractor Y is the line defined by $y = 0$, and $f|_Y$ is an automorphism of Y.Yang: To be revised.

Proposition 5. Suppose that $f|_Y$ is not of finite order and $f^{-1}(Y) \neq X$. Then there exists an affinoid subdomain $U \subseteq X$ such that for every point $x \in U$, the orbit $\{f^n(x)\}_{n\geq 0}$ is Zariski dense in X. Yang:

Appendix

Definition 6. Let A be a strictly $\mathbf{k}$-affinoid algebra. We say that A is *distinguished* if there exists a surjective morphism $\mathbf{k}\{x_1, \dots, x_n\} \to A$ such that the induced norm on A coincides with the spectral norm ρ_A. Yang:

Theorem 7. Let $\Bbbk$ be an algebraically closed non-Archimedean complete field. Then every strictly $\Bbbk$-affinoid algebra is distinguished. Yang:

References

[Xie25] Junyi Xie. “The existence of Zariski dense orbits for endomorphisms of projective surfaces”. In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).

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