

Item: Riemann Integral

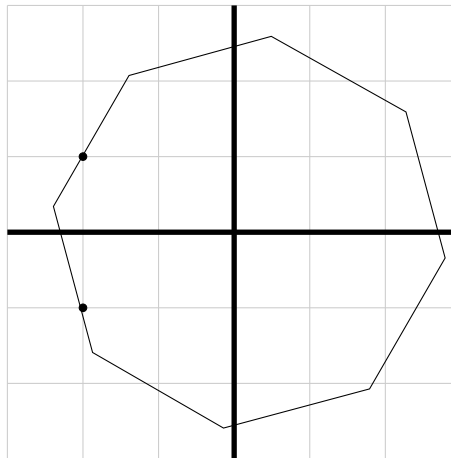
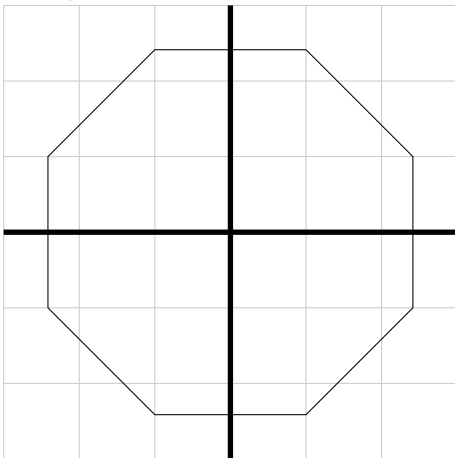
John D Mangual

In the math class we learn that Riemann sums converge to the Riemann integral.

$$\lim_{|\Delta x| \rightarrow 0} \sum_{i=0}^{N-1} f(x_i) \Delta x_i = \int_0^1 f(x) dx$$

where the interval $[0, 1]$ has been partitioned into small intervals $[x_i, x_{i+1}]$.

There are doubts that I have from when I learned Real Analysis. The most vanilla case of an integral is Area or Volume: any kind of **quadrature**.



Let us push that to an extreme: all integrals are areas; all integrals are quadratures.

Q1 As a warm-up. Does the rotated octagon (right) pass through the two marked points $(-2, \pm 1)$?

We are going to compute the octagon by decomposing into squares (and chunks of squares).

- homology
- scissors congruence

These are some buzzwords in the literature that correspond to this basic computation. This is not the original example I had in mind, which had to do with this formula:

$$\int_0^1 f(x) dx + \int_0^1 g(x) dx = \int_0^1 (f(x) + g(x)) dx$$

Our $f(x)$ is very well behaved $f(x) \equiv 1$ everywhere. So there is no issue yet.

References

(1) ...