$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$B \cdot B^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 3+8 \\ 3+8 & 9+16 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 11 \\ 11 & 21 \end{bmatrix}$$

Singular Matrix :

A singular Matrix is a square matrix whose determine is zero.

$$\det(A) = 0$$

properties

A^{-1} doesn't exist

Non - singular matrix

$$\det(A) \neq 0$$

$\therefore$ A^{-1} can be completed

Determinant of Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \det(A) = ad - cd$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$\det(A) = (2 \times 4 - 3 \times 1) = 8 - 3 = 5$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det(A) = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} - a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

where each 2×2 determinant is called a minor

#

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= 1(45-48) - 2(36-42) + 3(32-35)$$

$$= -3 + 12 - 9$$

$$= 0$$

Since $\det(A) = 0$ the matrix is singular (not invertible)

#

$$B = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 5 & -2 & 1 \end{bmatrix}$$

$$\det(B) = 2\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 4 \\ 5 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 3 \\ 5 & -2 \end{vmatrix}$$

$$= 2(3+8) + 1(1+20) + 0(-2-15)$$

$= 2 \times 11 + 1 \times (-19) + 0 \times (-17)$

$= 22 - 19 + 0$

$= 3$

The determinant of Matrix B is 3 which is non zero meaning B is invertible.