Minimal Model Program

Let $\Bbbk$ be an algebraically closed field. Unless otherwise specified, all varieties are assumed to be defined over $\Bbbk$.

1 Building block of varieties

Definition 1. A projective variety X is called *Fano* if its anticanonical divisor $-K_X$ is ample.

Definition 2. A projective variety X is called *Calabi-Yau* if its canonical divisor K_X is numerically trivial, i.e., $K_X \equiv 0$.

Definition 3. A projective variety X is called *of general type* if its canonical divisor K_X is big.

2 Pseudo-effectiveness of canonical divisor

Definition 4. A projective variety X is called *uniruled* if there exists a dominant rational map $\mathbb{P}^1 \times Y \dashrightarrow X$ for some variety Y with $\dim Y = \dim X - 1$.

Theorem 5 (ref. [BDPP12, Corollary 0.3])**.** Let X be a smooth projective variety over an algebraically closed field $\Bbbk$ of characteristic zero. Then the canonical divisor K_X is not pseudo-effective if and only if X is uniruled.

References

[BDPP12] Sébastien Boucksom et al. “The pseudo-effective cone of a compact Kähler manifold and varieties of negative Kodaira dimension”. In: *Journal of Algebraic Geometry* 22.2 (2012), pp. 201–248 (cit. on p. 1).

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