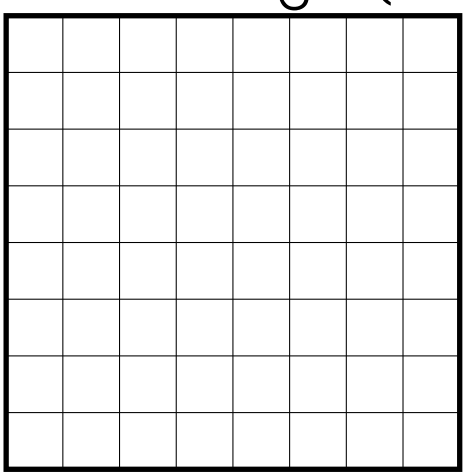
Proposal: Factorial

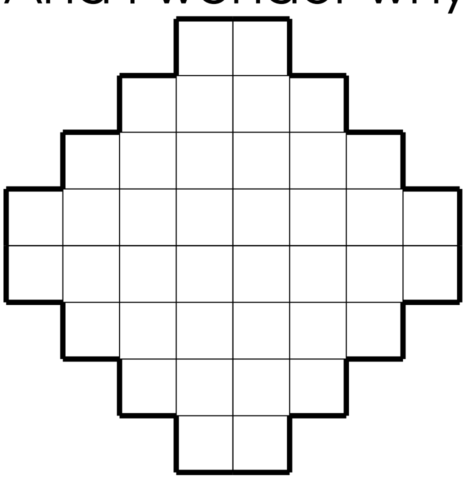
John D Mangual

Some clever person turned the theory domino tilings into a fundamental object of mathematics and of nature. For a long time there were really two shapes being studied.

The rectangle (here an 8×8 square):



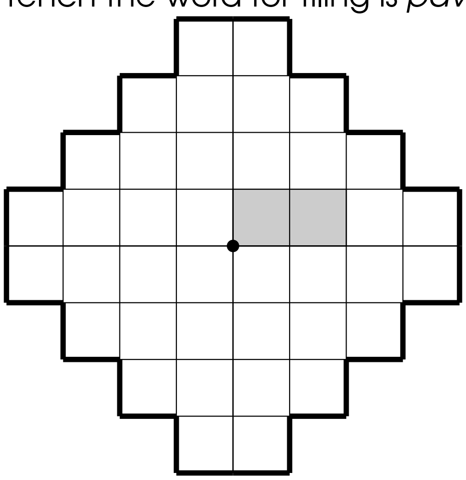
And I wonder why this particular shape is so essential:



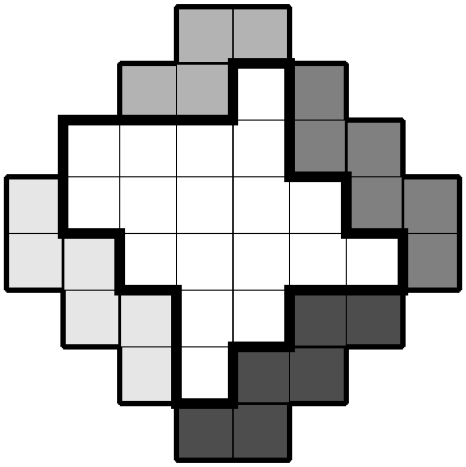
Pathetic Tutorial:

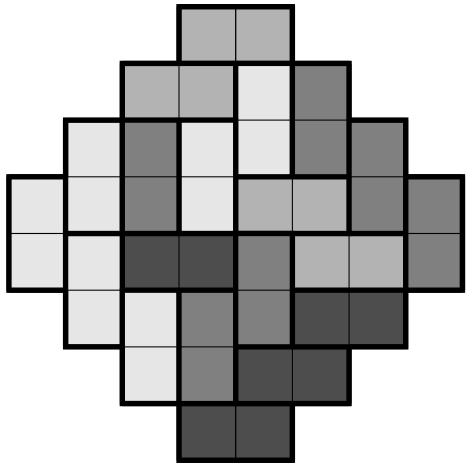
```
\begin{tikzpicture}[scale=0.75]

\foreach \a in {0,...,3}{

\draw[line width=2] (\a  ,4-\a)--(\a+1, 4-\a  );
\draw[line width=2] (\a+1,4-\a)--(\a+1, 4-\a-1);

\draw (   \a, 4-\a)--(   \a, \a-4);
\draw (-1*\a, 4-\a)--(-1*\a, \a-4);

\draw ( 4-\a,    \a)--(\a-4,    \a);
\draw ( 4-\a, -1*\a)--(\a-4, -1*\a);

\def \b {-1}
\def \c { 1}

\draw[line width=2] (\b*\a     ,\c*4-\c*\a)--(\b*\a+\b*1, \c*4-\c*\a     );
\draw[line width=2] (\b*\a+\b*1,\c*4-\c*\a)--(\b*\a+\b*1, \c*4-\c*\a-\c*1);

\def \b { 1}
\def \c {-1}

\draw[line width=2] (\b*\a     ,\c*4-\c*\a)--(\b*\a+\b*1, \c*4-\c*\a     );
\draw[line width=2] (\b*\a+\b*1,\c*4-\c*\a)--(\b*\a+\b*1, \c*4-\c*\a-\c*1);

\def \b {-1}
\def \c {-1}

\draw[line width=2] (\b*\a     ,\c*4-\c*\a)--(\b*\a+\b*1, \c*4-\c*\a     );
\draw[line width=2] (\b*\a+\b*1,\c*4-\c*\a)--(\b*\a+\b*1, \c*4-\c*\a-\c*1);

}

\end{tikzpicture}
```

In French the word for tiling is *pavage* – so literally we are **paving** the shapes with dominoes.

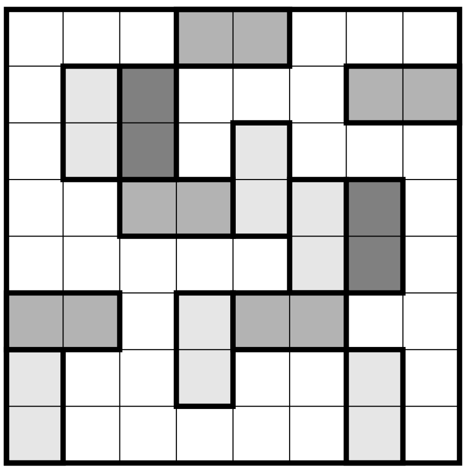


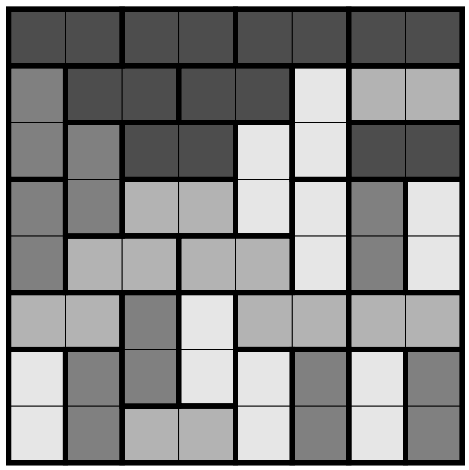
Let's put two reasonable tilings on the board. One strategy for this shape is to start from the corners and work indwards. And in this case we get lucky: it always works.





And for rectangle the case is even clearer. There are no intermediate stages. I mean, if you put enough tiles down there can be come question as to whether you put yourself in a corner yet.





There is also the lovely John Conway game of "Domineering" which is not related but you also place dominoes on a checkerboard. See **Winning Ways for Your Mathematical Plays** (Vol I).

Exercise Fill the rest of the tiling.

Answer There is no solution, but if you move the tiles slightly (actually quite a bit here...) you can get a problem with an answer.

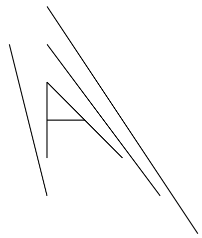
The number of tilings on the top looks atypical – just purely on a hunch. Where did that hunch come from? Is it right?

As with all hunches, it can be proven with hundreds of pages of equations this tiling is not likely to appear in a rectangles. We'll settle for somewhat simpler patterns.

References

(1) ...

The most amazing typo ever:



```
\begin{tikzpicture} [scale=0.5]
\foreach \a in {0,...,5}{
    \draw (\a, 5-\a)--(1, \a + 1);
}
\end{tikzpicture}
```