

vars, n, a, x, y, z, w, m, o

ivar, i, k, j, l

const, b

| | | | |
|-----------|-------|-----------------|------------------------------------|
| X, Y, Z | $::=$ | | |
| | | I | Unit |
| | | $X \supseteq Y$ | Associative Non-commutative tensor |
| | | $X \multimap Y$ | Left implication |
| | | $Y \multimap X$ | Right implication |
| | | $\mathsf{F} A$ | Right adjoint |

| | | | |
|-----------|-------|----------------------|------------------------------------|
| A, B, C | $::=$ | | |
| | | J | Unit |
| | | $A \triangleright B$ | Associative Non-commutative tensor |
| | | $A \multimap B$ | Left implication |
| | | $B \multimap A$ | Right implication |
| | | $\mathsf{G} X$ | Right adjoint |

| | | |
|----------|-------|----------------------|
| Γ | $::=$ | |
| | | \cdot |
| | | A |
| | | Γ_1, Γ_2 |
| | | (Γ) |
| | | Γ |

| | | |
|----------|-------|----------------------|
| Δ | $::=$ | |
| | | \cdot |
| | | X |
| | | Δ_1, Δ_2 |
| | | (Δ) |
| | | Δ |

$\boxed{\Delta \vdash_{\mathcal{A}} X}$

| | |
|---|----------------------|
| $\overline{X \vdash_{\mathcal{A}} X}$ | $\mathsf{A_VAR}$ |
| $\overline{\cdot \vdash_{\mathcal{A}} I}$ | $\mathsf{A_IR}$ |
| $\frac{\Delta \vdash_{\mathcal{A}} X}{\Delta, I \vdash_{\mathcal{A}} X}$ | $\mathsf{A_IL}$ |
| $\frac{\Delta_1 \vdash_{\mathcal{A}} X \quad \Delta_2 \vdash_{\mathcal{A}} Y}{\Delta_1, \Delta_2 \vdash_{\mathcal{A}} X \supseteq Y}$ | $\mathsf{A_TR}$ |
| $\frac{X, Y \vdash_{\mathcal{A}} Z}{X \supseteq Y \vdash_{\mathcal{A}} Z}$ | $\mathsf{A_TL}$ |
| $\frac{\Delta, X \vdash_{\mathcal{A}} Y}{\Delta \vdash_{\mathcal{A}} X \multimap Y}$ | $\mathsf{A_IRR}$ |
| $\frac{\Delta_1 \vdash_{\mathcal{A}} X \quad \Delta_2, Y \vdash_{\mathcal{A}} Z}{\Delta_1, \Delta_2, X \multimap Y \vdash_{\mathcal{A}} Z}$ | $\mathsf{A_IRL}$ |
| $\frac{\Delta, X, Y \vdash_{\mathcal{A}} Z \quad \Delta \neq \emptyset}{\Delta, X \supseteq Y \vdash_{\mathcal{A}} Z}$ | $\mathsf{A_ASSOCL}$ |

$$\frac{X, Y, \Delta \vdash_{\mathcal{A}} Z \quad \Delta \neq \emptyset}{X \supseteq Y, \Delta \vdash_{\mathcal{A}} Z} \text{ A_ASSOCR}$$

$$\frac{\Delta; \cdot \vdash_{\mathcal{L}} A}{\Delta \vdash_{\mathcal{A}} \mathbf{F} A} \text{ A_FR}$$

$$\boxed{\Delta; \Gamma \vdash_{\mathcal{L}} A}$$

$$\frac{}{\cdot; A \vdash_{\mathcal{L}} A} \text{ L_VAR}$$

$$\frac{}{\cdot; \cdot \vdash_{\mathcal{L}} J} \text{ L_JR}$$

$$\frac{\Delta; \Gamma \vdash_{\mathcal{L}} A}{\Delta; \Gamma, J \vdash_{\mathcal{L}} A} \text{ L_JL}$$

$$\frac{\Delta; \Gamma \vdash_{\mathcal{L}} A}{\Delta, I; \Gamma \vdash_{\mathcal{L}} A} \text{ L_IL}$$

$$\frac{\Delta_1; \Gamma_1 \vdash_{\mathcal{L}} A \quad \Delta_2; \Gamma_2 \vdash_{\mathcal{L}} B}{\Delta_1, \Delta_2; \Gamma_1, \Gamma_2 \vdash_{\mathcal{L}} A \triangleright B} \text{ L_TR}$$

$$\frac{\Delta; A, B \vdash_{\mathcal{L}} C}{\Delta; A \triangleright B \vdash_{\mathcal{L}} C} \text{ L_TL}$$

$$\frac{X, Y; \Gamma \vdash_{\mathcal{L}} C}{X \supseteq Y; \Gamma \vdash_{\mathcal{L}} C} \text{ L_ATL}$$

$$\frac{\Delta, X, Y; \Gamma \vdash_{\mathcal{L}} A \quad \Delta \neq \emptyset}{\Delta, X \supseteq Y; \Gamma \vdash_{\mathcal{L}} A} \text{ L_ASSOCL}$$

$$\frac{X, Y, \Delta; \Gamma \vdash_{\mathcal{L}} A \quad \Delta \neq \emptyset}{X \supseteq Y, \Delta; \Gamma \vdash_{\mathcal{L}} A} \text{ L_ASSOCR}$$

$$\frac{\Delta; \Gamma, A \vdash_{\mathcal{L}} B}{\Delta; \Gamma \vdash_{\mathcal{L}} A \multimap B} \text{ L_IRR}$$

$$\frac{\Delta_1; \Gamma_1 \vdash_{\mathcal{L}} A \quad \Delta_2; \Gamma_2, B \vdash_{\mathcal{L}} C}{\Delta_1, \Delta_2; \Gamma_1, \Gamma_2, A \multimap B \vdash_{\mathcal{L}} C} \text{ L_IRL}$$

$$\frac{\Delta_1 \vdash_{\mathcal{A}} X \quad \Delta_2, Y; \Gamma \vdash_{\mathcal{L}} A}{\Delta_1, \Delta_2, X \multimap Y; \Gamma \vdash_{\mathcal{L}} A} \text{ L_AIRL}$$

$$\frac{\Delta \vdash_{\mathcal{A}} X}{\Delta; \cdot \vdash_{\mathcal{L}} \mathbf{G} X} \text{ L_GR}$$

$$\frac{\Delta, X; \Gamma \vdash_{\mathcal{L}} A}{\Delta; \Gamma, \mathbf{G} X \vdash_{\mathcal{L}} A} \text{ L_GL}$$

$$\frac{\Delta; \Gamma, A \vdash_{\mathcal{L}} B}{\Delta, \mathbf{F} A; \Gamma \vdash_{\mathcal{L}} B} \text{ L_FL}$$

Definition rules: 25 good 0 bad

Definition rule clauses: 46 good 0 bad