Tune-Up: Rational Functions

There is so much semantics in this one, it's hard to decide where to begin.

Proposition IX.2.7 If two curves are birationally equivalent, they have the same geometric genus. The converse is true if $g = 0$: a curve is rational if and only if its geometric genus is zero.

This theorem sounds great if we can decide on the meaning of a few terms:

- "curve"
- birational equivalence
- "isomorphic"
- (geometric) genus
- "rational"

Are these close to the same terms we learned in high school? Let $x = \cos 5\theta$ and $y = 1 + \sin 3\theta$ find the equation $f(x, y) = 0$ or the ring $\mathbb{C}[x, y]$. This math-speak has gone too far.

References

[1] Daniel Perrin **Algebraic Geometry** (Universitext) Springer, 2008.