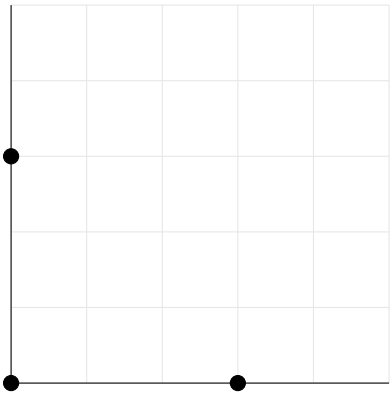
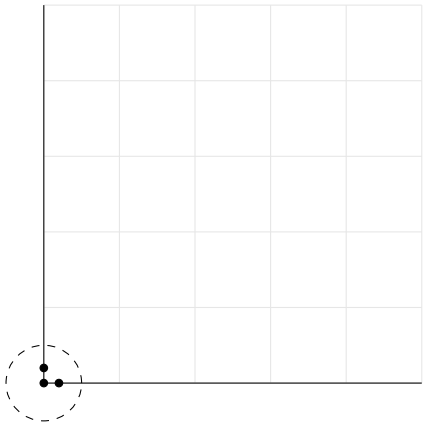
Scratchwork: Schemes

Let's list some of the examples in Chapter 2 of **Geometry fo Schemes**

- Spec R with $R = \mathbb{K}[x,y]/(x^2, xy, y^2, ax+by) \simeq K[t]/(t^2)$ This is a double-point.
- Spec$K[x]/(x^3) \not\simeq$ Spec$K[x,y]/(x^2, xy, y^2)$. These are examles of triple-points.
- $X_t = \{(0,0),(t,0),(0,t)\} \subset \mathbb{A}^2_K$ be three points in the affine plane $\mathbb{A}_K$. The limit scheme as $t \to 0$ is:
$$\lim_{t\to 0} X_t = X_0 = \text{Spec}K[x,y]/(x^2, xy, y^2)$$
which is a triple-point. There were three points to begin with and now they are infinitesimally close together.





Even more examples:

- $X = \text{Spec}K[x,y] = (x^2y, xy^2)$ the union of the x-axis and the y-axis. $\{x=0\} \cup \{y=0\}$.

Due to our lack of imagination, these are the minimum we can do. These arise as limiting situations in classical geometry and we are advised to look at a high-school textbook from here.

References

[1] Henri Cohen **Computational Number Theory in Relation with L-Functions** `arXiv:1809.10904`

[2] Hugh Montgomery, Robert Vaughan **Multiplicative Number Theory I: Classical Theory** (Cambridge Studies in Advanced Mathematics, #97) Cambridge University Press, 2010.

[3] Yitzhak Katznelson **An Introduction to Harmonic Analysis** (Cambridge Mathematical Library) Cambridge University Press, 2004.

References

[1] David Eisenbud, Joe Harris. **The Geometry of Schemes**. (GTM #197) Springer, 2000.

[2] Ravi Vakil **Foundations of Algebraic Geometry** (online) `http://math.stanford.edu/ vakil/216blog/`