

Problem: AMC 12A (2016)

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Problem What is the value of $\frac{11! - 10!}{9!}$?

(A) 99 (B) 100 (C) 110 (D) 121 (E) 132

Better get multiplying! And we don't have a calculator on this test! What is this?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11$$

I know that $2^{10} = 1024$ or is it 512? Not important right now:

$$1 \times 2 \times 3 \times 4 = 24$$

and then multiply by $5 \times 6 = 30$ so that makes

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 = 24 \times 30 = 720$$

Just five moroe numbers left.

$$7 \times 8 = 56 \quad 9 \times 10 \times 11 = 990$$

so what is 56×990 this is not getting us very far.

$11!$ is some very large number¹.

$$11! = 11 \times 10!$$

Then maybe we can subtract against the other $10!$

$$\frac{11! - 10!}{9!} = \frac{10! \times (11 - 1)}{9!} = 9 \times 10 = 90$$

That would be choice
and that was none of the choices ...

$$\frac{11 \times 10! - 10!}{9!} = (11 - 1) \times \frac{10 \times 9!}{9!} = 100$$

The answer is choice **(B)**.

Alternative If we took remainders upon division modulo 10. Forget it.

$$\frac{11! - 10!}{9!}$$

$9!$ is a very complicated thing. Since $2 < 9$ and $5 < 9$ we have that 10 divides $9!$ – this is not relevant.

$$\frac{11 \times 10 \times 9! - 10 \times 9!}{9!} = 11 \times 10 - 10 = 100$$

¹hopefully you know what the factorial symbol means