

Deriving exchange in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash F_{y_0} : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash F_{x_0} : FGA} \text{Fr}}{y_0 : GB, x_0 : GA; \cdot \vdash F_{y_0} \otimes F_{x_0} : FGB \otimes FGA} \text{TENR}}{x_1 : GA, y_1 : GB; \cdot \vdash \text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F_{y_0} \otimes F_{x_0}) : FGB \otimes FGA} \text{BETA}} \\
\frac{x_1 : GA, y_2 : FGB \vdash \text{let } y_2 : FGB \text{ be } F_{y_1} \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F_{y_0} \otimes F_{x_0})) : FGB \otimes FGA}{\vdash x_2 : FGA, y_2 : FGB \vdash \text{let } x_2 : FGA \text{ be } F_{x_1} \text{ in } (\text{let } y_2 : FGB \text{ be } F_{y_1} \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F_{y_0} \otimes F_{x_0}))) : FGB \otimes FGA} \text{FL}} \\
\frac{\vdash z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } F_{x_1} \text{ in } (\text{let } y_2 : FGB \text{ be } F_{y_1} \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F_{y_0} \otimes F_{x_0})))) : (FGB \otimes FGA)}{\vdash \cdot \vdash \lambda_{jz} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } F_{x_1} \text{ in } (\text{let } y_2 : FGB \text{ be } F_{y_1} \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (F_{y_0} \otimes F_{x_0})))) : (FGA \otimes FGB) \multimap (FGB \otimes FGA)} \text{IMPR}
\end{array}$$

Deriving right contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_1 : GA \vdash x_1 : GA} \text{AX}}{x_1 : GA; \cdot \vdash F_{x_1} : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash F_{y_0} : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash F_{x_0} : FGA} \text{Fr}}{y_0 : GB, x_0 : GA; \cdot \vdash F_{y_0} \otimes F_{x_0} : FGB \otimes FGA} \text{TENR}}{x_1 : GA, y_0 : GB, x_0 : GA; \cdot \vdash F_{x_1} \otimes (F_{y_0} \otimes F_{x_0}) : FGA \otimes (FGB \otimes FGA)} \text{TENR}} \\
\frac{\frac{\frac{}{y_0 : GB, x_2 : GA; \cdot \vdash \text{contrL } x_2 \text{ as } x_1, x_0 \text{ in } (F_{x_1} \otimes (F_{y_0} \otimes F_{x_0})) : FGA \otimes (FGB \otimes FGA)}{\vdash y_1 : FGB, x_3 : FGA \vdash \text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{contrL } x_2 \text{ as } x_1, x_0 \text{ in } (F_{x_1} \otimes (F_{y_0} \otimes F_{x_0}))) : FGA \otimes (FGB \otimes FGA)} \text{FL}}{\vdash y_1 : FGB, x_3 : FGA \vdash \text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{contrL } x_2 \text{ as } x_1, x_0 \text{ in } (F_{x_1} \otimes (F_{y_0} \otimes F_{x_0})))) : FGA \otimes (FGB \otimes FGA)} \text{FL}} \\
\frac{\vdash z : FGB \otimes FGA \vdash \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{contrL } x_2 \text{ as } x_1, x_0 \text{ in } (F_{x_1} \otimes (F_{y_0} \otimes F_{x_0})))) : FGA \otimes (FGB \otimes FGA)}{\vdash \cdot \vdash \lambda_{jz} : FGB \otimes FGA. \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{contrL } x_2 \text{ as } x_1, x_0 \text{ in } (F_{x_1} \otimes (F_{y_0} \otimes F_{x_0})))) : (FGB \otimes FGA) \multimap (FGA \otimes (FGB \otimes FGA))} \text{IMPR}
\end{array}$$

Deriving left contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash F_{x_0} : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash F_{y_0} : FGB} \text{Fr}}{x_0 : GA, y_0 : GB; \cdot \vdash F_{x_0} \otimes F_{y_0} : FGA \otimes FGB} \text{TENR} \quad \frac{\frac{\frac{}{x_1 : GA \vdash x_1 : GA} \text{AX}}{x_1 : GA; \cdot \vdash F_{x_1} : FGA} \text{Fr}}{x_0 : GA, y_0 : GB, x_1 : GA; \cdot \vdash (F_{x_0} \otimes F_{y_0}) \otimes F_{x_1} : (FGA \otimes FGB) \otimes FGA} \text{TENR}} \\
\frac{\frac{\frac{}{x_2 : GA, y_0 : GB; \cdot \vdash \text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F_{x_0} \otimes F_{y_0}) \otimes F_{x_1}) : (FGA \otimes FGB) \otimes FGA}{x_2 : GA, y_1 : FGB \vdash \text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F_{x_0} \otimes F_{y_0}) \otimes F_{x_1})) : (FGA \otimes FGB) \otimes FGA} \text{FL}}{\vdash x_3 : FGA, y_1 : FGB \vdash \text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F_{x_0} \otimes F_{y_0}) \otimes F_{x_1}))) : (FGA \otimes FGB) \otimes FGA} \text{FL}} \\
\frac{\vdash z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F_{x_0} \otimes F_{y_0}) \otimes F_{x_1})))) : (FGA \otimes FGB) \otimes FGA}{\vdash \cdot \vdash \lambda_{jz} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } F_{x_2} \text{ in } (\text{let } y_1 : FGB \text{ be } F_{y_0} \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((F_{x_0} \otimes F_{y_0}) \otimes F_{x_1})))) : (FGA \otimes FGB) \multimap ((FGA \otimes FGB) \otimes FGA)} \text{IMPR}
\end{array}$$

Deriving weakening in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{}{\cdot \vdash \text{triv} : \text{Unit}} \text{UNITR}}{x_0 : GA; \cdot \vdash \text{weak } x_0 \text{ in triv} : \text{Unit}} \text{WEAK} \\
\frac{x_1 : FGA; \cdot \vdash \text{let } x_1 : FGA \text{ be } F_{x_0} \text{ in weak } x_0 \text{ in triv} : \text{Unit}}{\vdash \cdot \vdash \lambda_{jx_1} : FGA. \text{let } x_1 : FGA \text{ be } F_{x_0} \text{ in weak } x_0 \text{ in triv} : (FGA) \multimap (\text{Unit})} \text{IMPR}
\end{array}$$

A Full Ott Spec

vars, n, a, x, y, z, w, m, o
ivar, i, k, j, l

$const, b$	
A, B, C	$::=$ $ \quad B$ $ \quad \text{Unit}$ $ \quad A \otimes B$ $ \quad A \multimap B$ $ \quad A \multimap B$ $ \quad FX$
X, Y, Z	$::=$ $ \quad B$ $ \quad \text{Unit}$ $ \quad X \otimes Y$ $ \quad X \multimap Y$ $ \quad X \multimap Y$ $ \quad GA$
T	$::=$ $ \quad A$ $ \quad X$
p	$::=$ $ \quad \star$ $ \quad x$ $ \quad \text{triv}$ $ \quad p \otimes p'$ $ \quad Fp$ $ \quad Gp$
s	$::=$ $ \quad x$ $ \quad b$ $ \quad \text{triv}$ $ \quad \text{let } s_1 : T \text{ be } p \text{ in } s_2$ $ \quad s_1 \otimes s_2$ $ \quad \lambda_l x : A. s$ $ \quad \lambda_r x : A. s$ $ \quad \lambda x : A. s$ $ \quad \text{app}_l s_1 s_2$ $ \quad \text{app}_r s_1 s_2$ $ \quad \text{app } s_1 s_2$ $ \quad \text{ex } x_1, x_2 \text{ with } s_1, s_2 \text{ in } s_3$ $ \quad \text{contrR } x_1 \text{ as } s_1, s_2 \text{ in } s_3$ $ \quad \text{contrL } x_1 \text{ as } s_1, s_2 \text{ in } s_3$

		weak x in s	
		(s)	S
		Ft	

t	::=		
		x	
		b	
		triv	
		let $t_1 : X$ be p in t_2	
		$t_1 \otimes t_2$	
		$\lambda_l x : X. t$	
		$\lambda_r x : X. t$	
		$\lambda x : X. t$	
		app _l $t_1 t_2$	
		app _r $t_1 t_2$	
		app $t_1 t_2$	
		ex x_1, x_2 with t_1, t_2 in t_3	
		contrR x_1 as t_1, t_2 in t_3	
		contrL x_1 as t_1, t_2 in t_3	
		weak x in t	
		(t)	S
		Gs	

$\Gamma, \Delta, \Phi, \Psi$::=		
		.	
		Γ_1, Γ_2	
		$x : A$	
		(Γ)	S
		$x : X$	

$\Gamma \vdash t : X$

$\frac{}{x : X \vdash x : X}$	T_VAR
$\frac{}{\Gamma, \Delta \vdash t : X}$	
$\frac{}{\Gamma, x : \text{Unit}, \Delta \vdash \text{let } x : \text{Unit} \text{ be triv in } t : X}$	T_UNITL
$\frac{}{\cdot \vdash \text{triv} : \text{Unit}}$	T_UNITR
$\frac{}{\Gamma, x : X, y : Y, \Delta \vdash t : Z}$	
$\frac{}{\Gamma, z : Y, w : X, \Delta \vdash \text{ex } w, z \text{ with } x, y \text{ in } t : Z}$	T_BETA
$\frac{}{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}$	
$\frac{}{\Gamma_1, \Gamma_2, z : X, \Gamma_3 \vdash \text{contrR } z \text{ as } x, y \text{ in } t : Y}$	T_CONTRR
$\frac{}{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}$	
$\frac{}{\Gamma_1, z : X, \Gamma_2, \Gamma_3 \vdash \text{contrL } z \text{ as } x, y \text{ in } t : Y}$	T_CONTRL
$\frac{}{\Gamma, \Delta \vdash t : Y \quad x \notin \Gamma, \Delta }$	
$\frac{}{\Gamma, x : X, \Delta \vdash \text{weak } x \text{ in } t : Y}$	T_WEAK

$$\begin{array}{c}
\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : X, \Delta_2 \vdash t_2 : Y}{\Delta_1, \Gamma, \Delta_2 \vdash [t_1/x]t_2 : Y} \quad \text{T_CUT} \\
\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : X \otimes Y, \Delta \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } t : Z} \quad \text{T_TENL} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta \vdash t_2 : Y}{\Gamma, \Delta \vdash t_1 \otimes t_2 : X \otimes Y} \quad \text{T_TEN} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, \Gamma, y : X \multimap Y, \Delta_2 \vdash [\text{app}_l y t_1/x]t_2 : Z} \quad \text{T_IMPLL} \\
\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, y : Y \multimap X, \Gamma, \Delta_2 \vdash [\text{app}_r y t_1/x]t_2 : Z} \quad \text{T_IMPL2} \\
\frac{\Gamma, x : X \vdash t : Y}{\Gamma \vdash \lambda_l x : X. t : X \multimap Y} \quad \text{T_IMPRL} \\
\frac{x : X, \Gamma \vdash t : Y}{\Gamma \vdash \lambda_r x : X. t : Y \multimap X} \quad \text{T_IMPRR} \\
\frac{\Gamma; \cdot \vdash s : A}{\Gamma \vdash \mathbf{Gs} : \mathbf{GA}} \quad \text{T_Gr}
\end{array}$$

$$\boxed{\Gamma; \Psi \vdash s : A}$$

$$\begin{array}{c}
\frac{}{\cdot; x : A \vdash x : A} \quad \text{S_AX} \\
\frac{\Gamma, \Delta; \Psi \vdash s : A}{\Gamma, x : \text{Unit}, \Delta; \Psi \vdash \text{let } x : \text{Unit} \text{ be } \text{triv} \text{ in } s : A} \quad \text{S_UNITL1} \\
\frac{\Gamma; \Psi, \Phi \vdash s : A}{\Gamma; \Psi, x : \text{Unit}, \Phi \vdash \text{let } x : \text{Unit} \text{ be } \text{triv} \text{ in } s : A} \quad \text{S_UNITL2} \\
\frac{}{\cdot; \cdot \vdash \text{triv} : \text{Unit}} \quad \text{S_UNITR} \\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : Y, w : X, \Delta; \Psi \vdash \text{ex } w, z \text{ with } x, y \text{ in } s : A} \quad \text{S_BETA} \\
\frac{\Gamma, \Delta; \Psi \vdash s : B \quad x \notin |\Gamma, \Delta, \Psi|}{\Gamma, x : X, \Delta; \Psi \vdash \text{weak } x \text{ in } s : B} \quad \text{S_WEAK} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash s : B}{\Gamma_1, \Gamma_2, z : X, \Gamma_3; \Psi \vdash \text{contrR } z \text{ as } x, y \text{ in } s : B} \quad \text{S_CONTRR} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash s : B}{\Gamma_1, z : X, \Gamma_2, \Gamma_3; \Psi \vdash \text{contrL } z \text{ as } x, y \text{ in } s : B} \quad \text{S_CONTRL} \\
\frac{\Gamma \vdash t : X \quad \Delta_1, x : X, \Delta_2; \Psi \vdash s : A}{\Delta_1, \Gamma, \Delta_2; \Phi \vdash [t/x]s : A} \quad \text{S_CUT1} \\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : A, \Phi_2 \vdash s_2 : B}{\Gamma, \Delta; \Phi_1, \Psi, \Phi_2 \vdash [s_1/x]s_2 : B} \quad \text{S_CUT2} \\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : X \otimes Y, \Delta; \Psi \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : A} \quad \text{S_TENL1}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Psi, x : A, y : B, \Phi \vdash s : A}{\Gamma; \Psi, z : A \otimes B, \Phi \vdash \text{let } z : A \otimes B \text{ be } x \otimes y \text{ in } s : A} \quad \text{S_TENL2} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi \vdash s_2 : B}{\Gamma, \Delta; \Psi, \Phi \vdash s_1 \otimes s_2 : A \otimes B} \quad \text{S_TENR} \\
\\
\frac{\Gamma \vdash t : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash s : A}{\Delta_1, \Gamma, y : X \multimap Y, \Delta_2; \Psi \vdash [\text{app}_l y t/x] s : A} \quad \text{S_IMPLL} \\
\\
\frac{\Gamma \vdash t : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash s : A}{\Delta_1, y : Y \multimap X, \Gamma, \Delta_2; \Psi \vdash [\text{app}_r y t/x] s : A} \quad \text{S_IMPL2} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash s_2 : A}{\Gamma, \Delta; \Phi_1, \Psi, y : A \multimap B, \Phi_2 \vdash [\text{app}_l y s_1/x] s_2 : A} \quad \text{S_IMPL3} \\
\\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash s_2 : A}{\Gamma, \Delta; \Phi_1, y : B \multimap A, \Psi, \Phi_2 \vdash [\text{app}_l y s_1/x] s_2 : A} \quad \text{S_IMPL4} \\
\\
\frac{\Gamma; \Psi, x : A \vdash s : B}{\Gamma; \Psi \vdash \lambda_l x : A.s : A \multimap B} \quad \text{S_IMPLR} \\
\\
\frac{\Gamma; x : A, \Psi \vdash s : B}{\Gamma; \Psi \vdash \lambda_r x : A.s : B \multimap A} \quad \text{S_IMPRR} \\
\\
\frac{\Gamma \vdash t : X}{\Gamma; \cdot \vdash Ft : FX} \quad \text{S_FR} \\
\\
\frac{\Gamma, x : X; \Psi \vdash s : A}{\Gamma; z : FX, \Psi \vdash \text{let } z : FX \text{ be } Fx \text{ in } s : A} \quad \text{S_FL} \\
\\
\frac{\Gamma; \Psi, x : A \vdash s : B}{z : GA, \Gamma; \Psi \vdash \text{let } z : GA \text{ be } Gx \text{ in } s : B} \quad \text{S_GL}
\end{array}$$