vars, *n, a, x, y, z, w, m, o*
ivar, *i, k, j, l*
const, *b*

A, B, C	::=		
	|	B	Base type
	|	Unit	Unit
	|	$A \otimes B$	Non-commutative tensor
	|	$A \rightharpoonup B$	Left implication
	|	$A \leftharpoonup B$	Right implication
	|	$w\,A$	
	|	$c\,A$	
	|	$e\,A$	

t	::=	
	|	x
	|	b
	|	unit
	|	let $t_1 : A$ be t_2 in t_3
	|	let t_1 be t_2 in t_3 S
	|	$t_1 \otimes t_2$
	|	$\lambda_l x : A.t$
	|	$\lambda_r x : A.t$
	|	$\text{app}_l\,t_1\,t_2$
	|	$\text{app}_r\,t_1\,t_2$
	|	$w\,t$
	|	$e\,t$
	|	$c\,t$
	|	$\text{weak}_l\,t_1$ in t_2
	|	$\text{weak}_r\,t_1$ in t_2
	|	$\text{con}_l\,x, y$ to t_1 in t_2
	|	$\text{con}_r\,x, y$ to t_1 in t_2
	|	$\text{ex}\,x_1, x_2$ with t_1, t_2 in t_3

$\Gamma, \Delta, \Phi, \Psi$	::=	
	|	$\cdot$
	|	Γ_1, Γ_2
	|	$x : A$
	|	(Γ) S

$$\boxed{\Gamma; \Psi; \Phi; \Delta \vdash t : A}$$

$$\frac{}{\cdot; \cdot; \cdot; x : A \vdash x : A} \ \text{L\textsc{ax}}$$

$$\frac{}{\cdot; \cdot; x : A; \cdot \vdash x : A} \ \text{W\textsc{ax}}$$

$$\frac{}{\cdot; x : A; \cdot; \cdot \vdash x : A} \ \text{C\textsc{ax}}$$

$$\frac{}{x : A; \cdot; \cdot; \cdot \vdash x : A} \ \text{E\textsc{ax}}$$

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$$\frac{}{\cdot\,;\,\cdot\,;\,\cdot\,;\,\cdot \vdash b : B}\ \text{CONST}$$

$$\frac{}{\cdot\,;\,\cdot\,;\,\cdot\,;\,\cdot \vdash \mathsf{unit} : \mathsf{Unit}}\ \text{UNIT}\mathrm{I}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : \mathsf{Unit} \quad \Gamma_2;\Psi_2;\Phi_1;\Delta_2 \vdash t_2 : C}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{let\,unit\,be}\,t_1\,\mathsf{in}\,t_2 : C}\ \text{UNIT}\mathrm{E}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : A \quad \Gamma_2;\Psi_2;\Phi_1;\Delta_2 \vdash t_2 : B}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash t_1 \otimes t_2 : A \otimes B}\ \text{TENSOR}\mathrm{I}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : A \otimes B \quad \Gamma_2;\Psi_2;\Phi_1;\Delta_2, x:A, y:B \vdash t_2 : B}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{let}\,x \otimes y : A \otimes B\,\mathsf{be}\,t_1\,\mathsf{in}\,t_2 : C}\ \text{TENSOR}\mathrm{E}$$

$$\frac{\Gamma;\Psi;\Phi;x:A,\Delta \vdash t : B}{\Gamma;\Psi;\Phi;\Delta \vdash \lambda_l x:A.t : A \rightharpoonup B}\ \text{LFUN}$$

$$\frac{\Gamma;\Psi;\Phi;\Delta, x:A \vdash t : B}{\Gamma;\Psi;\Phi;\Delta \vdash \lambda_r x:A.t : B \leftharpoonup A}\ \text{RFUN}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : A \rightharpoonup B \quad \Gamma_2;\Psi_2;\Phi_1;\Delta_2 \vdash t_2 : A}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{app}_l\,t_1\,t_2 : B}\ \text{LAPP}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : B \leftharpoonup A \quad \Gamma_2;\Psi_2;\Phi_1;\Delta_2 \vdash t_2 : A}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{app}_r\,t_1\,t_2 : B}\ \text{RAPP}$$

$$\frac{\cdot\,;\,\cdot\,;\,\Psi;\cdot \vdash t : A}{\cdot\,;\,\cdot\,;\,\Psi;\cdot \vdash \mathsf{w}\,t : \mathsf{w}\,A}\ \text{wI}$$

$$\frac{\cdot\,;\,\Phi;\cdot\,;\,\cdot \vdash t : A}{\cdot\,;\,\Phi;\cdot\,;\,\cdot \vdash \mathsf{c}\,t : \mathsf{c}\,A}\ \text{cI}$$

$$\frac{\Gamma;\cdot\,;\,\cdot\,;\,\cdot \vdash t : A}{\Gamma;\cdot\,;\,\cdot\,;\,\cdot \vdash \mathsf{e}\,t : \mathsf{e}\,A}\ \text{eI}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : \mathsf{w}\,A \quad \Gamma_2;\Psi_2;\Phi_1,x:A;\Delta_2 \vdash t_2 : B}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{let}\,\mathsf{w}\,x : \mathsf{w}\,A\,\mathsf{be}\,t_1\,\mathsf{in}\,t_2 : B}\ \text{wE}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : \mathsf{c}\,A \quad \Gamma_2;\Psi_2,x:A;\Phi_1;\Delta_2 \vdash t_2 : B}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{let}\,\mathsf{c}\,x : \mathsf{c}\,A\,\mathsf{be}\,t_1\,\mathsf{in}\,t_2 : B}\ \text{cE}$$

$$\frac{\Gamma_1;\Psi_1;\Phi_1;\Delta_1 \vdash t_1 : \mathsf{e}\,A \quad \Gamma_2,x:A;\Psi_2;\Phi_1;\Delta_2 \vdash t_2 : B}{(\Gamma_1,\Gamma_2);(\Psi_1,\Psi_2);(\Phi_1,\Phi_2);(\Delta_1,\Delta_2) \vdash \mathsf{let}\,\mathsf{e}\,x : \mathsf{e}\,A\,\mathsf{be}\,t_1\,\mathsf{in}\,t_2 : B}\ \text{eE}$$

$$\frac{\Gamma;\Psi;\Phi;\Delta \vdash t : B \quad x \notin \mathsf{FV}(t)}{\Gamma;\Psi;\Phi,x:A;\Delta \vdash \mathsf{weak}_r\,x\,\mathsf{in}\,t : B}\ \text{RWEAK}$$

$$\frac{\Gamma;\Psi;\Phi;\Delta \vdash t : B \quad x \notin \mathsf{FV}(t)}{\Gamma;\Psi;x:A,\Phi;\Delta \vdash \mathsf{weak}_l\,x\,\mathsf{in}\,t : B}\ \text{LWEAK}$$

$$\frac{\Gamma;\Psi_1,x:A,\Psi_2,y:A,\Psi_3;\Phi;\Delta \vdash t : B}{\Gamma;\Psi_1,z:A,\Psi_2,\Psi_3;\Phi;\Delta \vdash \mathsf{con}_l\,x,y\,\mathsf{to}\,z\,\mathsf{in}\,t : B}\ \text{LCON}$$

$$\frac{\Gamma;\Psi_1,x:A,\Psi_2,y:A,\Psi_3;\Phi;\Delta \vdash t : B}{\Gamma;\Psi_1,\Psi_2,z:A,\Psi_3;\Phi;\Delta \vdash \mathsf{con}_r\,x,y\,\mathsf{to}\,z\,\mathsf{in}\,t : B}\ \text{RCON}$$

$$\frac{\Gamma_1,x_1:A,x_2:B,\Gamma_2;\Psi;\Phi;\Delta \vdash t : C}{\Gamma,z_1:B,z_2:A;\Psi;\Phi;\Delta \vdash \mathsf{ex}\,x_1,x_2\,\mathsf{with}\,z_1,z_2\,\mathsf{in}\,t : C}\ \text{EX}$$