Algebraic stacks

1 Deligne-Mumford stacks

Definition 1. A *Deligne-Mumford stack* is an algebraic stack X such that there exists a scheme U and a representable étale surjective morphism $U \to X$.

2 Algebraic Stacks

Definition 2. An *algebraic stack* is an algebraic stack X such that there exists a scheme U and a representable smooth surjective morphism $U \to X$.

Example 3. Let $\Bbbk$ be a field. Consider the projective plane $\mathbb{P}^2_{\Bbbk}$ over $\Bbbk$ and all cubic curve $C \subseteq \mathbb{P}^2_{\Bbbk}$. Its moduli stack $\mathcal{M}$ of cubic curves is an algebraic stack. Yang: To be revised.

Appendix

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