Normal, Cohen-Macaulay, and regular schemes

1 Tangent spaces

There are many description of the tangent space of a scheme at a point. Here we give one of them.

Let X be a scheme over a field $\mathbf{k}$, and let $x \in X(\mathbf{k})$.

Proposition 1. Let $\operatorname{Spec} \mathbf{k}[\epsilon]/(\epsilon^2)$ be the spectrum of the ring of dual numbers over $\mathbf{k}$ with point $* : \operatorname{Spec} \mathbf{k} \to \operatorname{Spec} \mathbf{k}[\epsilon]/(\epsilon^2)$. The tangent space $T_x X$ is naturally isomorphic to the set of morphisms $\operatorname{Spec} \mathbf{k}[\epsilon]/(\epsilon^2) \to X$ that send $*$ to x, i.e.

$$T_x X \cong \{f : \operatorname{Spec} \mathbf{k}[\epsilon]/(\epsilon^2) \to X \mid f(*) = x\}.$$

Proof. Yang: To be filled. □