Reading: Approximate Groups

Lemma 2.5.1 Let G be a arbitrary group and let $A \subset G$ b a finite subset with $|A^2| \leq K|A|$. Then $|A^{-1}A| \leq K^2|A|$ and $|AA^{-1}| \leq K^2|A|$.

This is generalization to non-abelian groups, only for $m = 1$ and $n = 1$

Theorem 2.3.1 Let G be an abelian group and let A, B be finite subsets of G.

- suppose that $|A + B| \leq K|A|$ then $|mA - nA| \leq K^{m+n}|A|$.
- if $|A + A| \leq K|A|$ then $|mA - nA| \leq K^{m+n}|A|$.

for all non-negatve integers m, n.

These looking into the axioms of group theory. There are several instances of group theory that we encounter in other branches of mathemamtics:

- permutation groups
  ABCDE $\rightarrow$ BCDEA $\rightarrow$ CDEAB $\rightarrow$ DEABC $\rightarrow$ EABCD $\rightarrow$ $[\ldots]$
- groups of substutions, e.g. $x \mapsto 3x + 2y$ and $y \mapsto 4x + 3y$
- groups of transformations of physical objects (e.g. symmetries of square)

These symmetries in general were approximate since there was an enormous amount of work move and old the objects in a perfect evenly spaced circle.

References

[1] ...