

Scratchwork: Circle Through Three Points

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in \mathbb{R}^2$ be three points on a map, the Euclidean plane. We can find a single circle that passes through all three points by solving simultaneous equations.

$$\begin{vmatrix} x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \\ x_1 + y_1 & x & y & 1 \end{vmatrix} = 0$$

Why does this work? The equation for the circle should be of the form:

$$A(x^2 + y^2) + Bx + Cy + D = 0$$

Here $[A : B : C : D] \in \mathbb{R}P^3$, and given three data points we can so solve for thee three points. We can find the intersection of these three divisors.

Ex What is the center and radius of this circle?

Ex What happens if we move (x_1, y_1) to a nearby point $(x_1 + \epsilon, y_1 + \epsilon)$ what happens to the circle under this small change?

References

[1] ...