Using Quasi-Fermi-Level description for "Big Picture" device analysis – *pn* junction

Udayan Ganguly Department of Electrical Engineering IIT Bombay

The Trouble with Drift Diffusion Analysis

Drift current is due to electric field-$J_{drift} = q\mu nF$

Diffusion current is due to concentration gradient $J_{diff} = qD\frac{dn}{dx}$

Drift and diffusion are difficult to separate. There are two examples

Drift / Diffusion in a Metal rod:

For example, given a current driven through a metal rod by a potential difference V (Figure 1), what is the maximum current that a diffusion or drift current can push?

The maximum drift current by crude estimate is $J_{drift} = q\mu nF$ where $F = V/L$ i,e, voltage V applied across its length L.

The maximum diffusion current by crude estimate is $J_{diff} = qD\frac{dn}{dx} \approx qD\frac{n}{L}$ as shown in Figure 1; where $max\left(\frac{dn}{dx}\right) \approx \frac{n}{L}$ our estimate. It means if one end is emptied of carriers i.e. $n = 0$ while the other end retains its original carrier concentration, then the concentration gradient across the length L is given by n/L;

In this case, $J_{diff} \geq J_{drift}$ ➔ $qD\frac{n}{L} \geq q\mu nF$ ➔ $\frac{D}{\mu}/L = F$ ➔ $V_{Th} \geq FL = V_{app}$

i.e. ***Diffusion (for a linear gradient) dominates when applied bias V_{app} is less than thermal voltage V_{th} which is tiny*** (i.e. 26mV at room temperature)

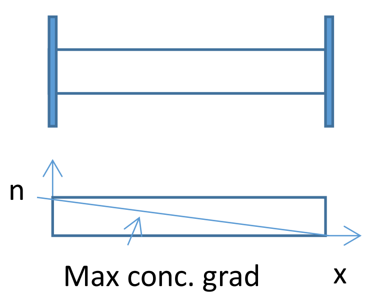

Figure 1: Metal Rod of Length (L) with carrier concentration (n) and applied bias V

Current in pn junction under equilibrium is zero implies Fermi level is flat:

in the depletion region of a pn junction the drift and diffusion balance each other under equilibrium.

$$J = J_{drift} + J_{diff} = 0$$

$$q\mu nF + qD\frac{dn}{dx} = 0$$

$$F = -\frac{D}{\mu}\frac{1}{n}\frac{dn}{dx}$$

$$F = -\frac{D}{\mu}\frac{dln(n)}{dx} = -V_{Th}\frac{dln(n)}{dx} \quad (1)$$

Where $V_{Th} = \frac{kT}{q}$ is the thermal voltage.

Thus for a drift by a finite field to be balanced by diffusion by a concentration gradient, the concentration gradient must be exponential.

Further, substituting Boltzmann distribution $\ln n = -\frac{E_c - E_F}{qV_{th}}$

$$V_{Th}\frac{dln(n(x))}{dx} = -F + \frac{1}{q}\frac{dE_F}{dx} \quad (2)$$

Substituting (2) in (1), equilibrium i.e. $J = 0$ implies $\frac{dE_F}{dx} = 0$; i.e. **Fermi level is Flat.**

Microscopic current balance based on Fermi level

Suppose at an energy ϵ, there are two locations left and right in the *pn* junction as shown inFigure 2. These locations have a difference number of states n_L and n_R (equivalent to different density of states within ϵ to $\epsilon + \delta\epsilon$). How can we show that if the Fermi level is flat, these two regions will not exchange net current?

Current going left to right is given by the product of three factors (i) number of filled states on the left ($n_L p_L$), (ii) number of empty states on the right ($n_R(1 - p_R)$) (iii) a proportionality constant (k).

$J_{L\to R} = kn_L p_L n_R(1 - p_R)$

Similarly

$$J_{R\to L} = kn_L(1 - p_L)n_R p_R$$

It is easy to show that if $J_{L\to R} = J_{R\to L}$, then it implies $p_L = P_R$

Now by Boltzmann's distribution or Fermi Dirac Distribution $p_L = p_R = f(\frac{\epsilon - E_F}{kT})$

Since ϵ and T are same on either side, then E_F must be the same. Hence, Fermi Level is flat.

Overall, another big observation is that ϵ can be any energy. Hence if E_F is flat, then at any energy ϵ, there is no net current. This is net zero current at any energy, anywhere.

Equilibrium band diagram

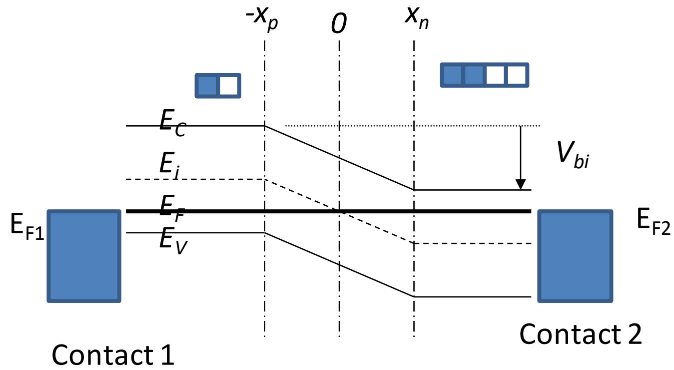

Figure 2 States of energy ϵ on the left (2 boxes) and right (4 boxes) in a pn junction.

So what is Quasi Fermi Level (compared to Fermi Level)?

Under equilibrium, the Fermi level determines simultaneous electron and hole occupation probability on the conduction and valance bands.

$$n = N_c \exp \frac{E_F - E_C}{kT}$$

$$p = N_v \exp \frac{E_V - E_F}{kT}$$

However, under applied bias (non-equilibrium), the electron and hole cannot be determined by a unique fermi level. Hence, a dedicated fermi level for electrons (E_{Fn}) and for holes (E_{Fp}) is used to determine n and p respectively. E_{Fn} has no effect on $p(x)$. It only determines $n(x)$ given $E_c(x)$. On the other hand, E_F affects both $n(x)$ and $p(x)$ simultaneously. This is the major difference between E_F and E_{Fn}.

$$n = N_c \exp \frac{E_{Fn} - E_C}{kT}$$

$$p = N_v \exp \frac{E_V - E_{Fp}}{kT}$$

Quasi Fermi Level to the Rescue for the Drift-Diffusion Dilemma

To avoid the determination of dilemma between drift and diffusion or a more complex mixed drift & diffusion transport, the QFL analysis is a welcome simplification.

$$J_n = J_{drift} + J_{diff} = q\mu nF + qD\frac{n}{L} = \mu n \frac{dE_{Fn}}{dx}$$

The quasi Fermi level based definition of current transport of electrons is that it is simply drift which is pushed by the gradient of quasi fermi level dE_{Fn}/dx as opposed to pure drift which is pushed by electric field.

An Analogy

It is always troublesome to think in forces though it may be intuitive. For example, consider a ball released with zero velocity at x=0 on a crazy frictionless surface. The question is that we need to find out what is the equation of motion. There are two approaches (i) force balance ($\vec{G}$ and $\vec{N}$, see figure below), which in 2D will require vectorial calculations (ii) energy balance (E_{tot} is conserved) which will be a scalar calculation. Hence, scalar calculation is preferred. Similarly, instead of balancing forces (drift and diffusion), an energy balance (QFL) is preferred.

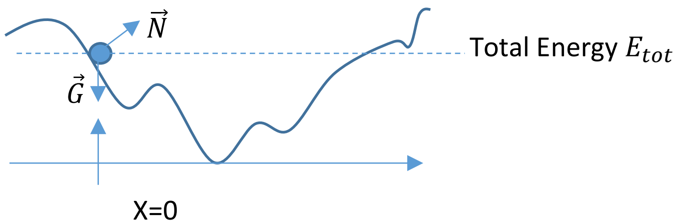

Figure 3: A ball released at zero velocity on an arbitrary surface under gravity will follow the surface. Local forces G and N are shown; total energy is conserved

Determination of QFL $E_{Fn}(x)$ in pn junction under forward bias

Step 1: Assume a small bias applied compared to equilibrium

Start by applying in small forward bias. Assume that the situation has not changed significantly compared to equilibrium where both FL was flat ($\frac{dE_F}{dx} = 0$) and electrostatics (i.e. $E_c(x)$) are known (Figure 4).

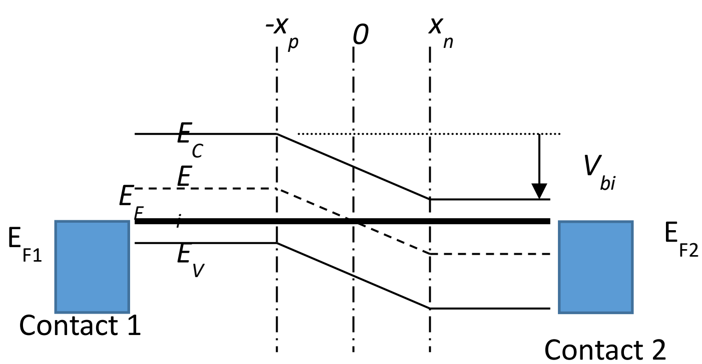

Figure 4: Band diagram in equilibrium where QFL is known and flat everywhere.

Step 2: Calculate n(x) and p(x)

Calculate the carrier profile n(x) and p(x) as shown in Figure 5 using

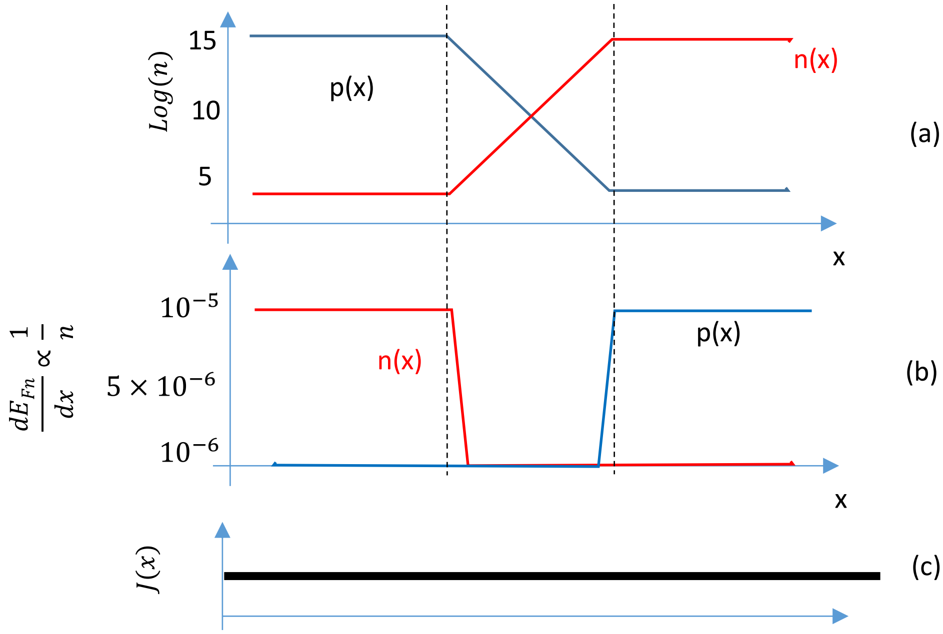

Figure 5: (a) Plot n(x) and p(x) in log scale and (b) translate to 1/n(x) and 1/p(x) in linear scale (c) given $J(x) = J_o$ is position independent constant, then we can use $\frac{1}{n(x)}$ and $\frac{1}{p(x)}$ to evaluate the gradient in E_{Fn} and E_{Fp} . This shows that E_{Fn} has maximum gradient in quasi-neutral minority carrier region while elsewhere the $\frac{dE_{Fn}}{dx} \approx 0$

Step 3: Determine the position where QFL is flat (and where it is not)

$J(x) = \mu n(x)\frac{dE_{Fn}(x)}{dx} = J_o$ for all x i.e. the current remains constant (Figure 5). This is equivalent to saying that in steady state the electron current going into the device, travels through the device and comes out without any loss. Otherwise, there will be charge storage and potential will become time dependent and hence violate steady state. This is current continuity, which is analogous to Kirchoff's Law in resistor networks. Thus

$$\frac{dE_{Fn}(x)}{dx} = \frac{J_o}{\mu n(x)}$$

As $n(x)$ is primarily (exponential) position dependent in RHS, given q ,$J_o$are constant, and μ may have a small dependence), it should contain all the position dependence of LHS.

We conclude that $\frac{dE_{Fn}}{dx}$ is non-zero & finite only in the minority carrier quasi-neutral region. Elsewhere it is negligible (i.e. zero). Hence the E_{Fn} will be flat everywhere except the minority carrier quasi-neutral region (Figure 6). Similar conclusions for E_{Fp} can be developed.

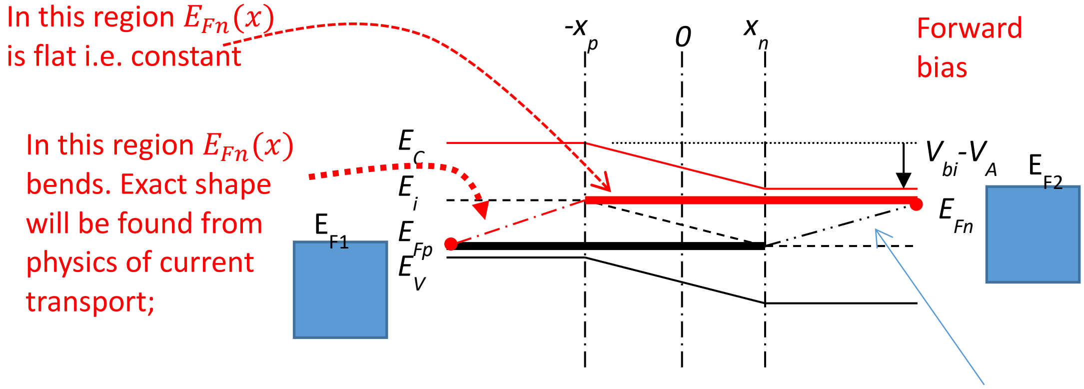

Figure 6: The $E_{Fn}(x)$ is flat everywhere in depletion and majority carrier quasi-neutral region as derived in Step 3. The only gradient is in minority carrier quasi neutral region. However the exact shape is not known yet. It emerges from solving the exact transport equation here. This happens to be diffusion as electric field $F = 0$ here.

Step 4. Perform detailed current calculation where QFL drops

The QLF drops in the minority carrier quasi-neutral region (see Figure 7). Thus, this region resists the current flow. Essentially, the contacts QFL are brought to the edges of this region.

Here the current needs to be accurately calculated. The detailed carrier profile can be now computed in the minority carrier quasi-neutral region. At the edge of the depletion region ($x = -x_p$), the carrier concentration is $n_p = n_{po}\exp\left(\frac{V}{V_{th}}\right)$; at the contact $n_p = n_{po}$; As there is no electric field, diffusion needs to be calculated.

$J_{diff}(x) = qD\frac{dn(x)}{dx} = J_o$ i.e. current is constant at all x in steady state. Hence $\frac{dn_p}{dx}$ is the only position dependent term for diffusion current. If $J(x) = J_o$ is independent of position, then $\frac{dn_p(x)}{dx}$ is constant or $n_p(x)$ in the minority carrier quasi-neutral region must be linear. Hence the diffusion current is given by the following.

$$J_{diff} = qD\frac{n_{po}\left(\exp\left(\frac{V_{app}}{V_{th}}\right) - 1\right)}{L}$$

This is akin to the diode current $I = I_o(\exp\left(\frac{V_{app}}{V_{th}}\right) - 1)$;

The linear $n_p(x)$ produces a logarithmic $E_{Fn}(x)$ given E_C is flat from Boltzmann distribution.

Step 5. Check self-consistency:

Given $E_{Fn}(x)$, re-calculate $n(x)$ and check if it is consistent earlier $n(x)$ as shown in Figure 7. For forward bias, this is true.

We will show that this is not true for reverse bias!

Such a method can be used to gently move from equilibrium to highly biased condition by gently updating the bias slowing iteratively like a computer program. These are called mental simulations.

Such a model will also work for reverse bias.

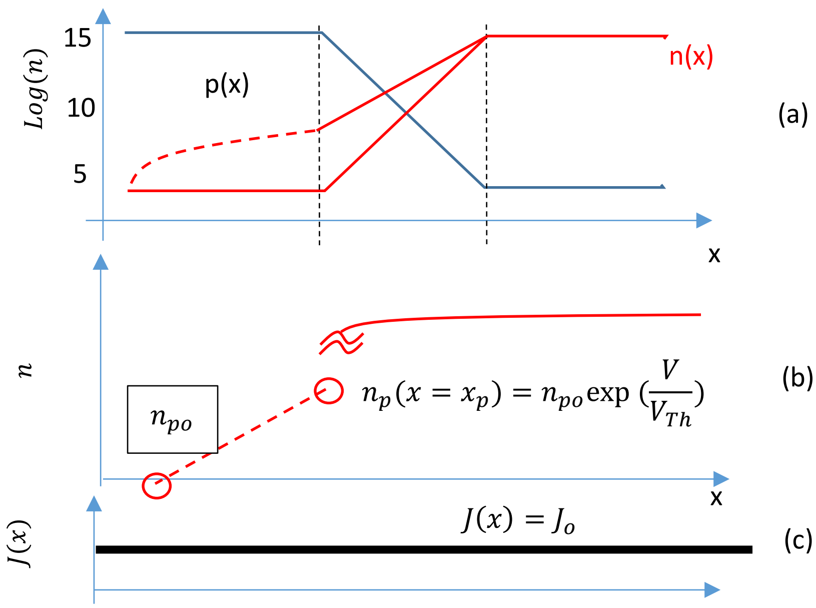

Figure 7 (a) Forward bias n(x) compared to equilibrium. The dashed portion is unknown yet – given $E_{Fn}(x)$ is not known in this region (minority carrier quasi neutral region) but $E_{Fn}(x)$ is flat elsewhere. (b) The carrier concentration at the edges of the minority carrier quasi neutral regions are known. It is also know that diffusion must occurs as the dominant transport mechanism as F=0. (c) Given J(x)=Jo constant, J(x)=Ddn/dx=constant; Hence n(x) is linear.

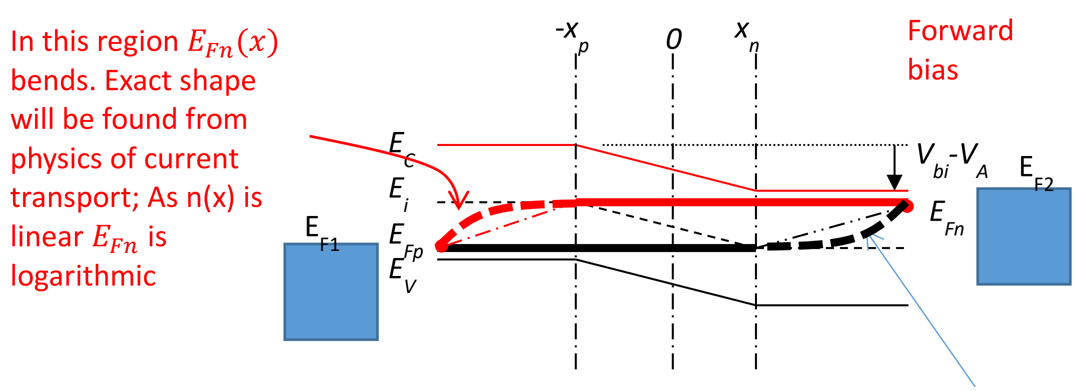

Figure 8: The unknown part of the $E_{Fn}(x)$ from Fig. 4 is filled in. Essentially $E_{Fn}(x)$ is now known (logarithmic in x), given $E_C(x)$ is flat and n(x) is linear.

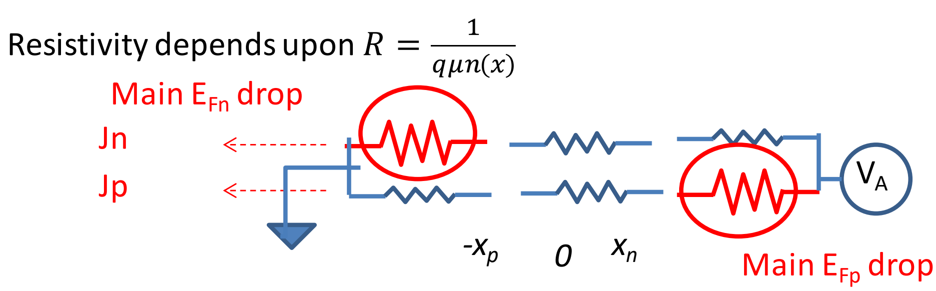

Figure 9: Given the drift based description for current transport by driven by gradient in QFL (akin to F), a resistor network equivalent can be drawn where local resistance depends upon n(x). This quickly shows that the QFL drops only in the minority

carrier quasi-neutral region- and requires detailed analysis. The rest of the regions are "shorts" and do not need any detailed analysis.

An analogy with a simple resistor network

The drift formalism of QFL based current flow reduces the differential equation based calculus to the simple resistor network where the local conductance $n(x)$ needs to be considered to identify the most resistive region where the QFL will drop (see Figure 9). This only the large red resistors are important. Rest are short to voltage source.

Exercise: Determination of QFL in pn junction under reverse bias

The reverse bias QFL and current derivation can proceed identical results! Check your answers with solutions below!

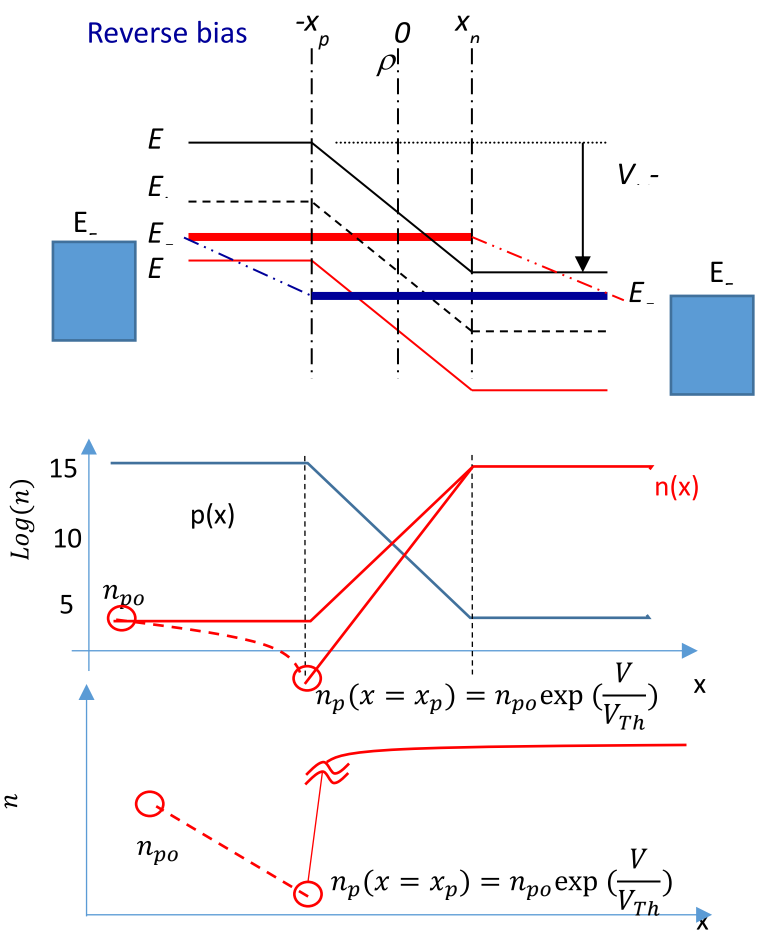

Figure 10: (a) Given the equilibrium n(x), $E_{Fn}(x)$ is flat everywhere except minority carrier quasi-neutral region (this analysis is same as forward bias except the sign of the applied bias). (b) new n(x) is calculated everywhere including edge of minority carrier

quasi-neutral region (c) dominance of diffusion current (given F=0) in minority carrier quasi-neutral region implies linear n(x). thus the dashed n(x) in updated in (b)

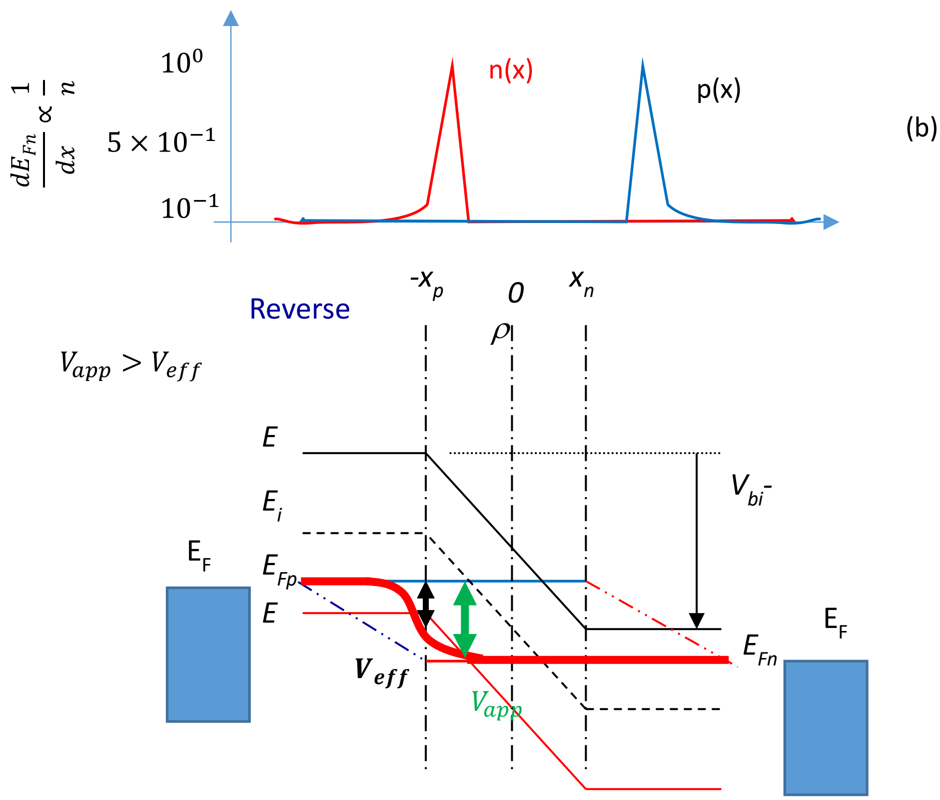

Figure 11 (a) $1/n(x)$ calculated from the carrier profile in Figure 10 is inconsistent with initial assumed $1/n(x)$ in Figure 5. (b) The new $1/n(x)$ show maximum gradient at the edge of depletion region which produces $E_{Fn}(x)$ as shown. This $E_{Fn}(x)$ is self consistent.

Does the error in $E_{Fn}(x)$ guess affect the current calculation in reverse bias?

Thus, the original $E_{Fn}(x)$ profile in Figure 10 will result in the current where $\frac{V_{app}}{V_{th}} \ll -1$

$$J_{diff} = qD\frac{n_{po}\left(\exp\left(\frac{V_{app}}{V_{th}}\right) - 1\right)}{L} \approx -qD\frac{n_{po}}{L}$$

where $\frac{V_{app}}{V_{th}} \ll -1$

However the new $E_{Fn}(x)$ profile in Figure 11 will result in the current

$$J_{diff} = qD\frac{n_{po}\left(\exp\left(\frac{V_{eff}}{V_{th}}\right) - 1\right)}{L} \approx -qD\frac{n_{po}}{L}$$

where $\frac{V_{eff}}{V_{th}} \ll -1$ even though $|V_{eff}|<|V_{app}|$

This error in the $E_{fn}(x)$ does not change the current magnitude as the J_{diff} is insensitive to the carrier concentration at the edge of the depletion region ($n_{po}\exp{(V/V_{th})}$) as long as it is negligible compared n_{po} i.e. $\frac{V}{V_{th}} \ll -1.$

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