Set & Subsets, Vector Space, Linear Independent, Norms

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1. Sets

Definition

A **set** is a collection of distinct objects.

$$A = \{1,2,3\}$$

Subset

A set $A$is a **subset** of Bif every element of $A$is also in B:

$$A \subseteq B$$

Types of Sets

- **Finite set**: $\{1, 2, 3\}$
- **Infinite set**: $\mathbb{R}$
- **Empty set**: $\emptyset$
- **Subset**: $A \subset B$

ML Connection

- Dataset → **set of samples**

$$X = \{x_1, x_2, \dots, x_n\}$$

- Feature space → set of all possible vectors

2. Vector Space

Definition

A **vector space** is a set of vectors where:

- Vector addition is possible
- Scalar multiplication is possible
- Certain axioms hold (closure, identity, etc.)

Example

$$\mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}$$

Properties

For vectors **u, v** and scalar **c**:

- $u + v \in V$
- $c \cdot u \in V$

ML Examples

ML Concept	Vector Space
Feature vector	$\mathbb{R}^n$
Word embedding	$\mathbb{R}^{300}$
Image (flattened)	$\mathbb{R}^{784}$

ML Insight:

Learning algorithms search for patterns **inside vector spaces**, not raw data.

3. Linear Independence

Definition

Vectors are **linearly independent** if:

$$c_1v_1 + c_2v_2 + \cdots + c_nv_n = 0 \Rightarrow c_1 = c_2 = \cdots = c_n = 0$$

Example

$$v_1 = (1,0)$$

$$v_2 = (0,1)$$

In $\mathbb{R}^2$ *are independent.*

$$v_1 = (1,2), v_2 = (2,4)$$

(1,2) and (2,4) are Dependent because (1,2) = 2 (1,2)

Why Important in ML?

- **Redundant features** → linearly dependent
- **PCA** removes dependent dimensions
- **Full-rank matrix** needed for invertibility

4. Norms

Definition

A **norm** measures the **length/magnitude** of a vector.

$$\| x \|$$

Common Norms

1. **L1 Norm (Manhattan)**

$$\| x \|_1 = \sum | x_i |$$

Used for sparsity and feature selection.

2. L2 Norm (Euclidean)

$$\| x \|_2 = \sqrt{\sum x_i^2}$$

Used for distance measurement and smooth optimization.

3. L∞ Norm

$$\| x \|_\infty = \max \ | x_i |$$

Used in robustness analysis.

ML Usage

Norm	Used In
L1	Lasso Regression (sparse features)
L2	Ridge Regression, SVM
L∞	Robust optimization

ML Flow

- Data → **Set**
- Features → **Vector Space**
- Feature quality → **Linear Independence**
- Optimization → **Norm minimization**

ML Algorithm Mapping

Core Topic	ML Algorithms Methods	Why It Is Used
Sets	Dataset handling, Train/Test split, Cross-validation	Data is treated as structured collections
Vector Space	Linear Regression, Logistic Regression, SVM, KNN	Models operate on vectors in $\mathbb{R}^n$
Subspace	PCA, LDA, Autoencoders	Reduce dimensionality while preserving information
Linear Independence	PCA, Linear Regression	Avoid redundant features, ensure numerical stability
Norms	Gradient Descent, SVM, Ridge/Lasso	Measure distance, loss, and regularization strength

2. Concept → Purpose in Machine Learning (Core Math View)

Concept	ML Purpose
Sets	Data organization and formal definition of datasets
Subsets	Train / Validation / Test splitting
Vector Space	Mathematical representation of features
Subspace	Dimensionality reduction and feature selection
Linear Independence	Feature quality and multicollinearity control
Norms	Distance, loss functions, regularization