Examples: Quadratic Reciprocity

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Analytic number theory is not my expertise. Here we work out some softball examples related to quadratic reciprocity.

Here's a question: **Can permutations be used to prove Artin reciprocity** or even parts of Class Field Theory?

The proof of quadratic reciprocity seems like a random hodge-podge of techniques.[1] Can we unify some of these arguments using:

- Geometry of Numbers
- Pigeonhole Principle

Gauss in his *Disquiciones Arithmeticae* uses Pigeonhole to prove that $a^p \equiv 0 \mod p$.

[1] This is great for a first class when I was 15 years old it is not so great when you in graduate school are trying to learn Class Field Theory.

Any other applications?

References

(1) Jared Weinstein. **Reciprocity laws and Galois representations: recent breakthroughs** Bull. Amer. Math. Soc. 53 (2016), 1-39

(2) David A Cox. **Primes of the Form $x^2 + ny^2$: Fermat, Class Field Theory, and Complex Multiplication** Wiley, 2013.

(3) **A prime ideal $\mathfrak{p}$ decomposes in $\mathbb{Q}(\zeta_{24})/\mathbb{Q}(\sqrt{-6})$ iff it is generated by $\alpha \in 1 + 2\mathbb{Z}[\sqrt{-6}]$** http://mathoverflow.net/q/234570/1358

(4) Roy L. Adler **Symbolic dynamics and Markov partitions** Bull. Amer. Math. Soc. 35 (1998), 1-56 http://www.ams.org/journals/bull/1998-35-01/S0273-0979-98-00737-X/