

# Category of sheaves of modules

## 1 Sheaves of modules, quasi-coherent and coherent sheaves

**Definition 1.** Let  $(X, \mathcal{O}_X)$  be a ringed space. An  $\mathcal{O}_X$ -module  $\mathcal{F}$  is called *quasi-coherent* if for every point  $x \in X$ , there exists an open neighborhood  $U$  of  $x$  such that  $\mathcal{F}|_U$  is isomorphic to the cokernel of a morphism of free  $\mathcal{O}_U$ -modules, i.e., there exists an exact sequence of sheaves of  $\mathcal{O}_U$ -modules

$$\mathcal{O}_U^{(I)} \rightarrow \mathcal{O}_U^{(J)} \rightarrow \mathcal{F}|_U \rightarrow 0,$$

where  $I, J$  are (possibly infinite) index sets.

**Definition 2.** Let  $(X, \mathcal{O}_X)$  be a ringed space. An  $\mathcal{O}_X$ -module  $\mathcal{F}$  is called *finitely generated* if for every point  $x \in X$ , there exists an open neighborhood  $U$  of  $x$  such that there exists a surjective morphism of sheaves of  $\mathcal{O}_U$ -modules

$$\mathcal{O}_U^n \rightarrow \mathcal{F}|_U \rightarrow 0.$$

**Definition 3.** Let  $(X, \mathcal{O}_X)$  be a ringed space. An  $\mathcal{O}_X$ -module  $\mathcal{F}$  is called *coherent* if it is finitely generated, and for every open set  $U \subseteq X$  and every morphism of sheaves of  $\mathcal{O}_U$ -modules  $\varphi : \mathcal{O}_U^n \rightarrow \mathcal{F}|_U$ , the kernel of  $\varphi$  is finitely generated.

## 2 As abelian categories

## 3 Relevant functors

## 4 Locally free sheaves and vector bundles

## 5 Cohomological theory