Separating Linear Modalities

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Abstract

TODO

1 Introduction

TODO [1]

1.1 Symmetric Monoidal Categories

Definition 1 A symmetric monoidal category (SMC) is a category, \mathcal{M} , with the following data:

- An object \top of \mathcal{M} ,
- A bi-functor \otimes : $\mathcal{M} \times \mathcal{M} \longrightarrow \mathcal{M}$,
- The following natural isomorphisms:

$$\lambda_A : \top \otimes A \longrightarrow A$$

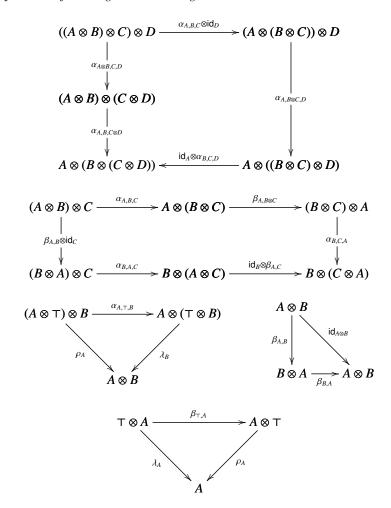
$$\rho_A : A \otimes \top \longrightarrow A$$

$$\alpha_{A,B,C} : (A \otimes B) \otimes C \longrightarrow A \otimes (B \otimes C)$$

• A symmetry natural transformation:

$$\beta_{A,B}: A \otimes B \longrightarrow B \otimes A$$

• Subject to the following coherence diagrams:



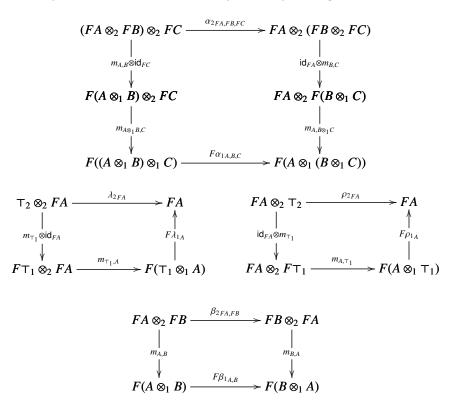
Definition 2 A symmetric monoidal closed category (SMCC) is a symmetric monoidal category, $(\mathcal{M}, \top, \otimes)$, such that, for any object B of M, the functor $-\otimes B : \mathcal{M} \longrightarrow \mathcal{M}$ has a specified right adjoint. Hence, for any objects A and C of M there is an object $B \multimap C$ of M and a natural bijection:

$$\operatorname{\mathsf{Hom}}_{\mathcal{M}}(A \otimes B, C) \cong \operatorname{\mathsf{Hom}}_{\mathcal{M}}(A, B \multimap C)$$

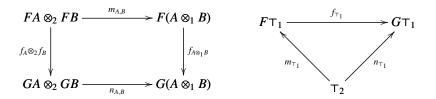
We call the functor \multimap : $\mathcal{M} \times \mathcal{M} \longrightarrow \mathcal{M}$ *the internal hom of* \mathcal{M} .

Definition 3 Suppose we are given two symmetric monoidal closed categories $(\mathcal{M}_1, \top_1, \otimes_1, \alpha_1, \lambda_1, \rho_1, \beta_1)$ and $(\mathcal{M}_2, \top_2, \otimes_2, \alpha_2, \lambda_2, \rho_2, \beta_2)$. Then a **symmetric monoidal** functor is a functor $F: \mathcal{M}_1 \longrightarrow \mathcal{M}_2$, a map $m_{\top_1}: \top_2 \longrightarrow F \top_1$ and a natural transfor-

mation $m_{A,B}$: $FA \otimes_2 FB \longrightarrow F(A \otimes_1 B)$ *subject to the following coherence conditions:*

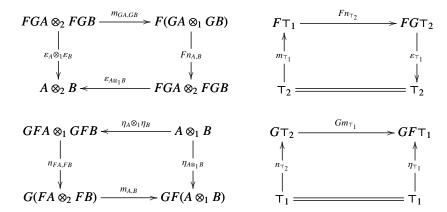


Definition 4 Suppose $(\mathcal{M}_1, \top_1, \otimes_1)$ and $(\mathcal{M}_2, \top_2, \otimes_2)$ are SMCs, and (F, m) and (G, n) are a symmetric monoidal functors between \mathcal{M}_1 and \mathcal{M}_2 . Then a symmetric monoidal natural transformation is a natural transformation, $f: F \longrightarrow G$, subject to the following coherence diagrams:

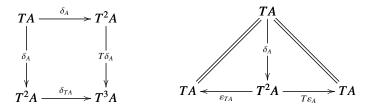


Definition 5 Suppose (M_1, \top_1, \otimes_1) and (M_2, \top_2, \otimes_2) are SMCs, and (F, m) is a symmetric monoidal functor between M_1 and M_2 and (G, n) is a symmetric monoidal functor between M_2 and M_1 . Then a **symmetric monoidal adjunction** is an ordinary adjunction $M_1: F \dashv G: M_2$ such that the unit, $\eta_A: A \to GFA$, and the counit, $\varepsilon_A: FGA \to A$, are symmetric monoidal natural transformations. Thus, the following

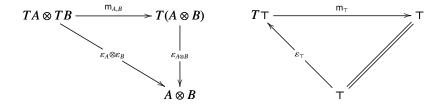
diagrams must commute:

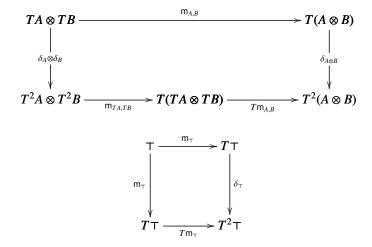


Definition 6 A symmetric monoidal comonad on a symmetric monoidal category C is a triple (T, ε, δ) , where (T, m) is a symmetric monoidal endofunctor on C, $\varepsilon_A : TA \longrightarrow A$ and $\delta_A : TA \to T^2A$ are symmetric monoidal natural transformations, which make the following diagrams commute:



The assumption that ε and δ are symmetric monoidal natural transformations amount to the following diagrams commuting:





2 Related Work

TODO

3 Conclusion

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References

[1] P. N. Benton. A mixed linear and non-linear logic: Proofs, terms and models (preliminary report). Technical Report UCAM-CL-TR-352, University of Cambridge Computer Laboratory, 1994. Accessible online at http://research.microsoft.com/en-us/um/people/nick/mixed3.ps.