

## John D Mangual

- Lagrange Interpolation
- Bezier Curves

- passing through points
- weaving around points

$$f(x) = \begin{cases} 1 & x \notin \mathbb{Q} \\ 0 & x \in \mathbb{Q} \end{cases}$$
$$\int_0^1 f(x) dx = 1 \times \left| \{x \notin \mathbb{Q}\} \cap [0, 1] \right| + 0 \times \left| \{x \in \mathbb{Q}\} \cap [0, 1] \right| = 1$$

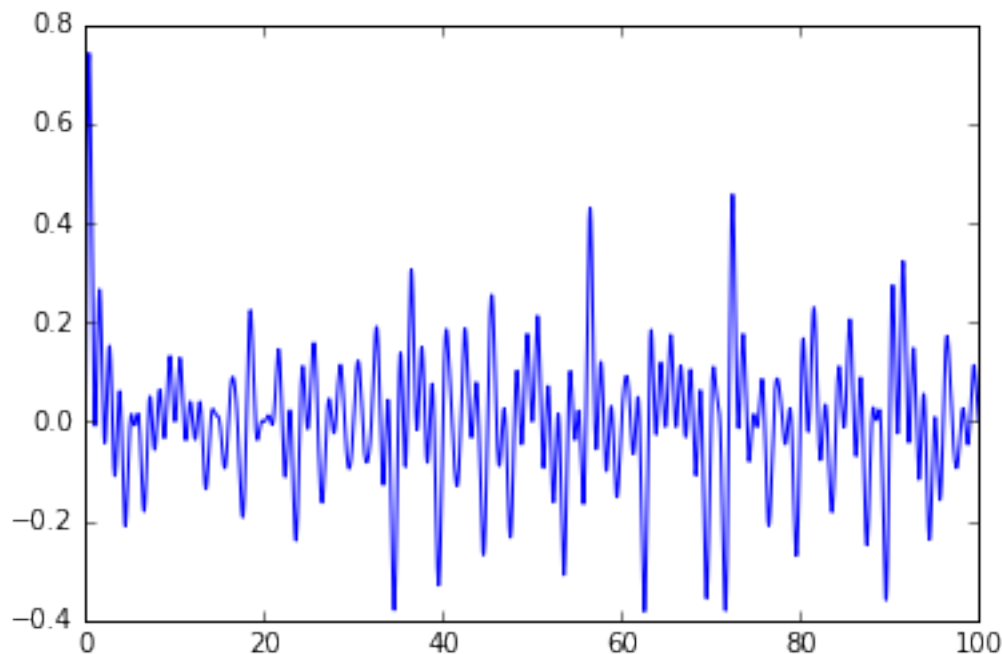
<sup>1</sup>Other times, theories comes out of the box, complete. I think such theories are unusuable, or they leave from for input from you and me. A comeback from that camp could be, “John, why are you so obsessed with this particular problem?” And I don’t have any good reason. Just because. Sometimes about me, and the time and place makes me interested. And I could be wrong!

This turns out to be one of the worse functions around, because it looks like 1 but isn't:

$$f(x) \approx 1 \quad \text{therefore} \quad \int f \approx \int 1$$

If I recall, we placed intervals around every single fraction  $\frac{a}{b} \in \mathbb{Q}$  we get an upper estimate for how large this integral could be, and that upper estimate  $\rightarrow 0$ .

How to deal with a function that often looks like  $f(x) \equiv 1$  but isn't?<sup>2</sup>



Here is a plot if we add all of the pure tones at all frequencies  $\frac{a}{b}$  with  $0 < a < b < 10$ . Theoretically we have found:

$$\frac{1}{28} \sum_{0 < a < b < 10} e^{2\pi i \frac{a}{b} t} \approx \frac{1}{c N^2} \sum_{0 < a < b < N} e^{2\pi i \frac{a}{b} t} = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

The constant in front is an oversimplification. The odds of two numbers being relatively primes is a famous one:

$$\phi(1) + \phi(2) + \cdots + \phi(N) = \#\{(a, b) : a < b \text{ and } \gcd(a, b) = 1\} \approx \frac{2}{\zeta(2)^2}$$

Analysis is the branch of math where we account for all the uses of the  $\approx$  symbol. What if I told you the function we just charted is  $\approx 0$ ?

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<sup>2</sup>One set of problems that plagued me was if we had two competing definition of limit, maybe one returns a number and the other does not. If we have two limiting procedures 1 and 2, maybe:

$$[\lim]_1 a_n = A \quad \text{implies} \quad [\lim]_2 a_n = A$$

Most of the time we don't really care how the limit procedure is defined. Who cares really?

$$N \gg 1 \quad \text{implies} \quad a_N \approx A$$

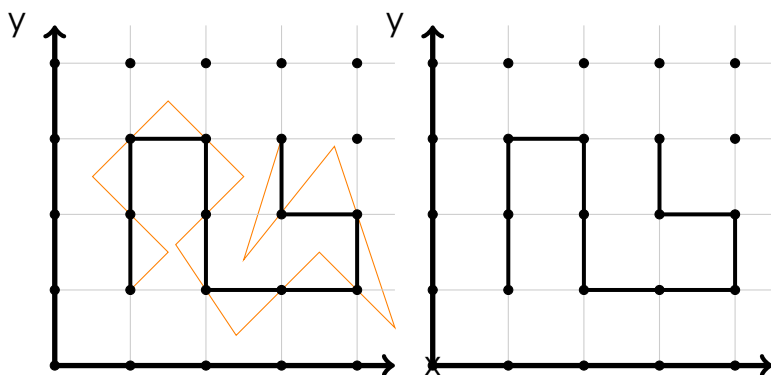
and the implication looks self-evident and most people won't question it. The only reason we remember the exception is because that particular conversation went on record.

When I read someone else's paper, a lot of my surprise stems from the author suggesting there is enough room in function space for something (very unlikely) to occur. Let's make a little problem set:

# 1 Show constant function on the rational numbers integrates to zero:

$$\int_{[0,1]} 1_{\mathbb{Q}} = 0$$

# 2 Find a curve that weaves around the obstacle course on page one:



With a pencil it's always possible to draw a curve that passes through all the dots. Our challenge is to find an equation that passes through all the loops. This is not so obvious it can always be done.

# 3 While I am remembering that my other example was going to be, we are going to solve the Prime Number Theorem.

$$\Lambda(n) = \begin{cases} \log p & \text{if } n = p \\ 0 & \text{not prime} \end{cases}$$

The prime number says that the density of primes is roughly  $\frac{1}{\# \text{ digits}}$  it can also be phrased as:

$$\sum_{n \leq x} \Lambda(n) = x + o(x)$$

where  $o(x)$  is a really small, unpredictable number<sup>3</sup> This won't be a review of number theory. Just ironing out one or two parts in a previous discussion. It's pretty hard.

# 4 Look at a real paper. There are one or two short papers of Bourgain that I try to get through from time to time.<sup>4</sup>

$$f(t) = \sum_{|x|,|y|,|z| < N} e^{2\pi i t (x^2 + y^2 - \sqrt{2}z^2)}$$

This is my favorite way to cause trouble and I get the sense this is the recommended way to start. A sense that comes from nowhere. .

<sup>3</sup>if you looked at my other projects, a great question to ask would be "how small", and "how unpredictable". We could ask even though  $o(x)$  is a small number, maybe it is bigger than 1 or 5 or 100.

<sup>4</sup>He does not write in a very forgiving way, and occasionally it's so bad, we may as well try to write the step ourselves.

## References

(1) ...

