

1a.

The current density equations

$$J_N = qn\mu_n\epsilon + qD_n \frac{\partial n}{\partial x}; \quad J_P = qp\mu_p\epsilon + qD_p \frac{\partial p}{\partial x}$$

The Poisson's equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{q}{\epsilon_{Si}}(N_D + p(x) - n(x) - N_A)$$

The Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_N}{\partial x} + G_N - R_N; \quad \frac{\partial p}{\partial t} = \frac{1}{q} \frac{\partial J_P}{\partial x} + G_P - R_P$$

The current density relation with the Quasi Fermi potential

$$J_N = n\mu_N \frac{\partial E_{FN}}{\partial x}; \quad J_P = p\mu_P \frac{\partial E_{FP}}{\partial x}$$

In steady state, the continuity equation reduces to

$$0 = \frac{1}{q} \frac{\partial J_N}{\partial x} + G_N - R_N; \quad 0 = \frac{1}{q} \frac{\partial J_P}{\partial x} + G_P - R_P$$

Since for every electron lost/generated due to recombination/generation, there is a corresponding hole lost/generated also,

$$G_N = G_P; \quad R_N = R_P$$

Therefore

$$\frac{1}{q} \frac{\partial (J_N + J_P)}{\partial x} = 0$$

or,

$$J_{Total} = J_N(x) + J_P(x) = constant$$

In other words, the net current flowing through the device is the same everywhere.

Since the current is the same everywhere, one can choose the region within the device for calculation of current-voltage characteristics.

All the holes that are injected at $x = -x_P$ reach the point $x = x_N$ so that

$$J_P(x_N) = J_P(-x_P)$$

Similarly all the electrons that are injected at $x = x_N$ reach the point $x = -x_P$, so that

$$J_N(x_N) = J_N(-x_P)$$

The minority current is largely diffusive, so

$$J_p(x) = J_p(\text{diff}) \text{ for } x \geq x_N$$

$$J_N(x) = J_N(\text{diff}) \text{ for } x \leq -x_P$$

So

$$J_{Total} = -qD_P \frac{\partial p}{\partial x / x=x_N} + qD_N \frac{\partial n}{\partial x / x=-x_P}$$

The task of computing the currents boils down to the computation of minority carrier profiles: p(x) in N-region and n(x) in P-region

We finally obtain from the minority carrier diatribution equation,

$$p_n(x) = P_{n0} + P_{n0} \left[\exp\left(\frac{eV}{kT} - 1\right) \right] \exp\left(\frac{x_n - x}{L_p}\right)$$

$$n_p(x) = n_{p0} + n_{p0} \left[\exp\left(\frac{eV}{kT} - 1\right) \right] \exp\left(\frac{x_p + x}{L_n}\right)$$

where $L_p = \sqrt{D_p \cdot \tau_p}$ and $L_n = \sqrt{D_n \cdot \tau_n}$

Using this, we calculate

$$J_p(x_n) = \frac{eD_p P_{n0}}{L_p} \left[\exp\left(\frac{eV}{kT} - 1\right) \right]$$

The total current is therefore

$$J_{Total} = J_P(x_N) + J_N(-x_P)$$

or,

$$\mathbf{J}_{Total} = \left[\frac{eD_p P_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right] \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

The reverse bias current is obtained similarly, where a negative value of V is introduced

$$p_n(x) = P_{n0} + P_{n0} \left[\exp\left(\frac{-eV}{kT} - 1\right) \right] \exp\left(\frac{x_n - x}{L_p}\right)$$

We have $\Delta p_n = -p_n$

Similarly, $\Delta n_p = -n_p$

Thus for a reverse bias of more than a few tenths of a volts, the minority carrier concentration at each edge of the transition regin becomes essentially 0 as the excess carrier concentration approaches the negative of the equilibrium concentration. this reverse bias depletion can be thought of as **minority carrier extraction**, analogous to the injection of forward bias.

In reverse bias, the quasi fermi levels split in the opposit sense than in the forward bias. In the depletion region, we have

$$pn = n_i^2 e^{\frac{E_{F_n} - E_{F_p}}{kT}} \approx 0$$

The Quasi fermi levels in reverse bias can go inside the bands.

The reverse current would therefore be:

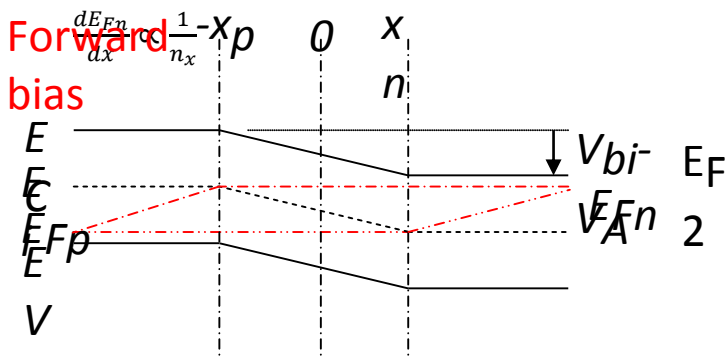
$$\mathbf{J}_{Total} = \left[\frac{eD_p P_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right] \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

or

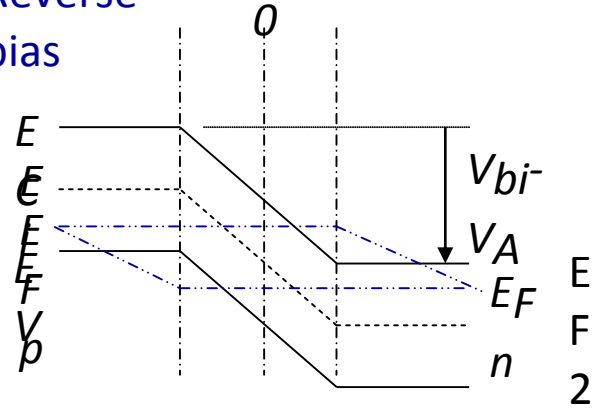
$$\mathbf{J}_{Total} \approx - \left[\frac{eD_p P_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right] \text{ or } \mathbf{J}_{Total} \approx -\mathbf{J}_s$$

1b. 2a. 2b.

Forward bias

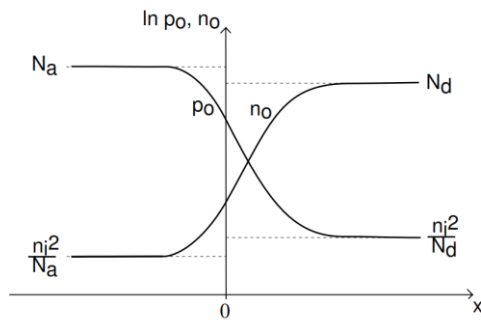


Reverse bias

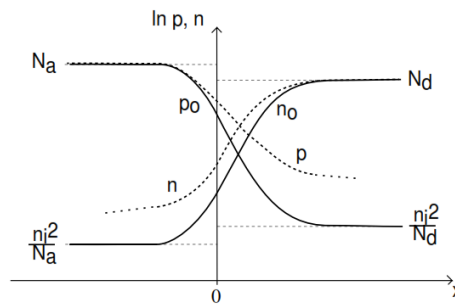


The fermi level according to charge carrier profile is:

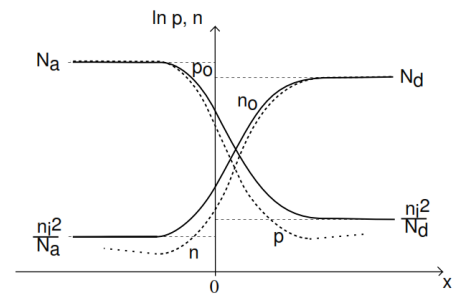
However the actual charge carrier profile is not consistent in the quasi neutral region which would result in a linear variation of fermi levels. Thus, the fermi levels need correction according to the actual charge carrier distribution.



Equilibrium



Forward Bias



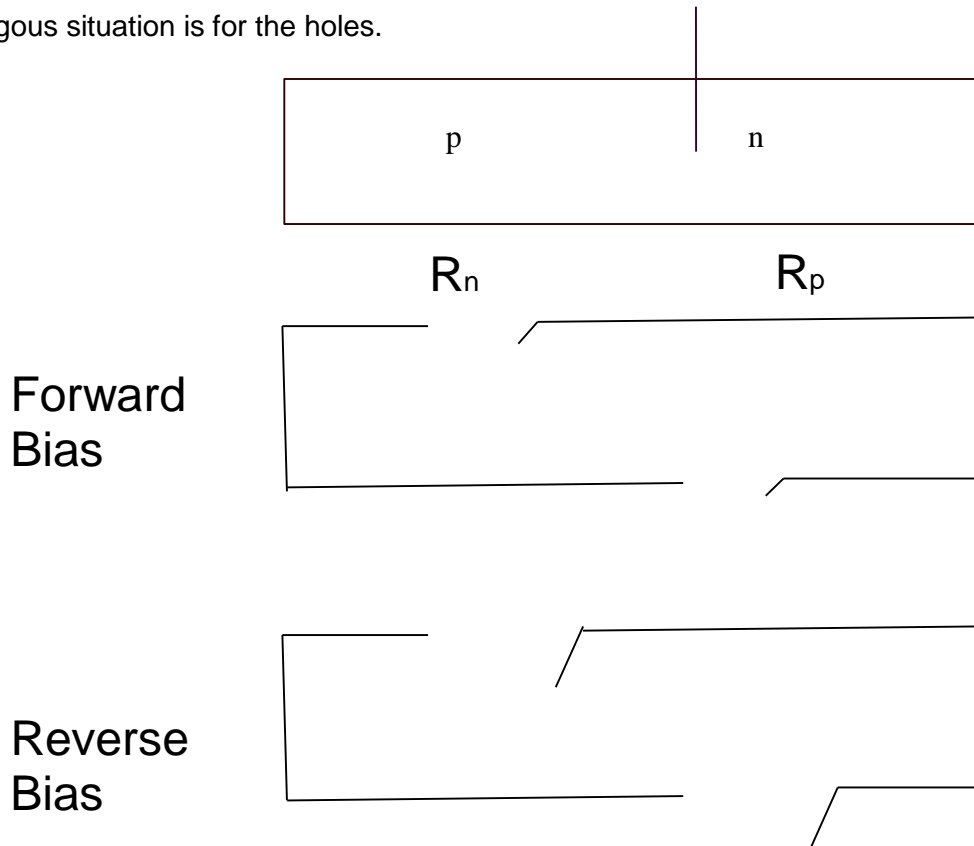
Reverse Bias

1c.

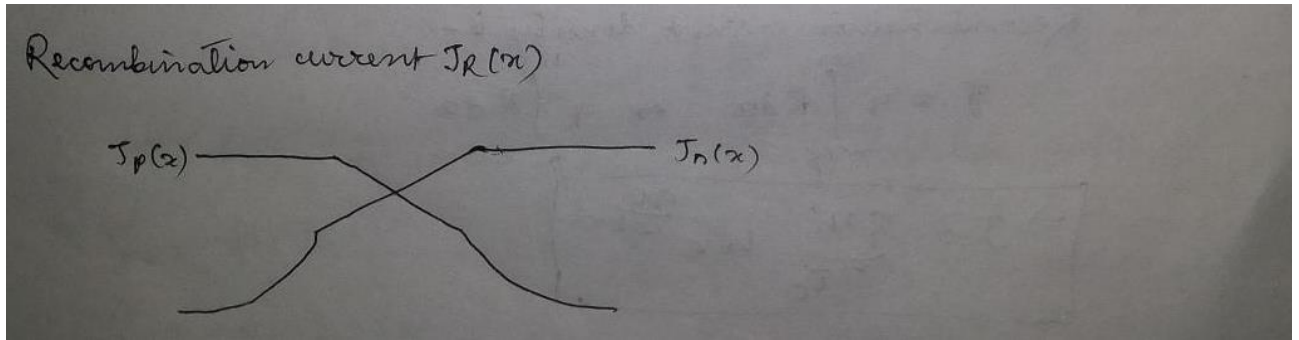
In the forward bias condition, electrons from n-side can diffuse easily to the p-side due to lowered potential barrier at depletion region. Electrons sweeping from p-side to n-side is significantly lower. Thus there is low resistance to electrons towards p-side.

In reverse bias, electrons at n-side are almost completely blocked from diffusing to the p-side. A minor current exists due to electrons from p-side being swept to n-side. In this case, there is higher resistance to electrons towards p-side.

Analogous situation is for the holes.



2c.



Recombination is maximum at centre of depletion region.

When the minority carriers begin to approach doping concentration, further increase in applied voltage gives rise to high level injection. This causes perturbation in majority carriers so as to achieve charge neutrality. This is observed as ' $2kT$ ' dependence of the current.

Obtaining Recombination Current:

$$R-G|_{\text{thermalSRH}} = -\frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

$$I_{R-G} \cong qA \int_{-x_p}^{x_n} (G-R)|_{\text{thermalSRH}} dx$$

In forward bias, $np \gg n_i^2$ (so recombination current)
 Recombination rate,
 $\therefore R \cong \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$

Also we have $np = n_i^2 (e^{qV/kT} - 1)$ (V = forward bias voltage)



At center point C, $n_C = p_C = n_i \sqrt{e^{qV/kT} - 1} \approx n_i e^{qV/2kT}$ (Law of junction)

Also, assuming the trap level situated at middle of the bandgap,

$$\begin{aligned} n_1 = p_1 = n_i & \quad a^2 - b^2 = (a+b)(a-b) \\ \Rightarrow R \cong \frac{n_i (e^{qV/2kT} - 1)}{2\tau_0 (e^{qV/2kT} + 1)} & \quad \text{where } \tau_0 = \frac{1}{2} \left(\tau_p \frac{n_1}{n_i} + \tau_n \frac{p_1}{n_i} \right) \\ \Rightarrow R \cong \frac{n_i}{2\tau_0} (e^{qV/2kT} - 1) & \quad = \frac{\tau_p + \tau_n}{2} \end{aligned}$$

$$\therefore R = \frac{n_i}{2\tau_0} e^{qV/2kT}$$

\therefore Recombination current density is -

$$J = q \int_{-x_p}^{x_n} R dx \quad \text{or} \quad q \int_{-x_p}^{x_n} R dx$$

$$\Rightarrow J = \frac{q n_i}{2\tau_0} W e^{qV/2kT}$$

Maximum recombination in forward bias is at the center C

When the recombination current is equal to ideal current

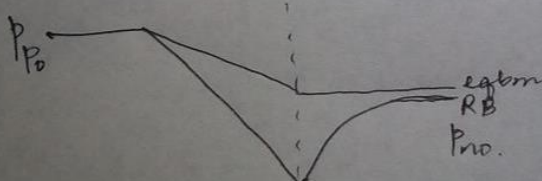
*
 Recombination current = Ideal current.

$$\Rightarrow \frac{q A n_i}{2 \tau_0} w e^{\frac{q V_A}{2 k T}} = I_0 e^{\frac{q V_A}{k T}}$$

$$\Rightarrow e^{\frac{q V_A}{2 k T}} = \frac{q A n_i w}{2 \tau_0 I_0}$$

$$\Rightarrow V_A = \frac{2 k T}{q} \ln \frac{q A n_i w}{2 \tau_0 I_0}$$

d) In reverse bias



Max. generation takes place in depletion region.

2d. Maximum generation occurs in depletion region.

Question 3.

Part a.

The fermi level

- p-doped side will be near the valance band
- n⁺-doped side near the conduction band
- Under no bias condition will be straight i.e. no bending as there is no net current (depends on gradient of fermi level)
- The rough quantitative estimate of the fermi level on both the sides can be obtained using Boltzmann approximation to the Fermi-Dirac statistics assuming that the acceptors/donors are completely ionised

On p-doped side

$$E_i - E_{fp} = k_B T \ln \left(\frac{N_a}{n_i} \right)$$

$$E_i - E_{fp} = 0.2879 \text{ eV}$$

On n-doped side

$$E_i - E_{fn} = k_B T \ln \left(\frac{N_d}{n_i} \right)$$

$$E_i - E_{fn} = 0.5757 \text{ eV}$$

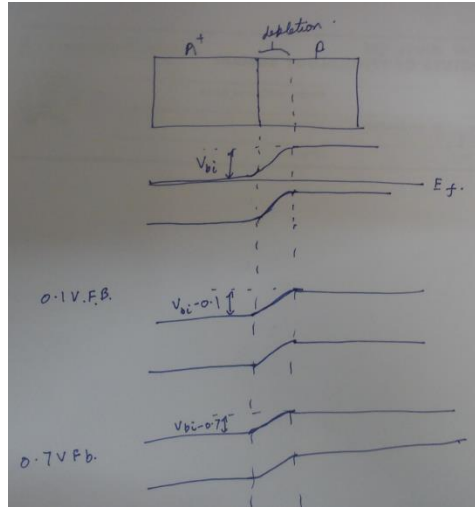
But for n-doped region $E_i - E_{fn} > \frac{E_g}{2}$, which is not correct. This means that our assumption that the donors are completely ionised is incorrect. But using this we can estimate that the fermi level in the n⁺ side almost touches the conduction band.

The built in voltage,

$$V_{bi} = \frac{k_B T}{q} \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$V_{bi} = 0.8636 \text{ eV}$$

Part b.

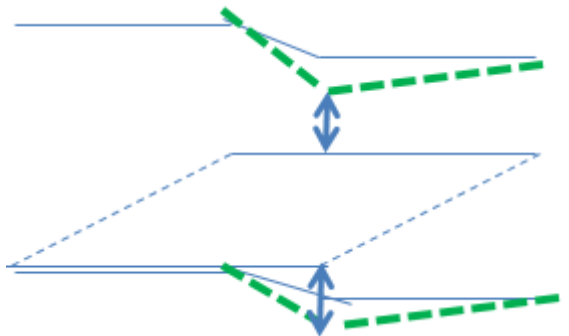


Part c.

- When an electric field is created the majority carriers will rearrange to cancel out the electric field in the quasi neutral region (concept of screening). In this case the holes (majority carriers in p-doped region) will be the responders. After the response of the carriers there will be no significant electric field.

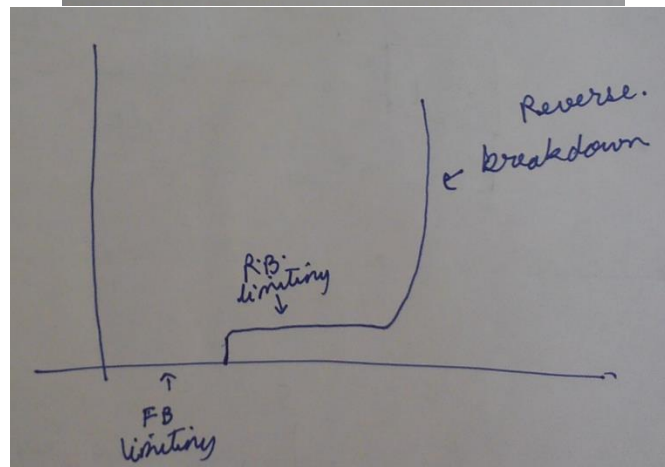
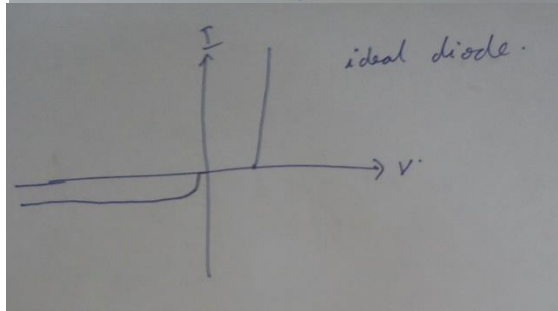
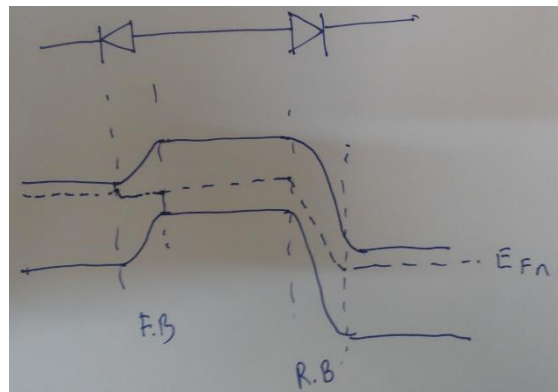
To significantly change the conduction band or fermi level we need to create a significant imbalance in the majority carriers. But in this case the imbalance is small because of the significantly small number of minority carriers injected as compared to the majority carriers, so no change.

- (refer to lecture slides lecture 13) In high bias the number of minority carriers will be large (high level injection), the screening effect will be similar to part a, but since this high level injection will create a significant imbalance the conduction band and fermi level may bend. The extent of bending will be determined by the amount of change in majority carrier because of screening.
 - As bandgap is constant the bending in conduction band will lead to bending of valence band.
 - This bending of bands will lead to lower number of minority carriers (responsible of current in F.B)
 - So the current will be lower than the ideal case that we normally use to derive current



Question 4.

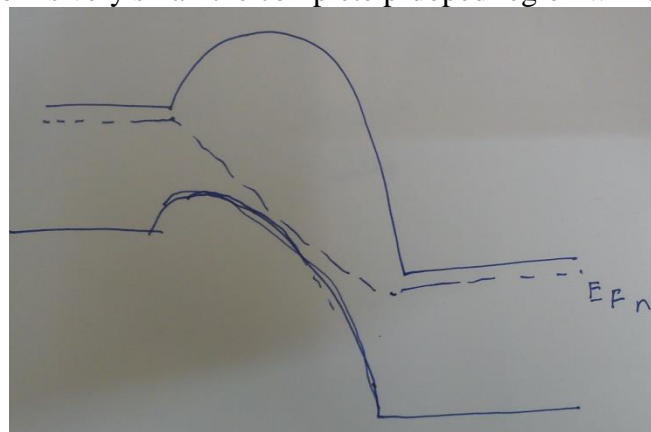
Part a.



- The primary drop in voltage will be across the reverse junction (think of it as nonlinear resistors). To get the IV characteristics we need to find the diode that will limit the current in each region and the total I-V characteristic will emulate the limiting diode.
- The relative difference in the fermi level at the two edges can be determined by the voltage drop across the junction (remember assumption - no voltage drop in the bulk)

Part b.

Now if the p-doped region is very small the complete p-doped region will be depleted.



- Since the voltage drop is across the entire P-doped region (depletion region) we cannot visualise this device as 2 diodes in series.
- the quasi fermi level will drop across the complete p region
- since the electron current source are the electrons above the barrier, the x dependence of electrons will be constant (visualise as, since the electrons cross the barrier the electric field in depletion will carry them forward)

