Controller design for FOC of IM

HCMUTE

Monna Dang (Dang Hoang Anh Chuong)

22/10/23

1.Gain margin and Phase margin

Gain margin: is amount of change in **the open-loop gain** needed to make a closed-loop system unstable. The gain margin is the difference between 0 dB and the gain at phase cross-over frequency (at phase = -180^0).
Phase margin: is amount of change in **the open-loop phase** needed to make a closed-loop system unstable. The phase margin is the difference between -180^0 and the phase at gain cross-over frequency (at gain = 0 dB).

A system will be stable if $\begin{cases} \left|G^o\right| > 0 \\ \angle G^o > 0 \end{cases}$

In general, the phase margin ~30-60^0 and the gain margin of 2-10 dB are desirable.
Large phase and gain margin -> stable but slow response.
Small phase and gain margin -> stable, fast response but oscillatory.

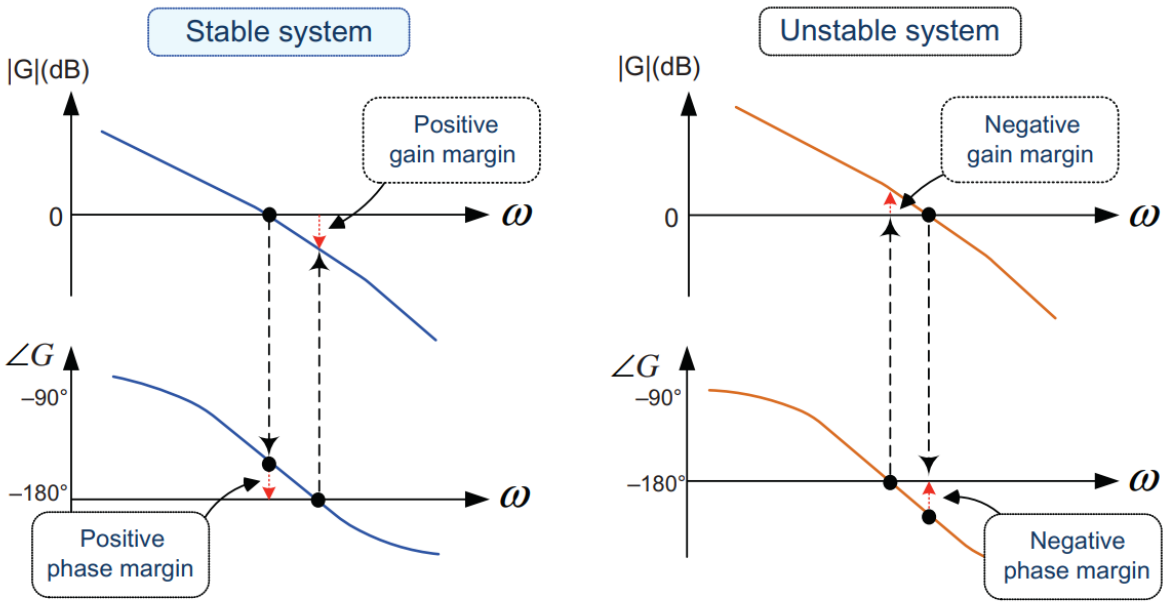

Fig 1: Bode plot of open-loop for stable and unstable system

2.Response time

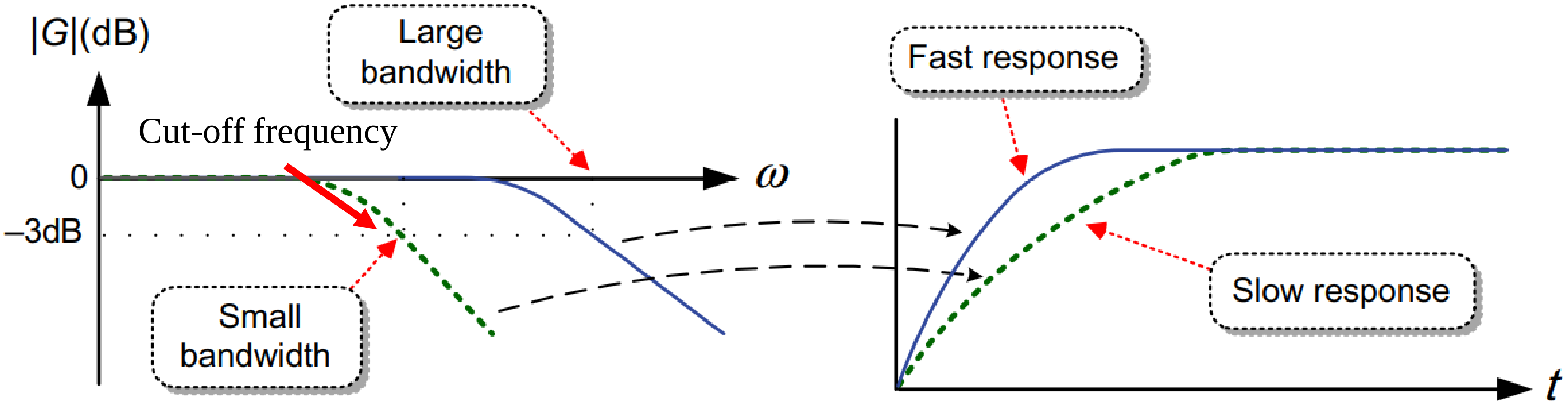

Fig 2: Bandwidth with response time

Cut-off frequency is the frequency at |G| = -3 dB.
Bandwidth is the range of frequencies for |G| > -3 dB (0 < f < cut-off frequency).
A system with wider bandwidth is faster response, but if bandwidth is too large, the system may be sensitive to noise and become unstable.

3. Mathematical description

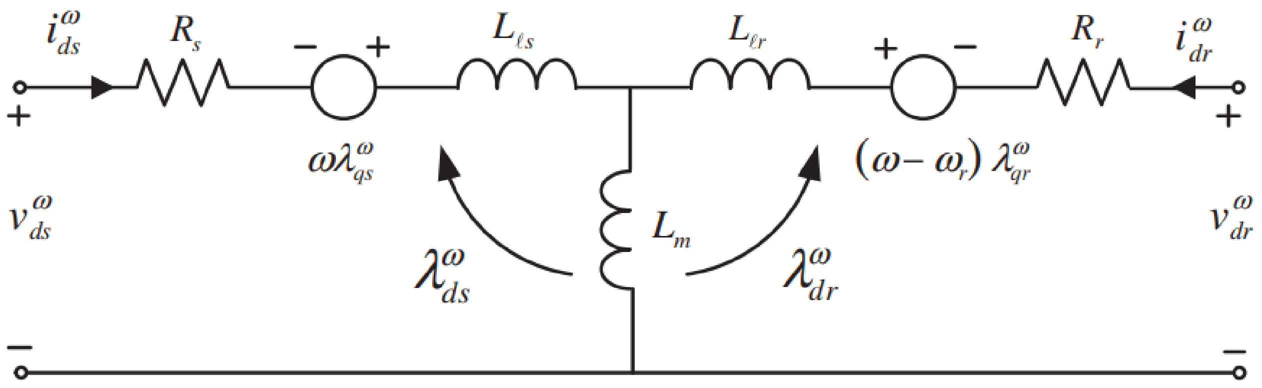

Fig 3.1: d-axe equivalent circuit of IM

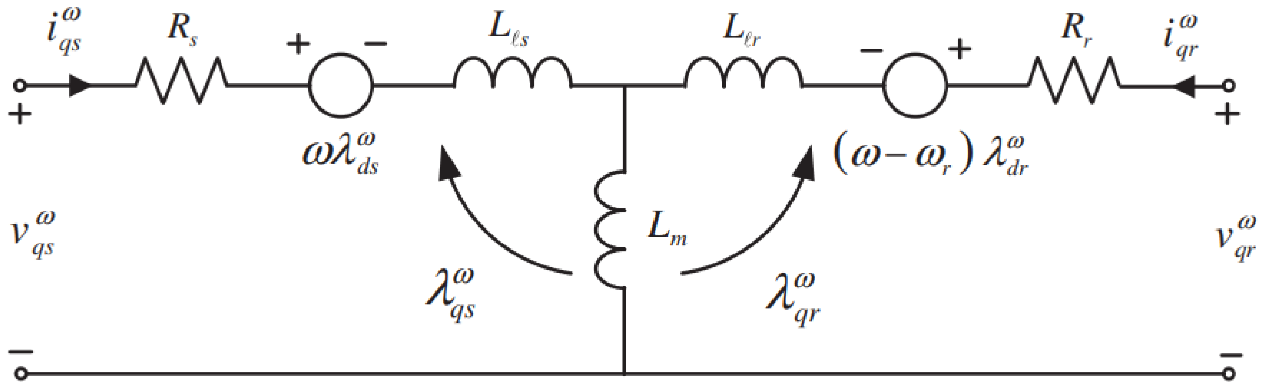

Fig 3.2: q-axe equivalent circuit of IM

Transform formula:

$$\begin{bmatrix} f_{ds} \\ f_{qs} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad (3.1)$$

Mathematical model:

Stationary frame:

$$v_{ds}^{s} = R_s i_{ds}^{s} + p\lambda_{ds}^{s} \quad (3.2) \qquad \lambda_{ds}^{s} = L_s i_{ds}^{s} + L_m i_{dr}^{s} \quad (3.6)$$

$$v_{qs}^{s} = R_s i_{qs}^{s} + p\lambda_{qs}^{s} \quad (3.3) \qquad \lambda_{qs}^{s} = L_s i_{qs}^{s} + L_m i_{qr}^{s} \quad (3.7)$$

$$0 = R_r i_{dr}^{s} + p\lambda_{dr}^{s} + \omega_r \lambda_{qr}^{s} \quad (3.4) \qquad \lambda_{dr}^{s} = L_r i_{dr}^{s} + L_m i_{ds}^{s} \quad (3.8)$$

$$0 = R_r i_{qr}^{s} + p\lambda_{qr}^{s} - \omega_r \lambda_{dr}^{s} \quad (3.5) \qquad \lambda_{qr}^{s} = L_r i_{qr}^{s} + L_m i_{qs}^{s} \quad (3.9)$$

3. Mathematical description

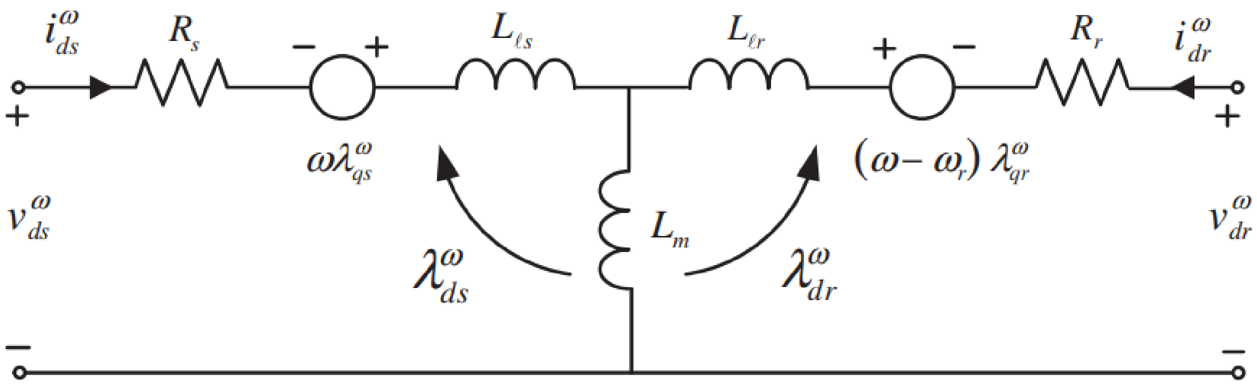

Fig 3.1: d-axe equivalent circuit of IM

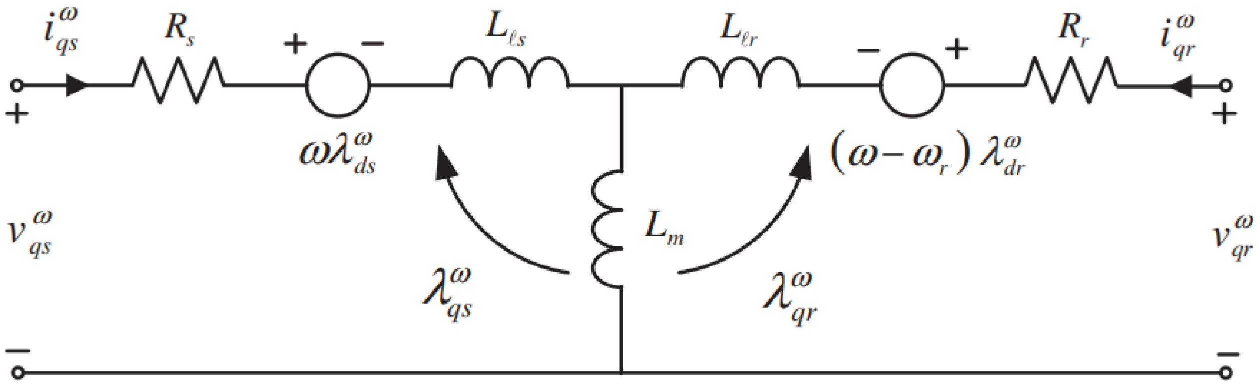

Fig 3.2: q-axe equivalent circuit of IM

Mathematical model:

Synchronous frame:

$$v_{ds}^{e} = R_s i_{ds}^{e} + p\lambda_{ds}^{e} - \omega_e \lambda_{qs}^{e} \quad (3.10) \qquad \lambda_{ds}^{e} = L_s i_{ds}^{e} + L_m i_{dr}^{e} \quad (3.14)$$

$$v_{qs}^{e} = R_s i_{qs}^{e} + p\lambda_{qs}^{e} + \omega_e \lambda_{ds}^{e} \quad (3.11) \qquad \lambda_{qs}^{e} = L_s i_{qs}^{e} + L_m i_{qr}^{e} \quad (3.15)$$

$$0 = R_r i_{dr}^{e} + p\lambda_{dr}^{e} - (\omega_e - \omega_r)\lambda_{qr}^{e} \quad (3.12) \qquad \lambda_{dr}^{e} = L_r i_{dr}^{e} + L_m i_{ds}^{e} \quad (3.16)$$

$$0 = R_r i_{qr}^{e} + p\lambda_{qr}^{e} + (\omega_e - \omega_r)\lambda_{dr}^{e} \quad (3.13) \qquad \lambda_{qr}^{e} = L_r i_{qr}^{e} + L_m i_{qs}^{e} \quad (3.17)$$

Rotor frame:

$$v_{ds}^{\omega} = R_s i_{ds}^{\omega} + \frac{d\lambda_{ds}^{\omega}}{dt} - \omega\lambda_{qs}^{\omega} \quad (3.18) \qquad v_{dr}^{\omega} = R_r i_{dr}^{\omega} + \frac{d\lambda_{dr}^{\omega}}{dt} - (\omega - \omega_r)\lambda_{qr}^{\omega} \quad (3.20)$$

$$v_{qs}^{\omega} = R_s i_{qs}^{\omega} + \frac{d\lambda_{qs}^{\omega}}{dt} + \omega\lambda_{ds}^{\omega} \quad (3.19) \qquad v_{qr}^{\omega} = R_r i_{qr}^{\omega} + \frac{d\lambda_{qr}^{\omega}}{dt} + (\omega - \omega_r)\lambda_{dr}^{\omega} \quad (3.21)$$

$$\lambda_{dr}^{\omega} = L_{lr} i_{dr}^{\omega} + L_m(i_{dr}^{\omega} + i_{ds}^{\omega}) = L_r i_{dr}^{\omega} + L_m i_{ds}^{\omega} \quad (3.22)$$

$$\lambda_{qr}^{\omega} = L_{lr} i_{qr}^{\omega} + L_m(i_{qr}^{\omega} + i_{qs}^{\omega}) = L_r i_{qr}^{\omega} + L_m i_{qs}^{\omega} \quad (3.23)$$

$$\lambda_{ds}^{\omega} = L_{ls} i_{ds}^{\omega} + L_m(i_{ds}^{\omega} + i_{dr}^{\omega}) = L_s i_{ds}^{\omega} + L_m i_{dr}^{\omega} \quad (3.24)$$

$$\lambda_{qs}^{\omega} = L_{ls} i_{qs}^{\omega} + L_m(i_{qs}^{\omega} + i_{qr}^{\omega}) = L_s i_{qs}^{\omega} + L_m i_{qr}^{\omega} \quad (3.25)$$

4. Field-oriented control of IM

Three requirement:

1. The space angle between the flux-producing current and the torque-producing current must be always be 90 degrees.
2. Both the flux-producing current and the torque-producing current should be controlled independently.
3. The torque-producing current can be controlled instantaneously.

For these reasons, flux vector is aligned to d-axe to producing constant flux, so we can control torque independently by changing q-axe current.

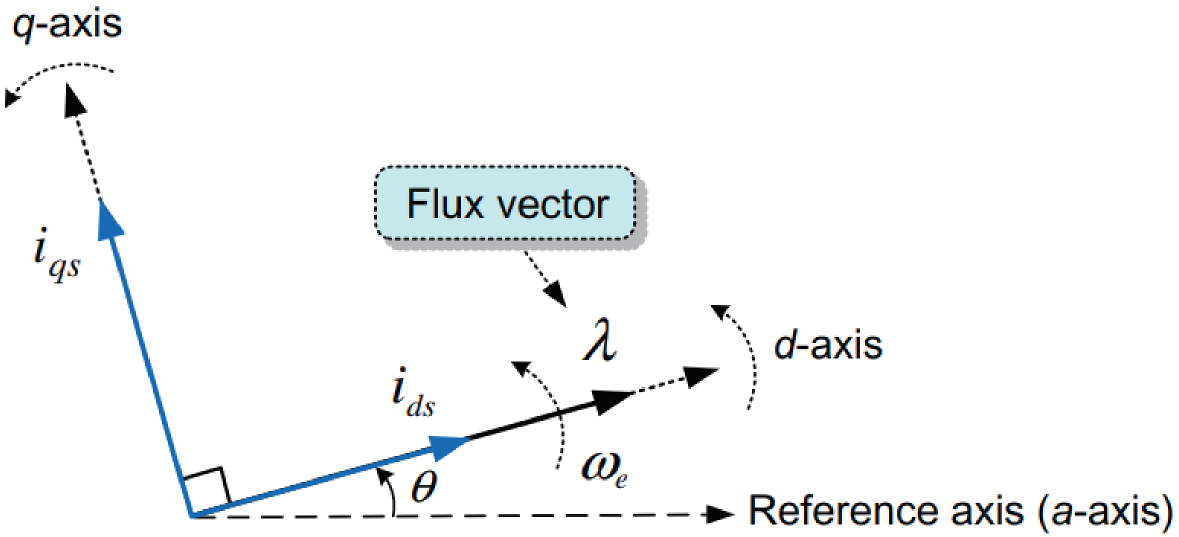

Fig 4.1: Assignment of d-q current

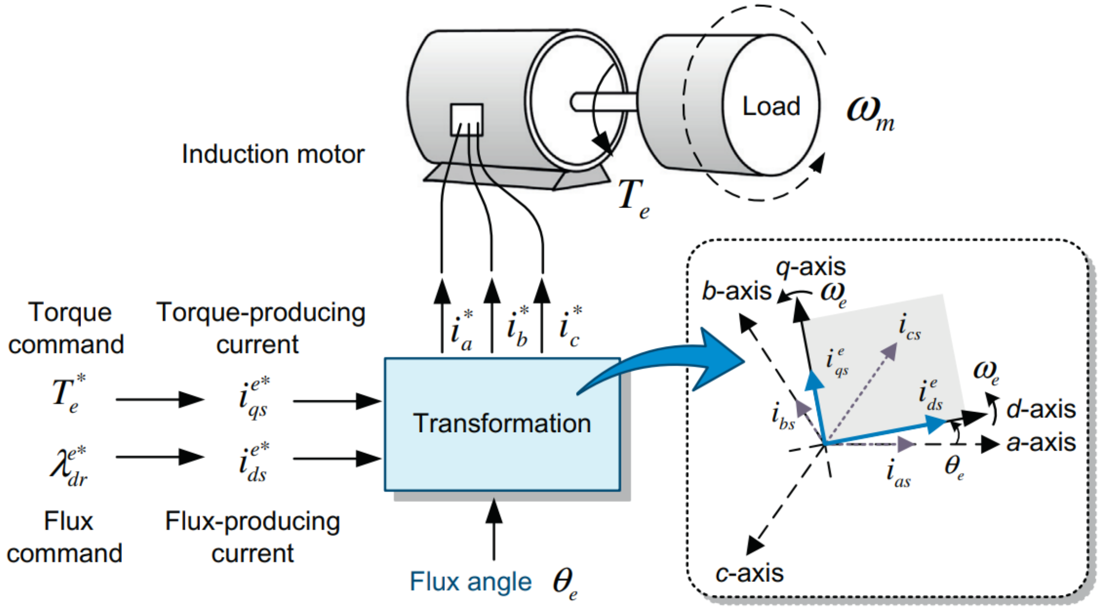

Fig 4.2: FOC of IM

5. Current controller

Leakage factor

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \qquad (5.1)$$

Substitute (3.16) and (5.1) into (3.14):

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e = L_s i_{ds}^e + L_m \left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r} \right) = \sigma L_s i_{ds}^e + \frac{L_m}{L_r} \lambda_{dr}^e \qquad (5.2)$$

Substitute (3.16) and (5.1) into (3.14):

$$\lambda_{qs}^e = \sigma L_s i_{qs}^e + \frac{L_m}{L_r} \lambda_{qr}^e \qquad (5.3)$$

For field-oriented control, q-axe flux equal to zero, thus (5.3) becomes: $\lambda_{qs}^e = \sigma L_s i_{qs}^e$ (5.4)

Substitute (5.2) and (5.4) into (3.10) and (3.11):

$$\begin{aligned} v_{ds}^e &= R_s i_{ds}^e + p\lambda_{ds}^e - \omega_e \lambda_{qs}^e \\ v_{qs}^e &= R_s i_{qs}^e + p\lambda_{qs}^e + \omega_e \lambda_{ds}^e \end{aligned} \quad \Rightarrow \quad \begin{aligned} v_{ds}^e &= R_s i_{ds}^e + \sigma L_s \frac{d}{dt} i_{ds}^e + \frac{L_m}{L_r} \frac{d}{dt} \lambda_{dr}^e - \omega_e \sigma L_s i_{qs}^e \\ v_{qs}^e &= R_s i_{qs}^e + \sigma L_s \frac{d}{dt} i_{qs}^e + \omega_e \frac{L_m}{L_r} \frac{d}{dt} \lambda_{dr}^e + \omega_e \sigma L_s i_{ds}^e \end{aligned}$$

$$\Rightarrow \quad v_{ds}^e = \left(R_s + R_r \frac{L_m^2}{L_r^2} \right) i_{ds}^e + \sigma L_s \frac{di_{ds}^e}{dt} \boxed{- \omega_e \sigma L_s i_{qs}^e - R_r \frac{L_m}{L_r^2} \lambda_{dr}^e} \qquad (5.6)$$

$$v_{qs}^e = \left(R_s + R_r \frac{L_m^2}{L_r^2} \right) i_{qs}^e + \sigma L_s \frac{di_{qs}^e}{dt} \boxed{+ \omega_e \sigma L_s i_{ds}^e + \omega_r \frac{L_m}{L_r} \lambda_{dr}^e} \qquad (5.5)$$

Feed-forward components

5. Current controller

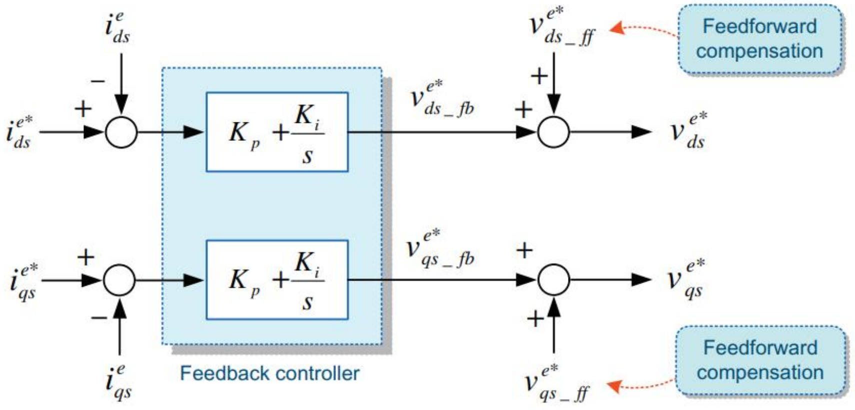

Fig 5.1: Current controller

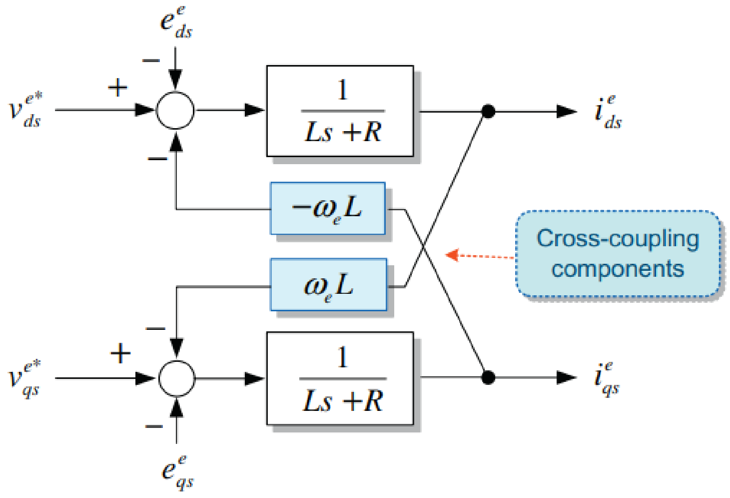

Fig 5.2: Coupling components

Let's assume feedforward components is ideally compensated, the current controller becomes:

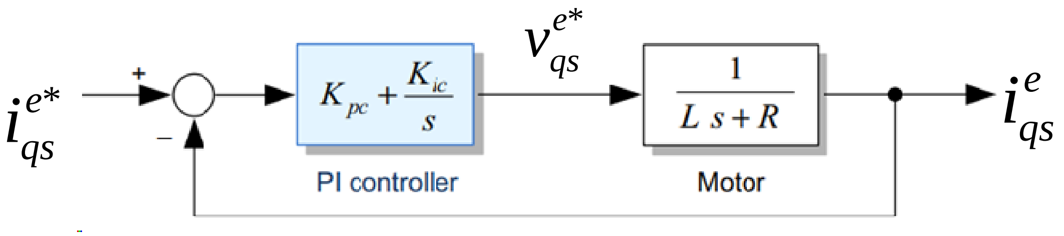

Fig 5.3: Simplified current controller

Open-loop transfer function of the system:

$$G^{o}(s) = K_{pc}\left(\frac{s+\frac{1}{\tau_{pi}}}{s}\right)\frac{1}{Ls+R} = K_{pc}\left(\frac{s+\frac{K_{ic}}{K_{pc}}}{s}\right)\frac{\frac{1}{L}}{s+\frac{R}{L}} \qquad (5.6)$$

Where: $R = R_s + R_r\left(\frac{L_m}{L_r}\right)^2$

$$L = \sigma L_s$$

5. Current controller

Open-loop transfer function of the system:

$$G^o(s) = K_{pc}\left(\frac{s+\dfrac{1}{\tau_{pi}}}{s}\right)\frac{1}{Ls+R} = K_{pc}\left(\frac{s+\dfrac{K_{ic}}{K_{pc}}}{s}\right)\frac{\dfrac{1}{L}}{s+\dfrac{R}{L}} \qquad (5.6)$$

From the eq. (5.6) if we choose:

$$\frac{1}{\tau_{pi}} = \frac{K_{ic}}{K_{pc}} = \frac{R}{L} \qquad (5.7)$$

And the eq. (5.6) becomes:

$$G^o(s) = \frac{1}{\dfrac{L}{K_{pc}}s} \qquad (5.8)$$

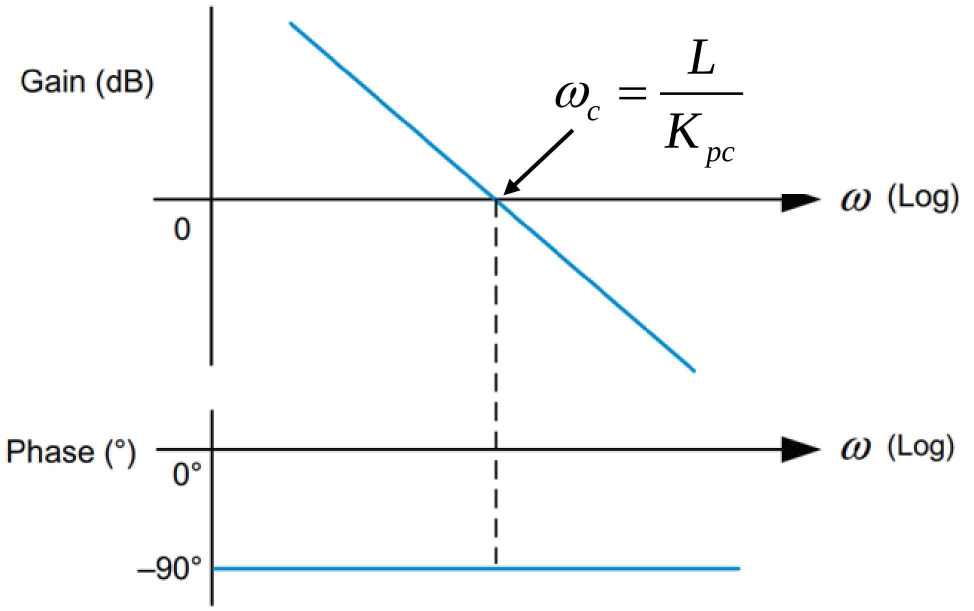

Fig 5.4: Bode plot of eq (5.8)

From the bode plot, we can see at the gain cross-over frequency, the phase is -90^0, this indicates that the system is table.
Now we can choose Kp and Ki:

Proportional gain: $K_p = \sigma L_s \cdot \omega_c$

Integral gain: $K_i = \left[R_s + R_r\left(\frac{L_m}{L_r}\right)^2\right]\cdot\omega_c$

6. Speed controller

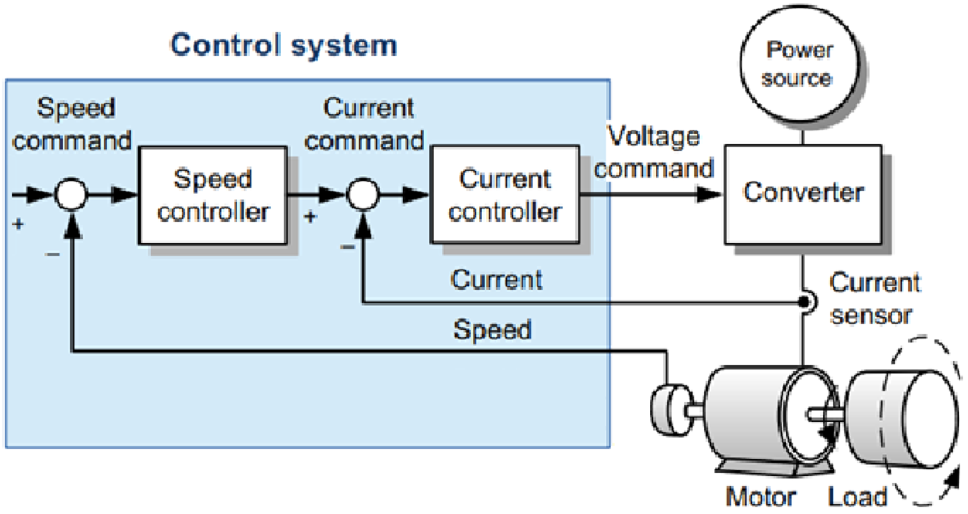

Fig 6.1: Motor control system

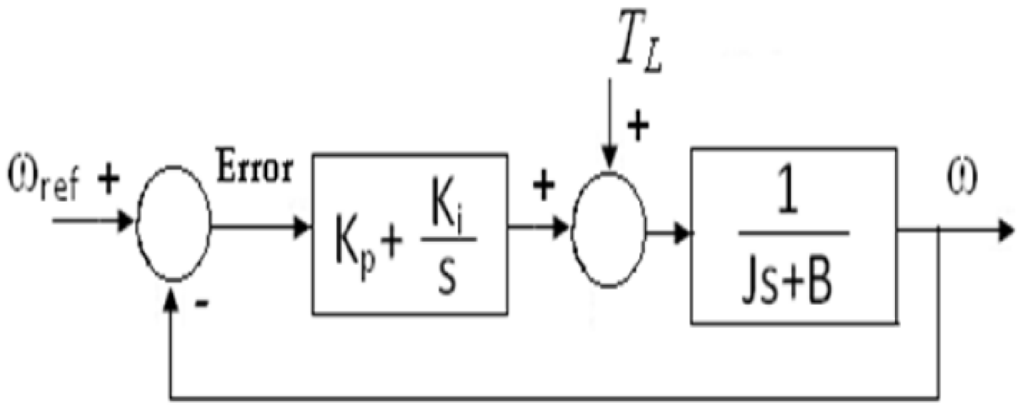

Fig 6.2: Simplified speed controller

Open-loop transfer function of the system:

$$\frac{\omega}{\omega_{ref}} = \frac{K_p s + K_i}{Js^2 + (B + K_p)s + K_i} \tag{6.1}$$

We consider a second-order polynomial d(s):

$$d(s) = s^2 + \frac{K_P + B}{J}s + \frac{K_I}{J} = s^2 + 2\xi\omega_n s + \omega_n^2 \tag{6.2}$$

Natural frequency and damping ratio of the system will be:

$$\Rightarrow \omega_n = \sqrt{\frac{K_I}{J}}, \quad \xi = \frac{K_P + B}{2J\omega_n}$$

Base on requirement of the system we can choose:

$$K_p = 2\xi\omega_n J - B \tag{6.3}$$

$$K_i = J\omega_n^2 \tag{6.4}$$

6. Speed controller

Some requirement of the system:

Overshoot formula:

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1, \qquad (6.5)$$

Settling time formula:

$$t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma} \qquad (6.6)$$

If we choose $\varsigma = 0.8$ and $\omega_n = 2\pi \times 5(rad/s)$

From eq. (6.5) and (6.6) Overshoot and settling time will be:

$$t_s = 0.18(s)$$

$$M_p = 15\%$$

Another technique is based on frequency response like current controller we can choose Kp and Kp for speed controller:

Proportional gain: $K_{ps} = \dfrac{J\omega_{cs}}{K_T}$ (6.7)

Integral gain: $K_{is} = K_{ps} \cdot \omega_{pi} = K_{ps} \cdot \dfrac{\omega_{cs}}{5} = \dfrac{J\omega_{cs}^2}{5K_T}$ (6.8)

Where:

$$K_T = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\lambda_{dr}^e \qquad (6.9)$$

And rotor d-axe flux can be calculated as:

$$I_{ds}^e = \frac{\sqrt{2}V_{phase}}{\sqrt{R_s^2 + X_s^2}} \qquad (6.10)$$

$$\lambda_{dr}^e = L_m I_{ds}^e \qquad (6.11)$$

7. Simulation result

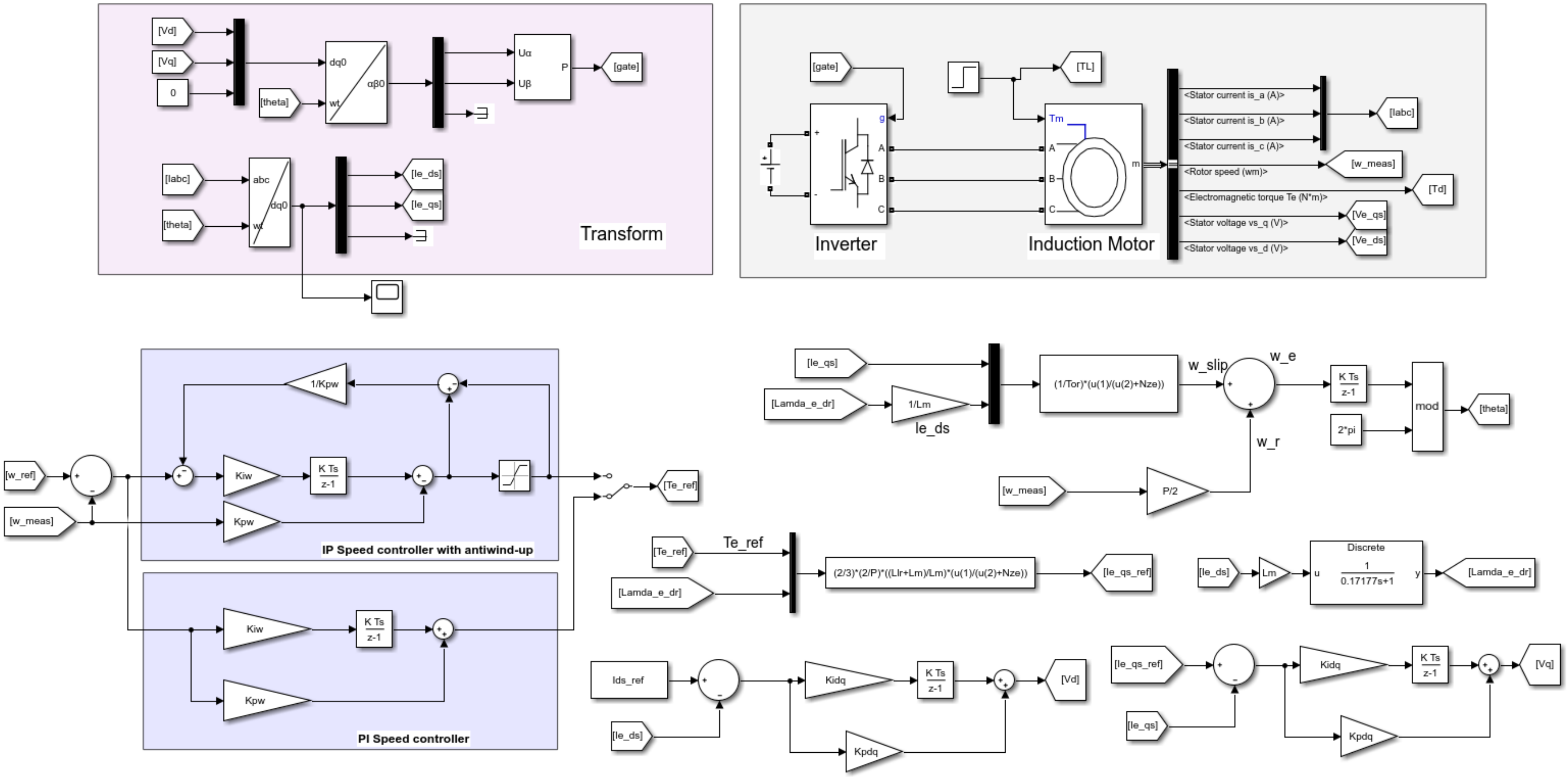

Fig 7.1: Simulink schematic

7. Simulation result

Parameters for simulation:

```
% Motor 2
%==============================
Pn = 7.5e3; Nn = 1440; Tn = 9.55*Pn/Nn;
Rs = 0.7384; Lls = 0.003045;
Rr = 0.7402; Llr = 0.003045;
Lm = 0.1241;
Ls = Lls + Lm;
Lr = Llr + Lm;
P = 4;  %poles
J = 0.0343;  B = 0.000503;
%==============================
%Motor 1
%==============================
%Pn = 15e3; Nn = 1460; Tn = 9.55*Pn/Nn;
%Rs = 0.2147; Lls = 0.000991;
%Rr = 0.2205; Llr = 0.000991;
%Lm = 0.06419;
%Ls = Lls + Lm;
%Lr = Llr + Lm;
%P = 4;
%J = 0.102;  B = 0.009541;
%==============================
```

```
Ts = 2e-5; Fsw = 2e3;
Tor = (Llr+Lm) /Rr;
sigma = 1 - Lm*Lm/(Ls*Lr);
Kpdq = (Rs+Rr*(Lm/Lr)^2)*2*pi*50;
Kidq  = sigma*Ls*2*pi*50;
Ids_ref = 220*sqrt(2)/sqrt(Rs^2+(2*pi*50*Ls)^2);
KT = (3/2)*(P/2)*(Lm^2/Lr)*Ids_ref;
%Kpw = J*2*pi*50/KT;
%Kiw = Kpw*2*pi*50/5;
Zt = 0.8; W_n = 2*pi*5;
Kpw = (2*Zt*W_n*J-B);
Kiw  = (J*W_n^2);
Nze = 1e-3;
```

7. Simulation result

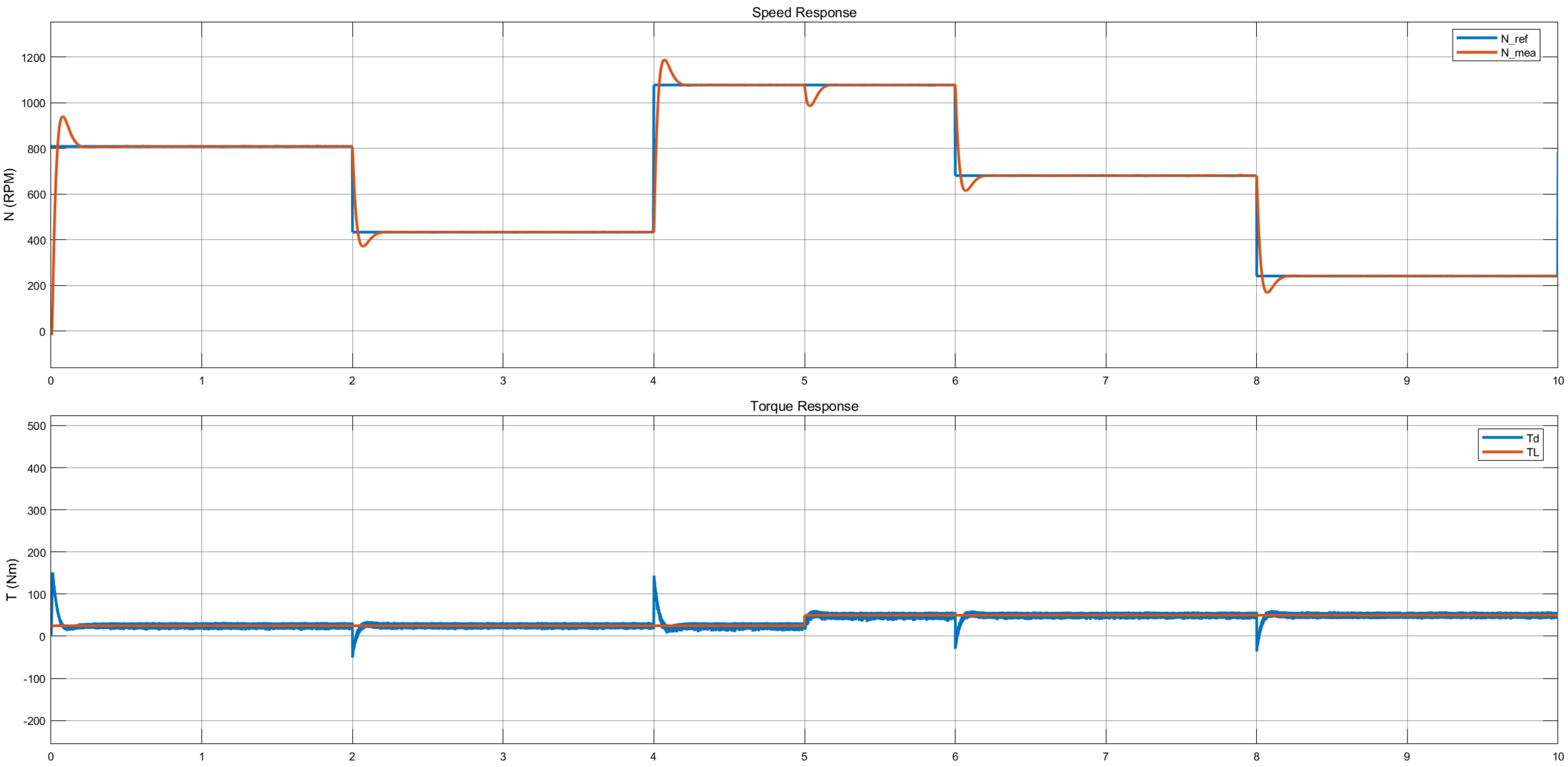

Fig 7.2: Speed response and torque response (PI controller)

7. Simulation result

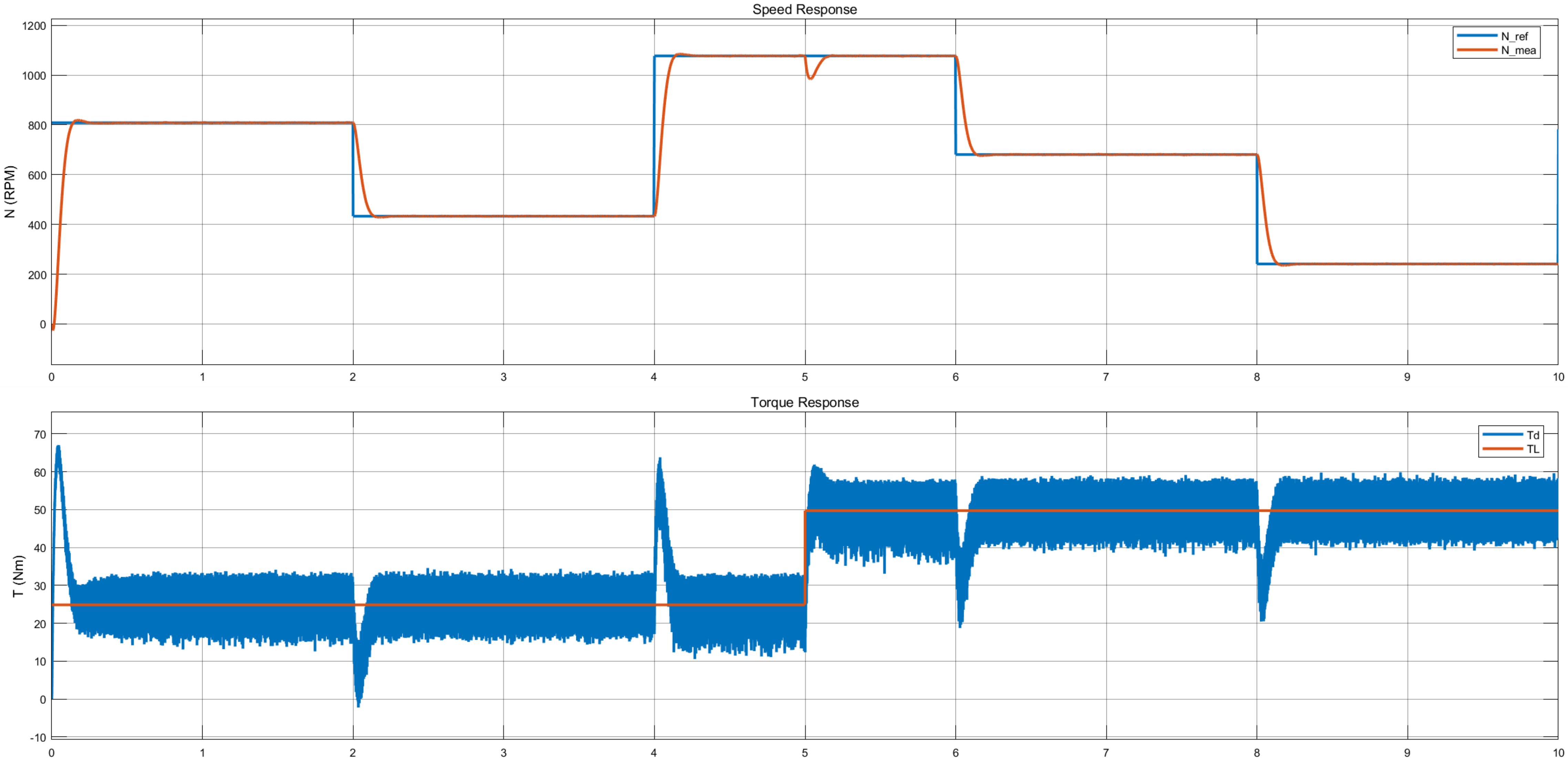

Fig 7.3: Speed response and torque response (IP controller with anti-windup)

8. Conclusion

In this presentation, we have explored various techniques for selecting parameters for a PI (Proportional-Integral) controller, including the renowned Ziegler-Nichols tuning method.

By conducting a comprehensive system analysis, I have illustrated an approach that simplifies the process of parameter selection for the PI controller.

My goal is to empower myself and you with the knowledge and insights needed to confidently determine the optimal parameters for your PI controller. While I have endeavored to provide accurate and valuable information, it's important to acknowledge that there may be certain aspects that could benefit from further refinement or clarification, owing to the limitations of my knowledge.

If you have any questions, concerns, or require additional information, please do not hesitate to reach out. I am here to assist and address any inquiries to the best of my knowledge and abilities.

Thank you for watching

8. Reference

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THANKS FOR WATCHING

HCMUTE

Monna Dang (Dang Hoang Anh Chuong)

22/10/23