Deriving exchange in Elle comonadicly:

$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{y_0 : GB \vdash y_0 : GB}\text{ AX}}{y_0 : GB; \cdot \vdash Fy_0 : FGB}\text{ F}_\text{R}
\qquad
\cfrac{\cfrac{}{x_0 : GA \vdash x_0 : GA}\text{ AX}}{x_0 : GA; \cdot \vdash Fx_0 : FGA}\text{ F}_\text{R}
}{
\cfrac{
\cfrac{
\cfrac{
\cfrac{y_0 : GB, x_0 : GA; \cdot \vdash Fy_0 \otimes Fx_0 : FGB \otimes FGA}{x_1 : GA, y_1 : GB; \cdot \vdash \text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0) : FGB \otimes FGA}\text{ BETA}
}{x_1 : GA; y_2 : FGB \vdash \text{let } y_2 : FGB \text{ be F } y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0)) : FGB \otimes FGA}\text{ F}_\text{L}
}{\cdot ; x_2 : FGA, y_2 : FGB \vdash \text{let } x_2 : FGA \text{ be F } x_1 \text{ in } (\text{let } y_2 : FGB \text{ be F } y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0))) : FGB \otimes FGA}\text{ F}_\text{L}
}{\cdot ; z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be F } x_1 \text{ in } (\text{let } y_2 : FGB \text{ be F } y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0)))) : (FGB \otimes FGA)}\text{ TENL}
}
}{\cdot ; \cdot \vdash \lambda_l z : FGA \otimes FGB.\text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be F } x_1 \text{ in } (\text{let } y_2 : FGB \text{ be F } y_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0)))) : (FGA \otimes FGB) \rightarrow (FGB \otimes FGA)}\text{ IMPR}
$$

Deriving right contraction in Elle comonadicly:

$$
\text{IMPR} \quad \text{TENL} \quad \text{F}_\text{L} \quad \text{F}_\text{L} \quad \text{CONTRR} \quad \text{TENR} \quad \text{TENR} \quad \text{F}_\text{R} \quad \text{F}_\text{R} \quad \text{F}_\text{R} \quad \text{AX} \quad \text{AX} \quad \text{AX}
$$

$$\cdot ; \cdot \vdash \lambda_l z : FGB \otimes FGA.\text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be F } y_0 \text{ in } (\text{let } x_3 : FGA \text{ be F } x_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0))))) : (FGB \otimes FGA) \rightarrow (FGA \otimes (FGB \otimes FGA))$$

Deriving left contraction in Elle comonadicly:

$$
\text{IMPR} \quad \text{TENL} \quad \text{F}_\text{L} \quad \text{F}_\text{L} \quad \text{CONTRR} \quad \text{TENR} \quad \text{TENR} \quad \text{F}_\text{R} \quad \text{F}_\text{R} \quad \text{F}_\text{R} \quad \text{AX} \quad \text{AX} \quad \text{AX}
$$

$$\cdot ; \cdot \vdash \lambda_l z : FGA \otimes FGB.\text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be F } x_3 \text{ in } (\text{let } y_1 : FGB \text{ be F } y_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } (Fx_0 \otimes (Fy_0 \otimes Fx_1))))) : (FGA \otimes FGB) \rightarrow ((FGA \otimes FGB) \otimes FGA)$$

Deriving weakening in Elle comonadicly:

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{\cfrac{}{\cdot \vdash \text{triv} : \text{Unit}}\text{ UNITR}}{x_0 : GA; \cdot \vdash \text{weak } x_0 \text{ in triv} : \text{Unit}}\text{ WEAK}
}{\cdot ; x_1 : FGA \vdash \text{let } x_1 : FGA \text{ be F } x_0 \text{ in weak } x_0 \text{ in triv} : \text{Unit}}\text{ F}_\text{L}
}{\cdot ; \cdot \vdash \lambda_l x_1 : FGA.\text{let } x_1 : FGA \text{ be F } x_0 \text{ in weak } x_0 \text{ in triv} : (FGA) \rightarrow (\text{Unit})}\text{ IMPR}
}{}
$$

GF is a monad:

- Deriving η:

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{
\cfrac{\cfrac{}{\cdot ; x_0 : FX \vdash x_0 : FX}\text{ AX}}{x_1 : GFX; \cdot \vdash \text{let } x_1 : GFX \text{ be G } x_0 \text{ in } x_0 : FX}\text{ G}_\text{L}
}{\cdot ; x_2 : FGFX \vdash \text{let } x_2 : FGFX \text{ be F } x_1 \text{ in } (\text{let } x_1 : GFX \text{ be G } x_0 \text{ in } x_0) : FX}\text{ F}_\text{L}
}{x_3 : GFGFX; \cdot \vdash \text{let } x_3 : GFGFX \text{ be G } x_2 \text{ in } (\text{let } x_2 : FGFX \text{ be F } x_1 \text{ in } (\text{let } x_1 : GFX \text{ be G } x_0 \text{ in } x_0)) : FX}\text{ G}_\text{L}
}{x_3 : GFGFX \vdash G(\text{let } x_3 : GFGFX \text{ be G } x_2 \text{ in } (\text{let } x_2 : FGFX \text{ be F } x_1 \text{ in } (\text{let } x_1 : GFX \text{ be G } x_0 \text{ in } x_0))) : GFX}\text{ G}_\text{R}
}{\cdot \vdash \lambda_l x_3 : GFGFX.G(\text{let } x_3 : GFGFX \text{ be G } x_2 \text{ in } (\text{let } x_2 : FGFX \text{ be F } x_1 \text{ in } (\text{let } x_1 : GFX \text{ be G } x_0 \text{ in } x_0))) : (GFGFX) \rightarrow (GFX)}\text{ IMPR}
}{}
$$

1

- Deriving μ:

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{\rule{2cm}{0.4pt}}{x : X \vdash x : X}\ \text{\scriptsize AX}
    }{x : X; \cdot \vdash Fx : FX}\ \text{\scriptsize FR}
  }{x : X \vdash GFx : GFX}\ \text{\scriptsize GR}
}{\cdot \vdash \lambda_l x : X.GFx : (X) \rightarrowtail (GFX)}\ \text{\scriptsize IMPR}
$$

The monad GF is strong:

- Deriving the tensorial strength τ:

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{
              \dfrac{
                \dfrac{\dfrac{\rule{1.5cm}{0.4pt}}{x_0 : X \vdash x_0 : X}\ \text{\scriptsize AX}\quad \dfrac{\rule{1.5cm}{0.4pt}}{y_0 : Y \vdash y_0 : Y}\ \text{\scriptsize AX}}{x_0 : X, y_0 : Y \vdash x_0 \otimes y_0 : X \otimes Y}\ \text{\scriptsize TENR}
              }{x_0 : X, y_0 : Y; \cdot \vdash F(x_0 \otimes y_0) : F(X \otimes Y)}\ \text{\scriptsize FR}
            }{x_0 : X; y_1 : FY \vdash \mathsf{let}\, y_1 : FY\, \mathsf{be}\, F\, y_0\, \mathsf{in}\, F(x_0 \otimes y_0) : F(X \otimes Y)}\ \text{\scriptsize FL}
          }{y_2 : GFY, x_0 : X; \cdot \vdash \mathsf{let}\, y_2 : GFY\, \mathsf{be}\, G\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : FY\, \mathsf{be}\, F\, y_0\, \mathsf{in}\, F(x_0 \otimes y_0)) : F(X \otimes Y)}\ \text{\scriptsize GL}
        }{y_2 : GFY, x_0 : X \vdash G(\mathsf{let}\, y_2 : GFY\, \mathsf{be}\, G\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : FY\, \mathsf{be}\, F\, y_0\, \mathsf{in}\, F(x_0 \otimes y_0))) : GF(X \otimes Y)}\ \text{\scriptsize GR}
      }{x_1 : X, y_3 : GFY \vdash \mathsf{ex}\, y_3, x_1\, \mathsf{with}\, y_2, x_0\, \mathsf{in}\, (G(\mathsf{let}\, y_2 : GFY\, \mathsf{be}\, G\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : FY\, \mathsf{be}\, F\, y_0\, \mathsf{in}\, F(x_0 \otimes y_0)))) : GF(X \otimes Y)}\ \text{\scriptsize BETA}
    }{z : X \otimes GFY \vdash \mathsf{let}\, z : X \otimes GFY\, \mathsf{be}\, x_1 \otimes y_3\, \mathsf{in}\, (\mathsf{ex}\, y_3, x_1\, \mathsf{with}\, y_2, x_0\, \mathsf{in}\, (G(\mathsf{let}\, y_2 : GFY\, \mathsf{be}\, G\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : FY\, \mathsf{be}\, F\, y_0\, \mathsf{in}\, F(x_0 \otimes y_0))))) : GF(X \otimes Y)}\ \text{\scriptsize TENL}
  }{\cdot \vdash \lambda_l z : X \otimes GFY.\mathsf{let}\, z : X \otimes GFY\, \mathsf{be}\, x_1 \otimes y_3\, \mathsf{in}\, (\mathsf{ex}\, y_3, x_1\, \mathsf{with}\, y_2, x_0\, \mathsf{in}\, (G(\mathsf{let}\, y_2 : GFY\, \mathsf{be}\, G\, y_1\, \mathsf{in}\, (\mathsf{let}\, y_1 : FY\, \mathsf{be}\, F\, y_0\, \mathsf{in}\, F(x_0 \otimes y_0))))) : (X \otimes GFY) \rightarrowtail (GF(X \otimes Y))}\ \text{\scriptsize IMPR}
$$

A Full Ott Spec

vars, n, a, x, y, z, w, m, o
ivar, i, k, j, l
const, b

A, B, C	::=	
	|	B
	|	Unit
	|	$A \otimes B$
	|	$A \rightharpoonup B$
	|	$A \leftharpoonup B$
	|	FX

X, Y, Z	::=	
	|	B
	|	Unit
	|	$X \otimes Y$
	|	$X \rightharpoonup Y$
	|	$X \leftharpoonup Y$
	|	GA

T	::=	
	|	A
	|	X

2

$$
\begin{array}{lll}
p & ::= \\
& \mid & \star \\
& \mid & x \\
& \mid & \text{triv} \\
& \mid & p \otimes p' \\
& \mid & \mathsf{F}\, p \\
& \mid & \mathsf{G}\, p \\
\\
s & ::= \\
& \mid & x \\
& \mid & b \\
& \mid & \text{triv} \\
& \mid & \text{let}\, s_1 : T\, \text{be}\, p\, \text{in}\, s_2 \\
& \mid & s_1 \otimes s_2 \\
& \mid & \lambda_l x : A.s \\
& \mid & \lambda_r x : A.s \\
& \mid & \lambda x : A.s \\
& \mid & (\lambda_l x : A.s_1)\, s_2 \\
& \mid & (\lambda_r x : A.s_1)\, s_2 \\
& \mid & (\lambda x : A.s_1)\, s_2 \\
& \mid & \text{app}_l\, s_1\, s_2 \\
& \mid & \text{app}_r\, s_1\, s_2 \\
& \mid & \text{app}\, s_1\, s_2 \\
& \mid & \text{ex}\, s_1, s_2\, \text{with}\, x_1, x_2\, \text{in}\, s_3 \\
& \mid & \text{contrR}\, x_1\, \text{as}\, s_1, s_2\, \text{in}\, s_3 \\
& \mid & \text{contrL}\, x_1\, \text{as}\, s_1, s_2\, \text{in}\, s_3 \\
& \mid & \text{weak}\, x\, \text{in}\, s \\
& \mid & (s) & \qquad\text{S} \\
& \mid & \mathsf{F} t \\
\\
t & ::= \\
& \mid & x \\
& \mid & b \\
& \mid & \text{triv} \\
& \mid & \text{let}\, t_1 : X\, \text{be}\, p\, \text{in}\, t_2 \\
& \mid & t_1 \otimes t_2 \\
& \mid & \lambda_l x : X.t \\
& \mid & \lambda_r x : X.t \\
& \mid & \lambda x : X.t \\
& \mid & (\lambda_l x : X.t_1)\, t_2 \\
& \mid & (\lambda_r x : X.t_1)\, t_2 \\
& \mid & (\lambda x : X.t_1)\, t_2 \\
& \mid & \text{app}_l\, t_1\, t_2 \\
\end{array}
$$

$$
\begin{array}{ll}
| & \mathsf{app}_r\, t_1\, t_2 \\
| & \mathsf{app}\, t_1\, t_2 \\
| & \mathsf{ex}\, t_1, t_2\, \mathsf{with}\, x_1, x_2\, \mathsf{in}\, t_3 \\
| & \mathsf{contrR}\, x_1\, \mathsf{as}\, t_1, t_2\, \mathsf{in}\, t_3 \\
| & \mathsf{contrL}\, x_1\, \mathsf{as}\, t_1, t_2\, \mathsf{in}\, t_3 \\
| & \mathsf{weak}\, x\, \mathsf{in}\, t \\
| & (t) \qquad\qquad\qquad \mathsf{S} \\
| & \mathsf{G}s
\end{array}
$$

$$
\begin{array}{lll}
\Gamma,\ \Delta,\ \Phi,\ \Psi & ::= & \\
& | & \cdot \\
& | & \Gamma_1, \Gamma_2 \\
& | & x : A \\
& | & (\Gamma) \qquad\qquad \mathsf{S} \\
& | & x : X
\end{array}
$$

$\boxed{\Gamma \vdash t : X}$

$$\frac{}{x : X \vdash x : X}\ \text{T\_VAR}$$

$$\frac{\Gamma, \Delta \vdash t : X}{\Gamma, x : \mathsf{Unit}, \Delta \vdash \mathsf{let}\, x : \mathsf{Unit}\, \mathsf{be}\, \mathsf{triv}\, \mathsf{in}\, t : X}\ \text{T\_UNITL}$$

$$\frac{}{\cdot \vdash \mathsf{triv} : \mathsf{Unit}}\ \text{T\_UNITR}$$

$$\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : Y, w : X, \Delta \vdash \mathsf{ex}\, w, z\, \mathsf{with}\, x, y\, \mathsf{in}\, t : Z}\ \text{T\_BETA}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}{\Gamma_1, \Gamma_2, z : X, \Gamma_3 \vdash \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, t : Y}\ \text{T\_CONTRR}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash t : Y}{\Gamma_1, z : X, \Gamma_2, \Gamma_3 \vdash \mathsf{contrL}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, t : Y}\ \text{T\_CONTRL}$$

$$\frac{\Gamma, \Delta \vdash t : Y \quad x \notin |\Gamma, \Delta|}{\Gamma, x : X, \Delta \vdash \mathsf{weak}\, x\, \mathsf{in}\, t : Y}\ \text{T\_WEAK}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : X, \Delta_2 \vdash t_2 : Y}{\Delta_1, \Gamma, \Delta_2 \vdash [t_1/x]t_2 : Y}\ \text{T\_CUT}$$

$$\frac{\Gamma, x : X, y : Y, \Delta \vdash t : Z}{\Gamma, z : X \otimes Y, \Delta \vdash \mathsf{let}\, z : X \otimes Y\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, t : Z}\ \text{T\_TENL}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta \vdash t_2 : Y}{\Gamma, \Delta \vdash t_1 \otimes t_2 : X \otimes Y}\ \text{T\_TEN}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, \Gamma, y : X \rightharpoonup Y, \Delta_2 \vdash [\mathsf{app}_l\, y\, t_1/x]t_2 : Z}\ \text{T\_IMPL1}$$

$$\frac{\Gamma \vdash t_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash t_2 : Z}{\Delta_1, y : Y \leftharpoonup X, \Gamma, \Delta_2 \vdash [\mathsf{app}_r\, y\, t_1/x]t_2 : Z}\ \text{T\_IMPL2}$$

$$\frac{\Gamma, x : X \vdash t : Y}{\Gamma \vdash \lambda_l x : X.t : X \rightharpoonup Y}\ \text{T\_IMPRL}$$

4

$$\frac{x : X, \Gamma \vdash t : Y}{\Gamma \vdash \lambda_r x : X.t : Y \leftharpoonup X} \quad \text{T\_IMPR}_\text{R}$$

$$\frac{\Gamma; \cdot \vdash s : A}{\Gamma \vdash \mathsf{G}s : \mathsf{G}A} \quad \text{T\_G}_\text{R}$$

$\boxed{\Gamma; \Psi \vdash s : A}$

$$\frac{}{\cdot; x : A \vdash x : A} \quad \text{S\_AX}$$

$$\frac{\Gamma, \Delta; \Psi \vdash s : A}{\Gamma, x : \mathsf{Unit}, \Delta; \Psi \vdash \mathsf{let}\, x : \mathsf{Unit}\, \mathsf{be}\, \mathsf{triv}\, \mathsf{in}\, s : A} \quad \text{S\_UNITL1}$$

$$\frac{\Gamma; \Psi, \Phi \vdash s : A}{\Gamma; \Psi, x : \mathsf{Unit}, \Phi \vdash \mathsf{let}\, x : \mathsf{Unit}\, \mathsf{be}\, \mathsf{triv}\, \mathsf{in}\, s : A} \quad \text{S\_UNITL2}$$

$$\frac{}{\cdot; \cdot \vdash \mathsf{triv} : \mathsf{Unit}} \quad \text{S\_UNITR}$$

$$\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : Y, w : X, \Delta; \Psi \vdash \mathsf{ex}\, w, z\, \mathsf{with}\, x, y\, \mathsf{in}\, s : A} \quad \text{S\_BETA}$$

$$\frac{\Gamma, \Delta; \Psi \vdash s : B \quad x \notin |\Gamma, \Delta, \Psi|}{\Gamma, x : X, \Delta; \Psi \vdash \mathsf{weak}\, x\, \mathsf{in}\, s : B} \quad \text{S\_WEAK}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash s : B}{\Gamma_1, \Gamma_2, z : X, \Gamma_3; \Psi \vdash \mathsf{contrR}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, s : B} \quad \text{S\_CONTRR}$$

$$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash s : B}{\Gamma_1, z : X, \Gamma_2, \Gamma_3; \Psi \vdash \mathsf{contrL}\, z\, \mathsf{as}\, x, y\, \mathsf{in}\, s : B} \quad \text{S\_CONTRL}$$

$$\frac{\Gamma \vdash t : X \quad \Delta_1, x : X, \Delta_2; \Psi \vdash s : A}{\Delta_1, \Gamma, \Delta_2; \Psi \vdash [t/x]s : A} \quad \text{S\_CUT1}$$

$$\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : A, \Phi_2 \vdash s_2 : B}{\Gamma, \Delta; \Phi_1, \Psi, \Phi_2 \vdash [s_1/x]s_2 : B} \quad \text{S\_CUT2}$$

$$\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash s : A}{\Gamma, z : X \otimes Y, \Delta; \Psi \vdash \mathsf{let}\, z : X \otimes Y\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, s : A} \quad \text{S\_TENL1}$$

$$\frac{\Gamma; \Psi, x : A, y : B, \Phi \vdash s : A}{\Gamma; \Psi, z : A \otimes B, \Phi \vdash \mathsf{let}\, z : A \otimes B\, \mathsf{be}\, x \otimes y\, \mathsf{in}\, s : A} \quad \text{S\_TENL2}$$

$$\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi \vdash s_2 : B}{\Gamma, \Delta; \Psi, \Phi \vdash s_1 \otimes s_2 : A \otimes B} \quad \text{S\_TENR}$$

$$\frac{\Gamma \vdash t : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash s : A}{\Delta_1, \Gamma, y : X \rightharpoonup Y, \Delta_2; \Psi \vdash [\mathsf{app}_l\, y\, t/x]s : A} \quad \text{S\_IMPL}_\text{L}$$

$$\frac{\Gamma \vdash t : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash s : A}{\Delta_1, y : Y \leftharpoonup X, \Gamma, \Delta_2; \Psi \vdash [\mathsf{app}_r\, y\, t/x]s : A} \quad \text{S\_IMPL2}$$

$$\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash s_2 : A}{\Gamma, \Delta; \Phi_1, \Psi, y : A \rightharpoonup B, \Phi_2 \vdash [\mathsf{app}_l\, y\, s_1/x]s_2 : A} \quad \text{S\_IMPL3}$$

$$\frac{\Gamma; \Psi \vdash s_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash s_2 : A}{\Gamma, \Delta; \Phi_1, y : B \leftharpoonup A, \Psi, \Phi_2 \vdash [\mathsf{app}_l\, y\, s_1/x]s_2 : A} \quad \text{S\_IMPL4}$$

$$\frac{\Gamma; \Psi, x : A \vdash s : B}{\Gamma; \Psi \vdash \lambda_l x : A.s : A \rightharpoonup B} \quad \text{S\_ImpR}_\text{L}$$

$$\frac{\Gamma; x : A, \Psi \vdash s : B}{\Gamma; \Psi \vdash \lambda_r x : A.s : B \leftharpoonup A} \quad \text{S\_ImpR}_\text{R}$$

$$\frac{\Gamma \vdash t : X}{\Gamma; \cdot \vdash \mathsf{F}t : \mathsf{F}X} \quad \text{S\_F}_\text{R}$$

$$\frac{\Gamma, x : X; \Psi \vdash s : A}{\Gamma; z : \mathsf{F}X, \Psi \vdash \mathsf{let}\, z : \mathsf{F}X \, \mathsf{be}\, \mathsf{F}\, x \, \mathsf{in}\, s : A} \quad \text{S\_F}_\text{L}$$

$$\frac{\Gamma; \Psi, x : A \vdash s : B}{z : \mathsf{G}A, \Gamma; \Psi \vdash \mathsf{let}\, z : \mathsf{G}A \, \mathsf{be}\, \mathsf{G}\, x \, \mathsf{in}\, s : B} \quad \text{S\_G}_\text{L}$$