Solution to 7B Midterm 2

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Fall 2022

1 Problem 2

1.1 (a)

Inside the cylinder, since it is a conductor, the electric field is zero [3 pts].

The charge Q will all flow to the boundary and distribute uniformly. We can then compute the electric field via the Gauss law:

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl.}}{\epsilon_0} \qquad \text{[1 pt]}. \tag{1}$$

Pick the Gaussian surface S to be a cylinder with length ℓ and radius r larger than R_0 [2 pts]. Notice that the surface area of S is $2\pi r\ell$ [1 pt] and that the electric field is in the radial direction (namely $\vec{E} = E_r\hat{r}$) by symmetry. We conclude that

$$E_r \cdot 2\pi r\ell = \tfrac{Q\ell}{L\epsilon_0} \tag{2}$$

$$\implies \vec{E} = \tfrac{Q}{2\pi\epsilon_0 Lr}\hat{r}, \qquad r > R_0 \quad \text{[1 pt]}. \tag{3}$$

1.2 (b)

Since the electric field inside the conductor must be zero, the charge on the inner surface must be $-q$ [3 pts].

1.3 (c)

By charge conservation, the charge on the outer surface will be $Q + q$, uniformly distributed [3 pts].

1.4 (d)

The total electric field is a superposition of three terms from the non-conducting sphere, the inner, spherical surface, and the outer surface respectively:

$$\vec{E} = \vec{E}_q + \vec{E}_{inner} + \vec{E}_{outer} \qquad \text{[2 pt]}. \tag{4}$$

Notice that the first two terms cancel out, so we are left with the last term. Repeating the same Gauss-law analysis as in part (a), we conclude that

$$\vec{E} = \frac{Q+q}{2\pi\epsilon_0 Lr}, \qquad r > R_0 \qquad \text{[4 pt]}. \tag{5}$$