"Tugas Pertemuan 10"

"Paulus Abraham Moung"

1/ $y = x^3 - 3x$ $\Rightarrow$ $-2, -1, 0, 1, 2$

$$m_{tan} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$$= \lim_{h \to 0} \frac{((c+h)^3 - 3(c+h)) - (c^3 - 3c)}{h}$$

$$= \lim_{h \to 0} \frac{c^3 + 3c^2h + 3ch^2 + h^3 - 3c - 3h - c^3 + 3c}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 3c^2h + 3ch^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(h^2 + 3c^2 + 3ch + 3)}{h}$$

$$= \lim_{h \to 0} h^2 + 3c^2 + 3ch + 3$$

$$= (0)^2 + 3c^2 + 3c(0) + 3$$

$$m_{tan} = 3c^2 + 3 \qquad \Rightarrow c = -2, -1, 0, 1, 2$$

$(\Rightarrow)$ $3(-2)^2 + 3 = 15$ $\Rightarrow$ titik koordinat $(-2, 15)$
$(\Rightarrow)$ $3(-1)^2 + 3 = 6$ $\Rightarrow$ titik koordinat $(-1, 6)$
$(\Rightarrow)$ $3(0)^2 + 3 = 3$ $\Rightarrow$ titik koordinat $(0, 3)$
$(\Rightarrow)$ $3(1)^2 + 3 = 6$ $\Rightarrow$ titik koordinat $(1, 6)$
$(\Rightarrow)$ $3(2)^2 + 3 = 15$ $\Rightarrow$ titik koordinat $(2, 15)$

2/ $y = \dfrac{1}{x-1}$ di titik $(0, -1)$

$$m_{tan} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{h-1} - \left(\frac{1}{-1}\right)}{h} = \lim_{h \to 0} \frac{\frac{1}{h-1} + 1}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1+h-1}{h-1}}{h} = \lim_{h \to 0} \frac{\frac{h}{h-1}}{h}$$

$$= \lim_{h \to 0} \frac{h}{h^2 - h} = \lim_{h \to 0} \frac{h}{h(h-1)}$$

$$= \lim_{h \to 0} \frac{1}{h-1} = \frac{1}{0-1} = -1 \; //$$

Singgung : $y - y_0 = m(x - x_0)$

$$\Rightarrow y - (-1) = -1(x-0)$$
$$y + 1 = -1(x-0) \; //$$

3/ $S = f(x) = t^2 + 1$ $\Rightarrow$ $t = 2$

$$V = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^2 + 1 - (2^2+1)}{h} = \frac{h^2 + 4h + 4 + 1 - 5}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 4h}{h} = \lim_{h \to 0} \frac{h(h+4)}{h}$$

$$= \lim_{h \to 0} h + 4 = 0 + 4$$

$V = 4$ $\Rightarrow$ Jadi kecepatan sesaat pada akhir 2 detik
          a/ 4 cm tiap detik.

4/ a/ $f(x) = 2x + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h) + 1 - (2x+1)}{h} = \lim_{h \to 0} \frac{2x + 2h + 1 - 2x - 1}{h}$$

$$f'(x) = \lim_{h \to 0} 2 = 2$$

A/ b/ $f(x) = x^3 + 2x + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ~~$(x+h)^3 + 2(x+h) + 1 - (x+h)$~~

$$= \lim_{h \to 0} \frac{((x+h)^3 + 2(x+h) + 1) - (x^2 + 2x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x + 2h + 1 - x^3 - 2x - 1}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 3x^2h + 3xh^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{h(h^2 + 3x^2 + 3xh + 2)}{h}$$

$$= \lim_{h \to 0} h^2 + 3x^2 + 3xh + 2$$

$$= (0)^2 + 3x^2 + 3x(0) + 2$$

$$f'(x) = 3x^2 + 2$$

5/ (a) $f(x) = 2x^{-2}$

$$= \frac{1}{2x^2}$$

$$f'(x) = \frac{1}{4x}$$

$$= 4x^{-1}$$

(b) $f(x) = x^{12} + 5x^2 - 3x^{-10}$

$$= 12x^{11} + 10x - \frac{1}{30x^9}$$

$$= 12x^{11} + 10x - 30x^{-9}$$

(c) $f(x) = (3x^2 + 2x)(x^4 - 3x + 1)$

$$\text{mis}: \quad u = 3x^2 + 2x \qquad v = x^4 - 3x + 1$$
$$u' = 6x + 2 \qquad v' = 4x^3 - 3$$

$$f'(x) = u'v + uv'$$
$$= (6x+2)(x^4 - 3x + 1) + (3x^2 + 2x)(4x^3 - 3)$$
$$= 6x^5 - 18x^2 + 6x + 2x^4 - 6x + 2 + 12x^5 - 9x^2 + 8x^4 - 6x$$
$$f'(x) = 18x^5 + 10x^4 - 27x^2 - 6x + 2$$

(d) $f(x) = \dfrac{5x^2 + 2x - 6}{3x - 1}$

$$\text{min}: \quad u = 5x^2 + 2x - 6 \qquad v = 3x - 1$$
$$u' = 10x + 2 \qquad v' = 3$$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{(10x+2)(3x-1) - (5x^2 + 2x - 6)(3)}{(3x-1)^2}$$

$$= \frac{30x^2 - 10x + 6x - 2 - 15x^2 - 6x + 18}{(3x-1)^2}$$

$$= \frac{15x^2 - 10x + 16}{9x^2 - 6x + 1}$$