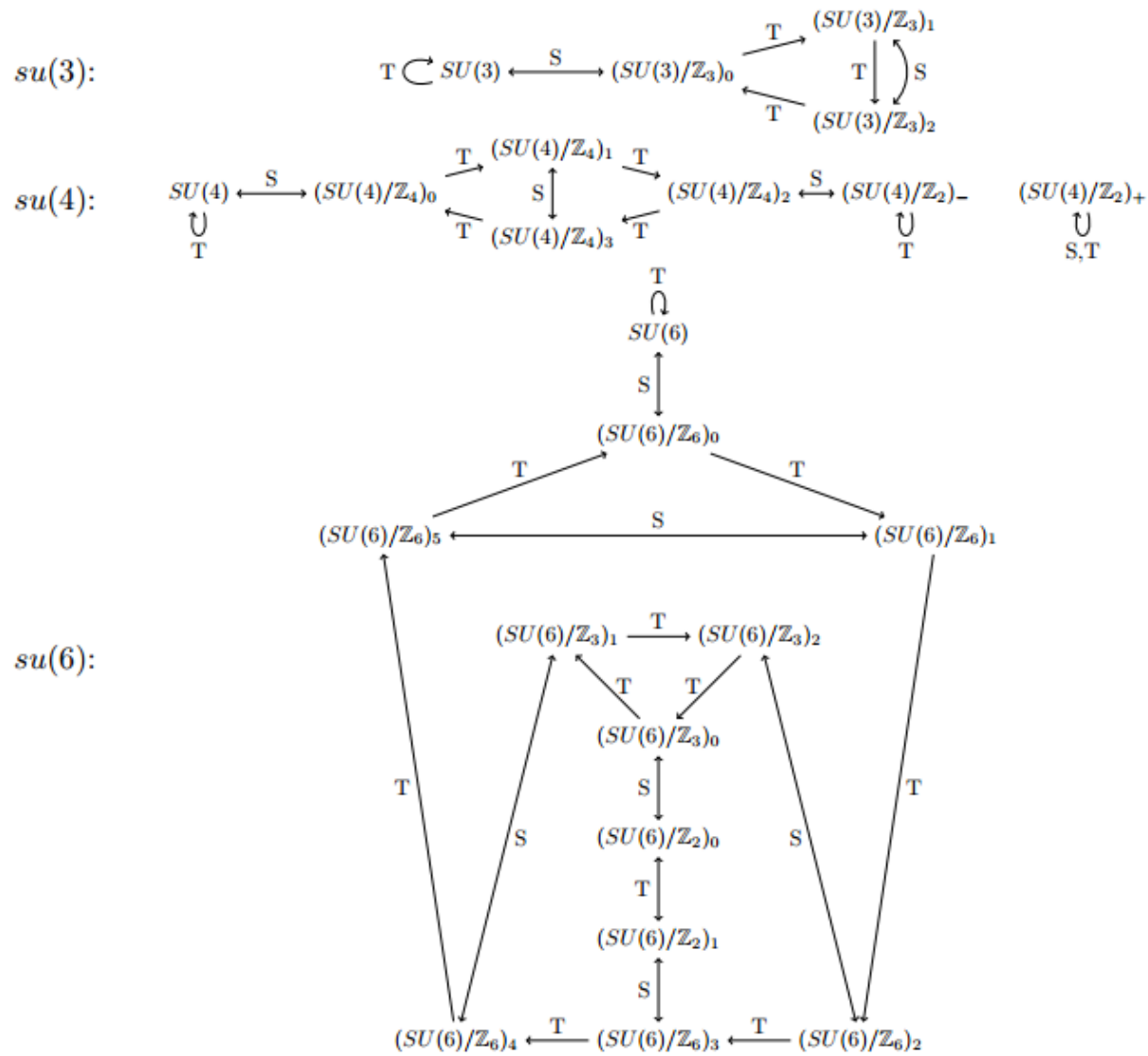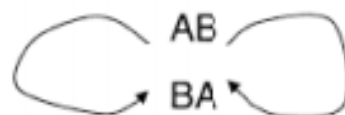# Rauzy-Veech Identities

John D Mangual

I was reading a string theory paper and checking the references. The formula didn't make much sense, and upon closer inspection the diagram from one set of papers I was reading looked exactly like th other.
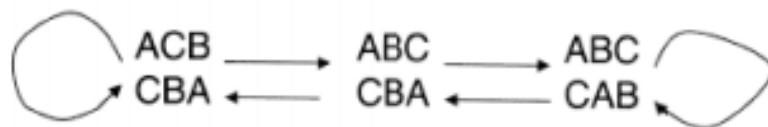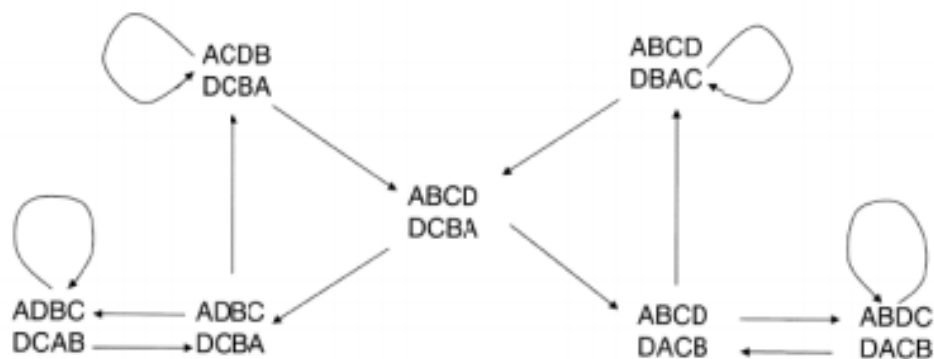
S-duality and T-duality

$su(3):$

$$T \subset SU(3) \xleftarrow{\quad S \quad} (SU(3)/\mathbb{Z}_3)_0 \xrightarrow{\quad T \quad} (SU(3)/\mathbb{Z}_3)_1$$

$(SU(3)/\mathbb{Z}_3)_1$

$T \downarrow \,\rangle S$

$(SU(3)/\mathbb{Z}_3)_2$

$su(4):$

$$SU(4) \xrightarrow{\quad S \quad} (SU(4)/\mathbb{Z}_4)_0$$

$(SU(4)/\mathbb{Z}_4)_1$

$(SU(4)/\mathbb{Z}_4)_2 \xleftarrow{\ S\ } (SU(4)/\mathbb{Z}_2)_-$

$(SU(4)/\mathbb{Z}_2)_+$

$\cup$ T

$(SU(4)/\mathbb{Z}_4)_3$

$\cup$ T

$\cup$ S,T

$su(6):$

$T \curvearrowright SU(6)$

$S \downarrow$

$(SU(6)/\mathbb{Z}_6)_0$

$(SU(6)/\mathbb{Z}_6)_5 \xleftarrow{\qquad\qquad S \qquad\qquad} (SU(6)/\mathbb{Z}_6)_1$

$(SU(6)/\mathbb{Z}_3)_1 \xrightarrow{\ T\ } (SU(6)/\mathbb{Z}_3)_2$

$(SU(6)/\mathbb{Z}_3)_0$

$S \downarrow$

$(SU(6)/\mathbb{Z}_2)_0$

$T \downarrow$

$(SU(6)/\mathbb{Z}_2)_1$

$S \downarrow$

$(SU(6)/\mathbb{Z}_6)_4 \xleftarrow{\ T\ } (SU(6)/\mathbb{Z}_6)_3 \xleftarrow{\ T\ } (SU(6)/\mathbb{Z}_6)_2$

2

Rauzy-Veech Induction

AB
BA

g=1, s=1,d=2

ACB ⟶ ABC ⟶ ABC
CBA ⟵ CBA ⟵ CAB

g=1,s=2,d=3

ACDB
DCBA

ABCD
DBAC

ABCD
DCBA

ADBC ⟵ ADBC
DCAB ⟶ DCBA

ABCD ⟶ ABDC
DACB ⟵ DACB

g=2,s=1,d=4

References

(1) Ofer Aharony, Nathan Seiberg, Yuji Tachikawa.
   Reading between the lines of four-dimensional gauge theories `arXiv:1305.0318`

(2) Artur Avila, Carlos Matheus, Jean-Christophe Yoccoz.
   Zorich conjecture for hyperelliptic Rauzy-Veech groups. `arXiv:1606.01227`

(3) JC Yoccoz **Interval exchange maps and translation surfaces** from
   Homogeneous Flows, Moduli Spaes and Arithmetic `http://www.claymath.org/library/proceedings/cmip010c.pdf`