Linear Systems

1 Sections, linear systems and morphisms to projective space

Theorem 1. Let A be a ring and X an A-scheme. Let $\mathcal{L}$ be a line bundle on X and $s_0, \dots, s_n \in \Gamma(X, \mathcal{L})$. Suppose that $\{s_i\}$ generate $\mathcal{L}$, i.e., $\bigoplus_i \mathcal{O}_X s_i \to \mathcal{L}$ is surjective. Then there is a unique morphism $f : X \to \mathbb{P}^n_A$ such that $\mathcal{L} \cong f^*\mathcal{O}(1)$ and $s_i = f^* x_i$, where x_i are the standard coordinates on $\mathbb{P}^n_A$.

Proof. Yang: To be continued. □

2 Asymptotic behavior of linear systems

3 Iitaka fibration

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