

Deriving exchange in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash Fy_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash Fx_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA; \cdot \vdash Fy_0 \otimes Fx_0 : FGB \otimes FGA} \text{TENR}}{x_1 : GA, y_1 : GB; \cdot \vdash \text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0) : FGB \otimes FGA} \text{BETA}} \\
\frac{x_1 : GA, y_2 : FGB \vdash \text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0)) : FGB \otimes FGA}{\vdash x_2 : FGA, y_2 : FGB \vdash \text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0))) : FGB \otimes FGA} \text{FL}} \\
\frac{\vdash z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0)))) : (FGB \otimes FGA)}{\vdash \cdot \vdash \lambda_{fz} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_2 \otimes y_2 \text{ in } (\text{let } x_2 : FGA \text{ be } Fx_1 \text{ in } (\text{let } y_2 : FGB \text{ be } Fy_1 \text{ in } (\text{ex } y_1, x_1 \text{ with } y_0, x_0 \text{ in } (Fy_0 \otimes Fx_0)))) : (FGA \otimes FGB) \rightarrow (FGB \otimes FGA)} \text{IMPR}
\end{array}$$

Deriving right contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_1 : GA \vdash x_1 : GA} \text{AX}}{x_1 : GA; \cdot \vdash Fx_1 : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash Fy_0 : FGB} \text{Fr} \quad \frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash Fx_0 : FGA} \text{Fr}}{y_0 : GB, x_0 : GA; \cdot \vdash Fy_0 \otimes Fx_0 : FGB \otimes FGA} \text{TENR}}{x_1 : GA, y_0 : GB, x_0 : GA; \cdot \vdash Fx_1 \otimes (Fy_0 \otimes Fx_0) : FGA \otimes (FGB \otimes FGA)} \text{TENR}} \\
\frac{y_0 : GB, x_2 : GA; \cdot \vdash \text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0)) : FGA \otimes (FGB \otimes FGA)}{y_0 : GB, x_3 : FGA \vdash \text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0))) : FGA \otimes (FGB \otimes FGA)} \text{FL}} \\
\frac{\vdash y_1 : FGB, x_3 : FGA \vdash \text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0)))) : FGA \otimes (FGB \otimes FGA)}{\vdash z : FGB \otimes FGA \vdash \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0)))) : FGA \otimes (FGB \otimes FGA)} \text{FL}} \\
\frac{\vdash \cdot \vdash \lambda_{fz} : FGB \otimes FGA. \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0)))) : (FGB \otimes FGA) \rightarrow (FGA \otimes (FGB \otimes FGA))}{\vdash \cdot \vdash \lambda_{fz} : FGB \otimes FGA. \text{let } z : FGB \otimes FGA \text{ be } y_1 \otimes x_3 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{contrR } x_2 \text{ as } x_1, x_0 \text{ in } (Fx_1 \otimes (Fy_0 \otimes Fx_0)))) : (FGB \otimes FGA) \rightarrow (FGA \otimes (FGB \otimes FGA))} \text{IMPR}
\end{array}$$

Deriving left contraction in Elle comonadically:

$$\begin{array}{c}
\frac{\frac{\frac{}{x_0 : GA \vdash x_0 : GA} \text{AX}}{x_0 : GA; \cdot \vdash Fx_0 : FGA} \text{Fr} \quad \frac{\frac{\frac{}{y_0 : GB \vdash y_0 : GB} \text{AX}}{y_0 : GB; \cdot \vdash Fy_0 : FGB} \text{Fr}}{x_0 : GA, y_0 : GB; \cdot \vdash Fx_0 \otimes Fy_0 : FGA \otimes FGB} \text{TENR} \quad \frac{\frac{\frac{}{x_1 : GA \vdash x_1 : GA} \text{AX}}{x_1 : GA; \cdot \vdash Fx_1 : FGA} \text{Fr}}{x_0 : GA, y_0 : GB, x_1 : GA; \cdot \vdash (Fx_0 \otimes Fy_0) \otimes Fx_1 : (FGA \otimes FGB) \otimes FGA} \text{TENR}} \\
\frac{x_2 : GA, y_0 : GB; \cdot \vdash \text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \otimes Fy_0) \otimes Fx_1) : (FGA \otimes FGB) \otimes FGA}{x_2 : GA, y_1 : FGB \vdash \text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \otimes Fy_0) \otimes Fx_1)) : (FGA \otimes FGB) \otimes FGA} \text{FL}} \\
\frac{\vdash x_3 : FGA, y_1 : FGB \vdash \text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \otimes Fy_0) \otimes Fx_1))) : (FGA \otimes FGB) \otimes FGA}{\vdash z : FGA \otimes FGB \vdash \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \otimes Fy_0) \otimes Fx_1)))) : (FGA \otimes FGB) \otimes FGA} \text{FL}} \\
\frac{\vdash \cdot \vdash \lambda_{fz} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \otimes Fy_0) \otimes Fx_1)))) : (FGA \otimes FGB) \rightarrow ((FGA \otimes FGB) \otimes FGA)}{\vdash \cdot \vdash \lambda_{fz} : FGA \otimes FGB. \text{let } z : FGA \otimes FGB \text{ be } x_3 \otimes y_1 \text{ in } (\text{let } x_3 : FGA \text{ be } Fx_2 \text{ in } (\text{let } y_1 : FGB \text{ be } Fy_0 \text{ in } (\text{contrL } x_2 \text{ as } x_0, x_1 \text{ in } ((Fx_0 \otimes Fy_0) \otimes Fx_1)))) : (FGA \otimes FGB) \rightarrow ((FGA \otimes FGB) \otimes FGA)} \text{IMPR}
\end{array}$$

Deriving weakening in Elle comonadically:

$$\begin{array}{c}
\frac{}{\cdot \vdash \text{triv} : \text{Unit}} \text{UNITR} \\
\frac{x_0 : GA; \cdot \vdash \text{weak } x_0 \text{ in triv} : \text{Unit}}{x_1 : FGA; \cdot \vdash \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in triv} : \text{Unit}} \text{WEAK} \\
\frac{\vdash \cdot \vdash \lambda_{fx_1} : FGA. \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in triv} : \text{Unit}}{\vdash \cdot \vdash \lambda_{fx_1} : FGA. \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in triv} : \text{Unit}} \text{FL} \\
\frac{}{\vdash \cdot \vdash \lambda_{fx_1} : FGA. \text{let } x_1 : FGA \text{ be } Fx_0 \text{ in weak } x_0 \text{ in triv} : \text{Unit}} \text{IMPR}
\end{array}$$

A Full Ott Spec

vars, n, a, x, y, z, w, m, o

ivar, i, k, j, l

$const, b$	
A, B, C	$::=$ $ \quad B$ $ \quad \text{Unit}$ $ \quad A \otimes B$ $ \quad A \multimap B$ $ \quad A \multimap B$ $ \quad FX$
X, Y, Z	$::=$ $ \quad B$ $ \quad \text{Unit}$ $ \quad X \otimes Y$ $ \quad X \multimap Y$ $ \quad X \multimap Y$ $ \quad GA$
T	$::=$ $ \quad A$ $ \quad X$
p	$::=$ $ \quad \star$ $ \quad x$ $ \quad \text{triv}$ $ \quad p \otimes p'$ $ \quad Fx$ $ \quad Gx$
t	$::=$ $ \quad x$ $ \quad b$ $ \quad \text{triv}$ $ \quad \text{let } t_1 : T \text{ be } p \text{ in } t_2$ $ \quad t_1 \otimes t_2$ $ \quad \lambda_l x : A. t$ $ \quad \lambda_r x : A. t$ $ \quad \lambda x : A. t$ $ \quad \text{app}_l t_1 t_2$ $ \quad \text{app}_r t_1 t_2$ $ \quad \text{app } t_1 t_2$ $ \quad \text{ex } x_1, x_2 \text{ with } t_1, t_2 \text{ in } t_3$ $ \quad \text{contrR } x_1 \text{ as } t_1, t_2 \text{ in } t_3$ $ \quad \text{contrL } x_1 \text{ as } t_1, t_2 \text{ in } t_3$

		weak x in t	
		(t)	S
		F_s	

s	::=		
		x	
		b	
		triv	
		let $s_1 : X$ be p in s_2	
		$s_1 \otimes s_2$	
		$\lambda_l x : X. s$	
		$\lambda_r x : X. s$	
		$\lambda x : X. s$	
		app _l $s_1 s_2$	
		app _r $s_1 s_2$	
		app $s_1 s_2$	
		ex x_1, x_2 with s_1, s_2 in s_3	
		contrR x_1 as s_1, s_2 in s_3	
		contrL x_1 as s_1, s_2 in s_3	
		weak x in s	
		(s)	S
		G_t	

$\Gamma, \Delta, \Phi, \Psi$::=		
		.	
		Γ_1, Γ_2	
		$x : A$	
		(Γ)	S
		$x : X$	

$\Gamma \vdash s : X$

$\frac{}{x : X \vdash x : X}$	S_VAR
$\frac{\Gamma, \Delta \vdash s : X}{\Gamma, x : \text{Unit}, \Delta \vdash \text{let } x : \text{Unit} \text{ be triv in } s : X}$	S_UNITL
$\frac{}{\cdot \vdash \text{triv} : \text{Unit}}$	S_UNITR
$\frac{\Gamma, x : X, y : Y, \Delta \vdash s : Z}{\Gamma, z : Y, w : X, \Delta \vdash \text{ex } w, z \text{ with } x, y \text{ in } s : Z}$	S_BETA
$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash s : Y}{\Gamma_1, \Gamma_2, z : X, \Gamma_3 \vdash \text{contrR } z \text{ as } x, y \text{ in } s : Y}$	S_CONTRR
$\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3 \vdash s : Y}{\Gamma_1, z : X, \Gamma_2, \Gamma_3 \vdash \text{contrL } z \text{ as } x, y \text{ in } s : Y}$	S_CONTRL
$\frac{\Gamma, \Delta \vdash s : Y \quad x \notin \Gamma, \Delta }{\Gamma, x : X, \Delta \vdash \text{weak } x \text{ in } s : Y}$	S_WEAK

$$\begin{array}{c}
\frac{\Gamma \vdash s_1 : X \quad \Delta_1, x : X, \Delta_2 \vdash s_2 : Y}{\Delta_1, \Gamma, \Delta_2 \vdash [s_1/x]s_2 : Y} \text{ S_CUT} \\
\frac{\Gamma, x : X, y : Y, \Delta \vdash s : Z}{\Gamma, z : X \otimes Y, \Delta \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } s : Z} \text{ S_TENL} \\
\frac{\Gamma \vdash s_1 : X \quad \Delta \vdash s_2 : Y}{\Gamma, \Delta \vdash s_1 \otimes s_2 : X \otimes Y} \text{ S_TEN} \\
\frac{\Gamma \vdash s_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash s_2 : Z}{\Delta_1, \Gamma, y : X \rightarrow Y, \Delta_2 \vdash [\text{app}_l y s_1/x]s_2 : Z} \text{ S_IMPLL} \\
\frac{\Gamma \vdash s_1 : X \quad \Delta_1, x : Y, \Delta_2 \vdash s_2 : Z}{\Delta_1, y : Y \leftarrow X, \Gamma, \Delta_2 \vdash [\text{app}_r y s_1/x]s_2 : Z} \text{ S_IMPL2} \\
\frac{\Gamma, x : X \vdash s : Y}{\Gamma \vdash \lambda_l x : X. s : X \rightarrow Y} \text{ S_IMPRL} \\
\frac{x : X, \Gamma \vdash s : Y}{\Gamma \vdash \lambda_r x : X. s : Y \leftarrow X} \text{ S_IMPRR} \\
\frac{\Gamma; \cdot \vdash t : A}{\Gamma \vdash \mathbf{G}t : \mathbf{G}A} \text{ S_GR}
\end{array}$$

$$\boxed{\Gamma; \Psi \vdash t : A}$$

$$\begin{array}{c}
\frac{}{\cdot; x : A \vdash x : A} \text{ L_AX} \\
\frac{\Gamma, \Delta; \Psi \vdash t : A}{\Gamma, x : \text{Unit}, \Delta; \Psi \vdash \text{let } x : \text{Unit} \text{ be } \text{triv} \text{ in } t : A} \text{ L_UNITL1} \\
\frac{\Gamma; \Psi, \Phi \vdash t : A}{\Gamma; \Psi, x : \text{Unit}, \Phi \vdash \text{let } x : \text{Unit} \text{ be } \text{triv} \text{ in } t : A} \text{ L_UNITL2} \\
\frac{}{\cdot; \cdot \vdash \text{triv} : \text{Unit}} \text{ L_UNITR} \\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash t : A}{\Gamma, z : Y, w : X, \Delta; \Psi \vdash \text{ex } w, z \text{ with } x, y \text{ in } t : A} \text{ L_BETA} \\
\frac{\Gamma, \Delta; \Psi \vdash t : B \quad x \notin |\Gamma, \Delta, \Psi|}{\Gamma, x : X, \Delta; \Psi \vdash \text{weak } x \text{ in } t : B} \text{ L_WEAK} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash t : B}{\Gamma_1, \Gamma_2, z : X, \Gamma_3; \Psi \vdash \text{contrR } z \text{ as } x, y \text{ in } t : B} \text{ L_CONTRR} \\
\frac{\Gamma_1, x : X, \Gamma_2, y : X, \Gamma_3; \Psi \vdash t : B}{\Gamma_1, z : X, \Gamma_2, \Gamma_3; \Psi \vdash \text{contrL } z \text{ as } x, y \text{ in } t : B} \text{ L_CONTRL} \\
\frac{\Gamma \vdash s : X \quad \Delta_1, x : X, \Delta_2; \Psi \vdash t : A}{\Delta_1, \Gamma, \Delta_2; \Phi \vdash [s/x]t : A} \text{ L_CUT1} \\
\frac{\Gamma; \Psi \vdash t_1 : A \quad \Delta; \Phi_1, x : A, \Phi_2 \vdash t_2 : B}{\Gamma, \Delta; \Phi_1, \Psi, \Phi_2 \vdash [t_1/x]t_2 : B} \text{ L_CUT2} \\
\frac{\Gamma, x : X, y : Y, \Delta; \Psi \vdash t : A}{\Gamma, z : X \otimes Y, \Delta; \Psi \vdash \text{let } z : X \otimes Y \text{ be } x \otimes y \text{ in } t : A} \text{ L_TENL1}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma; \Psi, x : A, y : B, \Phi \vdash t : A}{\Gamma; \Psi, z : A \otimes B, \Phi \vdash \text{let } z : A \otimes B \text{ be } x \otimes y \text{ in } t : A} \quad \text{L_TENL2} \\
\\
\frac{\Gamma; \Psi \vdash t_1 : A \quad \Delta; \Phi \vdash t_2 : B}{\Gamma, \Delta; \Psi, \Phi \vdash t_1 \otimes t_2 : A \otimes B} \quad \text{L_TENR} \\
\\
\frac{\Gamma \vdash s : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash t : A}{\Delta_1, \Gamma, y : X \multimap Y, \Delta_2; \Psi \vdash [\text{app}_l y s/x] t : A} \quad \text{L_IMPLL} \\
\\
\frac{\Gamma \vdash s : X \quad \Delta_1, x : Y, \Delta_2; \Psi \vdash t : A}{\Delta_1, y : Y \multimap X, \Gamma, \Delta_2; \Psi \vdash [\text{app}_r y s_1/x] t : A} \quad \text{L_IMPL2} \\
\\
\frac{\Gamma; \Psi \vdash t_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash t_2 : A}{\Gamma, \Delta; \Phi_1, \Psi, y : A \multimap B, \Phi_2 \vdash [\text{app}_l y t_1/x] t_2 : A} \quad \text{L_IMPL3} \\
\\
\frac{\Gamma; \Psi \vdash t_1 : A \quad \Delta; \Phi_1, x : B, \Phi_2 \vdash t_2 : A}{\Gamma, \Delta; \Phi_1, y : B \multimap A, \Psi, \Phi_2 \vdash [\text{app}_l y t_1/x] t_2 : A} \quad \text{L_IMPL4} \\
\\
\frac{\Gamma; \Psi, x : A \vdash t : B}{\Gamma; \Psi \vdash \lambda_l x : A. t : A \multimap B} \quad \text{L_IMPLR} \\
\\
\frac{\Gamma; x : A, \Psi \vdash t : B}{\Gamma; \Psi \vdash \lambda_r x : A. t : B \multimap A} \quad \text{L_IMPRR} \\
\\
\frac{\Gamma \vdash s : X}{\Gamma; \cdot \vdash \text{Fs} : \text{FX}} \quad \text{L_FR} \\
\\
\frac{\Gamma, x : X; \Psi \vdash t : A}{\Gamma; z : \text{FX}, \Psi \vdash \text{let } z : \text{FX} \text{ be } \text{Fx} \text{ in } t : A} \quad \text{L_FL} \\
\\
\frac{\Gamma; \Psi, x : A \vdash t : B}{z : \text{GA}, \Gamma; \Psi \vdash \text{let } z : \text{GA} \text{ be } \text{Gx} \text{ in } t : B} \quad \text{L_GL}
\end{array}$$