Scratchwork: Quaternionic Shimura Varieties

We are going to try to parse whatever we can in one of the latest editions of Advances in Mathematics. My theory is that a lot of these papers at least start off about things we are familiar with. Too much symbolic manipulation makes me a bit critical that we've lost track.

> The aim of this paper is to compare arithmetic intersection numbers on Shimura varieties attached to inner forms of GL_2 over a real quadratic field F.
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> To illustrate our approach, first consider the arithmetic volume of a Shimura variety attached to an inner form of GL_2 over $\mathbb{Q}$.
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> ...
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> We begin with a division quaternion algebra B over F whose discriminant $D_B := (p_1, \ldots, p_r) \cdot \mathcal{O}_F$ is a non-empty product of split rational primes.

While I may be grossly underestimating the difficulty of such a task, I feel there should be instances where the problem is easy to state and describe. Even if solving is a ton of work.[1] Even when they solve it, they understate the meaning so much it's just totally lost.

References

[1] Gerard Freixas i Montplet, Siddarth Sankaran "Twisted Hilbert modular surfaces, arithmetic intersections and the JacquetLanglands correspondence" Advances in Mathematics Volume 329, 30 April 2018, Pages 1-84

[1] Then I have a job.