Reading: Tilings and Metric Spaces

The authors discuss *metrics* and yet they also talk about *distance*

Def The *distance* between two tilings $R(T, T')$ to be the supremum of all radii r where T and T' can be translated by less than $\frac{1}{2r}$ to agree on a ball of radius r around the origin.

Example Rhombus tilings, Penrose tilings, cut-and-project tilings.

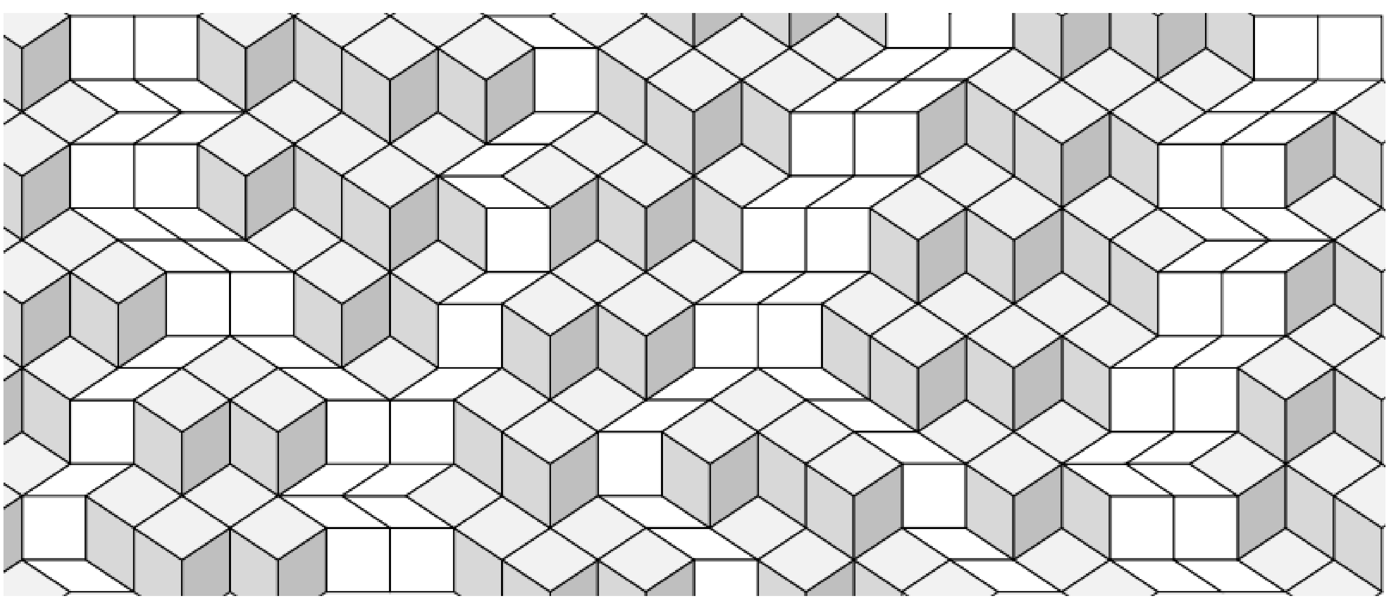
Example $\mathbb{R}$ with distance $d(x, y) = |x - y|$ is a metric space.

Example The space of d-planes (affine hyperplanes) in $\mathbb{R}^n$ with metric

$$d(E, F) = \max \left\{ \sup_{\vec{x} \in E \cap S} \inf\{||\vec{x} - \vec{y}|| : \vec{y} \in F\}, \sup_{\vec{x} \in E \cap S} \inf\{||\vec{x} - \vec{y}|| : \vec{y} \in E\} \right\}$$

is metric space. Here S is the unit sphere in $\mathbb{R}^n$, $\{||x|| = 1\}$.

Lemma This space is *compact*. It's rational points are planes with rational entires.



References

[1] Thomas Fernique, Mathieu Sablik. **Weak Colored Local Rules for Planar Tilings**