06. Determinant, Singular & Non-Singular Matrix

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1. Determinant of a Matrix

Definition

The **determinant** is a scalar value that can be computed from a **square matrix** $A \in \mathbb{R}^{n \times n}$.

$$\det(A) \text{ or } |A|$$

It provides information about:

- **Invertibility**
- **Scaling factor** of linear transformation
- **Volume change** in vector space

1.1 Example (2×2 matrix)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \det(A) = ad - bc$$

Interpretation:

- If $\det(A) = 0$, the **matrix is singular** (no inverse, collapses space).
- If $\det(A) \neq 0$, the **matrix is non-singular** (invertible).

1.2 Example (3×3 matrix)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

$$de\,t(A) = 1(5 \times 10 - 6 \times 8) - 2(4 \times 10 - 6 \times 7) + 3(4 \times 8 - 5 \times 7) = -3$$

1.3 Properties of Determinants

Property	Formula	Meaning
Product	$\det(AB) = \det(A)\det(B)$	Determinant multiplies
Transpose	$\det(A^T) = \det(A)$	Same determinant
Inverse	$\det(A^{-1}) = 1/\det(A)$	Only if non-zero
Triangular	$\det(A) = \prod_i a_{ii}$	Diagonal product for triangular matrices

2. Singular Matrix

Definition

A matrix $A$is **singular** if:

$$\det(A) = 0$$

Implications:

- Not invertible (A^{-1} does not exist)
- Rows or columns are **linearly dependent**
- The transformation collapses **volume** to zero

ML Example:

- In linear regression, $X^T X$being singular means **perfect multicollinearity** → cannot compute $w = (X^T X)^{-1} X^T y$

2.1 Example

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}, \det(A) = 2 \times 2 - 4 \times 1 = 0$$

Since det(A) = 0 the matrix is Singular (not invertible)

3. Non-Singular Matrix

Definition

A matrix $A$is **non-singular** if:

$$\det(A) \neq 0$$

Implications:

- Invertible (A^{-1} exists)
- Rows and columns are **linearly independent**
- Transformation preserves **volume scaling factor ≠ 0**

ML Example:

- Solving linear systems $Ax = b$
- Inverting covariance matrices in **PCA or Gaussian distributions**

3.1 Example

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \det(A) = 1 \times 4 - 2 \times 3 = -2 \neq 0$$

The determinant of matrix A is -2 which is non zero . So meaning A is Non-Singular matrix

Singular Matrix	Non-Singular Matrix
A square matrix is said to be a singular matrix if its determinant is zero, i.e., det A = 0.	A square matrix is said to be a non-singular matrix if its determinant is not zero, i.e., det A ≠ 0.
If a matrix is singular, then its inverse is not defined.	If a matrix is non-singular, then its inverse is defined.
The rank of a singular matrix will be less than the order of the matrix, i.e., Rank (A) < Order of A.	The rank of a non-singular matrix will be equal to the order of the matrix, i.e., Rank (A) = Order of A.
In a singular matrix, some rows and columns are linearly dependent.	In a non-singular matrix, all the rows and columns are linearly independent.
$A = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 1 & 2 \\ 3 & 7 & 9 \end{pmatrix}$	$B = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 0 & 8 \\ -1 & 4 & 0 \end{bmatrix}$

4. Python / NumPy Examples

```
import numpy as np

# 1. Determinant
A = np.array([[1, 2],
              [3, 4]])
det_A = np.linalg.det(A)
print("Determinant of A:", det_A)

# 2. Check singular or non-singular
if det_A == 0:
    print("Matrix A is Singular (Not Invertible)")
else:
    print("Matrix A is Non-Singular (Invertible)")

# 3. Singular example
B = np.array([[2, 4],
              [1, 2]])
det_B = np.linalg.det(B)
print("Determinant of B:", det_B)
print("Matrix B is", "Singular" if det_B==0 else "Non-Singular")

# 4. Inverse (only for non-singular)
if det_A != 0:
    A_inv = np.linalg.inv(A)
    print("Inverse of A:\n", A_inv)
```

5. ML & AI Applications

Concept	Use in ML/AI
Determinant	Measures volume change under transformation
Singular Matrix	Multicollinearity, non-invertible covariance
Non-Singular Matrix	Linear regression, PCA, Gaussian models
Inverse Matrix	Solving linear systems, computing weights