

HW-3 solutions (26/02/2015)

1. Carriers in Equilibrium: Assume a parabolic conduction band with $m^*=0.5m_0$ with band edge at $E=E_0$ at 300K;

- Please plot parabolic band $E(k)$;
- If $E_0-E_F=0.4\text{eV}$, please use MATLAB to plot E vs $f(E)$ i.e. probability function to compare Fermi Dirac vs Boltzmann distribution on the same plot;
- For same $E_0-E_F=0.5\text{eV}$, plot $N(E)$ Vs E , $f(E)*N(E)$ Vs E on same using FD distribution;
- Please repeat b-c for $E_0-E_F=0.05\text{eV}$;
- Compare $f(E)*N(E)$ plots using FD and MB distribution functions on same graph for 2 cases $E_0-E_F=0.5\text{eV}$ and $E_0-E_F=0.05\text{eV}$;
- Can you tell at what E_0-E_F is there a 10% difference in n between FD and Boltzmann Distributions?

Assume $E_0=0\text{eV}$ (bottom of Conduction band). $T=300\text{K}$

NOTE: All above plots to be made are for conduction band only; no need to plot valence band. Please ignore the parameter $n(E)$ in the question.

Use $N_c=3E19\text{cm}^{-3}$

Solution 1:

A. Parabolic conduction band:

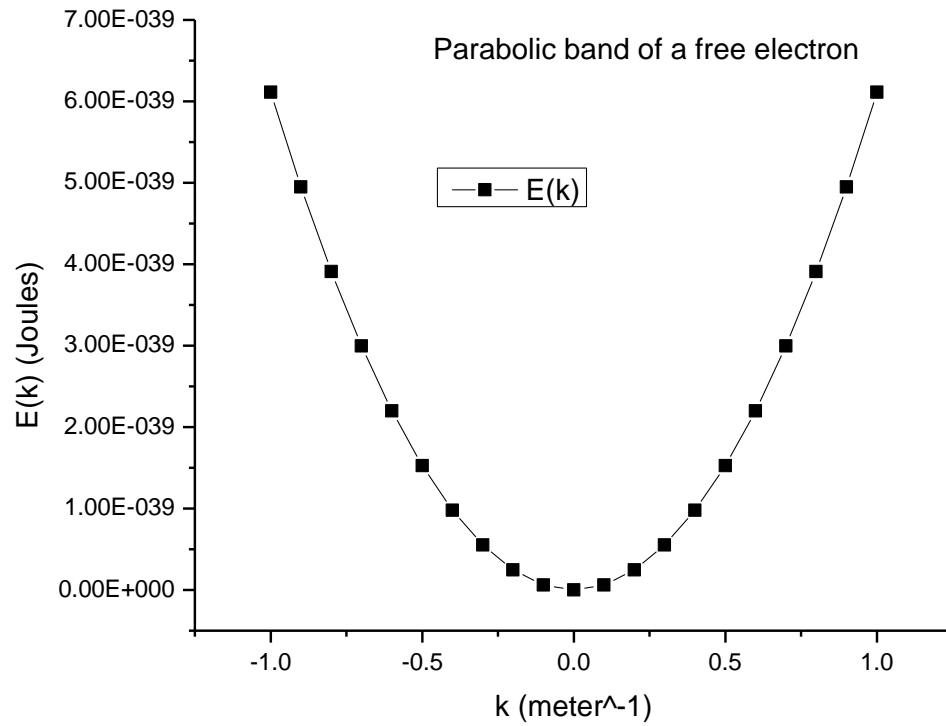
$$E(k) = \frac{\hbar^2 k^2}{2m^*}, \quad \text{where } m^* = 0.5m_0$$

$$\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$m_0 = 9.1 \times 10^{-31} \text{ kg}$$

Hence,

$$E(k) = 6.11 \times 10^{-39} k^2 \text{ (Joules)} \quad (k : \text{meter}^{-1})$$



- B. For the case of Fermi level E_F , $E_0 - E_F = 0.4\text{eV}$ at 300K, let E_0 (bottom of conduction band) = 0; hence $E_F = -0.4\text{eV}$; selecting range of $E = -1.2$ to 0.2eV covering the silicon conduction band;

Statistical Dist. functions :

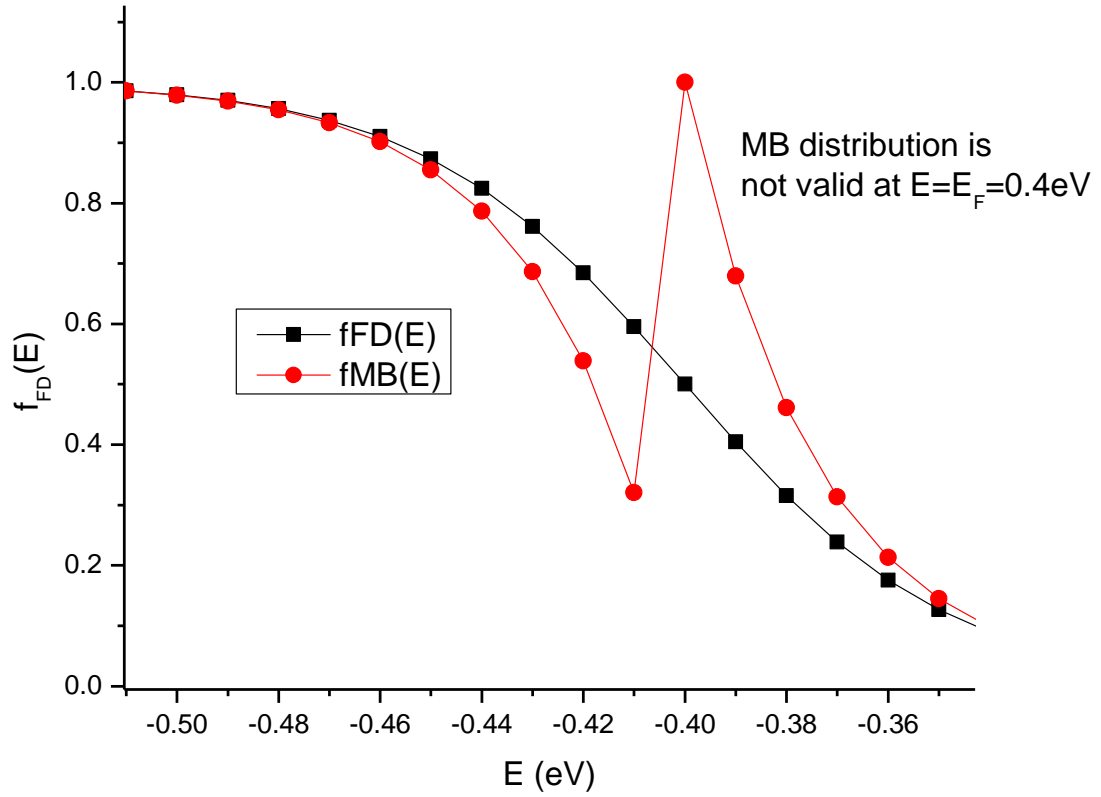
1. *FermiDirac function :*

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

2. *Maxwell Boltzmann dist function (valid when E_F is atleast $3kT$ away from band edges)*

$$f(E) = \exp\left(-\frac{(E - E_F)}{kT}\right) \text{ for } E > E_F$$

$$f(E) = 1 - \exp\left(\frac{(E - E_F)}{kT}\right) \text{ for } E < E_F$$



Note that the MB is discontinuous at EF.

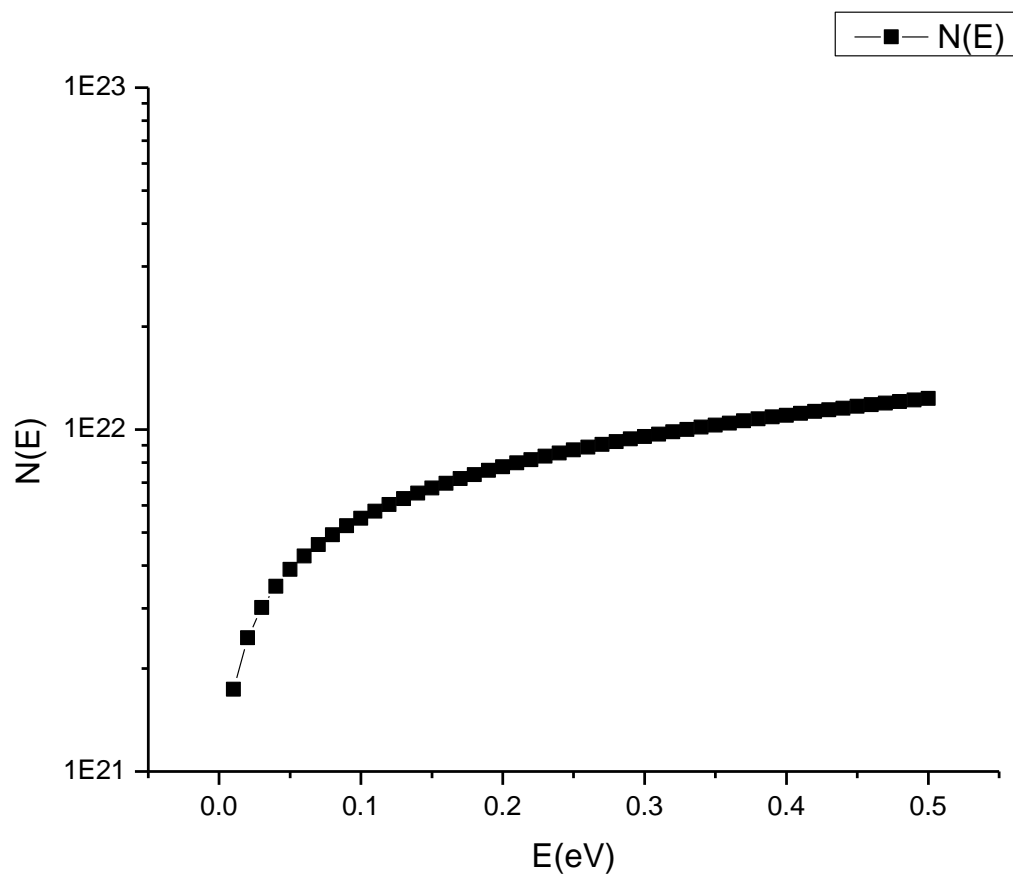
- C. For same $E_O - E_F = 0.5\text{eV}$, please plot $N(E) \cdot f(E)$ and calculate electron concentration by numerical integration; $m_l^* = 0.98m_0$; $m_t^* = 0.19m_0$; let bottom of CB = $E_c = 0\text{eV}$;

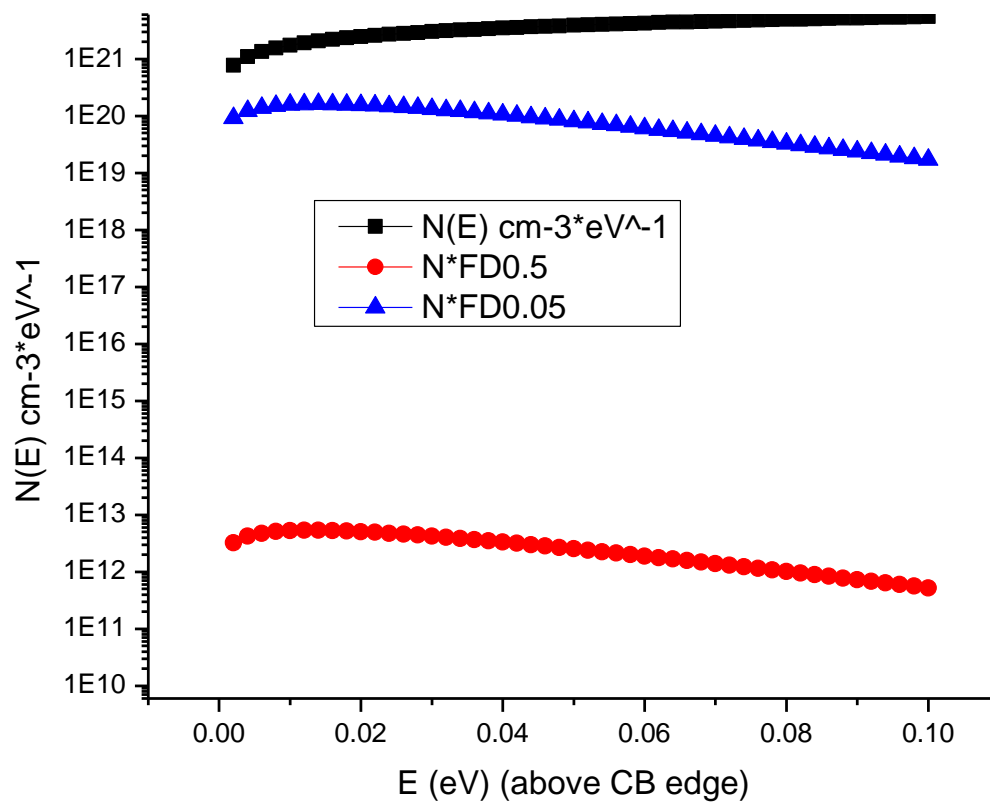
For CB,

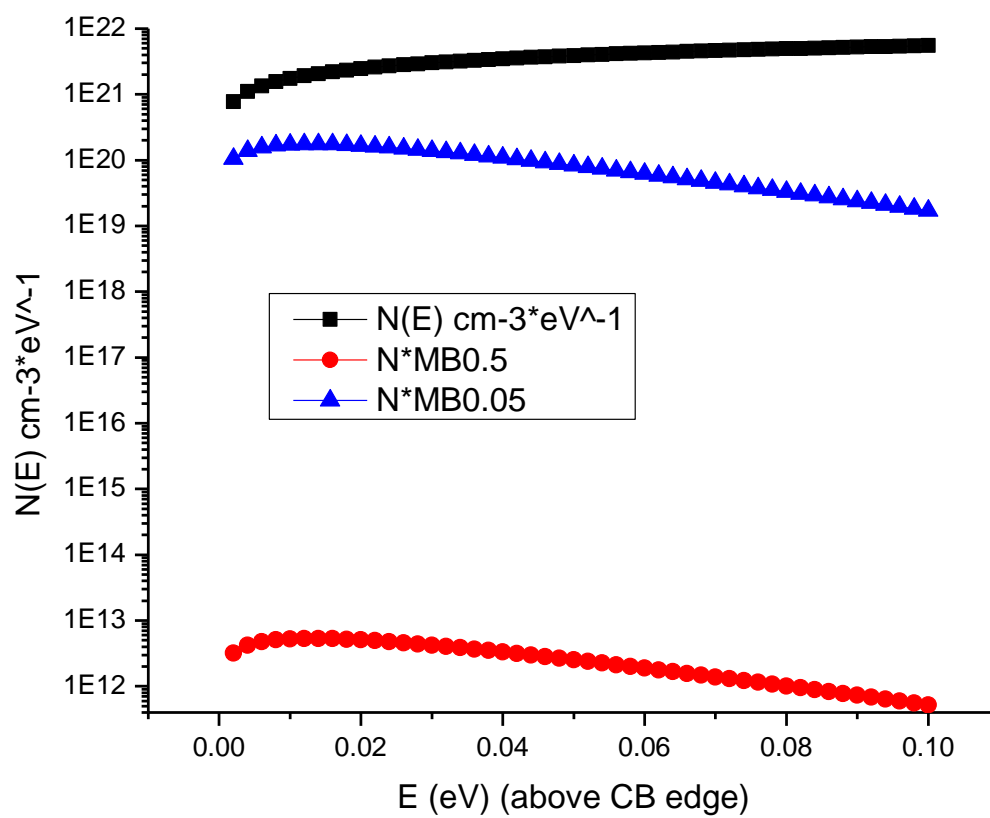
$$N(E) = \frac{8\pi g \sqrt{2(m_l^*)(m_t^*)^2}}{h^3} \sqrt{E - E_c}$$

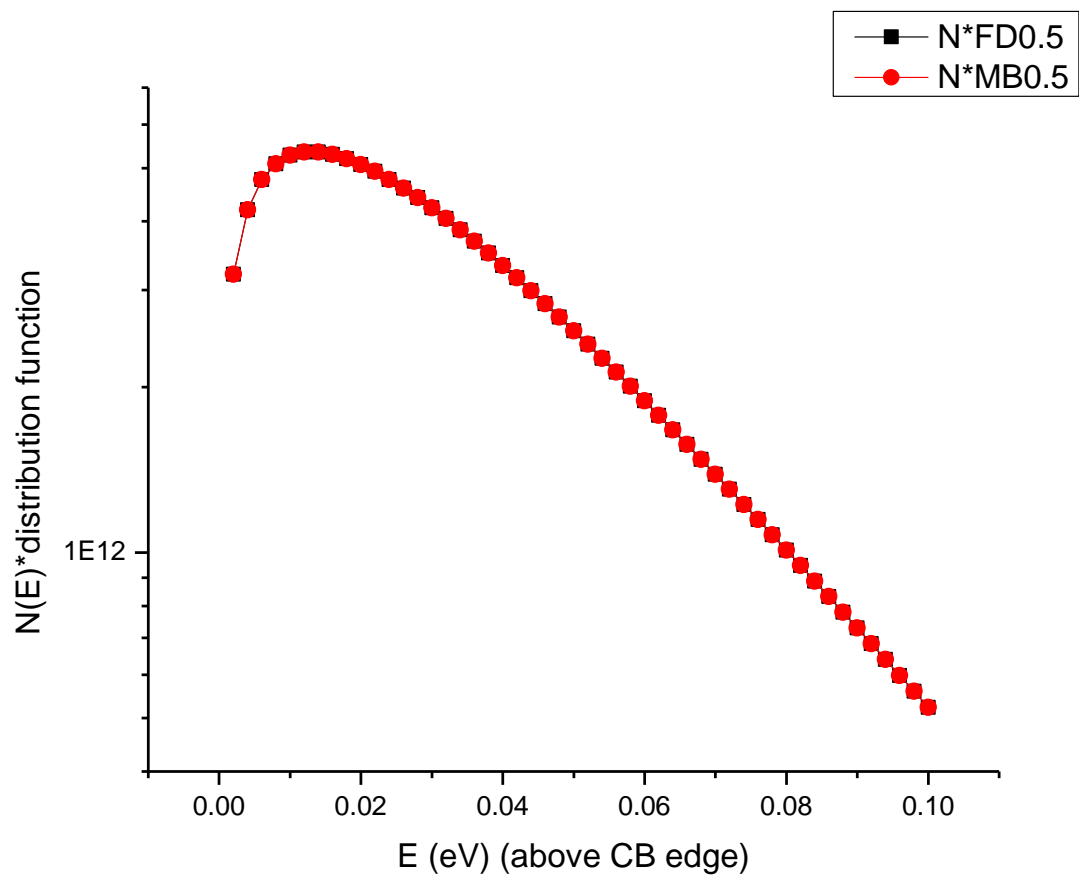
$$N(E) = 2.72E56 \sqrt{E} (m^{-3} J^{-1})$$

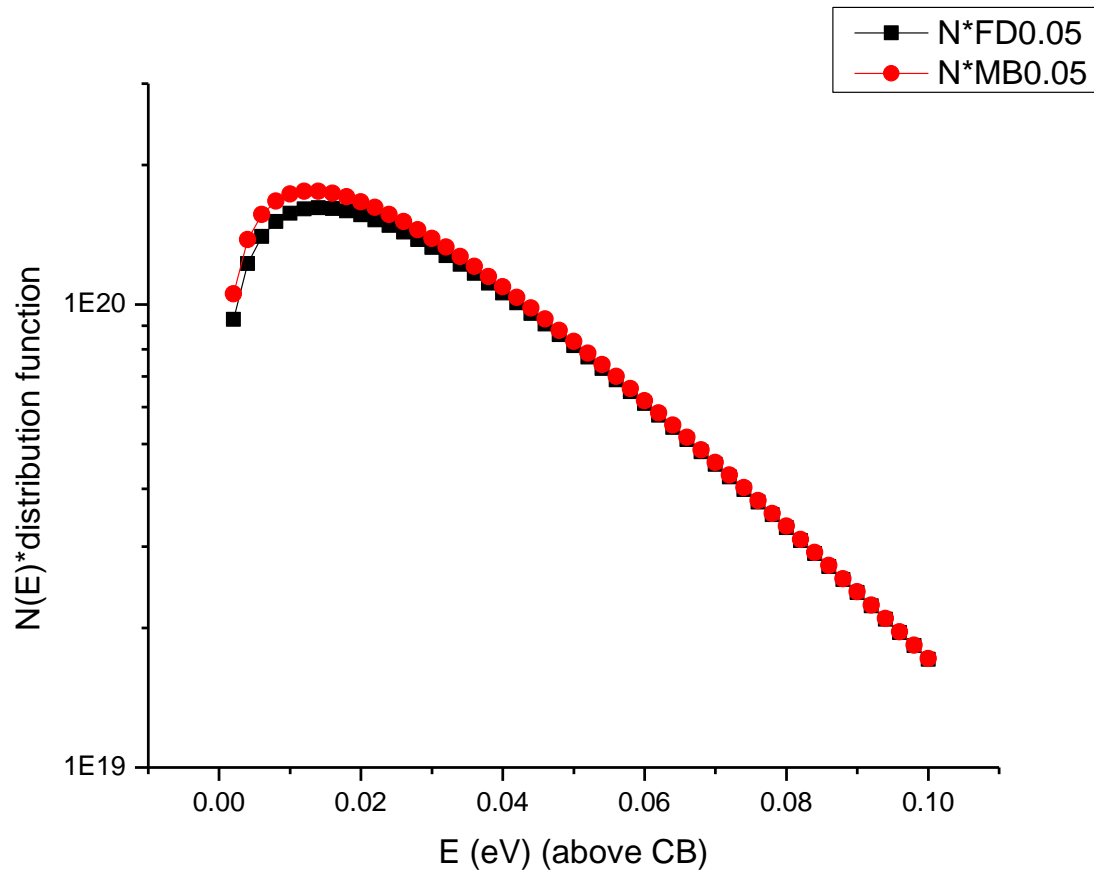
$$n(E) = N(E) f(E)$$





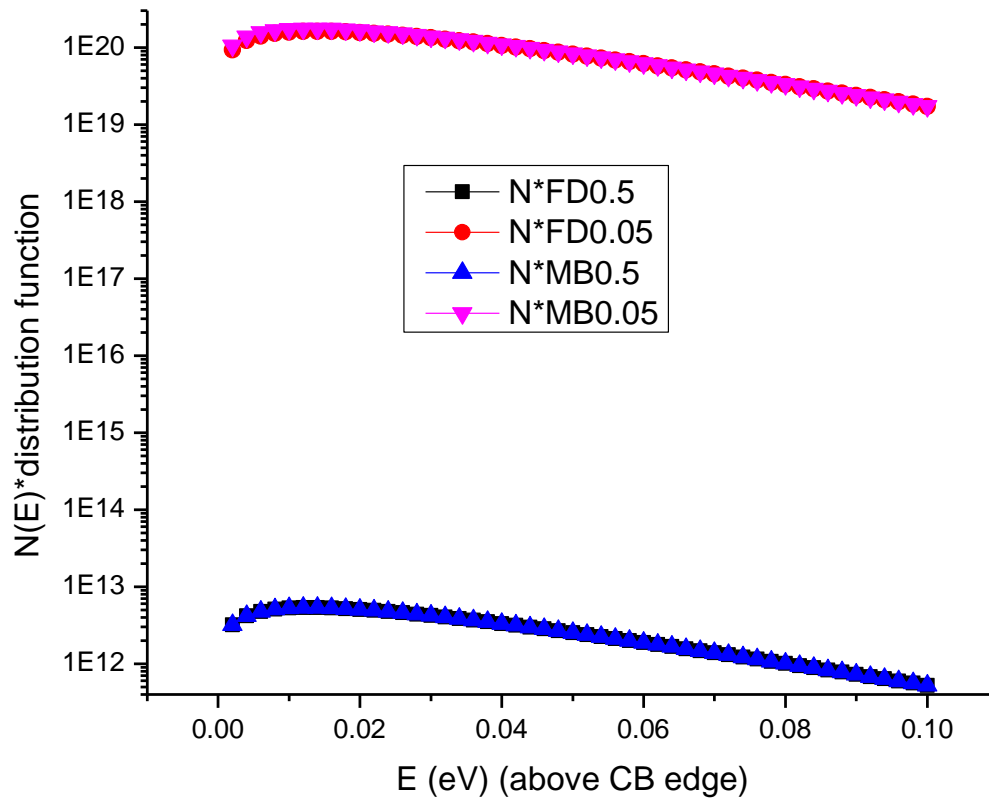






D. Please repeat b-c for $E_0 - E_F = 0.05$ eV,

E. Compare $f(E) \cdot N(E)$ plots using FD and MB distribution functions on same graph for 2 cases $E_0 - E_F = 0.5$ eV and $E_0 - E_F = 0.05$ eV;



F. Can you tell at what $E_O - E_F$ is there a 10% difference in electron concentration between FD and Boltzmann Distributions?

Note Vertical axis: Percent difference between electron concentration between FD and MB distributions; Horizontal axis : Fermi level;

