Markov Decision Process (BlackJack)

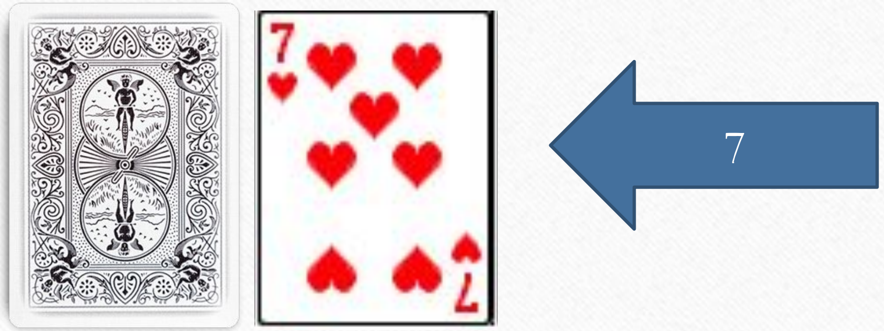
Jia Lin Hau (Monkie)

Introduction (Black Jack/ 21)

- Blackjack/21 is usually played in casino between several players and a dealer.
- Dealer decision is fixed (For example S17 rule), you know one of the dealer card.
- Player and dealer is given 2 cards to start with.
- Dealer get better odds because player lose immediately when they bust (exceed 21).

(Player card)

One of dealer card is visible for player to make decision



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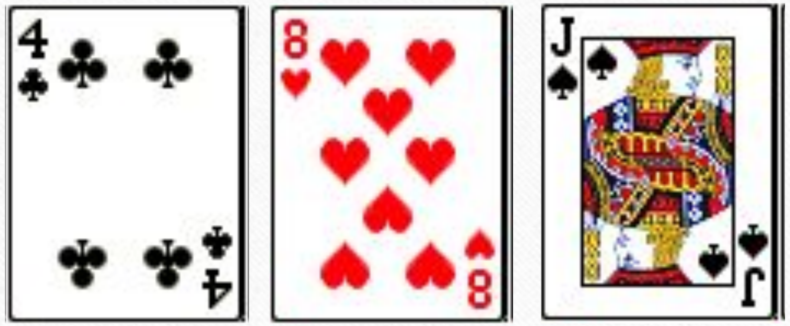

22 (bust)

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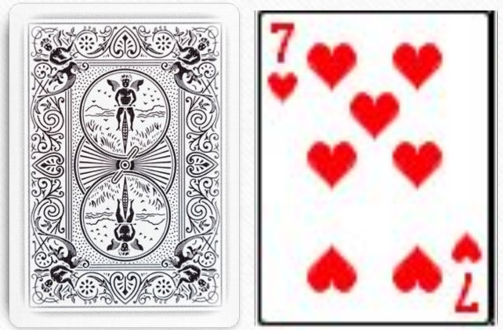

Dealer win

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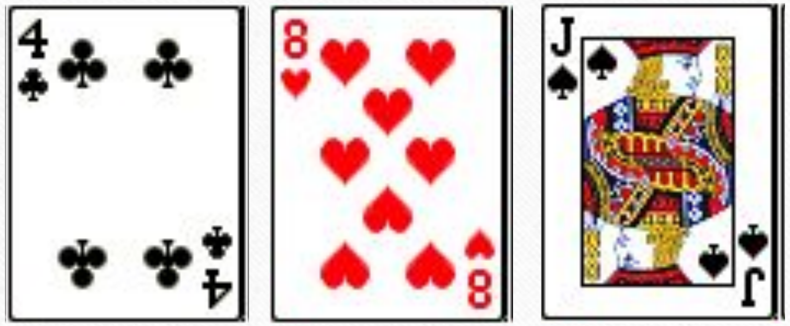
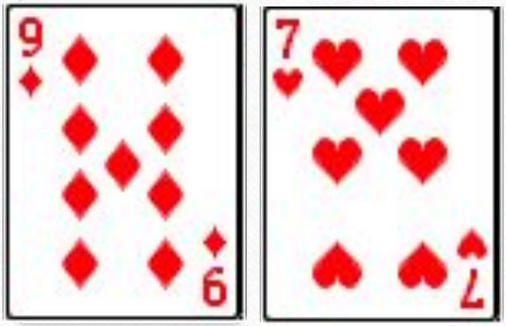


22 (bust)

Dealer 16



16

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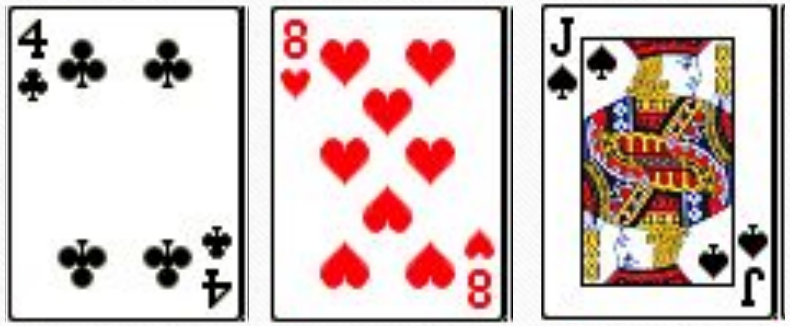
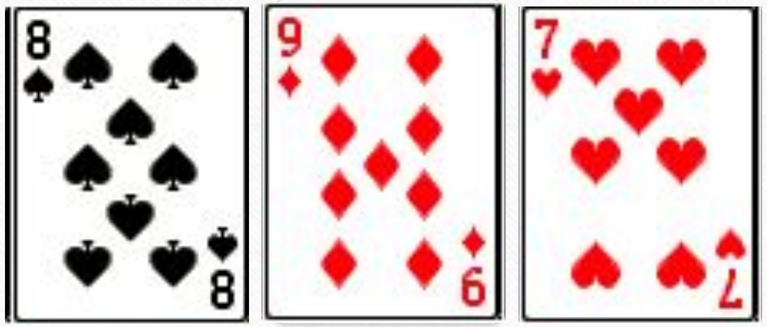


Dealer 24 bust



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Objective (how to win):

- Get 21 points on the player's first two cards (called a ”blackjack” or ”natural”), without a dealer 21. Ace and a face card or a 10. Ace is consider 11 when the sum of your cards is less than 21.
- Reach a final score higher than the dealer without exceeding 21.
- Let the dealer draw additional cards until their hand exceeds 21 (bust).

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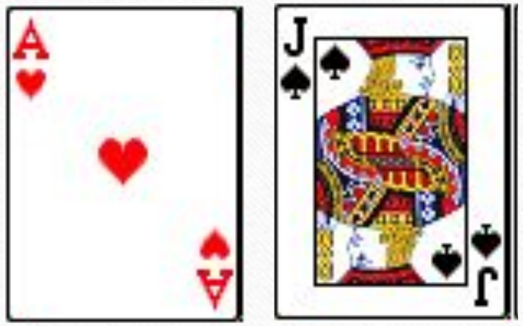
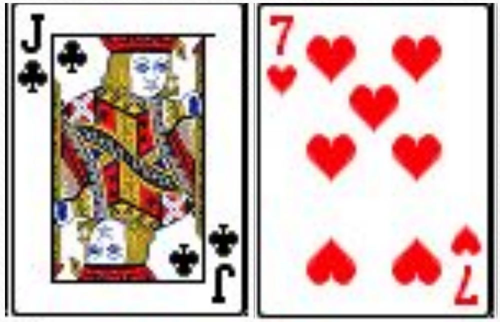


Player blackjack

Player win

One of dealer card is visible for player to make decision



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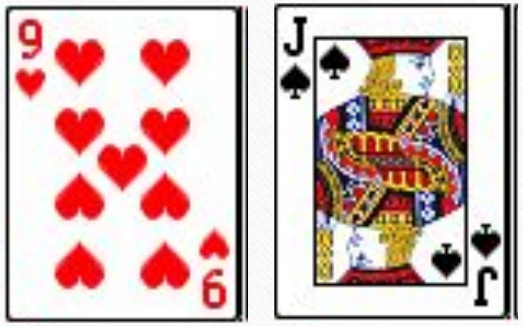
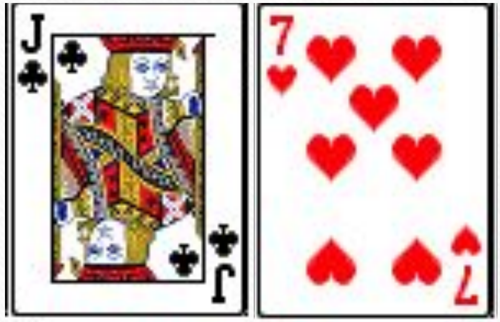


19>17

Player win

(Dealer card)



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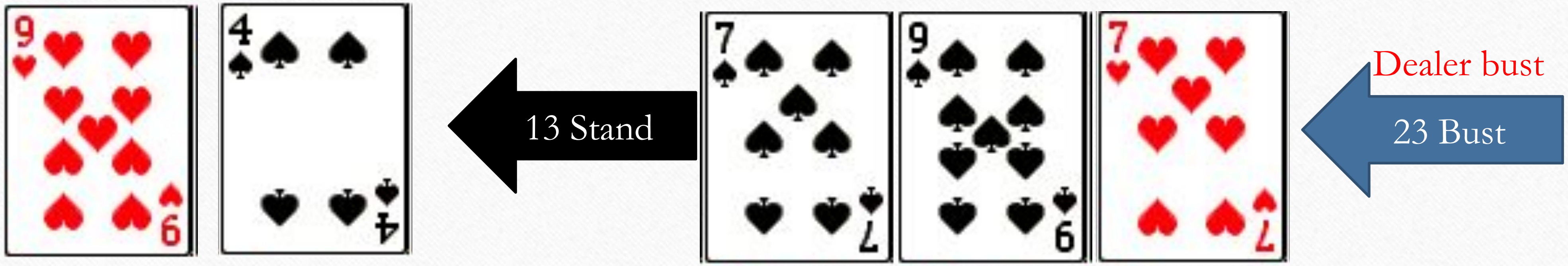


Why MDP (Black Jack/ 21)

Dealer Revealed Card

Player sum of Cards

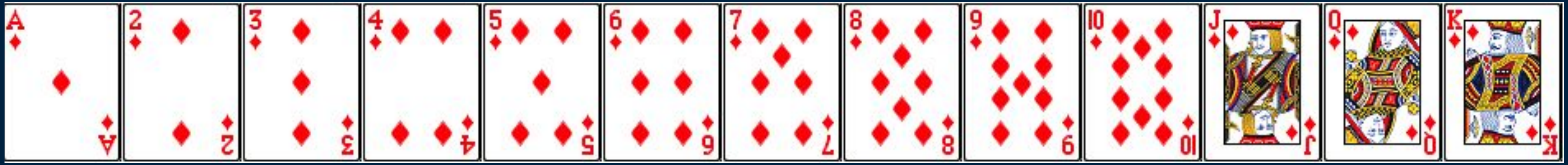
	2	3	4	5	6	7	8	9	10	A
4	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
5	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
6	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
7	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
8	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
9	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
10	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"
11	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"
12	"HIT"	"HIT"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
13	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
14	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"
15	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"
16	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"	"SURRENDER"
17	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"SURRENDER"
18	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
19	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s12	"HIT"	"HIT"	"HIT"	"HIT"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s13	"HIT"	"HIT"	"HIT"	"HIT"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s14	"HIT"	"HIT"	"HIT"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s15	"HIT"	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s16	"HIT"	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s17	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s18	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"
s19	"STAND"	"STAND"	"STAND"	"STAND"	"DOUBLE"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"

- Blackjack/21 can be solve with Markov Decision Process because your action only based on your current state, you don't care about the history.
- When an Ace(A) is treated as 11 then the card is called soft.

Assumptions

- In this paper, we will assume there is infinitely many decks.
- A double is allowed after a split.
- Blackjack get 1:1 pay.
- Surrender is only allowed before player done anything, surrender will lose half of the bet.
- Blackjack is equivalent as a s21 (blackjack is not superior than a regular 21).
- Dealer play a S17 rule. Hit when at S17 and smaller than 17 .

Dealer Transition (Soft 17)



1/13	**1/13**	**1/13**	**1/13**	**1/13**	**1/13**	1/13	1/13	1/13	4/13	Probability
12	13	14	15	16	17	18	19	20	21	Next State

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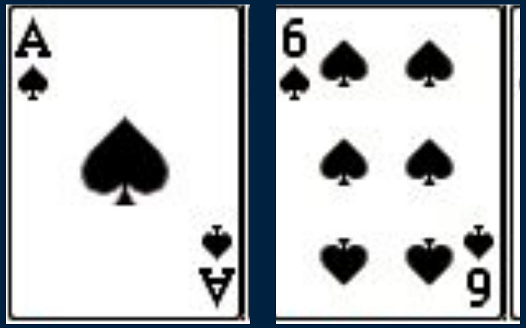
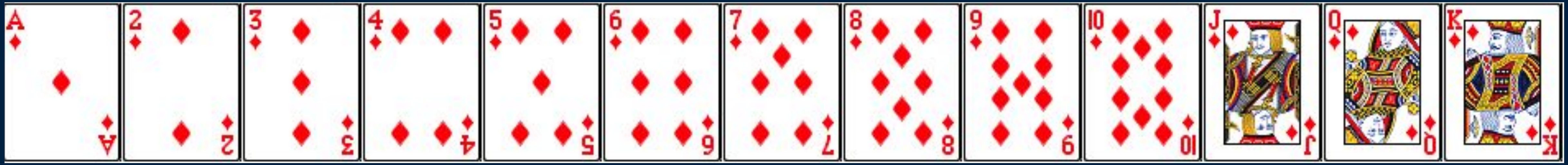


s17



1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	4/13	Probability
s18	s19	s20	s21	12	13	14	15	16	17	Next State

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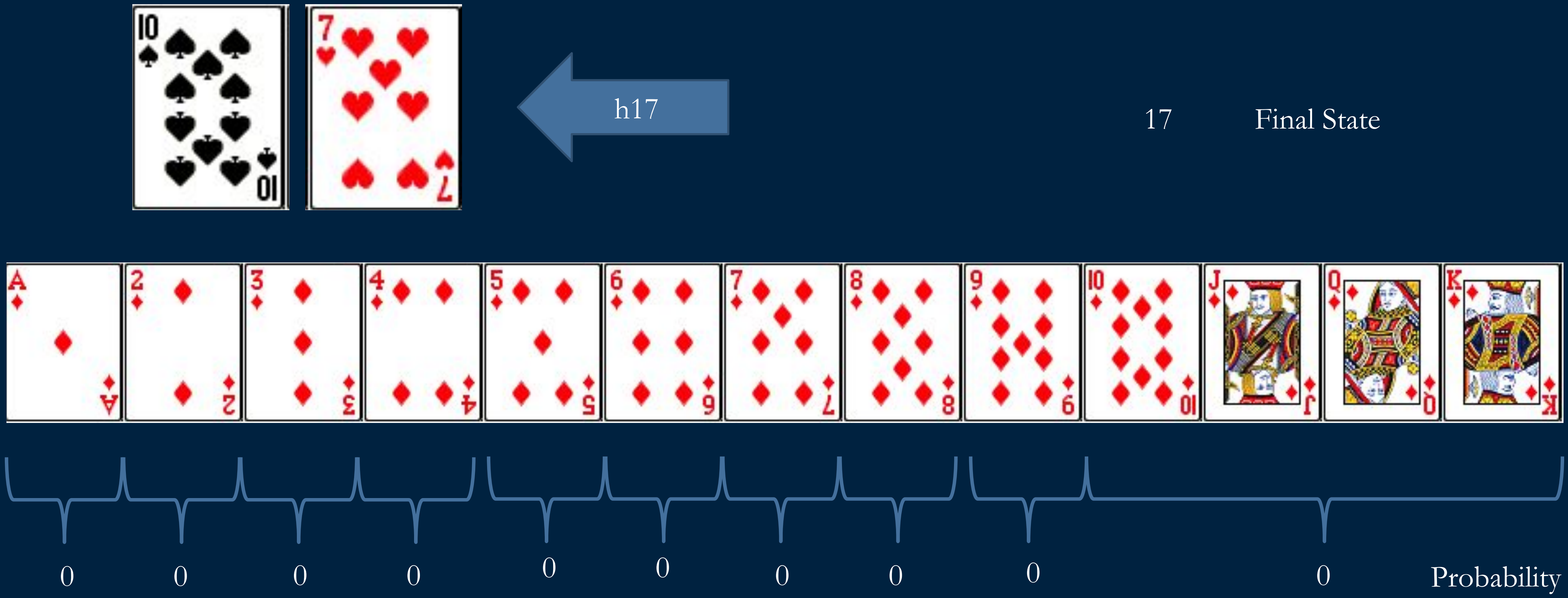


Dealer Transition Matrix (Markov Chain)

Dealer next state (s')

Dealer current state (s)

	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	0	0	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.30769231	0.00000000	0.00000000	0.00000000
3	0	0	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.30769231	0.00000000	0.00000000
4	0	0	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.30769231	0.00000000
5	0	0	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.30769231
6	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308
7	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308
8	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308
9	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308
10	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308
11	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308
12	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308
13	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308
14	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308
15	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
16	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
17	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
18	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
19	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
20	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
21	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
s11	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
s12	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.30769231	0.00000000	0.00000000	0.00000000
s13	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.30769231	0.00000000	0.00000000
s14	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.30769231	0.00000000
s15	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.30769231
s16	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308
s17	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308

	16	17	18	19	20	21	s11	s12	s13	s14	s15	s16	s17
2	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.07692308	0.00000000	0.00000000	0.00000000	0.00000000
3	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.07692308	0.00000000	0.00000000	0.00000000
4	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.07692308	0.00000000	0.00000000
5	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.00000000
6	0.30769231	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308
7	0.07692308	0.30769231	0.07692308	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
8	0.07692308	0.07692308	0.30769231	0.07692308	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
9	0.07692308	0.07692308	0.07692308	0.30769231	0.07692308	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
10	0.07692308	0.07692308	0.07692308	0.07692308	0.30769231	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
11	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.30769231	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
12	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
13	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
14	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
15	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
16	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
17	0.00000000	1.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
18	0.00000000	0.00000000	1.00000000	0.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
19	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
20	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
21	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
s11	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.30769231	0	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308
s12	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308
s13	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308
s14	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308
s15	0.00000000	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308	0.07692308
s16	0.30769231	0.00000000	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.07692308
s17	0.07692308	0.30769231	0.07692308	0.07692308	0.07692308	0.07692308	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000

Dealer Transition Matrix (Markov Chain)

Dealer.Transition.Matrix %^% 12

Only Extract row 2,3,4,5,6,7,8,9,10,s11

Dealer final state (s')

Dealer current state (s)

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	s11	s12	s13	s14	s15	s16	s17	bust
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.13013408	0.13654619	0.13129427	0.12566529	0.11963232	0	0	0	0	0	0	0	0.3567278
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12632803	0.13195701	0.12705522	0.12180330	0.11617432	0	0	0	0	0	0	0	0.3766821
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12240563	0.12730742	0.12275576	0.11785397	0.11260205	0	0	0	0	0	0	0	0.3970752
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11835894	0.12291060	0.11835894	0.11380728	0.10890549	0	0	0	0	0	0	0	0.4176588
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11483768	0.11483768	0.11483768	0.11028602	0.10573436	0	0	0	0	0	0	0	0.4394666
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.36856619	0.13779696	0.07862537	0.07862537	0.07407370	0	0	0	0	0	0	0	0.2623124
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12856654	0.35933578	0.12856654	0.06939495	0.06939495	0	0	0	0	0	0	0	0.2447412
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11999544	0.11999544	0.35076467	0.11999544	0.06082384	0	0	0	0	0	0	0	0.2284252
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11142434	0.11142434	0.11142434	0.34219357	0.11142434	0	0	0	0	0	0	0	0.2121091
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11142434	0.11142434	0.11142434	0.11142434	0.34219357	0	0	0	0	0	0	0	0.2121091
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10346546	0.10346546	0.10346546	0.10346546	0.10346546	0	0	0	0	0	0	0	0.4826727
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.09607507	0.09607507	0.09607507	0.09607507	0.09607507	0	0	0	0	0	0	0	0.5196247
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08921256	0.08921256	0.08921256	0.08921256	0.08921256	0	0	0	0	0	0	0	0.5539372
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08284024	0.08284024	0.08284024	0.08284024	0.08284024	0	0	0	0	0	0	0	0.5857988
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0	0	0	0	0	0	0.6153846
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	1.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	1.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0	0	0	0	0	0	0	0.0000000
s11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.05749325	0.14320428	0.14320428	0.14320428	0.37397351	0	0	0	0	0	0	0	0.1389204
s12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08294815	0.16253696	0.16253696	0.16253696	0.16253696	0	0	0	0	0	0	0	0.2669040
s13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08230213	0.15620603	0.15620603	0.15620603	0.15620603	0	0	0	0	0	0	0	0.2928737
s14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08132520	0.14995025	0.14995025	0.14995025	0.14995025	0	0	0	0	0	0	0	0.3188738
s15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08006792	0.14379118	0.14379118	0.14379118	0.14379118	0	0	0	0	0	0	0	0.3447674
s16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07857532	0.13774692	0.13774692	0.13774692	0.13774692	0	0	0	0	0	0	0	0.3704370
s17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.34219357	0.11142434	0.11142434	0.11142434	0.11142434	0	0	0	0	0	0	0	0.2121091
bust	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	1.0000000

Transition Matrix

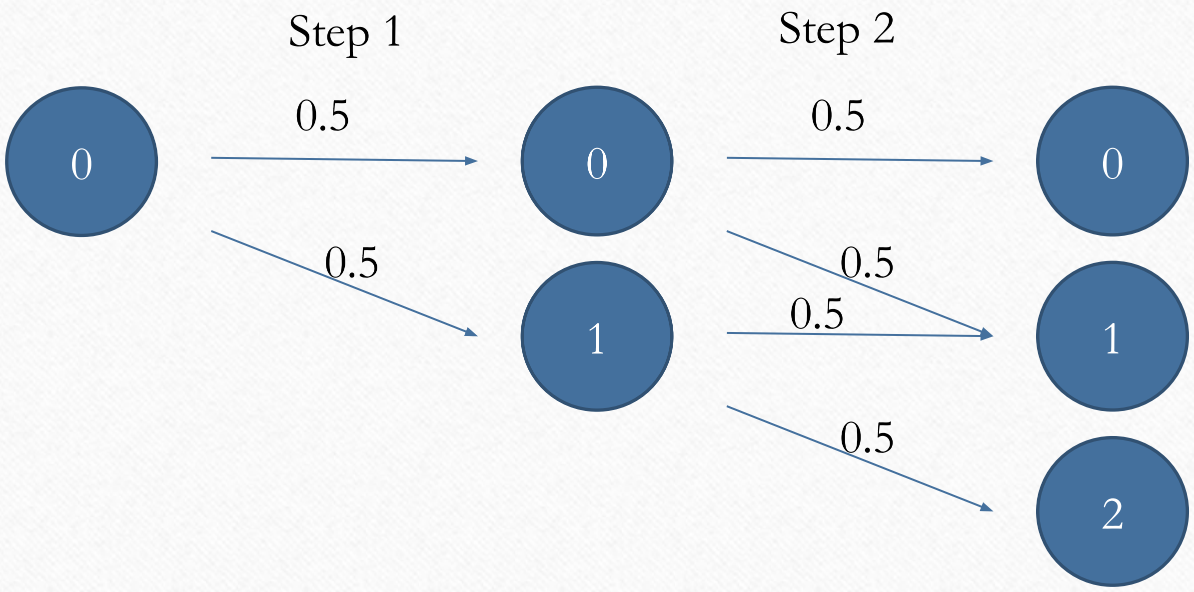
Note that the elements in a row always sum up to 1 cause is probability

next

current

state	0	1	2
0	0.5	0.5	0
1	0	0.5	0.5
2	0	0	1

For example you start at zero :



Probability after 2 steps

0.5*0.5=0.25

0.5*0.5+0.5*0.5=0.5

0.5*0.5=0.25

Transition Matrix

next

current

state	0	1	2
0	0.5	0.5	0
1	0	0.5	0.5
2	0	0	1

%*%

next

current

state	0	1	2
0	0.5	0.5	0
1	0	0.5	0.5
2	0	0	1

=

After 2 steps

current

state	0	1	2
0	0.25	0.5	0.25
1	0	0.25	0.75
2	0	0	1.0

After 2 steps:

Transition Matrix %^% 2

After n steps:

Transition Matrix %^% n

Dealer Transition Matrix (Markov Chain)

Dealer.Transition.Matrix %^% 12
Only Extract row 2,3,4,5,6,7,8,9,10,s11

Dealer final state (s')

Dealer current state (s)

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	s11	s12	s13	s14	s15	s16	s17	bust
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.13013408	0.13654619	0.13129427	0.12566529	0.11963232	0	0	0	0	0	0	0	0.3567278
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12632803	0.13195701	0.12705522	0.12180330	0.11617432	0	0	0	0	0	0	0	0.3766821
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12240563	0.12730742	0.12275576	0.11785397	0.11260205	0	0	0	0	0	0	0	0.3970752
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11835894	0.12291060	0.11835894	0.11380728	0.10890549	0	0	0	0	0	0	0	0.4176588
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11483768	0.11483768	0.11483768	0.11028602	0.10573436	0	0	0	0	0	0	0	0.4394666
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.36856619	0.13779696	0.07862537	0.07862537	0.07407370	0	0	0	0	0	0	0	0.2623124
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12856654	0.35933578	0.12856654	0.06939495	0.06939495	0	0	0	0	0	0	0	0.2447412
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11999544	0.11999544	0.35076467	0.11999544	0.06082384	0	0	0	0	0	0	0	0.2284252
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11142434	0.11142434	0.11142434	0.34219357	0.11142434	0	0	0	0	0	0	0	0.2121091
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11142434	0.11142434	0.11142434	0.11142434	0.34219357	0	0	0	0	0	0	0	0.2121091
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.10346546	0.10346546	0.10346546	0.10346546	0.10346546	0	0	0	0	0	0	0	0.4826727
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.09607507	0.09607507	0.09607507	0.09607507	0.09607507	0	0	0	0	0	0	0	0.5196247
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08921256	0.08921256	0.08921256	0.08921256	0.08921256	0	0	0	0	0	0	0	0.5539372
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08284024	0.08284024	0.08284024	0.08284024	0.08284024	0	0	0	0	0	0	0	0.5857988
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07692308	0.07692308	0.07692308	0.07692308	0.07692308	0	0	0	0	0	0	0	0.6153846
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	1.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	1.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.00000000	1.00000000	0.00000000	0	0	0	0	0	0	0	0.0000000
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	1.00000000	0	0	0	0	0	0	0	0.0000000
s11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.05749325	0.14320428	0.14320428	0.14320428	0.37397351	0	0	0	0	0	0	0	0.1389204
s12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08294815	0.16253696	0.16253696	0.16253696	0.16253696	0	0	0	0	0	0	0	0.2669040
s13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08230213	0.15620603	0.15620603	0.15620603	0.15620603	0	0	0	0	0	0	0	0.2928737
s14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08132520	0.14995025	0.14995025	0.14995025	0.14995025	0	0	0	0	0	0	0	0.3188738
s15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08006792	0.14379118	0.14379118	0.14379118	0.14379118	0	0	0	0	0	0	0	0.3447674
s16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07857532	0.13774692	0.13774692	0.13774692	0.13774692	0	0	0	0	0	0	0	0.3704370
s17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.34219357	0.11142434	0.11142434	0.11142434	0.11142434	0	0	0	0	0	0	0	0.2121091
bust	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	0	1.0000000

Dealer Final Probability Distribution

Dealer.Transition.Matrix %^% 12
Only Extract row 2,3,4,5,6,7,8,9,10,s11

	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	s12	s13	s14	s15	s16	s17	bust
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0.13013408	0.1365462	0.13129427	0.12566529	0.11963232	0	0	0	0	0	0	0.3567278
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12632803	0.1319570	0.12705522	0.12180330	0.11617432	0	0	0	0	0	0	0.3766821
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12240563	0.1273074	0.12275576	0.11785397	0.11260205	0	0	0	0	0	0	0.3970752
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11835894	0.1229106	0.11835894	0.11380728	0.10890549	0	0	0	0	0	0	0.4176588
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11483768	0.1148377	0.11483768	0.11028602	0.10573436	0	0	0	0	0	0	0.4394666
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0.36856619	0.1377970	0.07862537	0.07862537	0.07407370	0	0	0	0	0	0	0.2623124
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12856654	0.3593358	0.12856654	0.06939495	0.06939495	0	0	0	0	0	0	0.2447412
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11999544	0.1199954	0.35076467	0.11999544	0.06082384	0	0	0	0	0	0	0.2284252
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11142434	0.1114243	0.11142434	0.34219357	0.11142434	0	0	0	0	0	0	0.2121091
A	0	0	0	0	0	0	0	0	0	0	0	0	0	0.05749325	0.1432043	0.14320428	0.14320428	0.37397351	0	0	0	0	0	0	0.1389204

We can now compute the Stand.reward. For example:
You are standing when you have an 21 while the card dealer shows a #5. Thus, we know the stand.reward is
Winning – losing = Total.probability – tie – 2 * losing = 1 - 0.10890549 – 0 = 0.89709451

~The immediate reward for hitting is –P(bust)

~The immediate reward for standing is P(win)-P(lose)

~Transition for hitting matrix is similar to dealer transition matrix

~Transition for standing is zero

Value Iteration (backward induction)

- $$V_t(s) = \max_a \left(R(s,a) + \sum_{s'} T(s,a,s') \gamma V_{t+1}(s') \right)$$
- s : current state
- a : action {hit, stand}
- R : immediate reward
- T : transition matrix
- γ : di

$$V_t(s) = \max_a \left(R(s,a) + \sum_{s'} T(s,a,s') V_{t+1}(s') \right)$$

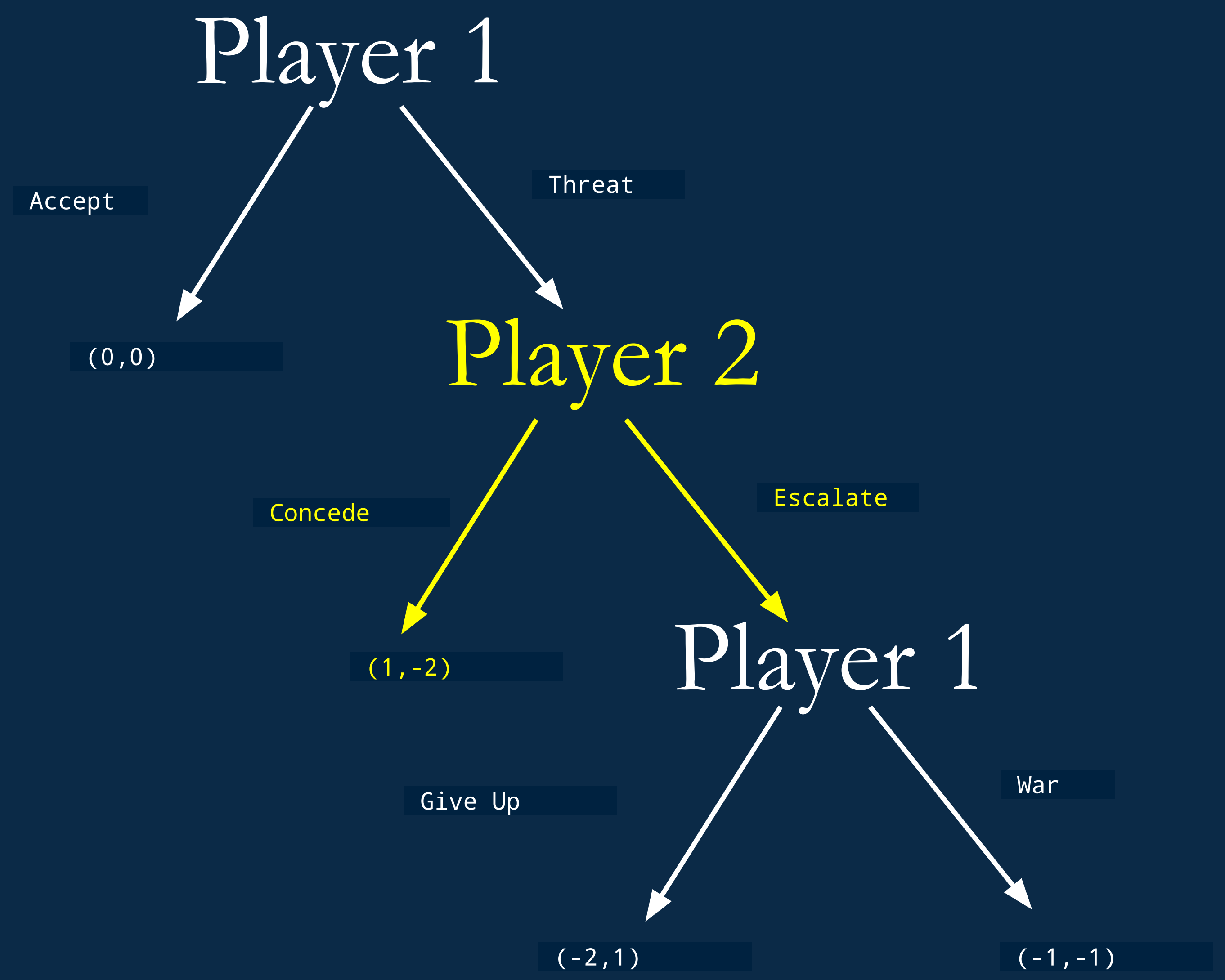
```
convergedvalue={}
for (n in 1:10){
  v=(rep(0,29))
  for (i in 1:20){
  v=pmax(Hit.Reward[1,]+Hit.Transition%*%v,(Stand.Reward[n,]))
  }
convergedvalue=cbind(convergedvalue,v)
}
result=(t(Hit.Reward)+Hit.Transition%*%convergedvalue)<t(Stand.Reward)
```

Dealer Revealed Card

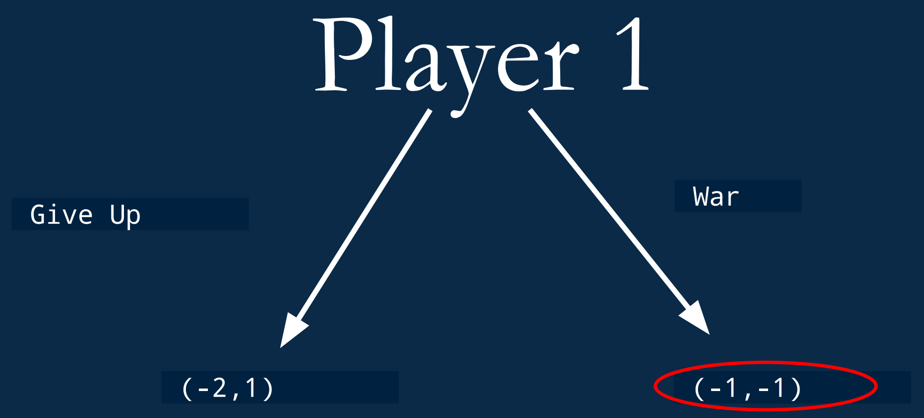
Player sum of Cards

	2	3	4	5	6	7	8	9	10	A
4	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
5	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
6	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
7	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
8	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
9	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
10	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
11	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
12	"HIT"	"HIT"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
13	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
14	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
15	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
16	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
17	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
18	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
19	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s12	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s13	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s14	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s15	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s16	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s17	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s18	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"
s19	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"

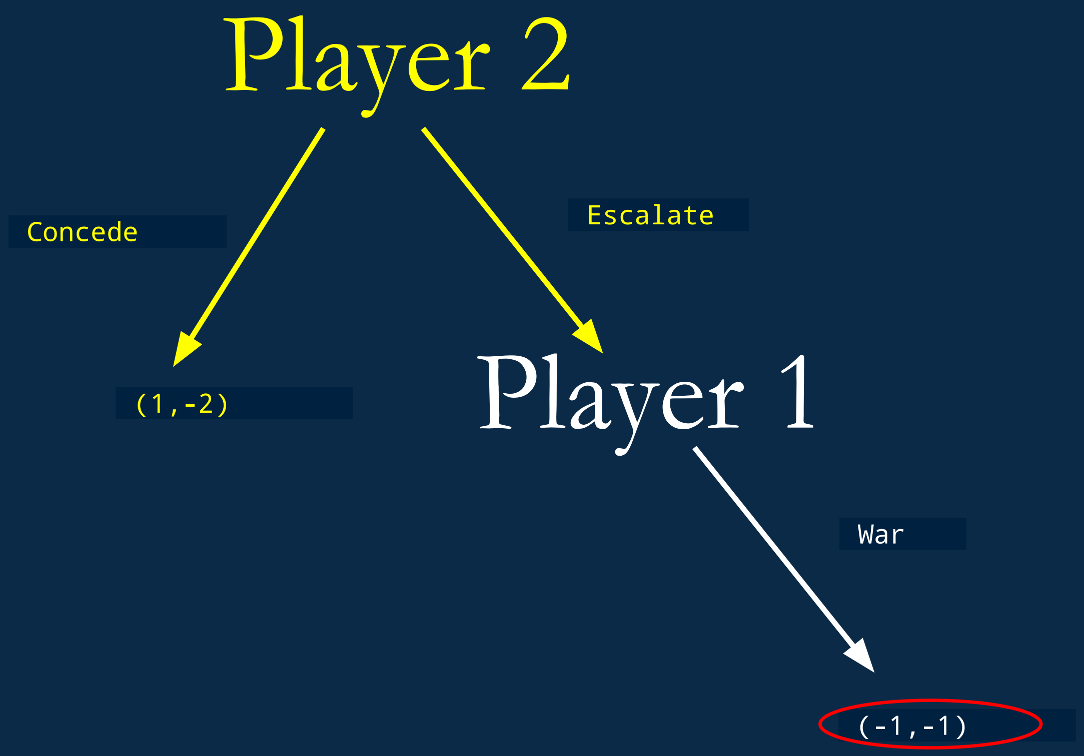
Backward Induction



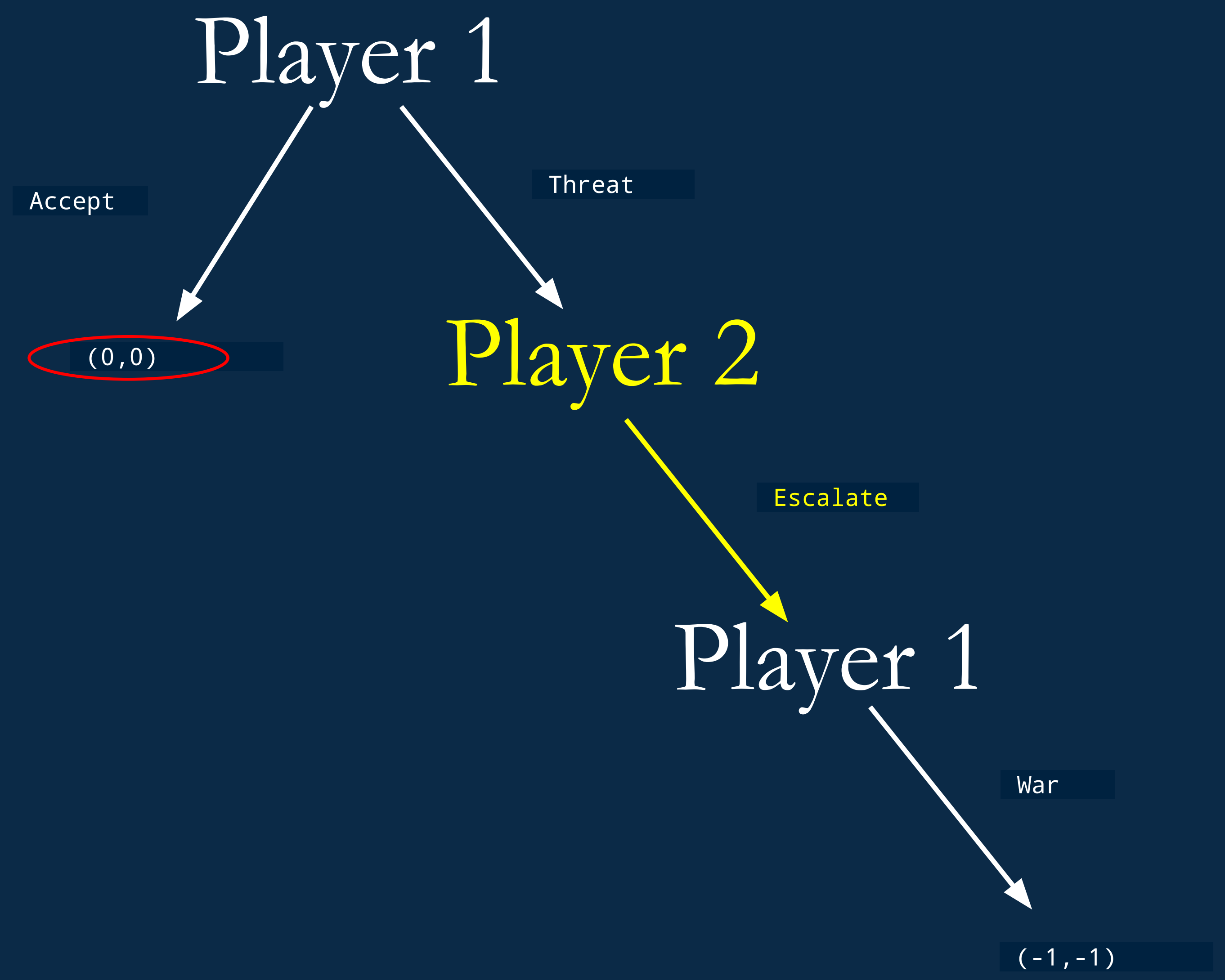
Backward Induction



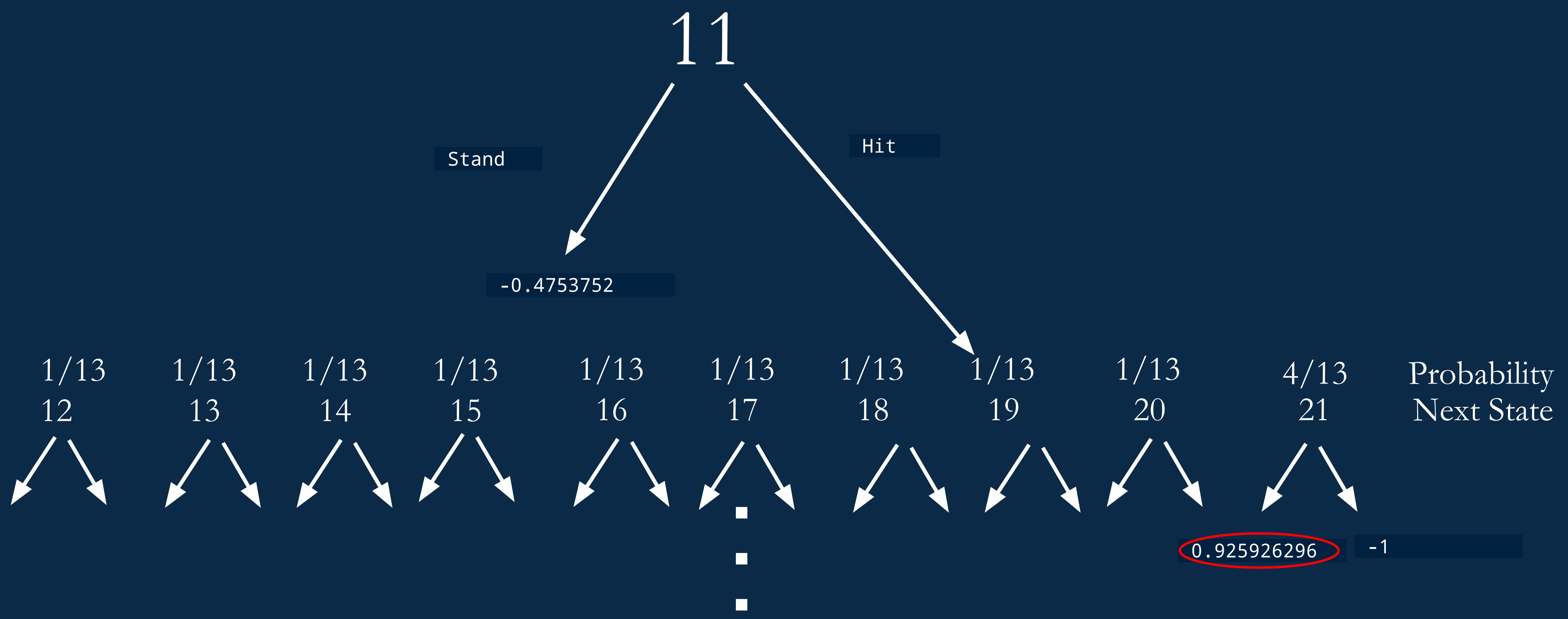
Backward Induction



Backward Induction



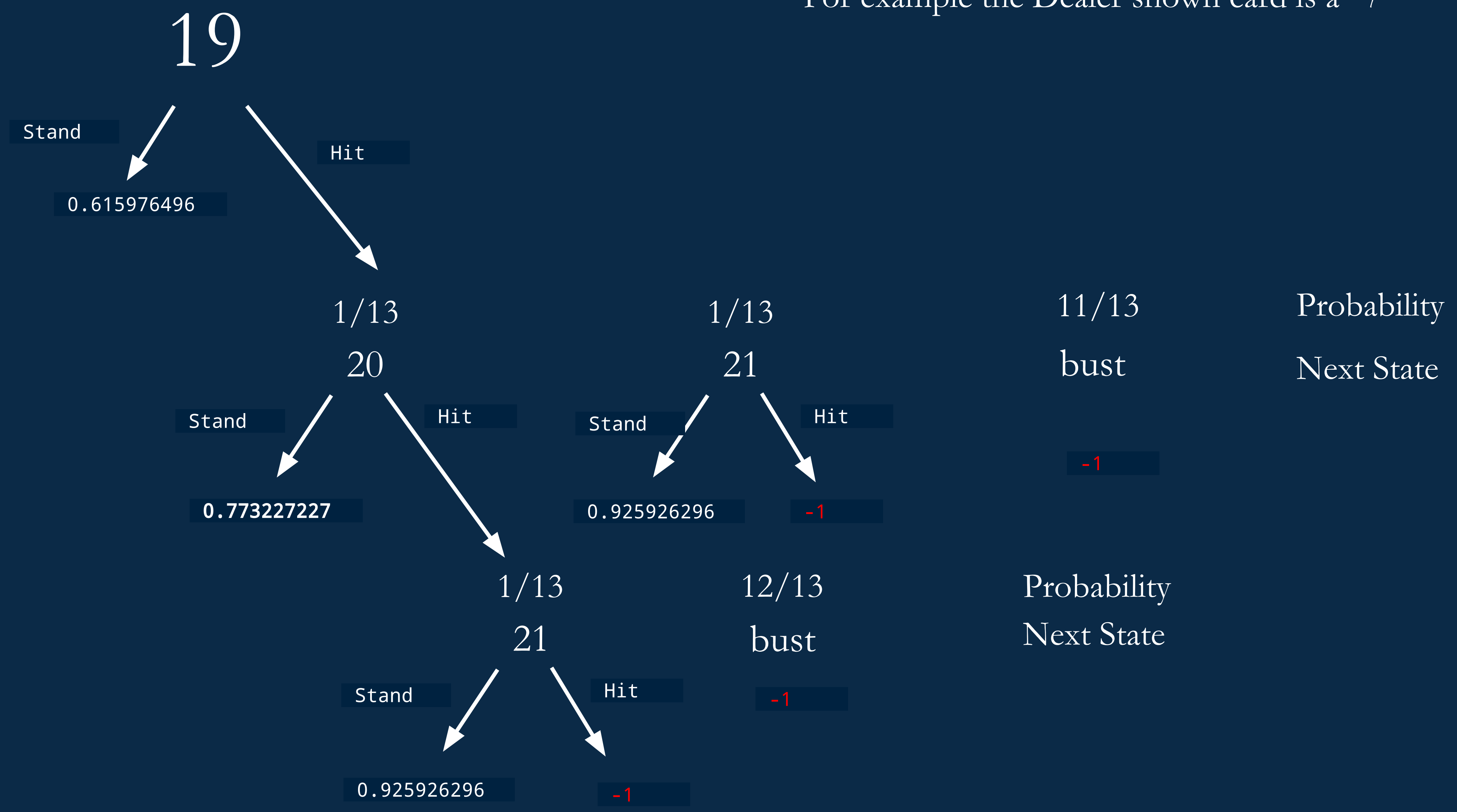
Value Iteration



Expected.Return = sum P(x)*Best.Reward(x)   for (x in A:10)

Value Iteration

For example the Dealer shown card is a “7”



Value Iteration

For example the Dealer shown card is a “7”

	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	s12	s13	s14	s15	s16	s17	bust
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0.13013408	0.1365462	0.13129427	0.12566529	0.11963232	0	0	0	0	0	0	0.3567278
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12632803	0.1319570	0.12705522	0.12180330	0.11617432	0	0	0	0	0	0	0.3766821
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12240563	0.1273074	0.12275576	0.11785397	0.11260205	0	0	0	0	0	0	0.3970752
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11835894	0.1229106	0.11835894	0.11380728	0.10890549	0	0	0	0	0	0	0.4176588
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11483768	0.1148377	0.11483768	0.11028602	0.10573436	0	0	0	0	0	0	0.4394666
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0.36856619	0.1377970	0.07862537	0.07862537	0.07407370	0	0	0	0	0	0	0.2623124
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12856654	0.3593358	0.12856654	0.06939495	0.06939495	0	0	0	0	0	0	0.2447412
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11999544	0.1199954	0.35076467	0.11999544	0.06082384	0	0	0	0	0	0	0.2284252
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0.11142434	0.1114243	0.11142434	0.34219357	0.11142434	0	0	0	0	0	0	0.2121091
A	0	0	0	0	0	0	0	0	0	0	0	0	0	0.05749325	0.1432043	0.14320428	0.14320428	0.37397351	0	0	0	0	0	0	0.1389204

21

Stand

Hit

1 - 0.0740437 = 0.925926296

-1

Value Iteration

For example the Dealer shown card is a “7”

Expected return of hitting at 20:
1/13 * 0.925926296 + 12/13 * (-1) = -0.8518518

$$\text{ExpectedReturn} = \sum_{x} p(x) * E^{*}(x)$$

20

Stand

0.773227227

Hit

1/13	12/13	Probability
21	bust	Next State

Stand

0.925926296

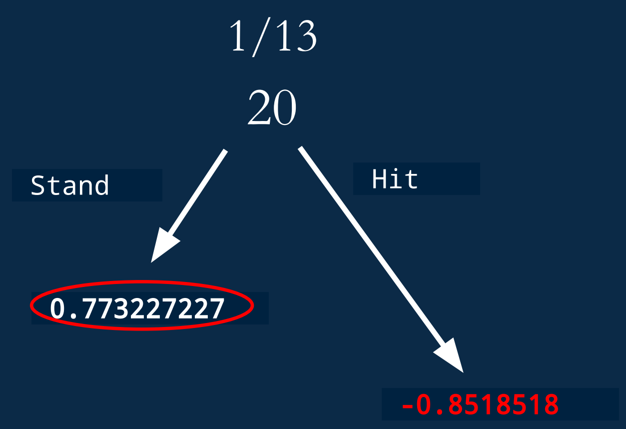
-1

Value Iteration

For example the Dealer shown card is a “7”

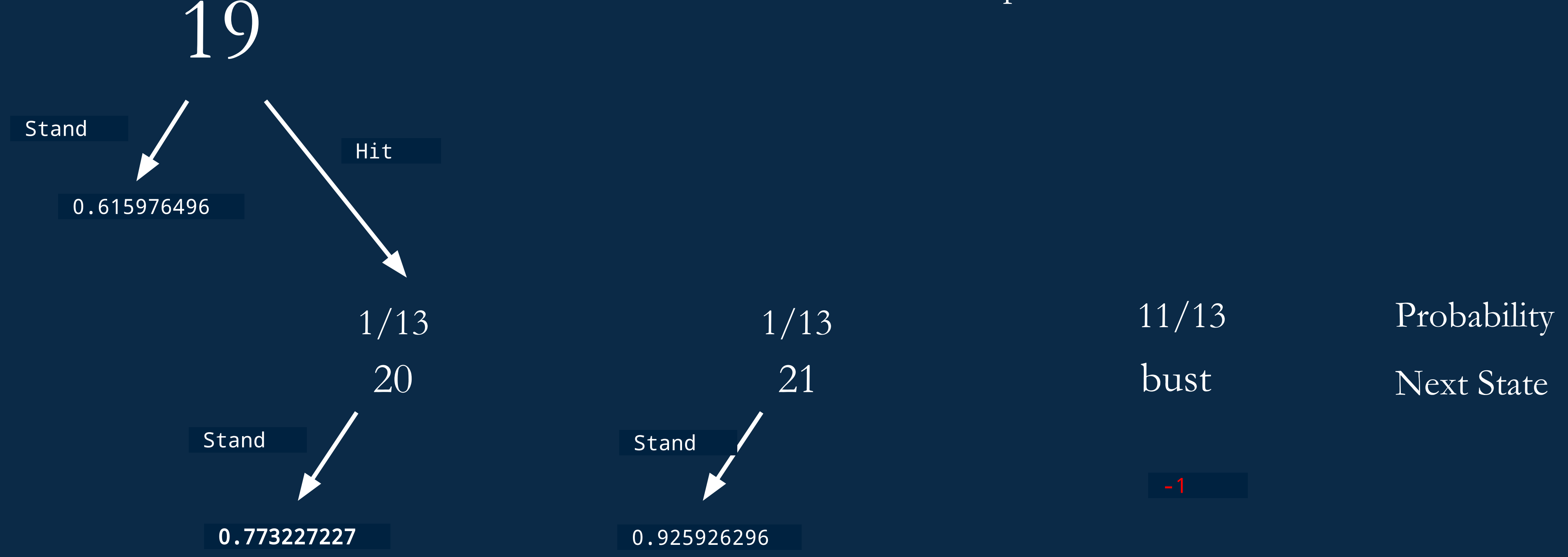
Expected return of hitting at 20:
$1/13 * 0.925926296 + 12/13 * (-1) =$ **-0.8518518**

$$\text{ExpectedReturn} = \sum_{x} p(x) * E^{*}(x)$$



Value Iteration

For example the Dealer shown card is a "7"



Expected return of hitting at 19:
1/13 * 0.773227227 + 1/13 * 0.925926296 + 11/13 * (-1)
= -0.7154497

$$\text{ExpectedReturn} = \sum_{x} p(x) * E^{*}(x)$$

Value Iteration

For example the Dealer shown card is a “7”

19

Stand

0.615976496

Hit

-0.7154497

Expected return of hitting at 19:
1/13 * 0.773227227 + 1/13 * 0.925926296 + 11/13 * (-1)
= -0.7154497

$$\text{ExpectedReturn} = \sum_{x} p(x) * E^{*}(x)$$

Value Iteration (backward induction)

- $$V_t(s) = \max_a \left(R(s,a) + \sum_{s'} T(s,a,s') \gamma V_{t+1}(s') \right)$$
- s : current state
- a : action {hit, stand}
- R : immediate reward
- T : transition matrix
- γ : di

$$V_t(s) = \max_a \left(R(s,a) + \sum_{s'} T(s,a,s') V_{t+1}(s') \right)$$

```
convergedvalue={}
for (n in 1:10){
  v=(rep(0,29))
  for (i in 1:20){
  v=pmax(Hit.Reward[1,]+Hit.Transition%*%v,(Stand.Reward[n,]))
  }
convergedvalue=cbind(convergedvalue,v)
}
result=(t(Hit.Reward)+Hit.Transition%*%convergedvalue)<t(Stand.Reward)
```

Dealer Revealed Card

Player sum of Cards

	2	3	4	5	6	7	8	9	10	A
4	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
5	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
6	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
7	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
8	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
9	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
10	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
11	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
12	"HIT"	"HIT"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
13	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
14	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
15	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
16	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
17	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
18	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
19	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s12	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s13	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s14	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s15	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s16	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s17	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s18	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"
s19	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"

Maximum Reward

Actions included in this game:

- 1. Hit - This action allow player to take an extra card.
- 2. Stand - This action indicate player want to end their turn.
- 3. Double - This action allow player to double its bet and end their turn by taking an extra card.
- 4. Surrender - This action allow player to give up in the game while losing only half of the bet before making any action.
- 5. Split - This action allow player to split into 2 games. This action is only required if player get 2 cards that has the same value at the beginning.

maxrew=pmax((t(Hit.Reward)+Hit.Transition%*%convergedvalue),t(Double.reward),t(surrender.reward),t(Stand.Reward),
t(Split.Reward))

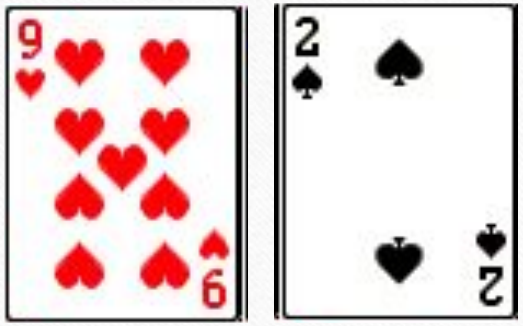
~The immediate reward for surrender is -0.5 cause you lose half of the bet
~The immediate reward for Double is 2*(P(win)-P(lose))
~The reward for splitting is 2*First.Card.Transition%*%maxrew
~Transition for surrender,double and splitting is considered zeroes

~We compute the element wise maximum reward as the best action because double, surrender and split only allow on the first move, thus we don't need backward induction.

Double

(Player card)

One of dealer card is visible for player to make decision



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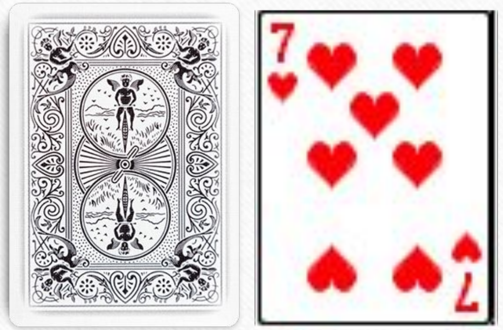
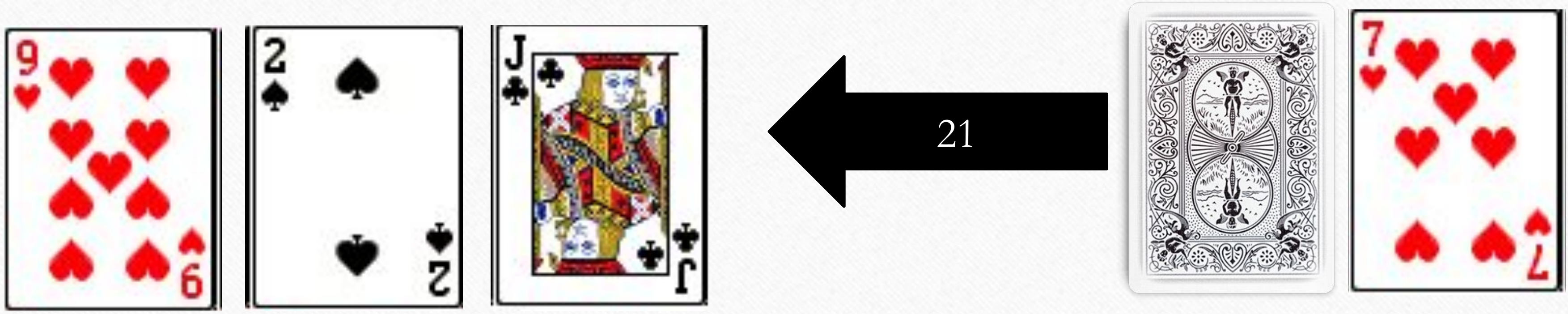


(bet=$1)

Double

(Player card)

One of dealer card is visible for player to make decision

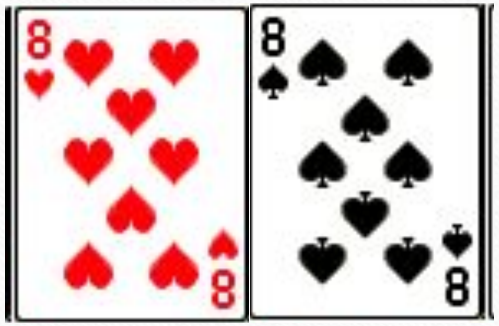


Double your bet and take a card ($2)

~The immediate reward for Double is, pick an extra card. Then, 2*(P(win)-P(lose)).

Split

(Player card)



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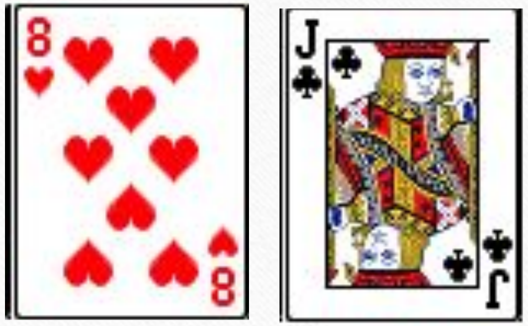
(bet=$1)

Split

(Player card)

(bet=$1)



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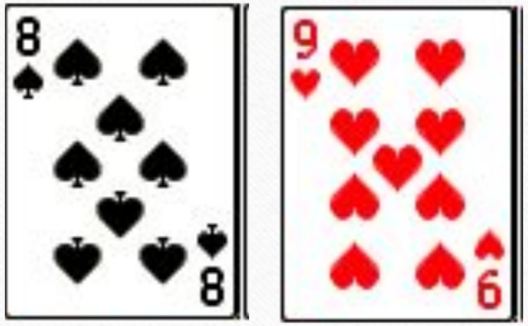
(bet=$1)



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Splitting Reward Explanation

		2	3	4	5	6	7	8	9	10	A
	4	-0.112855438	-0.080761553	-0.04696161	-0.011221572	0.026189020	-0.088279201	-0.15933415	-2.406662e-01	-0.33509987	-0.4749033
	5	-0.125854666	-0.093185805	-0.05886894	-0.022722051	0.015153620	-0.119447442	-0.18809330	-2.666151e-01	-0.35774345	-0.4928078
	6	-0.138073029	-0.104874041	-0.07007777	-0.033548870	0.004766509	-0.151932707	-0.21724188	-2.926407e-01	-0.38050766	-0.5000000
	7	-0.109577485	-0.076937568	-0.04282637	-0.007177267	0.030408566	-0.068807800	-0.21060477	-2.853654e-01	-0.36507790	-0.5000000
2x	8	-0.024506830	0.005567931	0.03701078	0.069950633	0.103858113	0.082207440	-0.05989828	-2.101863e-01	-0.30177739	-0.4687471
	9	0.072232809	0.118717086	0.17999961	0.242118664	0.304853450	0.171867860	0.09837622	-5.217805e-02	-0.21343169	-0.3749992
	10	0.356907198	0.407492012	0.45924220	0.511699535	0.564961696	0.392412456	0.28663572	1.443284e-01	-0.04499026	-0.2506490
	11	0.470121480	0.517327840	0.56560652	0.614490043	0.664663410	0.462888949	0.35069259	2.277834e-01	0.05969080	-0.1313003
	12	-0.253751471	-0.234016177	-0.20584969	-0.164682494	-0.121066850	-0.212847715	-0.27157481	-3.400133e-01	-0.42069619	-0.5000000
	13	-0.286544301	-0.246635774	-0.20584969	-0.164682494	-0.121066850	-0.269072878	-0.32360518	-3.871552e-01	-0.46207503	-0.5000000
	14	-0.286544301	-0.246635774	-0.20584969	-0.164682494	-0.121066850	-0.321281958	-0.37191909	-4.309298e-01	-0.50000000	-0.5000000
	15	-0.286544301	-0.246635774	-0.20584969	-0.164682494	-0.121066850	-0.369761817	-0.41678201	-4.715777e-01	-0.50000000	-0.5000000
	16	-0.286544301	-0.246635774	-0.20584969	-0.164682494	-0.121066850	-0.414778830	-0.45844044	-5.000000e-01	-0.50000000	-0.5000000
	17	-0.156410218	-0.120307743	-0.08344405	-0.046323555	-0.006229168	-0.106808989	-0.38195097	-4.231542e-01	-0.46435751	-0.5000000
	18	0.110270051	0.137977297	0.16626900	0.194945986	0.223446195	0.399554167	0.10595135	-1.831634e-01	-0.24150883	-0.4639684
	19	0.378110506	0.396989525	0.41633219	0.436215526	0.453121559	0.615976496	0.59385367	2.875968e-01	-0.01866015	-0.1775599
	20	0.635070067	0.645848047	0.65694192	0.668381744	0.678245261	0.773227227	0.79181516	7.583569e-01	0.43495775	0.1088487
	21	0.880367680	0.883825675	0.88739795	0.891094511	0.894265641	0.925926296	0.93060505	9.391762e-01	0.88857566	0.6260265
	s12	0.079806247	0.101680409	0.12678682	0.156574441	0.196048414	0.165472931	0.09511502	6.579068e-05	-0.12808280	-0.2978245
	s13	0.046611316	0.074096483	0.10302707	0.133627517	0.196048414	0.122385695	0.05405707	-3.769469e-02	-0.16080628	-0.3246016
	s14	0.022814486	0.051187036	0.08096445	0.127208347	0.196048414	0.079507489	0.01327722	-7.516319e-02	-0.19330354	-0.3511359
	s15	0.000717430	0.029913978	0.06103069	0.127208347	0.196048414	0.037028283	-0.02705478	-1.121888e-01	-0.22543993	-0.3773255
	s16	-0.019801265	0.010160424	0.06103069	0.127208347	0.196048414	-0.004890157	-0.06679485	-1.486435e-01	-0.25710121	-0.4030842
	s17	-0.001673173	0.053881647	0.11752560	0.181835550	0.249050421	0.053823464	-0.07291540	-1.497869e-01	-0.24941602	-0.4229116
	s18	0.114691805	0.173090127	0.23277778	0.293190723	0.355054435	0.399554167	0.10595135	-1.007443e-01	-0.20109793	-0.3828376
	s19	0.378110506	0.396989525	0.41633219	0.436215526	0.461058449	0.615976496	0.59385367	2.875968e-01	-0.01866015	-0.1775599
	s20	0.635070067	0.645848047	0.65694192	0.668381744	0.678245261	0.773227227	0.79181516	7.583569e-01	0.43495775	0.1088487
	s21	0.880367680	0.883825675	0.88739795	0.891094511	0.894265641	0.925926296	0.93060505	9.391762e-01	0.88857566	0.6260265
	bust	0.000000000	0.000000000	0.00000000	0.000000000	0.000000000	0.000000000	0.00000000	0.000000e+00	0.00000000	0.0000000

Maximum Reward

Actions included in this game:

- 1. Hit - This action allow player to take an extra card.
- 2. Stand - This action indicate player want to end their turn.
- 3. Double - This action allow player to double its bet and end their turn by taking an extra card.
- 4. Surrender - This action allow player to give up in the game while losing only half of the bet before making any action.
- 5. Split - This action allow player to split into 2 games. This action is only required if player get 2 cards that has the same value at the beginning.

maxrew=pmax((t(Hit.Reward)+Hit.Transition%*%convergedvalue),t(Double.reward),t(surrender.reward),t(Stand.Reward), t(Split.Reward))

~The immediate reward for surrender is -0.5 cause you lose half of the bet
~The immediate reward for Double is 2*(P(win)-P(lose))
~The reward for splitting is 2*First.Card.Transition%*%maxrew
~Transition for surrender,double and splitting is considered zeroes

~We compute the element wise maximum reward as the best action because double, surrender and split only allow on the first move, thus we don't need backward induction.

Final Decision Matrix

Dealer Revealed Card

Player sum of Cards

	2	3	4	5	6	7	8	9	10	A
4	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
5	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
6	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
7	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
8	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
9	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
10	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"
11	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"
12	"HIT"	"HIT"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
13	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
14	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"
15	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"
16	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"	"SURRENDER"
17	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"SURRENDER"
18	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
19	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s12	"HIT"	"HIT"	"HIT"	"HIT"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s13	"HIT"	"HIT"	"HIT"	"HIT"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s14	"HIT"	"HIT"	"HIT"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s15	"HIT"	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s16	"HIT"	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s17	"HIT"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
s18	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"STAND"	"STAND"	"HIT"	"HIT"	"HIT"
s19	"STAND"	"STAND"	"STAND"	"STAND"	"DOUBLE"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s20	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
s21	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"

	2	3	4	5	6	7	8	9	10	A
2,2	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"HIT"	"HIT"	"HIT"	"HIT"
3,3	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
4,4	"HIT"	"HIT"	"HIT"	"SPLIT"	"SPLIT"	"HIT"	"HIT"	"HIT"	"HIT"	"HIT"
5,5	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"DOUBLE"	"HIT"	"HIT"
6,6	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"HIT"	"HIT"	"HIT"	"HIT"	"SURRENDER"
7,7	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"HIT"	"HIT"	"SURRENDER"	"SURRENDER"
8,8	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SURRENDER"	"SURRENDER"
9,9	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"STAND"	"SPLIT"	"SPLIT"	"STAND"	"STAND"
10,10	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"	"STAND"
A,A	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"	"SPLIT"

Evaluation

- Optimal Strategy
  - Expectation return per game is: -0.02345701
- Same as dealer Strategy
  - Expectation return per game is: **-0.1172382**

Did You Know?

For some casinos: They pay 3:2 for Player blackjack, which increase the expected return of player by 0.02254823 on average.

Which result the expectation return for optimal strategy equal to -0.0009087813.

Can we do better ? Extra Fun Fact

- Yes, we are able to do better with counting cards.

- By knowing more information about the deck we can make better decision.
  - If we know the number of decks a casino using specifically and we implement the card count. We could do better than the casino.
  - However, to simulate the expected reward including the count could be difficult. One of the way to compute it is running a simulator a lot of times and compare the reward of each action on each state.

Fun Fact

- How does counting cards work.
  - When the count get high, we know there exist more (high cards)10,J,Q,K,A in the deck compare to (small cards) 2,3,4,5,6.
  - Some casino offer a 3:2 pay off for Player Blackjack. If there are more high cards in the deck, means there is more likely for someone to get blackjack.
  - When the deck has more high cards, the dealer is more likely to bust since they have a deterministic strategy.
  - Player is able to make a better STRATEGY against the dealer when they know more information about the deck of cards.

Thank You

- Any questions ?