Circle Inversions: A Guide

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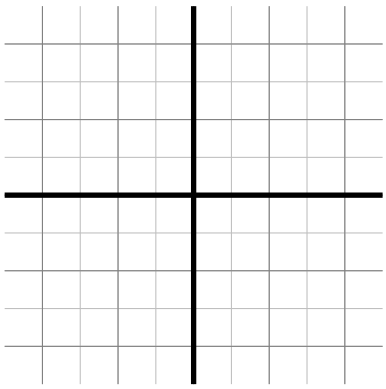
How does a circle map to another circle under the map $z \mapsto z + a$ and $z \mapsto -\frac{a}{z}$?

$$z \mapsto \frac{az + b}{cz + d}$$

Working our way up to the general transformation[1].

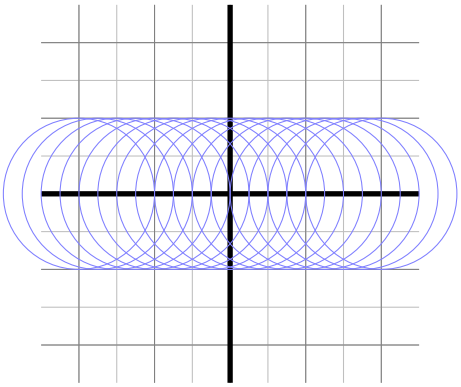


It doesn't make much sense to talk about theory. I am trying to draw the image of circles under fractional linear transformation.

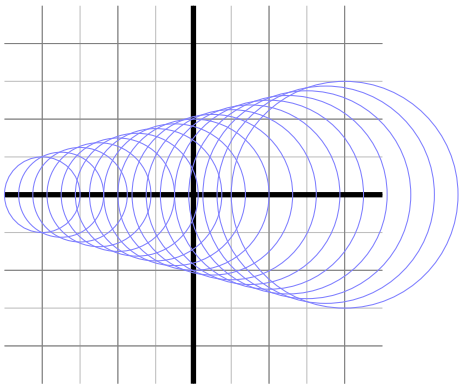
[1] To no avail, I have asked for explicit formulas for **inversions** - certain geometric transformations taking a circle to another circle. And I get referred from one book to another. And I will have to put together my own guide.

Shifts

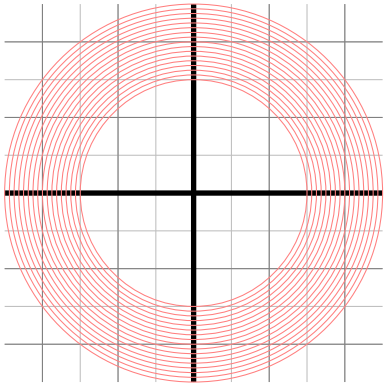
There are some cases that don't require a calculator. Such as $z \mapsto z+a$ and those maps are reasy to write down.



here is variation where we change the formula:



The other formula is $z \mapsto -\frac{a}{z}$ that minus sign preserves the orientation of the complex plane (it looks identical).



References

(1) Kurt Johansson **Discrete orthogonal polynomial ensembles and the Plancherel measure** arXiv:math/9906120

(2) Graham Lagarias Mallows Wilks Yan **Apollonian Circle Packings: Geometry and Group Theory I. The Apollonian Group** arXiv:math/0010298

(3) Caroline Series **The Modular Surface and Continued Fractions** J. London Math. Soc. (1985) s2-31 (1): 69-80. doi: 10.1112/jlms/s2-31.1.69