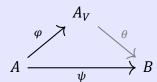
Affinoid domains

Definition 1. Let A be a **k**-affinoid algebra, and let $X = \mathcal{M}(A)$ be the associated affinoid space. A closed subset $V \subseteq X$ is called an *affinoid domain* if there exists a **k**-affinoid algebra A_V and a morphism of **k**-affinoid algebras $\varphi: A \to A_V$ satisfying the following universal property: for every bounded homomorphism of **k**-affinoid algebras $\psi: A \to B$ such that the induced map on spectra $\mathcal{M}(\psi): \mathcal{M}(B) \to X$ has its image contained in V, there exists a unique bounded homomorphism $\theta: A_V \to B$ such that the following diagram commutes:



In this case, we say that V is represented by the affinoid algebra A_V .

Slogan A closed subset $V \subset X$ is an affinoid domain if the functor "Mor(-,V)" is representable.

Yang: Why we consider closed subset rather that open subset?

Construction 2. Let $f = (f_1, ..., f_n), g = (g_1, ..., g_m)$ be two tuples of elements in A. Set $p = (p_1, ..., p_n)$ and $q = (q_1, ..., q_m)$ be two tuples of positive real numbers. We define the following closed subsets of X:

$$X\left(\underline{f/p},\underline{q/g}\right):=\left\{x\in X:\,|f_i(x)|\leq p_i,|g_j(x)|\geq q_j,1\leq i\leq n,1\leq j\leq m\right\}.$$

Such a closed subset is called a Weierstrass domain of X. Moreover, we can define a \mathbf{k} -affinoid algebra

$$A\left\langle \underline{f/p}, \underline{q/g}\right\rangle := A\left\langle \frac{f_1}{p_1}, \dots, \frac{f_n}{p_n}, \frac{q_1}{g_1}, \dots, \frac{q_m}{g_m}\right\rangle,$$

which is the quotient of the Tate algebra

$$A\langle T_1,\ldots,T_n,S_1,\ldots,S_m\rangle$$

by the ideal generated by the elements $p_i T_i - f_i$ for $1 \le i \le n$ and $g_j S_j - q_j$ for $1 \le j \le m$. There is a natural bounded homomorphism $\varphi: A \to A\langle \underline{f/p}, \underline{q/g} \rangle$ induced by the inclusion. It can be shown that the closed subset $X(\underline{f/p}, \underline{q/g})$ is an affinoid domain represented by the affinoid algebra $A\langle \underline{f/p}, \underline{q/g} \rangle$. Yang: To be checked

Construction 3. Let $f = (f_1, ..., f_n), g$ be elements in A such that the ideal generated by them is the whole algebra A. Set $p = (p_1, ..., p_n)$ be a tuple of positive real numbers. We define the following closed subset of X:

$$X(f/p,g) := \{x \in X : |f_i(x)| \le p_i|g(x)|, 1 \le i \le n\}.$$

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$$A\left\langle \underline{f/p}, g^{-1}\right\rangle := A\left\langle \frac{f_1}{p_1 g}, \dots, \frac{f_n}{p_n g}\right\rangle,$$

which is the quotient of the Tate algebra

$$A\langle T_1,\ldots,T_n\rangle$$

by the ideal generated by the elements $p_igT_i - f_i$ for $1 \le i \le n$. There is a natural bounded homomorphism $\varphi: A \to A\langle \underline{f/p}, g^{-1}\rangle$ induced by the inclusion. It can be shown that the closed subset $X(\underline{f/p}, g)$ is an affinoid domain represented by the affinoid algebra $A\langle \underline{f/p}, g^{-1}\rangle$. Yang: To be checked

Proposition 4. Let A be a **k**-affinoid algebra, and let $X = \mathcal{M}(A)$ be the associated affinoid space. Let $V \subseteq X$ be an affinoid domain represented by the **k**-affinoid algebra A_V . Then the natural bounded homomorphism $\varphi: A \to A_V$ is flat.

We have $\mathcal{M}(A_V) \cong V$.

Appendix

