

Examples: ABJM Theory

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There's not a whole lot to say until we write the formula:

$$\int \left(\prod_i e^{ik\pi(\theta_i^2 - \phi^2)} \right) \times \frac{\prod_{i \neq j} (2 \sinh \pi(\theta_i - \theta_j) 2 \sinh \pi(\phi_i - \phi_j))}{\prod_{i,j} (2 \cosh \pi(\theta_i - \phi_j))^2}$$

This integral could possibly be over $[0, 2\pi]^2$ but I am expecting it's over the real numbers:

$$(\theta, \phi) \in \mathbb{R}^N \times \mathbb{R}^N$$

since this the domain of integration of the reals. Here is the simplest one:

$$\int_{-\infty}^{\infty} d\theta e^{ik\pi\theta^2} = \sqrt{\frac{\pi}{8}}(1 + i)$$

I have not done that yet. If we set $N = 1$ or $N = 2$ and this integral is totally feasible.

ABJM theory could refer to two things:

- The $U(N) \times U(N)$ Chern-Simons-matter theory with $\mathcal{N} = 6$ superconformal symmetry. The first $U(N)$ is at level k and the second $U(N)$ is at level $-k$.
- The $U(N) \times U(N)$ Gaussian matrix integral the CS-theory localizes to.

Both of these are called **ABJM theory**. This is rather confusing.

Here is another Chern-Simons formula:

$$Z_{\text{CS}}(N, k) = \frac{1}{\sqrt{N+k}} \prod_{\alpha > 0} 2 \sin \frac{\pi \alpha \cdot \rho}{k+N}$$

where $\alpha_{ij} = e_i - e_j \in \mathbb{C}^N$ are vectors¹ and

$$\rho = \frac{1}{2} \sum_{\alpha > 0} \alpha = \sum_{i=1}^N \left(\frac{N+1}{2} - i \right) e_i$$

and also $k \in \mathbb{Z}$ is a positive integer, and so is $n \in \mathbb{Z}$.

Therefore our Chern-Simons partition function Z_{CS} should be a number.

¹This is called the **Cartan subalgebra** a term which should mean nothing right now.

References

- (1) Anton Kapustin, Brian Willett, Itamar Yaakov. **Exact Results for Wilson Loops in Superconformal Chern-Simons Theories with Matter**
<https://terrytao.wordpress.com/2009/08/23/determinantal-processes/>
- (2) Ofer Aharony, Oren Bergman, Daniel Louis Jafferis, Juan Maldacena **N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals** arXiv:0806.1218v4