

Deriving exchange in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash F y_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash F x_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA; \cdot \vdash F y_0 \otimes F x_0 : FGB \otimes FGA} \text{TENR}}{x_1 : GA, y_1 : GB; \cdot \vdash \text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F y_0 \otimes F x_0) : FGB \otimes FGA} \text{BETA}} \\
\frac{x_1 : GA; y_2 : FGB \vdash \text{let } y_2 : FGB \text{ be } F y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F y_0 \otimes F x_0)) : FGB \otimes FGA}{\vdash; x_2 : FGA, y_2 : FGB \vdash \text{let } x_2 : FGA \text{ be } F x_1 \text{ in } (\text{let } y_2 : FGB \text{ be } F y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F y_0 \otimes F x_0))) : FGB \otimes FGA} \text{FL}} \\
\frac{\vdash; z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } F x_1 \text{ in } (\text{let } y_2 : FGB \text{ be } F y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F y_0 \otimes F x_0)))) : (FGB \otimes FGA)}{\vdash; \cdot \vdash \lambda_{l,z} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } F x_1 \text{ in } (\text{let } y_2 : FGB \text{ be } F y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F y_0 \otimes F x_0)))) : (FGA \otimes FGB) \multimap (FGB \otimes FGA)} \text{IMP R}
\end{array}$$

Deriving right contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_1 : GA \vdash x_1 : GA} \text{AX}}{x_1 : GA; \cdot \vdash F x_1 : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash F y_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash F x_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA; \cdot \vdash F y_0 \otimes F x_0 : FGB \otimes FGA} \text{TENR}}{x_1 : GA, y_0 : GB, x_0 : GA; \cdot \vdash F x_1 \otimes (F y_0 \otimes F x_0) : FGA \otimes (FGB \otimes FGA)} \text{TENR}} \\
\frac{y_0 : GB, x_2 : GA; \cdot \vdash \text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (F x_1 \otimes (F y_0 \otimes F x_0)) : FGA \otimes (FGB \otimes FGA)}{\vdash; y_1 : FGB, x_3 : FGA \vdash \text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (F x_1 \otimes (F y_0 \otimes F x_0))) : FGA \otimes (FGB \otimes FGA)} \text{FL}} \\
\frac{\vdash; y_1 : FGB, x_3 : FGA \vdash \text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (F x_1 \otimes (F y_0 \otimes F x_0)))) : FGA \otimes (FGB \otimes FGA)}{\vdash; z : FGB \otimes FGA \vdash \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (F x_1 \otimes (F y_0 \otimes F x_0)))) : FGA \otimes (FGB \otimes FGA)} \text{TENL}} \\
\vdash; \cdot \vdash \lambda_{l,z} : FGB \otimes FGA. \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (F x_1 \otimes (F y_0 \otimes F x_0)))) : (FGB \otimes FGA) \multimap (FGA \otimes (FGB \otimes FGA)) \text{IMP R}
\end{array}$$

Deriving left contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash F x_0 : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash F y_0 : FGB} \text{Fr}}{x_0 : GA, y_0 : GB; \cdot \vdash F x_0 \otimes F y_0 : FGA \otimes FGB} \text{TENR} \quad \frac{\frac{\frac{}{x_1 : GA \vdash x_1 : GA} \text{AX}}{x_1 : GA; \cdot \vdash F x_1 : FGA} \text{Fr}}{x_1 : GA, y_0 : GB, x_1 : GA; \cdot \vdash (F x_0 \otimes F y_0) \otimes F x_1 : (FGA \otimes FGB) \otimes FGA} \text{TENR}} \\
\frac{x_2 : GA, y_0 : GB; \cdot \vdash \text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F x_0 \otimes F y_0) \otimes F x_1) : (FGA \otimes FGB) \otimes FGA}{x_2 : GA, y_1 : FGB \vdash \text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F x_0 \otimes F y_0) \otimes F x_1)) : (FGA \otimes FGB) \otimes FGA} \text{FL}} \\
\frac{\vdash; x_3 : FGA, y_1 : FGB \vdash \text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F x_0 \otimes F y_0) \otimes F x_1))) : (FGA \otimes FGB) \otimes FGA}{\vdash; z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F x_0 \otimes F y_0) \otimes F x_1)))) : (FGA \otimes FGB) \otimes FGA} \text{TENL}} \\
\vdash; \cdot \vdash \lambda_{l,z} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } F x_2 \text{ in } (\text{let } y_1 : FGB \text{ be } F y_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F x_0 \otimes F y_0) \otimes F x_1)))) : (FGA \otimes FGB) \multimap ((FGA \otimes FGB) \otimes FGA) \text{IMP R}
\end{array}$$

Deriving weakening in Elle comonadically:

$$\begin{array}{c}
\frac{}{\cdot \vdash \text{triv} : \text{Unit}} \text{UNIT R} \\
\frac{x_0 : GA; \cdot \vdash \text{weak } x_0 \text{ in triv} : \text{Unit}}{\vdash; x_1 : FGA \vdash \text{let } x_1 : FGA \text{ be } F x_0 \text{ in weak } x_0 \text{ in triv} : \text{Unit}} \text{FL} \\
\vdash; \cdot \vdash \lambda_{l,x_1} : FGA. \text{let } x_1 : FGA \text{ be } F x_0 \text{ in weak } x_0 \text{ in triv} : (FGA) \multimap (\text{Unit}) \text{IMP R}
\end{array}$$

GF is a monad:

- Deriving η :

$$\begin{array}{c}
\frac{\frac{}{\vdash; x_0 : FX \vdash x_0 : FX} \text{AX}}{x_1 : GFX; \cdot \vdash \text{let } x_1 : GFX \text{ be } G x_0 \text{ in } x_0 : FX} \text{GL}} \\
\frac{\vdash; x_2 : FGFX \vdash \text{let } x_2 : FGFX \text{ be } F x_1 \text{ in } (\text{let } x_1 : GFX \text{ be } G x_0 \text{ in } x_0) : FX}{x_3 : FGFX; \cdot \vdash \text{let } x_3 : FGFX \text{ be } G x_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } F x_1 \text{ in } (\text{let } x_1 : GFX \text{ be } G x_0 \text{ in } x_0)) : FX} \text{FL}} \\
\frac{x_3 : FGFX \vdash G(\text{let } x_3 : FGFX \text{ be } G x_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } F x_1 \text{ in } (\text{let } x_1 : GFX \text{ be } G x_0 \text{ in } x_0))) : GFX}{\cdot \vdash \lambda_{l,x_3} : FGFX. G(\text{let } x_3 : FGFX \text{ be } G x_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } F x_1 \text{ in } (\text{let } x_1 : GFX \text{ be } G x_0 \text{ in } x_0))) : (FGFX) \multimap (GFX)} \text{IMP R}
\end{array}$$

- Deriving μ :

$$\frac{\frac{\frac{}{x : X \vdash x : X} \text{AX}}{x : X; \cdot \vdash Fx : FX} \text{Fr}}{x : X \vdash GFx : GFX} \text{Gr}}{\cdot \vdash \lambda \mu x : X. GFx : (X) \rightarrow (GFX)} \text{IMPR}$$

The monad GF is strong:

- Deriving the tensorial strength τ :

$$\frac{\frac{\frac{\frac{\frac{}{x_0 : X \vdash x_0 : X} \text{AX}}{x_0 : X, y_0 : Y \vdash x_0 \otimes y_0 : X \otimes Y} \text{TENR}}{x_0 : X, y_0 : Y; \cdot \vdash F(x_0 \otimes y_0) : F(X \otimes Y)} \text{Fr}}{x_0 : X, y_1 : FY \vdash \text{let } y_1 : FY \text{ be } F_{y_0} \text{ in } F(x_0 \otimes y_0) : F(X \otimes Y)} \text{FL}}{\frac{\frac{y_2 : GFY, x_0 : X; \cdot \vdash \text{let } y_2 : GFY \text{ be } G_{y_1} \text{ in } (\text{let } y_1 : FY \text{ be } F_{y_0} \text{ in } F(x_0 \otimes y_0)) : F(X \otimes Y)} \text{GL}}{y_2 : GFY, x_0 : X \vdash G(\text{let } y_2 : GFY \text{ be } G_{y_1} \text{ in } (\text{let } y_1 : FY \text{ be } F_{y_0} \text{ in } F(x_0 \otimes y_0))) : GF(X \otimes Y)} \text{Gr}}{x_1 : X, y_3 : GFY \vdash \text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } G_{y_1} \text{ in } (\text{let } y_1 : FY \text{ be } F_{y_0} \text{ in } F(x_0 \otimes y_0)))) : GF(X \otimes Y)} \text{BETA}}{\frac{z : X \otimes GFY \vdash \text{let } z : X \otimes GFY \text{ be } x_1 \otimes y_3 \text{ in } (\text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } G_{y_1} \text{ in } (\text{let } y_1 : FY \text{ be } F_{y_0} \text{ in } F(x_0 \otimes y_0)))))) : GF(X \otimes Y)} \text{TENL}}{\cdot \vdash \lambda \tau z : X \otimes GFY. \text{let } z : X \otimes GFY \text{ be } x_1 \otimes y_3 \text{ in } (\text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } G_{y_1} \text{ in } (\text{let } y_1 : FY \text{ be } F_{y_0} \text{ in } F(x_0 \otimes y_0)))))) : (X \otimes GFY) \rightarrow (GF(X \otimes Y))} \text{IMPR}$$

A Full Ott Spec

vars, n, a, x, y, z, w, m, o
ivar, i, k, j, l
const, b

A, B, C ::=
 | B
 | Unit
 | $A \otimes B$
 | $A \multimap B$
 | $A \multimap B$
 | FX

X, Y, Z ::=
 | B
 | Unit
 | $X \otimes Y$
 | $X \multimap Y$
 | $X \multimap Y$
 | GA

T ::=
 | A
 | X

$$\begin{array}{lcl}
p & ::= & \\
& | & \star \\
& | & x \\
& | & \text{triv} \\
& | & p \otimes p' \\
& | & \mathsf{F}p \\
& | & \mathsf{G}p \\
\\
s & ::= & \\
& | & x \\
& | & b \\
& | & \text{triv} \\
& | & \text{let } s_1 : T \text{ be } p \text{ in } s_2 \\
& | & s_1 \otimes s_2 \\
& | & \lambda_l x : A. s \\
& | & \lambda_r x : A. s \\
& | & \lambda x : A. s \\
& | & \text{app}_l s_1 s_2 \\
& | & \text{app}_r s_1 s_2 \\
& | & \text{app } s_1 s_2 \\
& | & \text{ex } x_1, x_2 \text{ with } s_1, s_2 \text{ in } s_3 \\
& | & \text{contrR } x_1 \text{ as } s_1, s_2 \text{ in } s_3 \\
& | & \text{contrL } x_1 \text{ as } s_1, s_2 \text{ in } s_3 \\
& | & \text{weak } x \text{ in } s \\
& | & (s) \quad \text{S} \\
& | & \mathsf{F}t \\
\\
t & ::= & \\
& | & x \\
& | & b \\
& | & \text{triv} \\
& | & \text{let } t_1 : X \text{ be } p \text{ in } t_2 \\
& | & t_1 \otimes t_2 \\
& | & \lambda_l x : X. t \\
& | & \lambda_r x : X. t \\
& | & \lambda x : X. t \\
& | & \text{app}_l t_1 t_2 \\
& | & \text{app}_r t_1 t_2 \\
& | & \text{app } t_1 t_2 \\
& | & \text{ex } x_1, x_2 \text{ with } t_1, t_2 \text{ in } t_3 \\
& | & \text{contrR } x_1 \text{ as } t_1, t_2 \text{ in } t_3 \\
& | & \text{contrL } x_1 \text{ as } t_1, t_2 \text{ in } t_3 \\
& | & \text{weak } x \text{ in } t
\end{array}$$

$$\begin{array}{lcl}
& | & (t) \quad \mathbf{S} \\
& | & \mathbf{G}s \\
\Gamma, \Delta, \Phi, \Psi & ::= & \cdot \\
& | & \Gamma_1, \Gamma_2 \\
& | & x : A \\
& | & (\Gamma) \quad \mathbf{S} \\
& | & x : X
\end{array}$$

$$\boxed{\Gamma \vdash t : X}$$

$$\begin{array}{c}
\frac{}{x : X \vdash x : X} \quad \mathbf{T_VAR} \\
\frac{}{\Gamma, \Delta \vdash t : X} \quad \mathbf{T_UNITL} \\
\frac{}{\Gamma, x : \mathbf{Unit}, \Delta \vdash \text{let } x : \mathbf{Unit} \text{ be } \text{triv in } t : X} \quad \mathbf{T_UNITR} \\
\frac{}{\cdot \vdash \text{triv} : \mathbf{Unit}} \quad \mathbf{T_BETA} \\
\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : Y, w : X, \Delta \vdash \text{ex } w, z \text{ with } x, y \text{ in } t : Z} \quad \mathbf{T_CONTRR} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}{\Gamma_1, \Gamma_2, z : X, \Gamma_3 \vdash \text{contrR } z \text{ as } x, y \text{ in } t : Y} \quad \mathbf{T_CONTRL} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}{\Gamma_1, z : X, \Gamma_2, \Gamma_3 \vdash \text{contrL } z \text{ as } x, y \text{ in } t : Y} \quad \mathbf{T_WEAK} \\
\frac{\Gamma, \Delta \vdash t : Y \quad x \notin |\Gamma, \Delta|}{\Gamma, x : X, \Delta \vdash \text{weak } x \text{ in } t : Y} \quad \mathbf{T_CUT} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : X, \Delta_2 \vdash t_2 : Y}{\Delta_1, \Gamma, \Delta_2 \vdash [t_1/x]t_2 : Y} \quad \mathbf{T_TENL} \\
\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : X \otimes Y, \Delta \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } t : Z} \quad \mathbf{T_TEN} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta \vdash t_2 : Y}{\Gamma, \Delta \vdash t_1 \otimes t_2 : X \otimes Y} \quad \mathbf{T_IMPL1} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, \Gamma, y : X \multimap Y, \Delta_2 \vdash [\text{app}_l y t_1/x]t_2 : Z} \quad \mathbf{T_IMPL2} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, y : Y \multimap X, \Gamma, \Delta_2 \vdash [\text{app}_r y t_1/x]t_2 : Z} \quad \mathbf{T_IMPR1} \\
\frac{\Gamma, x : X \vdash t : Y}{\Gamma \vdash \lambda_l x : X. t : X \multimap Y} \quad \mathbf{T_IMPR2} \\
\frac{x : X, \Gamma \vdash t : Y}{\Gamma \vdash \lambda_r x : X. t : Y \multimap X} \quad \mathbf{T_GR} \\
\frac{\Gamma; \cdot \vdash s : A}{\Gamma \vdash \mathbf{G}s : \mathbf{GA}}
\end{array}$$

$$\boxed{\Gamma; \Psi \vdash s : A}$$

$$\begin{array}{c}
\frac{}{\vdash; x : A \vdash x : A} \text{S_AX} \\
\\
\frac{\Gamma, \Delta; \Psi \vdash s : A}{\Gamma, x : \text{Unit}, \Delta; \Psi \vdash \text{let } x : \text{Unit be triv in } s : A} \text{S_UNITL1} \\
\\
\frac{\Gamma; \Psi, \Phi \vdash s : A}{\Gamma; \Psi, x : \text{Unit}, \Phi \vdash \text{let } x : \text{Unit be triv in } s : A} \text{S_UNITL2} \\
\\
\frac{}{\vdash; \cdot \vdash \text{triv} : \text{Unit}} \text{S_UNITR} \\
\\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : Y, w : X, \Delta; \Psi \vdash \text{ex } w, z \text{ with } x, y \text{ in } s : A} \text{S_BETA} \\
\\
\frac{\Gamma, \Delta; \Psi \vdash s : B \quad x \notin |\Gamma, \Delta, \Psi|}{\Gamma, x : X, \Delta; \Psi \vdash \text{weak } x \text{ in } s : B} \text{S_WEAK} \\
\\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash s : B}{\Gamma_1, \Gamma_2, z : X, \Gamma_3; \Psi \vdash \text{contrR } z \text{ as } x, y \text{ in } s : B} \text{S_CONTRR} \\
\\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash s : B}{\Gamma_1, z : X, \Gamma_2, \Gamma_3; \Psi \vdash \text{contrL } z \text{ as } x, y \text{ in } s : B} \text{S_CONTRL} \\
\\
\frac{\Gamma \vdash t : X \quad \Delta_1, x : X, \Delta_2; \Psi \vdash s : A}{\Delta_1, \Gamma, \Delta_2; \Phi \vdash [t/x]s : A} \text{S_CUT1} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : A, \Phi_2 \vdash s_2 : B}{\Gamma, \Delta; \Phi_1, \Psi, \Phi_2 \vdash [s_1/x]s_2 : B} \text{S_CUT2} \\
\\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : X \otimes Y, \Delta; \Psi \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : A} \text{S_TENL1} \\
\\
\frac{\Gamma; \Psi, x : A, y : B, \Phi \vdash s : A}{\Gamma; \Psi, z : A \otimes B, \Phi \vdash \text{let } z : A \otimes B \text{ be } x \otimes y \text{ in } s : A} \text{S_TENL2} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi \vdash s_2 : B}{\Gamma, \Delta; \Psi, \Phi \vdash s_1 \otimes s_2 : A \otimes B} \text{S_TENR} \\
\\
\frac{\Gamma \vdash t : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash s : A}{\Delta_1, \Gamma, y : X \rightarrow Y, \Delta_2; \Psi \vdash [\text{app}_l y t/x]s : A} \text{S_IMPL1} \\
\\
\frac{\Gamma \vdash t : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash s : A}{\Delta_1, y : Y \leftarrow X, \Gamma, \Delta_2; \Psi \vdash [\text{app}_r y t/x]s : A} \text{S_IMPL2} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash s_2 : A}{\Gamma, \Delta; \Phi_1, \Psi, y : A \rightarrow B, \Phi_2 \vdash [\text{app}_l y s_1/x]s_2 : A} \text{S_IMPL3} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash s_2 : A}{\Gamma, \Delta; \Phi_1, y : B \leftarrow A, \Psi, \Phi_2 \vdash [\text{app}_l y s_1/x]s_2 : A} \text{S_IMPL4} \\
\\
\frac{\Gamma; \Psi, x : A \vdash s : B}{\Gamma; \Psi \vdash \lambda_l x : A. s : A \rightarrow B} \text{S_IMPR1} \\
\\
\frac{\Gamma; x : A, \Psi \vdash s : B}{\Gamma; \Psi \vdash \lambda_r x : A. s : B \leftarrow A} \text{S_IMPR2}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t : X}{\Gamma; \cdot \vdash Ft : FX} \quad \text{S_FR} \\
\\
\frac{\Gamma, x : X; \Psi \vdash s : A}{\Gamma; z : FX, \Psi \vdash \text{let } z : FX \text{ be } Fx \text{ in } s : A} \quad \text{S_FL} \\
\\
\frac{\Gamma; \Psi, x : A \vdash s : B}{z : GA, \Gamma; \Psi \vdash \text{let } z : GA \text{ be } Gx \text{ in } s : B} \quad \text{S_GL}
\end{array}$$