Worksheet: Exotic Number Systems

John D Mangual

Something truly exotic challenges your norms a little bit. . . makes one feel uneasy.

The goal of this project is to study new numbers and number-like things. We only try a small fraction of the ideas available. And there's no guarantee this will work.

Our decimal system is intimately related to the multiplication by ten shift map $\times 10$ we can envision a number as a sequence of decimal digits (kind of like a factory)

$$\pi = 3. \to 1 \to 4 \to 1 \to 5 \to 9 \to \dots$$

Each digit is connected to the last by one either by multplying by ten or shifting by 1:

- $T : x \mapsto (10 \times x) \pmod 1$ with $x \in \mathbb{R}$

- $T : (x_0, x_1, x_2, \dots) \mapsto (x_1, x_2, x_3, \dots)$ with $x \in \{0, 1\}^{\mathbb{N}}$

This framework kind of forgets that $\mathbb{R}$ is a ring $(\mathbb{R}, +, \times)$ or even a group $(\mathbb{R}, +)$. Regardless of our choice of T we can say that rings are isomorphic. Yet, in reality we must make a choice of T, and it can have nothing to do with the decimal system.

$$(x_0, x_1, x_2, \dots) \oplus (y_0, y_1, y_2, \dots) = ?$$

Our goal is to explain the different ways $\mathbb{R}$ could be partitioned and try to achieve an analogue of addition with carries. Some partitions are more ordinary, others are more unique.

References

(1) Michael Baake, Natalie Frank , Uwe Grimm , E. Arthur Robinson **Geometric properties of a binary non-Pisot inflation and absence of absolutely continuous diffraction** `arXiv:1706.03976`

(2) Roy L. Adler. **Symbolic dynamics and Markov partitions**
Bull. Amer. Math. Soc. 35 (1998), 1-56

(3) Carlos Matheus **New Numbers in M-L**
`https://matheuscmss.wordpress.com/2017/04/05/new-numbers-in-m-l/`