Kähler manifolds

1 Hermitian metric on complex manifolds

Definition 1. Let X be a complex manifold and $E \to X$ a holomorphic vector bundle. A *Hermitian metric* on E is a smoothly varying family of Hermitian inner products $\langle \cdot, \cdot \rangle_x$ on the fibers E_x for each $x \in X$, i.e., Yang: To be continued.

Example 2. Let $\mathbb{P}^n$ be the complex projective space. The *Fubini-Study metric* is a Hermitian metric on the tautological line bundle $\mathcal{O}_{\mathbb{P}^n}(-1)$ defined as follows: For a point $[z_0 : z_1 : \cdots : z_n] \in \mathbb{P}^n$ and a vector $v \in \mathcal{O}_{\mathbb{P}^n}(-1)_{[z]}$, we define

$$\langle v, v \rangle_{[z]} = \frac{|v|^2}{\sum_{i=0}^n |z_i|^2}.$$

Here, $|v|$ is the standard Hermitian norm on $\mathbb{C}^{n+1}$. Yang: To be continued.

2 the Kähler condition

Definition 3. A *Kähler manifold* is a complex manifold X equipped with a Hermitian metric h whose associated (1,1)-form ω is closed, i.e., $d\omega = 0$. Yang: To be checked.

Appendix