

Reading: Schemes

If we grab the easier parts of the algebraic geometry textbook, we suggest some geometric constructions....

Ex: This example has to do with **parabola** and the Affine plane (something like \mathbb{Q}^2):

$$\operatorname{Spec} \mathbb{Q}[x, y]/(y^2 - x, x) \simeq \operatorname{Spec} \mathbb{Q}[y]/(y^2)$$

Look like we can get the “pre-image of -1” and get the complex numbers:

$$\operatorname{Spec} \mathbb{Q}[x, y]/(y^2 - x, x + 1) \simeq \operatorname{Spec} \mathbb{Q}[y]/(y^2 + 1) \simeq \operatorname{Spec} \mathbb{Q}[i] = \operatorname{Spec} \mathbb{Q}(i)$$

So we are still trying to identify specific “points” in our geometric object (which we are describing with rings and equation). The pre-image over the “generic point”:

$$\operatorname{Spec} \mathbb{Q}[x, y]/(y^2 - x) \otimes_{\mathbb{Q}[x]} \mathbb{Q}(x) \simeq \operatorname{Spec} \mathbb{Q}[y] \otimes_{\mathbb{Q}[y^2]} \mathbb{Q}(y^2)$$

Ex ? What is the scheme-theoretic **fiber** of $\operatorname{Spec} \mathbb{Z}[i] \rightarrow \operatorname{Spec} \mathbb{Z}$ over the prime (p) ?

Ex Consider $\operatorname{Spec} k[\epsilon]/(\epsilon^2) \rightarrow \operatorname{Spec} k[x] = \mathbb{A}_k^1$ given by $x \mapsto \epsilon$. The image of the **fuzzy point**.

We'll consult with `math.Stackexchange` to see if these are correct.

References

[1] The Rising Sea: Foundations of Algebraic Geometry
<http://math.stanford.edu/~vakil/216blog/>

[2] ...