# Tune-Up: Times Tables

## Problem Statement

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Let's read their conjecture.

**Conjecture** The normalized counting measure on the finite set

$$\mathcal{J}_D = \left\{ (L, [L(\mathbb{Z})], [L^+(\mathbb{Z})], [\Lambda_{a_1(L)}, , [\Lambda_{a_2(L)}] : L \in \mathcal{R}_D \right\} \subseteq \mathsf{Gr}(\mathbb{R}) \times \mathcal{X}_2^4$$

equidistributes ot the uniform probability measure on $\mathsf{Gr}_{2\times 4}(\mathbb{R}) \times \mathcal{X}_2^4$ as $D \to \infty$ with $D \in \mathbb{D}$. That is:

$$\frac{1}{\mathcal{J}_D} \sum_{x \in \mathcal{J}_D} \delta_x \to m_{\mathsf{Gr}_{2\times 4} \times \mathcal{X}_2^4}$$

in the weak*-topology where $m_{\mathsf{Gr}_{2\times 4}}$ is the probability measure obtained from an $SO(4)$ invariant maesure on $\mathsf{Gr}_{2\times 4}(\mathbb{R})$ and an $\mathsf{SL}_2(\mathbb{R})$-invariant measure on $\mathbb{H}^2$

There's an infinite amount of geometric background here.

- E.g. what does weak* convergence of measures look like?
- $SO(4)$-invariant measureis usually given careful discussion in Lie algebra textbooks for some reason.

Here's what they actually prove (with congruence conditions). Resting on this conjecture (like a table) we could try to do a lot of numerical experimentation.

**Theorem** (Equidistribution) Let $p, q$ be any two odd primes. The normalized counting measure on the (finite set) $\mathcal{J}_D$ equidistributes to uniform probability measure on $\mathsf{Gr}_{2\times 4}(\mathbb{R}) \times \mathcal{X}_2^4$ as $D \in \mathbb{D}$ goes off to infiity while $D$ has the dditional conditions $-D \in (\mathbb{F}_P^\times)^2$ and $-D \in (\mathbb{F}_q^\times)^2$.

These innocent counting-measure problems have been argued using modular forms (in particular Maass forms). Results of this type have only been around since the 1950's. This is also a common story where result is "known" as far as academics are concerned which means almost nothing at this point.

# References


[1] Manfred Einsiedler, Elon Lindenstrauss **Joinings of Higher Rank Torus Actions on Homogeneous Spacs** `arXiv:1502.05133`

[2] Manfred Einsiedler, Manuel Luethi, Nimish Shah **Priitive Rational Points on Expanding Horocycles in Products of the Modular Surface with the Torus** `arXiv:1901.03078`

[3] Manny Aka, Manfred Einsiedler, Andreas Wieser **Planes in Four-Space and Four Associated CM Points** `arXiv:1901.05833`