SubOptimalCVaRexample

August 7, 2023

[1]:

```julia
using DataFrames
using Plots
```

[2]:

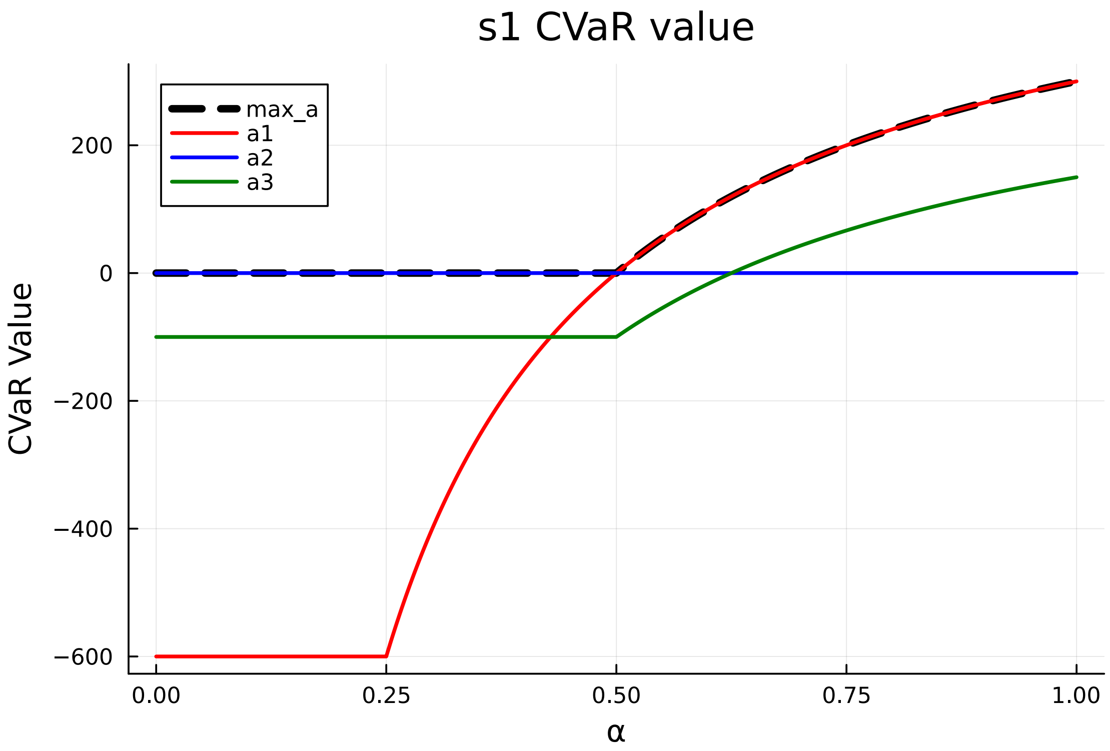
```julia
# O(Nlog(N)) with sort
function distribution(X,p)
    d = DataFrame(X = X, p = p)
    d = d[d.p .> 0,:]
    d = combine(groupby(d, ["X"]),df -> DataFrame(p = sum(df.p)) )
    sort!(d,["X"])
    return d
end

# This delta function is the inverse of cumsum
function delta(V)
    return [V[1];V[Not(1)]-V[Not(length(V))]]
end

function neat_CVaR(d,alpha) # O(N)
    if alpha == 0
        return minimum(d[d.p .> 0,:].X)
    else
        return (transpose(d.DeltaX)*max.(zeros(nrow(d)),alpha .- d.Psum .+ d.
 ↪p))/alpha
    end
end

# CVaR method to solve for multiple Alphas
function neat_CVaR_Vec(d,Alpha)
    d.DeltaX = delta(d.X)       # O(N)
    d.Psum = cumsum(d.p)    # O(N)
    return [neat_CVaR(d,alpha) for alpha in Alpha]
end

function CVaR2Distribution( cvar , lambda ; sig = 10)
    p = delta( lambda )
    X = round.( delta( lambda .* cvar ) ./ (p) ;sigdigits = sig)
    d = distribution(X,p)
    return d
end
```

[2]:
```
CVaR2Distribution (generic function with 1 method)
```

[3]:
```
s1a1 = DataFrame(X = [-600,600], p = [0.25,0.75])
s1a2 = DataFrame(X = [0], p = [1.0])
s1a3 = DataFrame(X = [-100,400], p = [0.5,0.5])
s2 = DataFrame(X = [200], p = [1.0])

Alpha = LinRange(0,1,1001)
s1a1CVaR = neat_CVaR_Vec(s1a1,Alpha);
s1a2CVaR = neat_CVaR_Vec(s1a2,Alpha);
s1a3CVaR = neat_CVaR_Vec(s1a3,Alpha);
s1optCVaR = max.(s1a1CVaR,s1a2CVaR,s1a3CVaR);
s1opt = CVaR2Distribution( s1optCVaR , Alpha);
```

[4]:
```
plot(
    Alpha,
    [s1optCVaR s1a1CVaR s1a2CVaR s1a3CVaR],
    title="s1 CVaR value",
    label=["max_a" "a1" "a2" "a3"],
    linestyle = [:dash :solid :solid :solid],
    lc = ["black" "red" "blue" "green"],
    linewidth=[ 4 2 2 2 ])
ylabel!("CVaR Value")
xlabel!(" ")
```

[4]:



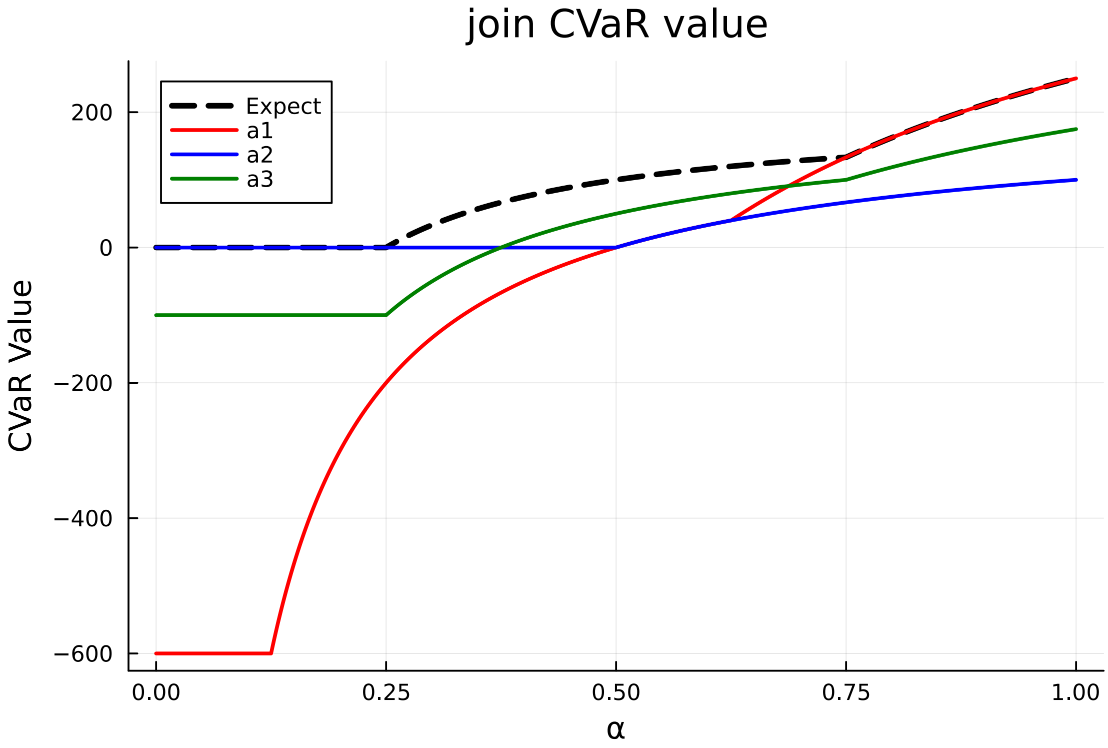
[5]:

```
jointopt = distribution([s1opt.X;s2.X],[s1opt.p .* 0.5;s2.p .* 0.5])
jointa1 = distribution([s1a1.X;s2.X],[s1a1.p .* 0.5;s2.p .* 0.5])
jointa2 = distribution([s1a2.X;s2.X],[s1a2.p .* 0.5;s2.p .* 0.5])
jointa3 = distribution([s1a3.X;s2.X],[s1a3.p .* 0.5;s2.p .* 0.5])
jointExpectedCVaR = neat_CVaR_Vec(jointopt,Alpha);
jointa1CVaR = neat_CVaR_Vec(jointa1,Alpha);
jointa2CVaR = neat_CVaR_Vec(jointa2,Alpha);
jointa3CVaR = neat_CVaR_Vec(jointa3,Alpha);
jointoptCVaR = max.(jointa1CVaR,jointa2CVaR,jointa3CVaR);
jointPerformCVaR = jointa2CVaR .* (abs.(jointExpectedCVaR .- jointa2CVaR) .<␣
 ↪1e-12) .+ jointa1CVaR .* (abs.(jointExpectedCVaR .- jointa2CVaR) .>= 1e-12);
```

[6]:

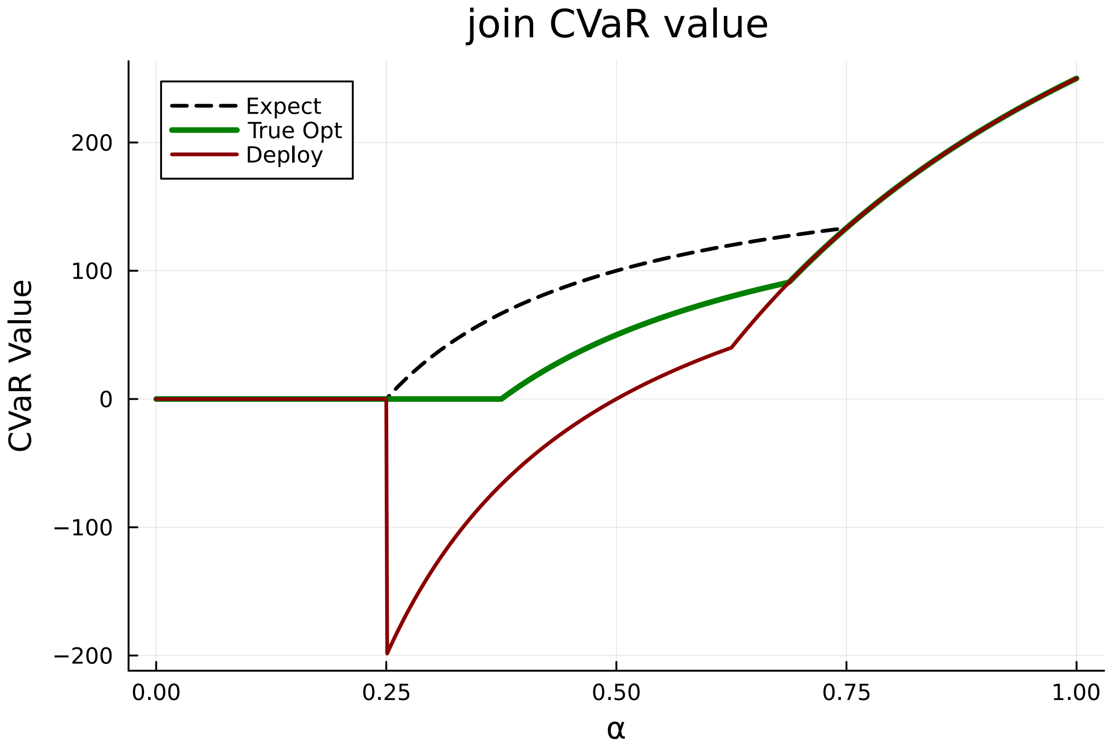
```
plot(
    Alpha,
    [jointExpectedCVaR jointa1CVaR jointa2CVaR jointa3CVaR],
    title="join CVaR value", label=["Expect" "a1" "a2" "a3"],
    linestyle = [:dash :solid :solid :solid],
    lc = ["black" "red" "blue" "green"],
    linewidth=[ 3 2 2 2 ])
ylabel!("CVaR Value")
xlabel!(" ")
```

[6]:



```
[7]: plot(
         Alpha,
         [jointExpectedCVaR jointoptCVaR jointPerformCVaR],
         title="join CVaR value",
         linestyle = [:dash :solid :solid],
         label=["Expect" "True Opt" "Deploy"],
         lc = ["black" "green" "darkred"],
         linewidth=[ 2 3 2 ])
     ylabel!("CVaR Value")
     xlabel!(" ")
```

[7]:

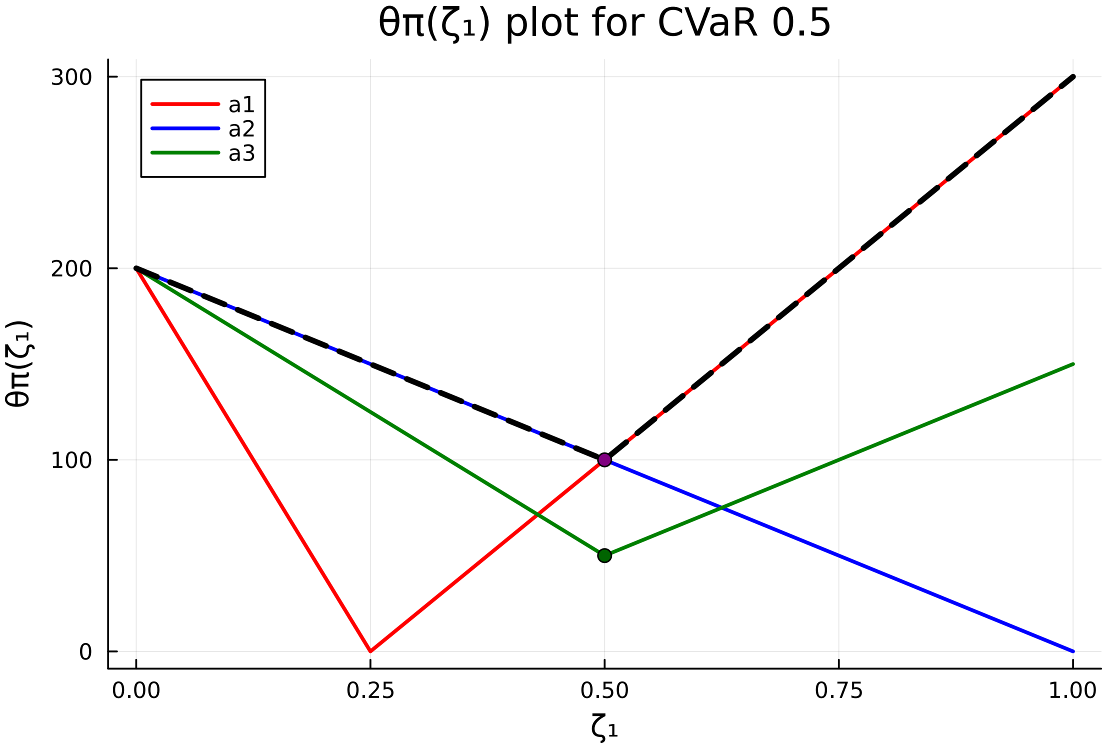


[8]:
```
Z = LinRange(0,1,10001)
Za1 = max.(s2.X[1] .+ (s1a1.X[1] - s2.X[1]) .* Z, (s2.X[1] + (s1a1.X[1] - s2.
 ↪X[1]) * s1a1.p[1]) .+ (s1a1.X[2] - s2.X[1]) .* (Z .- s1a1.p[1]))
Za2 = s2.X[1] .+ (s1a2.X[1] - s2.X[1]) .* Z
Za3 = max.( s2.X[1] .+ (s1a3.X[1] - s2.X[1]) .* Z , (s2.X[1] + (s1a3.X[1] - s2.
 ↪X[1]) * s1a3.p[1]) .+ (s1a3.X[2] - s2.X[1]) .* (Z .- s1a3.p[1]))
Zexp = max.(Za1,Za2,Za3)

expI = argmin(Zexp)
expVal = minimum(Zexp)

trueI = argmin(Za3)
trueVal = minimum(Za3);
```

[9]:
```
plot(
    Z,
    [Za1 Za2 Za3 Zexp],
    title=" ( ) plot for CVaR 0.5",
    label=["a1" "a2" "a3" ""],
    linestyle = [:solid :solid :solid :dash ],
    lc = ["red" "blue" "green" "black"],
    linewidth=[ 2 2 2 3 ])
ylabel!(" ( )")
```

```
xlabel!(" ")
plot!([Z[expI]], [expVal], seriestype=:scatter,color="purple",label="")
plot!([Z[trueI]], [trueVal], seriestype=:scatter,color="darkgreen",label="")
```

[9]:



For CVaR 50% in this case the the policy optimization risk decomposition would gives us the purple value 100. However, the true optimal action a3 has a value of 50. When one deploy the action suggested by the algorithm then they would receive 0 instead of the optimal 50 as their CVaR50%.