## Tune-Up: Pythagoras Triples

I seem to confuse two slightly different problems in Number Theory: pythagoras triples and primes as the sum of two squares. Without these, there's not much hope for anything else. The more we solve it, the more we can ask questions about the various methods, which seem a bit arbitrary. These advanced methods, seem to lump together entire classes of problems into a single bucket, without looking at any individual problem too carefully.

Ferma's Theorem says  $p=a^2+b^2$  if p=4k+1. This is true for  $p\in\mathbb{Z}$ . In order to do such a thing, we have actually called on *complex numbers* since we could say:

$$p = a^2 + b^2 = (a + bi)(a - bi) \in \mathbb{Z}[i]$$

In that case, we could define this as a statement of **ideals** in  $\mathbb{Z}[i]$ . We are trying to find the prime ideals  $\mathfrak{p} \subset \mathbb{Z}[i]$ .

$$[p\mathbb{Z}[i] : \mathbb{Z}[i]] = [p\mathbb{Z}[i] : (a+bi)\mathbb{Z}[i]][(a+bi)\mathbb{Z}[i] : \mathbb{Z}[i]]$$

So that  $p = \mathfrak{p}\overline{\mathfrak{p}}$ , with  $\mathfrak{p} = a + bi \in \mathbb{Z}[i]$ . We are counting the sizes of the ideals

A numerial example would to find a large prime number.

$$271 \times 271 + 476 \times 476 = 300017$$

and therefore we get a factorization of prime ideals.

$$\left[ 300017 \, \mathbb{Z}[i] : \mathbb{Z}[i] \right] = \left[ 300017 \, \mathbb{Z}[i] : (271 + 476i) \mathbb{Z}[i] \right] \left[ (271 + 476i) \mathbb{Z}[i] : \mathbb{Z}[i] \right]$$

How does this neat and tidy world translate to the messy world of empirical data and statistics where nothing is certain? Where  $\mathbb{R}$  is no longer adequate, since these are just limits of sequences of other measurements.

Computationally, how would we try to find numbers  $a,b\in\mathbb{Z}$  such that  $a^2+b^2=p=300017$  and how did we know it was a prime number?

• 
$$\sqrt{300017-178^2} \approx 518$$
 in that it's off by a small amount  $\sqrt{p-a_1^2} = [\dots] + 0.0087$ 

This is an approximation type problem which gives us one more close solution in addition to the exact answer.

## **Examples**

• 
$$6 \times 6 + 100 \times 100 = 10036$$

• 
$$\sqrt{10037 - 6 \times 6} \approx [...] + 0.005$$

• 
$$36 \times 36 + 94 \times 94 = 10132$$

• 
$$\sqrt{10133 - 36 \times 36} \approx [...] + 0.00532$$

• 
$$24 \times 24 + 98 \times 98 = 10180$$

• 
$$\sqrt{10181 - 24 \times 24} \approx [...] + 0.0051$$

• 
$$56 \times 56 + 84 \times 84 = 10192$$

• 
$$\sqrt{10193 - 56 \times 56} \approx [...] + 0.00595$$

• 
$$54 \times 54 + 86 \times 86 = 10312$$

• 
$$\sqrt{10313 - 54 \times 54} \approx [...] + 0.00581$$

• 
$$72 \times 72 + 72 \times 72 = 10368$$

• 
$$\sqrt{10369 - 72 \times 72} \approx [...] + 0.00694$$

• 
$$36 \times 36 + 96 \times 96 = 10512$$

• 
$$\sqrt{10513 - 36 \times 36} \approx [...] + 0.00521$$

• 
$$14 \times 14 + 102 \times 102 = 10600$$

• 
$$\sqrt{10601 - 14 \times 14} \approx [...] + 0.0049$$

• 
$$60 \times 60 + 84 \times 84 = 10656$$

• 
$$\sqrt{10657 - 60 \times 60} \approx [...] + 0.00595$$

• 
$$68 \times 68 + 78 \times 78 = 10708$$

• 
$$\sqrt{10709 - 68 \times 68} \approx [...] + 0.00641$$

• 
$$18 \times 18 + 102 \times 102 = 10728$$

• 
$$\sqrt{10729 - 18 \times 18} \approx [...] + 0.0049$$

• 
$$6 \times 6 + 104 \times 104 = 10852$$

• 
$$\sqrt{10853 - 6 \times 6} \approx [...] + 0.00481$$

• 
$$22 \times 22 + 102 \times 102 = 10888$$

• 
$$\sqrt{10889 - 22 \times 22} \approx [...] + 0.0049$$

• 
$$6 \times 6 + 106 \times 106 = 11272$$

• 
$$\sqrt{11273 - 6 \times 6} \approx [...] + 0.00472$$

• 
$$42 \times 42 + 98 \times 98 = 11368$$

• 
$$\sqrt{11369 - 42 \times 42} \approx [...] + 0.0051$$

• 
$$24 \times 24 + 104 \times 104 = 11392$$

• 
$$\sqrt{11393 - 24 \times 24} \approx [...] + 0.00481$$

• 
$$4 \times 4 + 108 \times 108 = 11680$$

• 
$$\sqrt{11681 - 4 \times 4} \approx [...] + 0.00463$$

• 
$$6 \times 6 + 108 \times 108 = 11700$$

• 
$$\sqrt{11701 - 6 \times 6} \approx [...] + 0.00463$$

• 
$$30 \times 30 + 104 \times 104 = 11716$$

• 
$$\sqrt{11717 - 30 \times 30} \approx [...] + 0.00481$$

• 
$$24 \times 24 + 106 \times 106 = 11812$$

• 
$$\sqrt{11813 - 24 \times 24} \approx [...] + 0.00472$$

• 
$$48 \times 48 + 98 \times 98 = 11908$$

• 
$$\sqrt{11909 - 48 \times 48} \approx [...] + 0.0051$$

• 
$$0 \times 0 + 110 \times 110 = 12100$$

• 
$$\sqrt{12101 - 0 \times 0} \approx [...] + 0.00455$$

• 
$$36 \times 36 + 104 \times 104 = 12112$$

• 
$$\sqrt{12113 - 36 \times 36} \approx [...] + 0.00481$$

• 
$$22 \times 22 + 108 \times 108 = 12148$$

• 
$$\sqrt{12149 - 22 \times 22} \approx [...] + 0.00463$$

• 
$$64 \times 64 + 90 \times 90 = 12196$$

• 
$$\sqrt{12197 - 64 \times 64} \approx [...] + 0.00556$$

• 
$$24 \times 24 + 108 \times 108 = 12240$$

• 
$$\sqrt{12241 - 24 \times 24} \approx [...] + 0.00463$$

• 
$$60 \times 60 + 94 \times 94 = 12436$$

• 
$$\sqrt{12437 - 60 \times 60} \approx [...] + 0.00532$$

• 
$$66 \times 66 + 90 \times 90 = 12456$$

• 
$$\sqrt{12457 - 66 \times 66} \approx [...] + 0.00556$$

• 
$$12 \times 12 + 112 \times 112 = 12688$$

• 
$$\sqrt{12689 - 12 \times 12} \approx [...] + 0.00446$$

• 
$$78 \times 78 + 82 \times 82 = 12808$$

- $\sqrt{12809 78 \times 78} \approx [...] + 0.0061$
- $34 \times 34 + 108 \times 108 = 12820$
- $\sqrt{12821 34 \times 34} \approx [...] + 0.00463$
- $54 \times 54 + 100 \times 100 = 12916$
- $\sqrt{12917 54 \times 54} \approx [...] + 0.005$
- $2 \times 2 + 114 \times 114 = 13000$
- $\sqrt{13001 2 \times 2} \approx [...] + 0.00439$
- $6 \times 6 + 114 \times 114 = 13032$
- $\sqrt{13033 6 \times 6} \approx [...] + 0.00439$
- $38 \times 38 + 108 \times 108 = 13108$
- $\sqrt{13109 38 \times 38} \approx [...] + 0.00463$
- $24 \times 24 + 112 \times 112 = 13120$
- $\sqrt{13121 24 \times 24} \approx [...] + 0.00446$
- $64 \times 64 + 96 \times 96 = 13312$
- $\sqrt{13313 64 \times 64} \approx [...] + 0.00521$
- $20 \times 20 + 114 \times 114 = 13396$
- $\sqrt{13397 20 \times 20} \approx [...] + 0.00439$
- $0 \times 0 + 116 \times 116 = 13456$
- $\sqrt{13457 0 \times 0} \approx [...] + 0.00431$
- $74 \times 74 + 90 \times 90 = 13576$
- $\sqrt{13577 74 \times 74} \approx [...] + 0.00556$
- $72 \times 72 + 92 \times 92 = 13648$
- $\sqrt{13649 72 \times 72} \approx [...] + 0.00543$
- $18 \times 18 + 116 \times 116 = 13780$
- $\sqrt{13781 18 \times 18} \approx [...] + 0.00431$
- $78 \times 78 + 88 \times 88 = 13828$
- $\sqrt{13829 78 \times 78} \approx [...] + 0.00568$
- $36 \times 36 + 112 \times 112 = 13840$
- $\sqrt{13841 36 \times 36} \approx [...] + 0.00446$
- $76 \times 76 + 90 \times 90 = 13876$

- $\sqrt{13877 76 \times 76} \approx [...] + 0.00556$
- $24 \times 24 + 116 \times 116 = 14032$
- $\sqrt{14033 24 \times 24} \approx [...] + 0.00431$
- $34 \times 34 + 114 \times 114 = 14152$
- $\sqrt{14153 34 \times 34} \approx [...] + 0.00439$
- $18 \times 18 + 118 \times 118 = 14248$
- $\sqrt{14249 18 \times 18} \approx [...] + 0.00424$
- $36 \times 36 + 114 \times 114 = 14292$
- $\sqrt{14293 36 \times 36} \approx [...] + 0.00439$
- $52 \times 52 + 108 \times 108 = 14368$
- $\sqrt{14369 52 \times 52} \approx [...] + 0.00463$
- $0 \times 0 + 120 \times 120 = 14400$
- $\sqrt{14401 0 \times 0} \approx [...] + 0.00417$
- $6 \times 6 + 120 \times 120 = 14436$
- $\sqrt{14437 6 \times 6} \approx [...] + 0.00417$
- $78 \times 78 + 92 \times 92 = 14548$
- $\sqrt{14549 78 \times 78} \approx [...] + 0.00543$
- $16 \times 16 + 120 \times 120 = 14656$
- $\sqrt{14657 16 \times 16} \approx [...] + 0.00417$
- $36 \times 36 + 116 \times 116 = 14752$
- $\sqrt{14753 36 \times 36} \approx [...] + 0.00431$
- $54 \times 54 + 110 \times 110 = 15016$
- $\sqrt{15017 54 \times 54} \approx [...] + 0.00455$
- $26 \times 26 + 120 \times 120 = 15076$
- $\sqrt{15077 26 \times 26} \approx [...] + 0.00417$
- $66 \times 66 + 104 \times 104 = 15172$
- $\sqrt{15173 66 \times 66} \approx [...] + 0.00481$
- $0 \times 0 + 124 \times 124 = 15376$
- $\sqrt{15377 0 \times 0} \approx [...] + 0.00403$
- $6 \times 6 + 124 \times 124 = 15412$
- $\sqrt{15413 6 \times 6} \approx [...] + 0.00403$

- $24 \times 24 + 122 \times 122 = 15460$
- $\sqrt{15461 24 \times 24} \approx [...] + 0.0041$
- $50 \times 50 + 114 \times 114 = 15496$
- $\sqrt{15497 50 \times 50} \approx [...] + 0.00439$
- $48 \times 48 + 116 \times 116 = 15760$
- $\sqrt{15761 48 \times 48} \approx [...] + 0.00431$
- $0 \times 0 + 126 \times 126 = 15876$
- $\sqrt{15877 0 \times 0} \approx [...] + 0.00397$
- $2 \times 2 + 126 \times 126 = 15880$
- $\sqrt{15881 2 \times 2} \approx [...] + 0.00397$
- $6 \times 6 + 126 \times 126 = 15912$
- $\sqrt{15913 6 \times 6} \approx [...] + 0.00397$
- $40 \times 40 + 120 \times 120 = 16000$
- $\sqrt{16001 40 \times 40} \approx [...] + 0.00417$
- $14 \times 14 + 126 \times 126 = 16072$
- $\sqrt{16073 14 \times 14} \approx [...] + 0.00397$
- $48 \times 48 + 118 \times 118 = 16228$
- $\sqrt{16229 48 \times 48} \approx [...] + 0.00424$
- $84 \times 84 + 96 \times 96 = 16272$
- $\sqrt{16273 84 \times 84} \approx [...] + 0.00521$
- $22 \times 22 + 126 \times 126 = 16360$
- $\sqrt{16361 22 \times 22} \approx [...] + 0.00397$
- $6 \times 6 + 128 \times 128 = 16420$
- $\sqrt{16421 6 \times 6} \approx [...] + 0.00391$
- $24 \times 24 + 126 \times 126 = 16452$
- $\sqrt{16453 24 \times 24} \approx [...] + 0.00397$
- $12 \times 12 + 128 \times 128 = 16528$
- $\sqrt{16529 12 \times 12} \approx [...] + 0.00391$
- $26 \times 26 + 126 \times 126 = 16552$
- $\sqrt{16553 26 \times 26} \approx [...] + 0.00397$

- $42 \times 42 + 122 \times 122 = 16648$
- $\sqrt{16649 42 \times 42} \approx [...] + 0.0041$
- $28 \times 28 + 126 \times 126 = 16660$
- $\sqrt{16661 28 \times 28} \approx [...] + 0.00397$
- $36 \times 36 + 124 \times 124 = 16672$
- $\sqrt{16673 36 \times 36} \approx [...] + 0.00403$
- $0 \times 0 + 130 \times 130 = 16900$
- $\sqrt{16901 0 \times 0} \approx [...] + 0.00385$
- $6 \times 6 + 130 \times 130 = 16936$
- $\sqrt{16937 6 \times 6} \approx [...] + 0.00385$
- $34 \times 34 + 126 \times 126 = 17032$
- $\sqrt{17033 34 \times 34} \approx [...] + 0.00397$
- $64 \times 64 + 114 \times 114 = 17092$
- $\sqrt{17093 64 \times 64} \approx [...] + 0.00439$
- $48 \times 48 + 122 \times 122 = 17188$
- $\sqrt{17189 48 \times 48} \approx [...] + 0.0041$
- $54 \times 54 + 120 \times 120 = 17316$
- $\sqrt{17317 54 \times 54} \approx [...] + 0.00417$
- $38 \times 38 + 126 \times 126 = 17320$
- $\sqrt{17321 38 \times 38} \approx [...] + 0.00397$
- $24 \times 24 + 130 \times 130 = 17476$
- $\sqrt{17477 24 \times 24} \approx [...] + 0.00385$
- $8 \times 8 + 132 \times 132 = 17488$
- $\sqrt{17489 8 \times 8} \approx [...] + 0.00379$
- $12 \times 12 + 132 \times 132 = 17568$
- $\sqrt{17569 12 \times 12} \approx [...] + 0.00379$
- $16 \times 16 + 132 \times 132 = 17680$
- $\sqrt{17681 16 \times 16} \approx [...] + 0.00379$
- $72 \times 72 + 112 \times 112 = 17728$
- $\sqrt{17729 72 \times 72} \approx [...] + 0.00446$
- $18 \times 18 + 132 \times 132 = 17748$

• 
$$\sqrt{17749 - 18 \times 18} \approx [...] + 0.00379$$

• 
$$22 \times 22 + 132 \times 132 = 17908$$

• 
$$\sqrt{17909 - 22 \times 22} \approx [...] + 0.00379$$

• 
$$0 \times 0 + 134 \times 134 = 17956$$

• 
$$\sqrt{17957 - 0 \times 0} \approx [...] + 0.00373$$

• 
$$42 \times 42 + 128 \times 128 = 18148$$

• 
$$\sqrt{18149 - 42 \times 42} \approx [...] + 0.00391$$

• 
$$48 \times 48 + 126 \times 126 = 18180$$

• 
$$\sqrt{18181 - 48 \times 48} \approx [...] + 0.00397$$

• 
$$96 \times 96 + 96 \times 96 = 18432$$

• 
$$\sqrt{18433 - 96 \times 96} \approx [...] + 0.00521$$

• 
$$66 \times 66 + 120 \times 120 = 18756$$

• 
$$\sqrt{18757 - 66 \times 66} \approx [...] + 0.00417$$

• 
$$76 \times 76 + 114 \times 114 = 18772$$

• 
$$\sqrt{18773 - 76 \times 76} \approx [...] + 0.00439$$

• 
$$54 \times 54 + 126 \times 126 = 18792$$

• 
$$\sqrt{18793 - 54 \times 54} \approx [...] + 0.00397$$

• 
$$38 \times 38 + 132 \times 132 = 18868$$

• 
$$\sqrt{18869 - 38 \times 38} \approx [...] + 0.00379$$

$$\bullet 90 \times 90 + 104 \times 104 = 18916$$

• 
$$\sqrt{18917 - 90 \times 90} \approx [...] + 0.00481$$

• 
$$56 \times 56 + 126 \times 126 = 19012$$

• 
$$\sqrt{19013 - 56 \times 56} \approx [...] + 0.00397$$

• 
$$24 \times 24 + 136 \times 136 = 19072$$

• 
$$\sqrt{19073 - 24 \times 24} \approx [...] + 0.00368$$

• 
$$6 \times 6 + 138 \times 138 = 19080$$

• 
$$\sqrt{19081 - 6 \times 6} \approx [...] + 0.00362$$

• 
$$84 \times 84 + 110 \times 110 = 19156$$

• 
$$\sqrt{19157 - 84 \times 84} \approx [...] + 0.00455$$

• 
$$16 \times 16 + 138 \times 138 = 19300$$

• 
$$\sqrt{19301 - 16 \times 16} \approx [...] + 0.00362$$

• 
$$60 \times 60 + 126 \times 126 = 19476$$

• 
$$\sqrt{19477 - 60 \times 60} \approx [...] + 0.00397$$

• 
$$46 \times 46 + 132 \times 132 = 19540$$

• 
$$\sqrt{19541 - 46 \times 46} \approx [...] + 0.00379$$

• 
$$36 \times 36 + 136 \times 136 = 19792$$

• 
$$\sqrt{19793 - 36 \times 36} \approx [...] + 0.00368$$

• 
$$64 \times 64 + 126 \times 126 = 19972$$

• 
$$\sqrt{19973 - 64 \times 64} \approx [...] + 0.00397$$

These examples are *abundant* yet they cost us time and resources. Should we look for more patterns? How do these compare to what we already have? What do we do about them? Just a quick-look one of them should go to a runoff:

$$19013 - 56^2 = 19973 - 64^2 \approx 0.0039682$$

This one was equality. We are looking for  $\approx$  or  $\approx$  and not really =. Maybe  $\equiv$ . And there is some typos.

## Even more patterns.

- 66 + 10001000 = 1000036
- $\sqrt{1000037 6 \times 6} \approx [...] + 0.0005$
- $\bullet$  9090 + 996996 = 1000116
- $\sqrt{1000117 90 \times 90} \approx [...] + 0.000502$
- 2424 + 10001000 = 1000576
- $\sqrt{1000577 24 \times 24} \approx [...] + 0.0005$
- 302302 + 954954 = 1001320
- $\sqrt{1001321 302 \times 302} \approx [...] + 0.000524$
- 182182 + 984984 = 1001380
- $\sqrt{1001381 182 \times 182} \approx [...] + 0.000508$
- 9898 + 996996 = 1001620
- $\sqrt{1001621 98 \times 98} \approx [...] + 0.000502$
- $\bullet$  262262 + 966966 = 1001800
- $\sqrt{1001801 262 \times 262} \approx [...] + 0.000518$
- 612612 + 792792 = 1001808
- $\sqrt{1001809 612 \times 612} \approx [...] + 0.000631$
- $\bullet$  462462 + 888888 = 1001988
- $\sqrt{1001989 462 \times 462} \approx [...] + 0.000563$
- 100100 + 996996 = 1002016
- $\sqrt{1002017 100 \times 100} \approx [...] + 0.000502$
- 672672 + 742742 = 1002148
- $\sqrt{1002149 672 \times 672} \approx [...] + 0.000674$
- $\bullet$  284284 + 960960 = 1002256
- $\sqrt{1002257 284 \times 284} \approx [...] + 0.000521$
- 386386 + 924924 = 1002772
- $\sqrt{1002773 386 \times 386} \approx [...] + 0.000541$
- 696696 + 720720 = 1002816
- $\sqrt{1002817 696 \times 696} \approx [...] + 0.000694$
- $\bullet$  186186 + 984984 = 1002852

- $\sqrt{1002853 186 \times 186} \approx [...] + 0.000508$
- 5454 + 10001000 = 1002916
- $\sqrt{1002917 54 \times 54} \approx [...] + 0.0005$
- 138138 + 992992 = 1003108
- $\sqrt{1003109 138 \times 138} \approx [...] + 0.000504$
- $\bullet$  216216 + 978978 = 1003140
- $\sqrt{1003141 216 \times 216} \approx [...] + 0.000511$
- $\bullet$  242242 + 972972 = 1003348
- $\sqrt{1003349 242 \times 242} \approx [...] + 0.000514$
- $\bullet$  286286 + 960960 = 1003396
- $\sqrt{1003397 286 \times 286} \approx [...] + 0.000521$
- $\bullet$  234234 + 974974 = 1003432
- $\sqrt{1003433 234 \times 234} \approx [...] + 0.000513$
- $\bullet$  258258 + 968968 = 1003588
- $\sqrt{1003589 258 \times 258} \approx [...] + 0.000517$
- 6060 + 10001000 = 1003600
- $\sqrt{1003601 60 \times 60} \approx [...] + 0.0005$
- $\bullet$  306306 + 954954 = 1003752
- $\sqrt{1003753 306 \times 306} \approx [...] + 0.000524$
- $\bullet$  154154 + 990990 = 1003816
- $\sqrt{1003817 154 \times 154} \approx [...] + 0.000505$
- $\bullet$  432432 + 904904 = 1003840
- $\sqrt{1003841 432 \times 432} \approx [...] + 0.000553$
- 126126 + 994994 = 1003912
- $\sqrt{1003913 126 \times 126} \approx [...] + 0.000503$
- 336336 + 944944 = 1004032
- $\sqrt{1004033 336 \times 336} \approx [...] + 0.00053$
- 110110 + 996996 = 1004116
- $\sqrt{1004117 110 \times 110} \approx [...] + 0.000502$
- $\bullet$  210210 + 980980 = 1004500

- $\sqrt{1004501 210 \times 210} \approx [...] + 0.00051$
- $\bullet$  112112 + 996996 = 1004560
- $\sqrt{1004561 112 \times 112} \approx [...] + 0.000502$
- $\bullet \ \ 268268 + 966966 = 1004980$
- $\sqrt{1004981 268 \times 268} \approx [...] + 0.000518$
- 114114 + 996996 = 1005012
- $\sqrt{1005013 114 \times 114} \approx [...] + 0.000502$
- 3232 + 10021002 = 1005028
- $\sqrt{1005029 32 \times 32} \approx [...] + 0.000499$
- 3434 + 10021002 = 1005160
- $\sqrt{1005161 34 \times 34} \approx [...] + 0.000499$
- $\bullet$  450450 + 896896 = 1005316
- $\sqrt{1005317 450 \times 450} \approx [...] + 0.000558$
- 704704 + 714714 = 1005412
- $\sqrt{1005413 704 \times 704} \approx [...] + 0.0007$
- 160160 + 990990 = 1005700
- $\sqrt{1005701 160 \times 160} \approx [...] + 0.000505$
- $\bullet$  474474 + 884884 = 1006132
- $\sqrt{1006133 474 \times 474} \approx [...] + 0.000566$
- 310310 + 954954 = 1006216
- $\sqrt{1006217 310 \times 310} \approx [...] + 0.000524$
- $\bullet$  4848 + 10021002 = 1006308
- $\sqrt{1006309 48 \times 48} \approx [...] + 0.000499$
- 418418 + 912912 = 1006468
- $\sqrt{1006469 418 \times 418} \approx [...] + 0.000548$
- 684684 + 734734 = 1006612
- $\sqrt{1006613 684 \times 684} \approx [...] + 0.000681$
- $\bullet$  264264 + 968968 = 1006720
- $\sqrt{1006721 264 \times 264} \approx [...] + 0.000517$
- $\bullet$  552552 + 838838 = 1006948

- $\sqrt{1006949 552 \times 552} \approx [...] + 0.000597$
- 138138 + 994994 = 1007080
- $\sqrt{1007081 138 \times 138} \approx [...] + 0.000503$
- 444444 + 900900 = 1007136
- $\sqrt{1007137 444 \times 444} \approx [...] + 0.000556$
- $\bullet$  456456 + 894894 = 1007172
- $\sqrt{1007173 456 \times 456} \approx [...] + 0.000559$
- 540540 + 846846 = 1007316
- $\sqrt{1007317 540 \times 540} \approx [...] + 0.000591$
- $\bullet \ 318318 + 952952 = 1007428$
- $\sqrt{1007429 318 \times 318} \approx [...] + 0.000525$
- $\bullet$  392392 + 924924 = 1007440
- $\sqrt{1007441 392 \times 392} \approx [...] + 0.000541$
- 336336 + 946946 = 1007812
- $\sqrt{1007813 336 \times 336} \approx [...] + 0.000529$
- $\bullet$  00 + 10041004 = 1008016
- $\sqrt{1008017 0 \times 0} \approx [...] + 0.000498$
- $\bullet$  294294 + 960960 = 1008036
- $\sqrt{1008037 294 \times 294} \approx [...] + 0.000521$
- 6464 + 10021002 = 1008100
- $\sqrt{1008101 64 \times 64} \approx [...] + 0.000499$
- $\bullet$  568568 + 828828 = 1008208
- $\sqrt{1008209 568 \times 568} \approx [...] + 0.000604$
- $\bullet$  274274 + 966966 = 1008232
- $\sqrt{1008233 274 \times 274} \approx [...] + 0.000518$
- $\bullet$  200200 + 984984 = 1008256
- $\sqrt{1008257 200 \times 200} \approx [...] + 0.000508$
- $\bullet$  128128 + 996996 = 1008400
- $\sqrt{1008401 128 \times 128} \approx [...] + 0.000502$
- $\bullet \ 144144 + 994994 = 1008772$
- $\sqrt{1008773 144 \times 144} \approx [...] + 0.000503$

- 594594 + 810810 = 1008936
- $\sqrt{1008937 594 \times 594} \approx [...] + 0.000617$
- 192192 + 986986 = 1009060
- $\sqrt{1009061 192 \times 192} \approx [...] + 0.000507$
- $\bullet$  560560 + 834834 = 1009156
- $\sqrt{1009157 560 \times 560} \approx [...] + 0.0006$
- 7272 + 10021002 = 1009188
- $\sqrt{1009189 72 \times 72} \approx [...] + 0.000499$
- $\bullet$  246246 + 974974 = 1009192

Even more skepitical than ever.

- $\sqrt{1009193 246 \times 246} \approx [...] + 0.000513$
- $\bullet$  254254 + 972972 = 1009300
- $\sqrt{1009301 254 \times 254} \approx [...] + 0.000514$
- 600600 + 806806 = 1009636
- $\sqrt{1009637 600 \times 600} \approx [...] + 0.00062$
- $\bullet$  4242 + 10041004 = 1009780
- $\sqrt{1009781 42 \times 42} \approx [...] + 0.000498$
- $\bullet$  204204 + 984984 = 1009872
- $\sqrt{1009873 204 \times 204} \approx [...] + 0.000508$

## References

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