

A Term Assignment for Natural Deduction Formulation of Elle

vars, n, a, x, y, z, w, m, o

ivar, i, k, j, l

const, b

A, B, C ::=
 | B
 | Unit
 | $A \otimes B$
 | $A \multimap B$
 | $A \multimap B$
 | FX

X, Y, Z ::=
 | B
 | Unit
 | $X \otimes Y$
 | $X \multimap Y$
 | $X \multimap Y$
 | GA

T ::=
 | A
 | X

p ::=
 | ★
 | x
 | triv
 | $p \otimes p'$
 | Fp
 | Gp

s ::=
 | x
 | b
 | triv
 | let $s_1 : T$ be p in s_2
 | $s_1 \otimes s_2$
 | $\lambda_l x : A. s$
 | $\lambda_r x : A. s$
 | $\lambda x : A. s$
 | $\text{app}_l s_1 s_2$
 | $\text{app}_r s_1 s_2$

		app $s_1 s_2$	
		ex x_1, x_2 with s_1, s_2 in s_3	
		contrR x_1 as s_1, s_2 in s_3	
		contrL x_1 as s_1, s_2 in s_3	
		weak x in s	
		derelict t	
		(s)	S
		F t	

t	::=		
		x	
		b	
		triv	
		let $t_1 : X$ be p in t_2	
		$t_1 \otimes t_2$	
		$\lambda_l x : X. t$	
		$\lambda_r x : X. t$	
		$\lambda x : X. t$	
		app _l $t_1 t_2$	
		app _r $t_1 t_2$	
		app $t_1 t_2$	
		ex x_1, x_2 with t_1, t_2 in t_3	
		contrR x_1 as t_1, t_2 in t_3	
		contrL x_1 as t_1, t_2 in t_3	
		weak x in t	
		(t)	S
		G s	

$\Gamma, \Delta, \Phi, \Psi$::=		
		\cdot	
		Γ_1, Γ_2	
		$x : A$	
		(Γ)	S
		$x : X$	

$\Gamma \vdash t : X$

$\frac{}{x : X \vdash x : X}$	T_IDENTITY
$\frac{}{\cdot \vdash \text{triv} : \text{Unit}}$	T_UNITI
$\frac{\Delta \vdash t_1 : \text{Unit} \quad \Gamma \vdash t_2 : Y}{\Gamma, \Delta \vdash \text{let } t_1 : \text{Unit} \text{ be triv in } t : Y}$	T_UNITE
$\frac{\Gamma \vdash t_1 : X \quad \Delta \vdash t_2 : Y}{\Gamma, \Delta \vdash t_1 \otimes t_2 : X \otimes Y}$	T_TENI
$\frac{\Gamma \vdash t_1 : X \otimes Y \quad \Delta, x : X, y : Y \vdash t_2 : Z}{\Gamma, \Delta \vdash \text{let } t_1 : X \otimes Y \text{ be } x \otimes y \text{ in } t_2 : Z}$	T_TENE

$$\begin{array}{c}
\frac{\Gamma, x : X \vdash t : Y}{\Gamma \vdash \lambda_l x : X. t : X \multimap Y} \quad \text{T_IMPLI} \\
\frac{\Gamma \vdash t_1 : X \multimap Y \quad \Delta \vdash t_2 : X}{\Gamma, \Delta \vdash \text{app}_l t_1 t_2 : Y} \quad \text{T_IMPLE} \\
\frac{x : X, \Gamma \vdash t : Y}{\Gamma \vdash \lambda_r x : X. t : Y \leftarrow X} \quad \text{T_IMPRI} \\
\frac{\Gamma \vdash t_1 : Y \leftarrow X \quad \Delta \vdash t_2 : X}{\Gamma, \Delta \vdash \text{app}_r t_1 t_2 : Y} \quad \text{T_IMPRE} \\
\frac{\Gamma; \cdot \vdash s : A}{\Gamma \vdash \text{Gs} : \text{GA}} \quad \text{T_GI}
\end{array}$$

$$\boxed{\Gamma; \Psi \vdash s : A}$$

$$\begin{array}{c}
\frac{}{\cdot; x : A \vdash x : A} \quad \text{S_IDENTITY} \\
\frac{}{\cdot; \cdot \vdash \text{triv} : \text{Unit}} \quad \text{S_UNITI} \\
\frac{\Delta; \Phi \vdash s_1 : \text{Unit} \quad \Gamma; \Psi \vdash s_2 : A}{\Gamma, \Delta; \Psi, \Phi \vdash \text{let } s_1 : \text{Unit} \text{ be } \text{triv} \text{ in } s_2 : A} \quad \text{S_UNIT E} \\
\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi \vdash s_2 : B}{\Gamma, \Delta; \Psi, \Phi \vdash s_1 \otimes s_2 : A \otimes B} \quad \text{S_TENI} \\
\frac{\Gamma \vdash z : X \otimes Y \quad \Delta, x : X, y : Y; \Psi \vdash s : A}{\Delta, \Gamma; \Psi \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : A} \quad \text{S_TENE1} \\
\frac{\Gamma; \Psi \vdash z : A \otimes B \quad \Delta; \Phi, x : A, y : B \vdash s : C}{\Gamma, \Delta; \Phi, \Psi \vdash \text{let } z : A \otimes B \text{ be } x \otimes y \text{ in } s : C} \quad \text{S_TENE2} \\
\frac{\Gamma; \Psi, x : A \vdash s : B}{\Gamma; \Psi \vdash \lambda_l x : A. s : A \multimap B} \quad \text{S_IMPLI} \\
\frac{\Gamma; \Psi \vdash s_1 : A \multimap B \quad \Delta; \Phi \vdash s_2 : A}{\Gamma, \Delta; \Psi, \Phi \vdash \text{app}_l s_1 s_2 : B} \quad \text{S_IMPLE} \\
\frac{\Gamma; x : A, \Psi \vdash s : B}{\Gamma; \Psi \vdash \lambda_r x : A. s : B \leftarrow A} \quad \text{S_IMPRI} \\
\frac{\Gamma; \Psi \vdash s_1 : B \leftarrow A \quad \Delta; \Phi \vdash s_2 : A}{\Gamma, \Delta; \Psi, \Phi \vdash \text{app}_r s_1 s_2 : B} \quad \text{S_IMPRE} \\
\frac{\Gamma \vdash t : X}{\Gamma; \cdot \vdash \text{Ft} : \text{FX}} \quad \text{S_FI} \\
\frac{\Gamma; \Psi \vdash y : \text{FX} \quad \Gamma, x : X; \Phi \vdash s_2 : A}{\Gamma; \Psi, \Phi \vdash \text{let } \text{Fx} : \text{FX} \text{ be } y \text{ in } s_2 : A} \quad \text{S_FE} \\
\frac{\Gamma \vdash t : \text{GA}}{\Gamma; \cdot \vdash \text{derelict } t : A} \quad \text{S_GE}
\end{array}$$