Deriving exchange in Elle comonadicly:

```
                                                   <<no parses (char 13):  y0 :  Gf B |- ***y0 :  Gf B >>
                                                 <<no parses (char 10):  y0 :  Gf B;*** .  |- F y0 :  F Gf
                                                   <<no parses (char 13):  x0 :  Gf A |- ***x0 :  Gf A >>
                                                 <<no parses (char 10):  x0 :  Gf A;*** .  |- F x0 :  F Gf
                                    <<no parses (char 22):  y0 :  Gf B, x0 :  Gf A ;*** .  |- h(F y0) (x) F x0 :
                              <<no parses (char 22):  x1 :  Gf A, y1 :  Gf B ;*** .  |- ex y1 , x1 with y0 , x0 in (h(F
                        <<no parses (char 11):  x1 :  Gf A ;*** y2 :  F Gf B |- let y2 :  F Gf B be F y1 in (ex y1 , x1 with
                <<no parses (char 3):  .  ;*** x2 :  F Gf A, y2 :  F Gf B |- let x2 :  F Gf A be F x1 in (let y2 :  F Gf B be F y1 in
     <<no parses (char 3):  .  ;*** z :  h(F Gf A) (x) F Gf B |- let z :  h(F Gf A) (x) F Gf B be x2 (x) y2 in (let x2 :  F Gf A be F x1 in (let y
<<no parses (char 3):  .  ;*** .  |- \l z :  h(F Gf A) (x) F Gf B . let z :  h(F Gf A) (x) F Gf B be x2 (x) y2 in (let x2 :  F Gf A be F x1 in (let y2 :  F
```

Deriving right contraction in Elle comonadicly:

```
                                                 <<no parses (char 13):  x1 :  Gf A |- ***x1 :  Gf A >>  AX
                                               <<no parses (char 10):  x1 :  Gf A;*** .  |- F x1 :  F Gf A >>  FR
                                                       <<no parses (char 13):  y0 :  Gf B |- ***y0 :
                                                     <<no parses (char 10):  y0 :  Gf B;*** .  |- F y0
                                                       <<no parses (char 13):  x0 :  Gf A |- ***x0 :
                                                     <<no parses (char 10):  x0 :  Gf A;*** .  |- F x0
                                           <<no parses (char 22):  y0 :  Gf B, x0 :  Gf A ;*** .  |- h(F y0) (
                                <<no parses (char 33):  x1 :  Gf A, y0 :  Gf B, x0 :  Gf A ;*** .  |- h(F x1) (x) (h(F
                              <<no parses (char 22):  y0 :  Gf B, x2 :  Gf A ;*** .  |- contrR x2 as x1, x0 in (h(F x1)
                        <<no parses (char 11):  y0 :  Gf B ;*** x3 :  F Gf A |- let x3 :  F Gf A be F x2 in (contrR x2 as x1,
                <<no parses (char 3):  .  ;*** y1 :  F Gf B, x3 :  F Gf A |- let y1 :  F Gf B be F y0 in (let x3 :  F Gf A be F x2 in
     <<no parses (char 3):  .  ;*** z :  h(F Gf B) (x) F Gf A |- let z :  h(F Gf B) (x) F Gf A be y1 (x) x3 in (let y1 :  F Gf B be F y0 in (let
<<no parses (char 3):  .  ;*** .  |- \l z :  h(F Gf B) (x) F Gf A . let z :  h(F Gf B) (x) F Gf A be y1 (x) x3 in (let y1 :  F Gf B be F y0 in (let x3 :  F
```

Deriving left contraction in Elle comonadicly:

```
                                                           <<no parses (char 13):  x0 :  Gf A |- ***x0 :
                                                         <<no parses (char 10):  x0 :  Gf A;*** .  |- F x0
                                                           <<no parses (char 13):  y0 :  Gf B |- ***y0 :
                                                         <<no parses (char 10):  y0 :  Gf B;*** .  |- F y0
                                             <<no parses (char 22):  x0 :  Gf A, y0 :  Gf B ;*** .  |- h(F x0) (x
                                               <<no parses (char 13):  x1 :  Gf A |- ***x1 :  Gf A >>  AX
                                             <<no parses (char 10):  x1 :  Gf A;*** .  |- F x1 :  F Gf A >>  FR
                                <<no parses (char 33):  x0 :  Gf A, y0 :  Gf B, x1 :  Gf A ;*** .  |- (h(F x0) (x) F
                              <<no parses (char 22):  x2 :  Gf A, y0 :  Gf B ;*** .  |- contrL x2 as x0, x1 in ((h(F x0
                        <<no parses (char 11):  x2 :  Gf A ;*** y1 :  F Gf B |- let y1 :  F Gf B be F y0 in (contrL x2 as x0
                <<no parses (char 3):  .  ;*** x3 :  F Gf A, y1 :  F Gf B |- let x3 :  F Gf A be F x2 in (let y1 :  F Gf B be F y0 i
      <<no parses (char 3):  .  ;*** z :  h(F Gf A) (x) F Gf B |- let z :  h(F Gf A) (x) F Gf B be x3 (x) y1 in (let x3 :  F Gf A be F x2 in (le
<<no parses (char 3):  .  ;*** .  |- \l z :  h(F Gf A) (x) F Gf B . let z :  h(F Gf A) (x) F Gf B be x3 (x) y1 in (let x3 :  F Gf A be F x3 in (let y1 :  F
```

Deriving weakening in Elle comonadicly:

```
----------------------------------------------------------------------- unitR
<<no parses (char 5):  .  |- ***triv :  Unit >>
----------------------------------------------------------------------- weak
<<no parses (char 11):  x0 :  Gf A ;*** .  |- weak x0 in triv :  Unit >>
----------------------------------------------------------------------- FL
<<no parses (char 3):  .  ;*** x1 :  F Gf A |- let x1 :  F Gf A be F x0 in weak x0 in triv :  Unit >>
----------------------------------------------------------------------- impR
<<no parses (char 3):  .  ;*** .  |- \l x1 :  F Gf A . let x1 :  F Gf A be F x0 in weak x0 in triv :  (F Gf A) -> (Unit) >>
```

GF is a monad:

- Deriving η:

```
----------------------------------------------------------------------- ax
<<no parses (char 3):  .  ;*** x0 :  F X |- x0 :  F X >>
----------------------------------------------------------------------- GL
<<no parses (char 13):  x1 :  Gf F X ;*** .  |- let x1 :  Gf F X be Gf x0 in x0 :  F X >>
-----------------------------------------------------------------------
<<no parses (char 3):  .  ;*** x2 :  F Gf F X |- let x2 :  F Gf F X be F x1 in (let x1 :  Gf F X be Gf x0 in x0) :
-----------------------------------------------------------------------
<<no parses (char 18):  x3 :  Gf F Gf F X ;*** .  |- let x3 :  Gf F Gf F X be Gf x2 in (let x2 :  F Gf F X be F x1 in (let x1 :  Gf F X
-----------------------------------------------------------------------
<<no parses (char 20):  x3 :  Gf F Gf F X |- ***Gf (let x3 :  Gf F Gf F X be Gf x2 in (let x2 :  F Gf F X be F x1 in (let x1 :  Gf F X
-----------------------------------------------------------------------
<<no parses (char 6):  .  |- ***\l x3 :  Gf F Gf F X . Gf (let x3 :  Gf F Gf F X be Gf x2 in (let x2 :  F Gf F X be F x1 in (let x1 :  Gf F X be Gf
```

- Deriving μ:

```
----------------------------------------------------------------------- ax
<<no parses (char 9):  x :  X |- ***x :  X >>
----------------------------------------------------------------------- FR
<<no parses (char 7):  x :  X ;*** .  |- F x :  F X >>
----------------------------------------------------------------------- GR
<<no parses (char 9):  x :  X |- ***Gf F x :  Gf F X >>
----------------------------------------------------------------------- impR
<<no parses (char 5):  .  |- ***\l x :  X . Gf F x :  (X) -> (Gf F X) >>
```

The monad GF is strong:

- Deriving the tensorial strength τ:

```
----------------------------------------------------------------------- ax  -----------------------------------------
<<no parses (char 10):  x0 :  X |- ***x0 :  X >>                               <<no parses (char 10):  y0 :  Y
-----------------------------------------------------------------------
<<no parses (char 19):  x0 :  X , y0 :  Y |- ***x0 (x) y0 :  h(X) (x)
-----------------------------------------------------------------------
<<no parses (char 17):  x0 :  X , y0 :  Y ;*** .  |- F (x0 (x) y0) :  F (h
-----------------------------------------------------------------------
<<no parses (char 8):  x0 :  X ;*** y1 :  F Y |- let y1 :  F Y be F y0 in F (x0 (x)
-----------------------------------------------------------------------
<<no parses (char 22):  y2 :  Gf F Y , x0 :  X ;*** .  |- let y2 :  Gf F Y be Gf y1 in (let y1 :  F Y
-----------------------------------------------------------------------
<<no parses (char 24):  y2 :  Gf F Y , x0 :  X |- ***Gf (let y2 :  Gf F Y be Gf y1 in (let y1 :  F Y b
-----------------------------------------------------------------------
<<no parses (char 24):  x1 :  X , y3 :  Gf F Y |- ***ex y3 , x1 with y2 , x0 in (Gf (let y2 :  Gf F Y be Gf y1 in (
-----------------------------------------------------------------------
<<no parses (char 23):  z :  h(X) (x) Gf F Y |- ***let z :  h(X) (x) Gf F Y be x1 (x) y3 in (ex y3 , x1 with y2 , x0 in (Gf (let y2 :
-----------------------------------------------------------------------
<<no parses (char 5):  .  |- ***\l z :  h(X) (x) Gf F Y . let z :  h(X) (x) Gf F Y be x1 (x) y3 in (ex y3 , x1 with y2 , x0 in (Gf (let y2 :  Gf F
```

A Full Ott Spec

vars, *n*, *a*, *x*, *y*, *z*, *w*, *m*, *o*
ivar, *i*, *k*, *j*, *l*
const, *b*

A, *B*, *C*	::=	
	|	B
	|	UnitS
	|	$A \triangleright B$

$$
\begin{array}{lcl}
 & | & A \rightharpoonup B \\
 & | & A \leftharpoonup B \\
 & | & \mathsf{F}X \\
 & & \\
X,\ Y,\ Z & ::= & \\
 & | & \mathsf{B} \\
 & | & \mathsf{UnitT} \\
 & | & X \otimes Y \\
 & | & X \multimap Y \\
 & | & \mathsf{G}A \\
 & & \\
T & ::= & \\
 & | & A \\
 & | & X \\
 & & \\
p & ::= & \\
 & | & \star \\
 & | & x \\
 & | & \mathsf{trivT} \\
 & | & \mathsf{trivS} \\
 & | & p \otimes p' \\
 & | & p \triangleright p' \\
 & | & \mathsf{F}\, p \\
 & | & \mathsf{G}\, p \\
 & & \\
s & ::= & \\
 & | & x \\
 & | & b \\
 & | & \mathsf{trivS} \\
 & | & \mathsf{let}\, s_1 : T\, \mathsf{be}\, p\, \mathsf{in}\, s_2 \\
 & | & \mathsf{let}\, t : T\, \mathsf{be}\, p\, \mathsf{in}\, s \\
 & | & s_1 \triangleright s_2 \\
 & | & \lambda_l x : A.s \\
 & | & \lambda_r x : A.s \\
 & | & \mathsf{app}_l\, s_1\, s_2 \\
 & | & \mathsf{app}_r\, s_1\, s_2 \\
 & | & \mathsf{ex}\, s_1, s_2\, \mathsf{with}\, x_1, x_2\, \mathsf{in}\, s_3 \\
 & | & \mathsf{contrR}\, x\, \mathsf{as}\, s_1, s_2\, \mathsf{in}\, s_3 \\
 & | & \mathsf{contrL}\, x\, \mathsf{as}\, s_1, s_2\, \mathsf{in}\, s_3 \\
 & | & \mathsf{weak}\, x\, \mathsf{in}\, s \\
 & | & (s) \qquad\qquad\qquad\qquad \mathsf{S} \\
 & | & \mathsf{F}t
\end{array}
$$

$$
\begin{array}{lcll}
t & ::= & & \\
 & | & x & \\
 & | & b & \\
 & | & \mathsf{trivT} & \\
 & | & \mathsf{let}\, t_1 : X\, \mathsf{be}\, p\, \mathsf{in}\, t_2 & \\
 & | & t_1 \otimes t_2 & \\
 & | & \lambda x : X.t & \\
 & | & \mathsf{app}\, t_1\, t_2 & \\
 & | & \mathsf{ex}\, t_1, t_2\, \mathsf{with}\, x_1, x_2\, \mathsf{in}\, t_3 & \\
 & | & \mathsf{contrR}\, x\, \mathsf{as}\, t_1, t_2\, \mathsf{in}\, t_3 & \\
 & | & \mathsf{contrR}\, x\, \mathsf{as}\, t_1, t_2\, \mathsf{in}\, t_3 & \\
 & | & \mathsf{weak}\, x\, \mathsf{in}\, t & \\
 & | & (t) & \mathsf{S} \\
 & | & \mathsf{G}s &
\end{array}
$$

$$
\begin{array}{lcll}
\Phi,\ \Psi & ::= & & \\
 & | & \cdot & \\
 & | & \Phi_1, \Phi_2 & \\
 & | & x : X & \\
 & | & (\Phi) & \mathsf{S}
\end{array}
$$

$$
\begin{array}{lcll}
\Gamma,\ \Delta & ::= & & \\
 & | & \cdot & \\
 & | & x : A & \\
 & | & \Phi & \\
 & | & \Gamma_1, \Gamma_2 & \\
 & | & (\Gamma) & \mathsf{S}
\end{array}
$$

$\boxed{\Phi \vdash_C t : X}$

$$\frac{}{x : X \vdash_C x : X}\quad \text{T\_VAR}$$

$$\frac{\Phi, \Psi \vdash_C t : X}{\Phi, x : \mathsf{UnitT}, \Psi \vdash_C \mathsf{let}\, x : \mathsf{UnitT}\, \mathsf{be}\, \mathsf{trivT}\, \mathsf{in}\, t : X}\quad \text{T\_UNITL}$$

$$\frac{}{\cdot \vdash_C \mathsf{trivT} : \mathsf{UnitT}}\quad \text{T\_UNITR}$$

$$\frac{\Phi, x : X, y : Y, \Psi \vdash_C t : Z}{\Phi, z : Y, w : X, \Psi \vdash_C \mathsf{ex}\, w, z\, \mathsf{with}\, x, y\, \mathsf{in}\, t : Z}\quad \text{T\_BETA}$$

$$\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash_C t : Y}{\Phi_1, \Phi_2, z : X, \Phi_3 \vdash_C \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, t : Y}\quad \text{T\_CONTRR}$$

$$\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash_C t : Y}{\Phi_1, z : X, \Phi_2, \Phi_3 \vdash_C \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, t : Y}\quad \text{T\_CONTRL}$$

$$\frac{\Phi, \Psi \vdash_C t : Y \quad x \notin |\Phi, \Psi|}{\Phi, x : X, \Psi \vdash_C \mathsf{weak}\, x\, \mathsf{in}\, t : Y}\quad \text{T\_WEAK}$$

$$\frac{\Phi \vdash_C t_1 : X \quad \Psi_1, x : X, \Psi_2 \vdash_C t_2 : Y}{\Psi_1, \Phi, \Psi_2 \vdash_C [t_1/x]t_2 : Y}\quad \text{T\_CUT}$$

$$\frac{\Phi, x : X, y : Y, \Psi \vdash_{\mathcal{C}} t : Z}{\Phi, z : X \otimes Y, \Psi \vdash_{\mathcal{C}} \mathsf{let}\, z : X \otimes Y\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, t : Z} \quad \text{T\_TENL}$$

$$\frac{\Phi \vdash_{\mathcal{C}} t_1 : X \quad \Psi \vdash_{\mathcal{C}} t_2 : Y}{\Phi, \Psi \vdash_{\mathcal{C}} t_1 \otimes t_2 : X \otimes Y} \quad \text{T\_TENR}$$

$$\frac{\Phi \vdash_{\mathcal{C}} t_1 : X \quad \Psi_1, x : Y, \Psi_2 \vdash_{\mathcal{C}} t_2 : Z}{\Psi_1, \Phi, y : X \multimap Y, \Psi_2 \vdash_{\mathcal{C}} [\mathsf{app}\, y\, t_1/x]t_2 : Z} \quad \text{T\_IMPL}$$

$$\frac{\Phi, x : X, \Psi \vdash_{\mathcal{C}} t : Y}{\Phi, \Psi \vdash_{\mathcal{C}} \lambda x : X.t : X \multimap Y} \quad \text{T\_IMPR}$$

$$\frac{\Phi \vdash_{\mathcal{L}} s : A}{\Phi \vdash_{\mathcal{C}} \mathsf{G}s : \mathsf{G}A} \quad \text{T\_GR}$$

$\boxed{\Gamma \vdash_{\mathcal{L}} s : A}$

$$\frac{}{x : A \vdash_{\mathcal{L}} x : A} \quad \text{S\_AX}$$

$$\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, x : \mathsf{UnitT}, \Delta \vdash_{\mathcal{L}} \mathsf{let}\, x : \mathsf{UnitT}\, \mathsf{be}\, \mathsf{trivT}\, \mathsf{in}\, s : A} \quad \text{S\_UNITL1}$$

$$\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, x : \mathsf{UnitS}, \Delta \vdash_{\mathcal{L}} \mathsf{let}\, x : \mathsf{UnitS}\, \mathsf{be}\, \mathsf{trivS}\, \mathsf{in}\, s : A} \quad \text{S\_UNITL2}$$

$$\frac{}{\cdot \vdash_{\mathcal{L}} \mathsf{trivS} : \mathsf{UnitS}} \quad \text{S\_UNITR}$$

$$\frac{\Gamma, x : X, y : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, z : Y, w : X, \Delta \vdash_{\mathcal{L}} \mathsf{ex}\, w, z\, \mathsf{with}\, x, y\, \mathsf{in}\, s : A} \quad \text{S\_BETA}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} s : A}{\Gamma_1, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, s : A} \quad \text{S\_CONTRR}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} s : A}{\Gamma_1, y : X, \Gamma_2, \Gamma_3 \vdash_{\mathcal{L}} \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, s : A} \quad \text{S\_CONTRL}$$

$$\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A \quad x \notin |\Gamma, \Delta|}{\Gamma, x : X, \Delta \vdash_{\mathcal{L}} \mathsf{weak}\, x\, \mathsf{in}\, s : B} \quad \text{S\_WEAK}$$

$$\frac{\Phi \vdash_{\mathcal{C}} t : X \quad \Gamma_1, x : X, \Gamma_2 \vdash_{\mathcal{L}} s : A}{\Gamma_1, \Phi, \Gamma_1 \vdash_{\mathcal{L}} [t/x]s : A} \quad \text{S\_CUT1}$$

$$\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta_1, x : A, \Delta_2 \vdash_{\mathcal{L}} s_2 : B}{\Delta_1, \Gamma, \Delta_2 \vdash_{\mathcal{L}} [s_1/x]s_2 : B} \quad \text{S\_CUT2}$$

$$\frac{\Gamma, x : X, y : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, z : X \otimes Y, \Delta \vdash_{\mathcal{L}} \mathsf{let}\, z : X \otimes Y\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, s : A} \quad \text{S\_TENL1}$$

$$\frac{\Gamma, x : A, y : B, \Delta \vdash_{\mathcal{L}} s : C}{\Gamma, z : A \triangleright B, \Delta \vdash_{\mathcal{L}} \mathsf{let}\, z : A \triangleright B\, \mathsf{be}\, x \triangleright y\, \mathsf{in}\, s : C} \quad \text{S\_TENL2}$$

$$\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta \vdash_{\mathcal{L}} s_2 : B}{\Gamma, \Delta \vdash_{\mathcal{L}} s_1 \triangleright s_2 : A \triangleright B} \quad \text{S\_TENR}$$

$$\frac{\Phi \vdash_{\mathcal{C}} t : X \quad \Gamma, x : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, \Phi, y : X \multimap Y, \Delta \vdash_{\mathcal{L}} [\mathsf{app}\, y\, t/x]s : A} \quad \text{S\_IMPL}$$

$$\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta, x : B \vdash_{\mathcal{L}} s_2 : C}{\Delta, \Gamma, y : A \rightharpoonup B \vdash_{\mathcal{L}} [\mathsf{app}_r\, y\, s_1/x]s_2 : C}\ \text{S\_IMPRL}$$

$$\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad x : B, \Delta \vdash_{\mathcal{L}} s_2 : C}{y : B \leftharpoonup A, \Gamma, \Delta \vdash_{\mathcal{L}} [\mathsf{app}_l\, y\, s_1/x]s_2 : C}\ \text{S\_IMPLL}$$

$$\frac{\Gamma, x : A \vdash_{\mathcal{L}} s : B}{\Gamma \vdash_{\mathcal{L}} \lambda_r x : A.s : A \rightharpoonup B}\ \text{S\_IMPRR}$$

$$\frac{x : A, \Gamma \vdash_{\mathcal{L}} s : B}{\Gamma \vdash_{\mathcal{L}} \lambda_l x : A.s : B \leftharpoonup A}\ \text{S\_IMPLR}$$

$$\frac{\Phi \vdash_{\mathcal{C}} t : X}{\Phi \vdash_{\mathcal{L}} \mathsf{F}t : \mathsf{F}X}\ \text{S\_FR}$$

$$\frac{\Gamma, x : X, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, y : \mathsf{F}X, \Delta \vdash_{\mathcal{L}} \mathsf{let}\, z : \mathsf{F}X\, \mathsf{be}\, \mathsf{F}\, x\, \mathsf{in}\, s : A}\ \text{S\_FL}$$

$$\frac{\Gamma, x : A, \Delta \vdash_{\mathcal{L}} s : B}{\Gamma, y : \mathsf{G}A, \Delta \vdash_{\mathcal{L}} \mathsf{let}\, z : \mathsf{G}A\, \mathsf{be}\, \mathsf{G}\, x\, \mathsf{in}\, s : B}\ \text{S\_GL}$$