

$vars, n, a, x, y, z, w, m, o$

$ivar, i, k, j, l$

$const, b$

A, B, C

$::=$

| B
| Unit
| $A \otimes B$
| $A \multimap B$
| $A \multimap_l B$
| wA
| cA
| eA

Base type

Unit

Non-commutative tensor

Left implication

Right implication

t

$::=$

| x
| b
| unit
| $\text{let } t_1 : A \text{ be } t_2 \text{ in } t_3$
| $\text{let } t_1 \text{ be } t_2 \text{ in } t_3$
| $t_1 \otimes t_2$
| $\lambda_l x : A. t$
| $\lambda_r x : A. t$
| $\text{app}_l t_1 t_2$
| $\text{app}_r t_1 t_2$
| $w t$
| $e t$
| $c t$
| $\text{weak}_l t_1 \text{ in } t_2$
| $\text{weak}_r t_1 \text{ in } t_2$
| $\text{con}_l x, y \text{ to } t_1 \text{ in } t_2$
| $\text{con}_r x, y \text{ to } t_1 \text{ in } t_2$
| $\text{ex } x_1, x_2 \text{ with } t_1, t_2 \text{ in } t_3$
| $\text{dist}_{\text{ecw}} t$
| $\text{dist}_{\text{cw}} t$
| $\text{dist}_{\text{ew}} t$
| $\text{dist}_{\text{ec}} t$

S

$\Gamma, \Delta, \Phi, \Psi$

$::=$

| \cdot
| Γ_1, Γ_2
| $x : A$
| (Γ)

S

$\boxed{\Gamma; \Psi; \Phi; \Delta \vdash t : A}$

$\frac{}{\cdot; \cdot; \cdot; x : A \vdash x : A} \text{LAX}$

$\frac{}{\cdot; \cdot; x : A; \cdot \vdash x : A} \text{WAX}$

$$\begin{array}{c}
\frac{}{\cdot; x : A; \cdot; \cdot \vdash x : A} \text{CAX} \\
\frac{}{x : A; \cdot; \cdot; \cdot \vdash x : A} \text{EAX} \\
\frac{}{\cdot; \cdot; \cdot; \cdot \vdash b : B} \text{CONST} \\
\frac{}{\cdot; \cdot; \cdot; \cdot \vdash \text{unit} : \text{Unit}} \text{UNITI} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : \text{Unit} \quad \Gamma_2; \Psi_2; \Phi_1; \Delta_2 \vdash t_2 : C}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{let unit be } t_1 \text{ in } t_2 : C} \text{UNIT E} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : A \quad \Gamma_2; \Psi_2; \Phi_1; \Delta_2 \vdash t_2 : B}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash t_1 \otimes t_2 : A \otimes B} \text{TENSOR I} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : A \otimes B \quad \Gamma_2; \Psi_2; \Phi_1; \Delta_2, x : A, y : B \vdash t_2 : B}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{let } x \otimes y : A \otimes B \text{ be } t_1 \text{ in } t_2 : C} \text{TENSOR E} \\
\frac{\Gamma; \Psi; \Phi; x : A, \Delta \vdash t : B}{\Gamma; \Psi; \Phi; \Delta \vdash \lambda_l x : A. t : A \rightarrow B} \text{LFUN} \\
\frac{\Gamma; \Psi; \Phi; \Delta, x : A \vdash t : B}{\Gamma; \Psi; \Phi; \Delta \vdash \lambda_r x : A. t : B \leftarrow A} \text{RFUN} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : A \rightarrow B \quad \Gamma_2; \Psi_2; \Phi_1; \Delta_2 \vdash t_2 : A}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{app}_l t_1 t_2 : B} \text{LAPP} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : B \leftarrow A \quad \Gamma_2; \Psi_2; \Phi_1; \Delta_2 \vdash t_2 : A}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{app}_r t_1 t_2 : B} \text{RAPP} \\
\frac{}{\cdot; \cdot; \Psi; \cdot \vdash t : A} \text{wI} \\
\frac{}{\cdot; \cdot; \Psi; \cdot \vdash \text{wt} : \text{w}A} \text{cI} \\
\frac{}{\cdot; \Phi; \cdot; \cdot \vdash t : A} \text{eI} \\
\frac{}{\cdot; \Phi; \cdot; \cdot \vdash \text{ct} : \text{c}A} \text{eI} \\
\frac{\Gamma; \cdot; \cdot; \cdot \vdash t : A}{\Gamma; \cdot; \cdot; \cdot \vdash \text{et} : \text{e}A} \text{eI} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : \text{w}A \quad \Gamma_2; \Psi_2; \Phi_1, x : A; \Delta_2 \vdash t_2 : B}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{let w } x : \text{w}A \text{ be } t_1 \text{ in } t_2 : B} \text{wE} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : \text{c}A \quad \Gamma_2; \Psi_2, x : A; \Phi_1; \Delta_2 \vdash t_2 : B}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{let c } x : \text{c}A \text{ be } t_1 \text{ in } t_2 : B} \text{cE} \\
\frac{\Gamma_1; \Psi_1; \Phi_1; \Delta_1 \vdash t_1 : \text{e}A \quad \Gamma_2, x : A; \Psi_2; \Phi_1; \Delta_2 \vdash t_2 : B}{(\Gamma_1, \Gamma_2); (\Psi_1, \Psi_2); (\Phi_1, \Phi_2); (\Delta_1, \Delta_2) \vdash \text{let e } x : \text{e}A \text{ be } t_1 \text{ in } t_2 : B} \text{eE} \\
\frac{\Gamma; \Psi; \Phi; \Delta \vdash t : B \quad x \notin \text{FV}(t)}{\Gamma; \Psi; \Phi, x : A; \Delta \vdash \text{weak}_r x \text{ in } t : B} \text{RWEAK} \\
\frac{\Gamma; \Psi; \Phi; \Delta \vdash t : B \quad x \notin \text{FV}(t)}{\Gamma; \Psi, x : A, \Phi; \Delta \vdash \text{weak}_l x \text{ in } t : B} \text{LWEAK} \\
\frac{\Gamma; \Psi_1, x : A, \Psi_2, y : A, \Psi_3; \Phi; \Delta \vdash t : B}{\Gamma; \Psi_1, z : A, \Psi_2, \Psi_3; \Phi; \Delta \vdash \text{con}_l x, y \text{ to } z \text{ in } t : B} \text{LCON} \\
\frac{\Gamma; \Psi_1, x : A, \Psi_2, y : A, \Psi_3; \Phi; \Delta \vdash t : B}{\Gamma; \Psi_1, \Psi_2, z : A, \Psi_3; \Phi; \Delta \vdash \text{con}_r x, y \text{ to } z \text{ in } t : B} \text{RCON}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma_1, x_1 : A, x_2 : B, \Gamma_2; \Psi; \Phi; \Delta \vdash t : C}{\Gamma, z_1 : B, z_2 : A; \Psi; \Phi; \Delta \vdash \text{ex } x_1, x_2 \text{ with } z_1, z_2 \text{ in } t : C} \text{ EX} \\
\\
\frac{\Gamma; \Psi; \Phi; \Delta \vdash t : \text{wc } A}{\Gamma; \Psi; \Phi; \Delta \vdash \text{dist}_{\text{cw}} t : \text{c w } A} \text{ DISTCW} \\
\\
\frac{\Gamma; \Psi; \Phi; \Delta \vdash t : \text{we } A}{\Gamma; \Psi; \Phi; \Delta \vdash \text{dist}_{\text{ew}} t : \text{e w } A} \text{ DISTEW} \\
\\
\frac{\Gamma; \Psi; \Phi; \Delta \vdash t : \text{ce } A}{\Gamma; \Psi; \Phi; \Delta \vdash \text{dist}_{\text{ec}} t : \text{e c } A} \text{ DISTEC} \\
\\
\frac{\Gamma; \Psi; \Phi; \Delta \vdash t : \text{wce } A}{\Gamma; \Psi; \Phi; \Delta \vdash \text{dist}_{\text{ecw}} t : \text{e c w } A} \text{ DISTECW}
\end{array}$$