Birational geometry on surfaces

Let $\Bbbk$ be an algebraically closed field of arbitrary characteristic. Unless otherwise specified, all varieties are defined over $\Bbbk$.

1 Castelnuovo's Theorem and Run the MMP

Theorem 1 (Castelnuovo's contractibility criterion)**.** Let X be a smooth projective surface over an algebraically closed field $\Bbbk$. Let $C \subseteq X$ be an irreducible curve. Then there exists a birational morphism $f : X \to Y$ contracting C to a smooth point if and only if $C \cong \mathbb{P}^1$ and $C^2 = -1$.

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