Affinoid algebras

1 The first properties

Definition 1. Let $\mathbf{k}$ be a non-archimedean field. A banach $\mathbf{k}$-algebra A is called a *affinoid* $\mathbf{k}$*-algebra* if there exists an admissible surjective homomorphism

$$\varphi : \mathbf{k}\{r_1^{-1}T_1, \dots, r_n^{-1}T_n\} \twoheadrightarrow A$$

for some $n \in \mathbb{N}$ and $r_1, \dots, r_n \in \mathbb{R}_{>0}$.
If one can choose $r_1 = \cdots = r_n = 1$, then we say that A is a *strict affinoid* $\mathbf{k}$*-algebra*.

Definition 2. Let $\mathbf{k}$ be a non-archimedean field. We define the *ring of restricted Laurent series* over $\mathbf{k}$ as

$$\mathbf{K}_r = \mathbf{L}_{\mathbf{k},r} = \left\{ \sum_{n\in\mathbb{Z}} a_n T^n : a_n \in \mathbf{k}, \lim_{|n|\to\infty} |a_n| r^n = 0 \right\}$$

equipped with the norm

$$\|f\| = \sup_{n\in\mathbb{Z}} |a_n| r^n.$$

Yang: Is $\mathbf{K}_r$ always a field? Yang: Do we have $\mathbf{L}_{\mathbf{k},r} = \mathrm{Frac}(\mathbf{k}\{T/r\})$?

Proposition 3. Let $\mathbf{k}$ be a non-archimedean field. If $r \notin \sqrt{|\mathbf{k}^\times|}$, then $\mathbf{K}_r$ is a complete non-archimedean field with non-trivial absolute value extending that of $\mathbf{k}$.

Proposition 4. Let A be an affinoid $\mathbf{k}$-algebra. Then A is noetherian, and every ideal of A is closed.

Proof. Yang: To be completed. ☐

Proposition 5. Let A be an affinoid $\mathbf{k}$-algebra. Then there exists a constant $C > 0$ and $N > 0$ such that for all $f \in A$ and $n \geq N$, we have

$$\|f^n\| \leq C\rho(f)^n.$$

Proof. Yang: To be completed. ☐

Proposition 6. Let A be an affinoid $\mathbf{k}$-algebra. If and only if $\rho(f) \in \sqrt{|\mathbf{k}|}$ for all $f \in A$, then A is strict. Yang: To be complete.

Proof. Yang: To be completed. ☐

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2 Noetherian normalization theorem

Theorem 7. Let A be an affinoid $\mathbf{k}$-algebra. Then there exists a finite injective homomorphism

$$\varphi : \mathbf{k}\{r_1^{-1}T_1, \dots, r_d^{-1}T_d\} \hookrightarrow A$$

for some $d \in \mathbb{N}$ and $r_1, \dots, r_d \in \mathbb{R}_{>0}$. Yang: To be checked.

3 Tate algebras and Weierstrass division

Theorem 8 (Weierstrass preparation theorem)**.** Let $\mathbf{k}$ be a complete non-archimedean field. Let $f \in \mathbf{k}\{\underline{T/r}\}$ be a restricted power series that is distinguished in the variable T_n of degree d, i.e.,

$$f = \sum_{\alpha \in \mathbb{N}^{n-1}} a_\alpha T^\alpha + \sum_{\alpha_n \geq 1} a_\alpha T^\alpha$$

with $a_{(0,\dots,0,d)}$ being a unit in $\mathbf{k}\{\underline{T/r}\}$ and $\|a_\alpha\| r^\alpha < \|a_{(0,\dots,0,d)}\| r_n^d$ for all $\alpha_n < d$. Then there exists a unique monic polynomial $P \in \mathbf{k}\{\underline{T/r}\}[T_n]$ of degree d in T_n and a unique unit $U \in \mathbf{k}\{\underline{T/r}\}$ such that

$$f = P \cdot U.$$

Yang: To be checked.

Theorem 9 (Weierstrass division theorem)**.** Let $\mathbf{k}$ be a complete non-archimedean field. Let $f \in \mathbf{k}\{\underline{T/r}\}$ be a restricted power series that is distinguished in the variable T_n of degree d. Then for every $g \in \mathbf{k}\{\underline{T/r}\}$, there exists a unique $Q \in \mathbf{k}\{\underline{T/r}\}$ and a unique polynomial $R \in \mathbf{k}\{\underline{T/r}\}[T_n]$ of degree less than d in T_n such that

$$g = Q \cdot f + R.$$

Yang: To be checked.

Proposition 10. Let $\mathbf{k}$ be a complete non-archimedean field and $r = (r_1, \dots, r_n) \in \mathbb{R}_+^n$. Then

$$\operatorname{Spec} \mathbf{k}\{\underline{T/r}\} = \{\},$$

where

4 Reduction

Definition 11. Let A be an affinoid $\mathbf{k}$-algebra. We define the *reduction* of A as

$$\widetilde{A} = A^\circ / A^{\circ\circ},$$

where

$$A^\circ = \{f \in A : \|f\|_{\sup} \leq 1\}, \quad A^{\circ\circ} = \{f \in A : \|f\|_{\sup} < 1\}.$$

Appendix