Holomorphic bundles

Definition 1. Let $E \xrightarrow{\pi} X$ be a complex vector bundle over a complex manifold X. We say E is a *holomorphic vector bundle* if there exists an open cover $\{U_\alpha\}$ of X and holomorphic trivializations

$$\phi_\alpha : \pi^{-1}(U_\alpha) \xrightarrow{\sim} U_\alpha \times \mathbb{C}^n$$

such that the transition maps

$$\phi_\beta \circ \phi_\alpha^{-1} : U_\alpha \cap U_\beta \times \mathbb{C}^n \to U_\alpha \cap U_\beta \times \mathbb{C}^n$$

are holomorphic for all α, β. Yang: To be checked.

Example 2. The holomorphic tangent bundle $T^{1,0}X$ of a complex manifold X is a holomorphic vector bundle.

Appendix

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