

An Adjoint Model for Process Trees with Sequential Composition

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Abstract

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1 Introduction

TODO [1]

2 Categorical Models

2.1 Lambek Categories

► **Definition 1.** A **monoidal category**, $(\mathcal{L}, \otimes, I, \lambda, \rho)$, is a category, \mathcal{L} , equipped with a bifunctor, $\otimes : \mathcal{L} \times \mathcal{L} \rightarrow \mathcal{L}$, called the tensor product, a distinguished object I of \mathcal{L} called the unit, and three natural isomorphisms $\lambda_A : I \otimes A \rightarrow A$, $\rho_A : A \otimes I \rightarrow A$, and $\alpha_{A,B,C} : A \otimes (B \otimes C) \rightarrow (A \otimes B) \otimes C$ called the left and right unitors and the associator respectively. Finally, these are subject to the following coherence diagrams:

$$\begin{array}{ccccc}
 ((A \otimes B) \otimes C) \otimes D & \xrightarrow{\alpha_{A,B,C} \otimes \text{id}_D} & (A \otimes (B \otimes C)) \otimes D & \xrightarrow{\alpha_{A,B \otimes C,D}} & A \otimes ((B \otimes C) \otimes D) \\
 \downarrow \alpha_{A \otimes B,C,D} & & & & \downarrow \text{id}_A \otimes \alpha_{B,C,D} \\
 (A \otimes B) \otimes (C \otimes D) & \xrightarrow{\alpha_{A,B,C \otimes D}} & & & A \otimes (B \otimes (C \otimes D))
 \end{array}$$

$$\begin{array}{ccc}
 (A \otimes I) \otimes B & \xrightarrow{\alpha_{A,I,B}} & A \otimes (I \otimes B) \\
 \downarrow \rho_A \otimes \text{id}_B & & \downarrow \text{id}_A \otimes \lambda_B \\
 A \otimes B & & A \otimes B
 \end{array}$$

► **Definition 2.** A **Lambek category** is a monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with two bifunctors $\multimap : \mathcal{L}^{\text{op}} \times \mathcal{L} \rightarrow \mathcal{L}$ and $\multimap : \mathcal{L} \times \mathcal{L}^{\text{op}} \rightarrow \mathcal{L}$ that are both right adjoint to the tensor product. That is, the following natural bijections hold:

$$\text{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \text{Hom}_{\mathcal{L}}(X, A \multimap B)$$

$$\text{Hom}_{\mathcal{L}}(A \otimes X, B) \cong \text{Hom}_{\mathcal{L}}(X, B \multimap A)$$



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One might call Lambek categories biclosed monoidal categories, but we name them in homage to Lambek for his contributions to non-commutative linear logic.

► **Definition 3.** A monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ is **symmetric** if there is a natural transformation $\beta_{A,B} : A \otimes B \longrightarrow B \otimes A$ such that $\beta_{B,A} \circ \beta_{A,B} = \text{id}_{A \otimes B}$ and the following commute:

$$\begin{array}{ccccc}
 (A \otimes B) \otimes C & \xrightarrow{\alpha_{A,B,C}} & A \otimes (B \otimes C) & \xrightarrow{\beta_{A,B \otimes C}} & (B \otimes C) \otimes A \\
 \downarrow \beta_{A,B} \otimes \text{id}_C & & & & \downarrow \alpha_{B,C,A} \\
 (B \otimes A) \otimes C & \xrightarrow{\alpha_{B,A,C}} & B \otimes (A \otimes C) & \xrightarrow{\text{id}_B \otimes \beta_{A,C}} & B \otimes (C \otimes A)
 \end{array}
 \quad
 \begin{array}{ccc}
 I \otimes A & \xrightarrow{\beta_{I,A}} & A \otimes I \\
 \searrow \lambda_A & & \swarrow \rho_A \\
 & A &
 \end{array}$$

► **Definition 4.** A symmetric monoidal category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \beta)$ is **closed** if it comes equipped with a bifunctor $\multimap : \mathcal{L}^{\text{op}} \times \mathcal{L} \longrightarrow \mathcal{L}$ that is right adjoint to the tensor product. That is, the following natural bijection $\text{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \text{Hom}_{\mathcal{L}}(X, A \multimap B)$ holds.

► **Definition 5.** A **Lambek category with weakening**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{weak})$, is a Lambek category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with a natural transformation $\text{weak}_A : A \longrightarrow I$.

► **Definition 6.** A **Lambek category with contraction**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{contraL}, \text{contraR})$, is a Lambek category $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ equipped with natural transformations:

$$\text{contraL}_{A,B} : (A \otimes B) \longrightarrow (A \otimes B) \otimes A \quad \text{contraR}_{A,B} : (B \otimes A) \longrightarrow A \otimes (B \otimes A)$$

Furthermore, the following diagrams must commute:

$$\begin{array}{ccc}
 A \otimes I & \xleftarrow{\rho_A^{-1}} & A \xrightarrow{\lambda_A^{-1}} I \otimes A \\
 \text{contraL}_{A,I} \downarrow & & \downarrow \text{contraR}_{A,I} \\
 (A \otimes I) \otimes A & \xrightarrow{\alpha_{A,I,A}} & A \otimes (I \otimes A)
 \end{array}$$

$$\begin{array}{ccccc}
 A \otimes A & \xrightarrow{\text{id}_A \otimes \rho_A^{-1}} & A \otimes (A \otimes I) & \xrightarrow{\text{id}_A \otimes \text{contraL}_{A,I}} & A \otimes ((A \otimes I) \otimes A) \\
 \downarrow \lambda_A^{-1} \otimes \text{id}_A & & & & \downarrow \text{id}_A \otimes (\rho_A \otimes \text{id}_A) \\
 (I \otimes A) \otimes A & \xrightarrow{\text{contraR}_{A,I} \otimes \text{id}_A} & (A \otimes (I \otimes A)) \otimes A & \xrightarrow{(\text{id}_A \otimes \lambda_A) \otimes \text{id}_A} & (A \otimes A) \otimes A \\
 & & & & \uparrow \alpha_{A,A,A} \\
 & & & & A \otimes (A \otimes A)
 \end{array}$$

► **Definition 7.** A **Lambek category with exchange**, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{ex})$, is a Lambek category, $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$, such that \mathcal{L} is symmetric monoidal, where $\text{ex}_{A,B} : A \otimes B \longrightarrow B \otimes A$ is the symmetry.

► **Lemma 8.** $(A \multimap B) \cong (B \multimap A)$

3 Related Work

TODO

4 Conclusion

TODO

References

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A Appendix