Varieties in more general settings

1 Varieties

Definition 1. A *variety* over an algebraically closed field $\mathbb{k}$ is an integral separated scheme of finite type over $\operatorname{Spec}\mathbf{k}$.

Yang: Suppose that $\mathbf{k}$ is not algebraically closed, let $\mathbf{k}'$ be an algebraic extension of $\mathbf{k}$. What is the relation between X, $X_{\mathbf{k}'}$, $X(\mathbf{k}')$ and $X_{\mathbf{k}'}(\mathbf{k}')$?

2 Geometric properties

Definition 2. Let $\mathbf{k}$ be a field and X a separated scheme of finite type over $\operatorname{Spec}\mathbf{k}$. We say that X has a *geometric property* p if $X_{\mathbb{k}}$ has the property p for the algebraic closure $\mathbb{k}$ of $\mathbf{k}$.

Definition 3. A *variety* over a field $\mathbf{k}$ is a separated geometrically integral scheme of finite type over $\operatorname{Spec}\mathbf{k}$.

3 Points in varieties

Proposition 4. Let k be a field and l an extension of k. Let X be a variety over k. Then we have the following:

(a) there is a natural bijection between $X(l)$ and $X_l(l)$;

(b) let m/l be an extension, then there is a natural inclusion $X(l) \subseteq X(m)$;

(c) suppose that $X = \operatorname{Spec} k[T_1, \dots, T_n]/I$ is an affine variety, then there is a natural bijection between $X(l)$ and the set $\{(x_1, \dots, x_n) \in l^n | f(x_1, \dots, x_n) = 0, \forall f \in I\}$.

Date: September 14, 2025, **Author**: Tianle Yang, My Homepage