Setup and the first examples

献出心脏吧！

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All schemes are assumed to be separated. For a "scheme" which is not separated, we will use the term "prescheme".

Let S be $\operatorname{Spec} \mathsf{k}$, $\operatorname{Spec} \mathcal{O}_K$ or an algebraic variety. An S-variety is an integral scheme X which is of finite type and flat over S. For an algebraic variety, we mean a k-variety.

We will use k, K to denote fields, and $\mathbf{k}, \mathbf{K}$ to denote their algebraically closure relatively.

Let X be an integral scheme. We denote by $\mathscr{K}(X)$ the function field of X. For a closed point $x \in X$, we denote by $\kappa(x)$ the residue field of x.

We denote the category of S-varieties by $\mathbf{Var}_S$. We denote by $X(T)$ the set of T-points of X, that is, the set of morphisms $T \to X$.

Let X be an algebraic variety over k. A geometrical point is referred a morphism $\operatorname{Spec} \mathbf{k} \to X$.

When refer a point (may not be closed) in a scheme, we will use the notation $\xi \in X$. We When we talk about a closed point on an algebraic variety, we will use the notation $x \in X(\mathbf{k})$.

Example 1. Let $\mathbf{k}$ be an algebraically closed field and A the localization of $\mathbf{k}[x]$ at (x). Let $S = \operatorname{Spec} A$ and $X = \operatorname{Spec} A[y]$. There are three types of points in X:

(i) closed points with residue field $\mathbf{k}$, like $p = (x, y - a)$;

(ii) closed points with residue field $\mathbf{k}(y)$, like $P = (xy - 1)$;

(iii) non-closed points, like $\eta_1 = (x), \eta_2 = (y), \eta_3 = (x - y)$.

1 Preparation in commutative algebra

Definition 2 (Associated prime ideals)**.** Let A be a noetherian ring and M a finitely generated A-module. The *associated prime ideals* of M are the prime ideals $\mathfrak{p}$ of form $\operatorname{Ann}(x)$ for some $x \in M$. The set of associated prime ideals of M is denoted by $\operatorname{Ass}(M)$.

Definition 3. Let A be a noetherian ring and M a finitely generated A-module. The *support* of M is the set of prime ideals $\mathfrak{p}$ of A such that $M_{\mathfrak{p}} \neq 0$.

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