04 Matrix Multiplication

Monower hossen

Introduction:

We've seen matrix addition and scalar multiplication, which behave similarly to real numbers. Naturally, we ask if two matrices can be multiplied. The answer is yes, but matrix multiplication is more complex than addition.

Definition:

If $A = [a_{ij}]_{m\times p}$ and $B = [b_{ij}]_{p\times n}$, their product $C = AB$ is an $m \times n$ matrix with entries:

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ip}b_{pj}$$

- $a_{i1}, a_{i2}, \ldots, a_{ip}$ are row i elements of A.
- $b_{1j}, b_{2j}, \ldots, b_{pj}$ are column j elements of B.
- Multiplication is defined only if the number of columns of A equals the number of rows of B.

Matrix Multiplication (2×2)

$$\mathrm{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } \mathrm{B} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$AB = \begin{bmatrix} (ap+br) & (aq+bs) \\ (cp+dr) & (cq+ds) \end{bmatrix}$$

Matrix Multiplication (3×3)

$$\mathrm{X} = \begin{bmatrix} \mathrm{x}_{11} & \mathrm{x}_{12} & \mathrm{x}_{13} \\ \mathrm{x}_{21} & \mathrm{x}_{22} & \mathrm{x}_{23} \\ \mathrm{x}_{31} & \mathrm{x}_{32} & \mathrm{x}_{33} \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

$$\text{XY} = \begin{bmatrix} (x_{11}y_{11}+x_{12}y_{21}+x_{13}y_{31}) & (x_{11}y_{12}+x_{12}y_{22}+x_{13}y_{32}) & (x_{11}y_{13}+x_{12}y_{23}+x_{13}y_{33}) \\ (x_{21}y_{11}+x_{22}y_{21}+x_{23}y_{31}) & (x_{21}y_{12}+x_{22}y_{22}+x_{23}y_{32}) & (x_{21}y_{13}+x_{22}y_{23}+x_{23}y_{33}) \\ (x_{31}y_{11}+x_{32}y_{21}+x_{33}y_{31}) & (x_{31}y_{12}+x_{32}y_{22}+x_{33}y_{32}) & (x_{31}y_{13}+x_{32}y_{23}+x_{33}y_{33}) \end{bmatrix}$$

Solved Question on Matrix Multiplication

Example 1.

Let $A = \begin{bmatrix} 1 & 8 & 3 \\ 9 & 4 & 5 \\ 6 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 & 4 \\ 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix}$ Find A×B?

Solution:

$$A \times B = \begin{bmatrix} 1 & 8 & 3 \\ 9 & 4 & 5 \\ 6 & 2 & 7 \end{bmatrix} \times \begin{bmatrix} 6 & 7 & 4 \\ 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times 6 + 8 \times 1 + 3 \times 5) & (1 \times 7 + 8 \times 3 + 3 \times 9) & (1 \times 4 + 8 \times 2 + 3 \times 8) \\ (9 \times 6 + 4 \times 1 + 5 \times 5) & (9 \times 7 + 4 \times 3 + 5 \times 9) & (9 \times 4 + 4 \times 2 + 5 \times 8) \\ (6 \times 6 + 2 \times 1 + 7 \times 5) & (6 \times 7 + 2 \times 3 + 7 \times 9) & (6 \times 4 + 2 \times 2 + 7 \times 8) \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 58 & 44 \\ 83 & 120 & 84 \\ 73 & 111 & 84 \end{bmatrix}$$

Example 2. Let $A = \begin{bmatrix} 1 & 5 & 4 \\ 9 & 3 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 1 & 3 \\ 5 & 9 \end{bmatrix}$. Find A×B?

Solution:

$$A \times B = \begin{bmatrix} (1 \times 6 + 5 \times 1 + 4 \times 5) & (1 \times 7 + 5 \times 3 + 4 \times 9) \\ (9 \times 6 + 3 \times 1 + 8 \times 5) & (9 \times 7 + 3 \times 3 + 8 \times 9) \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 58 \\ 97 & 144 \end{bmatrix}$$

Example 3. Let $A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$. Find **(AB + AC)?**

Solution:

$$A \times B = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (2 \times 3 + 0 \times (-1) + (-3) \times 4) & (2 \times 1 + 0 \times 0 + (-3) \times 2) \\ (1 \times 3 + 4 \times (-1) + 5 \times 4) & (1 \times 1 + 4 \times 0 + 5 \times 2) \end{bmatrix} = \begin{bmatrix} -6 & -4 \\ 19 & 11 \end{bmatrix}$$

$$A \times C = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (2 \times 4 + 0 \times 2 + (-3) \times 1) & (2 \times 7 + 0 \times 1 + (-3) \times (-1)) \\ (1 \times 4 + 4 \times 2 + 5 \times 1) & (1 \times 7 + 4 \times 1 + 5 \times (-1)) \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ 17 & 6 \end{bmatrix}$$

Now calculate (AB + AC)

$$= \begin{bmatrix} -6 & -4 \\ 19 & 11 \end{bmatrix} + \begin{bmatrix} 5 & 17 \\ 17 & 6 \end{bmatrix}$$

$$(AB + BC) = \begin{bmatrix} -1 & 13 \\ 36 & 17 \end{bmatrix}$$

Example 4. Let $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$, A^2 = pA, then find the value of **p?**

Solution:

Calculating, A^2

$A^2 = A \times A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \text{ X } \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} (2 \times 2 + (\text{-}2) \times (-2) & ((2((2 \times (-2) + (-2) \times 2) \\ ((2 \times (-2) + (-2) \times 2) & ((-2) \times (-2) + 2 \times 2) \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

Given,
$A^2 = pA$

Taking A^2 in the equation,

$$\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = p \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix}$$

Now,

- *$8 = 2p$*
- *$-8 = -2p$*

p = 4

*Thus, the value of p is **4***

Example 5: Find the value of 3P if $P = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 5 \\ 7 & -4 & 6 \end{bmatrix}$.

Solution:

$$3P = 3 \times \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 5 \\ 7 & -4 & 6 \end{bmatrix}$$

$$3P = \begin{bmatrix} 3 \times 2 & 3 \times -3 & 3 \times 4 \\ 3 \times 1 & 3 \times 0 & 3 \times 5 \\ 3 \times 7 & 3 \times -4 & 3 \times 6 \end{bmatrix}$$

$$3P = \begin{bmatrix} 6 & -9 & 12 \\ 3 & 0 & 15 \\ 21 & -12 & 18 \end{bmatrix}$$