Item: Riemann Integral

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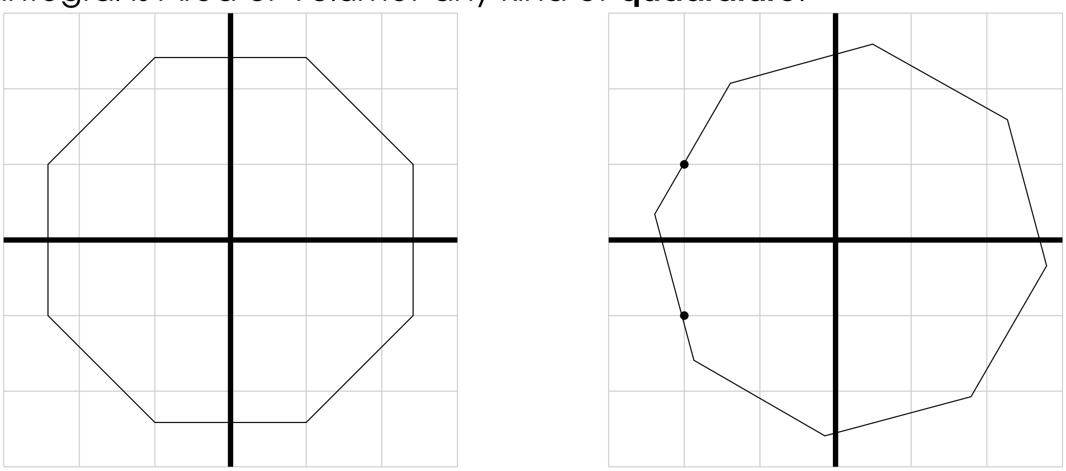
In the math class we learn that Riemann sums converge to the Riemann integral.

$$\lim_{|\Delta x| \to 0} \sum_{i=0}^{N-1} f(x_i)\, \Delta x_i = \int_0^1 f(x)\, dx$$

where the interval $[0, 1]$ has been partitioned into small intervals $[x_i, x_{i+1}]$.

There are doubts that I have from when I learned Real Analysis. The most vanilla case of an integral is Area or Volume: any kind of **quadrature**.



Let us push that to an extreme: all integrals are areas; all integrals are quadratures.

Q1 As a warm-up. Does the rotated octagon (right) pass through the two marks at $(-2, \pm 1)$?

Q2 No matter how I lay the octagon on the grid, there's always a tiny triangle!

We are going to compute the octagon by decomposing into squares (and chunks of squares).

- homology
- scissors congruence

These are some buzzwords in the literature.

Originally, I had been concerned about integrating series. Does $\int \Sigma = \Sigma \int$? Obviously:

$$\int_0^1 f(x)\, dx + \int_0^1 g(x)\, dx = \int_0^1 \Big(f(x) + g(x) \Big)\, dx$$

Our $f(x)$ is very well behaved $f(x) \equiv 1$ everywhere. So there is no issue just yet.

References

(1) ...