Iitaka fibration

Theorem 1 (Iitaka fibration, ref. [Laz04, Theorem2.1.33]). Let X be a normal projective variety, and L a line bundle on X such that $\kappa(X, L) > 0$. Then for all sufficiently large $k \in N(X, L)$, the rational mappings $\phi_k : X \to Y_k$ are birationally equivalent to a fixed algebraic fibre space

$$\phi_\infty : X_\infty \to Y_\infty$$

of normal varieties, and the restriction of L to a very general fibre of ϕ_∞ has Iitaka dimension $= 0$. More specifically, there exists for large $k \in N(X, L)$ a commutative diagram

$$\begin{tikzcd} X \arrow[r, dashed, "u_\infty"] \arrow[d, "\phi_k"'] & X_\infty \arrow[d, "\phi_\infty"] \\ Y_k \arrow[r, dashed, "\nu_k"'] & Y_\infty \end{tikzcd}$$

of rational maps and morphisms, where the horizontal maps are birational and u_∞ is a morphism. One has $\dim Y_\infty = \kappa(X, L)$. Moreover, if we set $L_\infty = u_\infty^* L$, and take $F \subseteq X_\infty$ to be a very general fibre of ϕ_∞, then

$$\kappa(F, L_\infty|F) = 0.$$

More precisely, the assertion is that the last displayed formula holds for the fibres of ϕ_∞ over all points in the complement of the union of countably many proper subvarieties of Y_∞.

Appendix

References

[Laz04] Robert Lazarsfeld. *Positivity in algebraic geometry. I*. Vol. 48. Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics]. Classical setting: line bundles and linear series. Springer-Verlag, Berlin, 2004, pp. xviii+387. ISBN: 3-540-22533-1. DOI: 10.1007/978-3-642-18808-4. URL: https://doi.org/10.1007/978-3-642-18808-4 (cit. on p. 1).