$termvar,\ x,\ y,\ z,\ f,\ r,\ ys$
$typevar,\ X,\ Y,\ Z$
$index,\ i,\ j,\ k$

$t,\ c,\ s \quad ::=$

| | x
| | triv
| | box
| | unbox
| | $error_A$
| | error
| | $\Lambda(X <: A).t$
| | $[A]t$
| | $\lambda(x : A).t$
| | $t_1\ t_2$
| | (t_1, t_2)
| | fst t
| | snd t
| | succ t
| | 0
| | case $t : A$ of $t_3 \to t_1, t_4 \to t_2$
| | $[]$
| | $t :: t'$
| | (t) \qquad\qquad S
| | squash
| | split

$n,\ m \quad ::=$

| | 0
| | succ n

$v \quad ::=$

| | triv
| | $[]$
| | $unbox_A$
| | $\Lambda(X <: A).t$
| | $\lambda(x : A).t$
| | n
| | case $t : A$ of $t_3 \to t_1, t_4 \to t_2$

$\mathcal{E} \quad ::=$

| | $\Box$
| | $\mathcal{E}\ t_2$
| | $unbox_A\ \mathcal{E}$
| | succ $\mathcal{E}$
| | fst $\mathcal{E}$
| | snd $\mathcal{E}$
| | $(\mathcal{E}, t)$
| | $(t, \mathcal{E})$
| | case $\mathcal{E} : A$ of $t_3 \to t_1, t_4 \to t_2$
| | $\mathcal{E} :: t_2$

$$
\begin{array}{ll}
| & t_1 :: \mathcal{E} \\
| & [A]\mathcal{E}
\end{array}
$$

$$
A,\ B,\ C,\ D,\ E,\ S,\ U,\ K,\ T \quad ::=
$$

$$
\begin{array}{ll}
| & X \\
| & \top \\
| & \text{List } A \\
| & \forall(X <: A).B \\
| & \mathbb{S} \\
| & \text{Unit} \\
| & \text{Nat} \\
| & ? \\
| & A_1 \to A_2 \\
| & A_1 \times A_2 \\
| & (A) \qquad\qquad \mathsf{S}
\end{array}
$$

$$
\Gamma \qquad\qquad\qquad\qquad ::=
$$

$$
\begin{array}{ll}
| & . \\
| & \Gamma, x : A \\
| & x : A \\
| & X <: A
\end{array}
$$

$$
\boxed{\Gamma \vdash A <: B}
$$

$$
\frac{}{\Gamma \vdash A <: A} \quad \text{S\_Refl}
$$

$$
\frac{}{\Gamma \vdash A <: \top} \quad \text{S\_Top}
$$

$$
\frac{X <: A \in \Gamma}{\Gamma \vdash X <: A} \quad \text{S\_Var}
$$

$$
\frac{}{\Gamma \vdash \top <: \mathbb{S}} \quad \text{S\_TopSL}
$$

$$
\frac{}{\Gamma \vdash \text{Nat} <: \mathbb{S}} \quad \text{S\_NatSL}
$$

$$
\frac{}{\Gamma \vdash \text{Unit} <: \mathbb{S}} \quad \text{S\_UnitSL}
$$

$$
\frac{\Gamma \vdash A <: \mathbb{S}}{\Gamma \vdash \text{List } A <: \mathbb{S}} \quad \text{S\_ListSL}
$$

$$
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \to B <: \mathbb{S}} \quad \text{S\_ArrowSL}
$$

$$
\frac{\Gamma \vdash A <: \mathbb{S} \quad \Gamma \vdash B <: \mathbb{S}}{\Gamma \vdash A \times B <: \mathbb{S}} \quad \text{S\_ProdSL}
$$

$$
\frac{\Gamma \vdash A <: B}{\Gamma \vdash \text{List } A <: \text{List } B} \quad \text{S\_List}
$$

$$
\frac{\Gamma \vdash A_1 <: A_2 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \times B_1 <: A_2 \times B_2} \quad \text{S\_Prod}
$$

$$
\frac{\Gamma \vdash A_2 <: A_1 \quad \Gamma \vdash B_1 <: B_2}{\Gamma \vdash A_1 \to B_1 <: A_2 \to B_2} \quad \text{S\_Arrow}
$$

$$\frac{\Gamma, X <: A \vdash B_1 <: B_2}{\Gamma \vdash \forall(X <: A).B_1 <: \forall(X <: A).B_2} \quad \text{S\_Forall}$$

$$\boxed{\Gamma_1 \sqsubseteq \Gamma_2}$$

$$\frac{}{\Gamma \sqsubseteq \Gamma} \quad \text{CtxP\_refl}$$

$$\frac{\Gamma_1 \sqsubseteq \Gamma_2 \quad A \sqsubseteq A' \quad \Gamma_3 \sqsubseteq \Gamma_4}{\Gamma_1, x : A, \Gamma_3 \sqsubseteq \Gamma_2, x : A', \Gamma_4} \quad \text{CtxP\_ext}$$

$$\boxed{A \sqsubseteq B}$$

$$\frac{\Gamma \vdash A <: \mathbb{S}}{A \sqsubseteq \,?} \quad \text{P\_U}$$

$$\frac{}{A \sqsubseteq A} \quad \text{P\_refl}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \to B) \sqsubseteq (C \to D)} \quad \text{P\_arrow}$$

$$\frac{A \sqsubseteq C \quad B \sqsubseteq D}{(A \times B) \sqsubseteq (C \times D)} \quad \text{P\_prod}$$

$$\frac{A \sqsubseteq B}{(\mathsf{List}\, A) \sqsubseteq (\mathsf{List}\, B)} \quad \text{P\_list}$$

$$\frac{B_1 \sqsubseteq B_2}{(\forall(X <: A).B_1) \sqsubseteq (\forall(X <: A).B_2)} \quad \text{P\_forall}$$

$$\boxed{\Gamma \vdash t \sqsubseteq t'}$$

$$\frac{x : A \,\in\, \Gamma}{\Gamma \vdash x \sqsubseteq x} \quad \text{TP\_var}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \mathsf{split}_{S_1} \sqsubseteq \mathsf{split}_{S_2}} \quad \text{TP\_split}$$

$$\frac{S_1 \sqsubseteq S_2}{\Gamma \vdash \mathsf{squash}_{S_1} \sqsubseteq \mathsf{squash}_{S_2}} \quad \text{TP\_squash}$$

$$\frac{}{\Gamma \vdash \mathsf{box} \sqsubseteq \mathsf{box}} \quad \text{TP\_box}$$

$$\frac{}{\Gamma \vdash \mathsf{unbox} \sqsubseteq \mathsf{unbox}} \quad \text{TP\_unbox}$$

$$\frac{}{\Gamma \vdash 0 \sqsubseteq 0} \quad \text{TP\_nat}$$

$$\frac{}{\Gamma \vdash \mathsf{triv} \sqsubseteq \mathsf{triv}} \quad \text{TP\_triv}$$

$$\frac{}{\Gamma \vdash [] \sqsubseteq []} \quad \text{TP\_empty}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{succ}\, t_1) \sqsubseteq (\mathsf{succ}\, t_2)} \quad \text{TP\_succ}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : \mathsf{Nat} \vdash t_3 \sqsubseteq t_6}{\Gamma \vdash (\mathsf{case}\, t_1 : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_2, (\mathsf{succ}\, x) \to t_3) \sqsubseteq (\mathsf{case}\, t_4 : \mathsf{Nat}\, \mathsf{of}\, 0 \to t_5, (\mathsf{succ}\, x) \to t_6)} \quad \text{TP\_Nate}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1, t_2) \sqsubseteq (t_3, t_4)} \quad \text{TP\_pair}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{fst}\ t_1) \sqsubseteq (\mathsf{fst}\ t_2)} \quad \text{TP\_FST}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\mathsf{snd}\ t_1) \sqsubseteq (\mathsf{snd}\ t_2)} \quad \text{TP\_SND}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1 :: t_2) \sqsubseteq (t_3 :: t_4)} \quad \text{TP\_CONS}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_4 \quad \Gamma \vdash t_2 \sqsubseteq t_5 \quad \Gamma, x : A_2, y : \mathsf{List}\ A_2 \vdash t_3 \sqsubseteq t_6 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\mathsf{case}\ t_1 : \mathsf{List}\ A_1\ \mathsf{of}\ [] \to t_2, (x :: y) \to t_3) \sqsubseteq (\mathsf{case}\ t_4 : \mathsf{List}\ A_2\ \mathsf{of}\ 0 \to t_5, (x :: y) \to t_6)} \quad \text{TP\_LISTE}$$

$$\frac{\Gamma, x : A_2 \vdash t_1 \sqsubseteq t_2 \quad A_1 \sqsubseteq A_2}{\Gamma \vdash (\lambda(x : A_1).t) \sqsubseteq (\lambda(x : A_2).t_2)} \quad \text{TP\_FUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_3 \quad \Gamma \vdash t_2 \sqsubseteq t_4}{\Gamma \vdash (t_1\ t_2) \sqsubseteq (t_3\ t_4)} \quad \text{TP\_APP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : ?}{\Gamma \vdash (\mathsf{unbox}_A\ t) \sqsubseteq t} \quad \text{TP\_UNBOXING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A}{\Gamma \vdash t \sqsubseteq (\mathsf{box}_A\ t)} \quad \text{TP\_BOXING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : ?}{\Gamma \vdash (\mathsf{split}_S\ t) \sqsubseteq t} \quad \text{TP\_SPLITING}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : S}{\Gamma \vdash t \sqsubseteq (\mathsf{squash}_S\ t)} \quad \text{TP\_SQUASHING}$$

$$\frac{\Gamma, X <: A \vdash t_1 \sqsubseteq t_2}{\Gamma \vdash (\Lambda(X <: A).t_1) \sqsubseteq (\Lambda(X <: A).t_2)} \quad \text{TP\_TFUN}$$

$$\frac{\Gamma \vdash t_1 \sqsubseteq t_2 \quad A \sqsubseteq B}{\Gamma \vdash [A]t_1 \sqsubseteq [B]t_2} \quad \text{TP\_TAPP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : B \quad A \sqsubseteq B}{\Gamma \vdash \mathsf{error}_A \sqsubseteq t} \quad \text{TP\_ERROR}$$

$$\boxed{\Gamma \vdash_{\mathsf{CG}} t : A}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{CG}} x : A} \quad \text{T\_VARP}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash_{\mathsf{CG}} x : A} \quad \text{T\_VAR}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box}_A : A \to ?} \quad \text{T\_BOX}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox}_A : ? \to A} \quad \text{T\_UNBOX}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{box} : \forall(X <: \mathbb{S}).(X \to ?)} \quad \text{T\_BOXP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{unbox} : \forall(X <: \mathbb{S}).(? \to X)} \quad \text{T\_UNBOXP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{squash}_S : S \to ?} \quad \text{T\_SQUASH}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{split}_S : ? \to S} \quad \text{T\_SPLIT}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{triv} : \mathsf{Unit}} \quad \text{T\_unitP}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} 0 : \mathsf{Nat}} \quad \text{T\_zeroP}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat}}{\Gamma \vdash_{\mathsf{CG}} \mathsf{succ}\, t : \mathsf{Nat}} \quad \text{T\_succ}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \mathsf{Nat} \quad \Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma, x : \mathsf{Nat} \vdash_{\mathsf{CG}} t_2 : A}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t : \mathsf{Nat}\, \mathsf{of}\, 0 \rightarrow t_1, (\mathsf{succ}\, x) \rightarrow t_2 : A} \quad \text{T\_ncase}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} [] : \forall(X <: \top).\mathsf{List}\, X} \quad \text{T\_empty}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \quad \Gamma \vdash_{\mathsf{CG}} t_2 : \mathsf{List}\, A}{\Gamma \vdash_{\mathsf{CG}} t_1 :: t_2 : \mathsf{List}\, A} \quad \text{T\_cons}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : B \quad \Gamma, x : A, y : \mathsf{List}\, A \vdash_{\mathsf{CG}} t_2 : B \quad \Gamma \vdash_{\mathsf{CG}} t : \mathsf{List}\, A}{\Gamma \vdash_{\mathsf{CG}} \mathsf{case}\, t : \mathsf{List}\, A\, \mathsf{of}\, [] \rightarrow t_1, (x :: y) \rightarrow t_2 : B} \quad \text{T\_lcase}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A_1 \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A_2}{\Gamma \vdash_{\mathsf{CG}} (t_1, t_2) : A_1 \times A_2} \quad \text{T\_pair}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{fst}\, t : A_1} \quad \text{T\_fst}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A_1 \times A_2}{\Gamma \vdash_{\mathsf{CG}} \mathsf{snd}\, t : A_2} \quad \text{T\_snd}$$

$$\frac{\Gamma, x : A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \lambda(x : A).t : A \rightarrow B} \quad \text{T\_lam}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t_1 : A \rightarrow B \quad \Gamma \vdash_{\mathsf{CG}} t_2 : A}{\Gamma \vdash_{\mathsf{CG}} t_1\, t_2 : B} \quad \text{T\_app}$$

$$\frac{\Gamma, X <: A \vdash_{\mathsf{CG}} t : B}{\Gamma \vdash_{\mathsf{CG}} \Lambda(X <: A).t : \forall(X <: A).B} \quad \text{T\_Lam}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : \forall(X <: B).C \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} [A]t : [A/X]C} \quad \text{T\_typeApp}$$

$$\frac{\Gamma \vdash_{\mathsf{CG}} t : A \quad \Gamma \vdash A <: B}{\Gamma \vdash_{\mathsf{CG}} t : B} \quad \text{T\_Sub}$$

$$\frac{}{\Gamma \vdash_{\mathsf{CG}} \mathsf{error}_A : A} \quad \text{T\_error}$$

$\boxed{t_1 \rightsquigarrow t_2}$ call by name

$$\frac{}{\mathsf{unbox}_A\, (\mathsf{box}_A\, t) \rightsquigarrow t} \quad \text{rd\_retracT}$$

$$\frac{A \neq B}{\mathsf{unbox}_A\, (\mathsf{box}_B\, t) \rightsquigarrow \mathsf{error}_A} \quad \text{rd\_retracTE}$$

$$\frac{}{\mathsf{split}_S\, (\mathsf{squash}_S\, t) \rightsquigarrow t} \quad \text{rd\_retractU}$$

$$\frac{x : B \vdash_{\mathsf{CG}} \mathcal{E}[x] : A}{\mathcal{E}[\mathsf{error}_B] \rightsquigarrow \mathsf{error}_A} \quad \text{rd\_error}$$

$$\overline{\mathsf{case}\, 0\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow t_1} \quad \text{RD\_NCASE0}$$

$$\overline{\mathsf{case}\, (\mathsf{succ}\, t)\colon \mathsf{Nat\ of}\ 0 \to t_1, (\mathsf{succ}\, x) \to t_2 \rightsquigarrow [t/x]t_2} \quad \text{RD\_NCASESUCC}$$

$$\overline{\mathsf{case}\, []\colon \mathsf{List}\ A\ \mathsf{of}\ [] \to t_1, (x :: y) \to t_2 \rightsquigarrow t_1} \quad \text{RD\_LCASEEMPTY}$$

$$\overline{\mathsf{case}\, (t_1 :: t_2)\colon \mathsf{List}\ A\ \mathsf{of}\ [] \to t_3, (x :: y) \to t_4 \rightsquigarrow [t_1/x][t_2/y]t_4} \quad \text{RD\_LCASECONS}$$

$$\overline{(\lambda(x : A_1).t_2)\, t_1 \rightsquigarrow [t_1/x]t_2} \quad \text{RD\_BETA}$$

$$\overline{\mathsf{fst}\, (t_1, t_2) \rightsquigarrow t_1} \quad \text{RD\_PROJ}1$$

$$\overline{\mathsf{snd}\, (t_1, t_2) \rightsquigarrow t_2} \quad \text{RD\_PROJ}2$$

$$\overline{[A](\Lambda(X <: B).t) \rightsquigarrow [A/X]t} \quad \text{RD\_TYPEBETA}$$

$$\frac{t_1 \rightsquigarrow t_2}{[A]t_1 \rightsquigarrow [A]t_2} \quad \text{RD\_TYPEAPP}$$

$$\frac{t_1 \rightsquigarrow t_2}{\mathcal{E}[t_1] \rightsquigarrow \mathcal{E}[t_2]} \quad \text{RD\_CONG}$$

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Definition rules:         83 good      0 bad
Definition rule clauses: 136 good      0 bad
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