MATAN 2 - 1. vježbe

1. Vektor smjera tangente na krivulju $\mathcal{C}$ u točki $(x(t), y(t), z(t))$ dan je s

$$\vec{s}(t) = x'(t)\vec{\imath} + y'(t)\vec{\jmath} + z'(t)\vec{k} = t^3\vec{\imath} + t^2\vec{\jmath} + t\vec{k}.$$

Iz uvjeta zadatka slijedi da taj vektor mora biti okomit na vektor normale zadane ravnine

$$\vec{s}(t) \perp \vec{n} \iff \vec{s}(t)\cdot\vec{n} = 0$$

$$\iff t^3 + 3t^2 + 2t = 0$$

$$\iff t(t+1)(t+2) = 0$$

Dakle, tražene točke na krivulji dobivamo za sljedeće vrijednosti parametra t:

$$t_1 = 0 \Rightarrow (0,0,0)$$

$$t_2 = -1 \Rightarrow \left(\frac{1}{4}, -\frac{1}{3}, \frac{1}{2}\right)$$

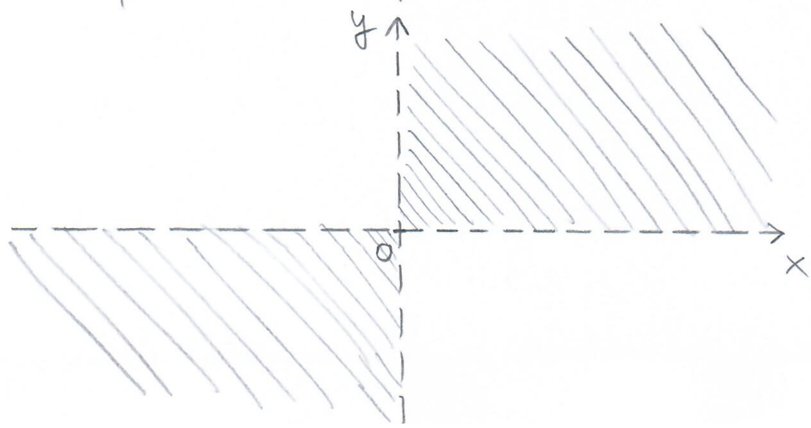
$$t_3 = -2 \Rightarrow \left(4, -\frac{8}{3}, 2\right)$$

2. (a) $f(x,y) = \ln(xy)$

Uvjet: $xy > 0$.

$x>0, y>0$ $\qquad$ $x<0, y<0$

$$\Rightarrow \mathcal{D}_f = \{(x,y)\in\mathbb{R}^2 \mid (x>0 \wedge y>0) \vee (x<0 \wedge y<0)\}$$



(b) $f(x,y)=\sqrt{\dfrac{x^2+y^2}{x^2-y^2}}$

Uvjeti:

1° $\dfrac{x^2+y^2}{x^2-y^2}\geq 0$

Zbog $x^2+y^2\geq 0$ za sve $(x,y)\in\mathbb{R}^2$ slijedi

$$x^2-y^2>0$$

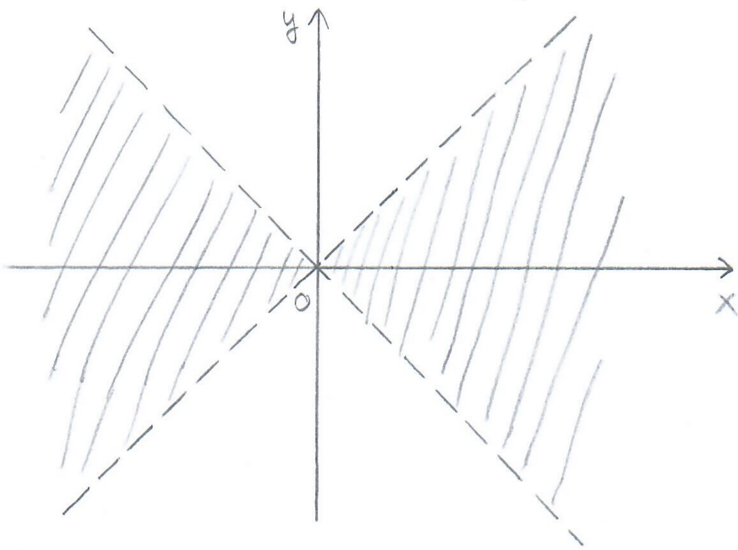
$$(x-y)(x+y)>0$$

$$\begin{cases} x-y>0 \Rightarrow y<x \\ x+y>0 \Rightarrow y>-x \end{cases} \qquad \begin{cases} x-y<0 \Rightarrow y>x \\ x+y<0 \Rightarrow y<-x \end{cases}$$

2° $x^2-y^2\neq 0$

Već uključeno u prethodni slučaj.

$\mathcal{D}_f=\{(x,y)\in\mathbb{R}^2 \mid -x<y<x \ \lor \ x<y<-x\}$



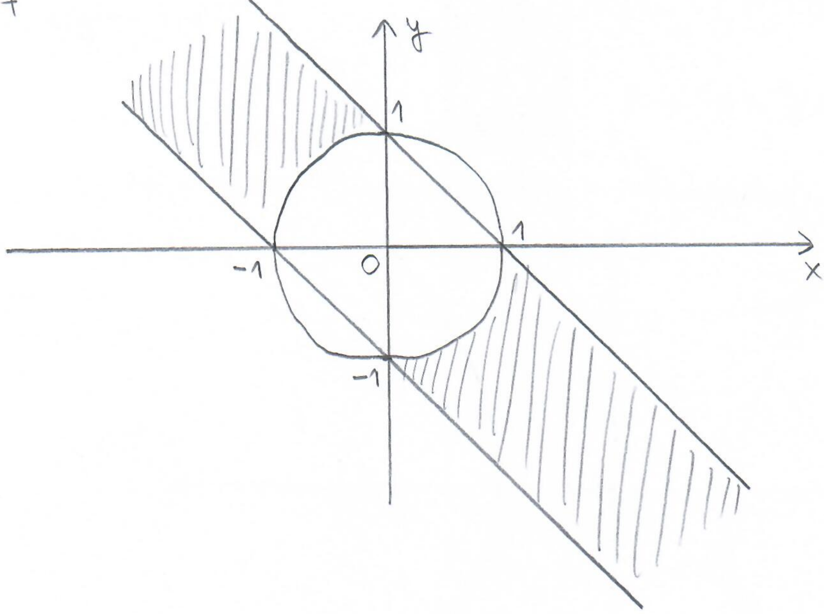
(c) $f(x,y) = 2^{\sqrt{x^2+y^2-1}} - \arcsin(x+y)$

Uvjeti:

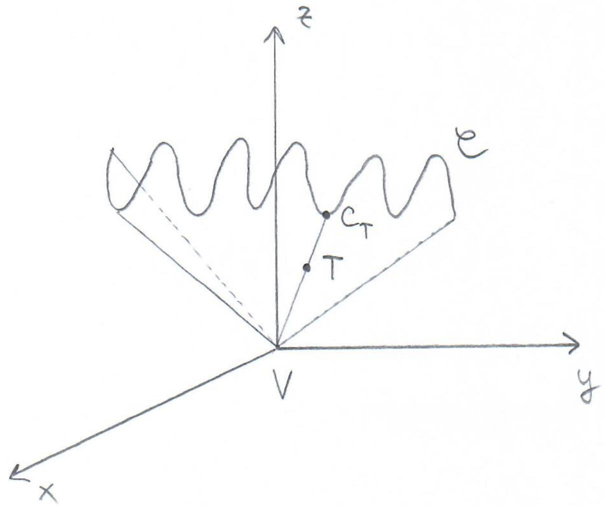
$1^\circ \quad x^2+y^2-1 \geq 0 \Rightarrow x^2+y^2 \geq 1$

$2^\circ \quad -1 \leq x+y \leq 1 \Rightarrow -x-1 \leq y \leq -x+1$

$\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \geq 1 \wedge -x-1 \leq y \leq -x+1\}$



3.



Neka je $T(x_0, y_0, z_0)$ proizvoljna točka. Tada T leži na zadanoj plohi ako i samo ako postoje $\lambda \in \mathbb{R}$ i točka $C_T(x_1, y_1, z_1) \in \mathcal{C}$ t.d.

$$\vec{VT} = \lambda \vec{VC_T}$$

$$\begin{cases} x_0 = \lambda x_1 \\ y_0 = \lambda y_1 \\ z_0 = \lambda \underbrace{z_1}_{=1} = \lambda \end{cases} \begin{aligned} &\Rightarrow x_0 = z_0 x_1 \Rightarrow x_1 = \frac{x_0}{z_0} \\ &\Rightarrow y_0 = z_0 y_1 = z_0 \sin x_1 = z_0 \sin \frac{x_0}{z_0} \end{aligned}$$

Zbog proizvoljnosti točke T slijedi da je jednadžba zadane plohe

$$y = z \sin \frac{x}{z}.$$

4. Ploha dobivena rotacijom krivulje zadane implicitno jednadžbom $F(y,z)=0$ oko osi Oz ima jednadžbu

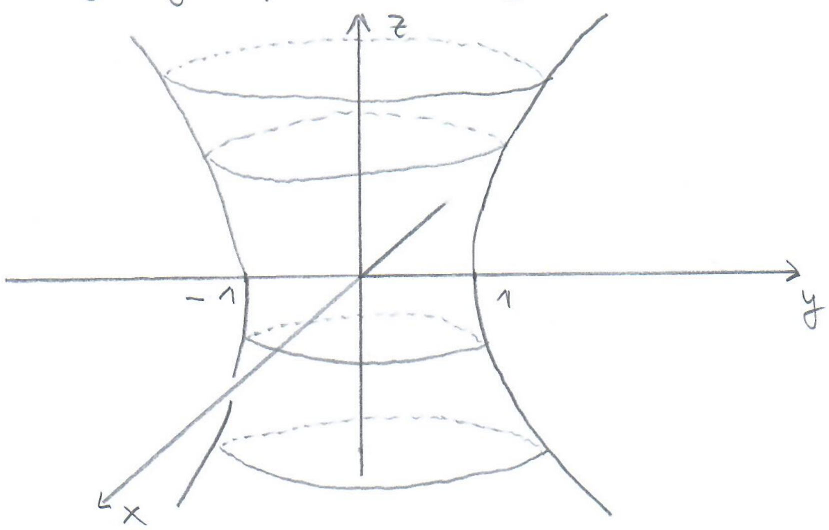
$$F\left(\sqrt{x^2+y^2},\, z\right)=0.$$

(a) Jednadžba tražene plohe je

$$\left(\sqrt{x^2+y^2}\right)^2 - z^2 = 1 \qquad \left(F(y,z)=y^2-z^2-1\right)$$

$$\Rightarrow \quad x^2+y^2-z^2=1$$

i riječ je jednoplošnom rotacijskom hiperboloidu:

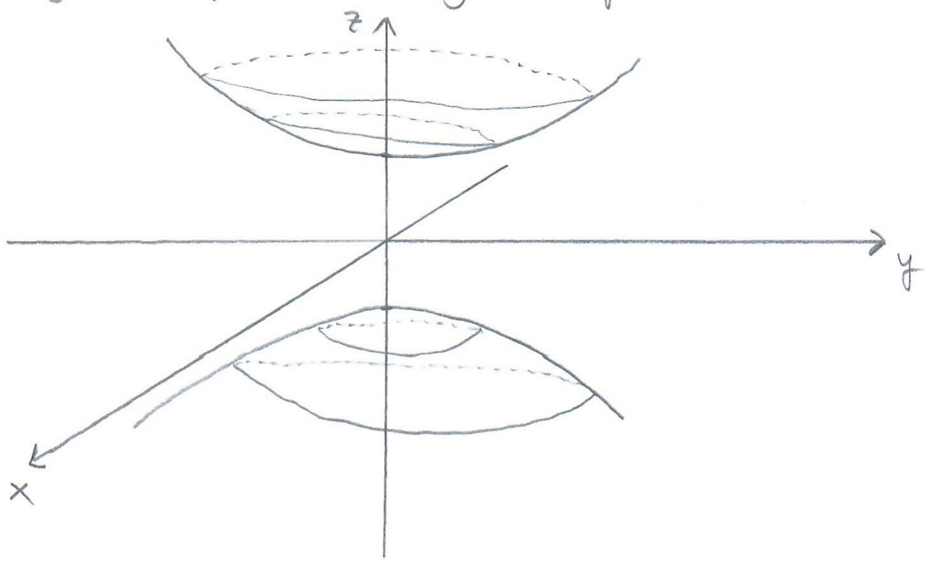


(b) Jednadžba tražene plohe je

$$z^2-\left(\sqrt{x^2+y^2}\right)^2 = 1 \qquad \left(F(y,z)=z^2-y^2-1\right)$$

$$\Rightarrow \quad z^2-x^2-y^2=1$$

i riječ je o dvoplošnom rotacijskom hiperboloidu:



5. (a) Neka je $c>0$. Određujemo krivulju zadanu implicitno s

$$e^{\frac{2x}{x^2+y^2}} = c$$

$$\Rightarrow \quad \frac{2x}{x^2+y^2} = \ln c$$

Posebno, za $c=1$ dobivamo $x=0$ pa je y-os jedna nivo-krivulja od f. Za $c \neq 1$ stavljanjem $d = \frac{1}{\ln c}$ $(d \in \mathbb{R}\setminus\{0\})$ dobivamo

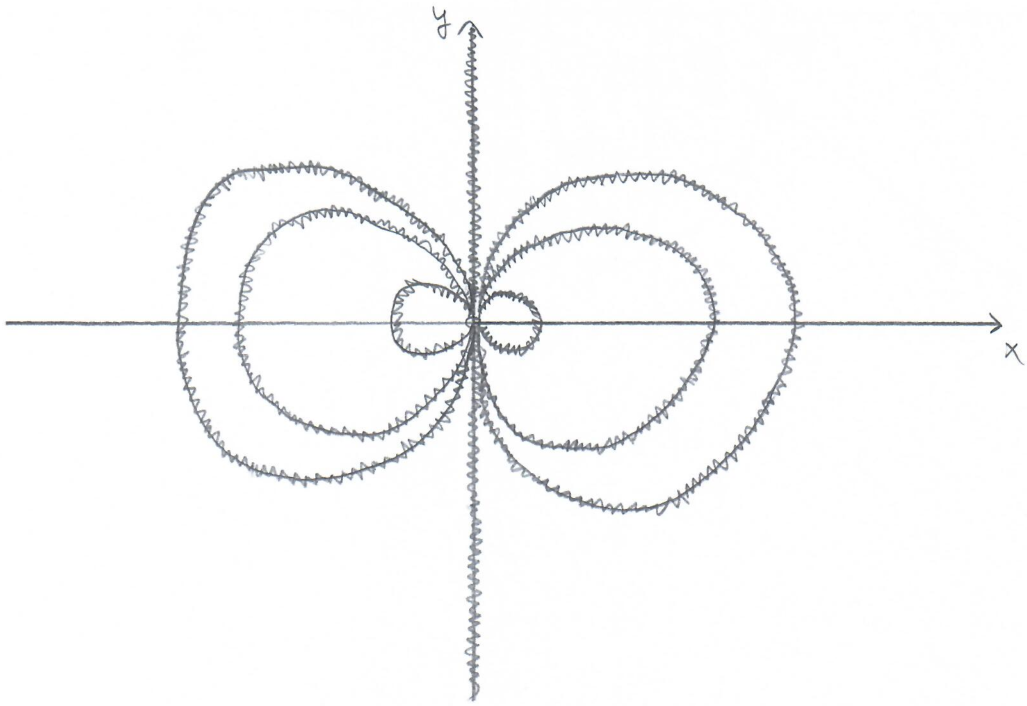
$$x^2+y^2 = 2dx$$

$$x^2-2dx+y^2 = 0$$

$$(x^2-2dx+d^2)+y^2 = d^2$$

$$(x-d)^2+y^2 = d^2$$

Dakle, preostale nivo-krivulje su kružnice sa središtem u $(d,0)$ radijusa $|d|$ (za $d \neq 0$).



(b) Određujemo plohu implicitno zadanu s

$$1-\sqrt{4x^2+4y^2+z^2}=c$$

$$\sqrt{4x^2+4y^2+z^2}=1-c \qquad (\text{uočimo da mora biti } c \leq 1)$$

$$4x^2+4y^2+z^2=1-c \qquad (\text{stavimo } d^2=1-c \geq 0)$$

$$\frac{x^2}{\frac{1}{4}d^2}+\frac{y^2}{\frac{1}{4}d^2}+\frac{z^2}{d^2}=1$$

Ovo je jednadžba rotacijskog elipsoida s poluosima duljina $\frac{1}{2}|d|$, $\frac{1}{2}|d|$, $|d|$ (posebno, u slučaju $d=0$ dobivamo točku $(0,0,0)$).