Adelic topology

The main reference of this section is [Xie25].

1 Basic adelic subset

Let $\mathbf{k}$ be a finitely generated field over $\mathbb{Q}$ and $\Bbbk$ be its algebraic closure.

Let $\mathcal{J}_{\mathbf{k}}$ be the set of all embeddings $\sigma : \mathbf{k} \to \mathbb{C}_\sigma$ over $\mathbf{k}$, where $\mathbb{C}_\sigma = \mathbb{C}_p$ or $\mathbb{C}$ according to whether v is non-archimedean or archimedean. For each $\sigma \in \mathcal{J}_{\mathbf{k}}$, set $\mathcal{J}_\sigma = \{\tau : \Bbbk \to \mathbb{C}_v \mid \tau|_{\mathbf{k}} = \sigma\}$. For every $\tau \in \mathcal{J}_{\Bbbk}$, we have an induced map $\phi_\tau : X(\Bbbk) \to X(\mathbb{C}_\tau)$.

On $X(\mathbb{C}_\tau)$, we have the analytic topology induced from $\mathbb{C}_\tau$.

Definition 1. Let $\sigma, \sigma_i \in \mathcal{J}_{\mathbf{k}}$ and let $U, U_i \subseteq X(\mathbb{C}_\sigma)$ be an open subset in the analytic topology. We define

$$X_{\mathbf{k}}(\sigma, U) := \bigcup_{\tau \in \mathcal{J}_\sigma} \phi_\tau^{-1}(U) \subseteq X(\Bbbk)$$

and

$$X_{\mathbf{k}}(\{\sigma_i, U_i\}_{i=1}^n) := \bigcap_{i=1}^n X_{\mathbf{k}}(\sigma_i, U_i) \subseteq X(\Bbbk).$$

The subset of form $X_{\mathbf{k}}(\{\sigma_i, U_i\}_{i=1}^n)$ is called a *basic adelic open subset* of $X(\Bbbk)$, where $U \subseteq X(\mathbb{C}_\sigma)$ is an open subset in the analytic topology.

Theorem 2 (ref. [Xie25, Proposition 3.9] cf. []). Suppose that X is irreducible. Then for any finite collection of places $\sigma_1, \ldots, \sigma_n \in M_{\mathbf{k}}$ and any basic adelic open subsets $X_{\mathbf{k}}(\sigma_i, U_i)$ for $i = 1, \ldots, n$, the intersection

$$\bigcap_{i=1}^n X_{\mathbf{k}}(\sigma_i, U_i)$$

is non-empty.

2 General adelic subset

3 Adelic topology

Yang: Is $\mathbb{A}^1(\mathbb{Q})$ adelic closed in $\mathbb{A}^1(\overline{\mathbb{Q}})$ with adelic topology? If so, why?

Yang: Describe all adelic closed subset in $\mathbb{A}^1(\overline{\mathbb{Q}})$.

Date: November 21, 2025, **Author**: Tianle Yang, My Homepage

Appendix

References

[Xie25] Junyi Xie. “The existence of Zariski dense orbits for endomorphisms of projective surfaces”. In: *Journal of the American Mathematical Society* 38.1 (2025), pp. 1–62 (cit. on p. 1).