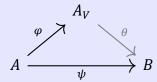
Affinoid domains

Definition 1. Let A be a **k**-affinoid algebra, and let $X = \mathcal{M}(A)$ be the associated affinoid space. A closed subset $V \subseteq X$ is called an *affinoid domain* if there exists a **k**-affinoid algebra A_V and a morphism of **k**-affinoid algebras $\varphi: A \to A_V$ satisfying the following universal property: for every bounded homomorphism of **k**-affinoid algebras $\psi: A \to B$ such that the induced map on spectra $\mathcal{M}(\psi): \mathcal{M}(B) \to X$ has its image contained in V, there exists a unique bounded homomorphism $\theta: A_V \to B$ such that the following diagram commutes:



In this case, we say that V is represented by the affinoid algebra A_V .

Slogan A closed subset $V \subset X$ is an affinoid domain if the functor "Mor(-,V)" is representable.

Yang: Why we consider closed subset rather that open subset?

Construction 2. Let $f = (f_1, ..., f_n)$ be a tuple of elements in A and $r = (r_1, ..., r_n)$ be a tuple of positive real numbers. Consider the closed subset of X:

$$X\left(\underline{f/r}\right):=\left\{x\in X:\, |f_i(x)|\leq r_i, 1\leq i\leq n\right\}.$$

Such a closed subset is called a Weierstrass domain of X. Moreover, we can define a \mathbf{k} -affinoid algebra

$$A\left\{\underline{f/r}\right\} := A\left\{f_1/r_1, \dots, f_n/r_n\right\}.$$

Yang: The domain X(f/r) is represented by $A\{f/r\}$.

Construction 3. Let $f = (f_1, ..., f_n), g = (g_1, ..., g_m)$ be two tuples of elements in A and $r = (r_1, ..., r_n), s = (s_1, ..., s_m)$ be two tuples of positive real numbers. Consider the following closed subset of X:

$$X\left(\underline{f/r}; \underline{g/s}^{-1}\right) := \left\{x \in X : |f_i(x)| \le r_i, |g_j(x)| \ge s_j, 1 \le i \le n, 1 \le j \le m\right\}.$$

Such a closed subset is called a Laurent domain of X. Moreover, we can define a k-affinoid algebra

$$A\left\{\underline{f/r};\underline{g/s}^{-1}\right\} := A\left\{f_1/r_1,\ldots,f_n/r_n,g_1^{-1}/s_1,\ldots,g_m^{-1}/s_m\right\}.$$

Yang: The domain $X(\underline{f/r}; \underline{g/s}^{-1})$ is represented by $A\{\underline{f/r}; \underline{g/s}^{-1}\}$.

Construction 4. Let $f = (f_1, ..., f_n), g$ be elements in A such that the ideal generated by them is the whole algebra A. Set $p = (p_1, ..., p_n)$ be a tuple of positive real numbers. We define the following

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closed subset of X:

$$X(f/p,g) := \{x \in X : |f_i(x)| \le p_i|g(x)|, 1 \le i \le n\}.$$

Such a closed subset is called a rational domain of X. Moreover, we can define a k-affinoid algebra

$$A\left\langle \underline{f/p}, g^{-1}\right\rangle := A\left\langle \frac{f_1}{p_1 g}, \dots, \frac{f_n}{p_n g}\right\rangle,$$

which is the quotient of the Tate algebra

$$A\langle T_1,\ldots,T_n\rangle$$

by the ideal generated by the elements $p_igT_i - f_i$ for $1 \le i \le n$. There is a natural bounded homomorphism $\varphi: A \to A\langle f/p, g^{-1}\rangle$ induced by the inclusion. It can be shown that the closed subset X(f/p,g) is an affinoid domain represented by the affinoid algebra $A\langle f/p,g^{-1}\rangle$. Yang: To be checked

Yang: We have a sequence of inclusion:

 $\{\text{Weierstrass domains}\}\subseteq \{\text{Laurent domains}\}\subseteq \{\text{Rational domains}\}\subseteq \{\text{Affinoid domains}\}.$

Proposition 5. Let A be a **k**-affinoid algebra, and let $X = \mathcal{M}(A)$ be the associated affinoid space. Let $V \subseteq X$ be an affinoid domain represented by the **k**-affinoid algebra A_V . Then the natural bounded homomorphism $\varphi: A \to A_V$ is flat.

We have $\mathcal{M}(A_V) \cong V$.

Appendix

