Forms and Currents

1 Differential forms

Let M be a complex manifold.

Denote by $\Omega^k_{\mathrm{sm}}(M)$ the space of smooth differential k-forms on M and by $\Omega^{p,q}_{\mathrm{sm}}(M)$ the space of smooth (p,q)-forms on M. Then $\Omega^k_{\mathrm{sm}}(M) = \bigoplus_{p+q=k} \Omega^{p,q}_{\mathrm{sm}}(M)$. Denote by $\Omega^k_{\mathrm{hol}}(M)$ the space of holomorphic differential k-forms on M. Then we have $\Omega^{k,0}_{\mathrm{sm}}(M) = \Omega^k_{\mathrm{hol}}(M) \otimes_{\mathcal{O}^{\mathrm{hol}}_M} \mathcal{O}^{\mathrm{sm}}_M$.

Recall that we have the exterior derivative

$$\mathrm{d} : \Omega^k_{\mathrm{sm}}(M) \to \Omega^{k+1}_{\mathrm{sm}}(M),$$

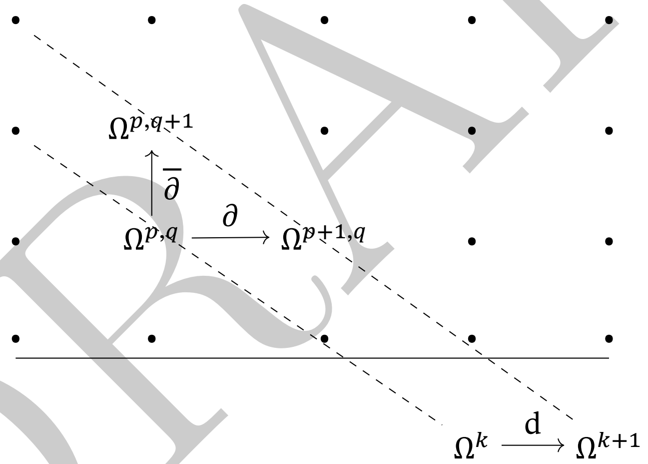
which can be decomposed as

$$\mathrm{d} = \partial + \overline{\partial},$$

where

$$\partial : \Omega^{p,q}_{\mathrm{sm}}(M) \to \Omega^{p+1,q}_{\mathrm{sm}}(M), \quad \overline{\partial} : \Omega^{p,q}_{\mathrm{sm}}(M) \to \Omega^{p,q+1}_{\mathrm{sm}}(M).$$

In a diagram, we have:



2 Currents

Appendix