Elementary functions

1 Elementary functions

Lemma 1. Let p be a prime number and $n \in \mathbb{N}$. We have $v_p(n!) =$.

Yang: Exponential, logarithmic, and the interpolation functions.

Fix a prime number p in the following and consider $\mathbf{k} = \mathbb{Q}_p, \mathbb{C}_p$, or Ω_p. Let $r_p := p^{-1/(p-1)}$.

Construction 2. The *exponential function* $\exp : \mathbf{k} \to \mathbf{k}$ is defined by the power series

$$\exp(x) := \sum_{n=0}^{+\infty} \frac{x^n}{n!}.$$

The *logarithmic function* $\log : 1 + \mathbf{k}^{\circ\circ} \to \mathbf{k}$ is defined by the power series

$$\log(1+x) := \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{x^n}{n}.$$

Yang: To be checked.

Definition 3. Let

Theorem 4. The series converges.

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