Scratchwork: Box Counting

Here's a nice question about Counting Boxes:

The number of plane partitions that fit inside an $a \times b \times c$ box is:

$$M(a,b,c) = \prod_{i=1}^{a}\prod_{j=1}^{b}\prod_{k=1}^{c} \frac{i+j+k-2}{i+j+k-1} = \frac{H(a)H(b)H(c)H(a+b+c)}{H(a+b)H(b+c)H(c+a)}$$

where $H(a) = 1!\,2!\,3!\,\ldots\,a!$ is the **hyperfactorial**.

In this branch of math, almost any enumerative question you can think of has been discussed and generalized several times over. The notation may be clumsy and difficult to read, but there is always an answer. This is true, because the boxes have a very very large symmetry attached to them, that we can use at any time. This algebra, at least the size of $\widehat{\mathfrak{gl}}(\infty)$.

Like all branches of mathematics, there degrees of specialization and many points of view.

1 2 3 $\cdots$

Given, there is so much activity on one part of the board, we can just assume there is going to be generalizatino there. Where could we play instead?

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Why does McMahon formula look like the inclusion-exclusion principle?

26

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The McMahon formula for the number of tilings of an $a \times b \times c$ hexagon by lozenges:

$$\Big[H(a)H(b)H(c)\Big]\Big[H(a+b)H(b+c)H(c+a)\Big]^{-1}\Big[H(a+b+c)\Big]$$

looks oddly like the inclusion-exclusion formula:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

Here $H(a) = 1!2!\ldots a!$ is the hyperfactorial.

Perhaps there is a more general explanation via Gelfand-Tsetlins or something?

http://research.microsoft.com/en-us/um/people/cohn/Graphics/hexagon.gif

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asked Sep 23 '15 at 18:11
john mangual
9,453 ● 2 ● 29 ● 111

bumped to the homepage by Community ♦ yesterday

This question has answers that may be good or bad; the system has marked it active so that they can be reviewed.

10 This is some shameless self-promotion, but it might be related to the observations in my preprint: arxiv.org/abs/1505.02717 where it turns out a lot of tableau-counting can be done with inclusion/exclusion. – Per Alexandersson Sep 23 '15 at 18:38

There is a very nice post on Gel'fand-Tsetlin patterns by Terry Tao here: terrytao.wordpress.com/tag/gelfand-tsetlin-patterns – Todd Trimble ♦ Oct 4 '15 at 12:56

The number of tilings is clearly symmetric in a, b, c, and also clearly vanishes when one of a, b, c is zero. So if you already somehow know that the answer is going to be a product of functions of $a, b, c, a+b, a+c, b+c, a+b+c$ it has to basically be of the above form (but with an unknown function H). – Terry Tao yesterday

There is a ``heuristic'' explanation of the hook-length formula counting Standard Young Tableaux (attributed to Knuth), where you ask for a random filling of the diagram what is the probability for each box that its entry is filled with the smallest value among all entries in its hook: this is one over the hook length; then, the events are not independent, but if you multiply all these probabilities together it turns out you get the right probability that they all are the smallest (namely, that the tableau is standard). Maybe some similar heuristic can explain the MacMahon (not "Mc") formula. – Sam Hopkins yesterday

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1 Answer

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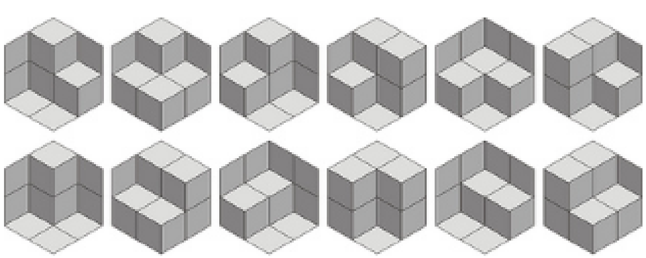
This is not an answer, but seeing as this is a fresh post, I would like to add:



References

[1] MathOverflow **Why does McMahon formula look like the inclusion-exclusion principle?** https://mathoverflow.net/q/219073/1358

[2] Alexei Borodin, Vadim Gorin **Shuffling algorithm for boxed plane partitions** arXiv:0804.3071