

Deriving exchange in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{y_0 : GB \vdash_C y_0 : GB} \text{AX}}{y_0 : GB \vdash_{\mathcal{L}} Fy_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash_C x_0 : GA} \text{AX}}{x_0 : GA \vdash_{\mathcal{L}} Fx_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA \vdash_{\mathcal{L}} Fy_0 \triangleright Fx_0 : FGB \triangleright FGA} \text{TENR}}{x_1 : GA, y_1 : GB \vdash_{\mathcal{L}} \text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0) : FGB \triangleright FGA} \text{BETA}} \\
\frac{x_1 : GA, y_2 : FGB \vdash_{\mathcal{L}} \text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0)) : FGB \triangleright FGA}{x_2 : FGA, y_2 : FGB \vdash_{\mathcal{L}} \text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0))) : FGB \triangleright FGA} \text{FL}} \\
\frac{z : FGA \triangleright FGB \vdash_{\mathcal{L}} \text{let } z : FGA \triangleright FGB \text{ be } x_2 \triangleright y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0)))) : (FGB \triangleright FGA)}{\cdot \vdash_{\mathcal{L}} \lambda_r z : FGA \triangleright FGB. \text{let } z : FGA \triangleright FGB \text{ be } x_2 \triangleright y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0)))) : (FGA \triangleright FGB) \multimap (FGB \triangleright FGA)} \text{IMPRR}
\end{array}$$

Deriving right contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_1 : GA \vdash_C x_1 : GA} \text{AX}}{x_1 : GA \vdash_{\mathcal{L}} Fx_1 : FGA} \text{Fr} \quad \frac{\frac{\frac{\frac{}{y_0 : GB \vdash_C y_0 : GB} \text{AX}}{y_0 : GB \vdash_{\mathcal{L}} Fy_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash_C x_0 : GA} \text{AX}}{x_0 : GA \vdash_{\mathcal{L}} Fx_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA \vdash_{\mathcal{L}} Fy_0 \triangleright Fx_0 : FGB \triangleright FGA} \text{TENR}}{x_1 : GA, y_0 : GB, x_0 : GA \vdash_{\mathcal{L}} Fx_1 \triangleright (Fy_0 \triangleright Fx_0) : FGA \triangleright (FGB \triangleright FGA)} \text{TENR}} \\
\frac{y_0 : GB, x_2 : GA \vdash_{\mathcal{L}} \text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)) : FGA \triangleright (FGB \triangleright FGA)}{\text{CONTRR}} \\
\frac{y_0 : GB, x_3 : FGA \vdash_{\mathcal{L}} \text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0))) : FGA \triangleright (FGB \triangleright FGA)}{\text{FL}} \\
\frac{y_1 : FGB, x_3 : FGA \vdash_{\mathcal{L}} \text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)))) : FGA \triangleright (FGB \triangleright FGA)}{\text{FL}} \\
\frac{z : FGB \triangleright FGA \vdash_{\mathcal{L}} \text{let } z : FGB \triangleright FGA \text{ be } y_1 \triangleright x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)))) : FGA \triangleright (FGB \triangleright FGA)}{\cdot \vdash_{\mathcal{L}} \lambda_r z : FGB \triangleright FGA. \text{let } z : FGB \triangleright FGA \text{ be } y_1 \triangleright x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)))) : (FGB \triangleright FGA) \multimap (FGA \triangleright (FGB \triangleright FGA))} \text{IMPRR}
\end{array}$$

Deriving left contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : GA \vdash_C x_0 : GA} \text{AX}}{x_0 : GA \vdash_{\mathcal{L}} Fx_0 : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash_C y_0 : GB} \text{AX}}{y_0 : GB \vdash_{\mathcal{L}} Fy_0 : FGB} \text{Fr}}{x_0 : GA, y_0 : GB \vdash_{\mathcal{L}} Fx_0 \triangleright Fy_0 : FGA \triangleright FGB} \text{TENR}} \\
\frac{x_1 : GA \vdash_C x_1 : GA}{x_1 : GA \vdash_{\mathcal{L}} Fx_1 : FGA} \text{Fr} \\
\frac{x_0 : GA, y_0 : GB, x_1 : GA \vdash_{\mathcal{L}} (Fx_0 \triangleright Fy_0) \triangleright Fx_1 : (FGA \triangleright FGB) \triangleright FGA}{x_2 : GA, y_0 : GB \vdash_{\mathcal{L}} \text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1) : (FGA \triangleright FGB) \triangleright FGA} \text{CONTRR}} \\
\frac{x_2 : GA, y_1 : FGB \vdash_{\mathcal{L}} \text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1)) : (FGA \triangleright FGB) \triangleright FGA}{x_3 : FGA, y_1 : FGB \vdash_{\mathcal{L}} \text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1))) : (FGA \triangleright FGB) \triangleright FGA} \text{FL}} \\
\frac{z : FGA \triangleright FGB \vdash_{\mathcal{L}} \text{let } z : FGA \triangleright FGB \text{ be } x_3 \triangleright y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1)))) : (FGA \triangleright FGB) \triangleright FGA}{\cdot \vdash_{\mathcal{L}} \lambda_r z : FGA \triangleright FGB. \text{let } z : FGA \triangleright FGB \text{ be } x_3 \triangleright y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1)))) : (FGA \triangleright FGB) \multimap ((FGA \triangleright FGB) \triangleright FGA)} \text{IMPRR}
\end{array}$$

Deriving weakening in Elle comonadically:

$$\begin{array}{c}
\frac{\cdot \vdash_{\mathcal{L}} \text{trivS} : \text{UnitS}}{x_0 : GA \vdash_{\mathcal{L}} \text{weak } x_0 \text{ in trivS} : \text{UnitS}} \text{WEAK} \\
\frac{x_1 : FGA \vdash_{\mathcal{L}} \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in trivS} : \text{UnitS}}{\cdot \vdash_{\mathcal{L}} \lambda_r x_1 : FGA. \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in trivS} : (FGA) \multimap (\text{UnitS})} \text{IMPRR}
\end{array}$$

$GF$  is a monad:

- Deriving  $\eta$ :

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : FX \vdash_{\mathcal{L}} x_0 : FX} \text{AX}}{x_1 : GFX \vdash_{\mathcal{L}} \text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0 : FX} \text{GL}}{x_2 : FGFX \vdash_{\mathcal{L}} \text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0) : FX} \text{FL}} \\
\frac{x_3 : FGFX \vdash_{\mathcal{L}} \text{let } x_3 : FGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0)) : FX}{x_3 : GFGFX \vdash_C G(\text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0))) : GFX} \text{GL}} \\
\frac{\cdot \vdash_{\mathcal{L}} \lambda_r x_3 : GFGFX. G(\text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0))) : (GFGFX) \multimap (GFX)}{\cdot \vdash_{\mathcal{L}} \lambda_r x_3 : GFGFX. G(\text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0))) : (GFGFX) \multimap (GFX)} \text{IMPRR}
\end{array}$$

- Deriving  $\mu$ :

$$\frac{\frac{\frac{}{x : X \vdash_C x : X} \text{AX}}{x : X \vdash_{\mathcal{L}} Fx : FX} \text{Fr}}{x : X \vdash_C GFx : GFX} \text{Gr}}{\vdash_C \lambda x : X. GFx : (X) \multimap (GFX)} \text{IMPRR}$$

The monad  $GF$  is strong:

- Deriving the tensorial strength  $\tau$ :

$$\frac{\frac{\frac{\frac{\frac{}{x_0 : X \vdash_C x_0 : X} \text{AX}}{x_0 : X, y_0 : Y \vdash_C x_0 \otimes y_0 : X \otimes Y} \text{TENR}}{x_0 : X, y_0 : Y \vdash_{\mathcal{L}} F(x_0 \otimes y_0) : F(X \otimes Y)} \text{Fr}}{x_0 : X, y_1 : FY \vdash_{\mathcal{L}} \text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0) : F(X \otimes Y)} \text{FL}}{\frac{y_2 : GFY, x_0 : X \vdash_{\mathcal{L}} \text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)) : F(X \otimes Y)}{y_2 : GFY, x_0 : X \vdash_C G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0))) : GF(X \otimes Y)} \text{Gr}} \text{GL}}{\frac{x_1 : X, y_3 : GFY \vdash_C \text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)))) : GF(X \otimes Y)}{z : X \otimes GFY \vdash_C \text{let } z : X \otimes GFY \text{ be } x_1 \otimes y_3 \text{ in } (\text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)))))) : GF(X \otimes Y)} \text{BETA}} \text{TENL}}{\vdash_C \lambda z : X \otimes GFY. \text{let } z : X \otimes GFY \text{ be } x_1 \otimes y_3 \text{ in } (\text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)))))) : (X \otimes GFY) \multimap (GF(X \otimes Y))} \text{IMPRR}$$

## A Full Ott Spec

*vars, n, a, x, y, z, w, m, o*

*ivar, i, k, j, l*

*const, b*

*A, B, C* ::=  
 | B  
 | UnitS  
 |  $A \triangleright B$   
 |  $A \multimap B$   
 |  $A \leftarrow B$   
 | FX

*X, Y, Z* ::=  
 | B  
 | UnitT  
 |  $X \otimes Y$   
 |  $X \multimap Y$   
 | GA

*T* ::=  
 | A  
 | X

*p* ::=

		★	
		$x$	
		trivT	
		trivS	
		$p \otimes p'$	
		$p \triangleright p'$	
		$F p$	
		$G p$	
$s$	::=		
		$x$	
		$b$	
		trivS	
		let $s_1 : T$ be $p$ in $s_2$	
		let $t : T$ be $p$ in $s$	
		$s_1 \triangleright s_2$	
		$\lambda_l x : A.s$	
		$\lambda_r x : A.s$	
		app <sub><math>l</math></sub> $s_1 s_2$	
		app <sub><math>r</math></sub> $s_1 s_2$	
		ex $s_1, s_2$ with $x_1, x_2$ in $s_3$	
		contrR $x$ as $s_1, s_2$ in $s_3$	
		contrL $x$ as $s_1, s_2$ in $s_3$	
		weak $x$ in $s$	
		( $s$ )	S
		$F t$	
$t$	::=		
		$x$	
		$b$	
		trivT	
		let $t_1 : X$ be $p$ in $t_2$	
		$t_1 \otimes t_2$	
		$\lambda x : X.t$	
		app $t_1 t_2$	
		ex $t_1, t_2$ with $x_1, x_2$ in $t_3$	
		contrR $x$ as $t_1, t_2$ in $t_3$	
		contrR $x$ as $t_1, t_2$ in $t_3$	
		weak $x$ in $t$	
		( $t$ )	S
		$G s$	
$\Phi, \Psi$	::=		

$$\begin{array}{c|c}
\Gamma, \Delta & ::= \\
\hline
& \cdot \\
& \Phi_1, \Phi_2 \\
& x : X \\
& (\Phi) \quad \text{S}
\end{array}$$

$$\boxed{\Phi \vdash_C t : X}$$

$$\begin{array}{c}
\frac{}{x : X \vdash_C x : X} \quad \text{T\_AX} \\
\\
\frac{\Phi, \Psi \vdash_C t : X}{\Phi, x : \text{UnitT}, \Psi \vdash_C \text{let } x : \text{UnitT} \text{ be } \text{trivT} \text{ in } t : X} \quad \text{T\_UNITL} \\
\\
\frac{}{\cdot \vdash_C \text{trivT} : \text{UnitT}} \quad \text{T\_UNITR} \\
\\
\frac{\Phi, x : X, y : Y, \Psi \vdash_C t : Z}{\Phi, z : Y, w : X, \Psi \vdash_C \text{ex } w, z \text{ with } x, y \text{ in } t : Z} \quad \text{T\_BETA} \\
\\
\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash_C t : Y}{\Phi_1, \Phi_2, z : X, \Phi_3 \vdash_C \text{contrR } z \text{ as } x, y \text{ in } t : Y} \quad \text{T\_CONTRR} \\
\\
\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash_C t : Y}{\Phi_1, z : X, \Phi_2, \Phi_3 \vdash_C \text{contrR } z \text{ as } x, y \text{ in } t : Y} \quad \text{T\_CONTRL} \\
\\
\frac{\Phi, \Psi \vdash_C t : Y \quad x \notin |\Phi, \Psi|}{\Phi, x : X, \Psi \vdash_C \text{weak } x \text{ in } t : Y} \quad \text{T\_WEAK} \\
\\
\frac{\Phi \vdash_C t_1 : X \quad \Psi_1, x : X, \Psi_2 \vdash_C t_2 : Y}{\Psi_1, \Phi, \Psi_2 \vdash_C [t_1/x]t_2 : Y} \quad \text{T\_CUT} \\
\\
\frac{\Phi, x : X, y : Y, \Psi \vdash_C t : Z}{\Phi, z : X \otimes Y, \Psi \vdash_C \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } t : Z} \quad \text{T\_TENL} \\
\\
\frac{\Phi \vdash_C t_1 : X \quad \Psi \vdash_C t_2 : Y}{\Phi, \Psi \vdash_C t_1 \otimes t_2 : X \otimes Y} \quad \text{T\_TENR} \\
\\
\frac{\Phi \vdash_C t_1 : X \quad \Psi_1, x : Y, \Psi_2 \vdash_C t_2 : Z}{\Psi_1, \Phi, y : X \multimap Y, \Psi_2 \vdash_C [\text{app } y t_1/x]t_2 : Z} \quad \text{T\_IMPL} \\
\\
\frac{\Phi, x : X, \Psi \vdash_C t : Y}{\Phi, \Psi \vdash_C \lambda x : X. t : X \multimap Y} \quad \text{T\_IMPR} \\
\\
\frac{\Phi \vdash_{\mathcal{L}} s : A}{\Phi \vdash_C \text{Gs} : \text{GA}} \quad \text{T\_GR}
\end{array}$$

$$\boxed{\Gamma \vdash_{\mathcal{L}} s : A}$$

$$\frac{}{x : A \vdash_{\mathcal{L}} x : A} \quad \text{S\_AX}$$

$$\begin{array}{c}
\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, x : \text{UnitT}, \Delta \vdash_{\mathcal{L}} \text{let } x : \text{UnitT} \text{ be } \text{trivT} \text{ in } s : A} \quad \text{S\_UNITL1} \\
\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, x : \text{UnitS}, \Delta \vdash_{\mathcal{L}} \text{let } x : \text{UnitS} \text{ be } \text{trivS} \text{ in } s : A} \quad \text{S\_UNITL2} \\
\frac{}{\cdot \vdash_{\mathcal{L}} \text{trivS} : \text{UnitS}} \quad \text{S\_UNITR} \\
\frac{\Gamma, x : X, y : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, z : Y, w : X, \Delta \vdash_{\mathcal{L}} \text{ex } w, z \text{ with } x, y \text{ in } s : A} \quad \text{S\_BETA} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} s : A}{\Gamma_1, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} \text{contrR } z \text{ as } x, y \text{ in } s : A} \quad \text{S\_CONTRR} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} s : A}{\Gamma_1, y : X, \Gamma_2, \Gamma_3 \vdash_{\mathcal{L}} \text{contrR } z \text{ as } x, y \text{ in } s : A} \quad \text{S\_CONTRL} \\
\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A \quad x \notin |\Gamma, \Delta|}{\Gamma, x : X, \Delta \vdash_{\mathcal{L}} \text{weak } x \text{ in } s : B} \quad \text{S\_WEAK} \\
\frac{\Phi \vdash_C t : X \quad \Gamma_1, x : X, \Gamma_2 \vdash_{\mathcal{L}} s : A}{\Gamma_1, \Phi, \Gamma_1 \vdash_{\mathcal{L}} [t/x]s : A} \quad \text{S\_CUT1} \\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta_1, x : A, \Delta_2 \vdash_{\mathcal{L}} s_2 : B}{\Delta_1, \Gamma, \Delta_2 \vdash_{\mathcal{L}} [s_1/x]s_2 : B} \quad \text{S\_CUT2} \\
\frac{\Gamma, x : X, y : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, z : X \otimes Y, \Delta \vdash_{\mathcal{L}} \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : A} \quad \text{S\_TENL1} \\
\frac{\Gamma, x : A, y : B, \Delta \vdash_{\mathcal{L}} s : C}{\Gamma, z : A \triangleright B, \Delta \vdash_{\mathcal{L}} \text{let } z : A \triangleright B \text{ be } x \triangleright y \text{ in } s : C} \quad \text{S\_TENL2} \\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta \vdash_{\mathcal{L}} s_2 : B}{\Gamma, \Delta \vdash_{\mathcal{L}} s_1 \triangleright s_2 : A \triangleright B} \quad \text{S\_TENR} \\
\frac{\Phi \vdash_C t : X \quad \Gamma, x : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, \Phi, y : X \multimap Y, \Delta \vdash_{\mathcal{L}} [\text{app } y t/x]s : A} \quad \text{S\_IMPL} \\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta, x : B \vdash_{\mathcal{L}} s_2 : C}{\Delta, y : A \multimap B, \Gamma \vdash_{\mathcal{L}} [\text{app}_r y s_1/x]s_2 : C} \quad \text{S\_IMPRL} \\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad x : B, \Delta \vdash_{\mathcal{L}} s_2 : C}{\Gamma, y : B \multimap A, \Delta \vdash_{\mathcal{L}} [\text{app}_l y s_1/x]s_2 : C} \quad \text{S\_IMPLL} \\
\frac{\Gamma, x : A \vdash_{\mathcal{L}} s : B}{\Gamma \vdash_{\mathcal{L}} \lambda_r x : A. s : A \multimap B} \quad \text{S\_IMPRR} \\
\frac{x : A, \Gamma \vdash_{\mathcal{L}} s : B}{\Gamma \vdash_{\mathcal{L}} \lambda_l x : A. s : B \multimap A} \quad \text{S\_IMPLR} \\
\frac{\Phi \vdash_C t : X}{\Phi \vdash_{\mathcal{L}} \text{F } t : \text{FX}} \quad \text{S\_FR} \\
\frac{\Gamma, x : X, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, y : \text{FX}, \Delta \vdash_{\mathcal{L}} \text{let } y : \text{FX} \text{ be } \text{F } x \text{ in } s : A} \quad \text{S\_FL} \\
\frac{\Gamma, x : A, \Delta \vdash_{\mathcal{L}} s : B}{\Gamma, y : \text{GA}, \Delta \vdash_{\mathcal{L}} \text{let } y : \text{GA} \text{ be } \text{G } x \text{ in } s : B} \quad \text{S\_GL}
\end{array}$$