What is Fourier Uniformity?

John D Mangual

I overheard that Fourier uniformity is a type of "size for sets" along with cardinality and dimension.[1] There was a recent big hoop-lah about the **capset bound**.

Any subset $A \subseteq \mathbb{F}_3^n$ with no arithmetic subset has size $< 2.756^n$

These problems clearly related to coding theory - the kind use to store movies and music in files and pass data over the internet[2]. Or maybe information theory can learn from math

[1] http://mathoverflow.net/a/43549/1358 " I have the vague feeling that ultimately, such notions of complexity should play as prominent a role in these sorts of combinatorial problems as existing notions of "size" for such sets, such as cardinality, dimension, or Fourier uniformity." - Terence Tao

[2] There is also result over $\mathbb{Z}_4$ that A w/o 3-term arithmetic sequence should have size $4^{0.926\,n}$.
https://quomodocumque.wordpress.com/2016/05/12/croot-lev-pach-on-ap-free-sets-in-z4zn/

Let $A \subset \mathbb{F}^n$

There is $V \leq \mathbb{F}^n$ with $\mathrm{codim} \ll \epsilon^{-1}$ and $x \in \mathbb{F}^n$ such that

$A \approx x + V$

The real definition of "$\approx$" is not so user-friendly. It says:

A is ϵ-uniform on the coset $x + V$ if $\sup_{r \notin V^\perp} |(1_A \mu_{x+V})\hat{}(r)| < \epsilon$

Google-searching[3] has turns the phrases **Fourier uniform** or **Fourier pseudorandom** or possibly **Fourier complexity**.

I also wonder where these subset $A \subset \mathbb{F}^n$ come from? My guess is they are related to computer programs[4].

---

[3]https://terrytao.wordpress.com/2010/04/08/254b-notes-2-roths-theorem/

[4]When I buy shoes on eBay, I have no predicting of the selection or availablity or the price. eBay has no prediction of where you are buying from - and share to ship it's products.

Hence, the title... **Fourier Uniformity... on Subspaces**

I have difficulty grasping the finer points of Fourier analysis over $\mathbb{R}$ or even $\mathbb{Z}$. Finite field fourier anlysis was taught to me as one of the simplest cases[5].

Green and Sanders write about Fourier analysis in several dimensions – in fact $n \to \infty$ dimensions.

Lastly, finite field's are not that simple, we could have $\mathbb{F}_{17}$ or $\mathbb{F}_{48112959837082048697}$ but the cases I think Sanders has in mind like:

$$(\mathbb{Z}/2\mathbb{Z})^{10^8}, (\mathbb{Z}/3\mathbb{Z})^{10^7}, (\mathbb{Z}/4\mathbb{Z})^{10^6}$$

Instead of infinity, let $n = 5$ be dimension of our space and $p = 101$.

What could be a reasonable choice of A?

[5] What could be simpler then $\mathbb{Z}_2$?

Not genuinely random

I got away in school (and occasionally in the real world) by approximating an unpredictable situation by a uniformly random situation[6]

Now I am reading that unpredictible $\neq$ random.

How much choice do I really have? Baskin-Robbins only has 31 flavors. If I don't want any of those... I am in trouble. Where does choice come from?

In real life there is a mix of choices we can make, and decisions made for us. How much choice did we have?

[6] I did! I should!

On to math[7]

Randomness extraction is an engineering term, and boolean functions are a highly theoretical way of looking at computer programs. Green-Sander's theorem basically says

any computer program over a data stream is essentially a line (or hyperplane)

How many tasks does a computer program has... it basically does **one** thing and hopefully it's pretty good.

[7] Theories of choice and fair division occupy thousands of pages in the Economics and Political theory literature, and we will ignore them all!

References

(1) Fourier uniformity on subspaces **Fourier Uniformity on Subspaces** arXiv:1510.08739

(2) What does $\operatorname{codim} V \ll_\delta 1$ mean if codimension can only be 0 or 1? http://math.stackexchange.com/q/1505341/4997