Deriving exchange in Elle comonadicly:

$$\overline{y_0 : \mathsf{G}B \vdash y_0 : \mathsf{G}B}\ \textsc{ax}$$

```
<<no parses (char 10):  y0 :  Gf B;*** .  |- F y0 :  F Gf
```

$$\overline{x_0 : \mathsf{G}A \vdash x_0 : \mathsf{G}A}\ \textsc{ax}$$

```
<<no parses (char 10):  x0 :  Gf A;*** .  |- F x0 :  F Gf
<<no parses (char 22):  y0 :  Gf B, x0 :  Gf A ;*** .  |- h(F y0) (x) F x0 :
<<no parses (char 22):  x1 :  Gf A, y1 :  Gf B ;*** .  |- ex y1 , x1 with y0 , x0 in (h(F
<<no parses (char 11):  x1 :  Gf A ;*** y2 :  F Gf B |- let y2 :  F Gf B be F y1 in (ex y1 , x1 with
<<no parses (char 3):  .  ;*** x2 :  F Gf A, y2 :  F Gf B |- let x2 :  F Gf A be F x1 in (let y2 :  F Gf B be F y1 in
<<no parses (char 3):  .  ;*** z :  h(F Gf A) (x) F Gf B |- let z :  h(F Gf A) (x) F Gf B be x2 (x) y2 in (let x2 :  F Gf A be F x1 in (let y
<<no parses (char 3):  .  ;*** .  |- \l z :  h(F Gf A) (x) F Gf B . let z :  h(F Gf A) (x) F Gf B be x2 (x) y2 in (let x2 :  F Gf A be F x1 in (let y2 :  F
```

Deriving right contraction in Elle comonadicly:

$$\overline{x_1 : \mathsf{G}A \vdash x_1 : \mathsf{G}A}\ \textsc{ax}$$

```
<<no parses (char 10):  x1 :  Gf A;*** .  |- F x1 :  F Gf A >>
```

$\textsc{Fr}$

$$\overline{y_0 : \mathsf{G}B \vdash y_0 : \mathsf{G}B}\ \textsc{ax}$$

```
<<no parses (char 10):  y0 :  Gf B;*** .  |- F y0
```

$$\overline{x_0 : \mathsf{G}A \vdash x_0 : \mathsf{G}A}\ \textsc{ax}$$

```
<<no parses (char 10):  x0 :  Gf A;*** .  |- F x0
<<no parses (char 22):  y0 :  Gf B, x0 :  Gf A ;*** .  |- h(F y0) (
<<no parses (char 33):  x1 :  Gf A, y0 :  Gf B, x0 :  Gf A ;*** .  |- h(F x1) (x) (h(F
<<no parses (char 22):  y0 :  Gf B, x2 :  Gf A ;*** .  |- contrR x2 as x1, x0 in (h(F x1)
<<no parses (char 11):  y0 :  Gf B ;*** x3 :  F Gf A |- let x3 :  F Gf A be F x2 in (contrR x2 as x1,
<<no parses (char 3):  .  ;*** y1 :  F Gf B, x3 :  F Gf A |- let y1 :  F Gf B be F y0 in (let x3 :  F Gf A be F x2 in
<<no parses (char 3):  .  ;*** z :  h(F Gf B) (x) F Gf A |- let z :  h(F Gf B) (x) F Gf A be y1 (x) x3 in (let y1 :  F Gf B be F y0 in (let
<<no parses (char 3):  .  ;*** .  |- \l z :  h(F Gf B) (x) F Gf A . let z :  h(F Gf B) (x) F Gf A be y1 (x) x3 in (let y1 :  F Gf B be F y0 in (let x3 :  F
```

Deriving left contraction in Elle comonadicly:

$$\overline{x_0 : \mathsf{G}A \vdash x_0 : \mathsf{G}A}\ \textsc{ax}$$

```
<<no parses (char 10):  x0 :  Gf A;*** .  |- F x0
```

$$\overline{y_0 : \mathsf{G}B \vdash y_0 : \mathsf{G}B}\ \textsc{ax}$$

```
<<no parses (char 10):  y0 :  Gf B;*** .  |- F y0
<<no parses (char 22):  x0 :  Gf A, y0 :  Gf B ;*** .  |- h(F x0) (x)
```

$$\overline{x_1 : \mathsf{G}A \vdash x_1 : \mathsf{G}A}\ \textsc{ax}$$

```
<<no parses (char 10):  x1 :  Gf A;*** .  |- F x1 :  F Gf A >>
```

$\textsc{Fr}$

```
<<no parses (char 33):  x0 :  Gf A, y0 :  Gf B, x1 :  Gf A ;*** .  |- (h(F x0) (x) F
<<no parses (char 22):  x2 :  Gf A, y0 :  Gf B ;*** .  |- contrL x2 as x0, x1 in ((h(F x0
<<no parses (char 11):  x2 :  Gf A ;*** y1 :  F Gf B |- let y1 :  F Gf B be F y0 in (contrL x2 as x0
<<no parses (char 3):  .  ;*** x3 :  F Gf A, y1 :  F Gf B |- let x3 :  F Gf A be F x2 in (let y1 :  F Gf B be F y0 i
<<no parses (char 3):  .  ;*** z :  h(F Gf A) (x) F Gf B |- let z :  h(F Gf A) (x) F Gf B be x3 (x) y1 in (let x3 :  F Gf A be F x2 in (le
<<no parses (char 3):  .  ;*** .  |- \l z :  h(F Gf A) (x) F Gf B . let z :  h(F Gf A) (x) F Gf B be x3 (x) y1 in (let x3 :  F Gf A be F x3 in (let y1 :  F
```

Deriving weakening in Elle comonadicly:

```
<<no parses (char 10):  .  |- triv ***:  Unit >>   UNITR
<<no parses (char 11):  x0 :  Gf A ;*** .  |- weak x0 in triv :  Unit >>   WEAK
<<no parses (char 3):  .  ;*** x1 :  F Gf A |- let x1 :  F Gf A be F x0 in weak x0 in triv :  Unit >>   FL
<<no parses (char 3):  .  ;*** .  |- \l x1 :  F Gf A . let x1 :  F Gf A be F x0 in weak x0 in triv :  (F Gf A) -> (Unit) >>   IMPR
```

GF is a monad:

- Deriving η:

```
<<no parses (char 3):  .  ;*** x0 :  F X |- x0 :  F X >>   AX
<<no parses (char 13):  x1 :  Gf F X ;*** .  |- let x1 :  Gf F X be Gf x0 in x0 :  F X >>   GL
<<no parses (char 3):  .  ;*** x2 :  F Gf F X |- let x2 :  F Gf F X be F x1 in (let x1 :  Gf F X be Gf x0 in x0) :
<<no parses (char 18):  x3 :  Gf F Gf F X ;*** .  |- let x3 :  Gf F Gf F X be Gf x2 in (let x2 :  F Gf F X be F x1 in (let x1 :  Gf F
```

$$x_3 : \mathsf{GFGF}X \vdash \mathsf{G}(\mathsf{let}\, x_3 : \mathsf{GFGF}X\, \mathsf{be}\, \mathsf{G}\, x_2\, \mathsf{in}\, (\mathsf{let}\, x_2 : \mathsf{FGF}X\, \mathsf{be}\, \mathsf{F}\, x_1\, \mathsf{in}\, (\mathsf{let}\, x_1 : \mathsf{GF}X\, \mathsf{be}\, \mathsf{G}\, x_0\, \mathsf{in}\, x_0))) : \mathsf{GF}X$$

```
<<no parses (char 15):  .  |- \l x3 :  G***f F Gf F X . Gf (let x3 :  Gf F Gf F X be Gf x2 in (let x2 :  F Gf F X be F x1 in (let x1 :  Gf F X be G
```

- Deriving μ:

$$\dfrac{}{x : X \vdash x : X}\ \textsc{ax}$$

```
<<no parses (char 7):  x :  X ;*** .  |- F x :  F X >>   FR
```

$$x : X \vdash \mathsf{GF}x : \mathsf{GF}X \quad \textsc{Gr}$$

```
<<no parses (char 13):  .  |- \l x :  X*** .  Gf F x :  (X) -> (Gf F X) >>   IMPR
```

The monad GF is strong:

- Deriving the tensorial strength τ:

$$\dfrac{\dfrac{}{x_0 : X \vdash x_0 : X}\ \textsc{ax} \quad \dfrac{}{y_0 : Y \vdash y_0 : Y}\ \textsc{ax}}{x_0 : X, y_0 : Y \vdash x_0 \otimes y_0 : X \otimes Y}\ \textsc{tenR}$$

```
<<no parses (char 17):  x0 :  X , y0 :  Y ;*** .  |- F (x0 (x) y0) :  F (
<<no parses (char 8):  x0 :  X ;*** y1 :  F Y |- let y1 :  F Y be F y0 in F (x0 (x)
<<no parses (char 22):  y2 :  Gf F Y , x0 :  X ;*** .  |- let y2 :  Gf F Y be Gf y1 in (let y1 :  F Y
```

$$y_2 : \mathsf{GF}Y, x_0 : X \vdash \mathsf{G}(\mathsf{let}\, y_2 : \mathsf{GF}Y\, \mathsf{be}\, \mathsf{G}\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : \mathsf{F}Y\, \mathsf{be}\, \mathsf{F}\, y_0\, \mathsf{in}\, \mathsf{F}(x_0 \otimes y_0)))$$

$$x_1 : X, y_3 : \mathsf{GF}Y \vdash \mathsf{ex}\, y_3, x_1\, \mathsf{with}\, y_2, x_0\, \mathsf{in}\, (\mathsf{G}(\mathsf{let}\, y_2 : \mathsf{GF}Y\, \mathsf{be}\, \mathsf{G}\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : \mathsf{F}Y\, \mathsf{be}\, \mathsf{F}\, y_0\, \mathsf{in}$$

$$z : X \otimes \mathsf{GF}Y \vdash \mathsf{let}\, z : X \otimes \mathsf{GF}Y\, \mathsf{be}\, x_1 \otimes y_3\, \mathsf{in}\, (\mathsf{ex}\, y_3, x_1\, \mathsf{with}\, y_2, x_0\, \mathsf{in}\, (\mathsf{G}(\mathsf{let}\, y_2 : \mathsf{GF}Y\, \mathsf{be}\, \mathsf{G}\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1$$

```
<<no parses (char 15):  .  |- \l z :  h(X***) (x) Gf F Y . let z :  h(X) (x) Gf F Y be x1 (x) y3 in (ex y3 , x1 with y2 , x0 in (Gf (let y2 :  Gf F
```

A Full Ott Spec

vars, n, a, x, y, z, w, m, o
ivar, i, k, j, l
const, b

A, B, C	::=	
	|	B
	|	UnitS
	|	$A \triangleright B$

	$\vert$	$A \rightharpoonup B$	
	$\vert$	$A \leftharpoonup B$	
	$\vert$	$\mathsf{F}X$	
$X,\ Y,\ Z$	$::=$		
	$\vert$	B	
	$\vert$	UnitT	
	$\vert$	$X \otimes Y$	
	$\vert$	$X \multimap Y$	
	$\vert$	$\mathsf{G}A$	
T	$::=$		
	$\vert$	A	
	$\vert$	X	
p	$::=$		
	$\vert$	$\star$	
	$\vert$	x	
	$\vert$	trivT	
	$\vert$	trivS	
	$\vert$	$p \otimes p'$	
	$\vert$	$p \triangleright p'$	
	$\vert$	$\mathsf{F}\, p$	
	$\vert$	$\mathsf{G}\, p$	
s	$::=$		
	$\vert$	x	
	$\vert$	b	
	$\vert$	trivS	
	$\vert$	$\mathsf{let}\, s_1 : T\, \mathsf{be}\, p\, \mathsf{in}\, s_2$	
	$\vert$	$\mathsf{let}\, t : T\, \mathsf{be}\, p\, \mathsf{in}\, s$	
	$\vert$	$s_1 \triangleright s_2$	
	$\vert$	$\lambda_l x : A.s$	
	$\vert$	$\lambda_r x : A.s$	
	$\vert$	$\mathsf{app}_l\, s_1\, s_2$	
	$\vert$	$\mathsf{app}_r\, s_1\, s_2$	
	$\vert$	$\mathsf{ex}\, s_1, s_2\, \mathsf{with}\, x_1, x_2\, \mathsf{in}\, s_3$	
	$\vert$	$\mathsf{contrR}\, x\, \mathsf{as}\, s_1, s_2\, \mathsf{in}\, s_3$	
	$\vert$	$\mathsf{contrL}\, x\, \mathsf{as}\, s_1, s_2\, \mathsf{in}\, s_3$	
	$\vert$	$\mathsf{weak}\, x\, \mathsf{in}\, s$	
	$\vert$	(s)	S
	$\vert$	$\mathsf{F}t$	

t	::=		
	$\mid$	x	
	$\mid$	b	
	$\mid$	trivT	
	$\mid$	$\mathsf{let}\, t_1 : X\, \mathsf{be}\, p\, \mathsf{in}\, t_2$	
	$\mid$	$t_1 \otimes t_2$	
	$\mid$	$\lambda x : X.t$	
	$\mid$	$\mathsf{app}\, t_1\, t_2$	
	$\mid$	$\mathsf{ex}\, t_1, t_2\, \mathsf{with}\, x_1, x_2\, \mathsf{in}\, t_3$	
	$\mid$	$\mathsf{contrR}\, x\, \mathsf{as}\, t_1, t_2\, \mathsf{in}\, t_3$	
	$\mid$	$\mathsf{contrR}\, x\, \mathsf{as}\, t_1, t_2\, \mathsf{in}\, t_3$	
	$\mid$	$\mathsf{weak}\, x\, \mathsf{in}\, t$	
	$\mid$	(t)	S
	$\mid$	$\mathsf{G}s$	

$\Gamma,\ \Delta,\ \Phi,\ \Psi$	::=		
	$\mid$	$\cdot$	
	$\mid$	Γ_1, Γ_2	
	$\mid$	$x : A$	
	$\mid$	(Γ)	S
	$\mid$	$x : X$	

$\boxed{\Gamma \vdash t : X}$

$$\frac{}{x : X \vdash x : X}\ \textsc{T\_var}$$

$$\frac{\Gamma, \Delta \vdash t : X}{\Gamma, x : \mathsf{UnitT}, \Delta \vdash \mathsf{let}\, x : \mathsf{UnitT}\, \mathsf{be}\, \mathsf{trivT}\, \mathsf{in}\, t : X}\ \textsc{T\_unitL}$$

$$\frac{}{\cdot \vdash \mathsf{trivT} : \mathsf{UnitT}}\ \textsc{T\_unitR}$$

$$\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : Y, w : X, \Delta \vdash \mathsf{ex}\, w, z\, \mathsf{with}\, x, y\, \mathsf{in}\, t : Z}\ \textsc{T\_beta}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}{\Gamma_1, \Gamma_2, z : X, \Gamma_3 \vdash \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, t : Y}\ \textsc{T\_contrR}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}{\Gamma_1, z : X, \Gamma_2, \Gamma_3 \vdash \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, t : Y}\ \textsc{T\_contrL}$$

$$\frac{\Gamma, \Delta \vdash t : Y \quad x \notin |\Gamma, \Delta|}{\Gamma, x : X, \Delta \vdash \mathsf{weak}\, x\, \mathsf{in}\, t : Y}\ \textsc{T\_weak}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : X, \Delta_2 \vdash t_2 : Y}{\Delta_1, \Gamma, \Delta_2 \vdash [t_1/x]t_2 : Y}\ \textsc{T\_cut}$$

$$\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : X \otimes Y, \Delta \vdash \mathsf{let}\, z : X \otimes Y\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, t : Z}\ \textsc{T\_tenL}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta \vdash t_2 : Y}{\Gamma, \Delta \vdash t_1 \otimes t_2 : X \otimes Y}\ \textsc{T\_ten}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, \Gamma, y : X \multimap Y, \Delta_2 \vdash [\mathsf{app}\, y\, t_1/x]t_2 : Z}\ \textsc{T\_impL}$$

$$\frac{\Gamma, x : X \vdash t : Y}{\Gamma \vdash \lambda x : X.t : X \multimap Y}\ \text{T\_IMPR}$$

$$\frac{\Gamma \vdash s : A}{\Gamma \vdash \mathsf{G}s : \mathsf{G}A}\ \text{T\_GR}$$

$\boxed{\Psi \vdash s : A}$

$$\frac{}{x : A \vdash x : A}\ \text{S\_AX}$$

$$\frac{\Phi, \Psi \vdash s : A}{\Phi, x : \mathsf{UnitT}, \Psi \vdash \mathsf{let}\, x : \mathsf{UnitT}\, \mathsf{be}\, \mathsf{trivT}\, \mathsf{in}\, s : A}\ \text{S\_UNITL1}$$

$$\frac{\Phi, \Psi \vdash s : A}{\Phi, x : \mathsf{UnitS}, \Psi \vdash \mathsf{let}\, x : \mathsf{UnitS}\, \mathsf{be}\, \mathsf{trivS}\, \mathsf{in}\, s : A}\ \text{S\_UNITL2}$$

$$\frac{}{\cdot \vdash \mathsf{trivS} : \mathsf{UnitS}}\ \text{S\_UNITR}$$

$$\frac{\Phi, x : X, y : Y, \Psi \vdash s : A}{\Phi, z : Y, w : X, \Psi \vdash \mathsf{ex}\, w, z\, \mathsf{with}\, x, y\, \mathsf{in}\, s : A}\ \text{S\_BETA}$$

$$\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash s : A}{\Phi_1, \Phi_2, y : X, \Phi_3 \vdash \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, s : A}\ \text{S\_CONTRR}$$

$$\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash s : A}{\Phi_1, y : X, \Phi_2, \Phi_3 \vdash \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, s : A}\ \text{S\_CONTRL}$$

$$\frac{\Phi, \Psi \vdash s : A \quad x \notin |\Phi, \Psi|}{\Phi, x : X, \Psi \vdash \mathsf{weak}\, x\, \mathsf{in}\, s : B}\ \text{S\_WEAK}$$

$$\frac{\Gamma \vdash t : X \quad \Phi_1, x : X, \Phi_2 \vdash s : A}{\Phi_1, \Gamma, \Phi_1 \vdash [t/x]s : A}\ \text{S\_CUT1}$$

$$\frac{\Psi \vdash s_1 : A \quad \Phi_1, x : A, \Phi_2 \vdash s_2 : A}{\Phi_1, \Psi, \Phi_2 \vdash [s_1/x]s_2 : A}\ \text{S\_CUT2}$$

$$\frac{\Phi, x : X, y : Y, \Psi \vdash s : A}{\Phi, z : X \otimes Y, \Psi \vdash \mathsf{let}\, z : X \otimes Y\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, s : A}\ \text{S\_TENL1}$$

$$\frac{\Phi, x : A, y : B, \Psi \vdash s : C}{\Phi, z : A \triangleright B, \Psi \vdash \mathsf{let}\, z : A \triangleright B\, \mathsf{be}\, x \triangleright y\, \mathsf{in}\, s : C}\ \text{S\_TENL2}$$

$$\frac{\Phi \vdash s_1 : A \quad \Psi \vdash s_2 : B}{\Phi, \Psi \vdash s_1 \triangleright s_2 : A \triangleright B}\ \text{S\_TENR}$$

$$\frac{\Gamma \vdash t : X \quad \Phi, x : Y, \Psi \vdash s : A}{\Phi, \Gamma, y : X \multimap Y, \Psi \vdash [\mathsf{app}\, y\, t/x]s : A}\ \text{S\_IMPL}$$

$$\frac{\Phi \vdash s_1 : A \quad \Psi, x : B \vdash s_2 : C}{\Psi, \Phi, y : A \rightharpoonup B \vdash [\mathsf{app}_r\, y\, s_1/x]s_2 : C}\ \text{S\_IMPRL}$$

$$\frac{\Phi \vdash s_1 : A \quad x : B, \Psi \vdash s_2 : C}{y : B \leftharpoonup A, \Phi, \Psi \vdash [\mathsf{app}_l\, y\, s_1/x]s_2 : C}\ \text{S\_IMPLL}$$

$$\frac{\Phi, x : A \vdash s : B}{\Phi \vdash \lambda_r x : A.s : A \rightharpoonup B}\ \text{S\_IMPRR}$$

$$\frac{x : A, \Phi \vdash s : B}{\Phi \vdash \lambda_l x : A.s : B \leftharpoonup A}\ \text{S\_IMPLR}$$

$$\frac{\Gamma \vdash t : X}{\Gamma \vdash \mathsf{F}t : \mathsf{F}X}\ \text{S\_F}_{\text{R}}$$

$$\frac{\Phi, x : X, \Psi \vdash s : A}{\Phi, y : \mathsf{F}X, \Psi \vdash \mathsf{let}\, z : \mathsf{F}X\, \mathsf{be}\, \mathsf{F}\, x\, \mathsf{in}\, s : A}\ \text{S\_F}_{\text{L}}$$

$$\frac{\Phi, x : A, \Psi \vdash s : A}{\Phi, y : \mathsf{G}A, \Psi \vdash \mathsf{let}\, z : \mathsf{G}A\, \mathsf{be}\, \mathsf{G}\, x\, \mathsf{in}\, s : A}\ \text{S\_G}_{\text{L}}$$