Scratchwork: Algebraic Curves

9/23 Can we think of an algebraic curve besides the circle? Let's try a polar coordinates circle:

$$\begin{aligned} x &= \cos\theta \\ y &= \sin\theta \\ z &= 0 \end{aligned}$$

What if we add another circle, moving around the plane generated by the height and the radius of the circle. $e_{\vec{r}} = e_{\vec{x}} \cos\theta + e_{\vec{y}} \sin\theta$ This is a moving coordinate plane, and we can draw a circle that moving plane.

$$\begin{aligned} r &= \epsilon \, \cos\phi \\ z &= \epsilon \, \sin\phi \end{aligned}$$

where $\epsilon \ll 1$. Then we can put the moving plane back into a 3D ambient space. The new coordinate equation are

$$\begin{aligned} x &= \cos\theta + \epsilon \, \cos\theta \, \cos\phi \\ y &= \sin\theta + \epsilon \, \sin\theta \, \cos\phi \\ z &= \quad 0 + \epsilon \, \sin\phi \end{aligned}$$

We claim this is an "algebraic" curve. If we let $\phi = 3\theta$ or anything like that, we could solve for one (or a few) equations $f \in \mathbb{R}[x, y, z]$ such that $f(x, y, z) = 0$.

Unfortunately, algebraic geometry requres that $x, y, z \in \mathbb{C}$ meaning the curve lives in six dimensional space. And we'll ignore them and ask for points on this curve with $x, y, z \in \mathbb{Q}$. So now this is an arithemtic scheme.

`[draw picture]`

References

[1] ...