Section name

1 Fonts in math mode

We use the unicode-math package to support Unicode math symbols. Below is a list of common math symbols, grouped by font/style, with their names:

- **Greek letters:** $A, B, \Gamma, \Delta, E, Z, H, \Theta, I, K, \Lambda, M, N, \Xi, O, \Pi, P, \Sigma, T, \Upsilon, \Phi, X, \Psi, \Omega$;

- $\alpha, \beta, \gamma, \delta, \epsilon, \varepsilon, \zeta, \eta, \theta, \iota, \kappa, \lambda, \mu, \nu, \xi, o, \pi, \rho, \sigma, \tau, \upsilon, \phi, \varphi, \chi, \psi, \omega$;

- **Numbers and operators:** $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, \times, \div, =, <, >, \leq, \geq$

- **Common math symbols:** $\infty, \partial, \nabla, \forall, \exists, \neg, \wedge, \vee, \implies, \iff, \subset, \subseteq, \supset, \supseteq, \cup, \cap, \backslash, \varnothing, \hookrightarrow, \twoheadrightarrow, \dashrightarrow$, $\varprojlim, \varinjlim$;

- **Musical symbols:** $\sharp, \flat, \natural$;

- **math italic:** $A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z$;

- $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z$;

- **Calligraphic:** $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}, \mathcal{I}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \mathcal{M}, \mathcal{N}, \mathcal{O}, \mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{S}, \mathcal{T}, \mathcal{U}, \mathcal{V}, \mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z}$,

- $a, b, c, d, e, f, g, h, i, j, k, \ell, m, n, o, p, q, r, s, t, u, v, w, x, y, z$

- **Blackboard bold:** $\mathbb{A}, \mathbb{B}, \mathbb{C}, \mathbb{D}, \mathbb{E}, \mathbb{F}, \mathbb{G}, \mathbb{H}, \mathbb{I}, \mathbb{J}, \mathbb{K}, \mathbb{L}, \mathbb{M}, \mathbb{N}, \mathbb{O}, \mathbb{P}, \mathbb{Q}, \mathbb{R}, \mathbb{S}, \mathbb{T}, \mathbb{U}, \mathbb{V}, \mathbb{W}, \mathbb{X}, \mathbb{Y}, \mathbb{Z}$;

- $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z$;

- **Fraktur:** $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{E}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H}, \mathfrak{I}, \mathfrak{J}, \mathfrak{K}, \mathfrak{L}, \mathfrak{M}, \mathfrak{N}, \mathfrak{O}, \mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, \mathfrak{S}, \mathfrak{T}, \mathfrak{U}, \mathfrak{V}, \mathfrak{W}, \mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}$;

- $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z$;

- **Script:** $\mathscr{A}, \mathscr{B}, \mathscr{C}, \mathscr{D}, \mathscr{E}, \mathscr{F}, \mathscr{G}, \mathscr{H}, \mathscr{I}, \mathscr{J}, \mathscr{K}, \mathscr{L}, \mathscr{M}, \mathscr{N}, \mathscr{O}, \mathscr{P}, \mathscr{Q}, \mathscr{R}, \mathscr{S}, \mathscr{T}, \mathscr{U}, \mathscr{V}, \mathscr{W}, \mathscr{X}, \mathscr{Y}, \mathscr{Z}$;

- $a, b, c, d, e, f, g, h, i, j, k, \ell, m, n, o, p, q, r, s, t, u, v, w, x, y, z$;

- **Upright:** $\mathrm{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}$;

- $\mathrm{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}$;

- **Bold :** $\mathbf{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}$;

- $\mathbf{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}$.

- **Sans-serif:** $\mathsf{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}$;

- $\mathsf{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}$;

- **Roman:** $\mathrm{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z}$;

- $\mathrm{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}$;

2 Theorems and definitions

There are two types of theorem environments, one is with background color, the other is without background color. The following is a list of theorem environments supported by this template:

Slogan *This is a slogan environment, it is used to highlight important slogans or quotes.*

Definition 1 (this is a definition)**.** A *locally ringed space* is a pair $(X, \mathcal{O}_X)$, where X is a topological space, and $\mathcal{O}_X$ is a sheaf of rings on X, such that for every point $x \in X$, the stalk $\mathcal{O}_{X,x}$ is a local ring.

Construction 2 (this is a construction)**.** test

Proposition 3 (this is a proposition)**.** test

Proof. This is a proof environment, it is used to prove theorems, propositions, lemmas, corollaries, etc. We allow to use step environments inside the proof environment, such as:

Step 1. This is a step environment, it is used to break down the proof into smaller steps.

Step 2. This is another step environment, it is used to break down the proof into smaller steps.

And the step environment should be used inside the proof environment. The proof environment will automatically end with a square box. □

Theorem 4 (this is a theorem)**.** test

Proof. This is a proof environment. The step environment is labelled in the proof environment. A new proof environment will refresh the step environment counter.

Step 1. Goal 1.

Proof of Goal 1.

Step 2. Goal 2.

Proof of Goal 2.

Here we test the hyperlink to the step environment Step 1.

You can also use the claim environment to make a claim in the proof environment, such as:

Claim 5. This is a claim environment, it is used to make a claim in the proof environment.

And the claim environment should be used inside the proof environment. □

Proof of Claim 5. This is a proof for the Claim 5. Here we test the case environment.

Case 1. This is a case environment, it is used to break down the proof into smaller cases.

Case 2. This is another case environment, it is used to break down the proof into smaller cases.

And the case environment should be used inside the proof environment. □

Lemma 6 (this is a lemma)**.** test

Proof. Here we test case environment again.

Case 1. This is a case environment, it is used to break down the proof into smaller cases.

Case 2. This is another case environment, it is used to break down the proof into smaller cases.

□

Corollary 7 (this is a corollary). test here we test the hyperlink to the code environment Code 1.

Question 8 (this is a question). test

Conjecture 9 (this is a conjecture). test

Example 10 (this is an example). test

Exercise 11 (this is an exercise). test

Remark 12 (this is a remark). test Here we test the hyperlink to the case environment Case 1.

this is a proof. test, here we test Section 1 and Subsection 2. □

Here we test the code block environment:

```python
def hello():
    print("Hello, world!")

```

Listing 1: Python code example

```python
import numpy as np
from numpy.polynomial import Polynomial

# Example: create two integer matrices
A = np.array([[1,1,0], [4,0,1], [1,0,0]], dtype=int)
B = np.array([[2,1,0], [4,1,1],[1,0,1]], dtype=int)

# Matrix addition

# Matrix multiplication
def M(i, j):
    return np.linalg.matrix_power(A, i) @ np.linalg.matrix_power(B, j)

mul_result = M(1, 1)

def rho(A):
    eigvals = np.linalg.eigvals(A)
    return np.max(np.abs(eigvals))

# Spectral radius
results = {}
for i in range(-10, 11):
    for j in range(-10, 11):
        m = M(i, j)
        results[(i, j)] = rho(m)

# Print results
# for key in sorted(results.keys()):
#     print(f"rho(M({key[0]}, {key[1]})) = {results[key]}")

print("A =" , A)
print("B =" , B)
```

```python
33  print("A * B =" , mul_result)
34  print("B * A =" , np.matmul(B, A))
35
36  print("Eigenvalues of A:", np.linalg.eigvals(A))
37  print("Eigenvalues of B:", np.linalg.eigvals(B))
38
39  def char_poly(matrix):
40      # Get coefficients of the characteristic polynomial
41      coeffs = np.poly(matrix)
42      # Format as a string
43      terms = []
44      deg = len(coeffs) - 1
45      for i, c in enumerate(coeffs):
46          power = deg - i
47          if power == 0:
48              terms.append(f"{c:.2f}")
49          elif power == 1:
50              terms.append(f"{c:.2f}*x")
51          else:
52              terms.append(f"{c:.2f}*x^{power}")
53      return " + ".join(terms)
54
55  print("Characteristic polynomial of A:")
56  print(char_poly(A))
57  print("Characteristic polynomial of B:")
58  print(char_poly(B))
```

Listing 2: Python code example from file

3   sectionlevel=section

In this mode, the section is the highest level, and usually there is only one section in the document. There is no title page, no table of contents, and no cover image. All theorem and definition environments are labelled with a unique number, and the numbering is continuous throughout the document.