Slide 1: Title (25s)

Thank you very much for the introduction and the invitation from the organizers. It is my pleasure to give a talk here.

Today I will talk about some recent work in *algebraic dynamics*.

Slide 2: Kawaguchi–Silverman Conjecture (1min10s)

Let X be a smooth projective variety defined over $\mathbb{Q}$, and let $f : X \to X$ be a surjective endomorphism. Fix an ample line bundle H on X and a height function h associated to H.

Kawaguchi and Silverman conjectured that if a point $x \in X$ has a Zariski dense orbit, then the dynamical degree δ_f coincides with the arithmetic degree $\alpha_f(x)$.

The first invariant is the *dynamical degree* δ_f, which is the exponential growth rate of intersection numbers.

The second one is the *arithmetic degree* $\alpha_f(x)$, which is the exponential growth rate of heights along the orbit of x.

The Kawaguchi–Silverman Conjecture can be viewed as an example of the slogan:

Geometry controls arithmetic.

Slide 3: An Example (2min)

Let me illustrate this conjecture with a concrete example.

On the slide, I have drawn the first few points in the orbit of a point.

Since $f^*H \sim 2H$, one can compute that the dynamical degree is equal to 2.

I also list the heights of the points in the orbit.

The ratio of successive heights is approximately 2, so the arithmetic degree is also expected to be 2, in agreement with the conjecture.

Slide 4: Known Cases (2min25s)

The Kawaguchi–Silverman Conjecture is still open in general, but it has been verified in many important cases.

For example, it is known in lower dimensions, for certain special classes of varieties, and for certain special classes of endomorphisms.

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Slide 5: Main Results (3min25s)

Now let me explain where our work fits into this picture.

We study a special Fano fibration case. This setting comes from the Minimal Model Program and can be viewed as one of its final outputs.

Explicitly, we consider a smooth Fano fibration

$$\pi : X \to Y$$

with relative Picard number one, where the base Y is either an abelian variety, or a smooth projective variety of Picard number one.

In this setting, we show that the Kawaguchi–Silverman Conjecture holds for X.

Since some subcases are already known, we further impose certain dynamical conditions.

Under these assumptions, we construct a new equivariant fibration which preserves the dynamical degree, and whose base is polarized.

This reduces the Kawaguchi–Silverman Conjecture for X to the conjecture for the base, which is already known.

Slide 6: Sketch of Proof (4min25s)

Let me very briefly explain the idea of the proof.

We work in the special Fano fibration case, under the dynamical condition that the dynamical degree strictly decreases.

First, using this dynamical restriction, we obtain a decomposition of the pseudo-effective cone of divisors on X.

Using this decomposition together with the geometry of the base Y, we show that the ramification divisor R_f is an eigenvector of f^*.

This implies that the induced dynamics on the base are polarized.

Next, we establish a criterion for toric bundles using dynamical methods.

Applying this criterion, we show that $\kappa(X, R_f) > 0$, which implies that the fibration we construct is non-trivial.

Finally, using dynamical Iitaka theory, we combine these steps to produce the desired equivariant fibration.

Slide 7: Conclusion (5min)

To conclude, we use the dynamics to restrict geometry and get new structures.

Thank you very much for your attention.