Tune-Up: Pythagoras Triples

I seem to confuse two slightly different problems in Number Theory: pythagoras triples and primes as the sum of two squares. Without these, there's not much hope for anything else. The more we solve it, the more we can ask questions about the various methods, which seem a bit arbitrary. These advanced methods, seem to lump together entire classes of problems into a single bucket, without looking at any individual problem too carefully.

Ferma's Theorem says $p = a^2 + b^2$ if $p = 4k + 1$. This is true for $p \in \mathbb{Z}$. In order to do such a thing, we have actually called on *complex numbers* since we could say:

$$p = a^2 + b^2 = (a + bi)(a - bi) \in \mathbb{Z}[i]$$

In that case, we could define this as a statement of **ideals** in $\mathbb{Z}[i]$. We are trying to find the prime ideals $\mathfrak{p} \subset \mathbb{Z}[i]$.

$$[p\mathbb{Z}[i] : \mathbb{Z}[i]] = [p\mathbb{Z}[i] : (a + bi)\mathbb{Z}[i]][(a + bi)\mathbb{Z}[i] : \mathbb{Z}[i]]$$

So that $p = \mathfrak{p}\overline{\mathfrak{p}}$, with $\mathfrak{p} = a + bi \in \mathbb{Z}[i]$. We are counting the sizes of the ideals

A numerial example would to find a large prime number.

$$271 \times 271 + 476 \times 476 = 300017$$

and therefore we get a factorization of prime ideals.

$$\big[300017\,\mathbb{Z}[i] : \mathbb{Z}[i]\big] = \big[300017\,\mathbb{Z}[i] : (271 + 476i)\mathbb{Z}[i]\big]\ \big[(271 + 476i)\mathbb{Z}[i] : \mathbb{Z}[i]\big]$$

How does this neat and tidy world translate to the messy world of empirical data and statistics where nothing is certain? Where $\mathbb{R}$ is no longer adequate, since these are just limits of sequences of other measurements.

Computationally, how would we try to find numbers $a, b \in \mathbb{Z}$ such that $a^2 + b^2 = p = 300017$ and how did we know it was a prime number?

- $\sqrt{300017 - 178^2} \approx 518$ in that it's off by a small amount $\sqrt{p - a_1^2} = [\dots] + 0.0087$

This is an *approximation* type problem which gives us one more close solution in addition to the exact answer.

Examples

- $6 \times 6 + 100 \times 100 = 10036$
- $\sqrt{10037 - 6 \times 6} \approx [...] + 0.005$
- $36 \times 36 + 94 \times 94 = 10132$
- $\sqrt{10133 - 36 \times 36} \approx [...] + 0.00532$
- $24 \times 24 + 98 \times 98 = 10180$
- $\sqrt{10181 - 24 \times 24} \approx [...] + 0.0051$
- $56 \times 56 + 84 \times 84 = 10192$
- $\sqrt{10193 - 56 \times 56} \approx [...] + 0.00595$
- $54 \times 54 + 86 \times 86 = 10312$
- $\sqrt{10313 - 54 \times 54} \approx [...] + 0.00581$
- $72 \times 72 + 72 \times 72 = 10368$
- $\sqrt{10369 - 72 \times 72} \approx [...] + 0.00694$
- $36 \times 36 + 96 \times 96 = 10512$
- $\sqrt{10513 - 36 \times 36} \approx [...] + 0.00521$
- $14 \times 14 + 102 \times 102 = 10600$
- $\sqrt{10601 - 14 \times 14} \approx [...] + 0.0049$
- $60 \times 60 + 84 \times 84 = 10656$
- $\sqrt{10657 - 60 \times 60} \approx [...] + 0.00595$
- $68 \times 68 + 78 \times 78 = 10708$
- $\sqrt{10709 - 68 \times 68} \approx [...] + 0.00641$
- $18 \times 18 + 102 \times 102 = 10728$
- $\sqrt{10729 - 18 \times 18} \approx [...] + 0.0049$
- $6 \times 6 + 104 \times 104 = 10852$
- $\sqrt{10853 - 6 \times 6} \approx [...] + 0.00481$
- $22 \times 22 + 102 \times 102 = 10888$
- $\sqrt{10889 - 22 \times 22} \approx [...] + 0.0049$
- $6 \times 6 + 106 \times 106 = 11272$
- $\sqrt{11273 - 6 \times 6} \approx [...] + 0.00472$
- $42 \times 42 + 98 \times 98 = 11368$
- $\sqrt{11369 - 42 \times 42} \approx [...] + 0.0051$
- $24 \times 24 + 104 \times 104 = 11392$
- $\sqrt{11393 - 24 \times 24} \approx [...] + 0.00481$
- $4 \times 4 + 108 \times 108 = 11680$
- $\sqrt{11681 - 4 \times 4} \approx [...] + 0.00463$
- $6 \times 6 + 108 \times 108 = 11700$
- $\sqrt{11701 - 6 \times 6} \approx [...] + 0.00463$
- $30 \times 30 + 104 \times 104 = 11716$
- $\sqrt{11717 - 30 \times 30} \approx [...] + 0.00481$
- $24 \times 24 + 106 \times 106 = 11812$
- $\sqrt{11813 - 24 \times 24} \approx [...] + 0.00472$
- $48 \times 48 + 98 \times 98 = 11908$
- $\sqrt{11909 - 48 \times 48} \approx [...] + 0.0051$
- $0 \times 0 + 110 \times 110 = 12100$
- $\sqrt{12101 - 0 \times 0} \approx [...] + 0.00455$
- $36 \times 36 + 104 \times 104 = 12112$
- $\sqrt{12113 - 36 \times 36} \approx [...] + 0.00481$
- $22 \times 22 + 108 \times 108 = 12148$
- $\sqrt{12149 - 22 \times 22} \approx [...] + 0.00463$
- $64 \times 64 + 90 \times 90 = 12196$
- $\sqrt{12197 - 64 \times 64} \approx [...] + 0.00556$
- $24 \times 24 + 108 \times 108 = 12240$
- $\sqrt{12241 - 24 \times 24} \approx [...] + 0.00463$
- $60 \times 60 + 94 \times 94 = 12436$
- $\sqrt{12437 - 60 \times 60} \approx [...] + 0.00532$
- $66 \times 66 + 90 \times 90 = 12456$
- $\sqrt{12457 - 66 \times 66} \approx [...] + 0.00556$
- $12 \times 12 + 112 \times 112 = 12688$
- $\sqrt{12689 - 12 \times 12} \approx [...] + 0.00446$
- $78 \times 78 + 82 \times 82 = 12808$

- $\sqrt{12809 - 78 \times 78} \approx [...] + 0.0061$
- $34 \times 34 + 108 \times 108 = 12820$
- $\sqrt{12821 - 34 \times 34} \approx [...] + 0.00463$
- $54 \times 54 + 100 \times 100 = 12916$
- $\sqrt{12917 - 54 \times 54} \approx [...] + 0.005$
- $2 \times 2 + 114 \times 114 = 13000$
- $\sqrt{13001 - 2 \times 2} \approx [...] + 0.00439$
- $6 \times 6 + 114 \times 114 = 13032$
- $\sqrt{13033 - 6 \times 6} \approx [...] + 0.00439$
- $38 \times 38 + 108 \times 108 = 13108$
- $\sqrt{13109 - 38 \times 38} \approx [...] + 0.00463$
- $24 \times 24 + 112 \times 112 = 13120$
- $\sqrt{13121 - 24 \times 24} \approx [...] + 0.00446$
- $64 \times 64 + 96 \times 96 = 13312$
- $\sqrt{13313 - 64 \times 64} \approx [...] + 0.00521$
- $20 \times 20 + 114 \times 114 = 13396$
- $\sqrt{13397 - 20 \times 20} \approx [...] + 0.00439$
- $0 \times 0 + 116 \times 116 = 13456$
- $\sqrt{13457 - 0 \times 0} \approx [...] + 0.00431$
- $74 \times 74 + 90 \times 90 = 13576$
- $\sqrt{13577 - 74 \times 74} \approx [...] + 0.00556$
- $72 \times 72 + 92 \times 92 = 13648$
- $\sqrt{13649 - 72 \times 72} \approx [...] + 0.00543$
- $18 \times 18 + 116 \times 116 = 13780$
- $\sqrt{13781 - 18 \times 18} \approx [...] + 0.00431$
- $78 \times 78 + 88 \times 88 = 13828$
- $\sqrt{13829 - 78 \times 78} \approx [...] + 0.00568$
- $36 \times 36 + 112 \times 112 = 13840$
- $\sqrt{13841 - 36 \times 36} \approx [...] + 0.00446$
- $76 \times 76 + 90 \times 90 = 13876$
- $\sqrt{13877 - 76 \times 76} \approx [...] + 0.00556$
- $24 \times 24 + 116 \times 116 = 14032$
- $\sqrt{14033 - 24 \times 24} \approx [...] + 0.00431$
- $34 \times 34 + 114 \times 114 = 14152$
- $\sqrt{14153 - 34 \times 34} \approx [...] + 0.00439$
- $18 \times 18 + 118 \times 118 = 14248$
- $\sqrt{14249 - 18 \times 18} \approx [...] + 0.00424$
- $36 \times 36 + 114 \times 114 = 14292$
- $\sqrt{14293 - 36 \times 36} \approx [...] + 0.00439$
- $52 \times 52 + 108 \times 108 = 14368$
- $\sqrt{14369 - 52 \times 52} \approx [...] + 0.00463$
- $0 \times 0 + 120 \times 120 = 14400$
- $\sqrt{14401 - 0 \times 0} \approx [...] + 0.00417$
- $6 \times 6 + 120 \times 120 = 14436$
- $\sqrt{14437 - 6 \times 6} \approx [...] + 0.00417$
- $78 \times 78 + 92 \times 92 = 14548$
- $\sqrt{14549 - 78 \times 78} \approx [...] + 0.00543$
- $16 \times 16 + 120 \times 120 = 14656$
- $\sqrt{14657 - 16 \times 16} \approx [...] + 0.00417$
- $36 \times 36 + 116 \times 116 = 14752$
- $\sqrt{14753 - 36 \times 36} \approx [...] + 0.00431$
- $54 \times 54 + 110 \times 110 = 15016$
- $\sqrt{15017 - 54 \times 54} \approx [...] + 0.00455$
- $26 \times 26 + 120 \times 120 = 15076$
- $\sqrt{15077 - 26 \times 26} \approx [...] + 0.00417$
- $66 \times 66 + 104 \times 104 = 15172$
- $\sqrt{15173 - 66 \times 66} \approx [...] + 0.00481$
- $0 \times 0 + 124 \times 124 = 15376$
- $\sqrt{15377 - 0 \times 0} \approx [...] + 0.00403$
- $6 \times 6 + 124 \times 124 = 15412$
- $\sqrt{15413 - 6 \times 6} \approx [...] + 0.00403$

- $24 \times 24 + 122 \times 122 = 15460$
- $\sqrt{15461 - 24 \times 24} \approx [...] + 0.0041$
- $50 \times 50 + 114 \times 114 = 15496$
- $\sqrt{15497 - 50 \times 50} \approx [...] + 0.00439$
- $48 \times 48 + 116 \times 116 = 15760$
- $\sqrt{15761 - 48 \times 48} \approx [...] + 0.00431$
- $0 \times 0 + 126 \times 126 = 15876$
- $\sqrt{15877 - 0 \times 0} \approx [...] + 0.00397$
- $2 \times 2 + 126 \times 126 = 15880$
- $\sqrt{15881 - 2 \times 2} \approx [...] + 0.00397$
- $6 \times 6 + 126 \times 126 = 15912$
- $\sqrt{15913 - 6 \times 6} \approx [...] + 0.00397$
- $40 \times 40 + 120 \times 120 = 16000$
- $\sqrt{16001 - 40 \times 40} \approx [...] + 0.00417$
- $14 \times 14 + 126 \times 126 = 16072$
- $\sqrt{16073 - 14 \times 14} \approx [...] + 0.00397$
- $48 \times 48 + 118 \times 118 = 16228$
- $\sqrt{16229 - 48 \times 48} \approx [...] + 0.00424$
- $84 \times 84 + 96 \times 96 = 16272$
- $\sqrt{16273 - 84 \times 84} \approx [...] + 0.00521$
- $22 \times 22 + 126 \times 126 = 16360$
- $\sqrt{16361 - 22 \times 22} \approx [...] + 0.00397$
- $6 \times 6 + 128 \times 128 = 16420$
- $\sqrt{16421 - 6 \times 6} \approx [...] + 0.00391$
- $24 \times 24 + 126 \times 126 = 16452$
- $\sqrt{16453 - 24 \times 24} \approx [...] + 0.00397$
- $12 \times 12 + 128 \times 128 = 16528$
- $\sqrt{16529 - 12 \times 12} \approx [...] + 0.00391$
- $26 \times 26 + 126 \times 126 = 16552$
- $\sqrt{16553 - 26 \times 26} \approx [...] + 0.00397$
- $42 \times 42 + 122 \times 122 = 16648$
- $\sqrt{16649 - 42 \times 42} \approx [...] + 0.0041$
- $28 \times 28 + 126 \times 126 = 16660$
- $\sqrt{16661 - 28 \times 28} \approx [...] + 0.00397$
- $36 \times 36 + 124 \times 124 = 16672$
- $\sqrt{16673 - 36 \times 36} \approx [...] + 0.00403$
- $0 \times 0 + 130 \times 130 = 16900$
- $\sqrt{16901 - 0 \times 0} \approx [...] + 0.00385$
- $6 \times 6 + 130 \times 130 = 16936$
- $\sqrt{16937 - 6 \times 6} \approx [...] + 0.00385$
- $34 \times 34 + 126 \times 126 = 17032$
- $\sqrt{17033 - 34 \times 34} \approx [...] + 0.00397$
- $64 \times 64 + 114 \times 114 = 17092$
- $\sqrt{17093 - 64 \times 64} \approx [...] + 0.00439$
- $48 \times 48 + 122 \times 122 = 17188$
- $\sqrt{17189 - 48 \times 48} \approx [...] + 0.0041$
- $54 \times 54 + 120 \times 120 = 17316$
- $\sqrt{17317 - 54 \times 54} \approx [...] + 0.00417$
- $38 \times 38 + 126 \times 126 = 17320$
- $\sqrt{17321 - 38 \times 38} \approx [...] + 0.00397$
- $24 \times 24 + 130 \times 130 = 17476$
- $\sqrt{17477 - 24 \times 24} \approx [...] + 0.00385$
- $8 \times 8 + 132 \times 132 = 17488$
- $\sqrt{17489 - 8 \times 8} \approx [...] + 0.00379$
- $12 \times 12 + 132 \times 132 = 17568$
- $\sqrt{17569 - 12 \times 12} \approx [...] + 0.00379$
- $16 \times 16 + 132 \times 132 = 17680$
- $\sqrt{17681 - 16 \times 16} \approx [...] + 0.00379$
- $72 \times 72 + 112 \times 112 = 17728$
- $\sqrt{17729 - 72 \times 72} \approx [...] + 0.00446$
- $18 \times 18 + 132 \times 132 = 17748$

- $\sqrt{17749 - 18 \times 18} \approx [...] + 0.00379$
- $22 \times 22 + 132 \times 132 = 17908$
- $\sqrt{17909 - 22 \times 22} \approx [...] + 0.00379$
- $0 \times 0 + 134 \times 134 = 17956$
- $\sqrt{17957 - 0 \times 0} \approx [...] + 0.00373$
- $42 \times 42 + 128 \times 128 = 18148$
- $\sqrt{18149 - 42 \times 42} \approx [...] + 0.00391$
- $48 \times 48 + 126 \times 126 = 18180$
- $\sqrt{18181 - 48 \times 48} \approx [...] + 0.00397$
- $96 \times 96 + 96 \times 96 = 18432$
- $\sqrt{18433 - 96 \times 96} \approx [...] + 0.00521$
- $66 \times 66 + 120 \times 120 = 18756$
- $\sqrt{18757 - 66 \times 66} \approx [...] + 0.00417$
- $76 \times 76 + 114 \times 114 = 18772$
- $\sqrt{18773 - 76 \times 76} \approx [...] + 0.00439$
- $54 \times 54 + 126 \times 126 = 18792$
- $\sqrt{18793 - 54 \times 54} \approx [...] + 0.00397$
- $38 \times 38 + 132 \times 132 = 18868$
- $\sqrt{18869 - 38 \times 38} \approx [...] + 0.00379$
- $90 \times 90 + 104 \times 104 = 18916$
- $\sqrt{18917 - 90 \times 90} \approx [...] + 0.00481$
- $56 \times 56 + 126 \times 126 = 19012$
- $\sqrt{19013 - 56 \times 56} \approx [...] + 0.00397$
- $24 \times 24 + 136 \times 136 = 19072$
- $\sqrt{19073 - 24 \times 24} \approx [...] + 0.00368$
- $6 \times 6 + 138 \times 138 = 19080$
- $\sqrt{19081 - 6 \times 6} \approx [...] + 0.00362$
- $84 \times 84 + 110 \times 110 = 19156$
- $\sqrt{19157 - 84 \times 84} \approx [...] + 0.00455$
- $16 \times 16 + 138 \times 138 = 19300$
- $\sqrt{19301 - 16 \times 16} \approx [...] + 0.00362$
- $60 \times 60 + 126 \times 126 = 19476$
- $\sqrt{19477 - 60 \times 60} \approx [...] + 0.00397$
- $46 \times 46 + 132 \times 132 = 19540$
- $\sqrt{19541 - 46 \times 46} \approx [...] + 0.00379$
- $36 \times 36 + 136 \times 136 = 19792$
- $\sqrt{19793 - 36 \times 36} \approx [...] + 0.00368$
- $64 \times 64 + 126 \times 126 = 19972$
- $\sqrt{19973 - 64 \times 64} \approx [...] + 0.00397$

These examples are *abundant* yet they cost us time and resources. Should we look for more patterns? How do these compare to what we already have? What do we do about them? Just a quick-look one of them should go to a runoff:

$$19013 - 56^2 = 19973 - 64^2 \approx 0.0039682$$

This one was equality. We are looking for $\approx$ or $\asymp$ and not really $=$. Maybe $\equiv$. And there is some typos.

Even more patterns.

- $6 \times 6 + 1000 \times 1000 = 1000036$
- $\sqrt{1000037 - 6 \times 6} \approx [...] + 0.0005$
- $90 \times 90 + 996 \times 996 = 1000116$
- $\sqrt{1000117 - 90 \times 90} \approx [...] + 0.0005$
- $24 \times 24 + 1000 \times 1000 = 1000576$
- $\sqrt{1000577 - 24 \times 24} \approx [...] + 0.0005$
- $302 \times 302 + 954 \times 954 = 1001320$
- $\sqrt{1001321 - 302 \times 302} \approx [...] + 0.00052$
- $182 \times 182 + 984 \times 984 = 1001380$
- $\sqrt{1001381 - 182 \times 182} \approx [...] + 0.00051$
- $98 \times 98 + 996 \times 996 = 1001620$
- $\sqrt{1001621 - 98 \times 98} \approx [...] + 0.0005$
- $262 \times 262 + 966 \times 966 = 1001800$
- $\sqrt{1001801 - 262 \times 262} \approx [...] + 0.00052$
- $612 \times 612 + 792 \times 792 = 1001808$
- $\sqrt{1001809 - 612 \times 612} \approx [...] + 0.00063$
- $462 \times 462 + 888 \times 888 = 1001988$
- $\sqrt{1001989 - 462 \times 462} \approx [...] + 0.00056$
- $100 \times 100 + 996 \times 996 = 1002016$
- $\sqrt{1002017 - 100 \times 100} \approx [...] + 0.0005$
- $672 \times 672 + 742 \times 742 = 1002148$
- $\sqrt{1002149 - 672 \times 672} \approx [...] + 0.00067$
- $284 \times 284 + 960 \times 960 = 1002256$
- $\sqrt{1002257 - 284 \times 284} \approx [...] + 0.00052$
- $386 \times 386 + 924 \times 924 = 1002772$
- $\sqrt{1002773 - 386 \times 386} \approx [...] + 0.00054$
- $696 \times 696 + 720 \times 720 = 1002816$
- $\sqrt{1002817 - 696 \times 696} \approx [...] + 0.00069$
- $186 \times 186 + 984 \times 984 = 1002852$
- $\sqrt{1002853 - 186 \times 186} \approx [...] + 0.00051$
- $54 \times 54 + 1000 \times 1000 = 1002916$
- $\sqrt{1002917 - 54 \times 54} \approx [...] + 0.0005$
- $138 \times 138 + 992 \times 992 = 1003108$
- $\sqrt{1003109 - 138 \times 138} \approx [...] + 0.0005$
- $216 \times 216 + 978 \times 978 = 1003140$
- $\sqrt{1003141 - 216 \times 216} \approx [...] + 0.00051$
- $242 \times 242 + 972 \times 972 = 1003348$
- $\sqrt{1003349 - 242 \times 242} \approx [...] + 0.00051$
- $286 \times 286 + 960 \times 960 = 1003396$
- $\sqrt{1003397 - 286 \times 286} \approx [...] + 0.00052$
- $234 \times 234 + 974 \times 974 = 1003432$
- $\sqrt{1003433 - 234 \times 234} \approx [...] + 0.00051$
- $258 \times 258 + 968 \times 968 = 1003588$
- $\sqrt{1003589 - 258 \times 258} \approx [...] + 0.00052$
- $60 \times 60 + 1000 \times 1000 = 1003600$
- $\sqrt{1003601 - 60 \times 60} \approx [...] + 0.0005$
- $306 \times 306 + 954 \times 954 = 1003752$
- $\sqrt{1003753 - 306 \times 306} \approx [...] + 0.00052$
- $154 \times 154 + 990 \times 990 = 1003816$
- $\sqrt{1003817 - 154 \times 154} \approx [...] + 0.00051$
- $432 \times 432 + 904 \times 904 = 1003840$
- $\sqrt{1003841 - 432 \times 432} \approx [...] + 0.00055$
- $126 \times 126 + 994 \times 994 = 1003912$
- $\sqrt{1003913 - 126 \times 126} \approx [...] + 0.0005$
- $336 \times 336 + 944 \times 944 = 1004032$
- $\sqrt{1004033 - 336 \times 336} \approx [...] + 0.00053$
- $110 \times 110 + 996 \times 996 = 1004116$
- $\sqrt{1004117 - 110 \times 110} \approx [...] + 0.0005$
- $210 \times 210 + 980 \times 980 = 1004500$

- $\sqrt{1004501 - 210 \times 210} \approx [...] + 0.00051$
- $112 \times 112 + 996 \times 996 = 1004560$
- $\sqrt{1004561 - 112 \times 112} \approx [...] + 0.0005$
- $268 \times 268 + 966 \times 966 = 1004980$
- $\sqrt{1004981 - 268 \times 268} \approx [...] + 0.00052$
- $114 \times 114 + 996 \times 996 = 1005012$
- $\sqrt{1005013 - 114 \times 114} \approx [...] + 0.0005$
- $32 \times 32 + 1002 \times 1002 = 1005028$
- $\sqrt{1005029 - 32 \times 32} \approx [...] + 0.0005$
- $34 \times 34 + 1002 \times 1002 = 1005160$
- $\sqrt{1005161 - 34 \times 34} \approx [...] + 0.0005$
- $450 \times 450 + 896 \times 896 = 1005316$
- $\sqrt{1005317 - 450 \times 450} \approx [...] + 0.00056$
- $704 \times 704 + 714 \times 714 = 1005412$
- $\sqrt{1005413 - 704 \times 704} \approx [...] + 0.0007$
- $160 \times 160 + 990 \times 990 = 1005700$
- $\sqrt{1005701 - 160 \times 160} \approx [...] + 0.00051$
- $474 \times 474 + 884 \times 884 = 1006132$
- $\sqrt{1006133 - 474 \times 474} \approx [...] + 0.00057$
- $310 \times 310 + 954 \times 954 = 1006216$
- $\sqrt{1006217 - 310 \times 310} \approx [...] + 0.00052$
- $48 \times 48 + 1002 \times 1002 = 1006308$
- $\sqrt{1006309 - 48 \times 48} \approx [...] + 0.0005$
- $418 \times 418 + 912 \times 912 = 1006468$
- $\sqrt{1006469 - 418 \times 418} \approx [...] + 0.00055$
- $684 \times 684 + 734 \times 734 = 1006612$
- $\sqrt{1006613 - 684 \times 684} \approx [...] + 0.00068$
- $264 \times 264 + 968 \times 968 = 1006720$
- $\sqrt{1006721 - 264 \times 264} \approx [...] + 0.00052$
- $552 \times 552 + 838 \times 838 = 1006948$
- $\sqrt{1006949 - 552 \times 552} \approx [...] + 0.0006$
- $138 \times 138 + 994 \times 994 = 1007080$
- $\sqrt{1007081 - 138 \times 138} \approx [...] + 0.0005$
- $444 \times 444 + 900 \times 900 = 1007136$
- $\sqrt{1007137 - 444 \times 444} \approx [...] + 0.00056$
- $456 \times 456 + 894 \times 894 = 1007172$
- $\sqrt{1007173 - 456 \times 456} \approx [...] + 0.00056$
- $540 \times 540 + 846 \times 846 = 1007316$
- $\sqrt{1007317 - 540 \times 540} \approx [...] + 0.00059$
- $318 \times 318 + 952 \times 952 = 1007428$
- $\sqrt{1007429 - 318 \times 318} \approx [...] + 0.00053$
- $392 \times 392 + 924 \times 924 = 1007440$
- $\sqrt{1007441 - 392 \times 392} \approx [...] + 0.00054$
- $336 \times 336 + 946 \times 946 = 1007812$
- $\sqrt{1007813 - 336 \times 336} \approx [...] + 0.00053$
- $0 \times 0 + 1004 \times 1004 = 1008016$
- $\sqrt{1008017 - 0 \times 0} \approx [...] + 0.0005$
- $294 \times 294 + 960 \times 960 = 1008036$
- $\sqrt{1008037 - 294 \times 294} \approx [...] + 0.00052$
- $64 \times 64 + 1002 \times 1002 = 1008100$
- $\sqrt{1008101 - 64 \times 64} \approx [...] + 0.0005$
- $568 \times 568 + 828 \times 828 = 1008208$
- $\sqrt{1008209 - 568 \times 568} \approx [...] + 0.0006$
- $274 \times 274 + 966 \times 966 = 1008232$
- $\sqrt{1008233 - 274 \times 274} \approx [...] + 0.00052$
- $200 \times 200 + 984 \times 984 = 1008256$
- $\sqrt{1008257 - 200 \times 200} \approx [...] + 0.00051$
- $128 \times 128 + 996 \times 996 = 1008400$
- $\sqrt{1008401 - 128 \times 128} \approx [...] + 0.0005$
- $144 \times 144 + 994 \times 994 = 1008772$
- $\sqrt{1008773 - 144 \times 144} \approx [...] + 0.0005$

- $594 \times 594 + 810 \times 810 = 1008936$
- $\sqrt{1008937 - 594 \times 594} \approx [...] + 0.00062$
- $192 \times 192 + 986 \times 986 = 1009060$
- $\sqrt{1009061 - 192 \times 192} \approx [...] + 0.00051$
- $560 \times 560 + 834 \times 834 = 1009156$
- $\sqrt{1009157 - 560 \times 560} \approx [...] + 0.0006$
- $72 \times 72 + 1002 \times 1002 = 1009188$
- $\sqrt{1009189 - 72 \times 72} \approx [...] + 0.0005$
- $246 \times 246 + 974 \times 974 = 1009192$
- $\sqrt{1009193 - 246 \times 246} \approx [...] + 0.00051$
- $254 \times 254 + 972 \times 972 = 1009300$
- $\sqrt{1009301 - 254 \times 254} \approx [...] + 0.00051$
- $600 \times 600 + 806 \times 806 = 1009636$
- $\sqrt{1009637 - 600 \times 600} \approx [...] + 0.00062$
- $42 \times 42 + 1004 \times 1004 = 1009780$
- $\sqrt{1009781 - 42 \times 42} \approx [...] + 0.0005$
- $204 \times 204 + 984 \times 984 = 1009872$
- $\sqrt{1009873 - 204 \times 204} \approx [...] + 0.00051$

Even more skepitical than ever.

References

[1] Yann Bugeaud **Effective simultaneous rational approximation to pairs of real quadratic numbers** `1907.10253`

[2] Ivan Nourdin, Giovanni Peccatti, Maurizia Rossi **Nodal Statistics of Planar Random Waves** `arXiv:1708.02281`

[3] Zeev Rudnick, Ezra Waxman **Angles of Gaussian Primes** `arXiv:1705.07498`

[4] Alexander Logunov, Eugenia Malinnikova **Nodal Sets of Laplace Eigenfunctions: Estimates of the Hausdorff Measure in Dimension Two and Three** `arXiv:1605.02595`