Examples: L^p spaces

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L^2 spaces are somewhat natural because we have Pythagoras theorem:

$$\big|\big|(a,b)\big|\big|^2 = a^2 + b^2$$

I have trouble understanding the meaning of the L^p norms:

$$\big(a^p + b^p\big)^{1/p}$$

I have no interpretation for them. There's no picture I can draw.

There are a few starting points. Let $p, q \in (1, \infty)$ be related by

$$\frac{1}{p} + \frac{1}{q} = 1$$

Then all measurable functions $f, g \geq 0$ satisfy:

$$\int f g \, d\mu \leq \big|\big|f\big|\big|_p \, \big|\big|g\big|\big|_q$$

so for some reason this relationship of fractions gets promoted to a relationship of measurable functions.

There is also Minkowski inequality:

$$||f+g||_p \leq ||f||_p + ||g||_p$$

and it's downhill from there. I have read in some places these have their origins in **convex gometry** in high dimensional space $\mathbb{R}^n$.

All the arguments I'm finding are pretty clumsy and not very geometric. Instead we drown in a morass of

- poor disorganized writing
- clumsy notation
- non-visual thinking

I am concluding this subject is simply too difficult. And going for my bike ride.

References