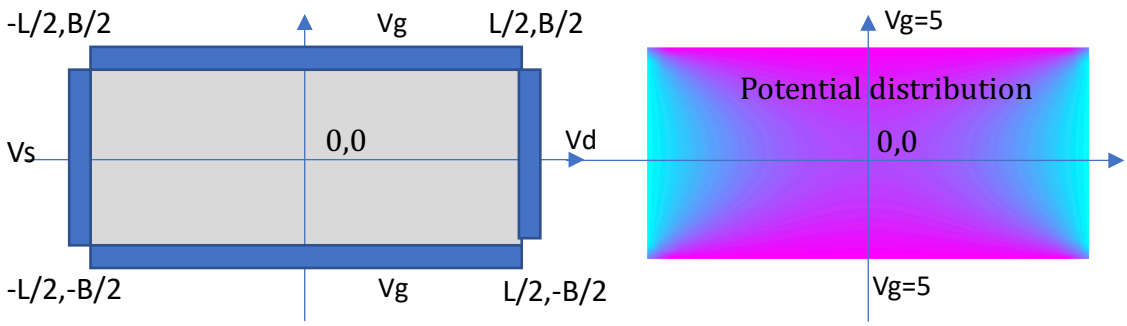
1.



(a) Strategy to determine potential at any (x,y) point (10 marks)

a. Potential at any point is given by V(x,y)
b. Laplace is true as no free charge: $\nabla^2 V(x,y) = 0$
c. Assuming the potential component in the x and y directions are independent
d. To solve for potential using separation of variables to Laplace in single variable differential equations.
   i. $V(x,y) = V(x)V(y) = X(x)\,Y(y)$
   ii. $\nabla^2 V(x,y) = \nabla^2\big(V(x)V(y)\big) = \nabla^2 XY = 0$
   iii. $\nabla^2 XY = Y\nabla^2 X + X\nabla^2 Y = 0$
   iv. $\frac{\nabla^2 X}{X} + \frac{\nabla^2 Y}{Y} = 0$
   v. Eq (iv) will be zero if both terms are equal and opposite
   vi. $\frac{\nabla^2 X}{X} = -k^2;\ \frac{\nabla^2 Y}{Y} = k^2$
   vii. Solving (vi);
   $X = A\cos k_n x + B\sin k_n x$;
   $Y = C\cosh k_n y + D\sinh k_n y$

The boundary conditions:

$X(x, y=B/2) = Vg = 5V$; $X(x, y=-B/2) = Vg = 5V$
$Y(x=L/2, y) = 0V$; $Y(x=-L/2, y) = 0V$

The boundary conditions are symmetric on both axis, so,

$X = A\cos k_n x$ and $Y = C\cosh k_n y$

$V(x,y) = \sum_{n=0}^{\infty} G_n \cos k_n x \cosh k_n y$; $k_n = (2n+1)\frac{\pi}{2}$

Multiply both sides by $\cos k'_n x$ and integrate from x=-L/2 to x=L/2

LHS: $\frac{2V_g}{k'_n}\left(\sin(2n+1)\frac{\pi}{2}\right) = (-1)^n (2V_g/k'_n)$

RHS: Only terms with $k_n = k_n'$ remains

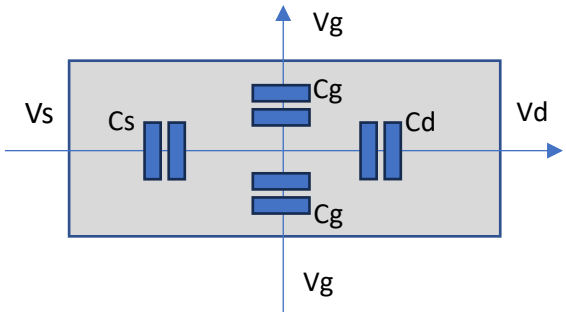
$$G_n \frac{L}{2}\cosh(k_n B/2)$$

Equating LHS and RHS

$$G_n = (-1)^n \frac{4V_g}{Lk_n \cosh(k_n B/2)}$$

$$V(x,y) = \sum_{n=0}^{\infty}(-1)^n \frac{4V_g}{Lk_n \cosh(k_n B/2)} \cos k_n x \ \cosh k_n y$$

1. (b)
   (i) The first term (n=0) will be dominant. The value of the function is maximum at this point and decreases as n increases.
   (ii) Capacitive equivalent circuit:

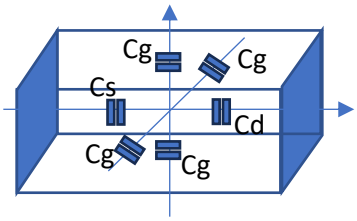


$$V_b = \frac{C_s V_s + C_d V_d + 2C_g V_g}{C_s + C_d + 2C_g}$$

$$\frac{V_b}{V_g} = \frac{2C_g}{C_s + C_d + 2C_g}$$

   (iii) $SS = \frac{dV_b}{dV_g}$
   (iv) SS is independent of the dielectric constant. No change in SS.

1. (c) Diagram = 5 marks ; Similarities = 5 marks; Differences = 5 marks



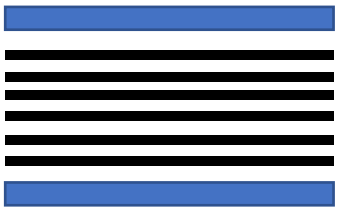
Similarities:

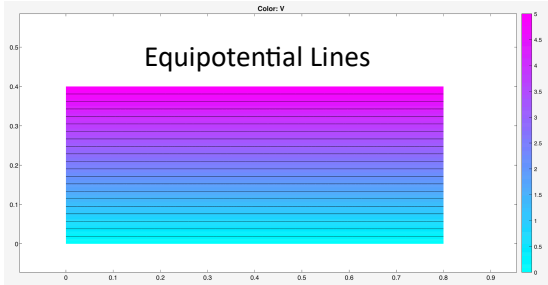
- 1D capacitive model equivalent valid
- The maximum Barrier position is the same
- Barrier voltage is dependent on the geometry

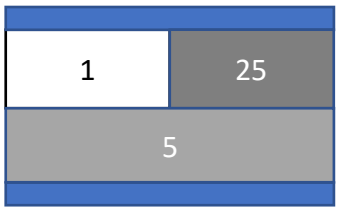
Differences:

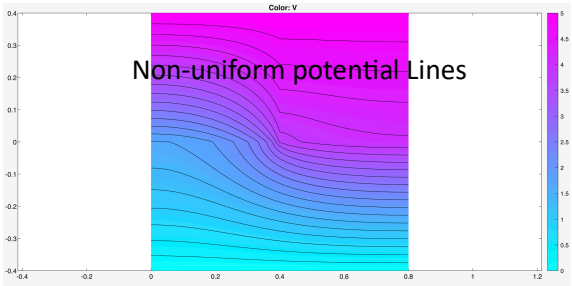
- Gate capacitance contribution is higher in 3D NW
- 3D NW has better gate control over 2D NR
- Magnitude of barrier voltage will be different

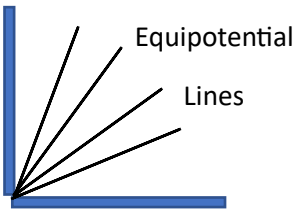
2. (a) First and Third structures can be simplified into smaller capacitors
    - First will be 2 capacitors in parallel
    - Second structure cannot be simplified
    - Third will be capacitors in series

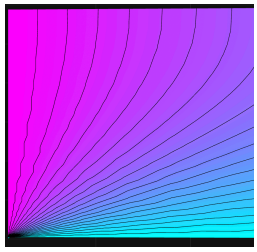






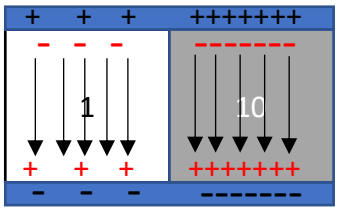


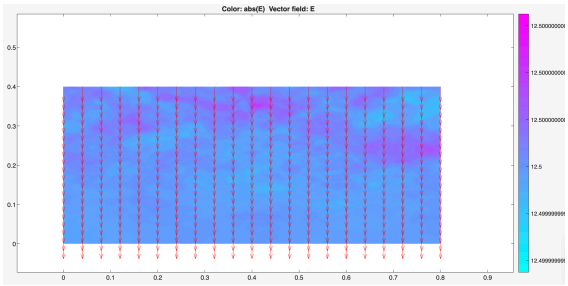




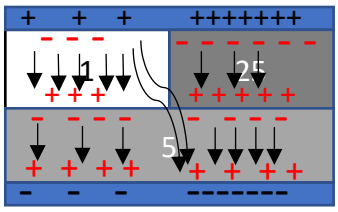
(b)

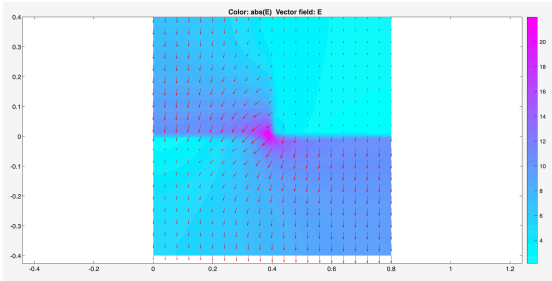
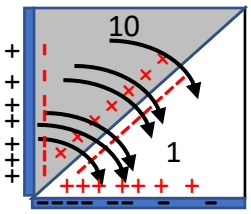
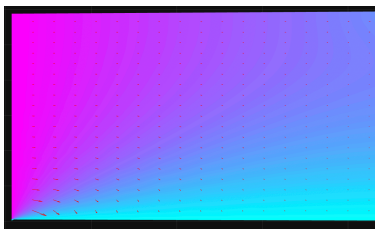
(i) Uniform EF as E=V/d





(ii) Bound charges at the interface; Non-uniform EF





(iii) Bound charges