EE 669 VLSI TECHNOLOGY

ASSIGNMENT 3:
SOLUTION OF EXERCISE QUESTIONS LECTURE 9 - 13

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L9_11- S6

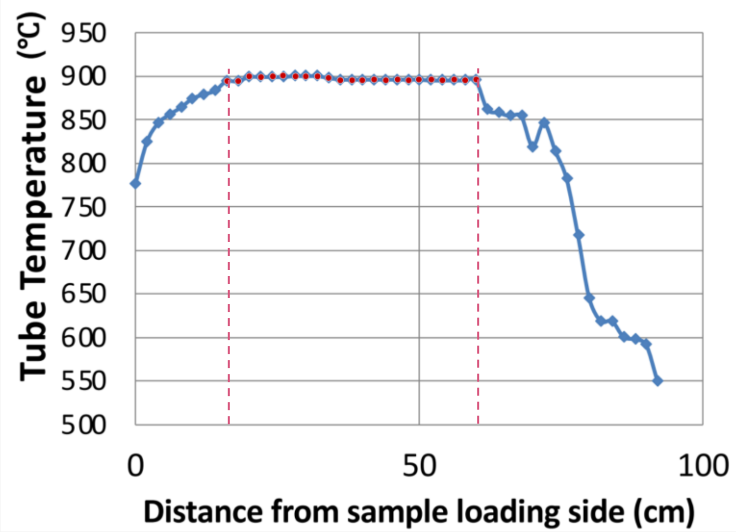
Density of SiO_2 (quartz) is approximately 2.65 g/cm^3

Therefore mass of SiO_2 in 2.2 cm^3 $= 2.2 * 2.65 = 5.83\ gm$

Molar mass of SiO_2 $= 28.085 \frac{g}{mol} (for\ Si) + 2 * 15.9994 \frac{g}{mol} (for\ O) = 60.0843 \frac{g}{mol}$

Hence, number of silicon atoms $= \frac{5.83\ gm}{60.0843 \frac{gm}{mol}} * 6.022 * 10^{23} = \mathbf{5.84 * 10^{22}}\ \boldsymbol{atoms}$

L9_11- S9



I used webplot digitizer (https://automeris.io) to extract the distances from sample loading side where the flat zone starts and ends. Data extracted is shown on the right.

Flat-zone length = 60.39 – 16.48 = 43.91 cm

The answer can have an error of ± 0.5 cm.

Distance	Temperature
16.48	894.59
18.42	894.59
20.36	900
22.44	899.1
24.52	900
26.32	900.9
28.53	900
30.33	900
32.41	900
34.35	898.2
36.42	895.5
38.5	895.5
40.3	895.5
42.38	896.4
44.46	895.5
46.53	896.4
48.47	895.5
50.27	896.4
52.49	896.4
54.43	895.5
56.51	896.4
58.45	895.5
60.39	896.4

EXERCISE ANSWERS

L9_11- S11

No, silicon atoms do not diffuse to the surface. Rather the O2 atoms diffuse through the growing layer of oxide and react with the silicon. Transport through the oxide takes place through diffusion. The oxygen enters the oxide through diffusion at the surface but this diffusion is not rate-limiting to the process.

As the SiO2 layer is formed silicon atoms get pushed deeper into the substrate due to pressure from the growing oxide and they fill up vacancies and interstitial sites in the bulk of the substrate.

L9_11- S15

N is defined as the number of oxidant consumed per unit volume of the film grown

For O2 (Dry oxidation)

- Chemical reaction : $Si + O_2 \rightarrow SiO_2$
- 1 mole of O2 forms 1 mole of SiO2
- Molar volume of SiO2 $= \frac{Molar\ mass}{Density} = \frac{60.08}{2.2} = 27.31 \frac{\mathrm{cm}^3}{\mathrm{mol}}$
- Therefore, number of O2 molecules per unit volume SiO2

$$\frac{6.022 * 10^{23} * 1}{27.31} \approx 2.20 * 10^{22}\ molecules$$

- Hence $\boldsymbol{N_{O_2} = 2.20 * 10^{22}}\ \frac{\textbf{molecules}}{\boldsymbol{Volume_{\{SiO2\}}}}$

For H2O (Wet oxidation)

- Chemical reaction : $Si + 2H_2O \rightarrow SiO_2 + H_2$
- 2 moles of H2O forms 1 mole of SiO2
- Molar volume of SiO2 $= \frac{Molar\ mass}{Density} = \frac{60.08}{2.2} = 27.31 \frac{\mathrm{cm}^3}{\mathrm{mol}}$
- Therefore, number of O2 molecules per unit volume SiO2

$$\frac{6.022 * 10^{23} * 2}{27.31} \approx 4.40 * 10^{22}\ molecules$$

Hence $\boldsymbol{N_{H_2O} = 4.40 * 10^{22}}\ \frac{\textbf{molecules}}{\boldsymbol{Volume_{\{SiO2\}}}}$

L12- S7

- The given equation is : $\mathrm{C(x,t) = C(0,t) * e}^{-\frac{x^2}{L_d^2}}$, $C(0,t) = \frac{Q}{2\sqrt{\pi Dt}}$, $L_d = 2\sqrt{Dt}$;
- Fick's second law of diffusion states : $\frac{\partial C}{\partial t} = \frac{\partial}{dx}\left(D\frac{\partial C}{\partial x}\right)$
- Therefore , giving the equation as input to the law we get:

- LHS = $\frac{\partial\, C(0,t)e^{-\frac{x^2}{L_d^2}}}{\partial t} = \frac{\partial\left[\frac{Q}{2\sqrt{\pi Dt}}\, e^{-\frac{x^2}{4Dt}}\right]}{\partial t} = \frac{Q}{2\sqrt{\pi Dt}} * \frac{x^2}{4Dt^2} * e^{-\frac{x^2}{4Dt}} - \frac{Q}{2\sqrt{\pi D}} * \frac{1}{2t\sqrt{t}} * e^{-\frac{x^2}{4Dt}} = \boldsymbol{\frac{Q}{4t\sqrt{\pi Dt}}\left(\frac{x^2}{2Dt} - 1\right) e^{-\frac{x^2}{4Dt}}}$

- RHS = $\frac{\partial}{\partial x}\left(D\,\frac{\partial\left[\frac{Q}{2\sqrt{\pi Dt}}\, e^{-\frac{x^2}{4Dt}}\right]}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{-Q}{4t\sqrt{\pi Dt}} * x\, e^{-\frac{x^2}{4Dt}}\right) = \frac{-Q}{4t\sqrt{\pi Dt}}\left(e^{-\frac{x^2}{4Dt}} - x * \frac{2x}{4Dt} * e^{-\frac{x^2}{4Dt}}\right) = \boldsymbol{\frac{Q}{4t\sqrt{\pi Dt}}\left(\frac{x^2}{2Dt} - 1\right) e^{-\frac{x^2}{4Dt}}}$

Hence LHS = RHS , the given equation satisfies Fick's second law of diffusion.

L12- S10

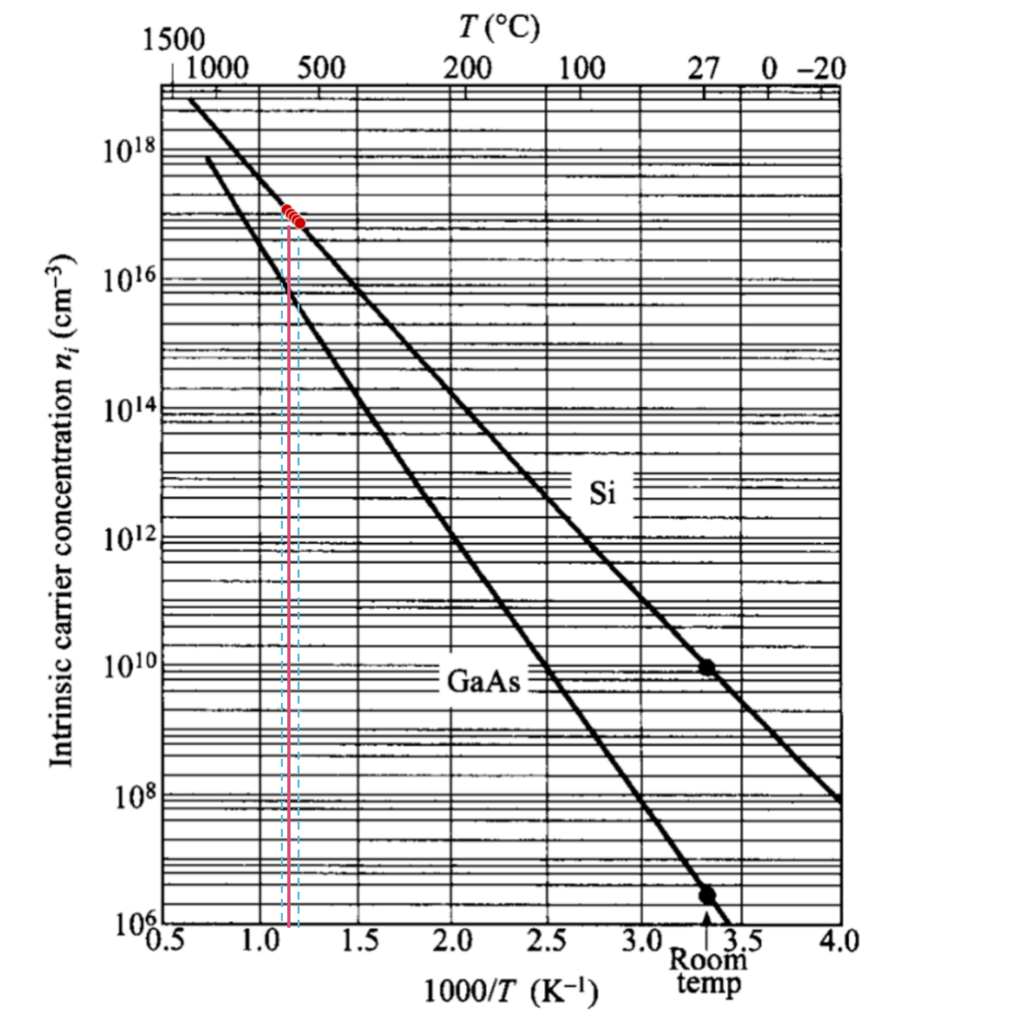
I will solve for a system with infinite source at surface of semiconductor with concentration at source C_0 :

- Fick's second law of diffusion states : $\frac{\partial C}{\partial t} = \frac{\partial}{dx}\left(D\frac{\partial C}{\partial x}\right) \rightarrow \frac{\partial C}{\partial t} = D\frac{\partial^2 C}{dx^2}$
- Boundary conditions :
  - At time t=0, the concentration is zero everywhere except at the source: $C(x,0) = 0 \ \forall\, x > 0$
  - At x=0, the concentration is constant : $C(0,t) = \mathrm{C}_0 \ \ \forall\, t > 0$
  - As x approaches infinity, the concentration approaches zero : $C(x,t) \ \rightarrow 0 \ \ for\ x \rightarrow \infty$
- Consider a solution : $C(x,t) = C_0 f\left(\frac{x}{\sqrt{4Dt}}\right)$ $where$ f(η) is a function of the similarity variable η= $\frac{x}{\sqrt{4Dt}}$
- Substituting this into the diffusion equation: $\frac{\partial C}{\partial t} = C_0\left(\frac{\partial f}{d\eta} * \frac{\partial \eta}{dt}\right) = C_0\left(\frac{\partial f}{d\eta} * \left(-\frac{x}{2\sqrt{4Dt^3}}\right)\right)$; $\frac{\partial^2 C}{dx^2} = C_0 * \frac{\partial^2 f}{d\eta^2} * \frac{1}{4Dt}$
- Substituting these into the diffusion equation : $C_0 * \left(-\frac{x}{2\sqrt{4Dt^3}} * \frac{\partial f}{d\eta}\right) = D * \frac{\partial^2 f}{d\eta^2} * \frac{C_0}{4Dt} \Rightarrow \ -2\eta\frac{\partial f}{d\eta} = \frac{\partial^2 f}{d\eta^2}$
- This is a 2nd ordinary differential equation (ODE). General solution is : $f(\eta) = A * \mathrm{erf}(\eta) + B$ $where$ $\mathrm{erf}(\eta)$ $is\ the\ error\ function$
- Using boundary condition:
  - At x=0, η=0, $C(0,t) = \mathrm{C}_0 \quad \Rightarrow \quad \mathrm{f}(0) = 1 \quad \Rightarrow \quad \mathrm{f}(0) = \mathrm{A} * \mathrm{erf}(0) + \mathrm{B} = \mathrm{B} \quad \Rightarrow \quad \mathrm{B} = 1$
  - As $x \rightarrow \infty, \eta \rightarrow \infty$, $C(\mathrm{x},t) = 0 \quad \Rightarrow \quad \mathrm{f}(\eta) = 0 \quad \Rightarrow \quad \mathrm{f}(\infty) = \mathrm{A} * \mathrm{erf}(\infty) + 1 = 0 \quad \Rightarrow \quad \mathrm{A} = -1$
- Therefore the solution is

$$\boxed{C(x,t) = C_0\left(1 - \mathrm{erf}\left(\frac{x}{\sqrt{4Dt}}\right)\right)}$$

EXERCISE ANSWERS

L12- S11



I assumed completion ionization of dopant atoms. So the doping concentration would be 10^17 cm^-3. Using webplot digitizer I found the approximate point where we get this concentration.

Temperature = 1000/1.19 = 840.336 K

The answer can have an error of $\pm$3 K

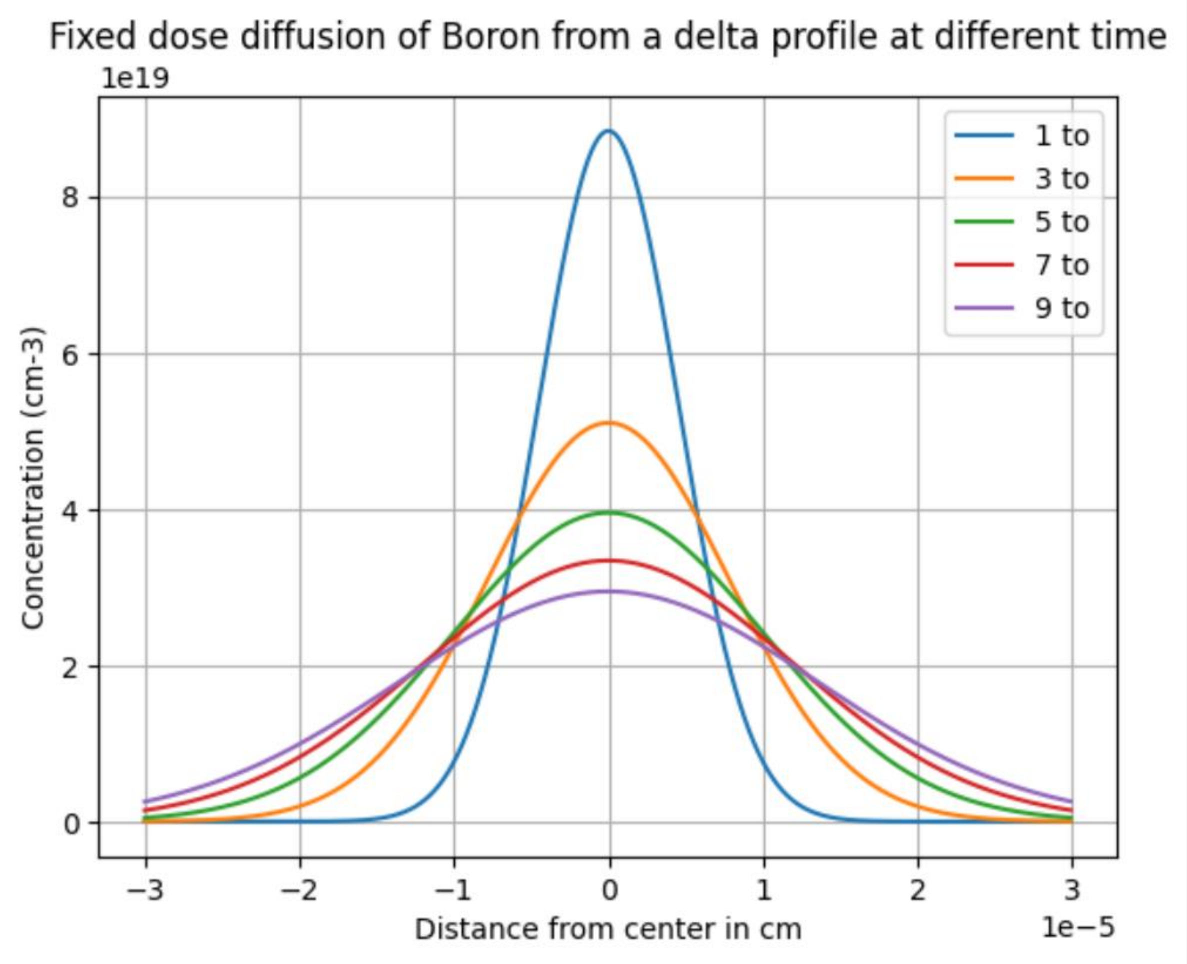
1000/T	Conc(cm-3)
1.199,	16.985
1.184,	17.031
1.169,	17.077
1.154,	17.123
1.132,	17.192

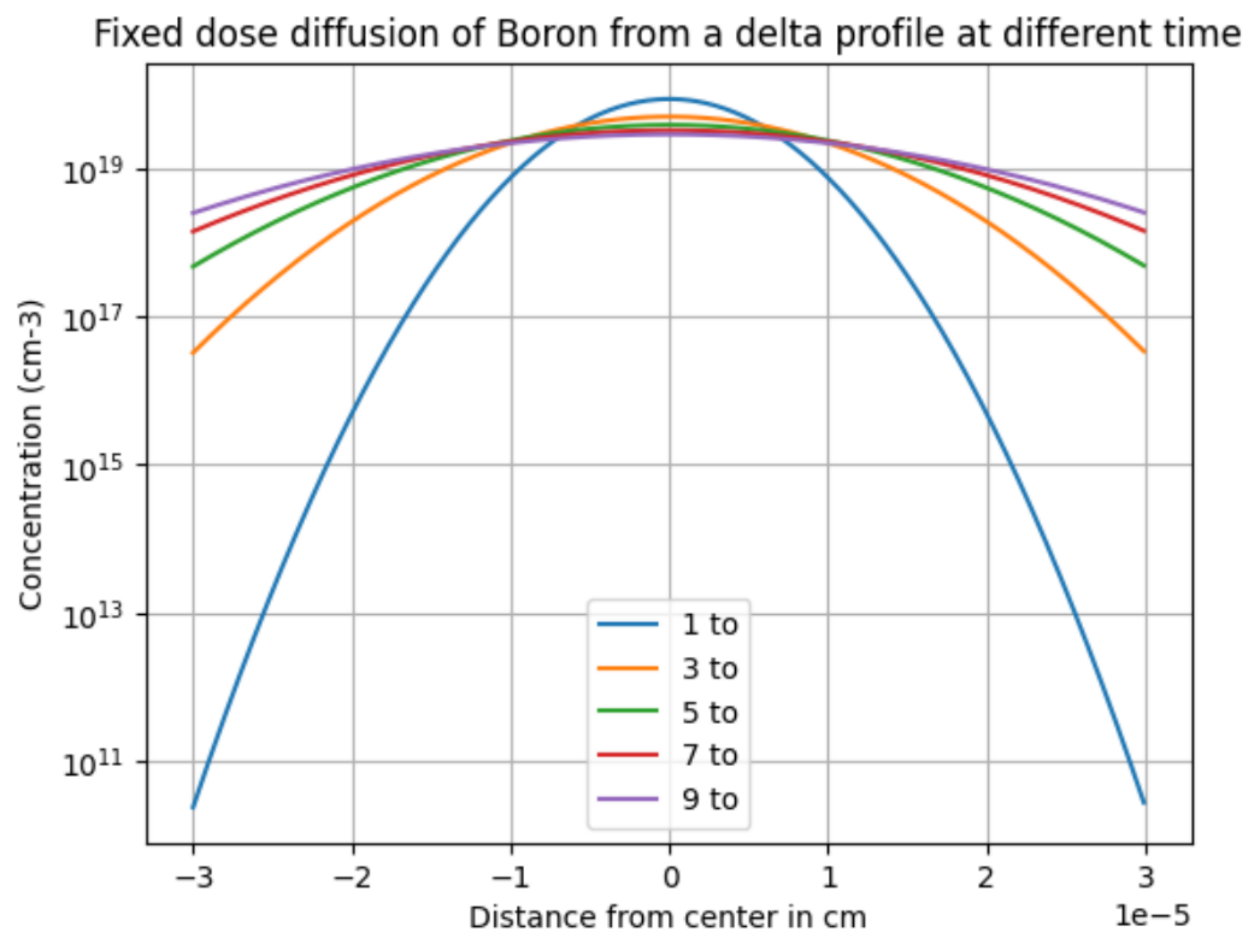
EXERCISE ANSWERS

L12- S13

1) The equation is given as : $C(x,t) = \frac{Q}{L_d\sqrt{\pi}} * e^{-\frac{x^2}{L_d^2}}$ and parameters are given: Q = 1e15 cm-2, T = 1000℃, t = 30 min

Therefore D0 = 1.0, Ea = 3.5 eV, D = 5.67e-15, t0 = 1800 sec, Ld = 6.39e-6 cm





EXERCISE ANSWERS

L12- S13

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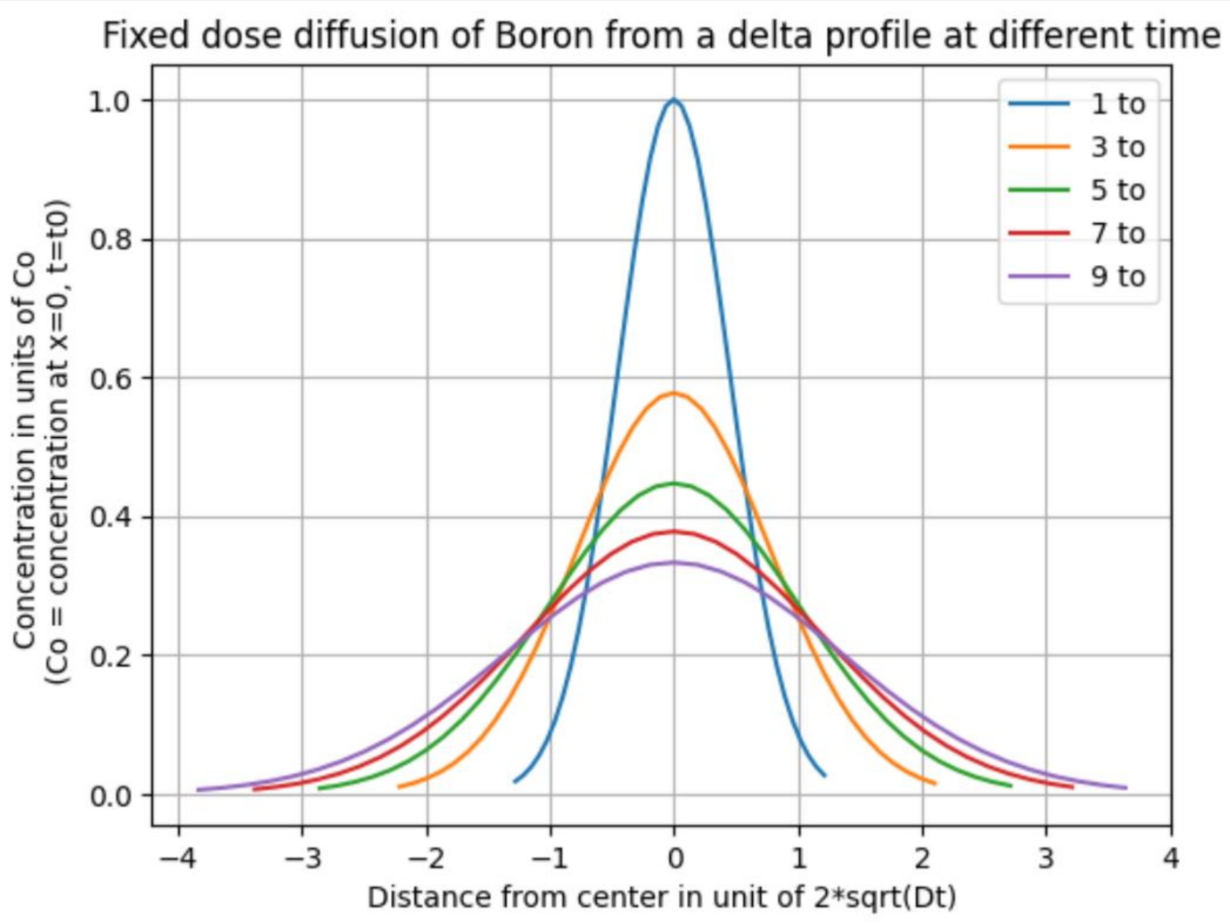
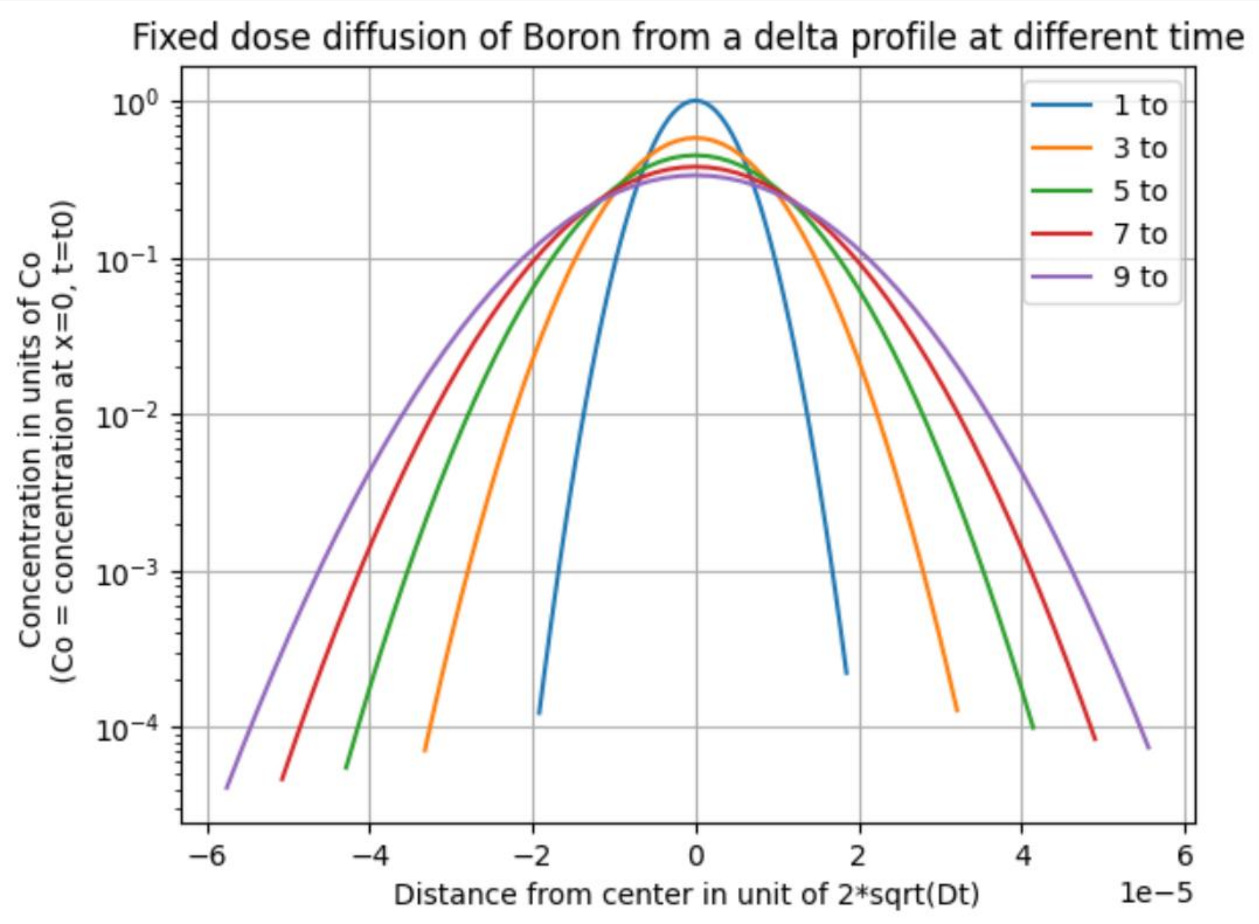
Normalized plots :





EXERCISE ANSWERS

L12- S13

2)

We know that $D = D_0 e^{\left(-\frac{E_a}{kT}\right)}$

Now Ea = 3.5 eV, T = 900 C, therefore kT = 0.025 * 1173 / 298 = 0.098 eV and D0 = 1

Therefore $D = D_0 e^{\left(-\frac{E_a}{kT}\right)} = 1 * e^{-\frac{3.5}{0.098}} = 3.09 * 10^{-16}\ cm^2/s$

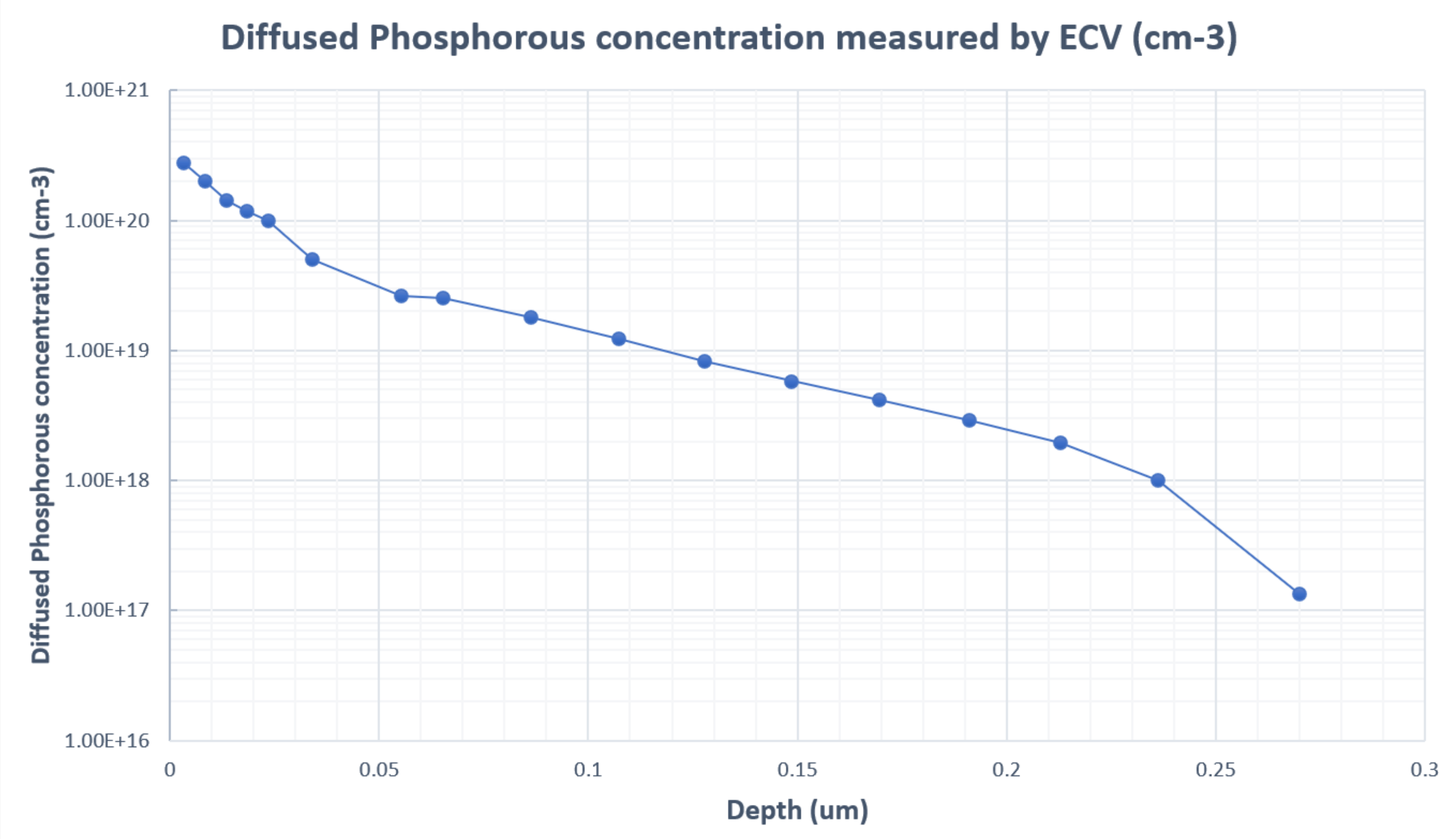
Hence for the same thermal budget Dt should be same

$$D_1 t_1 = D_2 t_2 \Rightarrow t_2 = \frac{D_1 t_1}{D_2} = \frac{\left(5.76 * 10^{-15} * 1800\right)}{3.09 * 10^{-16}} = \mathbf{33533}\ \boldsymbol{s} = \mathbf{559.22\ min} = \mathbf{9.32}\ \boldsymbol{hrs}$$

The diffusion time is 9.32 hrs.

EXERCISE ANSWERS

L13- S29 : Additional problem : Concept of sheet resistance



Depth (um)	Diffused Phosphorous concentration (cm-3)
0.0032	2.74E+20
0.0085	2.01E+20
0.0135	1.41E+20
0.0184	1.17E+20
0.0236	9.96E+19
0.034	5.00E+19
0.0552	2.60E+19
0.0653	2.51E+19
0.0863	1.81E+19
0.1072	1.23E+19
0.1277	8.25E+18
0.1486	5.80E+18
0.1696	4.16E+18
0.191	2.92E+18
0.2128	1.94E+18
0.2362	1.00E+18
0.27	1.34E+17

EXERCISE ANSWERS

L13- S29 : Additional problem : Concept of sheet resistance

The sheet resistance is calculated in two cases, one with background doping and one without background doping
The background dopant is Boron with a constant concentration of 1e16/cm3 along the depth of the substrate and provides resistivity of 1.45 Ω.cm

Model	Sheet resistance without background dopant (Ω/sq)	Sheet resistance without background dopant (Ω/sq)
Klaassen1992	107.97	108.03
Arora1982	89.09	89.13
Dorkel1981	96.33	96.38