Numbers and Entropy

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1 Motive Galois Group

Goal: To define and study a Galois Theory of Feynman Amplitudes.

- Broadhurst-Kreimer (1995) found multi-ζ values as amplitudes of massless ϕ^4 theory.
- Deligne-Ihara-Drinfield (1989) Motivic Galois Group, $MT(\mathbb{Z})$
- Cartier (1998) Is there a "cosmic" Galois Group.
- Kontsevich (1998) Counting points over finite fields.
- Connes-Marcolli (2004) Cosmic Galois Group related to renormalization.
- Belkale-Brosner (2003) Graph hypersurfaces of general type. Physically unrealistic counterexamples.
- Bloch-Enault-Kreimer (2006) Defined motive on "primitive" graphs.
- co-workers: Dorin, Panzer, Schentz, Yates. Exist amplitudes which are **not** multiple-zeta values. **not all** multiple zeta values appear as amplitudes.

NB motives play no role. Retrieve as special cases, the "symbol" of an amplitude. Renormalization group (Connes-Kreimer Hopf algebra). Hidden recursive structure on amplitudes; contraints.

1.1 Feynman Graphs

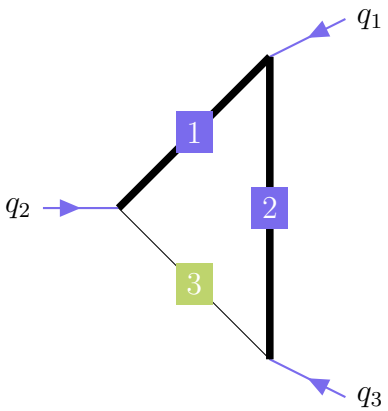
Scalar QFT in Euclidean space, $\mathbb{R}^d$. A **Feynman graph** is a connected graph.

$$G = (V, E_G, E_G^{ext})$$

Vertices, internal Edges and external half-Edges. "kinematic" data.

- A particle mass $m_e : E_G \to \mathbb{R}$
- Incoming momentum $q : E_G^{ext} \to \mathbb{R}^d$
- Conservation of momentum $\sum_{E_G^{ext}} q_i = 0$. Draw massive edges as **thickened** lines.

Ex



Two massive particles $m_1, m_2 \neq 0$ and one masslesss particle $m_3 = 0$. Momentum conservation says $m_1 + m_2 = 0$. Why are mass and momentum identified???

`Over 90 minutes of lecture remain...`

References

[1] Francis Brown. Iterated Integrals in Quantum Field Theory. `http://www.ihes.fr/~brown/ColombiaNotes7.pdf`

[2] Francis Brown. *Irrationality proofs for zeta values, moduli spaces and dinner parties* `arXiv:1412.6508`

[3] Francis Brown. *Modular forms in Quantum Field Theory* `arXiv:1304.5342`