

A Term Assignment for Natural Deduction Formulation of Elle

vars, n, a, x, y, z, w, m, o

ivar, i, k, j, l

const, b

$A, B, C \quad ::=$

- | B
- | Unit
- | $A \triangleright B$
- | $A \multimap B$
- | $A \multimap B$
- | FX

$X, Y, Z \quad ::=$

- | B
- | Unit
- | $X \otimes Y$
- | $X \multimap Y$
- | GA

$T \quad ::=$

- | A
- | X

$p \quad ::=$

- | \star
- | x
- | triv
- | $p \otimes p'$
- | $p \triangleright p'$
- | $\text{F}p$
- | $\text{G}p$

$s \quad ::=$

- | x
- | b
- | triv
- | $\text{let } s_1 : T \text{ be } p \text{ in } s_2$
- | $s_1 \triangleright s_2$
- | $\lambda_l x : A. s$
- | $\lambda_r x : A. s$
- | $\text{app}_l s_1 s_2$
- | $\text{app}_r s_1 s_2$
- | $\text{derelict } t$

		$\text{ex } s_1, s_2 \text{ with } x_1, x_2 \text{ in } s_3$	
		(s)	S
		$\text{F}t$	

t	::=		
		x	
		b	
		triv	
		$\text{let } t_1 : X \text{ be } p \text{ in } t_2$	
		$t_1 \otimes t_2$	
		$\lambda x : X. t$	
		$\text{app } t_1 t_2$	
		$\text{ex } t_1, t_2 \text{ with } x_1, x_2 \text{ in } t_3$	
		(t)	S
		$\text{G}s$	

$\Gamma, \Delta, \Phi, \Psi$::=		
		\cdot	
		Γ_1, Γ_2	
		$x : A$	
		(Γ)	S
		$x : X$	

$\Gamma \vdash t : X$

$\frac{}{x : X \vdash x : X}$	T_IDENTITY
$\frac{}{\cdot \vdash \text{triv} : \text{Unit}}$	T_UNITI
$\frac{\Delta \vdash t_1 : \text{Unit} \quad \Gamma \vdash t_2 : Y}{\Gamma, \Delta \vdash \text{let } t_1 : \text{Unit} \text{ be } \text{triv} \text{ in } t_2 : Y}$	T_UNITE
$\frac{\Gamma \vdash t_1 : X \quad \Delta \vdash t_2 : Y}{\Gamma, \Delta \vdash t_1 \otimes t_2 : X \otimes Y}$	T_TENI
$\frac{\Gamma \vdash t_1 : X \otimes Y \quad \Delta, x : X, y : Y \vdash t_2 : Z}{\Gamma, \Delta \vdash \text{let } t_1 : X \otimes Y \text{ be } x \otimes y \text{ in } t_2 : Z}$	T_TENE
$\frac{\Gamma, x : X \vdash t : Y}{\Gamma \vdash \lambda x : X. t : X \multimap Y}$	T_IMPI
$\frac{\Gamma \vdash t_1 : X \multimap Y \quad \Delta \vdash t_2 : X}{\Gamma, \Delta \vdash \text{app } t_1 t_2 : Y}$	T_IMPE
$\frac{\Gamma; \cdot \vdash s : A}{\Gamma \vdash \text{G}s : \text{GA}}$	T_GI
$\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : Y, w : X, \Delta \vdash \text{ex } w, z \text{ with } x, y \text{ in } t : Z}$	T_BETA

$\Gamma; \Psi \vdash s : A$

$$\begin{array}{c}
\frac{}{\vdash; x : A \vdash x : A} \text{S_IDENTITY} \\
\frac{}{\vdash; \cdot \vdash \text{triv} : \text{Unit}} \text{S_UNITI} \\
\frac{\Delta; \Phi \vdash s_1 : \text{Unit} \quad \Gamma; \Psi \vdash s_2 : A}{\Gamma, \Delta; \Psi, \Phi \vdash \text{let } s_1 : \text{Unit} \text{ be } \text{triv in } s_2 : A} \text{S_UNITE} \\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi \vdash s_2 : B}{\Gamma, \Delta; \Psi, \Phi \vdash s_1 \triangleright s_2 : A \triangleright B} \text{S_TENI} \\
\frac{\Gamma \vdash z : X \otimes Y \quad \Delta, x : X, y : Y; \Psi \vdash s : A}{\Delta, \Gamma; \Psi \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : A} \text{S_TENE1} \\
\frac{\Gamma; \Psi \vdash z : A \triangleright B \quad \Delta; \Phi, x : A, y : B \vdash s : C}{\Gamma, \Delta; \Phi, \Psi \vdash \text{let } z : A \triangleright B \text{ be } x \triangleright y \text{ in } s : C} \text{S_TENE2} \\
\frac{\Gamma; \Psi, x : A \vdash s : B}{\Gamma; \Psi \vdash \lambda_r x : A. s : A \rightarrow B} \text{S_IMPLI} \\
\frac{\Gamma; \Psi \vdash s_1 : A \rightarrow B \quad \Delta; \Phi \vdash s_2 : A}{\Gamma, \Delta; \Psi, \Phi \vdash \text{app}_r s_1 s_2 : B} \text{S_IMPLE} \\
\frac{\Gamma; x : A, \Psi \vdash s : B}{\Gamma; \Psi \vdash \lambda_l x : A. s : B \leftarrow A} \text{S_IMPRI} \\
\frac{\Gamma; \Psi \vdash s_1 : B \leftarrow A \quad \Delta; \Phi \vdash s_2 : A}{\Gamma, \Delta; \Psi, \Phi \vdash \text{app}_l s_1 s_2 : B} \text{S_IMPRE} \\
\frac{\Gamma \vdash t : X}{\Gamma; \cdot \vdash F t : F X} \text{S_FI} \\
\frac{\Gamma; \Psi \vdash y : F X \quad \Gamma, x : X; \Phi \vdash s_2 : A}{\Gamma; \Psi, \Phi \vdash \text{let } F x : F X \text{ be } y \text{ in } s_2 : A} \text{S_FE} \\
\frac{\Gamma \vdash t : G A}{\Gamma; \cdot \vdash \text{derelict } t : A} \text{S_GE} \\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : Y, w : X, \Delta; \Psi \vdash \text{ex } w, z \text{ with } x, y \text{ in } s : A} \text{S_BETA}
\end{array}$$

$$\boxed{t_1 \rightsquigarrow t_2}$$

$$\begin{array}{c}
\frac{}{\text{let } \text{triv} : \text{Unit} \text{ be } \text{triv in } t \rightsquigarrow t} \text{TRED_LETU} \\
\frac{}{\text{let } t_1 \otimes t_2 : X \otimes Y \text{ be } x \otimes y \text{ in } t_3 \rightsquigarrow [t_1/x][t_2/y]t_3} \text{TRED_LETT} \\
\frac{}{\text{app } (\lambda x : X. t_1) t_2 \rightsquigarrow [t_2/x]t_1} \text{TRED_LAM} \\
\frac{t_1 \rightsquigarrow t'_1}{\text{app } t_1 t_2 \rightsquigarrow \text{app } t'_1 t_2} \text{TRED_APP1} \\
\frac{t_2 \rightsquigarrow t'_2}{\text{app } t_1 t_2 \rightsquigarrow \text{app } t_1 t'_2} \text{TRED_APP2}
\end{array}$$

$$\boxed{s_1 \rightsquigarrow s_2}$$

$$\begin{array}{c}
\frac{}{\text{let } \text{triv} : \text{Unit} \text{ be } \text{triv} \text{ in } s \rightsquigarrow s} \text{SRED_LETU} \\
\frac{}{\text{let } s_1 \triangleright s_2 : A \triangleright B \text{ be } x \triangleright y \text{ in } s_3 \rightsquigarrow [s_1/x][s_2/y]s_3} \text{SRED_LET T} \\
\frac{}{\text{let } F_t : FX \text{ be } F.x \text{ in } s \rightsquigarrow [t/x]s} \text{SRED_LET F} \\
\frac{}{\text{app}_l (\lambda_l x : A.s_1) s_2 \rightsquigarrow [s_2/x]s_1} \text{SRED_LAM L} \\
\frac{}{\text{app}_r (\lambda_r x : A.s_1) s_2 \rightsquigarrow [s_2/x]s_1} \text{SRED_LAM R} \\
\frac{s_1 \rightsquigarrow s'_1}{\text{app}_l s_1 s_2 \rightsquigarrow \text{app}_l s'_1 s_2} \text{SRED_APPL1} \\
\frac{s_2 \rightsquigarrow s'_2}{\text{app}_l s_1 s_2 \rightsquigarrow \text{app}_l s_1 s'_2} \text{SRED_APPL2} \\
\frac{s_1 \rightsquigarrow s'_1}{\text{app}_r s_1 s_2 \rightsquigarrow \text{app}_r s'_1 s_2} \text{SRED_APPR1} \\
\frac{s_2 \rightsquigarrow s'_2}{\text{app}_r s_1 s_2 \rightsquigarrow \text{app}_r s_1 s'_2} \text{SRED_APPR2} \\
\frac{}{\text{derelict } Gs \rightsquigarrow s} \text{SRED_DERELICT}
\end{array}$$