Numbers and Entropy

John Mangual

1 Motive Galois Group

Goal: To define and study a Galois Theory of Feynman Amplitudes.

- Broadhurst-Kreimer (1995) found multi-ζ values as amplitudes of massless ϕ^4 theory.
- Deligne-Ihara-Drinfield (1989) Motivic Galois Group, $MT(\mathbb{Z})$
- Cartier (1998) Is there a "cosmic" Galois Group.
- Kontsevich (1998) Counting points over finite fields.
- Connes-Marcolli (2004) Cosmic Galois Group related to renormalization.
- Belkale-Brosner (2003) Graph hypersurfaces of general type. Physically unrealistic counterexamples.
- Bloch-Enault-Kreimer (2006) Defined motive on "primitive" graphs.
- co-workers: Dorin, Panzer, Schentz, Yates. Exist amplitudes which are **not** multiple-zeta values. **not all** multiple zeta values appear as amplitudes.

NB motives play no role. Retrieve as special cases, the "symbol" of an amplitude. Renormalization group (Connes-Kreimer Hopf algebra). Hidden recursive structure on amplitudes; contraints.

1.1 Feynman Graphs

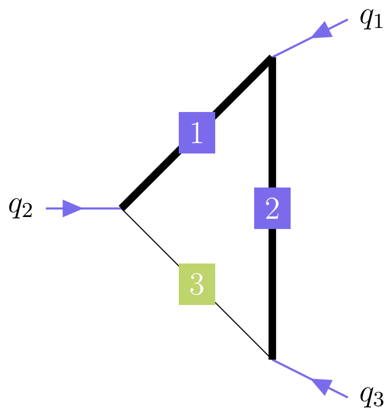
Scalar QFT in Euclidean space, $\mathbb{R}^d$. A **Feynman graph** is a connected graph.

$$G = (V, E_G, E_G^{ext})$$

Vertices, internal Edges and external half-Edges. "kinematic" data.

- A particle mass $m_e : E_G \to \mathbb{R}$
- Incoming momentum $q : E_G^{ext} \to \mathbb{R}^d$
- Conservation of momentum $\sum_{E_G^{ext}} q_i = 0$. Draw massive edges as **thickened** lines.

Ex



Two massive particles $m_1, m_2 \neq 0$ and one masslesss particle $m_3 = 0$. Momentum conservation says $q_1 + q_2 + q_3 = 0$. Why are mass and momentum identified????

As a short hand usually replace several incoming arrow with a single arrow representing the sum of momenta.

View Feynman amplitude $I_G(m, q)$ as a function (**multi-valued** depending on choice of covering space, **infinite** possibly everywhere) function of affine variety

$$\mathcal{M}_G^V = \left\{ (m,q) : \begin{array}{ll} m_e \in \mathbb{A}^1 \backslash \{0\} & e \notin V_m \\ m_e = 0 & e \in V_m \end{array} \middle| \begin{array}{ll} q_i \in \mathbb{A}^1 \backslash \{0\} & i \notin V_q \\ q_i = 0 & i \in V_q \end{array} \right\} \cap \left\{ \sum q_i = 0 \right\}$$

The **Euclidean Region** is $\mathcal{M}_G^V(\mathbb{R})$.

1.2 Feynman Amplitudes

1960's presentation using **graph polynomials**.

A **spanning** k**-tree** $T = T_1 \cup T_2 \cup \cdots \cup T_n \subseteq G$ is a subgraph with k connected components T_i such that $V_G = \dot{\bigcup} V_{T_i}$ to each $e \in E_g$ we associate "Schwinger parameter"...

Kirkhoff Polynomial ψ_G "1st Symanzik" polynomial

$$\psi_G = \sum_{T \subseteq G} \prod_{e \in T} \alpha_e$$

References

[1] Francis Brown. Iterated Integrals in Quantum Field Theory. `http://www.ihes.fr/~brown/ColombiaNotes7.pdf`

[2] Francis Brown. *Irrationality proofs for zeta values, moduli spaces and dinner parties* `arXiv:1412.6508`

[3] Francis Brown. *Modular forms in Quantum Field Theory* `arXiv:1304.5342`

[4] Chris Godsil, Gordon Royle. **Algebraic Graph Theory** (Graduate Texts in Mathematics 207) Springer, 2001.