Curves

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Contents

1 The First Properties of Curves

Let $\Bbbk$ be an algebraically closed field. Unless otherwise specified, everything is defined over $\Bbbk$. A *curve* is a one-dimensional variety.

1.1 Riemann-Roch Theorem for Curves

Theorem 1.1 (Riemann-Roch Theorem for Curves)**.** Let C be a smooth proper curve of genus g over $\Bbbk$. Then for any divisor D on C, we have

$$h^0(D) - h^1(D) = \deg D + 1 - g.$$

That is, the number $\deg D + \chi(\mathcal{O}_C(D))$ is independent of D.

Proof. Yang: To be filled. □

1.2 Classification of Curves

1.3 Hurwitz's Formula

Theorem 1.2 (Hurwitz's Formula)**.** Yang: To be filled.

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1.4 Positivity on Curves

Theorem 1.3. Let C be a smooth proper curve of genus g over $\Bbbk$ and D a divisor on C.

(a) If $\deg D \geq 2g$, then D is base point free.

(b) If $\deg D \geq 2g + 1$, then D is very ample.

Proof. Yang: To be filled. □

2 Elliptic Curve

Let $\Bbbk$ be an algebraically closed field. Unless otherwise specified, everything is defined over $\Bbbk$.

2.1 Elliptic curves are cubic curves

2.2 Group structure on elliptic curves

2.3 As Riemannian surfaces

3 Curves of Higher Genus

3.1 Hyperelliptic Curves