1. Flowchart of the Paper:

Background and challenge:

According to Moore's law CMOS device dimensions are scaling down more and more to achieve more device inside a chip during past three decades. Now the technology has come to a state that the device dimensions cannot be scaled more as Short Channel Effects come into account and oxide thickness cannot be smaller as there the problem of gate leakage for tunneling will rise. So, one solution is the device geometry should be changed such a way that gate length can be reduced with thicker oxide. In order to optimize the performance of double-gate devices, self-aligned processes and structures are proposed, with FinFET being one of the most promising.

Problem statement and Proposal:

Due to the asymmetrical three-dimensional (3-D) structure of FinFET, the coupling between gates that surround the channel cannot be neglected in aggressively scaled device dimensions. Hence, 3-D analyses are necessary to obtain reasonable prediction of device performance and structure optimization. In the regime when both fin height and fin thickness have control over SCE, the dependence of SCE on device dimensions is not well characterized. In order to establish the functional expressions of SCE, analytical solution of 3-D Laplace's equation is used to derive the design equations for the subthreshold behavior of FinFET. Based on the 3-D electrostatic potential distribution in the subthreshold region, the subthreshold swing and the threshold voltage roll-off are estimated by considering the source barrier changes in the most leaky channel path.

Validation:

3-D FinFET with undoped channel, n polysilicon gate, uniform 1.5 nm gate oxide, abrupt source-to-channel and drain-to-channel junctions has been simulated in FIELDAY. The dependence of subthreshold swing and Threshold Voltage roll-off on Fin thickness and Fin height are shown from simulation. The electrostatic potential in the subthreshold region can be described by 3-D Laplace's equation if the potential perturbation due to the carrier concentration and generation are neglected (which is the case for undoped fin). From the analytical expressions of potentials the subthreshold swing and the roll-off in threshold voltage are calculated and compared with the simulation.

Impact of solution:

The six boundary conditions which are used in the solution of the analytical expressions for potential distribution in FinFet can be modified to get the solutions for FDFET, DGFET and SGFET only changing the boundary conditions and applying into the solutions. In this way the comparisons are drawn from the analytical solutions for different types of FETs.

2. Write down the assumptions of the analytical model proposed and comment on their validity.

Assumption 1: The buried oxide is assumed to be thick enough that any finite potential across the buried oxide leads to a negligible electric field.

Assumption 2: The boundaries between gate oxide and silicon fin are eliminated by replacing the physical dimensions with effective dimensions. The whole region is treated as homogeneous silicon with effective thickness (T_{eff}), effective channel length (L_{eff}) and effective height (H_{eff}). Oxide is replaced with an equivalent region of Si modified by $\epsilon_{Si}/\epsilon_{ox}$. The simplification is based on the fact that the normal component of the electric field changes by a factor of $\frac{\epsilon_{Si}}{\epsilon_{ox}}$ across the silicon-oxide boundary.

So, considering front gate oxide and back gate oxide , the new effective thickness of the FinFet is

$$T_{eff} = T_{fin} + \frac{2\epsilon_{Si}}{\epsilon_{ox}} T_{ox}$$

Considering the top gate oxide:

$$H_{eff} = H_{fin} + \frac{\epsilon_{Si}}{\epsilon_{ox}} H_{ox}$$

For DGFETs,

$$T_{eff} = \sqrt{T_{fin}(T_{fin} + \frac{4\epsilon_{Si}}{\epsilon_{ox}} T_{ox})}$$

And

$$H_{eff} = \sqrt{H_{fin}(H_{fin} + \frac{2\epsilon_{Si}}{\epsilon_{ox}} H_{ox})}$$

Assumption 3: While calculating the subthreshold slope, we have assumed that leakage current is dominated by the current with the minimum source barrier, the most leaky path. The assumption is based on the exponential dependence of the drain current on source barrier in subthreshold region.

3.

Proof of Solution:

Laplace equation: $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=0$

Solution to Laplace equation is of form:

$$\psi(x,y,z)=C(sinh\, or cosh)*(\sin or\, cos)*(\sin or\ \ cos)$$

A) <u>Top Gate:</u>

Potential function when a gate voltage V_G is applied to top gate only = ψ_{tg}

- Boundary conditions:

$$\psi(x,y,H_{eff})=V_g-V_{fb}$$
$$\psi(0,y,z)=0$$
$$\psi(T_{eff},y,z)=0$$
$$\psi(x,0,z)=0$$
$$\psi(x,L_{eff},z)=0$$
$$\psi(x,y,-H_{eff})=V_g-V_{fb}$$

It can be easily seen that in x and y directions, sine function would be present as the boundary conditions are 0. In the z direction, it will be a cosh function as both boundary conditions are V_g-V_{fb}.

So, we can write down the potential function as :

$$\psi(x,y,z)=\sum_{m,n=1}^{\infty}C_{mn}\,cosh(k_3z)*\sin(k_1x)*\sin(k_2y)$$

$\sin(k_1T_{eff})=0 \Rightarrow k_1=\frac{n\prime\pi}{T_{eff}}$

$\sin(k_2L_{eff})=0 \Rightarrow k_2=\frac{m\prime\pi}{L_{eff}}$

$$V_g-V_{fb}=\sum_{m,n=1}^{\infty}C_{mn}\,cosh(k_3H_{eff})*\sin(k_1x)*\sin(k_2y)$$

$$C_{nm}cosh(k_3H_{eff})=\frac{4}{L_{eff}T_{eff}}\iint_0^{T_{eff},L_{eff}}(V_g-V_{fb})\sin(k_1x)\sin(k_2y)dxdy$$

$$C_{nm}=\frac{16(V_g-V_{fb})}{\pi^2n'm'(cosh(k_zH_{eff}))}\ for\ odd\ m',n'$$

Let $m=2m'-1\ and\ n=2n'-1$

$$\psi(x,y,z)=\sum_{m,n=1}^{\infty}\frac{16(V_g-V_{fb})}{\pi^2(2n-1)(2m-1)}\left(\frac{cosh(k_3z)}{cosh(k_3H_{eff})}\right)*\sin\left(\frac{(2n-1)\pi x}{T_{eff}}\right)*\sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)$$

Now , $k_3=\sqrt{k_1^2+k_2^2}=\pi\sqrt{\left(\frac{2n-1}{T_{eff}}\right)^2+\left(\frac{2m-1}{L_{eff}}\right)^2}$

The authors of the paper have assumed $k_z=\frac{k_3H_{eff}}{\pi}$

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So,

$$\boxed{\psi(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})}{\pi^2(2n-1)(2m-1)} \left(\frac{cosh\left(\frac{k_z \pi}{H_{eff}} z\right)}{cosh(k_z \pi)} \right) * \sin\left(\frac{(2n-1)\pi x}{T_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)}$$

B: Front Gate:

Potential function when a gate voltage V_G is applied to front gate only = ψ_{fg}

- Boundary conditions:

$$\psi(x,y,H_{eff}) = 0$$
$$\psi(0,y,z) = 0$$
$$\psi(T_{eff},y,z) = V_g - V_{fb}$$
$$\psi(x,0,z) = 0$$
$$\psi(x,L_{eff},z) = 0$$
$$\psi(x,y,-H_{eff}) = 0$$

It can be easily seen that in y and z directions, sine function and cosine function respectively would be present as the boundary conditions are 0. Sine in y direction because the conditions are 0 to L_{eff}. Cosine in z direction because the conditions are $-H_{eff}$ to H_{eff}. In the x direction, it will be a sinh function as only one boundary condition is V_g-V_{fb}.

So, we can write down the potential function as :

$$\psi(x,y,z) = \sum_{m,n=1}^{\infty} C_{mn}\, sinh(k_3 x) * \sin(k_1 y) * \cos(k_2 z)$$

$$\cos(\pm k_2 H_{eff}) = 0 => k_2 = \frac{(2n-1)\pi}{2H_{eff}} \quad \sin(k_1 L_{eff}) = 0 => k_1 = \frac{m'\pi}{L_{eff}}$$

$$V_g - V_{fb} = \sum_{m,n=1}^{\infty} C_{mn}\, sinh(k_3 T_{eff}) * \sin(k_1 y) * \cos(k_2 z)$$

$$C_{nm} sinh(k_3 T_{eff}) = \frac{4}{2H_{eff}T_{eff}} \iint_{0,-H_{eff}}^{L_{eff},H_{eff}} (V_g - V_{fb}) \sin(k_1 y) \cos(k_2 z)\, dz dy$$

$$C_{nm} = \frac{4(V_g - V_{fb})(1-\cos(m'\pi))(2\sin((2n-1)\pi))}{\pi^2(2n-1)m'(sinh(k_3 T_{eff}))}\ for\ all\ m', n$$

$$C_{nm} = \frac{16(V_g - V_{fb})\sin((2n-1)\pi)}{\pi^2(2n-1)m'(sinh(k_3 T_{eff}))}\ for\ odd\ m'$$

$$C_{nm} = \frac{16(V_g - V_{fb})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)(sinh(k_3 T_{eff}))}\ for\ all\ m, n$$

$$\psi(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh(k_3 x)}{sinh(k_3 T_{eff})}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)$$

$$k_3 = \sqrt{k_1^2 + k_2^2} = \pi\sqrt{\left(\frac{2n-1}{2H_{eff}}\right)^2 + \left(\frac{2m-1}{L_{eff}}\right)^2}$$

The authors of the paper have assumed $k_x = \frac{k_3 T_{eff}}{\pi}$

So,

$$\boxed{\psi(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})(-1)^{\mathrm{n+1}}}{\pi^2(2n-1)(2m-1)}\left(\frac{sinh\left(\frac{k_x\pi}{T_{eff}}x\right)}{sinh(k_x\pi)}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)}$$

C: Back Gate:

Potential function when a gate voltage V_G is applied to front gate only = ψ_{bg}

- Boundary conditions:

$$\psi(x,y,H_{eff}) = 0$$
$$\psi(0,y,z) = V_g - V_{fb}$$
$$\psi(T_{eff},y,z) = 0$$
$$\psi(x,0,z) = 0$$
$$\psi(x,L_{eff},z) = 0$$
$$\psi(x,y,-H_{eff}) = 0$$

It can be easily seen that in y and z directions, sine function and cosine function respectively would be present as the boundary conditions are 0. Sine in y direction because the conditions are 0 to L_{eff}. Cosine in z direction because the conditions are $-H_{eff}$ to H_{eff}. In the x direction, it will be a sinh function as only one boundary condition is V_g-V_{fb}.
By symmetry, this is exactly like applying a potential to the front gate, except the x axis is translated by an amount T_{eff}. **So, the potential function will be exactly the same except that the x axis is translated by an amount T_{eff}.**

$$\boxed{\psi(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})(-1)^{\mathrm{n+1}}}{\pi^2(2n-1)(2m-1)}\left(\frac{sinh\left(\frac{k_x\pi}{T_{eff}}(T_{eff}-x)\right)}{sinh(k_x\pi)}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)}$$

D: Source

Potential function considering the metal-semiconductor contact at source only = ψ_s

- Boundary conditions:

$$\psi(x,y,H_{eff}) = 0$$
$$\psi(0,y,z) = 0$$
$$\psi(T_{eff},y,z) = 0$$
$$\psi(x,0,z) = -\varphi_{ms}$$
$$\psi(x,L_{eff},z) = 0$$
$$\psi(x,y,-H_{eff}) = 0$$

It can be easily seen that in x and z directions, sine function and cosine function respectively would be present as the boundary conditions are 0. Sine in y direction because the conditions are 0 to L_{eff}. cosine in z direction because the conditions are $-H_{eff}$ to H_{eff}. In the y direction, it will be a sinh function as only one boundary condition is $-\varphi_{ms}$. We have to translate the y axis by an amount L_{eff} in order to satisfy the boundary conditions.

New boundary conditions:

$$\psi(x, -L_{eff}, z) = -\varphi_{ms}$$
$$\psi(x, 0, z) = 0$$

$$\psi(x, y, z) = \sum_{m,n=1}^{\infty} C_{mn}\, sinh(k_3(y - L_{eff})) * \sin(k_1 x) * \cos(k_2 z)$$

$$\cos(\pm k_2 H_{eff}) = 0 => k_2 = \frac{(2n-1)\pi}{2H_{eff}}$$

$$\sin(k_1 T_{eff}) = 0 => k_1 = \frac{m'\pi}{T_{eff}}$$

$$-\varphi_{ms} = \sum_{m,n=1}^{\infty} C_{mn}\, sinh(k_3 L_{eff}) * \sin(k_1 x) * \cos(k_2 z)$$

$$-C_{nm} sinh(k_3 L_{eff}) = \frac{4}{2H_{eff}T_{eff}} \iint_{0,-H_{eff}}^{T_{eff},H_{eff}} -\varphi_{ms} \sin(k_1 x)\cos(k_2 z)dxdz$$

$$C_{nm} = \frac{-4\varphi_{ms}(1-\cos(m'\pi))\big(2\sin((2n-1)\pi)\big)}{-\pi^2(2n-1)m'(sinh(k_3 L_{eff}))}\ for\ all\ m', n$$

$$C_{nm} = \frac{-16\varphi_{ms}\sin((2n-1)\pi)}{-\pi^2(2n-1)m'(sinh(k_3(L_{eff})))}\ for\ odd\ m'$$

$$C_{nm} = \frac{-16\varphi_{ms}(-1)^{\mathrm{n}+1}}{-\pi^2(2n-1)(2m-1)(sinh(k_3(L_{eff})))}\ for\ all\ m, n$$

$$\psi(x, y, z) = \sum_{m,n=1}^{\infty} \frac{-16\varphi_{ms}(-1)^{\mathrm{n}+1}}{\pi^2(2n-1)(2m-1)}\left(\frac{sinh(k_3(L_{eff}-y))}{sinh(k_3(L_{eff}))}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)$$

$$k_3 = \sqrt{k_1^2 + k_2^2} = \pi\sqrt{\left(\frac{2n-1}{2H_{eff}}\right)^2 + \left(\frac{2m-1}{T_{eff}}\right)^2}$$

The authors of the paper have assumed $k_y = \frac{k_3 L_{eff}}{\pi}$

$$\boxed{\psi(x, y, z) = \sum_{m,n=1}^{\infty} \frac{-16\varphi_{ms}(-1)^{\mathrm{n}+1}}{\pi^2(2n-1)(2m-1)}\left(\frac{sinh\left(\frac{k_y\pi}{L_{eff}}(L_{eff}-y)\right)}{sinh(k_y\pi)}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{T_{eff}}\right)}$$

E: Drain

Potential function considering the metal-semiconductor contact at source only = ψ_d

- Boundary conditions:

$$\psi(x, y, H_{eff}) = 0$$
$$\psi(0, y, z) = 0$$
$$\psi(T_{eff}, y, z) = 0$$
$$\psi(x, L_{eff}, z) = -\varphi_{ms} + V_{ds}$$
$$\psi(x, 0, z) = 0$$
$$\psi(x, y, -H_{eff}) = 0$$

It can be easily seen that in x and z directions, sine function and cosine function respectively would be present as the boundary conditions are 0. Sine in x direction because the conditions are 0 to T_{eff}. cosine in z direction because the conditions are $-H_{eff}$ to H_{eff}. In the y direction, it will be a sinh function as only one boundary condition is $-\varphi_{ms} - V_{ds}$.

So, we can write down the potential function as :

$$\psi(x, y, z) = \sum_{m,n=1}^{\infty} C_{mn}\, sinh(k_3 y) * \sin(k_1 x) * \cos(k_2 z)$$

$$\cos(\pm k_2 H_{eff}) = 0 => k_2 = \frac{(2n-1)\pi}{2H_{eff}}$$

$$\sin(k_1 L_{eff}) = 0 => k_1 = \frac{m'\pi}{T_{eff}}$$

$$-\varphi_{ms} + V_{ds} = \sum_{m,n=1}^{\infty} C_{mn}\, sinh(k_3 L_{eff}) * \sin(k_1 y) * \cos(k_2 z)$$

$$C_{nm} sinh(k_3 L_{eff}) = \frac{4}{2H_{eff}T_{eff}} \iint_{0,-H_{eff}}^{T_{eff},H_{eff}} (-\varphi_{ms} + V_{ds}) \sin(k_1 x) \cos(k_2 z) dx dz$$

$$C_{nm} = \frac{4(-\varphi_{ms} + V_{ds})(1 - \cos(m'\pi))(2\sin((2n-1)\pi))}{\pi^2(2n-1)m'(sinh(k_3 L_{eff}))} \; for\ all\ m', n$$

$$C_{nm} = \frac{16(-\varphi_{ms} + V_{ds}) \sin((2n-1)\pi)}{\pi^2(2n-1)m'(sinh(k_3 L_{eff}))} \; for\ odd\ m'$$

$$C_{nm} = \frac{16(-\varphi_{ms} + V_{ds})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)(sinh(k_3 T_{eff}))} \; for\ all\ m, n$$

$$\psi(x, y, z) = \sum_{m,n=1}^{\infty} \frac{-16(\varphi_{ms} - V_{ds})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh(k_3 y)}{sinh(k_3 L_{eff})}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi x}{T_{eff}}\right)$$

$$k_3 = \sqrt{k_1^2 + k_2^2} = \pi \sqrt{\left(\frac{2n-1}{2H_{eff}}\right)^2 + \left(\frac{2m-1}{T_{eff}}\right)^2}$$

The authors of the paper have assumed $k_y = \frac{k_3 L_{eff}}{\pi}$

$$\boxed{\psi(x, y, z) = \sum_{m,n=1}^{\infty} \frac{-16(\varphi_{ms} - V_{ds})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh\left(\frac{k_y \pi}{L_{eff}} x\right)}{sinh(k_y \pi)}\right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{T_{eff}}\right)}$$

F: Buried Oxide Layer

The effect of Buried oxide layer has already been taken into consideration while calculating the potential function of top gate. The bottom buried-oxide boundary condition is replaced with bottom-gate boundary condition in (20). Due to the symmetrical structure in the direction, the buried oxide boundary condition is implicit in the model and the same solutions can be reached.

So, $\psi_{BO} = 0$

Summary:

Top gate: $\psi_{tg}(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})}{\pi^2(2n-1)(2m-1)} \left(\frac{cosh\left(\frac{k_z\pi}{H_{eff}}z\right)}{cosh(k_z\pi)} \right) * \sin\left(\frac{(2n-1)\pi x}{T_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)$

Front gate: $\psi_{fg}(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh\left(\frac{k_x\pi}{T_{eff}}x\right)}{sinh(k_x\pi)} \right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)$

Back gate: $\psi_{bg}(x,y,z) = \sum_{m,n=1}^{\infty} \frac{16(V_g - V_{fb})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh\left(\frac{k_x\pi}{T_{eff}}(T_{eff}-x)\right)}{sinh(k_x\pi)} \right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{L_{eff}}\right)$

Source: $\psi_s(x,y,z) = \sum_{m,n=1}^{\infty} \frac{-16\varphi_{ms}(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh\left(\frac{k_y\pi}{L_{eff}}(L_{eff}-y)\right)}{sinh(k_y\pi)} \right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{T_{eff}}\right)$

Drain: $\psi_d(x,y,z) = \sum_{m,n=1}^{\infty} \frac{-16(\varphi_{ms} - V_{ds})(-1)^{n+1}}{\pi^2(2n-1)(2m-1)} \left(\frac{sinh\left(\frac{k_y\pi}{L_{eff}}y\right)}{sinh(k_y\pi)} \right) * \cos\left(\frac{(2n-1)\pi z}{2H_{eff}}\right) * \sin\left(\frac{(2m-1)\pi y}{T_{eff}}\right)$

Buried Oxide Layer: $\psi_{BO} = 0$

$\psi(x,y,z) = \psi_{tg}(x,y,z) + \psi_{fg}(x,y,z) + \psi_{bg}(x,y,z) + \psi_s(x,y,z) + \psi_d(x,y,z) + \psi_{BO}$

Where,

$$k_x = \sqrt{\left(\frac{2n-1}{2H_{eff}}\right)^2 + \left(\frac{2m-1}{L_{eff}}\right)^2} \times T_{eff}$$

$$k_y = \sqrt{\left(\frac{2n-1}{2H_{eff}}\right)^2 + \left(\frac{2m-1}{T_{eff}}\right)^2} \times L_{eff}$$

$$k_z = \sqrt{\left(\frac{2n-1}{T_{eff}}\right)^2 + \left(\frac{2m-1}{L_{eff}}\right)^2} \times H_{eff}$$

Subthreshold Slope:

$$\frac{1}{S}=\frac{\partial(logI_D)}{\partial V_g}=\frac{q}{2.3kT}\frac{\partial\psi_s}{\partial V_g}|(x_c,y_c,z_c)=\frac{q}{2.3kT}\frac{\partial(\psi_{fg}+\psi_{bg}+\psi_{tg})}{\partial V_g}$$

$$\frac{1}{S}=\frac{q}{2.3kT}\sum_{m,n=1}^{\infty}\frac{16\sin\left(\frac{(2m-1)\pi y_c}{L_{eff}}\right)}{\pi^2(2n-1)(2m-1)}\left(\sin\left(\frac{(2n-1)\pi x_c}{T_{eff}}\right)\left(\frac{cosh\left(\frac{k_Z\pi}{H_{eff}}z_c\right)}{cosh(k_Z\pi)}\right)+\right.$$

$$\left.\cos\left(\frac{(2n-1)\pi z_c}{2H_{eff}}\right)(-1)^{n+1}\left(\left(\frac{sinh\left(\frac{k_X\pi}{T_{eff}}x_c\right)}{\sinh(k_X\pi)}\right)+\left(\frac{sinh\left(\frac{k_X\pi}{T_{eff}}(T_{eff}-x_c)\right)}{\sinh(k_X\pi)}\right)\right)\right)$$

Here, (x_c,y,z_c) is the most leaky path.

Let us consider m=1,n=1

$$k_y=\sqrt{\left(\frac{1}{2H_{eff}}\right)^2+\left(\frac{1}{T_{eff}}\right)^2}\times L_{eff}$$

Let $$\frac{L_{eff}}{k_y}=\frac{1}{\sqrt{\left(\frac{1}{2H_{eff}}\right)^2+\left(\frac{1}{T_{eff}}\right)^2}}=L_D$$

The y-coordinate of the most leaky path can be found out proceeding in the positive y-direction. Here source potential is $-\varphi_{ms}$ and drain potential is $-\varphi_{ms}+V_{ds}$. So, the potential rises towards drain. We move from the source side and from the drain side, and we meet at the highest point at which

$$\psi_s(x,y_c,z)=\psi_d(x,y_c,z)$$

Considering only n=1,m=1 (which contributes to the largest term)

$$\left(\frac{sinh\left(\frac{k_y\pi}{L_{eff}}(L_{eff}-y_c)\right)}{\sinh(k_y\pi)}\right)(-\varphi_{ms})=\left(\frac{sinh\left(\frac{k_y\pi}{L_{eff}}y_c\right)}{\sinh(k_y\pi)}\right)(-\varphi_{ms}+V_{ds})$$

$$\left(sinh\left(\pi(L_{eff}-y_c)/L_D\right)\right)(-\varphi_{ms})=\left(sinh\left(\frac{\pi}{L_D}y_c\right)\right)(-\varphi_{ms}+V_{ds})$$

As we are moving along positive y, the positive y component of sinh will dominate and hence,

$$\frac{\exp\left(\pi\frac{L_{eff}-y_c}{L_D}\right)}{\exp\left(\frac{\pi}{L_D}y_c\right)}=\frac{(-\varphi_{ms}+V_{ds})}{-\varphi_{ms}}$$

$$\exp\left(\frac{\pi L_{eff}}{L_D}-\frac{2\pi y_c}{L_D}\right)=\frac{(-\varphi_{ms}+V_{ds})}{-\varphi_{ms}}$$

$$\frac{\pi L_{eff}}{L_D}-\frac{2\pi y_c}{L_D}=\ln\left(\frac{(-\varphi_{ms}+V_{ds})}{-\varphi_{ms}}\right)$$

$$y_c=\frac{L_{eff}}{2}+\frac{L_D}{2\pi}ln\left(\frac{-\varphi_{ms}}{-\varphi_{ms}+V_{ds}}\right)$$

V_t roll off:

$$\Delta V_{th} = \frac{\partial V_{gs}}{\partial \psi_c}\Delta\psi_c(V_{ds})$$

Or, $\Delta V_{th} = \frac{Sq}{2.3kT}\Delta\psi_c(V_{ds})$

At $V_{ds} = V_{ds}$

$$\psi_c = \frac{-16\varphi_{ms}}{\pi^2}\sin\left(\frac{\pi x_c}{T_{eff}}\right)\cos\left(\frac{\pi z_c}{2H_{eff}}\right)\frac{\sinh(\frac{k_y\pi}{L_{eff}}(L_{eff}-y_c))}{\sinh(k_y\pi)}$$

$$where, y_c = \frac{L_{eff}}{2}+\frac{L_D}{2\pi}ln\left(\frac{-\varphi_{ms}}{-\varphi_{ms}+V_{ds}}\right)$$

$$L_{eff}-y_c = \frac{L_{eff}}{2}-\frac{L_D}{2\pi}ln\left(\frac{-\varphi_{ms}}{-\varphi_{ms}+V_{ds}}\right)$$

Now, as we are moving along positive y direction, positive component of sinh will dominate:

$$\exp\left(k_y\pi\left(L_{eff}-y_c\right)\right) = \exp\left(\frac{\pi}{L_d}\left(\frac{L_{eff}}{2}-\frac{L_D}{2\pi}ln\left(\frac{-\varphi_{ms}}{-\varphi_{ms}+V_{ds}}\right)\right)\right)$$

$$= \exp\left(\frac{\pi L_{eff}}{2L_d}\right)\exp(-\frac{1}{2}\ln(\left(\frac{-\varphi_{ms}}{-\varphi_{ms}+V_{ds}}\right)))$$

$$= \exp\left(\frac{\pi L_{eff}}{2L_d}\right)\sqrt{(\varphi_{ms}-V_{ds})/\varphi_{ms}}$$

So,

$$\frac{\sinh(\frac{k_y\pi}{L_{eff}}(L_{eff}-y_c))}{\sinh(k_y\pi)} = \frac{\exp\left(\frac{\pi L_{eff}}{2L_d}\right)\sqrt{\frac{\varphi_{ms}-V_{ds}}{\varphi_{ms}}}}{\exp\left(\frac{L_{eff}\pi}{L_d}\right)} = \exp\left(-\frac{\pi L_{eff}}{2L_d}\right)\sqrt{(\varphi_{ms}-V_{ds})/\varphi_{ms}}$$

$$\psi_c = \frac{-16\varphi_{ms}}{\pi^2}\sin\left(\frac{\pi x_c}{T_{eff}}\right)\cos\left(\frac{\pi z_c}{2H_{eff}}\right)\exp\left(-\frac{\pi L_{eff}}{2L_d}\right)\sqrt{(\varphi_{ms}-V_{ds})/\varphi_{ms}}$$

$$\psi_c = \frac{-16}{\pi^2}\sin\left(\frac{\pi x_c}{T_{eff}}\right)\cos\left(\frac{\pi z_c}{2H_{eff}}\right)\exp\left(-\frac{\pi L_{eff}}{2L_d}\right)\sqrt{\varphi_{ms}(\varphi_{ms}-V_{ds})}$$

At $V_{ds} = 0$

$$\psi_c(V_{ds}=0) = \frac{-16}{\pi^2}\sin\left(\frac{\pi x_c}{T_{eff}}\right)\cos\left(\frac{\pi z_c}{2H_{eff}}\right)\exp\left(-\frac{\pi L_{eff}}{2L_d}\right)\sqrt{\varphi_{ms}^2}$$

$\Delta\psi_c(V_{ds}) = \frac{-32}{\pi^2}\sin\left(\frac{\pi x_c}{T_{eff}}\right)\cos\left(\frac{\pi z_c}{2H_{eff}}\right)\exp\left(-\frac{\pi L_{eff}}{2L_d}\right)(\sqrt{\varphi_{ms}(\varphi_{ms}-V_{ds})}-|\varphi_{ms}|)$

$So,$

$$\boxed{\Delta V_{th} = -\frac{Sq}{2.3kT}\,\frac{32}{\pi^2}\sin\left(\frac{\pi x_c}{T_{eff}}\right)\cos\left(\frac{\pi z_c}{2H_{eff}}\right)\exp\left(-\frac{\pi L_{eff}}{2L_d}\right)(\sqrt{\varphi_{ms}(\varphi_{ms}-V_{ds})}-|\varphi_{ms}|)}$$

4. Benefits of the Paper:

When Fin thickness is much larger than Fin height or when top gate oxide is much thinner than the front and back oxides, FinFET can be approximately treated as single-gate fully depleted SOI MOSFET (FDFET) as long as the silicon fin remains fully depleted. On the other hand, when Fin height is much larger than Fin thickness or top gate oxide is much thicker than the front and back oxides, FinFET can be approximately treated as DGFET. So from the analytical solutions of the FinFET only changing the boundary conditions we can get the potential distribution in the DGFET and FDFET.

Also from the analytical solutions the Subthreshold swing and voltage roll-off is calculated which are the measure of Short Channel Effects in the devices and they are compared.

5. Extent of understanding of the paper based on concepts covered in class:

In the class we have studied Electrostatics and with the help of Electrostatics how to go through the device problems. In case of FinFET, large Fin height was assumed and the 3 dimensional problem was converted into a 2 dimensional problem. Assuming no free charges inside the semiconductor the potential distribution inside the device was derived from the solution of Laplace equation. From that analytical solution the effect of drain and Gate was achieved by simple mathematics. Now for 3 dimensional case the height is finite and 3 dimensional Laplace equation is solved to achieve the potential profile inside the device. And again from that solution the subthreshold slope and threshold voltage roll-off is calculated in the paper. Changing the boundary conditions for DGFET and FDFET the potential profile and Short Channel Effects was calculated and compared using the same analogy.

ADDITIONAL QUESTION:

What?	The paper demonstrates the design consideration of 3D FinFET device by simulation and analytical modelling. And elaborates scaling effects from SCE in FinFET device design. FDFET, DGFET, FinFET, and SGFET are compared in basis of their advantages for selection from the analytical modelling.
Goal	Which device geometry can be used to reduce short channel effects and achieve better performance with miniaturization without any more changing the oxide thickness?
Methods	3-D FinFET with undoped channel, n polysilicon gate, uniform 1.5 nm gate oxide, abrupt source-to-channel and drain-to-channel junctions has been simulated in *FIELDAY*. The dependence of subthreshold swing and Threshold Voltage roll-off on Fin thickness and Fin height are shown from simulation. The electrostatic potential in the subthreshold region can be described by 3-D Laplace's equation if the potential perturbation due to the carrier concentration and generation are neglected (which is the case for undoped fin). From the analytical expressions of potentials the subthreshold swing and the roll-off in threshold voltage are calculated and compared with the simulation.
Experience	We can do the analytical modelling of any 3D device by simple electrostatics and some understanding of the device geometry and commonsense. The analytical model can be compared with the conducted simulation results to see its validity. We can extend this understanding to the other devices which can be easily achieved by only changing certain boundary conditions.