Nilsequences

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Terence Tao sees a lot of things, but he writes in an obfuscated way, and I think he misses a lot of things. About 10 years ago, I was introduced to the topic of **nilsequences** in a course of Dynamics and Number Theory. I did nothing with it. Let's read Terry's latest blog on this topic[1].

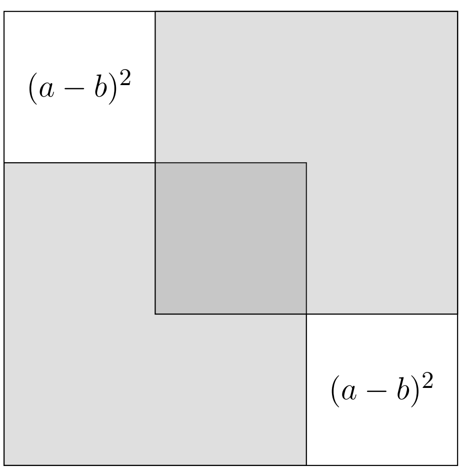
Part of it is like. . . where do fractions come from?

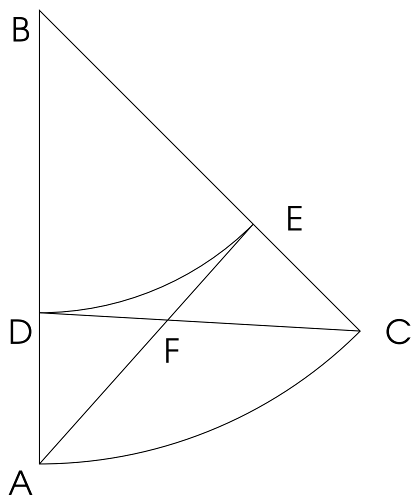
If we take scientific measurement, there's quite a bit error that obstructs us from observing the most delicate patterns. In fact, shielding us completely from finding them (or protecting us).

$\sqrt{2} = 1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503$

If you examine the digits carefully[2] we can prove the decimals do not exhibit any pattern in the decimal expension. However, if we use continued fractions:

$\sqrt{2} = [1; 2, 2, 2, 2, 2, \ldots]$ and this is a nicer system since we have exponential convergence of the the number. Here the error is 10^{-6} (microscopic).





These pictures lead to two different proofs that $\sqrt{2} \notin \mathbb{Q}$. They are both geometric proofs, and argue by infinite descent. Therefore, there must have been an ètale cohomology.

I have no idea what ètale cohomology and the books do not simplify it enough for me. Forget it. One way to get meaningful numbers to arbitrary accuracy is to observe a dynamical system over time and take detailed measurements. And you will see.

[1] `https://terrytao.wordpress.com/2017/04/28/notes-on-nilcharacters-and-their-symbols/`

[2] `http://www.gutenberg.org/files/129/129.txt`

Here's a reading list. I will leave in the Class Field Theory book since even though don't need it any way (we are solving over $\mathbb{Z}$), in fact we may need it anyway.

References

(1) Nancy Childress **Class Field Theory** (Universitext). Spinger, 2009.

(2) Ben Green, Terence Tao, Tamar Ziegler. **An inverse theorem for the Gowers $U^{s+1}[N]$-norm** `arXiv:1009.3998`

(3) Ben Green **Approximate algebraic structure** `arXiv:1404.0093`