Tune-Up: Curves

Let me draw a sequence of points and a path through them. $x[0]$, $x[1]$ and $x[2]$. The curve as an edge at $x[1]$ how can we make the thing "smoother"? There are infinintey many curves $\phi : [0,1] \to \mathbb{R}^2$ that start with $\phi(0) = x[0]$ and $\phi(1) = x[1]$. Here is an example:

$$\phi_0(t) = x[0] \cdot (1-t)^3 + A \cdot 3(1-t)^2 t + B \cdot 3(1-t)t^2 + x[1] \cdot t^3$$

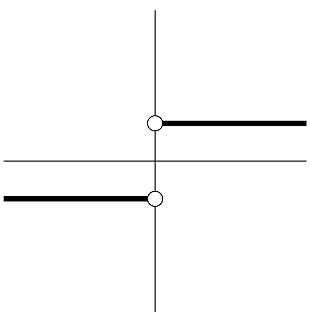
Then we have a second curve connecting $x[1]$ and $x[2]$ of the same "cubic" type.

$$\phi_1(t) = x[1] \cdot (2-t)^3 + A_1 \cdot 3(2-t)^2(t-1) + B_1 \cdot 3(2-t)(t-1)^2 + x[2] \cdot (t-1)^3$$

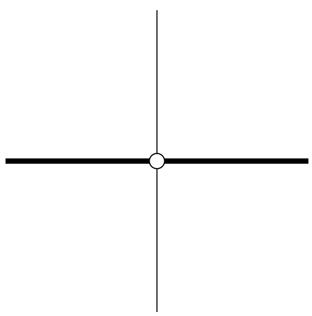
and we observee that $\phi_1(1) = x[1]$ and $\phi_1(2) = x[2]$. Then let's "glue" the two functions together:

$$\phi(t) = \begin{cases} \phi_0(t) & t \in [0,1] \\ \phi_1(t) & t \in [1,2] \end{cases}$$

We have that $\phi(0^+) = \phi(0^-)$. This is a great example to draw since it appears both in Calculus class and also in Topology class.



vs.



Here we've drawn the picture suggestively that we can join the two matchsticks together at the origin $(0,0)$ to obtain a copy of the number line (such as $\mathbb{R}$) this is not necessarily true we could have joined the two segments any way we wanted. The decision to the value of $\phi(0)$ or the "shape" of our number line depends on the choice of functions we want to use.

In our case, we want $\phi'(0^+) = \phi'(0^-)$ and maybe even $\phi''(0^+) = \phi''(0^-)$. How can we compute the values of A and B in this case? Let's do the subtraction problem:

$$\phi'(0^+) - \phi'(0^-) = 0 \text{ and } \phi''(0^+) - \phi''(0^-) = 0 \text{ and } \phi \text{ is cubic}$$

The only solution I can think of is $\phi(t) = C \cdot t^3$. If we set the "jump" at $t = 1$ there is $\phi(t) \approx C \cdot (t-1)^3$ for $t \approx 1$. Then $\phi(2) = C \cdot (2-1)^3 = C$.

Exercise We have three points $A = (1, 2)$, $B = (2, 5)$ and $C = (3, -2)$ let's try to find a cubic curve passing through them.

References

[1] ...