An Adjoint Model for Process Trees with **Sequential Composition**

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Introduction

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Categorical Models

Lambek Categories

▶ **Definition 1.** A monoidal category, $(\mathcal{L}, \otimes, I, \lambda, \rho)$, is a category, \mathcal{L} , equipped with a bifunctor, $\otimes: \mathcal{L} \times \mathcal{L} \longrightarrow \mathcal{L}$, called the tensor product, a distinguished object I of \mathcal{L} called the unit, and three natural isomorphisms $\lambda_A: I \otimes A \longrightarrow A$, $\rho_A: A \otimes I \longrightarrow A$, and $\alpha_{A,B,C}: A \otimes (B \otimes C) \longrightarrow (A \otimes B) \otimes C$ called the left and right unitors and the associator respectively. Finally, these are subject to the following coherence diagrams:

$$((A \otimes B) \otimes C) \otimes D \xrightarrow{\alpha_{A,B,C} \otimes \mathrm{id}_{D}} A \otimes (B \otimes C)) \otimes D \xrightarrow{\alpha_{A,B \otimes C,D}} A \otimes ((B \otimes C) \otimes D)$$

$$\downarrow \qquad \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad$$

▶ **Definition 2.** A **Lambek category** is a monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with two bifunctors $\rightharpoonup: \mathcal{L}^{op} \times \mathcal{L} \longrightarrow \mathcal{L}$ and $\vdash: \mathcal{L} \times \mathcal{L}^{op} \longrightarrow \mathcal{L}$ that are both right adjoint to the tensor product. That is, the following natural bijections hold:

$$\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \rightharpoonup B) \qquad \qquad \mathsf{Hom}_{\mathcal{L}}(A \otimes X, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, B \leftharpoonup A)$$



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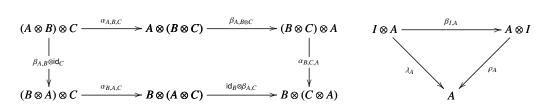
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One might call Lambek categories biclosed monoidal categories, but we name them in homage to Lambek for his contributions to non-commutative linear logic.

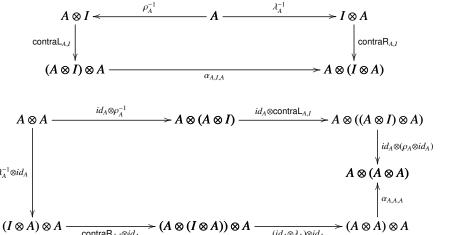
▶ **Definition 3.** A monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ is **symmetric** if there is a natural transformation $\beta_{A,B} : A \otimes B \longrightarrow B \otimes A$ such that $\beta_{B,A} \circ \beta_{A,B} = \mathrm{id}_{A \otimes B}$ and the following commute:



- ▶ **Definition 4.** A symmetric monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \beta)$ is **closed** if it comes equipped with a bifunctor \multimap : $\mathcal{L}^{op} \times \mathcal{L} \longrightarrow \mathcal{L}$ that is right adjoint to the tensor product. That is, the following natural bijection $\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \multimap B)$ holds.
- ▶ **Definition 5.** A **Lambek category with weakening**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{weak})$, is a Lambek category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with a natural transformation $\text{weak}_A : A \longrightarrow I$.
- ▶ **Definition 6.** A **Lambek category with contraction**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{contraL}, \text{contraR})$, is a Lambek category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with natural transformations:

$$\mathsf{contral}_{A,B} : (A \otimes B) {\:\longrightarrow\:} (A \otimes B) \otimes A \qquad \mathsf{contral}_{A,B} : (B \otimes A) {\:\longrightarrow\:} A \otimes (B \otimes A)$$

Furthermore, the following diagrams must commute:



- ▶ **Definition 7.** A **Lambek category with exchange**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, ex)$, is a Lambek category, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$, such that \mathcal{L} is symmetric monoidal, where $ex_{A,B} : A \otimes B \longrightarrow B \otimes A$ is the symmetry.
- ▶ Lemma 8. Let A and B be two objects in a Lambek category with exchange. Then

$$(A \rightharpoonup B) \cong (B \leftharpoonup A)$$

Proof. For any object *C* in the category, we have

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Hom[C, A \rightarrow B]

\cong Hom[C \otimes A, B] \mathcal{L} is a Lambek category

\cong Hom[A \otimes C, B] By the symmetry ex_{C,A}

\cong Hom[C, B \leftarrow A] \mathcal{L} is a Lambek category
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Similarly, the function $Hom[D, A \rightharpoonup B] \rightarrow Hom[C, A \rightharpoonup B]$ is isomorphic to the function $Hom[D, B \leftharpoonup A] \rightarrow Hom[C, B \leftharpoonup A]$.

Thus, $Y_C(A \to B) \cong Y_C(B \leftarrow A)$, where Y_C is a Yoneda embedding. So $A \to B \cong B \leftarrow A$ by Yoneda Lemma.

3 Related Work

TODO

4 Conclusion

TODO

- References -

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A Appendix