The First Properties of Curves

Let $\Bbbk$ be an algebraically closed field. Unless otherwise specified, everything is defined over $\Bbbk$. A *curve* is a one-dimensional variety.

1 Riemann-Roch Theorem for Curves

Theorem 1 (Riemann-Roch Theorem for Curves)**.** Let C be a smooth proper curve of genus g over $\Bbbk$. Then for any divisor D on C, we have

$$h^0(D) - h^1(D) = \deg D + 1 - g.$$

That is, the number $\deg D + \chi(\mathcal{O}_C(D))$ is independent of D.

Proof. Yang: To be filled. ☐

2 Classification of Curves

3 Hurwitz's Formula

Theorem 2 (Hurwitz's Formula)**.** Yang: To be filled.

4 Positivity on Curves

Theorem 3. Let C be a smooth proper curve of genus g over $\Bbbk$ and D a divisor on C.

(a) If $\deg D \geq 2g$, then D is base point free.

(b) If $\deg D \geq 2g + 1$, then D is very ample.

Proof. Yang: To be filled. ☐

Appendix