SU1: 9. DOMAĆA ZADAĆA

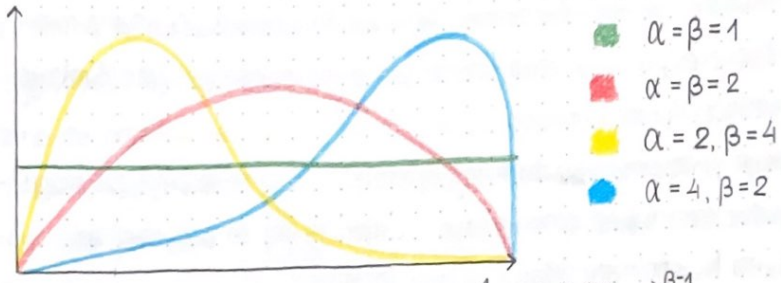
V14 – zadaci za učenje

5. a) $\hat{\theta}_{MAP} = \arg\max_\theta p(\theta|\mathcal{D}) = \arg\max_\theta p(\mathcal{D}|\theta)\,p(\theta)$

Procjenitelj MAP bolji je od procjenitelja MLE jer kombinira apriorno znanje i informacije dobivene iz podataka.

b) Da su dvije distribucije konjugatne znači da su iste vrste, a konjugatna apriorna distribucija je ona koja, kada se pomnoži izglednošću, daje distribuciju koja je iste vrste kao i aposteriorna distribucija. Svojstvo konjugatnosti bitno nam je jer nam omogućava da radimo tzv. „online" učenje.

c)



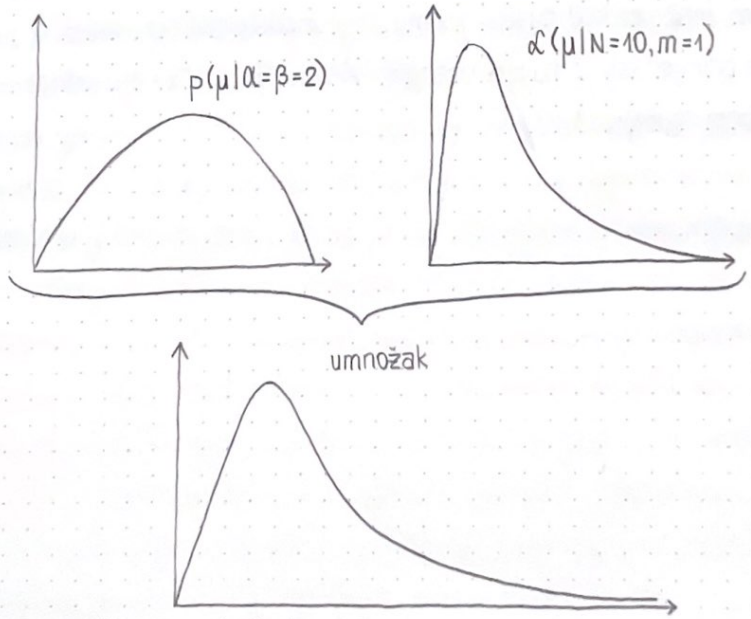
d) $p(\mathcal{D}|\mu) = \mu^m(1-\mu)^{N-m}$ izglednost

$p(\mu|\alpha,\beta) = \frac{1}{B(\alpha,\beta)}\mu^{\alpha-1}(1-\mu)^{\beta-1}$ apriorna vjerojatnost

$$p(\mu|\mathcal{D},\alpha,\beta) = p(\mu|N,m,\alpha,\beta) = \mu^m(1-\mu)^{N-m}\frac{1}{B(\alpha,\beta)}\mu^{\alpha-1}(1-\mu)^{\beta-1}\frac{1}{p(\mathcal{D})}$$

$$= \mu^{m+\alpha-1}(1-\mu)^{N-m+\beta-1}\frac{1}{B(\alpha,\beta)\,p(\mathcal{D})}$$

e)



f) $\hat{\mu}_{MAP} = \frac{m+\alpha-1}{\alpha+N+\beta-2} = \frac{1+2-1}{2+10+2-2} = \frac{2}{12} = 0.167$

$\hat{\mu}_{MLE} = \frac{1}{10} = 0.1$

Porastom broja primjera N, razlika između $\hat{\mu}_{MAP}$ i $\hat{\mu}_{MLE}$ se smanjuje jer sve više vjerujemo podacima.

g) Ako za parametre beta distribucije odaberemo $\alpha=\beta=2$, MAP procjenitelj svodi se na $\hat{\mu}_{MAP}=\frac{m+1}{N+2}$, što zovemo Laplaceovim procjeniteljem.

V15 - Zadaci za učenje

2. $P(y=1)=0.2$ $P(x|y=1)=0.8$ $h_1(x)=P(x|y=1)P(y=1)=0.16$

$P(y=2)=0.05$ $P(x|y=2)=0.5$ $h_2(x)=P(x|y=2)P(y=2)=0.025$

$P(y=3)=0.75$ $P(x|y=3)=0.5$ $h_3(x)=P(x|y=3)P(y=3)=0.375 \Rightarrow$ MAP hipoteza

$P(x)=\sum_{y=1}^{3} P(x|y)P(y)=0.56$

$P(y=1|x)=\frac{h_1(x)}{P(x)}=0.286$

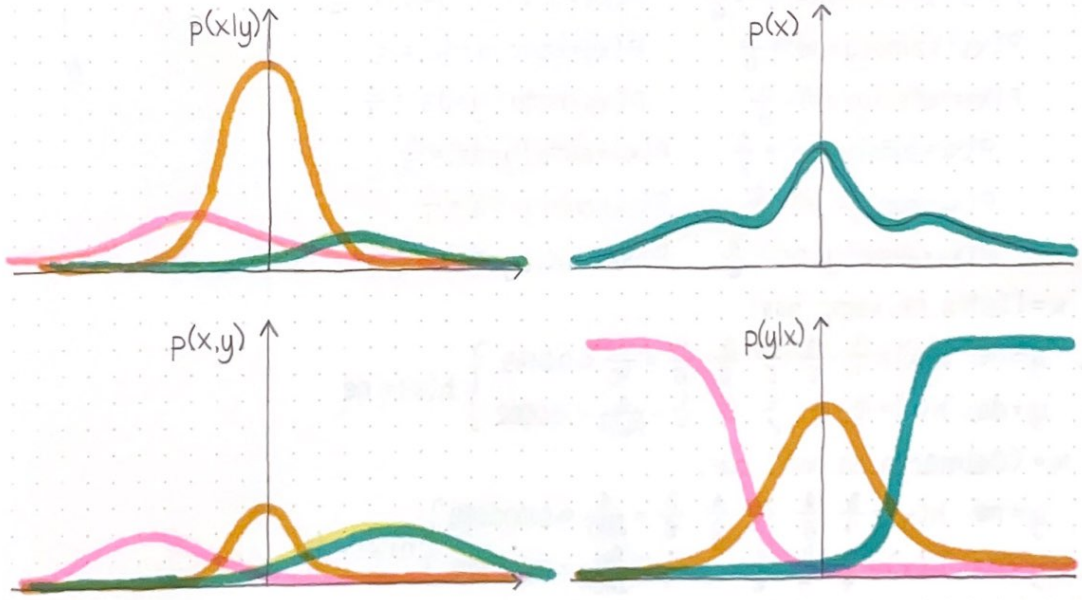
$P(y=2|x)=\frac{h_2(x)}{P(x)}=0.446$

$P(y=3|x)=\frac{h_3(x)}{P(x)}=0.670$

3. $P(y=1)=0.3$ $\mu_1=-5$ $\sigma_1^2=5$

$P(y=2)=0.2$ $\mu_2=0$ $\sigma_2^2=1$

$P(y=3)=0.5$ $\mu_3=5$ $\sigma_3^2=10$



V16 - Zadaci za učenje

2. a) $P(x_1=\text{Istra}|y=\text{ne})=0$ $P(x_1=\text{Istra}|y=\text{da})=\frac{1}{2}$

$P(x_1=\text{Dalmacija}|y=\text{ne})=\frac{1}{3}$ $P(x_1=\text{Dalmacija}|y=\text{da})=\frac{1}{2}$

$P(x_1=\text{Kvarner}|y=\text{ne})=\frac{2}{3}$ $P(x_1=\text{Kvarner}|y=\text{da})=0$

$P(x_2=\text{ne}|y=\text{ne})=1$ $P(x_2=\text{ne}|y=\text{da})=\frac{1}{4}$

$P(x_2=\text{da}|y=\text{ne})=0$ $P(x_2=\text{da}|y=\text{da})=\frac{3}{4}$

$P(x_3=\text{privatni}|y=\text{ne})=\frac{1}{3}$ $P(x_3=\text{privatni}|y=\text{da})=\frac{1}{2}$

$P(x_3=\text{kamp}|y=\text{ne})=\frac{2}{3}$ $P(x_3=\text{kamp}|y=\text{da})=0$

$P(x_3=\text{hotel}|y=\text{ne})=0$ $P(x_3=\text{hotel}|y=\text{da})=\frac{1}{2}$

$P(x_4=\text{auto}|y=\text{ne})=0$ $P(x_4=\text{auto}|y=\text{da})=\frac{3}{4}$

$P(x_4=\text{bus}|y=\text{ne})=\frac{2}{3}$ $P(x_4=\text{bus}|y=\text{da})=0$

$P(x_4=\text{avion}|y=\text{ne})=\frac{1}{3}$ $P(x_4=\text{avion}|y=\text{da})=\frac{1}{4}$

$h(\text{Istra, ne, kamp, bus}) = \arg\max_y P(y)\prod_{k=1}^{n} P(x_k|y)$

$y=\text{ne}: h=\frac{3}{7}\cdot 0\cdot 1\cdot\frac{2}{3}\cdot\frac{2}{3}=0$

$y=\text{da}: h=\frac{4}{7}\cdot\frac{1}{2}\cdot\frac{1}{4}\cdot 0\cdot 0=0$

} ne možemo klasificirati primjer na ovaj način

$h(\text{Dalmacija, da, hotel, bus}) = \arg\max_y P(y)\prod_{k=1}^{n} P(x_k|y)$

$y=\text{ne}: h=\frac{1}{3}\cdot 0\cdot 0\cdot\frac{2}{3}=0$

$y=\text{da}: h=\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot 0=$

} ne možemo klasificirati primjer na ovaj način

b) $P(x_1=\text{Istra}|y=\text{ne})=\frac{1}{6}$ $P(x_1=\text{Istra}|y=\text{da})=\frac{3}{7}$

$P(x_1=\text{Dalmacija}|y=\text{ne})=\frac{2}{6}$ $P(x_1=\text{Dalmacija}|y=\text{da})=\frac{3}{7}$

$P(x_1=\text{Kvarner}|y=\text{ne})=\frac{3}{6}$ $P(x_1=\text{Kvarner}|y=\text{da})=\frac{1}{7}$

$P(x_2=\text{ne}|y=\text{ne})=\frac{4}{5}$ $P(x_2=\text{ne}|y=\text{da})=\frac{2}{6}$

$P(x_2=\text{da}|y=\text{ne})=\frac{1}{5}$ $P(x_2=\text{da}|y=\text{da})=\frac{4}{6}$

$P(x_3=\text{privatni}|y=\text{ne})=\frac{2}{6}$ $P(x_3=\text{privatni}|y=\text{da})=\frac{3}{7}$

$P(x_3=\text{kamp}|y=\text{ne})=\frac{3}{6}$ $P(x_3=\text{kamp}|y=\text{da})=\frac{1}{7}$

$P(x_3=\text{hotel}|y=\text{ne})=\frac{1}{6}$ $P(x_3=\text{hotel}|y=\text{da})=\frac{3}{7}$

$P(x_4=\text{auto}|y=\text{ne})=\frac{1}{6}$ $P(x_4=\text{auto}|y=\text{da})=\frac{4}{7}$

$P(x_4=\text{bus}|y=\text{ne})=\frac{3}{6}$ $P(x_4=\text{bus}|y=\text{da})=\frac{1}{7}$

$P(x_4=\text{avion}|y=\text{ne})=\frac{2}{6}$ $P(x_4=\text{avion}|y=\text{da})=\frac{2}{7}$

$x=(\text{Istra, ne, kamp, bus})$

$y=\text{ne}: h(x)=\frac{3}{7}\cdot\frac{1}{6}\cdot\frac{4}{5}\cdot\frac{3}{6}\cdot\frac{3}{6}=\frac{1}{70}\approx 0.0143$

$y=\text{da}: h(x)=\frac{4}{7}\cdot\frac{3}{7}\cdot\frac{2}{6}\cdot\frac{1}{7}\cdot\frac{1}{7}=\frac{4}{2401}\approx 0.0002$

} $h(x)=\text{ne}$

$x=(\text{Dalmacija, da, hotel, bus})$

$y=\text{ne}: h(x)=\frac{3}{7}\cdot\frac{2}{6}\cdot\frac{1}{5}\cdot\frac{1}{6}\cdot\frac{3}{6}=\frac{1}{420}\approx 0.000238$

$y=\text{da}: h(x)=\frac{4}{7}\cdot\frac{3}{7}\cdot\frac{4}{6}\cdot\frac{3}{7}\cdot\frac{1}{7}=\frac{96}{2401}\approx 0.039983$

} $h(x)=\text{da}$

3. a) ENTROPIJA: $H(P)=-\sum_x P(x)\ln P(x)$ UNAKRSNA ENTROPIJA: $H(P,Q)=-\sum_x P(x)\ln Q(x)$

RELATIVNA ENTROPIJA: $H(P,Q)-H(P)=-\sum_x P(x)\ln Q(x)-(-\sum_x P(x)\ln P(x))$

$=-\sum_x P(x)\ln Q(x)+\sum_x P(x)\ln P(x)$

$=\sum_x P(x)\ln\frac{P(x)}{Q(x)}=D_{KL}(P\|Q)$

$D_{KL}(P(x,y)\|P(x)P(y))=\sum_{xy} P(x,y)\ln\frac{P(x,y)}{P(x)P(y)}=I(x,y)$ UZAJAMNA INFORMACIJA

b) $P(1,1)=0.2$ $P(1,2)=0.05$ $P(1,3)=0.3$

$P(2,1)=0.05$ $P(2,2)=0.3$ $P(2,3)=0.1$

$P(x=1)=0.55$ $P(x=2)=0.45$

$P(y=1)=0.25$ $P(y=2)=0.35$ $P(y=3)=0.4$

} $I(x,y)=P(1,1)\ln\frac{P(1,1)}{P(x=1)P(y=1)}+\ldots$

$+P(2,3)\ln\frac{P(2,3)}{P(x=2)P(y=3)}$

$=0.194563$

Varijable nisu nezavisne.