PHYSICS 7B – Fall 2022
Midterm 2, A. Lanzara
Thursday, Nov. 3, 7-9 pm

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- ***Rules:*** *You may work on this exam from 7:10-9:10 pm PT. This midterm is closed book and closed notes and you are* **not** *allowed to use any electronic device including calculator. You are* **not** *allowed to communicate with any other students or discuss the content of the exam with anyone besides the Physics 7B teaching staff. Any violations of this policy will be considered a breach of academic integrity.*

- ***Honors Statement:*** *It is expected that during this examination, as with any examination that they undertake at the university, students adhere to the usual standards of academic integrity at the University of California at Berkeley as outlined on by the Center for Teaching and Learning. Therefore, by submitting your exam, you are affirming the following statement:*

  "I swear on my honor that I have neither given nor received aid on this exam. In addition, I abided by all the examination policies as outlined above."

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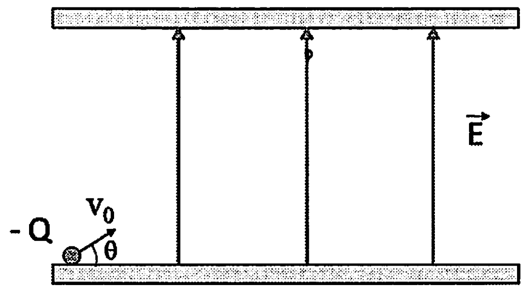
Problem 1 (15pts):
A particle of mass M and charge -Q (where Q > 0) enters a region of constant electric field **E** with speed v_0 and at an angle θ, as shown in the figure below. Neglect gravity and assume that the plates have infinite lengths.

a) (6pts) How far away from where the charge enters, will the charge strike the bottom plate?

b) (3pts) Does the electric potential energy of the charge increase or decrease as the charged particle reaches the maximum distance from the bottom plate? Explain and show your answer.



c) (3pts) Using your results from part a, sketch the shape of the motion of the particle if it has a charge of +Q instead of -Q?

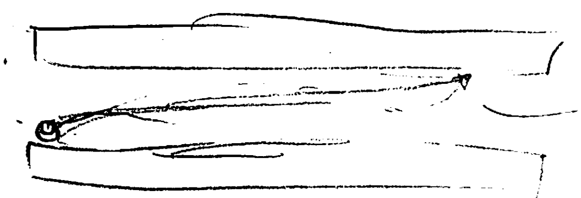
d) (3pts) What happens to the electric potential energy in the case stated in part (c)?

a) $\Delta x = vt$ $\quad \Delta x = V_0 t + \frac{1}{2} a t^2$ —> we need this

$F = qE = ma \Rightarrow \frac{qE}{m} = a \qquad E = k\frac{Q}{r^2}$

$\Delta x = V_0 t + \frac{1}{2} \frac{qE}{m} t^2$

b) The electric PE as the particle increases as the particle reaches the max distance from the bottom plate. If we were to express electric potential energy as $\frac{kQ}{r} = V$ then as

c) 

The positive charge would cause the charge to be pushed up into the top plate.

d) The electric potential energy in this case decreases as the particle gets closer to the negatively charged plate.

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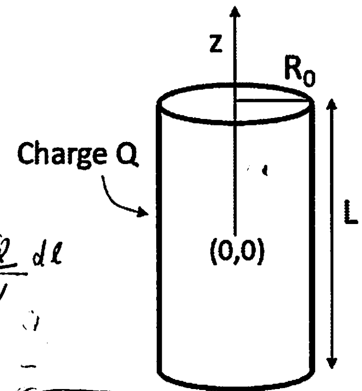
Problem 2 (20pts):
A very long solid conducting cylinder of radius R_0, length $L >> R_0$ and charge Q is centered at the origin (0,0) of a cylindrical coordinate system.



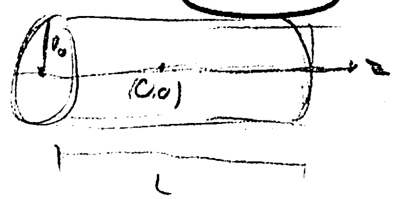
a) (8pts) Determine the electric field inside and outside the cylinder at a point r, $|z| << L$.

inside the cylinder

$r = ((R_0 - x) + y)^2$ → in relation to (0, 0) $\frac{Q}{L} dl$

$dQ = \lambda dl$ $\frac{Q}{A} dl$

$E = \int dE = \int \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2}$

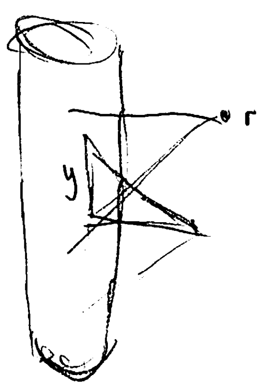


$dQ = \lambda dl$

$\lambda = \frac{Q}{\pi R_0^2 L}$

Outside the cylinder

$r^2 = ((R_0 + x)^2 + (\frac{L}{2})^2)$



$E = \int dE = \int \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2} \Rightarrow \int \frac{1}{4\pi\epsilon_0} \frac{dQ}{((R_0+x)^2 + (\frac{L}{2})^2)} \cos\theta$ $\frac{R_0 + x}{((R_0+x)^2 + (\frac{L}{2})^2)^{1/2}}$

$\int \frac{1}{4\pi\epsilon_0} \frac{dQ}{((R_0+x)^2 + (\frac{L}{2})^2)^{3/2}} (R_0 + x)$

$\frac{(R_0 + x)}{4\pi\epsilon_0 ((R_0+x)^2 + (\frac{L}{2})^2)^{3/2}} \int_{-\frac{L}{2}}^{L/2} dl$

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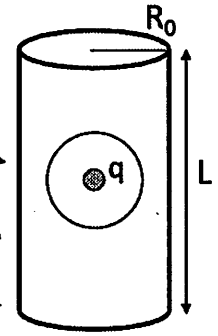
We now place a non-conducting sphere of charge q and radius $R_1 \ll R_0$ inside the cylinder and we let the charge rest at the center of the hollow spherical cavity (see figure below).



b) (3pts) What is the charge on the surface of the hollow spherical space? Show your answer

c) (3pts) What is the charge on the outside surface of the cylinder? Show your answer.

$dQ = \lambda \, dr$

$\lambda = \frac{Q}{A}$

d) (6pts) What is the total electric field outside the cylinder?

b) $E = \frac{Q_{enc}}{\epsilon_0 A}$ $\quad E = \frac{q}{4\pi R_1^2} \rightarrow \frac{q}{\text{surface area of sphere}}$

c) $E = \frac{Q_{enc}}{\pi R_0^2 + 2\pi R_0 L} \rightarrow \frac{q}{\text{surface area of cylinder}}$

d) $\frac{1}{4\pi\epsilon_0}$

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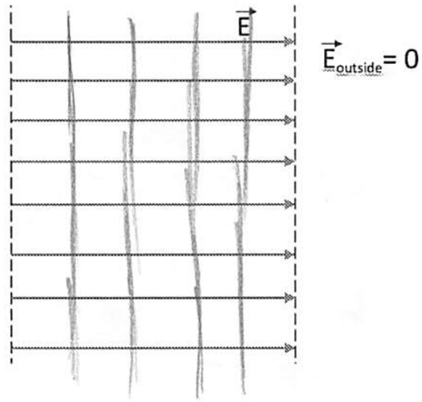
Problem 3 (23pts):
In a confined region in space (delineated by the dashed line in the figure below) we turn on a uniform electric field **E**. The field is horizontal pointing to the right inside the region and zero outside.

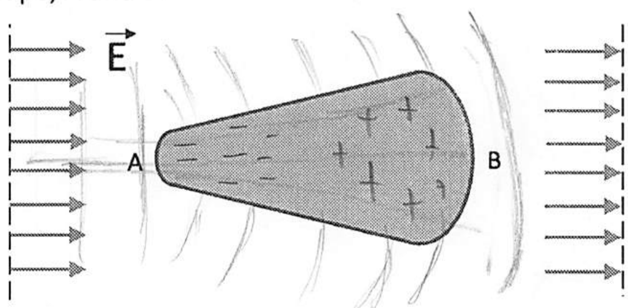
a) (2pts) Draw the equipotential surfaces associated with this electric field.



We now place an uncharged conductor of irregular shape in the center of the region where the field is nonzero (see figure below). As we discussed in class, when a non-symmetric shaped conductor is placed in this region of space, the E field is modified.

b) (2pts) Draw the distribution of charge on the conductor (focus mainly on the two arc sections, A and B)

c) (4pts) What can you tell about the equipotential lines in this case? Draw them in the proximity of the two extrema of the shape, A and B.



c) The equipotential lines are perpendicular to the electric field lines and since the conductor warps the E-field, the equipotential lines are now curved as well instead of being straight as they were in part a.

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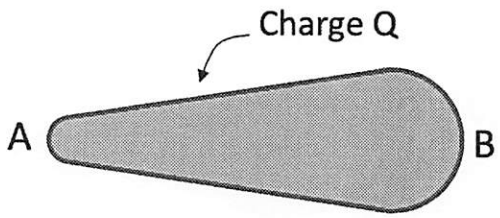
In class we used this irregular conducting shape to study how the electric field is modified in the presence of a tip. We charged the conductor with a total charge Q, and assumed that the radius of curvature of arc A is R_1 and of arc B is R_2, with $R_2 \gg R_1$. If we turn off the electric field,



d) (4pts) How does the electric potential in arc A compares to the electric potential in arc B? Explain.

e) (6pts) Is the charge uniformly distributed? If not, calculate how the charge density in A compares to the charge density in B. Show your answer

f) (5pts) Is the electric field right outside B larger or smaller than the field right outside A and if so, by how much? Show your answer.

d) ~~The electric~~ electric potential in arc A is greater than the electric potential in arc B since the E-field lines are closer together which should mean the electric potential is greater.

e) The charge is not uniformly distributed. since A is much narrower than B.

f) The electric field right outside B is ~~larger~~ smaller than the field right outside A.

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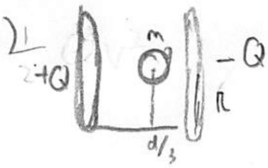
Problem 4 (21pts):
In class we discussed how an old-style doorbell works. We approximated the doorbell as a capacitor made out of two discs of radius R and an uncharged metallic pendulum of mass M in between (see figure below). In its vertical position the pendulum is off center and slightly closer to the right plate of the capacitor, with its center of mass being at 1/3 d, where d is the distance between the two capacitor plates.

The first thing we did was to charge the capacitor, by placing a charge +Q on plate A and -Q on plate B of the capacitor. Neglect any fringe field.

a) (4pts) What is the total work needed to charge the capacitor from Q=0 to Q?

b) (3pts) What is the capacitance of the capacitor (without the pendulum)?

a) $\frac{1}{2}$

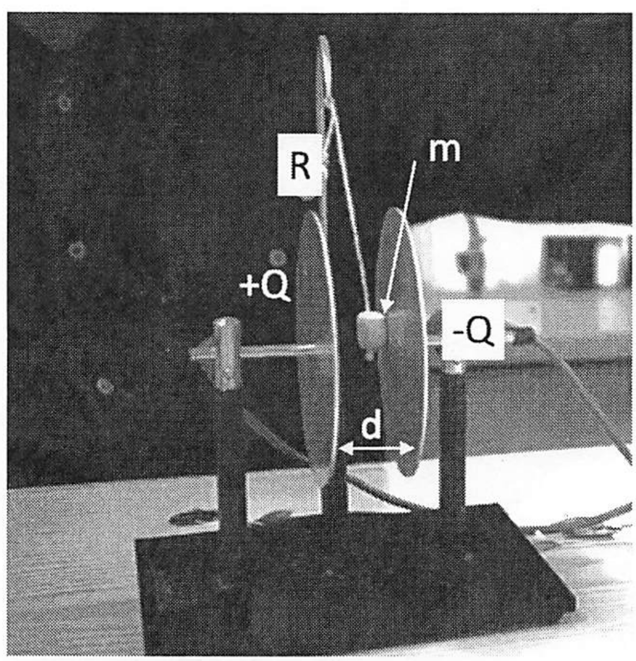


B) $C = \frac{\varepsilon_0}{A}$

Soon after the capacitor is charged, the pendulum starts swinging.

c) (5pts) Describe the mechanism that allows the uncharged pendulum to first swing to the right and eventually hit the right plate of the capacitor. Be very specific in describing every step of the process, making sure to <u>draw</u> charges, field, force and anything else that can help your argument.

(Assuming the pendulum is metallic, when the left plate is positively charged the E-field points to the right and induces a charge on the pendulum where the pendulum becomes positively charged on one end (the right). Because it is closer to the right, the force



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d) (5pts) What happens in the instant in which the pendulum hit the right plate? Be specific in your answer.

When the pendulum hits the right plate, it's positively

Let's now treat the pendulum as a rectangular slab of dielectric constant k:

e) (4pts) Is the total capacitance higher or lower than the capacitance of the doorbell before the insertion of the pendulum? Show your answer

e) The total capacitance should be higher than the capacitance of the door bell before the insertion of the pendulum.

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Problem 5 (21pts):

A total charge +Q is distributed over an anulus in the (y, z) plane. The anulus has an internal radius R_1 (R_2) and external radius R_2 (R_1). The center of the anulus is at the origin of a coordinate system.

a) (10pts) Find the electric field at a point P along the x axis away from the center at a distance a.

b) (8pts) Find the electric potential at P.

c) (3pts) If the material is not plastic but a conductor, what will be the electric field at (0,0)? Justify your answer.

arc length $= r\,d\theta$

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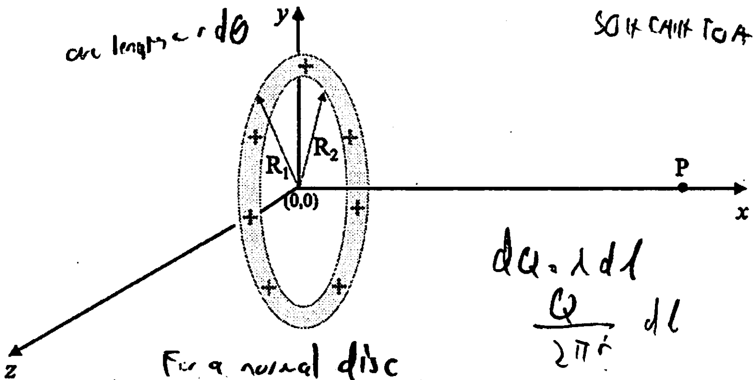


$dQ = \lambda\, dl$

$\frac{Q}{2\pi r}\, dl$

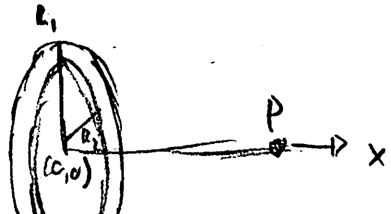
For a normal disc

$E = \int dE = \int \frac{1}{4\pi\varepsilon_0} \frac{dQ}{r^2}$ $\quad \cos\theta = \frac{x}{(x^2+a^2)^{1/2}}$

$\frac{1x}{4\pi\varepsilon_0} \frac{\lambda\, dl}{(x^2+r^2)^{3/2}}$ $\quad \frac{Q}{\ }$

$\frac{x\lambda}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}} \int_0^{2\pi r} dl = \frac{2\pi x}{2\pi\, 4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$

a)



Treat this like a ~~disc~~ solid disc instead

$\frac{\lambda}{r}\, dr$

$dE = \frac{x\, dQ}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$

$\int_{R_2}^{R_1} \frac{x\, dQ}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}} \Rightarrow$

b) $V = \frac{kQ}{r}$ $\quad V = \frac{k\left(\frac{E r^2}{k}\right)}{r} \Rightarrow V = Er$

$E = \frac{kQ}{r^2}$ $\quad Q = \frac{E r^2}{k}$

c) The electric field @ (0,0) should be (0,0) as the electric fields at that point should cancel out.

Equation Sheets

Integration

$$\int \sin^2 x\, dx = \int \frac{1 - \cos 2x}{2}\, dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]$$

$$\int \cos^2 x\, dx = \int \frac{1 + \cos 2x}{2}\, dx = \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]$$

$$\int \sin x \cos x\; dx = \int \frac{1}{2}\sin 2x\; dx = \frac{1}{2}\left[-\frac{1}{2}\cos 2x\right]$$

$$\int \frac{dx}{c_1 x + c_2} = \frac{1}{c_1}\ln\left(c_1 x + c_2\right)$$

$$\int \frac{x\, dx}{x + c} = \int \frac{(x + c) - c}{x + c} dx = \int \left[1 - \frac{c}{x + c}\right] dx = x - c\ln\left(x + c\right)$$

$$\int \frac{dx}{x^n} = \frac{x^{-n+1}}{-n + 1}, \quad n > 1$$

$$\int x^n\, dx = \frac{x^{n+1}}{n + 1}, \quad n > 0$$

$$\int \frac{dx}{\sqrt{x^2 + y^2}} = \ln\left(\frac{\sqrt{x^2 + y^2} + x}{y}\right)$$

$$\int \frac{x\, dx}{\sqrt{x^2 + y^2}} = \sqrt{x^2 + y^2}$$

Taylor Series

$$f(x) \approx f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f^{(3)}(a)}{3!}(x - a)^3 + \cdots$$

Then, for $x \ll 1$, $a = 0$:

$$\frac{1}{1 + x} \approx 1 - x + x^2 - x^3 + \cdots$$

$$\frac{1}{(1 + x)^2} \approx 1 - 2x + 3x^2 - 4x^3 + \cdots$$

$$\frac{1}{(1 + x)^3} \approx 1 - 3x + 6x^2 - 10x^3 + \cdots$$

$$\ln(1 + x) \approx x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$$

Kinematics:

Linear Motion at Constant Speed: $\Delta x = vt$

Linear Motion at Constant Acceleration: $\Delta x = v_{0x}t + \frac{1}{2}at^2$