Examples: The Matrix-Tree Theorem

John D Mangual

On YouTube there are some nice videos of Francis Brown lecturing on the "Cosmic Galois Group". I know a little bit about a few of these things:

- periods (integrals of stuff)
- the matrix-tree theorem
- Feynman Diagrams
- $\zeta(2) = \sum \frac{1}{n^2} = \frac{\pi^2}{6}$
  - the Feynman diagram evaluates to some zeta functions
  - the zeta function can be represented as a period
- Things I like such as $\zeta(-1) = 1 + 2 + 3 + \cdots = -\frac{1}{12}$

Before we get too exicted let's discuss the questions at hand.

Problem #1 – What?? Don't we know how to compute Feynman diagrams already?

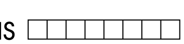
Problem #2 – Which QFT is Brown[1] referring to?

Brown refers to the Matrix Theorem while defining is Feynman Amplitudes – but *which Feynman amplitudes* – it is not obvious to me this is plain-old ϕ^4 theory.

- ϕ^4 theory is what my Physics teacher glossed over, a trivial and generate case and an endless source of homework problems. So obviously we know all about that.

- After some research there is the thesis of Eric Panzer who refers to the **Schwinger Trick** – basically a Mellin transform – turning Feynman diagrams into sums over spanning trees. **which Schwinger trick**? [2]

So...comparing Brown's approach to Chapter 4 of Peskin and Schroeder leads to a lot of confusion which we hopefully can resolve.

Also, Brown is looking to evaluate all diagrams I will only look at few such as  and 

[1] or Panzer or Schnetz or Block or Broadhurst or Kreimer

[2] Due to my limited knowledge of QFT. Depending on who teaches the course, you get a different version of the story.

References

(1) Francis Brown **Feynman Amplitudes and Cosmic Galois group** `arXiv:1512.06409`

(2) Francis Brown **Notes on Motivic Periods** `arXiv:1512.06410`

(3) Michael Peskin, Daniel Schroeder **Quantum Field Theory** (Student Economy Edition), 2015