

Deriving exchange in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{y_0 : GB \vdash_C y_0 : GB} \text{AX}}{y_0 : GB \vdash_{\mathcal{L}} Fy_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash_C x_0 : GA} \text{AX}}{x_0 : GA \vdash_{\mathcal{L}} Fx_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA \vdash_{\mathcal{L}} Fy_0 \triangleright Fx_0 : FGB \triangleright FGA} \text{TENR}}{x_1 : GA, y_1 : GB \vdash_{\mathcal{L}} \text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0) : FGB \triangleright FGA} \text{BETA}} \\
\frac{x_1 : GA, y_2 : FGB \vdash_{\mathcal{L}} \text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0)) : FGB \triangleright FGA}{x_2 : FGA, y_2 : FGB \vdash_{\mathcal{L}} \text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0))) : FGB \triangleright FGA} \text{FL}} \\
\frac{z : FGA \triangleright FGB \vdash_{\mathcal{L}} \text{let } z : FGA \triangleright FGB \text{ be } x_2 \triangleright y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0)))) : (FGB \triangleright FGA)}{\cdot \vdash_{\mathcal{L}} \lambda_r z : FGA \triangleright FGB. \text{let } z : FGA \triangleright FGB \text{ be } x_2 \triangleright y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \triangleright Fx_0)))) : (FGA \triangleright FGB) \rightarrow (FGB \triangleright FGA)} \text{IMPRR}
\end{array}$$

Deriving right contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_1 : GA \vdash_C x_1 : GA} \text{AX}}{x_1 : GA \vdash_{\mathcal{L}} Fx_1 : FGA} \text{Fr} \quad \frac{\frac{\frac{\frac{}{y_0 : GB \vdash_C y_0 : GB} \text{AX}}{y_0 : GB \vdash_{\mathcal{L}} Fy_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash_C x_0 : GA} \text{AX}}{x_0 : GA \vdash_{\mathcal{L}} Fx_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA \vdash_{\mathcal{L}} Fy_0 \triangleright Fx_0 : FGB \triangleright FGA} \text{TENR}}{x_1 : GA, y_0 : GB, x_0 : GA \vdash_{\mathcal{L}} Fx_1 \triangleright (Fy_0 \triangleright Fx_0) : FGA \triangleright (FGB \triangleright FGA)} \text{TENR}} \\
\frac{y_0 : GB, x_2 : GA \vdash_{\mathcal{L}} \text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)) : FGA \triangleright (FGB \triangleright FGA)}{\text{CONTRR}} \\
\frac{y_0 : GB, x_3 : FGA \vdash_{\mathcal{L}} \text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0))) : FGA \triangleright (FGB \triangleright FGA)}{\text{FL}} \\
\frac{y_1 : FGB, x_3 : FGA \vdash_{\mathcal{L}} \text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)))) : FGA \triangleright (FGB \triangleright FGA)}{\text{FL}} \\
\frac{z : FGB \triangleright FGA \vdash_{\mathcal{L}} \text{let } z : FGB \triangleright FGA \text{ be } y_1 \triangleright x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)))) : FGA \triangleright (FGB \triangleright FGA)}{\cdot \vdash_{\mathcal{L}} \lambda_r z : FGB \triangleright FGA. \text{let } z : FGB \triangleright FGA \text{ be } y_1 \triangleright x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \triangleright (Fy_0 \triangleright Fx_0)))) : (FGB \triangleright FGA) \rightarrow (FGA \triangleright (FGB \triangleright FGA))} \text{IMPRR}
\end{array}$$

Deriving left contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : GA \vdash_C x_0 : GA} \text{AX}}{x_0 : GA \vdash_{\mathcal{L}} Fx_0 : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash_C y_0 : GB} \text{AX}}{y_0 : GB \vdash_{\mathcal{L}} Fy_0 : FGB} \text{Fr}}{x_0 : GA, y_0 : GB \vdash_{\mathcal{L}} Fx_0 \triangleright Fy_0 : FGA \triangleright FGB} \text{TENR}} \\
\frac{x_1 : GA \vdash_C x_1 : GA}{x_1 : GA \vdash_{\mathcal{L}} Fx_1 : FGA} \text{Fr} \\
\frac{x_0 : GA, y_0 : GB, x_1 : GA \vdash_{\mathcal{L}} (Fx_0 \triangleright Fy_0) \triangleright Fx_1 : (FGA \triangleright FGB) \triangleright FGA}{x_2 : GA, y_0 : GB \vdash_{\mathcal{L}} \text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1) : (FGA \triangleright FGB) \triangleright FGA} \text{CONTRR}} \\
\frac{x_2 : GA, y_1 : FGB \vdash_{\mathcal{L}} \text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1)) : (FGA \triangleright FGB) \triangleright FGA}{x_3 : FGA, y_1 : FGB \vdash_{\mathcal{L}} \text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1))) : (FGA \triangleright FGB) \triangleright FGA} \text{FL}} \\
\frac{z : FGA \triangleright FGB \vdash_{\mathcal{L}} \text{let } z : FGA \triangleright FGB \text{ be } x_3 \triangleright y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1)))) : (FGA \triangleright FGB) \triangleright FGA}{\cdot \vdash_{\mathcal{L}} \lambda_r z : FGA \triangleright FGB. \text{let } z : FGA \triangleright FGB \text{ be } x_3 \triangleright y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \triangleright Fy_0) \triangleright Fx_1)))) : (FGA \triangleright FGB) \rightarrow ((FGA \triangleright FGB) \triangleright FGA)} \text{IMPRR}
\end{array}$$

Deriving weakening in Elle comonadically:

$$\begin{array}{c}
\frac{\cdot \vdash_{\mathcal{L}} \text{trivS} : \text{UnitS}}{x_0 : GA \vdash_{\mathcal{L}} \text{weak } x_0 \text{ in trivS} : \text{UnitS}} \text{WEAK} \\
\frac{x_1 : FGA \vdash_{\mathcal{L}} \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in trivS} : \text{UnitS}}{\cdot \vdash_{\mathcal{L}} \lambda_r x_1 : FGA. \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in trivS} : (FGA) \rightarrow (\text{UnitS})} \text{IMPRR}
\end{array}$$

GF is a monad:

- Deriving η :

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : FX \vdash_{\mathcal{L}} x_0 : FX} \text{AX}}{x_1 : GFX \vdash_{\mathcal{L}} \text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0 : FX} \text{GL}}{x_2 : FGFX \vdash_{\mathcal{L}} \text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0) : FX} \text{FL}} \\
\frac{x_3 : GFGFX \vdash_{\mathcal{L}} \text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0)) : FX}{x_3 : GFGFX \vdash_C G(\text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0))) : GFX} \text{GL}} \\
\frac{\cdot \vdash_{\mathcal{L}} \lambda_r x_3 : GFGFX. G(\text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0))) : (GFGFX) \rightarrow (GFX)}{\cdot \vdash_{\mathcal{L}} \lambda_r x_3 : GFGFX. G(\text{let } x_3 : GFGFX \text{ be } Gx_2 \text{ in } (\text{let } x_2 : FGFX \text{ be } Fx_1 \text{ in } (\text{let } x_1 : GFX \text{ be } Gx_0 \text{ in } x_0))) : (GFGFX) \rightarrow (GFX)} \text{IMPRR}
\end{array}$$

- Deriving μ :

$$\frac{\frac{\frac{}{x : X \vdash_C x : X} \text{AX}}{x : X \vdash_{\mathcal{L}} Fx : FX} \text{Fr}}{x : X \vdash_C GFx : GFX} \text{Gr}}{\vdash_C \lambda x : X. GFx : (X) \multimap (GFX)} \text{IMPRR}$$

The monad GF is strong:

- Deriving the tensorial strength τ :

$$\frac{\frac{\frac{\frac{\frac{}{x_0 : X \vdash_C x_0 : X} \text{AX}}{x_0 : X, y_0 : Y \vdash_C x_0 \otimes y_0 : X \otimes Y} \text{TENR}}{x_0 : X, y_0 : Y \vdash_{\mathcal{L}} F(x_0 \otimes y_0) : F(X \otimes Y)} \text{Fr}}{x_0 : X, y_1 : FY \vdash_{\mathcal{L}} \text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0) : F(X \otimes Y)} \text{FL}}{\frac{y_2 : GFY, x_0 : X \vdash_{\mathcal{L}} \text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)) : F(X \otimes Y)}{y_2 : GFY, x_0 : X \vdash_C G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0))) : GF(X \otimes Y)} \text{Gr}} \text{GL}}{\frac{x_1 : X, y_3 : GFY \vdash_C \text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)))) : GF(X \otimes Y)}{z : X \otimes GFY \vdash_C \text{let } z : X \otimes GFY \text{ be } x_1 \otimes y_3 \text{ in } (\text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)))))) : GF(X \otimes Y)} \text{BETA}} \text{TENL}}{\vdash_C \lambda z : X \otimes GFY. \text{let } z : X \otimes GFY \text{ be } x_1 \otimes y_3 \text{ in } (\text{ex } y_3, x_1 \text{ with } y_2, x_0 \text{ in } (G(\text{let } y_2 : GFY \text{ be } Gy_1 \text{ in } (\text{let } y_1 : FY \text{ be } Fy_0 \text{ in } F(x_0 \otimes y_0)))))) : (X \otimes GFY) \multimap (GF(X \otimes Y))} \text{IMPRR}$$

A Full Ott Spec

vars, n, a, x, y, z, w, m, o

ivar, i, k, j, l

const, b

A, B, C ::=
 | B
 | UnitS
 | $A \triangleright B$
 | $A \multimap B$
 | $A \leftarrow B$
 | FX

X, Y, Z ::=
 | B
 | UnitT
 | $X \otimes Y$
 | $X \multimap Y$
 | GA

T ::=
 | A
 | X

p ::=

		★	
		x	
		trivT	
		trivS	
		$p \otimes p'$	
		$p \triangleright p'$	
		$F p$	
		$G p$	
s	::=		
		x	
		b	
		trivS	
		let $s_1 : T$ be p in s_2	
		let $t : T$ be p in s	
		$s_1 \triangleright s_2$	
		$\lambda_l x : A.s$	
		$\lambda_r x : A.s$	
		app _{l} $s_1 s_2$	
		app _{r} $s_1 s_2$	
		ex s_1, s_2 with x_1, x_2 in s_3	
		contrR x as s_1, s_2 in s_3	
		contrL x as s_1, s_2 in s_3	
		weak x in s	
		(s)	S
		$F t$	
t	::=		
		x	
		b	
		trivT	
		let $t_1 : X$ be p in t_2	
		$t_1 \otimes t_2$	
		$\lambda x : X.t$	
		app $t_1 t_2$	
		ex t_1, t_2 with x_1, x_2 in t_3	
		contrR x as t_1, t_2 in t_3	
		contrR x as t_1, t_2 in t_3	
		weak x in t	
		(t)	S
		$G s$	
Φ, Ψ	::=		

$$\begin{array}{c|c}
\Gamma, \Delta & ::= \\
\hline
& \cdot \\
& \Phi_1, \Phi_2 \\
& x : X \\
& X \\
& (\Phi) \quad S
\end{array}$$

$$\boxed{\Phi \vdash_C t : X}$$

$$\begin{array}{c}
\frac{}{x : X \vdash_C x : X} \quad \text{T_AX} \\
\\
\frac{\Psi \vdash_C t : X}{x : \text{UnitT}, \Psi \vdash_C \text{let } x : \text{UnitT} \text{ be } \text{trivT} \text{ in } t : X} \quad \text{T_UNITL} \\
\\
\frac{}{\cdot \vdash_C \text{trivT} : \text{UnitT}} \quad \text{T_UNITR} \\
\\
\frac{\Phi, x : X, y : Y, \Psi \vdash_C t : Z}{\Phi, z : Y, w : X, \Psi \vdash_C \text{ex } w, z \text{ with } x, y \text{ in } t : Z} \quad \text{T_BETA} \\
\\
\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash_C t : Y}{\Phi_1, \Phi_2, z : X, \Phi_3 \vdash_C \text{contrR } z \text{ as } x, y \text{ in } t : Y} \quad \text{T_CONTRR} \\
\\
\frac{\Phi_1, x : X, \Phi_2, y : X, \Phi_3 \vdash_C t : Y}{\Phi_1, z : X, \Phi_2, \Phi_3 \vdash_C \text{contrR } z \text{ as } x, y \text{ in } t : Y} \quad \text{T_CONTRL} \\
\\
\frac{\Phi, \Psi \vdash_C t : Y \quad x \notin |\Phi, \Psi|}{\Phi, x : X, \Psi \vdash_C \text{weak } x \text{ in } t : Y} \quad \text{T_WEAK} \\
\\
\frac{\Phi \vdash_C t_1 : X \quad \Psi_1, x : X, \Psi_2 \vdash_C t_2 : Y}{\Psi_1, \Phi, \Psi_2 \vdash_C [t_1/x]t_2 : Y} \quad \text{T_CUT} \\
\\
\frac{\Phi, x : X, y : Y, \Psi \vdash_C t : Z}{\Phi, z : X \otimes Y, \Psi \vdash_C \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } t : Z} \quad \text{T_TENL} \\
\\
\frac{\Phi \vdash_C t_1 : X \quad \Psi \vdash_C t_2 : Y}{\Phi, \Psi \vdash_C t_1 \otimes t_2 : X \otimes Y} \quad \text{T_TENR} \\
\\
\frac{\Phi, x : X \vdash_C t : Y}{\Phi \vdash_C \lambda x : X. t : X \multimap Y} \quad \text{T_IMPR} \\
\\
\frac{\Phi \vdash_C t_1 : X \quad \Psi_1, x : Y, \Psi_2 \vdash_C t_2 : Z}{\Psi_1, y : X \multimap Y, \Phi, \Psi_2 \vdash_C [\text{app } y t_1/x]t_2 : Z} \quad \text{T_IMPL} \\
\\
\frac{\Phi \vdash_{\mathcal{L}} s : A}{\Phi \vdash_C \text{Gs} : \text{GA}} \quad \text{T_GR}
\end{array}$$

$$\boxed{\Gamma \vdash_{\mathcal{L}} s : A}$$

$$\begin{array}{c}
\frac{}{x : A \vdash_{\mathcal{L}} x : A} \text{S_AX} \\
\\
\frac{\Delta \vdash_{\mathcal{L}} s : A}{x : \text{UnitT}, \Delta \vdash_{\mathcal{L}} \text{let } x : \text{UnitT} \text{ be } \text{trivT} \text{ in } s : A} \text{S_UNITL1} \\
\\
\frac{\Delta \vdash_{\mathcal{L}} s : A}{x : \text{UnitS}, \Delta \vdash_{\mathcal{L}} \text{let } x : \text{UnitS} \text{ be } \text{trivS} \text{ in } s : A} \text{S_UNITL2} \\
\\
\frac{}{\cdot \vdash_{\mathcal{L}} \text{trivS} : \text{UnitS}} \text{S_UNITR} \\
\\
\frac{\Gamma, x : X, y : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, z : Y, w : X, \Delta \vdash_{\mathcal{L}} \text{ex } w, z \text{ with } x, y \text{ in } s : A} \text{S_BETA} \\
\\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} s : A}{\Gamma_1, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} \text{contrR } z \text{ as } x, y \text{ in } s : A} \text{S_CONTRR} \\
\\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash_{\mathcal{L}} s : A}{\Gamma_1, y : X, \Gamma_2, \Gamma_3 \vdash_{\mathcal{L}} \text{contrR } z \text{ as } x, y \text{ in } s : A} \text{S_CONTRL} \\
\\
\frac{\Gamma, \Delta \vdash_{\mathcal{L}} s : A \quad x \notin |\Gamma, \Delta|}{\Gamma, x : X, \Delta \vdash_{\mathcal{L}} \text{weak } x \text{ in } s : B} \text{S_WEAK} \\
\\
\frac{\Phi \vdash_C t : X \quad \Gamma_1, x : X, \Gamma_2 \vdash_{\mathcal{L}} s : A}{\Gamma_1, \Phi, \Gamma_1 \vdash_{\mathcal{L}} [t/x]s : A} \text{S_CUT1} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta_1, x : A, \Delta_2 \vdash_{\mathcal{L}} s_2 : B}{\Delta_1, \Gamma, \Delta_2 \vdash_{\mathcal{L}} [s_1/x]s_2 : B} \text{S_CUT2} \\
\\
\frac{\Gamma, x : X, y : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, z : X \otimes Y, \Delta \vdash_{\mathcal{L}} \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : A} \text{S_TENL1} \\
\\
\frac{\Gamma, x : A, y : B, \Delta \vdash_{\mathcal{L}} s : C}{\Gamma, z : A \triangleright B, \Delta \vdash_{\mathcal{L}} \text{let } z : A \triangleright B \text{ be } x \triangleright y \text{ in } s : C} \text{S_TENL2} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta \vdash_{\mathcal{L}} s_2 : B}{\Gamma, \Delta \vdash_{\mathcal{L}} s_1 \triangleright s_2 : A \triangleright B} \text{S_TENR} \\
\\
\frac{\Phi \vdash_C t : X \quad \Gamma, x : Y, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, y : X \multimap Y, \Phi, \Delta \vdash_{\mathcal{L}} [\text{app } y t/x]s : A} \text{S_IMPL} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta_1, x : B, \Delta_2 \vdash_{\mathcal{L}} s_2 : C}{\Delta_1, y : A \multimap B, \Gamma, \Delta_2 \vdash_{\mathcal{L}} [\text{app}_r y s_1/x]s_2 : C} \text{S_IMPRL} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} s_1 : A \quad \Delta_1, x : B, \Delta_2 \vdash_{\mathcal{L}} s_2 : C}{\Delta_1, \Gamma, y : B \multimap A, \Delta_2 \vdash_{\mathcal{L}} [\text{app}_l y s_1/x]s_2 : C} \text{S_IMPLL} \\
\\
\frac{\Gamma, x : A \vdash_{\mathcal{L}} s : B}{\Gamma \vdash_{\mathcal{L}} \lambda_r x : A. s : A \multimap B} \text{S_IMPRR} \\
\\
\frac{x : A, \Gamma \vdash_{\mathcal{L}} s : B}{\Gamma \vdash_{\mathcal{L}} \lambda_l x : A. s : B \multimap A} \text{S_IMPLR} \\
\\
\frac{\Phi \vdash_C t : X}{\Phi \vdash_{\mathcal{L}} \text{F } t : \text{FX}} \text{S_FR} \\
\\
\frac{\Gamma, x : X, \Delta \vdash_{\mathcal{L}} s : A}{\Gamma, y : \text{FX}, \Delta \vdash_{\mathcal{L}} \text{let } y : \text{FX} \text{ be } \text{F } x \text{ in } s : A} \text{S_FL}
\end{array}$$

$$\frac{\Gamma, x : A, \Delta \vdash_{\mathcal{L}} s : B}{\Gamma, y : \mathbf{GA}, \Delta \vdash_{\mathcal{L}} \text{let } y : \mathbf{GA} \text{ be } G.x \text{ in } s : B} \text{ S_GL}$$

$$\boxed{\Phi \vdash_C X}$$

$$\begin{array}{c} \overline{X \vdash_C X} \quad \text{TTY_AX} \\ \frac{\Phi \vdash_C X}{\text{UnitT}, \Phi \vdash_C X} \quad \text{TTY_UNITL} \\ \frac{}{\cdot \vdash_C \text{UnitT}} \quad \text{TTY_UNITR} \\ \frac{\Phi, X, Y, \Psi \vdash_C Z}{\Phi, Y, X, \Psi \vdash_C Z} \quad \text{TTY_BETA} \\ \frac{\Phi \vdash_C X \quad \Psi_1, X, \Psi_2 \vdash_C Y}{\Psi_1, \Phi, \Psi_2 \vdash_C Y} \quad \text{TTY_CUT} \\ \frac{\Phi, X, Y, \Psi \vdash_C Z}{\Phi, X \otimes Y, \Psi \vdash_C Z} \quad \text{TTY_TENL} \\ \frac{\Phi \vdash_C X \quad \Psi \vdash_C Y}{\Phi, \Psi \vdash_C X \otimes Y} \quad \text{TTY_TENR} \\ \frac{\Phi, X \vdash_C Y}{\Phi \vdash_C X \multimap Y} \quad \text{TTY_IMPR} \\ \frac{\Phi \vdash_C X \quad \Psi_1, Y, \Psi_2 \vdash_C Z}{\Psi_1, X \multimap Y, \Phi, \Psi_2 \vdash_C Z} \quad \text{TTY_IMPL} \\ \frac{\Phi \vdash_{\mathcal{L}} A}{\Phi \vdash_C \mathbf{GA}} \quad \text{TTY_GR} \end{array}$$

$$\boxed{\Gamma \vdash_{\mathcal{L}} A}$$

$$\begin{array}{c} \overline{A \vdash_{\mathcal{L}} A} \quad \text{STY_AX} \\ \frac{\Delta \vdash_{\mathcal{L}} A}{\text{UnitT}, \Delta \vdash_{\mathcal{L}} A} \quad \text{STY_UNITL1} \\ \frac{\Delta \vdash_{\mathcal{L}} A}{\text{UnitS}, \Delta \vdash_{\mathcal{L}} A} \quad \text{STY_UNITL2} \\ \frac{}{\cdot \vdash_{\mathcal{L}} \text{UnitS}} \quad \text{STY_UNITR} \\ \frac{\Gamma, X, Y, \Delta \vdash_{\mathcal{L}} A}{\Gamma, Y, X, \Delta \vdash_{\mathcal{L}} A} \quad \text{STY_BETA} \\ \frac{\Phi \vdash_C X \quad \Gamma_1, X, \Gamma_2 \vdash_{\mathcal{L}} A}{\Gamma_1, \Phi, \Gamma_2 \vdash_{\mathcal{L}} A} \quad \text{STY_CUT1} \\ \frac{\Gamma \vdash_{\mathcal{L}} A \quad \Delta_1, A, \Delta_2 \vdash_{\mathcal{L}} B}{\Delta_1, A, \Delta_2 \vdash_{\mathcal{L}} B} \quad \text{STY_CUT2} \\ \frac{\Gamma, X, Y, \Delta \vdash_{\mathcal{L}} A}{\Gamma, X \otimes Y, \Delta \vdash_{\mathcal{L}} A} \quad \text{STY_TENL1} \end{array}$$

$$\begin{array}{c}
\frac{\Gamma, A, B, \Delta \vdash_{\mathcal{L}} C}{\Gamma, A \triangleright B, \Delta \vdash_{\mathcal{L}} C} \text{ STY_TENL2} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} A \quad \Delta \vdash_{\mathcal{L}} B}{\Gamma, \Delta \vdash_{\mathcal{L}} A \triangleright B} \text{ STY_TENR} \\
\\
\frac{\Phi \vdash_C X \quad \Gamma, Y, \Delta \vdash_{\mathcal{L}} A}{\Gamma, X \multimap Y, \Phi, \Delta \vdash_{\mathcal{L}} A} \text{ STY_IMPL} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} A \quad \Delta_1, B, \Delta_2 \vdash_{\mathcal{L}} C}{\Delta_1, A \multimap B, \Gamma, \Delta_2 \vdash_{\mathcal{L}} C} \text{ STY_IMPLR} \\
\\
\frac{\Gamma \vdash_{\mathcal{L}} A \quad \Delta_1, B, \Delta_2 \vdash_{\mathcal{L}} C}{\Delta_1, \Gamma, B \multimap A, \Delta_2 \vdash_{\mathcal{L}} C} \text{ STY_IMPLL} \\
\\
\frac{\Gamma, A \vdash_{\mathcal{L}} B}{\Gamma \vdash_{\mathcal{L}} A \multimap B} \text{ STY_IMPRR} \\
\\
\frac{A, \Gamma \vdash_{\mathcal{L}} B}{\Gamma \vdash_{\mathcal{L}} B \multimap A} \text{ STY_IMPLR} \\
\\
\frac{\Phi \vdash_C X}{\Phi \vdash_{\mathcal{L}} \text{FX}} \text{ STY_FR} \\
\\
\frac{\Gamma, X, \Delta \vdash_{\mathcal{L}} A}{\Gamma, \text{FX}, \Delta \vdash_{\mathcal{L}} A} \text{ STY_FL} \\
\\
\frac{\Gamma, A, \Delta \vdash_{\mathcal{L}} B}{\Gamma, \text{GA}, \Delta \vdash_{\mathcal{L}} B} \text{ STY_GL}
\end{array}$$