Hermitian line bundles on rings of integers

Let K be a number field, and $\mathcal{O}_K$ be its ring of integers.

Definition 1. A *hermitian line bundle* $\overline{\mathcal{L}}$ on $\operatorname{Spec}\mathcal{O}_K$ consists of

- a line bundle $\mathcal{L}$ on $\operatorname{Spec}\mathcal{O}_K$, i.e., a finitely generated projective $\mathcal{O}_K$-module of rank 1;
- for each embedding $\sigma : K \hookrightarrow \mathbb{C}$, a Hermitian metric $\|\cdot\|_\sigma$ on the complex line $\mathcal{L}_\sigma := \mathcal{L} \otimes_\sigma \mathbb{C}$.

Yang: To be revised.

Definition 2. Let $\overline{\mathcal{L}}$ be a hermitian line bundle on $\operatorname{Spec}\mathcal{O}_K$. The *degree* of $\overline{\mathcal{L}}$ is defined as

$$\deg\overline{\mathcal{L}} := \log\#(\mathcal{L}/\mathcal{O}_K s) - \sum_{\sigma:K\hookrightarrow\mathbb{C}} \log\|s\|_\sigma,$$

where s is any non-zero section of $\mathcal{L}$. Yang: To be revised

Appendix

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