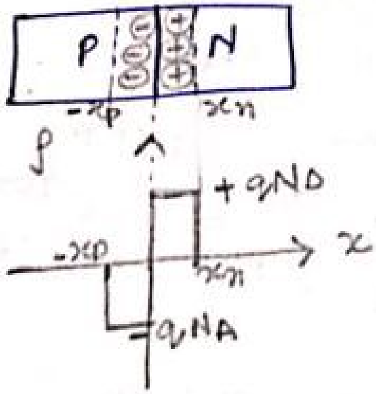
Q5. $N_D = N_A = 10^{17}/cc$

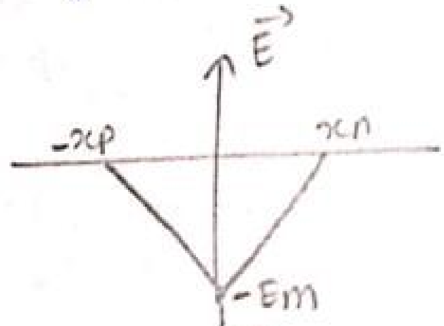


a. for depletion approximation, there are no free carriers

$$\rho = q[N_D - N_A + \underbrace{p - n}_{0}]$$

0 for depletion approx.

$\rho_{net} = q[N_D - N_A]$ [for entire PN diode]



For n side

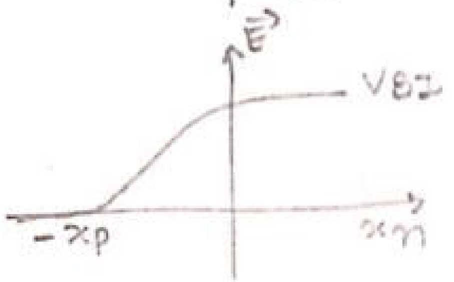
$\rho = qN_D \quad 0 < x < x_n$

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon} N_D \quad 0 < x < x_n$$

Integrating : $E = \frac{q}{\varepsilon} N_D x + C$

$$\boxed{E = \frac{q}{\varepsilon} N_D [x - x_n]}$$

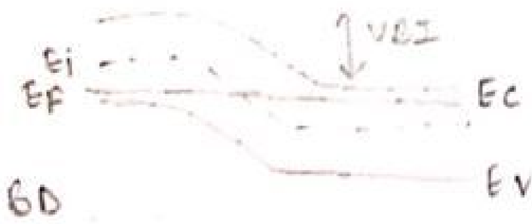
(@ $x = x_n$ $E = 0$)



For P side

$\rho = -qN_A \quad -x_p < x < 0$

$$\frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon} = -\frac{qN_A}{\varepsilon} \quad -x_p < x < 0$$



Integrating : $E = -\frac{qN_A}{\varepsilon} x + C$

$$\boxed{E = -\frac{q}{\varepsilon} N_A [+x + x_p]}$$

(@ $x = -x_p$ $E = 0$)

At $x = 0$ $E_{n side} = E_{p side}$

given $N_A = N_D$ $\therefore$ $\boxed{x_n = x_p}$ $|E_m| = \frac{qN_D}{\varepsilon} x_n = \frac{qN_A}{\varepsilon} x_p$

$V_{BI} = \left(\frac{1}{2}\right)(2x_n) E_m$

$W = x_n + x_p$

$$\boxed{V_{BI} \Rightarrow \frac{1}{2} W E_m}$$

b) Exact solution of Poisson Equation:-

we will be considering free carriers

Poisson equation;

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_s}$$

$$\boxed{q\phi(x) = -E_c(x) + \text{constant}}$$

Poisson eq in terms of conduction band;

$$\frac{d^2E_c(x)}{dx^2} = \frac{q\rho(x)}{\varepsilon_s}$$

$$\boxed{\rho(x) = q[N_D - n(x)]}$$

c) $n(x) = N_C \exp\left[-\frac{E_c(x) - E_F}{kT}\right]$

For neutral region there is no band bending:

$$n(x\to\infty) = N_D = n_{no} = N_C \exp\left[-\frac{E_c(+\infty) - E_F}{kT}\right]$$

$$n(x) = N_D \exp\left[\underbrace{-\underbrace{\frac{E_c(x) - E_c(x\to\infty)}{kT}}_{\psi_n(x)}}_{\phi(x)}\right]$$

$$\boxed{\mathcal{E} = -\frac{d\phi}{dx} = \frac{kT}{q}\frac{d\psi_n}{dx}}$$

$$V_T = \left(\frac{kT}{q}\right)$$

$$\therefore \rho(x) = qN_D\left[1 - e^{\frac{\phi(x)}{V_T}}\right]$$

$$\frac{d^2\phi}{dx^2} = -\frac{qN_D}{\varepsilon_s}\left[1 - e^{\phi/V_T}\right]$$

multiply both sides with $\frac{d\phi}{dx}$

$$\int_{\frac{d\phi}{dx}}^{0} \frac{d}{dx}\left[\frac{d\phi}{dx}\right]\frac{d\phi}{dx} = -\int_{\phi}^{0}\frac{qN_D}{\varepsilon_s}\left[1 - e^{\phi/V_T}\right] d\phi$$

$$+\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 = +\frac{qN_D}{\varepsilon_s}\left[-V_T - \phi + e^{\phi/V_T}V_T\right]$$

$$\frac{d\phi}{dx} = \left[\frac{2qN_D}{\varepsilon_s}\left[e^{\phi/V_T} - \frac{\phi}{V_T} - 1\right]V_T\right]^{\frac{1}{2}}$$

$$\boxed{\mathcal{E} = \left[\frac{2qN_D}{\varepsilon_s}\left[e^{\phi/V_T} - \frac{\phi}{V_T} - 1\right]V_T\right]^{\frac{1}{2}}}$$

IIIly for p-type side can be calculated.

for $x = \infty$, band bending is zero $\therefore E = 0$

Debye length, $L_D = \sqrt{\frac{\epsilon_S KT}{q^2 N_D}}$

$$E = \left[\frac{2q^2 N_D}{\epsilon_S KT}\left[V_T\left[\frac{KT}{qV}\right]\right]\left[e^{\phi/V_T} - \frac{\phi}{V_T} - 1\right]\right]^{\frac{1}{2}}$$

$$\boxed{E = \left[\frac{2}{L_D^2} V_T^2 \left[e^{\phi/V_T} - \frac{\phi}{V_T} - 1\right]\right]^{\frac{1}{2}}}$$

① If $\phi < 0$ $\left|\frac{\phi}{V_T}\right| >> 0$

$$E(x) = \left[\left(\frac{2V_T^2}{L_D^2}\right)\left[-\frac{\phi}{V_T} - 1\right]\right]^{\frac{1}{2}}$$

② If $\phi < 0$ $\left|\frac{\phi}{V_T}\right| \sim 0$, opening e^{ϕ/V_T} with binomial expansion

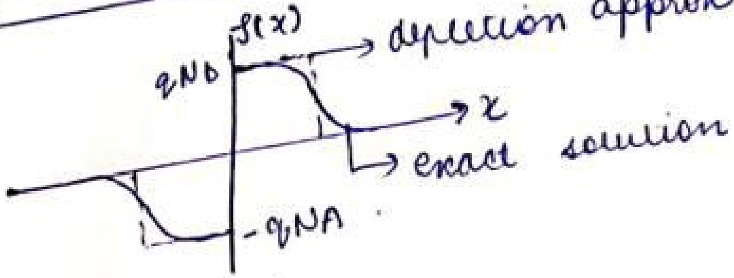
$$E = \left[\left(\frac{2V_T^2}{L_D^2}\right)\left[\cancel{-\frac{\phi}{V_T}} + \cancel{1} + \cancel{\frac{\phi}{V_T}} + \frac{\phi^2}{2V_T^2} \cancel{-1}\right]\right]^{\frac{1}{2}}$$

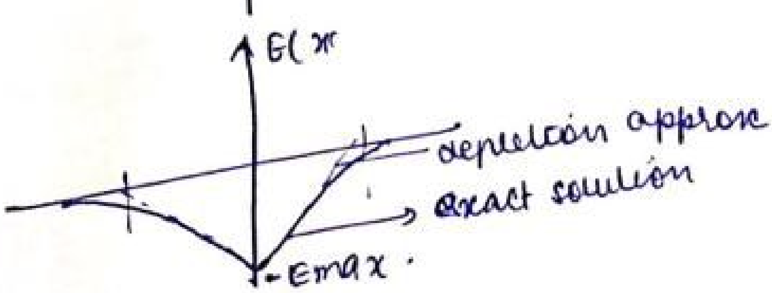
$$E \Rightarrow \frac{\phi}{L_D}$$

$$-\frac{d\phi}{dx} = \frac{\phi}{L_D} \Rightarrow -\frac{1}{\phi}d\phi = \frac{1}{L_D}dx \Rightarrow \ln\phi = -x/L_D$$

$$\phi = e^{-x/L_D}$$

$\boxed{\phi \propto e^{-x/L_D}}$ exponential dependence of ϕ on x.





For step by step numerical solution :-

1) Initial guess voltage $V = V^{old}$ is taken

2) ~~Consi~~ In second iteration V_{new} is taken which is

$$V^{new} = V^{old} + \delta$$

δ is small potential value added to old potential value as an increment.

3) Poissoneq is given as:

$$\frac{d^2 V^{new}}{dx^2} = -\frac{q n_i}{\varepsilon}\left[e^{-V^{old}/V_T} - e^{V^{old}/V_T} + C/n_i\right] + \frac{q n_i \delta}{\varepsilon}\left[e^{-V^{old}/V_T} + e^{V^{old}/V_T}\right].$$

Bringing V^{new} terms on 1 side and solve the differential eq.

- $\delta = V^{new} - V^{old}$
- δ being small $\quad e^{\pm\delta/V_T} = \left(1 \pm \delta/V_T\right)$

4) Finite elements i.e. the differential equation is solved ~~using~~ by discretizing the differential equation in k components.

5) Normalisation of variables is done, so consider the ~~equation~~ variables unitless.

so $\left[\frac{x}{L_D} \rightarrow X\right]$ where X is unitless
L_D is debye length

n is change to n/n_i $\quad$ p to p/n_i
V is changed to V/V_T.

6) finite difference form of linearized 1D Poisson eq is then written as:

$$\frac{\partial^2 V}{\partial x^2} = \frac{V_{i+1} - 2V_i + V_{i-1}}{\Delta^2} = a_i V_{i-1} + b_i V_i + c_i V_{i+1} = f_i$$

This gives 1 run of V

Matrix form of V can be written which will be in a ~~tin~~ diagonal form :

7) with LU decomposition method [V] matrix can be determined .

8) So we start with some guess Vold and calculate Vnew with iterations .

The solution will converge if the error

$$V_{i+1} - V_i < \text{preset error (tolerance)}$$ for all k points

Q5.d. Solved for n side :

$$C = \left|\frac{dQ}{dV_a}\right| = \varepsilon\left|\frac{dE}{dV_a}\right|$$

$$= \varepsilon\left[\frac{2V_T^2}{L_D^2}\right]^{\frac{1}{2}}\left|\frac{\left[\frac{e^{\phi/V_T}}{V_T} - \frac{1}{V_T}\right]}{\left[e^{\phi/V_T} - \frac{\phi}{V_T} - 1\right]^{\frac{1}{2}}}\right|.$$

$$\Rightarrow \left|\frac{\sqrt{2}\,\varepsilon}{L_D}\frac{\left[e^{\phi/V_T} - 1\right]}{2\left[e^{\phi/V_T} - \frac{\phi}{V_T} - 1\right]^{\frac{1}{2}}}\right|.$$

e. $\phi_s = -\phi_i + V_a$

very small because $\phi_s < 0$

$e^{-ve} \to$ very small

$$C = \frac{\sqrt{2}\,\varepsilon}{L_D}\frac{\left[e^{\frac{-\phi_i}{V_T}}e^{\frac{V_a}{V_T}} - 1\right]}{2\left[\underbrace{e^{\frac{-\phi_i}{V_T}}e^{\frac{V_a}{V_T}}}_{\text{very small}} - \underbrace{\frac{(-\phi_i + V_a)}{V_T}}_{\text{very large}} - 1\right]^{\frac{1}{2}}}$$

$\phi_s < 0$ and $\frac{|\phi_s|}{V_T} >> 1$

$\Rightarrow$

$$C = \frac{\sqrt{2}\,\varepsilon}{2L_D}\sqrt{\frac{+1}{+\left[\frac{V_a - \phi_i}{V_T} + 1\right]}}$$

$$\Rightarrow \frac{\varepsilon}{\sqrt{2}\,L_D}\sqrt{\frac{V_T}{(V_a - \phi_i + V_T)}}$$

$$C = \sqrt{\frac{q^2 N_D \varepsilon^2 kT}{2\,kT\,\varepsilon\,q\,(V_a - \phi_i + V_T)}}.$$

$$C \Rightarrow \sqrt{\frac{\varepsilon q N_D}{2(V_a - \phi_i + V_T)}}$$

for depeletion app

$C_{dep} = \frac{\varepsilon}{W}$

$= \sqrt{\frac{\varepsilon q N_D}{2(\phi_i - V_a)}}$

Both are same approx.