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# Complexes and Homology

**Definition 1.** Let  $A_\bullet$  and  $B_\bullet$  be two complexes in  $\mathcal{A}$  and  $\varphi_\bullet, \psi_\bullet : A_\bullet \rightarrow B_\bullet$  be two morphisms of complexes. A *homotopy* between  $\varphi_\bullet$  and  $\psi_\bullet$  is a collection of morphisms  $h_n : A_n \rightarrow B_{n-1}$  such that

$$\varphi_n - \psi_n = d_{B_{n+1}} \circ h_n + h_{n-1} \circ d_{A_n}.$$

In diagram, we have

$$\begin{array}{ccccccc} \cdots & \longrightarrow & A_{n+1} & \longrightarrow & A_n & \xrightarrow{d_{A_n}} & A_{n-1} & \longrightarrow & \cdots \\ & & \nearrow h_n & & \downarrow \psi_n & & \downarrow \varphi_n & & \nearrow h_{n-1} \\ \cdots & \longrightarrow & B_{n+1} & \xrightarrow{d_{B_{n+1}}} & B_n & \longrightarrow & B_{n-1} & \longrightarrow & \cdots \end{array}$$