Modular Form

阿巴阿巴阿巴阿巴阿巴阿巴!

Definition and Examples

In this section and all the books, we denote by $\mathcal{H}$ the upper half plane, and let $\mathrm{SL}_2(\mathbb{Z})$ act on $\mathcal{H}$ in the natural way.

Definition 1. Let $\Gamma < \mathrm{SL}_2(\mathbb{Z})$ be a subgroup of finite index. A *modular form* of *weight* k and *level* Γ is a holomorphic function $f : \mathcal{H} \to \mathbb{C}$ such that:

(a) (Automorphy) For all $\gamma = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \Gamma$, we have $f(\gamma(z)) = (cz+d)^k f(z)$.

(b) (Holomorphic at the cusps) For every $\gamma \in \Gamma$, the function $(cz+d)^{-k} f(\gamma(z))$ is bounded as $\mathrm{Im}(z) \to +\infty$.

Date: July 10, 2025, **Author**: Tianle Yang, My Website