QUALITATIVE MODELLING AND REASONING

Ivan Bratko
Faculty of Computer and Information Science
Ljubljana University

CONTENTS

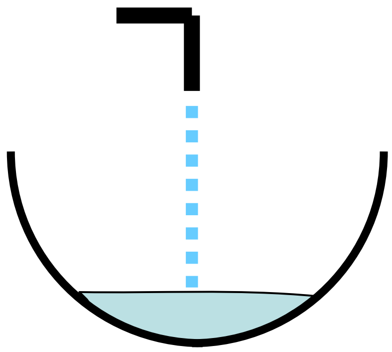
- Common sense, qualitative reasoning and naive physics
- Why qualitative reasoning and modelling
- Qualitative reasoning about static systems
- Qualitative reasoning about dynamic systems
- Qualitative differential equations (QDEs) and QSIM
- Learning qualitative models

QUALITATIVE REASONING, NAIVE PHYSICS

- Describe physical processes *qualitatively*, without numbers or exact numerical relations
- Close to common sense descriptions
- This is sometimes called “naïve physics” which people use in everyday life, as opposed to proper physics

EXAMPLE: BATH TUB

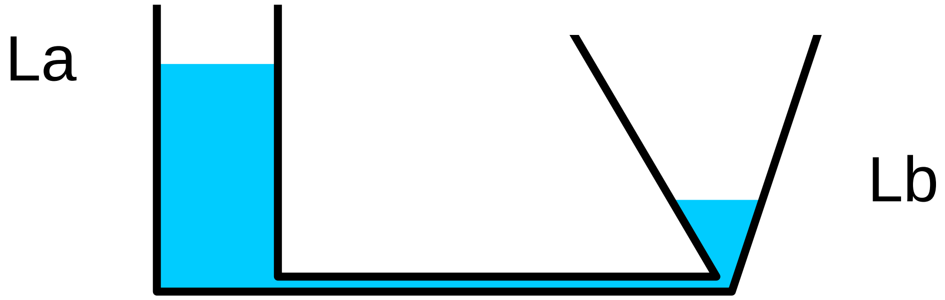
- What will happen?
- The physicist's answer, the commonsense answer?



Amount A of water will keep increasing,
so will level L,
until the level reaches the top.

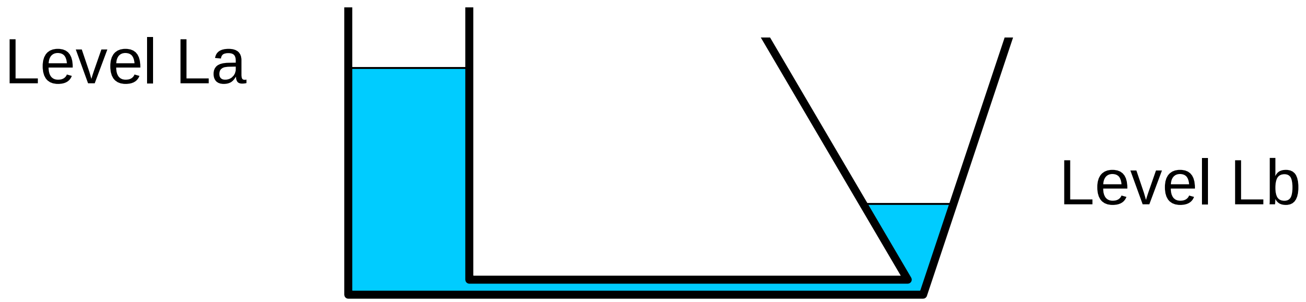
EXAMPLE: U-TUBE

- What will happen?



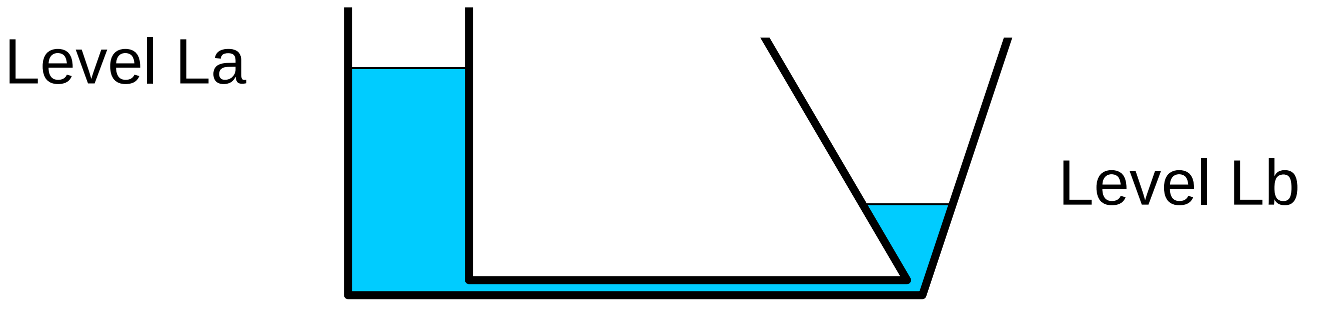
Level La will be decreasing,
and Lb increasing,
until La = Lb

QUALITATIVE REASONING ABOUT U-TUBE



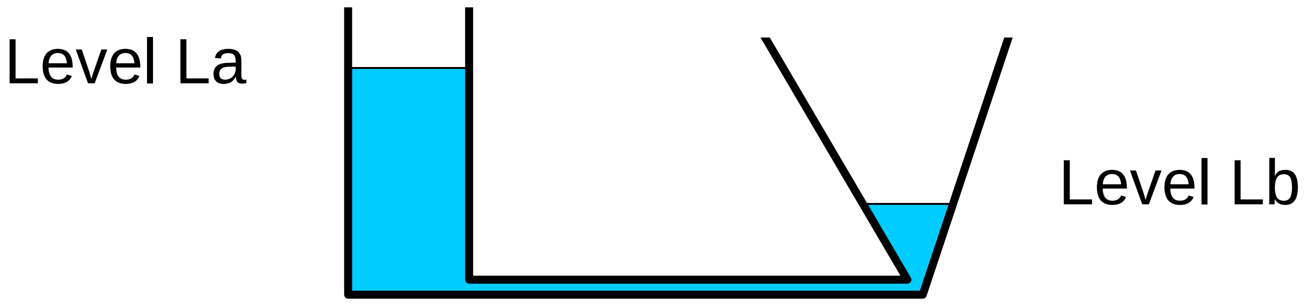
- **Total amount of water in system is constant**
- **If La > Lb then flow from A to B**
- **Flow causes amount in A to decrease**
- **Flow causes amount in B to increase**
- **All changes in time happen *continuously and smoothly***

QUALITATIVE REASONING ABOUT U-TUBE

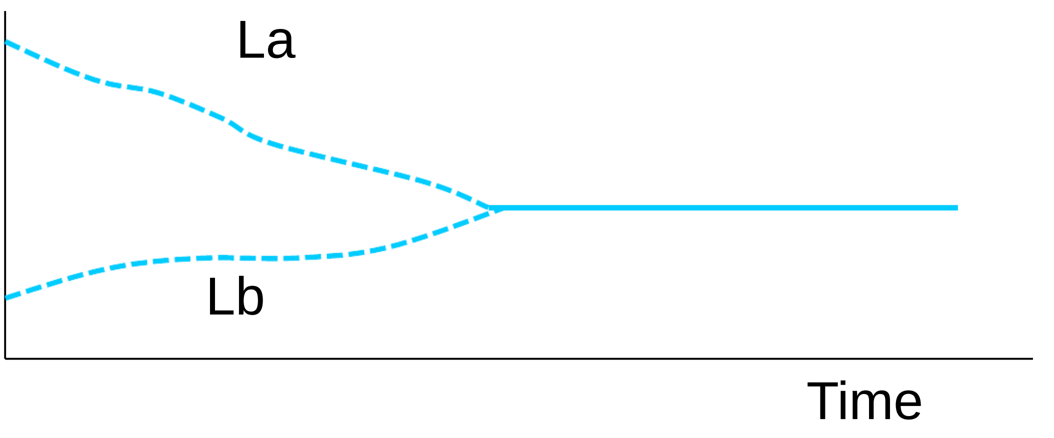


- **In any container: the greater the amount, the greater the level**
- **So, La will keep decreasing, Lb increasing**

QUALITATIVE REASONING ABOUT U-TUBE

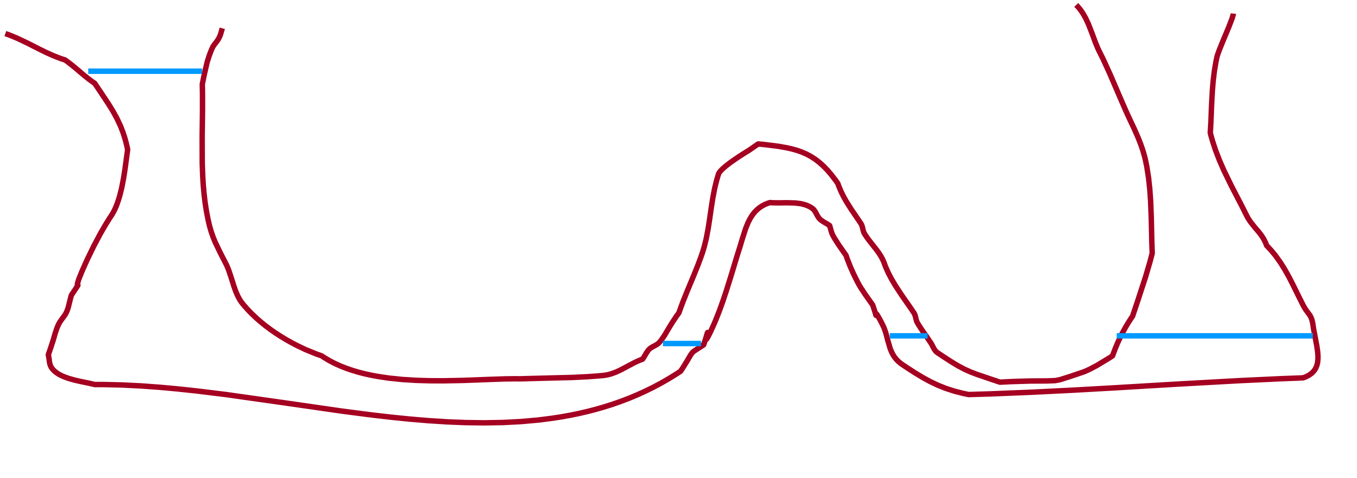


- **La will keep decreasing, Lb increasing, until they become equal**

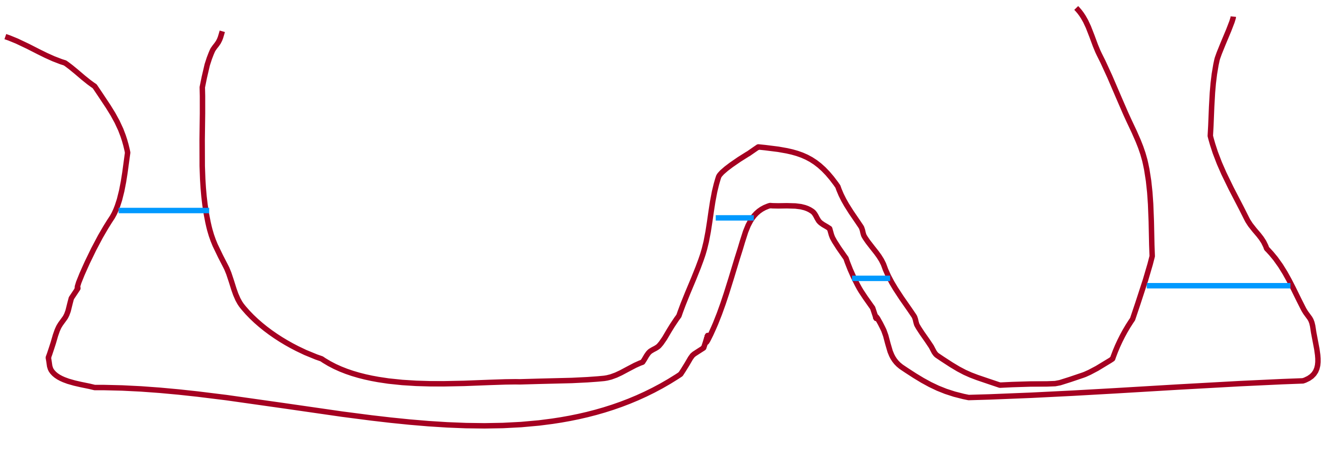


THIS REASONING IS VALID FOR *ALL* CONTAINERS
OF ***ANY* SHAPE AND SIZE**,
REGARDLESS OF ACTUAL NUMBERS!

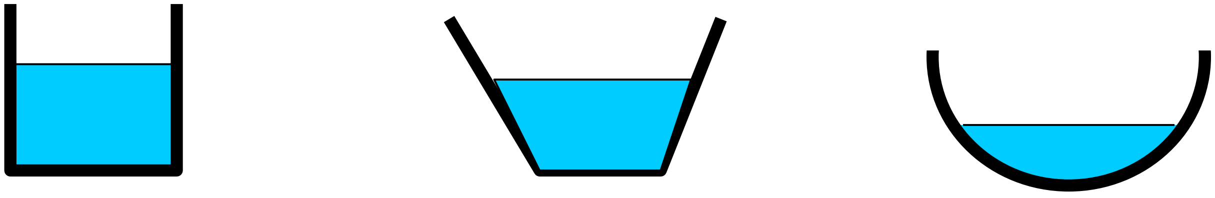
HOW ABOUT THIS ONE?



CAN YOU INFER QUALITATIVELY THIS FINAL STATE?



RELATION BETWEEN AMOUNT AND LEVEL



- The greater the amount, the greater the level

$$A = M^+(L)$$

- **A** is a *monotonically increasing function* of **L**

WHY REASON QUALITATIVELY?

- Because it is easier than quantitatively
- Because it is easy to understand -
  facilitates explanation,
  relevant to Explainable AI
- We want to exploit these advantages in ML

COMPARISON WITH NUMERICAL, QUANTITATIVE MODELLING

- Traditional modelling and simulation:
  - Differential equations
  - Numerical methods
  - Arithmetic computation
  - The world is represented by numbers

- But: Representing the world with numbers is only a special case of describing the world

QUALITATIVE REASONING

- Alternative representations, reasoning
- Common sense reasoning
- Naïve physics, as opposed to "proper physics“
- Qualitative modelling, as opposed to quantitative modelling

QUALITATIVE ABSTRACTIONS OF QUANTITATIVE INFORMATION

Quantitative statement	**Qualitative statement**
Level(3.2 sec) = 2.6 cm	Level(t1) = pos
Level(3.2 sec) = 2.6 cm	Level(t1) = zero..top
d/dt Level(3.2 sec) = 0.12 m/sec	Level(t1) increasing
Amount = Level * (Level + 5.7)	M^+(Amount, Level)

QUALITATIVE ABSTRACTIONS

- numbers --> symbolic values and intervals
- time derivatives --> directions of change
- functions --> monotonic relations
- monotonic sequence of values in time -->
  one symbolic value + direction of change

WHY QUALITATIVE MODELLING?

- Exact relations may not be known, e.g. in physiology
- Numerical parameters may be hard to measure
- Quantitative model may be computationally complex
- Qualitative models facilitate:
  - Explanation – how system works?
  - Diagnostic reasoning for fault diagnosis
  - Interpreting alarms in process control
  - Functional reasoning
  - Structural synthesis, invention from first principles

SOME APPROACHES TO QUALITATIVE REASONING AND MODELLING

- Qualitative Physics Theory, QPT;

  notion of "process" (Forbus 1984)
- Qualitative simulation, QSIM;

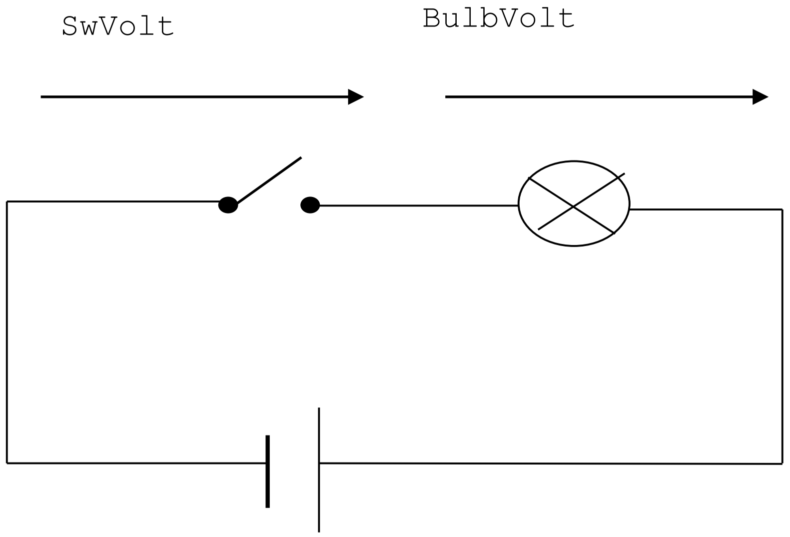
  abstraction of differential equations into qualitative differential equations (QDE) (Kuipers 1986)
- Envisionment; based on "confluences", a kind of QDE (de Kleer and Brown, 1984)
- KARDIO, qualitative model of the heart;

  symbolic descriptions in logic (Bratko, Mozetič, Lavrač 1989)

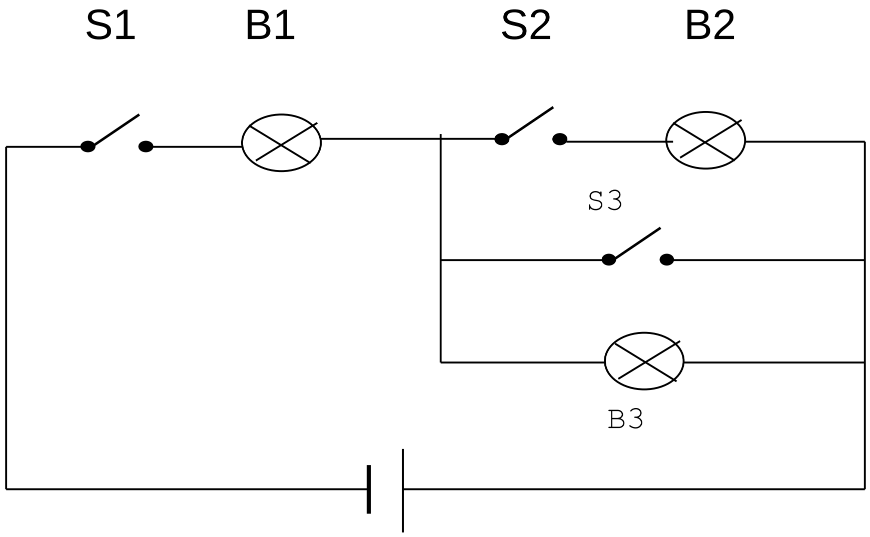
QUALITATIVE REASONING ABOUT STATIC SYSTEMS

EXAMPLE: ELECTRIC CIRCUITS

- Circuit 1



CIRCUIT 2



A diagnostic question about circuit 1:
If the switch on and the bulb dark, what is state of the bulb?

A diagnostic question about circuit 2:
Seeing that bulb 2 is light and bulb 3 is dark, can we conclude that bulb 3 is blown (without knowing switch positions)?

Abstracting real number X into qualitative value

- if X > 0 then "pos"
- if X = 0 then "zero"
- if X < 0 then "neg"

QUALITATIVE SUMMATION

```
%  qsum( Q1, Q2, Q3):
%    Q3 = Q1 + Q2, qualitative sum over domain [pos,zero,neg]

qsum( pos, pos, pos).           % pos + pos = pos
qsum( pos, zero, pos).
qsum( pos, neg, pos).           % pos + neg  may be pos
qsum( pos, neg, zero).          % pos + neg  may be zero
qsum( pos, neg, neg).           % pos + neg  may be neg
qsum( zero, pos, pos).
...
```

ELECTRIC COMPONENTS IN LOGIC

```
%  Definition of the switch
%  switch( SwitchPosition, Voltage, Current)
%  SwitchPosition in { on, off}
%  Voltage, Current in { pos, neg, zero}
```

switch(on, zero, AnyCurrent). % Switch on: zero voltage

switch(off, AnyVoltage, zero). % Switch off: zero current

Predicate

Variable (upper case initial)

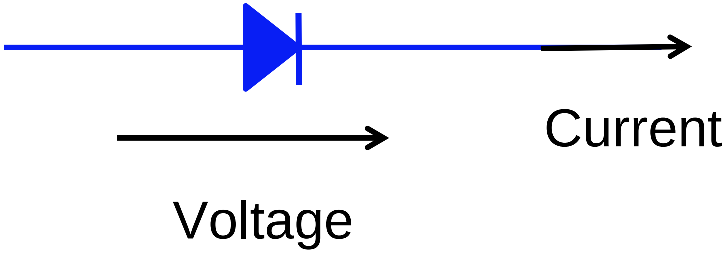
Constants (lower case initial)

ELECTRIC COMPONENTS, CTD.

```
% Definition of bulb
% bulb( BulbState, Lightness, Voltage, Current)
% BulbState in { ok, blown}
% Lightness in { light, dark}
% Current, Voltage in { pos, neg, zero}

bulb( blown, dark, AnyVoltage, zero).

bulb( ok, light, pos, pos).

bulb( ok, light, neg, neg).

bulb( ok, dark, zero, zero).
```

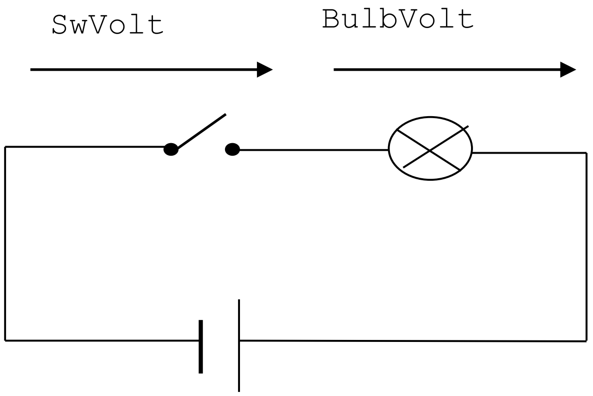
EXERCISE

- Define a qualitative model of the diode



- The diode only allows the current in the forward direction

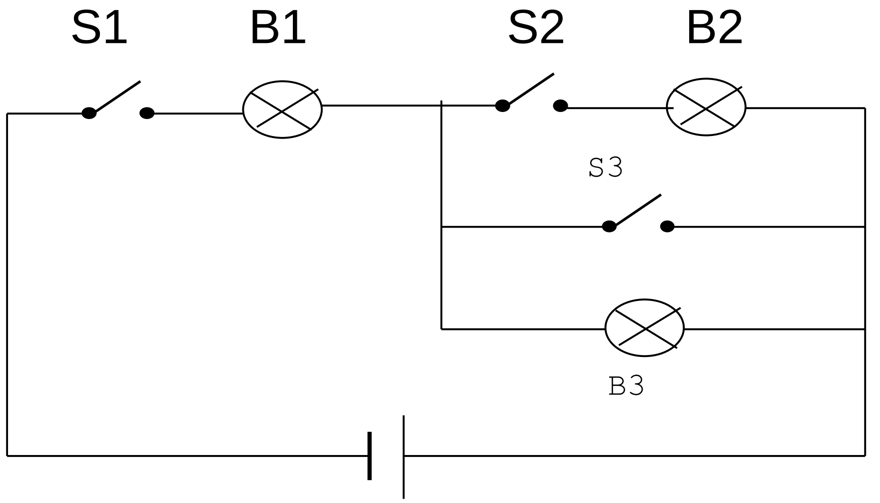
DEFINITION OF CIRCUIT 1




```
circuit1( SwitchPos, BulbState, Lightness)  :-
   switch( SwitchPos, SwVolt, Current),
   bulb( BulbState, Lightness, BulbVolt, Current),
   qsum( SwVolt, BulbVolt, pos).          % Battery voltage = pos
% The last constraint corresponds to the  Kirchhoff's law about voltages
```

CIRCUIT 2

```
circuit2( Sw1, Sw2, Sw3, B1, B2, B3, L1, L2, L3)  :-
   switch( Sw1, VSw1, C1),
   bulb( B1, L1, VB1, C1),
   switch( Sw2, VSw2, C2),
   bulb( B2, L2, VB2, C2),
   qsum( VSw2, VB2, V3),
   switch( Sw3, V3, CSw3),
   bulb( B3, L3, V3, CB3),
   qsum( VSw1, VB1, V1),
   qsum( V1, V3, pos),
   qsum( CSw3, CB3, C3),
   qsum( C2, C3, C1).
```



SOME QUESTIONS

- **A diagnostic question: Seeing that bulb 2 is light and bulb 3 is dark, what are the state of the bulbs?**

?- circuit2(_, _, _, B1, B2, B3, light, _, dark).

B1 = ok, B2 = ok, B3 = blown

- **A control-type question: If we want bulb 3 to be light, what are the positions of the switches?**

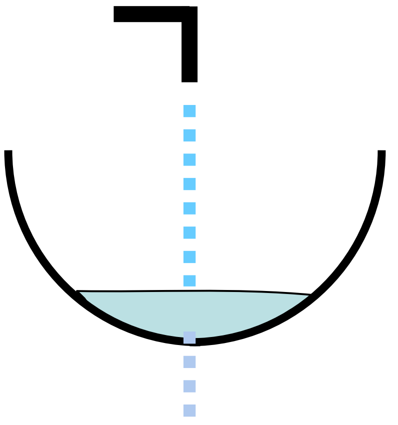
?- circuit2(SwPos1, SwPos2, SwPos3, ok, ok, ok, _, _, light).

SwPos1 = on, SwPos2 = on, SwPOs3 = off;

Swpos1 = on, SwPos2 = off, SwPos3 = off

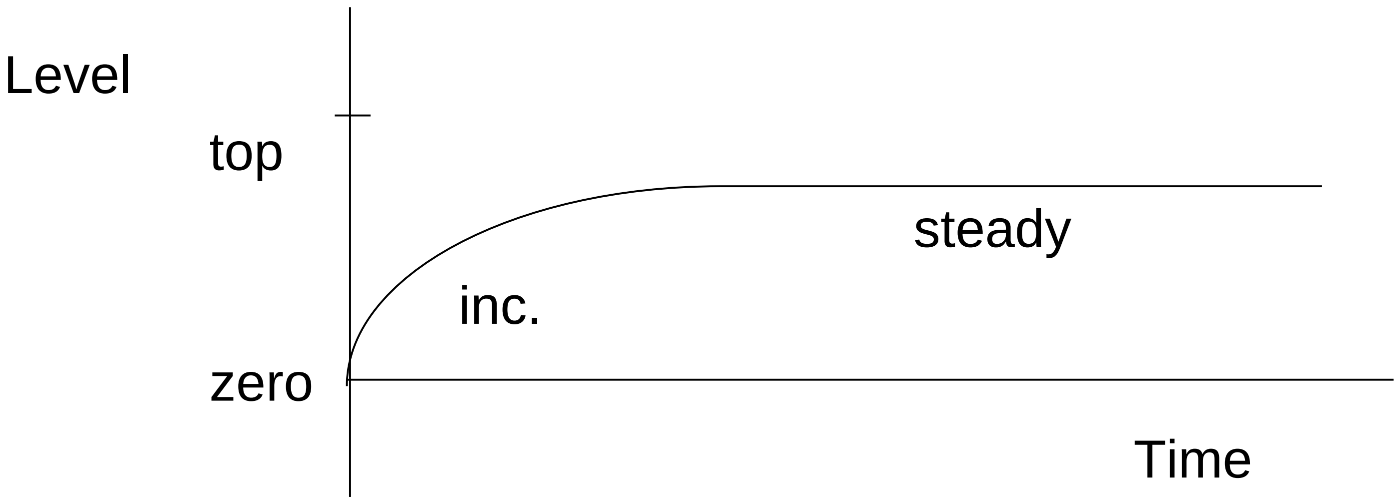
QUALITATIVE REASONING
ABOUT
DYNAMIC SYSTEMS

BATH TUB, AN EXAMPLE OF DYNAMIC SYSTEM



- Bath tub with open drain and constant input flow.

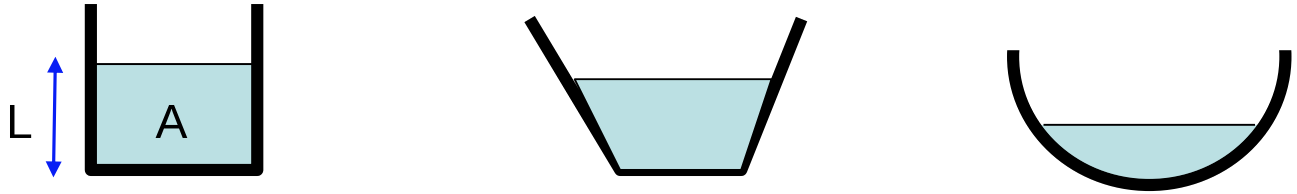
WATER LEVEL IN TIME



- Qualitatively

Time	Level
t0	zero / inc
(t0, t1)	zero..top / inc
t1	zero..top / std

RELATION BETWEEN AMOUNT AND LEVEL



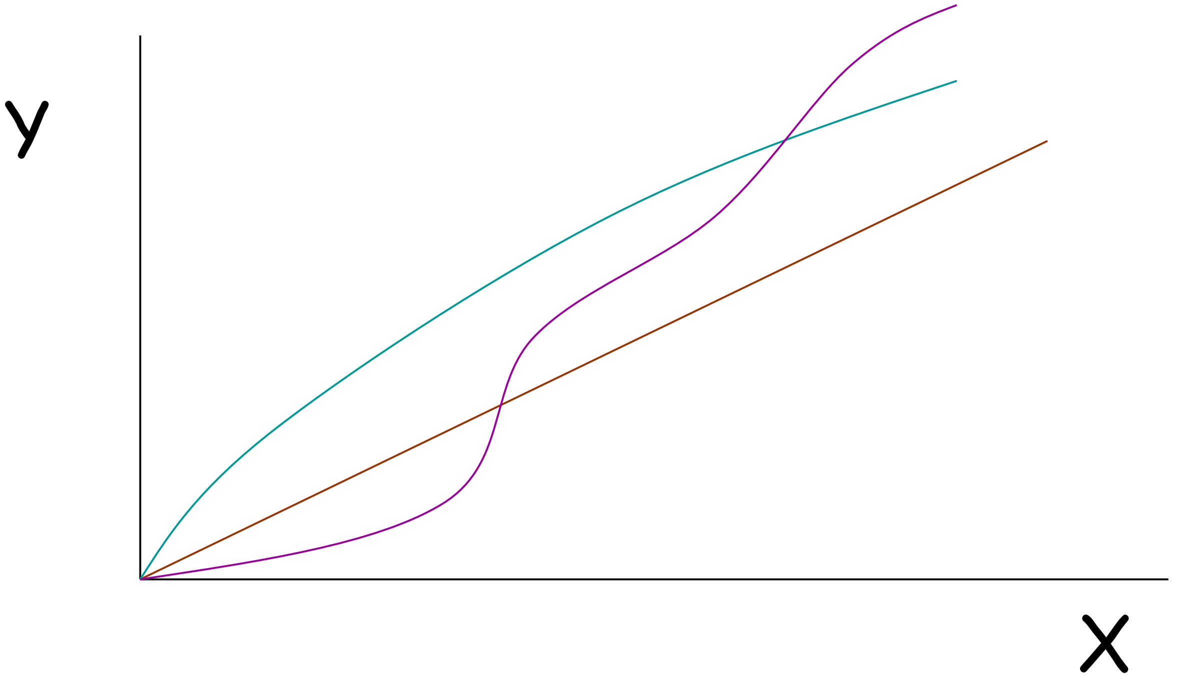
- The greater the amount, the greater the level

$$A = M^{+}(L)$$

- A is a *monotonically increasing function* of L

MONOTONIC FUNCTIONS

$Y = M^{+}(X)$ specifies a family of functions



TYPES OF QUALITATIVE CONSTRAINTS

- $M^+(X,Y)$ Y is a monotonically increasing function of X
- $M^-(X,Y)$ Y is a monotonically decreasing function of X
- sum(X,Y,Z) $Z = X+Y$
- minus(X,Y) $Y = -X$
- mult(X,Y,Z) $Z = X*Y$
- deriv(X,Y) $Y = dX/dt$ (Y is time derivative of X)

QUALITATIVE MODEL OF BATH TUB

Quantities:

- Level = level of water
- Amount = amount of water
- Pressure = pressure at drain
- Inflow = input flow
- Outflow = output flow
- Netflow = net flow (Netflow = Inflow – Outflow)

LANDMARK VALUES

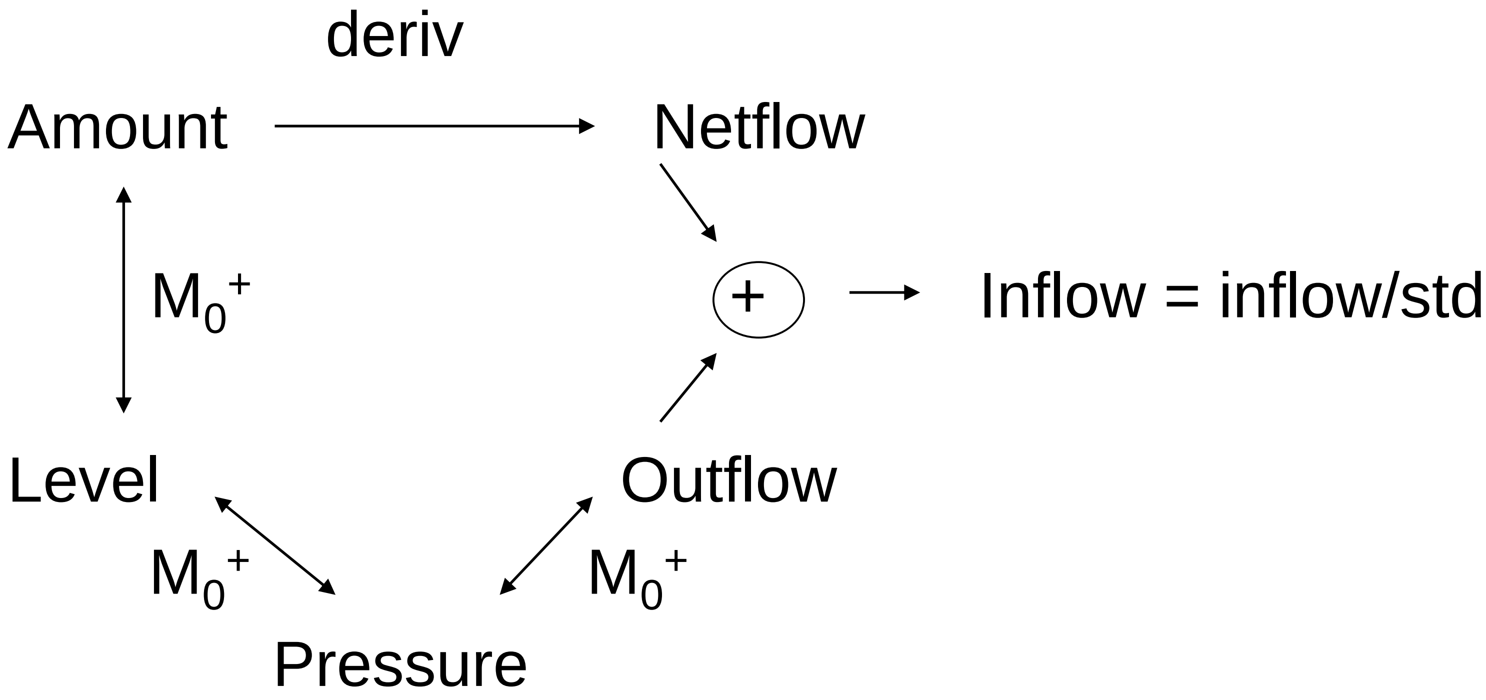
- Standard landmarks, for all variables: minf, zero, inf
- User may define additional landmarks
- Level: minf, zero, top, inf
- Amount: minf, zero, full, inf

QUALITATIVE MODEL OF BATH TUB

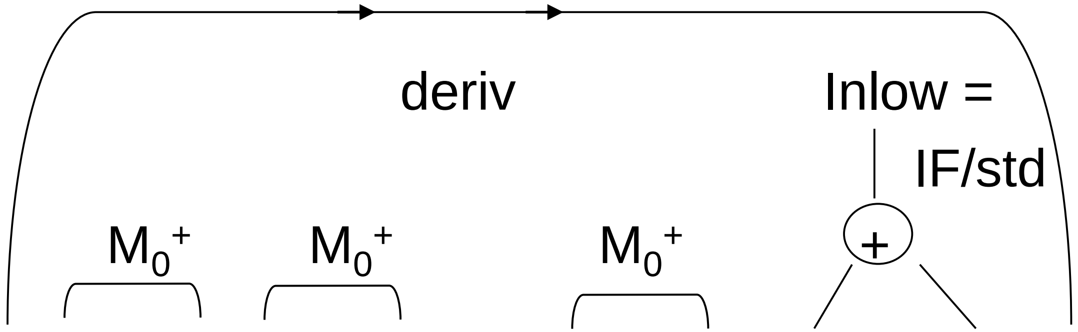
Constraints:

- M_0^+(Amount, Level), correspond: (zero,zero), (full,top)
- M_0^+(Level, Pressure)
- M_0^+(Pressure, Outflow)
- sum(Outflow, Netflow, Inflow)
- deriv(Amount, Netflow)
- Inflow = constant = inflow/std

GRAPHICAL REPRESENTATION OF BATH-TUB MODEL



QUALITATIVE SIMULATION - EXAMPLE



	Time	Amount	Level	Pressure	Outflow	Netflow
	to	0/inc	0/inc	0..inf/inc	0/inc	IF/dec
	to..t1	0..full/inc	0..top/inc	0..inf/inc	0..IF/inc	0..IF/dec
(a)	t1	full/inc	top/inc	0..inf/inc	0..IF/inc	0..IF/dec
(b)	t1	0..full/std	0..top/std	0..inf/std	IF/std	0/std
(c)	t1	full/std	top/std	0..inf/std	IF/std	0/std

QUALITATIVE STATE TRANSITIONS

Main principle:

Assume all variables are continuous and smooth in time.

CONTINUITY AND SMOOTHNESS

- Therefore in single transition step:

  a variable can either stay unchanged or change between neighbour landmarks; e.g.:

  Landmark1 ---> Landmark1 .. Landmark2

  Landmark1 .. Landmark2 ---> Landmark2

- variable's direction of change can stay unchanged, or change to its neighbour value; e.g.:

  dec ---> std

  std ---> inc

  However, dec ---> inc is impossible.

TABLE OF POSSIBLE STATE TRANSITIONS

Let L1 and L2 be two successive landmarks.

Possible state transitions are:

- L1 / std ---> L1 / std
- L1 / std ---> L1 .. L2 / inc
- L1 .. L2 / inc ---> L1 .. L2 / std
- L1 .. L2 / inc ---> L2 / inc
- L1 .. L2 / inc ---> L2 / std
- etc.

Variable cannot jump over a landmark.

SKETCH OF SIMULATION ALGORITHM

Given:

A model M as a set of constraints, and initial state S0

Initialise current state S := S0

While transition possible to next state do

begin

Find a next state NextS (using state transition table)

such that

NextS satisfies the constraints in model M;

S := NextS

end

Note: This algortihm non-deterministically generates possible behaviours.

PROBLEM

Consider a dynamic system with just one variable x. The time behaviour of x is:

$$x(t) = (t - 1) * (t - 1)$$

(a) Give the qualitative behaviour of x in time (i.e. sequence of qualitative states of x), starting with time t0 = 0. Assume the landmarks for x are: minf, zero, inf.

PROBLEM, CTD.

Now consider a dynamic system with three variables: x, y and z. The landmarks for all these variables are: minf, zero, inf. The time behaviours of these variables is:

$x(t) = (t - 1) * (t - 1)$

$y(t) = c*t$ (c is a constant, $c > 0$)

$z(t) = x(t) - y(t)$

(b) How many different qualitative behaviours of this system are possible (starting with $t0 = 0$)?

(c) Give all possible qualitative behaviours in time of this system (starting with $t0 = 0$).

PROBLEM

Consider a qualitative model given by the following qualitative constraints:

$M_0^+(X,Y)$
$M_0^-(Z,T)$
sum(X,Y,Z)

The lamdmarks for all the variables in the model are:

minf, zero, inf

Let the current state of T be: T = zero..inf/inc.

What are all the possible qualitative values of variables X, Y and Z at the same time so that the constraints above are satisfied?

PROBLEM

A qualitative model of a system contains variables x, y and z, and the constraints:

$M_0^+(x,y)$ % Monotonically incr. function

plus(x,y,z) % $x + y = z$

The landmarks for the three variables are:

x, y: minf, 0, inf

z: minf, 0, landz, inf

"minf" stands for "minus infinity", "inf" stands for "infinity".

PROBLEM, CTD.

At time t0, the qualitative value of x is x(t0) = 0/dec.

(a) What are the qualitative values y(t0) and z(t0)?

(b) What are the possible qualitative values of x, y and z at time interval (t0,t1)?

(c) What are the possible qualitative values of x, y and z at time t1?

PROBLEM

A qualitative model of a system contains variables x, y and z, and the constraints:

$M_0^+(x,y)$ % Monotonically incr. function
plus(x,z,y) % x + z = y
deriv(x,z) % dx/dt = z

The landmarks for the three variables are:

x: minf, 0, x0, inf
y: minf, 0, y0, inf
z: minf, 0, inf

"minf" stands for "minus infinity", "inf" stands for "infinity".

PROBLEM, CTD.

At time t0, the qualitative values of x and y are x(t0) = x0/dec and y(t0) = y0/dec.

(a) What are all the possible qualitative values z(t0) compatible with the above constraints?

(b) What are the possible values of x and y at time interval (t0,t1)?

(c) What are all the possible values of x at time point t1, compatible with the constraints in the model?

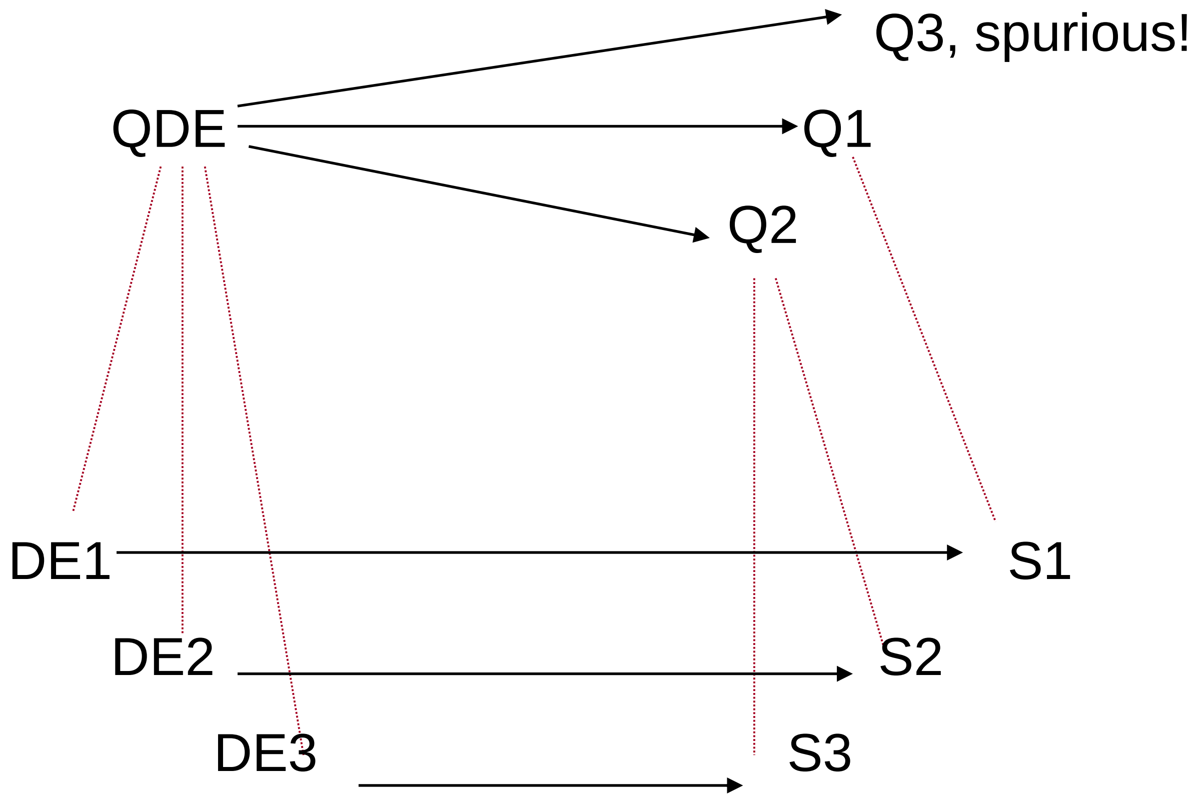
(d) Now suppose that in the initial state at t0, x is 0..x0 and y is 0..y0. What are all possible behaviours at t0..t1 and at t1?

SPURIOUS BEHAVIOURS

QSIM algorithm has the following mathematical properties:

- 1. QSIM is *complete*: it is guaranteed to generate *all* possible qualitative behaviours of a simulated system.
- 2. QSIM is *not sound*: it may generate qualitative behaviours that the simulated system cannot exhibit. These are called *spurious* behaviours. Unfortunately this deficiency seems to be very difficult to fix.

SPURIOUS SOLUTIONS



EXAMPLE

- Block and spring, block sliding on ice
- Qual. model:
  - $A = M_0^-(x)$
  - deriv(V, A)
  - deriv(X, V)

- QSIM produces spurious behaviours (increasing/dec. oscillation)
- Energy conservation follows from Model:
- Model ==> Energy=const (so steady oscillation only possible)

EXAMPLE CTD.

- Logically therefore:

  Model =\=> Inc. oscillation

- But:

  Model $|\!\!-\!\!-_{QSIM}$ Inc. oscillation

However:

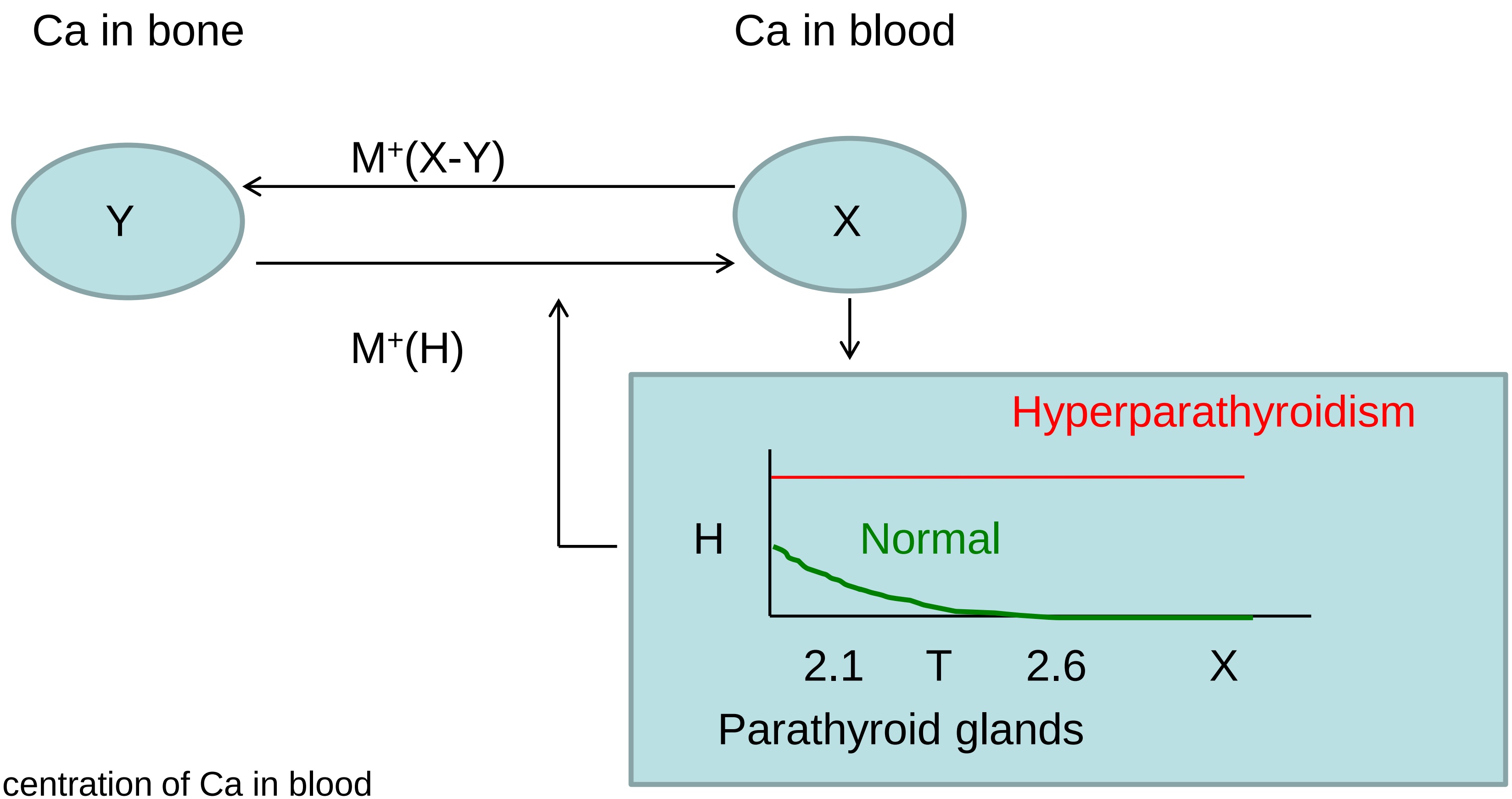
Model & Energy=const $|\!\!-\!\!/\!\!-_{QSIM}$ Inc. oscillation

IDEA OF PREVENTING SPURIOUS BEHAVIOURS

- How can we automatically make QDE models robust against spurious behaviours?

- Can we use ILP?

REGULATION OF Ca IN BLOOD AND BONE



X = concentration of Ca in blood

Y = concentration of Ca in bone

H = level of parathyroid hormone