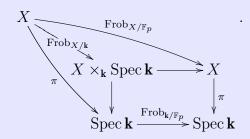
Bend and Break

1 Preliminary

Definition 1 (Frobinius morphism). Let X be a variety over a field \mathbf{k} of characteristic p > 0. Denote the structure morphism by $\pi : X \to \operatorname{Spec} \mathbf{k}$. The absolute Frobenius morphism is the morphism given by $\mathcal{O}_X \to \mathcal{O}_X$, $f \mapsto f^p$, denoted by $\operatorname{Frob}_{X/\mathbb{F}_p}$. The relative Frobenius morphism is the morphism $\operatorname{Frob}_{X/\mathbf{k}}$ given by the following commutative diagram:



We usually denote $X \times_{\mathbf{k}} \operatorname{Spec} \mathbf{k}$ appearing above by $X^{(p)}$.

Proposition 2. Let X be a variety of dimension d over a field \mathbf{k} of characteristic p > 0. Then the relative Frobenius morphism $\operatorname{Frob}_{X/\mathbf{k}} : X \to X^{(p)}$ is a finite morphism of degree p^d over \mathbf{k} .

2 Deformation of curves

Theorem 3 (ref. [Kol96, Chapter II, Theorem 1.2]). Let C be a projective smooth curve of genus g and X a smooth projective variety of dimension n. Let $f: C \to X$ be a non-constant morphism. Then every irreducible component of Mor(C, X) containing f has dimension at least

$$-K_Y \cdot f(C) + (1-g)n.$$

3 Find rational curves

Theorem 4. Let X be a smooth Fano variety. Then for any $x \in X(\mathbf{k})$, there is a rational curve C passing through x with

$$0 < -C \cdot K_X \le \dim X + 1.$$

Theorem 5. Let X be a smooth projective variety such that $K_X \cdot C < 0$ for some irreducible curve $C \subset X$. Let H be an ample divisor on X. Then there exists a rational curve Γ such that

$$-(K_X \cdot C) \cdot \frac{H \cdot \Gamma}{H \cdot C} \le -K_X \cdot \Gamma \le \dim X + 1.$$

Date: July 20, 2025, Author: Tianle Yang, My Website

References

[Kol96] János Kollár. Rational Curves on Algebraic Varieties. Vol. 32. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge / A Series of Modern Surveys in Mathematics. Berlin, Heidelberg: Springer-Verlag, 1996, p. 320. ISBN: 978-3-540-60168-5. DOI: 10.1007/978-3-662-03276-3. URL: https://doi.org/10.1007/978-3-662-03276-3.