Domino Tilings

John D Mangual

It is time to review that classic problem, domino tilings¹.

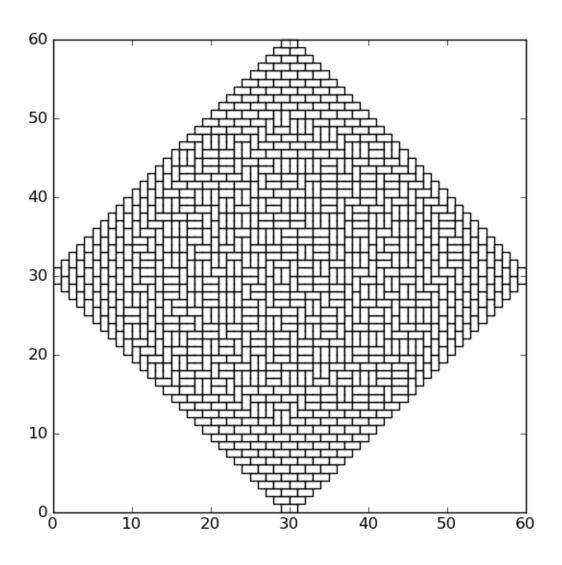
I liked Solomon Golomb's book on Polyominoes. We can do a bit if we focus on just one type of piece such as an L or a T.

¹There are possibly two tracks we can do. A coding track and a theory track. In this iteration of review we will focus on **theory** with a mimimum of coding. Certain steps, which for that community are quite clear, I'll take a lot more work to understand. So I will expend a lot of effort re-writing existing arguments in a way that hopefully makes sense to everybody. In my defense, my teacher was David Speyer – who is an expert in domino tilings – and even he wasn't aware of certain things I was telling him. Indeed, we will take many detours that look somewhat frivolous or purely for my own curiosity... because the lecture notes are for me.



 $^{^2}$ Figure should be an 8×8 checkboard with two corners removed. Can you tile with 2×1 rectangles? Depending on your choice of corners, answer is **YES** or **NO**.

Q2: Is that Really a Circle in the Middle?



References

- (1) William Thurston Conway's Tiling Groups http://www.jstor.org/stable/2324578
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- (3) Alexei Borodin, Leonid Petrov
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- (4) Alexei Borodin, Vadim Gorin Lectures on integrable probability arXiv:1212.3351
- (5) Kurt Johansson. Discrete orthogonal polynomial ensembles and the Plancherel measure arXiv:math/9906120
- (6) Michael Freedman, Matthew Headrick Bit threads and holographic entanglement arXiv:1604.00354