The Quot functor

1 Grassmannian

Definition 1. Let S be a noetherian scheme and $\mathcal{E}$ a vector bundle of rank n on S. The *Grassmannian functor* $\mathfrak{Grass}_{\mathcal{E},r} : \mathbf{Sch}_S^{\mathrm{op}} \to \mathbf{Set}$ is defined as

$$T \mapsto \{\mathcal{E}_T \to \mathcal{Q} \to 0 \mid \mathcal{Q} \text{ locally free of rank } r \text{ on } T\}/\sim,$$

where two quotients $\mathcal{E}_T \to \mathcal{Q} \to 0$ and $\mathcal{E}_T \to \mathcal{Q}' \to 0$ are equivalent if there is an isomorphism $\mathcal{Q} \cong \mathcal{Q}'$ making the obvious diagram commute. Yang: To be revised.

2 Hilbert functor and scheme

Definition 2. Let S be a noetherian scheme and X a projective scheme over S. Fix a relatively very ample line bundle $\mathcal{O}_X(1)$ on X over S. For a polynomial $P \in \mathbb{Q}[t]$, the *Hilbert functor* $\mathfrak{Hilb}_{X/S,P} : \mathbf{Sch}_S^{\mathrm{op}} \to \mathbf{Set}$ is defined as

$$T \mapsto \left\{Y \subseteq X_T \;\middle|\; Y \to T \text{ is flat and for all } t \in T, P_{\mathcal{O}_{Y_t}} = P\right\},$$

where Y_t is the fiber of Y over the point t and $P_{\mathcal{O}_{Y_t}}$ is the Hilbert polynomial of $\mathcal{O}_{Y_t}$ with respect to $\mathcal{O}_X(1)$. Yang: To be revised.

3 Quot functor and scheme

Definition 3. Let S be a noetherian scheme, X a projective scheme over S, and $\mathcal{E}$ a vector bundle on X. Fix a relatively very ample line bundle $\mathcal{O}_X(1)$ on X over S. The *Quot functor* $\mathfrak{Quot}_{\mathcal{E}/X/S} : \mathbf{Sch}_S^{\mathrm{op}} \to \mathbf{Set}$ is defined as

$$T \mapsto \{\mathcal{E}_T \to \mathcal{Q} \to 0 \mid \mathcal{Q} \text{ locally free of rank } r \text{ on } T\}/\sim,$$

where two quotients $\mathcal{E}_T \to \mathcal{Q} \to 0$ and $\mathcal{E}_T \to \mathcal{Q}' \to 0$ are equivalent if there is an isomorphism $\mathcal{Q} \cong \mathcal{Q}'$ making the obvious diagram commute. Yang: To be revised.

Appendix

Date: November 6, 2025, **Author**: Tianle Yang, My Homepage