# EE 724: Electrostatics L4

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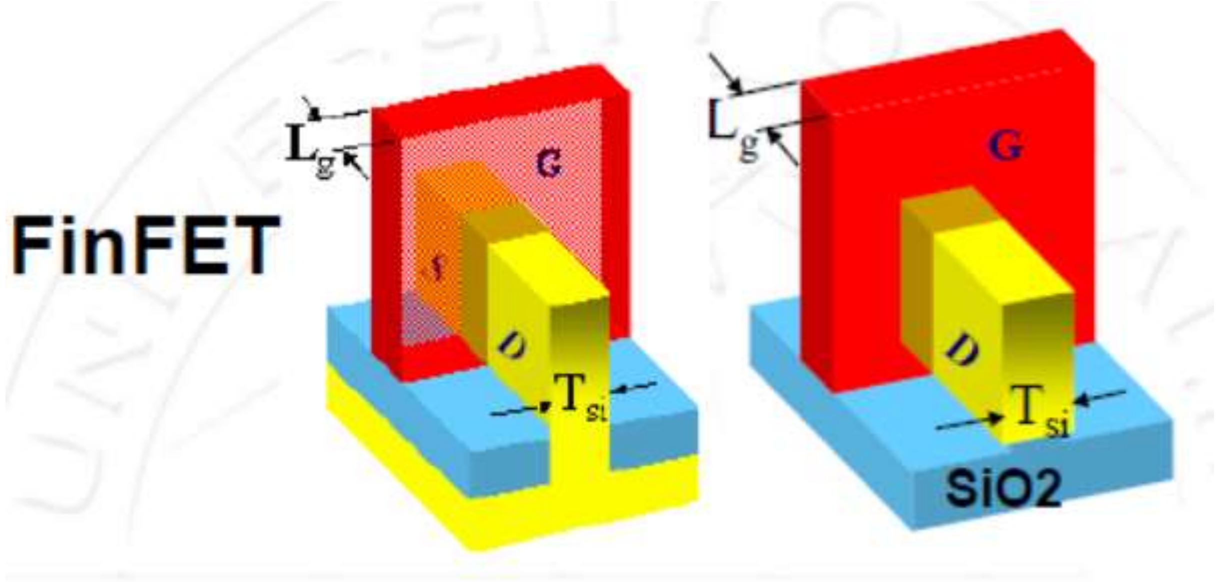
Based on Griffiths “Electrodynamics” Ch 2-4.

# Is FinFET a good switch?




FinFET on bulk Si and SOI

- What is  good switch?
- Only gate controlled non-linear resistor
- What is the efficiency of gate bias controlling the channel current?
- High
- What is the efficiency of drain bias controlling the channel current?
- Low

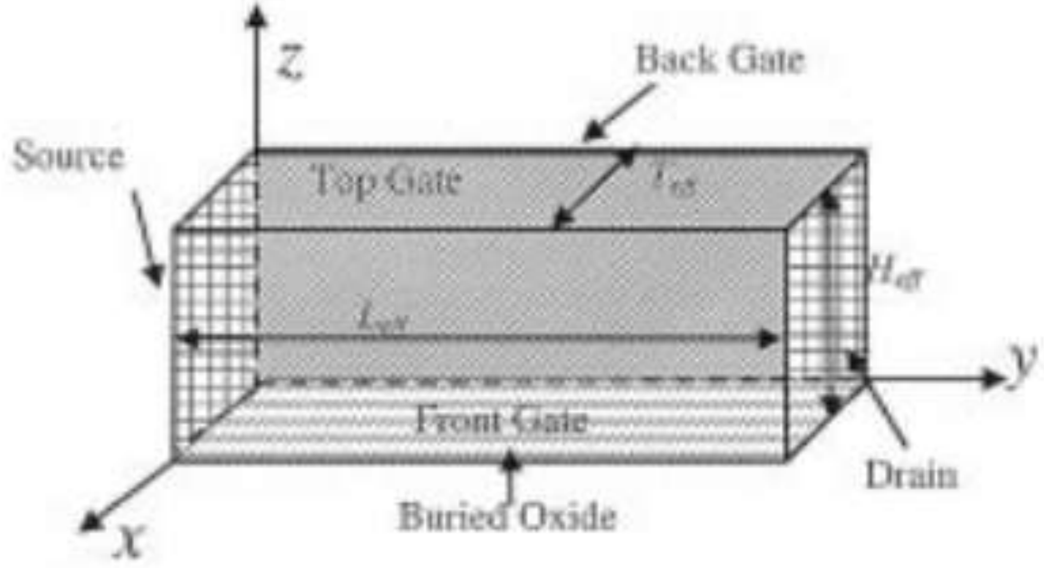
# Can we Solve Electrostatics?

Can we use Laplace instead of Poisson Equation?

- Yes if free charge is negligible ➔ off state

Can we solve in 2D?

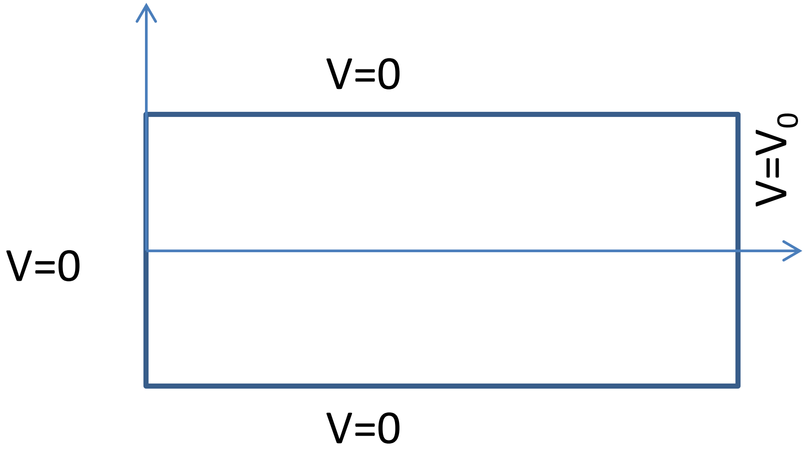
- Yes, if z direction is infinite then solve in XY plane



# Laplace Equation

- Review Ch 3.3 Griffiths Electrodynamics
- Laplace Equation is $\nabla^2 V = 0$
- Use Separation of Variable V(x,y)=X(x)Y(y)
- $Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} = 0$
- Dividing by XY
- $1/X\frac{d^2X}{dx^2} + 1/Y\frac{d^2Y}{dy^2} = 0$
- Equating each terms to $k^2$ and $-k^2$
- $\frac{d^2X}{dx^2}=k^2X$ ➔ solution is sinh (kx) and cosh (kx);
- $\frac{d^2Y}{dy^2}=-k^2Y$ ➔ Solution is sin(ky), cos(ky)

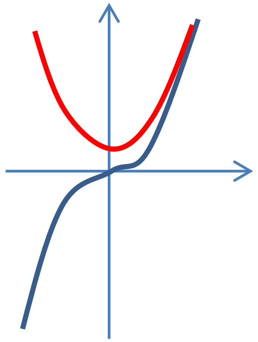
# Using boundary conditions



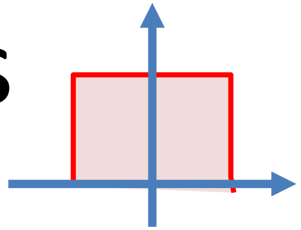

- V(x=0,y)=V(x,y=±a)=0
- V(x=b,y)=Vo

- Based on V(x,y=±a)=0
  - Due to symmetry about x-axis, cos (kx) is a solution (in comparison if antisymmetric it would be sin (kx))
  - Where $k_n a = \frac{\pi}{2}(2n+1)$ where n=0,1,2,…

- Based on V(x=0,y)=0 and V(x=b,y)=Vo
  - Due to asymmetry we can chose sinh (kx)



# Determining coefficients



- $V(x,y) = \sum_{n=0}^{inf} C_n cos(k_n y)\sinh(k_n x)$

To meet the b.c. V(x=b,y)=Vo & V(x=b,y=±a)=0 we use the Fourier decomposition

- $V(x,y) = \sum_{n=0}^{inf} B_n cos(k_n y)$ where $C_n \sinh(k_n x) = B_n$
- Multiplying $\int_{-a}^{a} cos(k_n{}' y)dy$ to either side
- RHS= $\int_{-a}^{a} V(x=b,y)cos(k_{n'}y)dy = \frac{V_o}{k_n} 2sink_n'a = (-1)^n \frac{2V_o}{k_n{}'}$
- LHS= $\int_{-a}^{a} B_n cos(k_n\ y)cos(k_{n'}y)dy$ =0 when$k_n \neq k_n'$

  =$\int_{-a}^{a} B_n cos^2(k_{n'}y)dy = \int_{-a}^{a} \frac{B_n}{2}(cos(2k_{n'}y) + 1)dy = B_n a$
- Hence $C_n = \frac{2Vo}{ak_n{}'\sinh(k_n b)}$

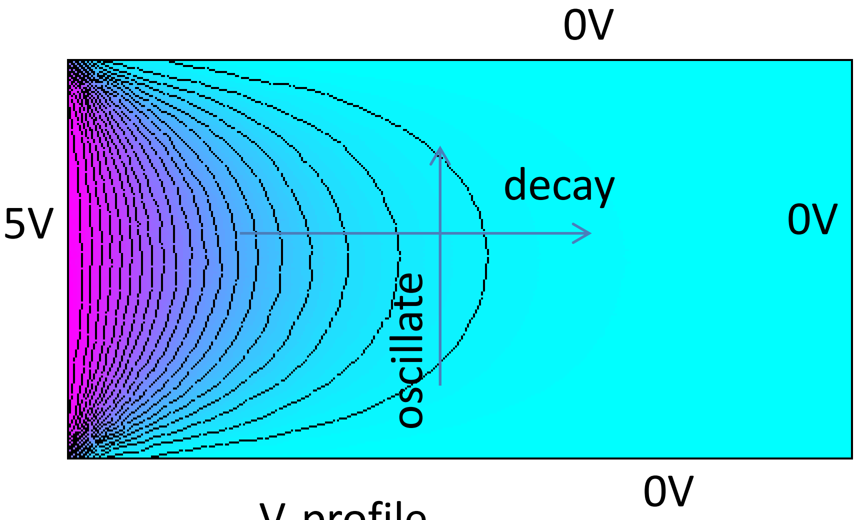
# Solution

$$V(x,y) = \sum_{n=0}^{inf} (-1)^n \frac{2Vo}{ak_n} cos(k_n y) \frac{\sinh(k_n x)}{\sinh(k_n b)}$$

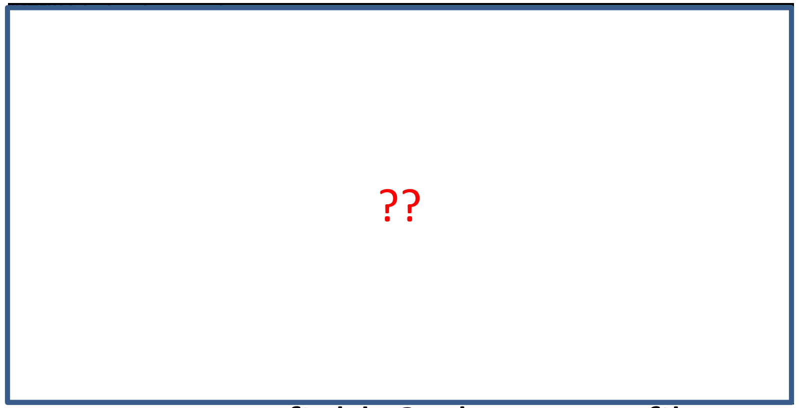
- Which is the most dominant terms in terms of furthest effect from the "active" electrode?

- Why?

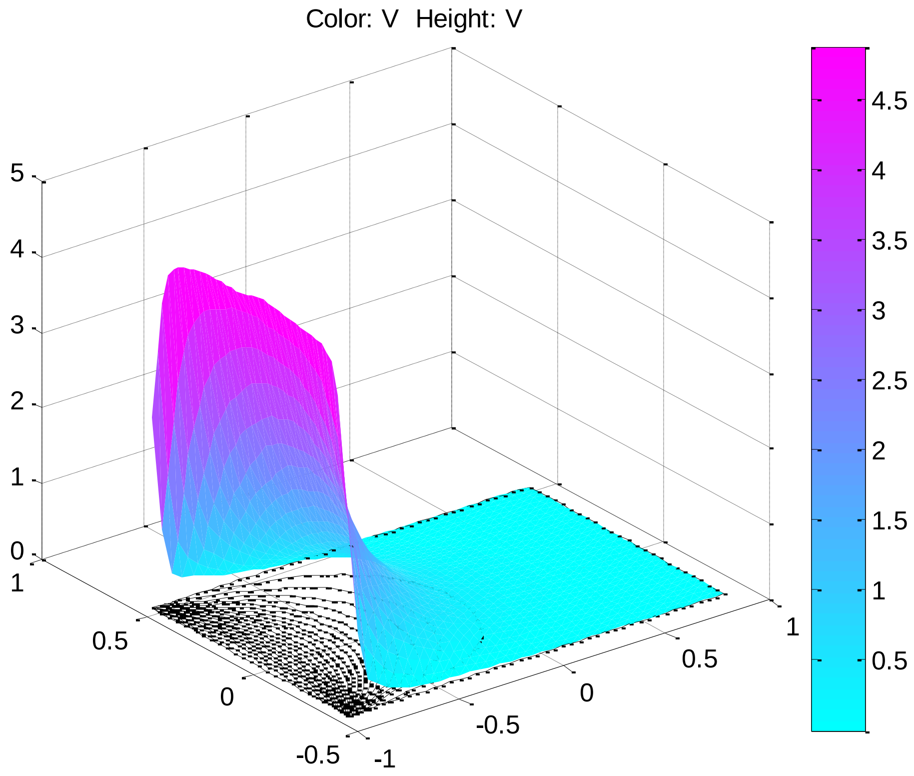
# Draw the V, E , Field lines & $\rho$ profile




V-profile



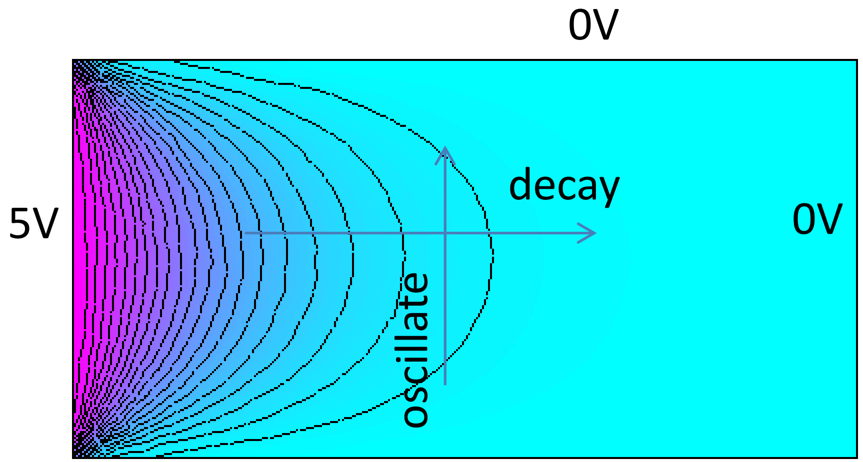

E-field & charge profile



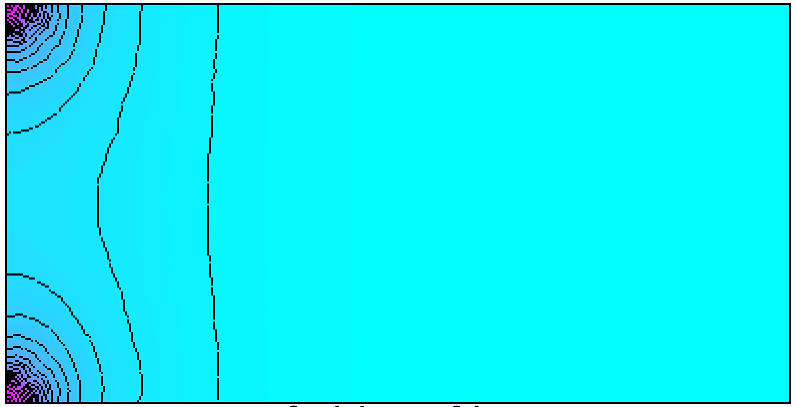
# Solution

- $V(x,y) = \sum_{n=0}^{inf}(-1)^n \frac{2Vo}{ak_n} cos(k_n y) \frac{\sinh(k_n x)}{\sinh(k_n b)}$
- Which is the most dominant terms in terms of furthest effect from the "active" electrode?
- It's the n=0 term
- Why?
- It has shortest spatial frequency $k_n = \frac{\pi(2n+1)}{2a}$ i.e. $k_o = = \frac{\pi}{2a}$ or wavelength of $1/k = \frac{2a}{\pi(2n+1)}$ with which the sinh function decays; So other terms decay exponentially and with shorter length-scales
- It has the largest prefactor
  - Because prefactor $2V_o/k_n$ is largest for smallest $k_n$
  - The cos term oscillates and is max at y=0
  - The sinh function is normalized to 1 by dividing it with its max value at x=b

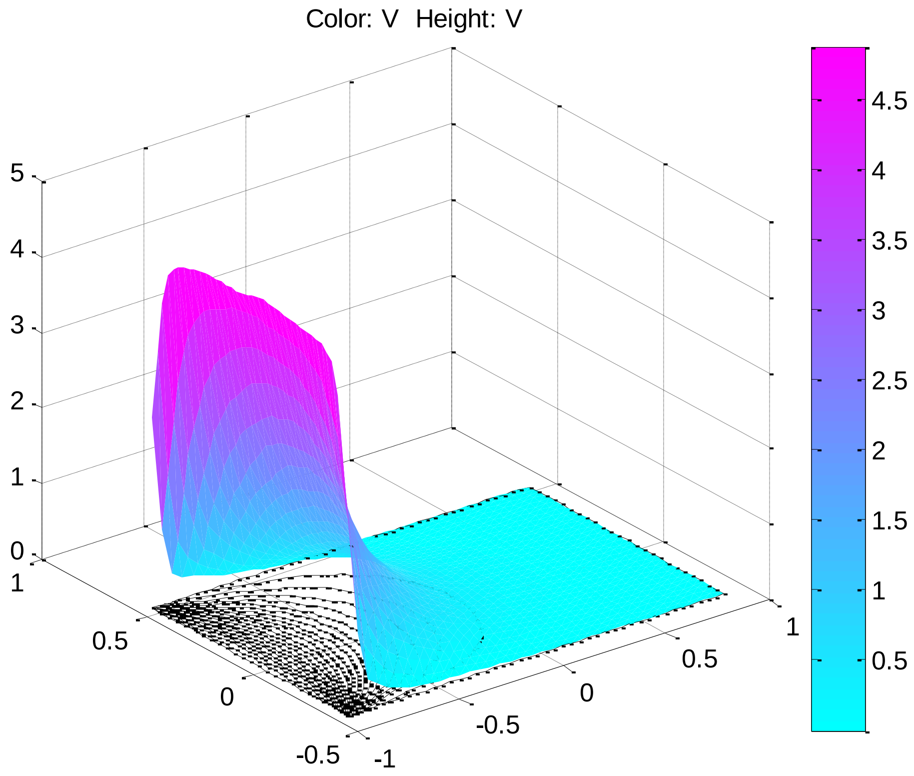
# Draw the V, E , Field lines & $\rho$ profile




V-profile

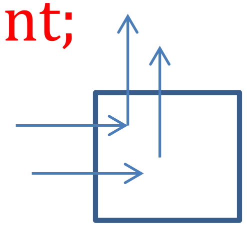


E-field profile



# What does Laplace Equation physically mean?

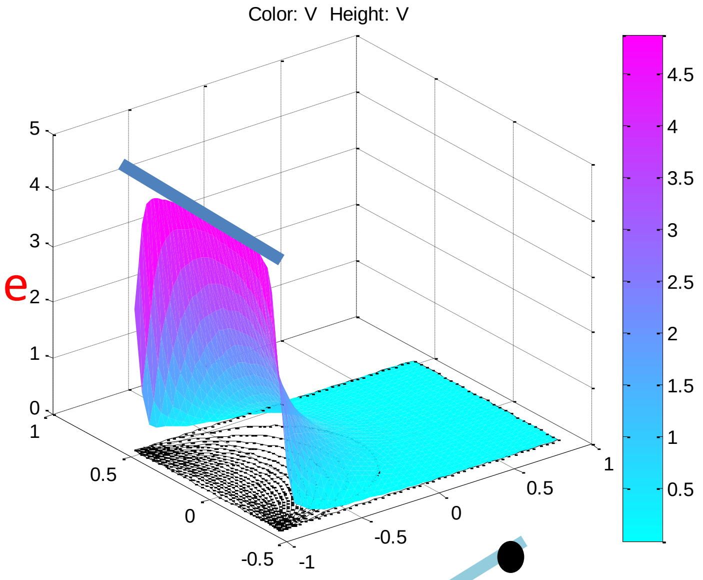
- In 1D – Laplace Equation ensure constant E field when charge is zero
- In 2D – even without charge E field is position dependent; Why

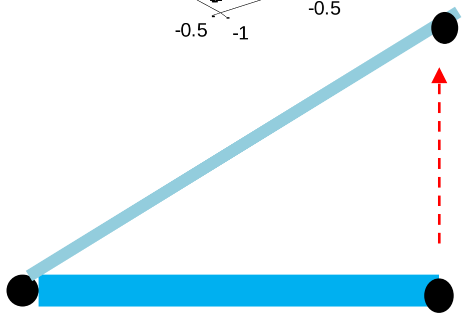


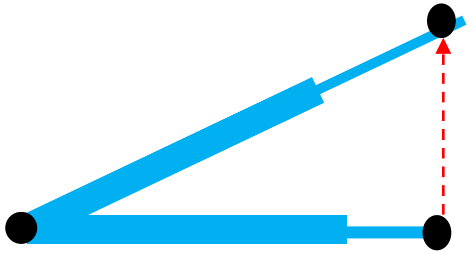
- $\nabla^2 V = \nabla E = 0$ ➔ Net divergence in E-field is zero ➔ that there is no charge
- In 1D, Laplace equation $\frac{dE}{dx} = 0$ implies that E is constant
- In 2D, $\left(\hat{\imath}\frac{d}{dx} + \hat{\jmath}\frac{d}{dy}\right) E = 0$ does not imply that E is constant.
- Instead it implies that flux coming into infinitesimal box the x direction may go out in the y direction.
  - This conserves flux (as there is not net charge in the box)
  - However E field in x direction has some divergence which should get canceled by divergence in the y direction
  - This also leads to opposite curvature in potential in x and y directions which produces a **saddle point**

# Analogy

- Can you think of another physical system, that would behave equivalently.
- If a square membrane is held by 4 sticks at the edge and one is moved up in z; should we get the same image as below?
- Yes
- What is the reason? Why does Laplace hold for stress-strain?
- Because stress is balanced at sticks
- Analogous to charge balance at electrode
- Can you also create a membrane analogy in 1D for a 1D capacitor?
- What if there are two dielectrics?
- At the interface, the force is balanced; but one elongates more; equivalent to fractions of voltage drops



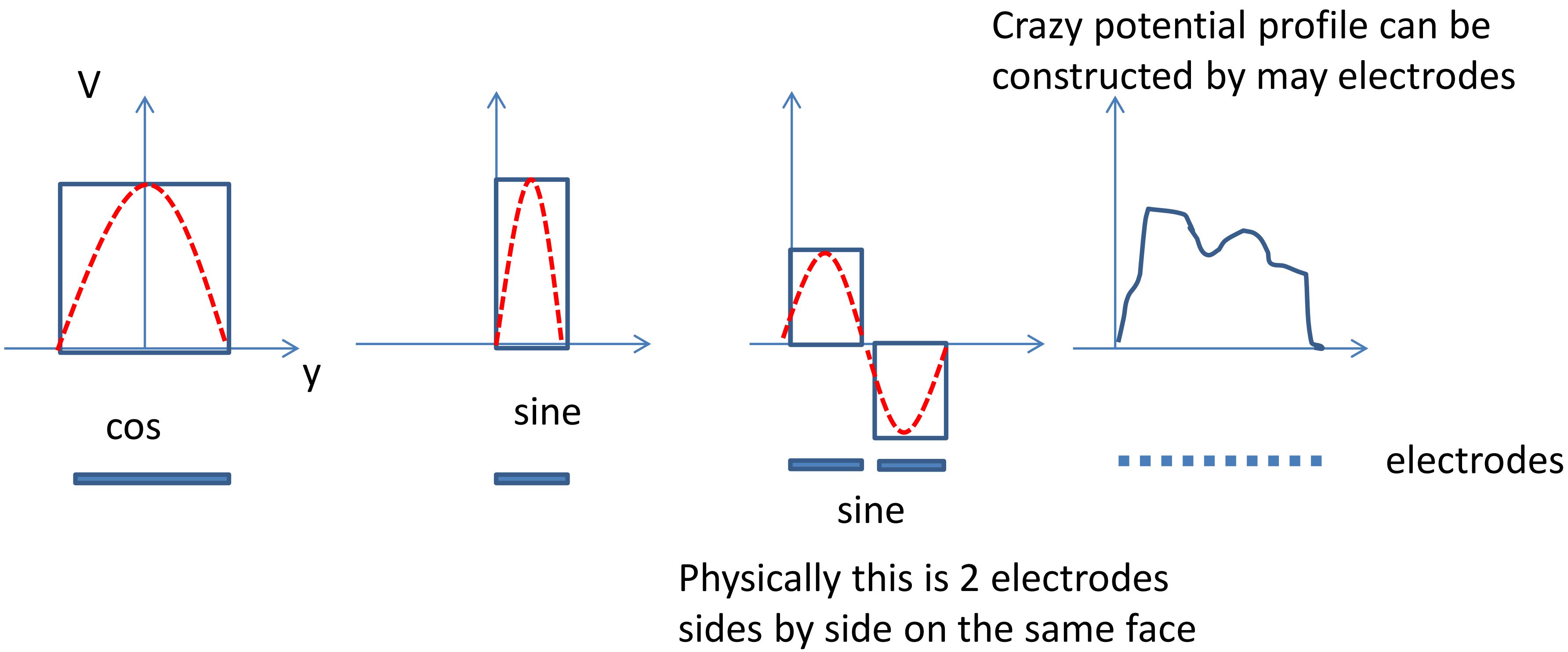





I wonder where this analogy will fail ☺

# Oscillating Part

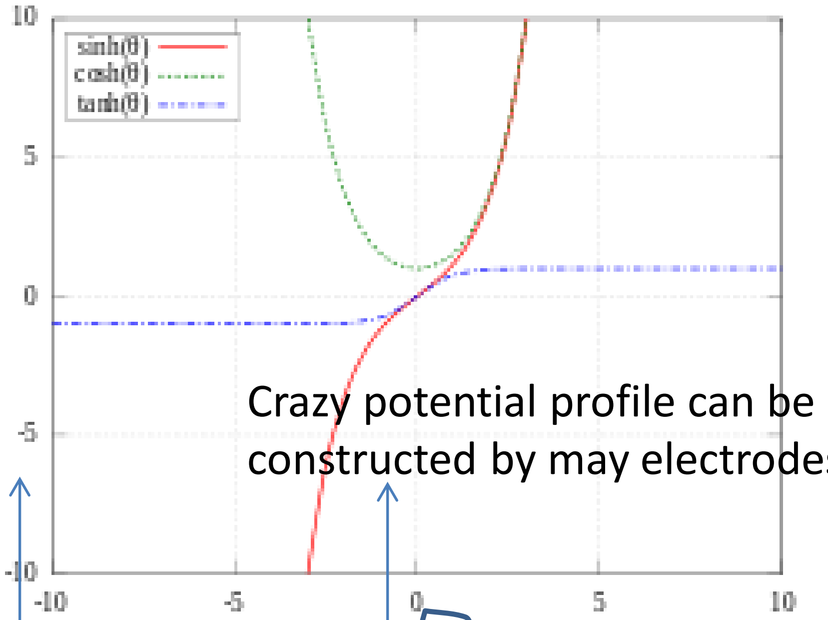
- Choice of solutions depends upon boundary conditions
  - Weighted sum of Cosine or sine need to form the V(x=0, y)= V(y) by Fourier integration



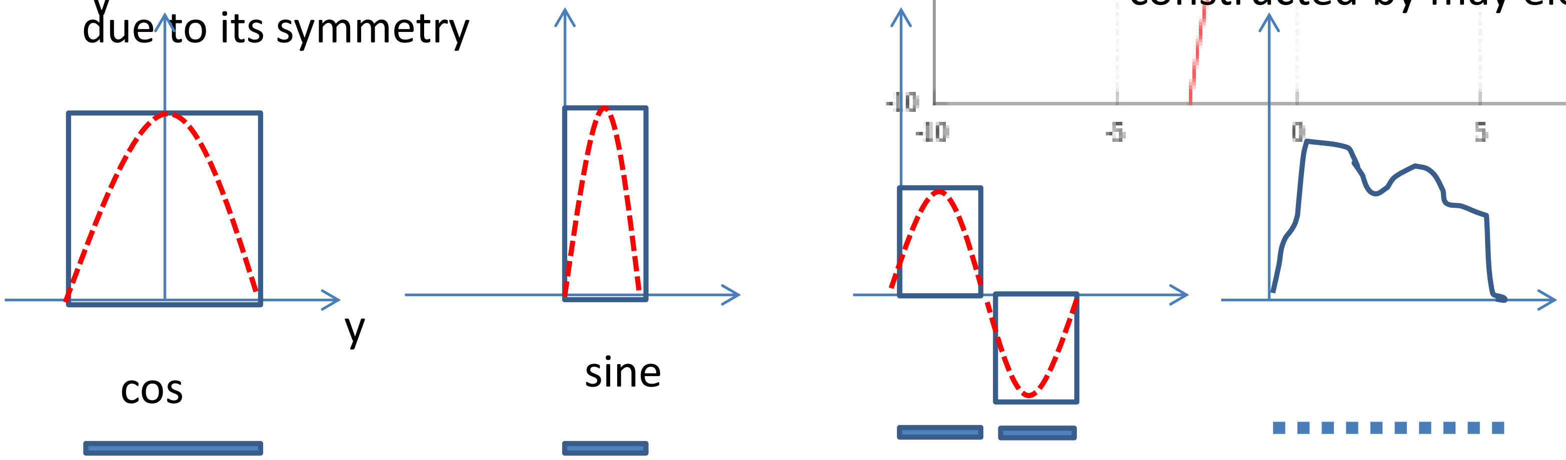
# The decaying part

If the other terminals are grounded, then a simple sinh function is nice as it is always zero at origin.

If two opposite terminals are at same potential, then cosh function may be nicer to use due to its symmetry




Crazy potential profile can be constructed by may electrodes




electrodes

Which one would decay the longest in the perpendicular direction?

# Points to remember

- Laplace solution is valid only when free charge is negligible.
- The general shapes of the field lines can be guessed for Laplace solution even without detailed math for simple potential profiles.
- Fourier trick can be used to solve for any potential profile at the boundary for a rectangular geometry
- Solution is always a combination of decaying term and oscillating terms creating a saddle point