Affinoid algebras

1 The first properties

Definition 1. Let $\mathbf{k}$ be a non-archimedean field. A banach $\mathbf{k}$-algebra A is called a *affinoid* $\mathbf{k}$*-algebra* if there exists an admissible surjective homomorphism

$$\varphi : \mathbf{k}\{\underline{T/r}\} \twoheadrightarrow A$$

for some $r = (r_1, \dots, r_n) \in \mathbb{R}^n_{>0}$.
If one can choose $r_1 = \dots = r_n = 1$, then we say that A is a *strict affinoid* $\mathbf{k}$*-algebra*.

Example 2. Yang: To be added.

Example 3. Let A be an affinoid $\mathbf{k}$-algebra and $f_i, g_i \in A$. Then the normed localization

$$A\{(f_i/g_i)/r_i\}_{i=1}^n = A \otimes_{\mathbf{k}\{\underline{T}\}} \mathbf{k}\{\underline{T/r}, \underline{S}\}/(g_i T_i - f_i)_{i=1}^n$$

is again an affinoid $\mathbf{k}$-algebra. Yang: To be added.

Proposition 4. Let A be an affinoid $\mathbf{k}$-algebra. Then A is noetherian, and every ideal of A is closed.

Proof. Yang: To be completed. $\square$

Proposition 5. Let A be an affinoid $\mathbf{k}$-algebra. Then there exists a constant $C > 0$ and $N > 0$ such that for all $f \in A$ and $n \geq N$, we have

$$\|f^n\| \leq C\rho(f)^n.$$

In particular, $\mathrm{Qnil}(A) = \mathrm{nil}(A)$.
Furthermore, if A is reduced, we have

$$\|f\| \leq C\rho(f)$$

for all $f \in A$.

Proof. Yang: To be completed. $\square$

Proposition 6. Let A be an affinoid $\mathbf{k}$-algebra. If and only if $\rho(f) \in \sqrt{|\mathbf{k}|} \cup \{0\}$ for all $f \in A$, then A is strict. Yang: To be complete.

Proof. Yang: To be completed. $\square$

Definition 7. Let $\mathbf{k}$ be a non-archimedean field. We define the *ring of restricted Laurent series* over $\mathbf{k}$ as

$$\mathbf{K}_r = \mathbf{L}_{\mathbf{k},r} = \left\{ \sum_{n \in \mathbb{Z}} a_n T^n : a_n \in \mathbf{k}, \lim_{|n| \to \infty} |a_n| r^n = 0 \right\}$$

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equipped with the norm
$$\|f\| = \sup_{n\in\mathbb{Z}} |a_n| r^n.$$

Yang: Is $\mathbf{K}_r$ always a field? Yang: Do we have $\mathbf{L}_{\mathbf{k},r} = \mathrm{Frac}(\mathbf{k}\{T/r\})$?

Proposition 8. Let $\mathbf{k}$ be a non-archimedean field. If $r \notin \sqrt{|\mathbf{k}^\times|}$, then $\mathbf{K}_r$ is a complete non-archimedean field with non-trivial absolute value extending that of $\mathbf{k}$.

Yang: Tensor with $\mathbf{K}_r$.

Theorem 9. Let A be a strict affinoid $\mathbf{k}$-algebra. Then there exists a finite injective admissible homomorphism
$$\varphi : \mathbf{k}\{\underline{T}\} \hookrightarrow A$$
for some $d \in \mathbb{N}$ and $r_1, \dots, r_d \in \mathbb{R}_{>0}$. Yang: To be checked.

2 Finite modules over affinoid algebras

There are three different categories of finite modules over an affinoid algebra A:

- The category $\mathbf{Banmod}_A$ of finite banach A-modules with A-linear maps as morphisms.
- The category $\mathbf{Banmod}_A^b$ of finite banach A-modules with bounded A-linear maps as morphisms.
- The category $\mathbf{mod}_A$ of finite A-modules with all A-linear maps as morphisms.

Theorem 10. Let A be an affinoid $\mathbf{k}$-algebra. Then the category of finite banach A-modules with bounded A-linear maps as morphisms is equivalent to the category of finite A-modules with A-linear maps as morphisms. Yang: To be revised.

For simplicity, we will just write mod_A to denote the category of finite banach A-modules with bounded A-linear maps as morphisms.

Appendix