# An Adjoint Model for Process Trees with **Sequential Composition**

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**TODO** 

1998 ACM Subject Classification TODO

Keywords and phrases TODO

Digital Object Identifier 10.4230/LIPIcs...

### Introduction

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# **Categorical Models**

### **Lambek Categories**

▶ **Definition 1.** A monoidal category,  $(\mathcal{L}, \otimes, I, \lambda, \rho)$ , is a category,  $\mathcal{L}$ , equipped with a bifunctor,  $\otimes: \mathcal{L} \times \mathcal{L} \longrightarrow \mathcal{L}$ , called the tensor product, a distinguished object I of  $\mathcal{L}$  called the unit, and three natural isomorphisms  $\lambda_A: I \otimes A \longrightarrow A$ ,  $\rho_A: A \otimes I \longrightarrow A$ , and  $\alpha_{A,B,C}: A \otimes (B \otimes C) \longrightarrow (A \otimes B) \otimes C$  called the left and right unitors and the associator respectively. Finally, these are subject to the following coherence diagrams:

$$((A \otimes B) \otimes C) \otimes D \xrightarrow{\alpha_{A,B,C} \otimes \mathrm{id}_{D}} A \otimes (B \otimes C)) \otimes D \xrightarrow{\alpha_{A,B \otimes C,D}} A \otimes ((B \otimes C) \otimes D)$$

$$\downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \qquad \downarrow \qquad$$

▶ **Definition 2.** A **Lambek category** is a monoidal category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  equipped with two bifunctors  $\rightharpoonup: \mathcal{L}^{op} \times \mathcal{L} \longrightarrow \mathcal{L}$  and  $\vdash: \mathcal{L} \times \mathcal{L}^{op} \longrightarrow \mathcal{L}$  that are both right adjoint to the tensor product. That is, the following natural bijections hold:

$$\mathsf{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, A \rightharpoonup B) \qquad \qquad \mathsf{Hom}_{\mathcal{L}}(A \otimes X, B) \cong \mathsf{Hom}_{\mathcal{L}}(X, B \leftharpoonup A)$$



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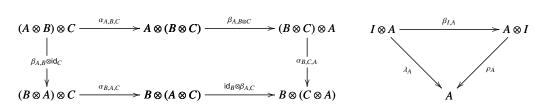
Leibniz International Proceedings in Informatics

LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

#### **XX:2** An Adjoint Model for Process Trees with Sequential Composition

One might call Lambek categories biclosed monoidal categories, but we name them in homage to Lambek for his contributions to non-commutative linear logic.

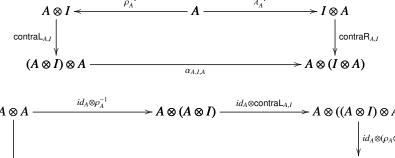
▶ **Definition 3.** A monoidal category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  is **symmetric** if there is a natural transformation  $\beta_{A,B}: A \otimes B \longrightarrow B \otimes A$  such that  $\beta_{B,A} \circ \beta_{A,B} = id_{A \otimes B}$  and the following commute:



- ▶ **Definition 4.** A symmetric monoidal category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \beta)$  is **closed** if it comes equipped with a bifunctor  $\multimap: \mathcal{L}^{op} \times \mathcal{L} \longrightarrow \mathcal{L}$  that is right adjoint to the tensor product. That is, the following natural bijection  $\operatorname{Hom}_{\mathcal{L}}(X \otimes A, B) \cong \operatorname{Hom}_{\mathcal{L}}(X, A \multimap B)$  holds.
- ▶ **Definition 5.** A **Lambek category with weakening**,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{weak})$ , is a Lambek category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  equipped with a natural transformation weak<sub>A</sub> :  $A \longrightarrow I$ .
- ▶ **Definition 6.** A **Lambek category with contraction**,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, \text{contraL}, \text{contraR})$ , is a Lambek category  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$  equipped with natural transformations:

$$\mathsf{contraL}_{A,B} : (A \otimes B) \longrightarrow (A \otimes B) \otimes A \qquad \mathsf{contraR}_{A,B} : (B \otimes A) \longrightarrow A \otimes (B \otimes A)$$

Furthermore, the following diagrams must commute:



$$A \otimes A \xrightarrow{id_{A} \otimes \rho_{A}^{-1}} \rightarrow A \otimes (A \otimes I) \xrightarrow{id_{A} \otimes \operatorname{contraL}_{A,I}} \rightarrow A \otimes ((A \otimes I) \otimes A)$$

$$\downarrow id_{A} \otimes (\rho_{A} \otimes id_{A})$$

$$\downarrow id_{A} \otimes (\rho_{A} \otimes id_{A})$$

$$\downarrow A \otimes (A \otimes A)$$

$$\uparrow \alpha_{A,A,A}$$

$$(I \otimes A) \otimes A \xrightarrow{\operatorname{contraR}_{A,I} \otimes id_{A}} \rightarrow (A \otimes (I \otimes A)) \otimes A \xrightarrow{(id_{A} \otimes \lambda_{A}) \otimes id_{A}} \rightarrow (A \otimes A) \otimes A$$

▶ Definition 7. A Lambek category with exchange,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha, ex)$ , is a Lambek category,  $(\mathcal{L}, \otimes, I, \lambda, \rho, \alpha)$ , such that  $\mathcal{L}$  is symmetric monoidal, where  $ex_{A,B} : A \otimes B \longrightarrow B \otimes A$  is the symmetry.

### **Related Work**

TODO

# Conclusion

**TODO** 

## - References -

P. N. Benton. A mixed linear and non-linear logic: Proofs, terms and models (preliminary report). Technical Report UCAM-CL-TR-352, University of Cambridge Computer Laboratory, 1994. Accessible online at http://research.microsoft.com/en-us/um/people/nick/mixed3.ps.

# A Appendix