

▣ Addition and subtraction

▣ Addition

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{3 \times 2}$$

$$B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}_{3 \times 2}$$

$$A + B = \begin{bmatrix} 1+7 & 2+8 & 3+9 \\ 4+10 & 5+11 & 6+12 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

▣ Subtraction

$$A - B = \begin{bmatrix} 1-7 & 2-8 & 3-9 \\ 4-10 & 5-11 & 6-12 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -6 & -6 \\ -6 & -6 & -6 \end{bmatrix}$$

Scaler Broadcasting

$$A + S = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + S$$

$$= \begin{bmatrix} a_{11} + S & a_{12} + S \\ a_{21} + S & a_{22} + S \end{bmatrix}$$

Scaler addition

$$A = \begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}$$

$$A + 3 = \begin{bmatrix} 2 + 3 & 5 + 3 \\ 7 + 3 & 1 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 \\ 10 & 4 \end{bmatrix}$$

Trace Calculation

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{tr}(A) = 1 + 5 + 9 = 15$$

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 & 1 \\ 6 & 4 & 3 \\ 7 & 8 & 8 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 7 & 6 & 7 \\ 7 & 7 & 8 \\ 14 & 17 & 17 \end{bmatrix}$$

$$\therefore \text{tr}(A+B) = 7 + 7 + 17 = 31$$

$$A-B = \begin{bmatrix} -3 & 2 & 5 \\ -5 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \text{tr}(A-B) = (-3) + (-1) + 1 = -3$$

Matrix Multiplication

$$A_{m \times n} \times B_{n \times p} = C_{m \times p}$$

1 row matrix \times 2 matrix column

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \quad \downarrow \text{Column}$$

$\xrightarrow{\text{Row}}$

$$C_{ij} = \sum (\text{Row of } A \times \text{Column of } B)$$

$$C = \begin{bmatrix} (1 \times 5 + 2 \times 7) & (1 \times 6 + 2 \times 8) \\ (3 \times 5 + 4 \times 7) & (3 \times 6 + 4 \times 8) \end{bmatrix}$$

$$C = \begin{bmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$