# An Adjoint Model for Process Trees with **Sequential Composition**

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#### Introduction

TODO [1]

### **Categorical Models**

#### **Lambek Categories**

▶ **Definition 1.** A monoidal category,  $(\mathcal{L}, \otimes, I, \lambda, \rho)$ , is a category,  $\mathcal{L}$ , equipped with a bifunctor,  $\otimes: \mathcal{L} \times \mathcal{L} \longrightarrow \mathcal{L}$ , called the tensor product, a distinguished object I of  $\mathcal{L}$  called the unit, and three natural isomorphisms  $\lambda_A: I \otimes A \longrightarrow A$ ,  $\rho_A: A \otimes I \longrightarrow A$ , and  $\alpha_{A,B,C}: A \otimes (B \otimes C) \longrightarrow (A \otimes B) \otimes C$  called the left and right unitors and the associator respectively. Finally, these are subject to the following coherence diagrams:

$$((A \otimes B) \otimes C) \otimes D \xrightarrow{\alpha_{A,B,C} \otimes \mathrm{id}_{D}} A \otimes (B \otimes C)) \otimes D \xrightarrow{\alpha_{A,B \otimes C,D}} A \otimes ((B \otimes C) \otimes D)$$

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▶ **Definition 2.** A **Lambek category** is a monoidal category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  equipped with two bifunctors  $\rightharpoonup: \mathcal{L}^{op} \times \mathcal{L} \longrightarrow \mathcal{L}$  and  $\vdash: \mathcal{L} \times \mathcal{L}^{op} \longrightarrow \mathcal{L}$  that are both right adjoint to the tensor product. That is, the following natural bijections hold:

$$\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \rightharpoonup B) \qquad \qquad \mathsf{Hom}_{\mathcal{L}}(A \otimes X, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, B \leftharpoonup A)$$



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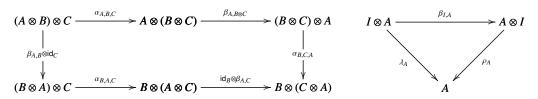
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#### XX:2 An Adjoint Model for Process Trees with Sequential Composition

One might call Lambek categories biclosed monoidal categories, but we name them in homage to Lambek for his contributions to non-commutative linear logic.

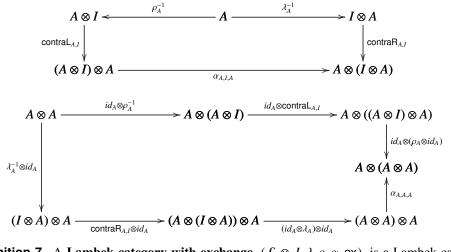
▶ **Definition 3.** A monoidal category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  is **symmetric** if there is a natural transformation  $\beta_{A,B} : A \otimes B \longrightarrow B \otimes A$  such that  $\beta_{B,A} \circ \beta_{A,B} = \mathrm{id}_{A \otimes B}$  and the following commute:



- ▶ **Definition 4.** A symmetric monoidal category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \beta)$  is **closed** if it comes equipped with a bifunctor  $\multimap$ :  $\mathcal{L}^{op} \times \mathcal{L} \longrightarrow \mathcal{L}$  that is right adjoint to the tensor product. That is, the following natural bijection  $\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \multimap B)$  holds.
- ▶ **Definition 5.** A **Lambek category with weakening**,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{weak})$ , is a Lambek category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  equipped with a natural transformation  $\text{weak}_A : A \longrightarrow I$ .
- ▶ **Definition 6.** A **Lambek category with contraction**,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{contraL}, \text{contraR})$ , is a Lambek category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  equipped with natural transformations:

$$\mathsf{contral}_{A,B} : (A \otimes B) {\:\longrightarrow\:} (A \otimes B) \otimes A \qquad \mathsf{contral}_{A,B} : (B \otimes A) {\:\longrightarrow\:} A \otimes (B \otimes A)$$

Furthermore, the following diagrams must commute:



- ▶ **Definition 7.** A **Lambek category with exchange**,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, ex)$ , is a Lambek category,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ , such that  $\mathcal{L}$  is symmetric monoidal, where  $ex_{A,B} : A \otimes B \longrightarrow B \otimes A$  is the symmetry.
- ▶ Lemma 8.  $(A \rightarrow B) \cong (B \leftarrow A)$

#### 3 Related Work

**TODO** 

#### 4 Conclusion

**TODO** 

#### - References -

P. N. Benton. A mixed linear and non-linear logic: Proofs, terms and models (preliminary report). Technical Report UCAM-CL-TR-352, University of Cambridge Computer Laboratory, 1994. Accessible online at http://research.microsoft.com/en-us/um/people/nick/mixed3.ps.

## A Appendix