Scratchwork: Miraculous Cancellations

Let's explore the "miraculous cancellation" identites, proven by Francois Bonahon and Helen Wong. These authors know a lot about 2D surfaces. One the one hand, 2D surfaces are classfied by their genus, so why is a surface to these people a more complicated object? Bonahon's latest argument shows these cancellation type arguments in a purely algebraic framework, withouth any discussion – separate from the geometry of surfaces.

Proposition Let $XY = qYX$ with $q^n = 1$. Then $(X + Y)^n = X^n + Y^n$.

Lemma Let $q^n = 1$ be "primitive" root of unity, with $q^k \neq 1$. The "quantum binomial coefficent" $\binom{n}{k}_q = 0$, for $1 < k < n$.

Since we are researchers and not historians, I have to – without any prior knowledge of the subject – identify the unknown problems / conjecture to "work on".

References

[1] Francis Bonahon **Miraculous cancellations for quantum** SL_2 `arXiv:1708.07617`

[2] Francis Bonahon, Helen Wong
   Representations of the Kauffman bracket skein algebra I: invariants and miraculous cancellations `arXiv:1206.1638`
   Representations of the Kauffman bracket skein algebra II: punctured surfaces `arXiv:1206.1639`
   Representations of the Kauffman bracket skein algebra III: closed surfaces and naturality `arXiv:1505.01522`