Setup and the first examples

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Let S be a locally noetherian and separated scheme. An S-variety is a separated scheme X which is of finite type and flat over S.

We will use k, K to denote fields, and $\mathbf{k}, \mathbf{K}$ to denote their algebraically closure relatively. Let X be an integral scheme. We denote by $\mathsf{K}(X)$ the function field of X. For a closed point $x \in X$, we denote by $\mathsf{k}(x)$ the residue field of x. We denote the category of S-varieties by $\mathbf{Var}_S$. We denote by $X(T)$ the set of T-points of X, that is, the set of morphisms $T \to X$.

Example 1. Let $\mathbf{k}$ be an algebraically closed field and A the localization of $\mathbf{k}[x]$ at (x). Let $S = \operatorname{Spec} A$ and $X = \operatorname{Spec} A[y]$. There are three types of points in X:

(i) closed points with residue field $\mathbf{k}$, like $p = (x, y - a)$;

(ii) closed points with residue field $\mathbf{k}(y)$, like $P = (xy - 1)$;

(iii) non-closed points, like $\eta_1 = (x), \eta_2 = (y), \eta_3 = (x - y)$.

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