Algebraic Dynamics and Dynamical Iitaka Theory

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base on the joint work with Sheng Meng and Long Wang

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Kawaguchi-Silverman Conjecture

Work over $\overline{\mathbb{Q}}$. X: smooth projective variety, $f : X \to X$: surjective endomorphism. H: ample divisor on X. $h : X(\overline{\mathbb{Q}}) \to \mathbb{R}_{\geq 1}$: a height function associated to H.

Conjecture: Kawaguchi-Silverman Conjecture = KSC

If the orbit $O_f(x) := \{f^n(x) \mid n \geq 0\}$ is Zariski dense in X, then

$$\alpha_f(x) = \delta_f.$$

here,

$$\alpha_f(x) := \lim_{n\to\infty} h(f^n(x))^{1/n},$$

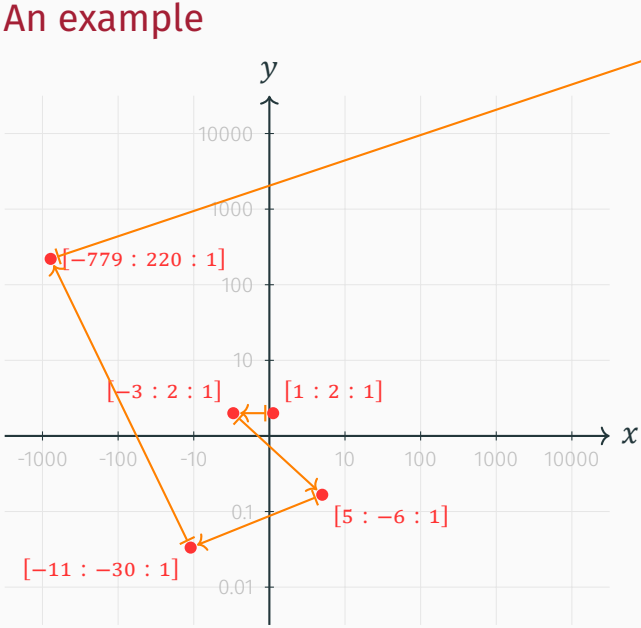
arithmetic invariant at x,

$$\delta_f := \lim_{n\to\infty} ((f^n)^* H \cdot H^{\dim X - 1})^{1/n},$$

geometric invariant of f.

Three orbit conjecture

An example



- $X = \mathbb{P}^2$,
  $f : [x : y : z] \mapsto [x^2 - y^2 : xy : z^2]$,
  $x = [1 : 2 : 1]$.
- $f^*H \sim 2H \Rightarrow \delta_f = 2$.

n	$h(f^n(x))$	
0	$\log 2$	≈ 0.7
1	$\log 3$	≈ 1.1
2	$\log 6$	≈ 1.8
3	$\log 30$	≈ 3.4
4	$\log 779$	≈ 6.7
5	$\log 558441$	≈ 13.2

- It is expected that $\alpha_f(x) = 2$.

Dynamical Iitaka Theory

Settings

Main results

Strategies and Techniques

Further problem

Thank You!