

Setup and the first examples



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Let S be $\text{Spec } \mathbf{k}$, $\text{Spec } \mathcal{O}_K$ or an algebraic variety. An S -variety is a separated and integral scheme X which is of finite type and flat over S .

We will use \mathbf{k}, \mathbf{K} to denote fields, and \mathbf{k}, \mathbf{K} to denote their algebraically closure relatively.

Let X be an integral scheme. We denote by $\mathcal{H}(X)$ the function field of X . For a closed point $x \in X$, we denote by $\kappa(x)$ the residue field of x .

We denote the category of S -varieties by \mathbf{Var}_S . We denote by $X(T)$ the set of T -points of X , that is, the set of morphisms $T \rightarrow X$.

Let X be an algebraic variety over \mathbf{k} . A geometrical point is refer a morphism $\text{Spec } \mathbf{k} \rightarrow X$.

When refer a point (may not be closed) in a scheme, we will use the notation $\xi \in X$. When we talk about a closed point on an algebraic variety, we will use the notation $x \in X(\mathbf{k})$.

Example 1. Let \mathbf{k} be an algebraically closed field and A the localization of $\mathbf{k}[x]$ at (x) . Let $S = \text{Spec } A$ and $X = \text{Spec } A[y]$. There are three types of points in X :

- (i) closed points with residue field \mathbf{k} , like $p = (x, y - a)$;
- (ii) closed points with residue field $\mathbf{k}(y)$, like $P = (xy - 1)$;
- (iii) non-closed points, like $\eta_1 = (x), \eta_2 = (y), \eta_3 = (x - y)$.