An Adjoint Model for Process Trees with Sequential Composition

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Abstract

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1 Introduction

TODO [1]

2 Categorical Models

2.1 Lambek Categories

▶ **Definition 1.** A **monoidal category**, $(\mathcal{L}, \otimes, I, \lambda, \rho)$, is a category, $\mathcal{L}$, equipped with a bifunctor, $\otimes : \mathcal{L} \times \mathcal{L} \longrightarrow \mathcal{L}$, called the tensor product, a distinguished object I of $\mathcal{L}$ called the unit, and three natural isomorphisms $\lambda_A : I \otimes A \longrightarrow A$, $\rho_A : A \otimes I \longrightarrow A$, and $\alpha_{A,B,C} : A \otimes (B \otimes C) \longrightarrow (A \otimes B) \otimes C$ called the left and right unitors and the associator respectively. Finally, these are subject to the following coherence diagrams:

$$\begin{tikzcd}
((A \otimes B) \otimes C) \otimes D \arrow[r, "\alpha_{A,B,C} \otimes \mathsf{id}_D"] \arrow[d, "\alpha_{A \otimes B,C,D}"'] & (A \otimes (B \otimes C)) \otimes D \arrow[r, "\alpha_{A,B \otimes C,D}"] & A \otimes ((B \otimes C) \otimes D) \arrow[d, "\mathsf{id}_A \otimes \alpha_{B,C,D}"] \\
(A \otimes B) \otimes (C \otimes D) \arrow[rr, "\alpha_{A,B,C \otimes D}"] & & A \otimes (B \otimes (C \otimes D))
\end{tikzcd}$$

$$\begin{tikzcd}
(A \otimes I) \otimes B \arrow[rr, "\alpha_{A,I,B}"] \arrow[dr, "\rho_A \otimes id_B"'] & & A \otimes (I \otimes B) \arrow[dl, "id_A \otimes \lambda_B"] \\
& A \otimes B &
\end{tikzcd}$$

▶ **Definition 2.** A **Lambek category** is a monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with two bifunctors $\rightharpoonup : \mathcal{L}^{\mathsf{op}} \times \mathcal{L} \longrightarrow \mathcal{L}$ and $\leftharpoonup : \mathcal{L} \times \mathcal{L}^{\mathsf{op}} \longrightarrow \mathcal{L}$ that are both right adjoint to the tensor product. That is, the following natural bijections hold:

$$\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \rightharpoonup B) \qquad \mathsf{Hom}_{\mathcal{L}}(A \otimes X, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, B \leftharpoonup A)$$

One might call Lambek categories biclosed monoidal categories, but we name them in homage to Lambek for his contributions to non-commutative linear logic.

▶ **Definition 3.** A monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ is **symmetric** if there is a natural transformation $\beta_{A,B} : A \otimes B \longrightarrow B \otimes A$ such that $\beta_{B,A} \circ \beta_{A,B} = \mathsf{id}_{A\otimes B}$ and the following commute:

$$
\begin{tikzcd}
(A \otimes B) \otimes C \arrow[r, "\alpha_{A,B,C}"] \arrow[d, "\beta_{A,B}\otimes\mathsf{id}_C"'] & A \otimes (B \otimes C) \arrow[r, "\beta_{A,B\otimes C}"] & (B \otimes C) \otimes A \arrow[d, "\alpha_{B,C,A}"] \\
(B \otimes A) \otimes C \arrow[r, "\alpha_{B,A,C}"] & B \otimes (A \otimes C) \arrow[r, "\mathsf{id}_B\otimes\beta_{A,C}"] & B \otimes (C \otimes A)
\end{tikzcd}
\qquad
\begin{tikzcd}
I \otimes A \arrow[rr, "\beta_{I,A}"] \arrow[dr, "\lambda_A"'] & & A \otimes I \arrow[dl, "\rho_A"] \\
& A &
\end{tikzcd}
$$

▶ **Definition 4.** A symmetric monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \beta)$ is **closed** if it comes equipped with a bifunctor $\multimap : \mathcal{L}^{\mathsf{op}} \times \mathcal{L} \longrightarrow \mathcal{L}$ that is right adjoint to the tensor product. That is, the following natural bijection $\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \multimap B)$ holds.

▶ **Definition 5.** A **Lambek category with weakening**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \mathsf{weak})$, is a Lambek category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with a natural transformation $\mathsf{weak}_A : A \longrightarrow I$.

▶ **Definition 6.** A **Lambek category with contraction**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \mathsf{contraL}, \mathsf{contraR})$, is a Lambek category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with natural transformations:

$$\mathsf{contraL}_{A,B} : (A \otimes B) \longrightarrow (A \otimes B) \otimes A \qquad \mathsf{contraR}_{A,B} : (B \otimes A) \longrightarrow A \otimes (B \otimes A)$$

Furthermore, the following diagrams must commute:

$$
\begin{tikzcd}
A \otimes I \arrow[d, "\mathsf{contraL}_{A,I}"'] & A \arrow[l, "\rho_A^{-1}"'] \arrow[r, "\lambda_A^{-1}"] & I \otimes A \arrow[d, "\mathsf{contraR}_{A,I}"] \\
(A \otimes I) \otimes A \arrow[rr, "\alpha_{A,I,A}"'] & & A \otimes (I \otimes A)
\end{tikzcd}
$$

$$
\begin{tikzcd}
A \otimes A \arrow[r, "id_A\otimes\rho_A^{-1}"] \arrow[dd, "\lambda_A^{-1}\otimes id_A"'] & A \otimes (A \otimes I) \arrow[r, "id_A\otimes\mathsf{contraL}_{A,I}"] & A \otimes ((A \otimes I) \otimes A) \arrow[d, "id_A\otimes(\rho_A\otimes id_A)"] \\
& & A \otimes (A \otimes A) \\
(I \otimes A) \otimes A \arrow[r, "\mathsf{contraR}_{A,I}\otimes id_A"'] & (A \otimes (I \otimes A)) \otimes A \arrow[r, "(id_A\otimes\lambda_A)\otimes id_A"'] & (A \otimes A) \otimes A \arrow[u, "\alpha_{A,A,A}"']
\end{tikzcd}
$$

▶ **Definition 7.** A **Lambek category with exchange**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \mathsf{ex})$, is a Lambek category, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$, such that $\mathcal{L}$ is symmetric monoidal, where $\mathsf{ex}_{A,B} : A \otimes B \longrightarrow B \otimes A$ is the symmetry.

▶ **Lemma 8.** *Let A and B be two objects in a Lambek category with exchange. Then*

$$(A \rightharpoonup B) \cong (B \leftharpoonup A)$$

.

Proof. For any object C in the category, we have

$$
\begin{aligned}
& Hom[C, A \rightharpoonup B] \\
& \cong Hom[C \otimes A, B] && \mathcal{L} \text{ is a Lambek category} \\
& \cong Hom[A \otimes C, B] && \text{By the symmetry } \mathsf{ex}_{C,A} \\
& \cong Hom[C, B \leftharpoonup A] && \mathcal{L} \text{ is a Lambek category}
\end{aligned}
$$

Similarly, the function $Hom[D, A \rightharpoonup B] \to Hom[C, A \rightharpoonup B]$ is isomorphic to the function $Hom[D, B \leftharpoonup A] \to Hom[C, B \leftharpoonup A]$.
Thus, $Y_C(A \rightharpoonup B) \cong Y_C(B \leftharpoonup A)$, where Y_C is a Yoneda embedding. So $A \rightharpoonup B \cong B \leftharpoonup A$ by Yoneda Lemma. ◀

3 Related Work

TODO

4 Conclusion

TODO

References

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A Appendix