An Inversion Problem

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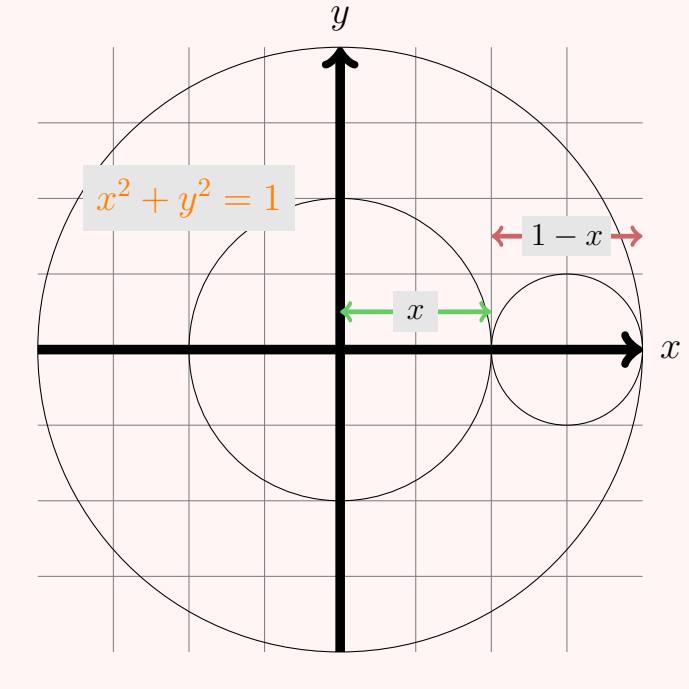
I ask around for a solution to an inversion problem. Everyone could show me **how** to solve it but nobody wanted put the solution[1]



Let $a = \frac{1}{3}$. I would like the image of these circles under the map:

$$z \mapsto \frac{z - a}{\overline{a}z - 1}$$

[1] I didn't ask "how would you solve it" – I was asking for an explicity answer, with a center and a radius. Nobody wanted to. If you do it neatly takes about a page (or less). If you don't know algebra it takes 10 pages and you get nowhere. This was a skill in textbook in 19th century – and in fact all of my resources come from that time period.

A - the Easy Way

This particular layout of circle is symmetric about the x axis — so we might[2] find an easier solution!

[2]The small circle $|z-3|=\frac{1}{4}$ is moving in between $|z|=1$ and the image of $|z|=\frac{1}{2}$. What happens (under inversion) if I rotate this figure? My question is what the image of the circle is under the map $z \mapsto \frac{z-a}{\overline{a}z-1}$ and also $a = e^{i\theta}\frac{1}{3}$ and $\theta \in [0, 2\pi]$.

B - from Old Textbooks

References

(1) Curtis McMullen. **Uniformly Diophantine Fixed Numbers in a Real Quadratic Field**

(2) Jean Bourgain, Alex Kontorovich. **Beyond Expansion II: Traces of Thin Semigroups** arXiv:1310.7190v1