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VECTOR \rightarrow mathematical object that encodes a length and direction

Vector in \mathbb{R}^n

$$v = [v_1, v_2, \dots, v_n] \in \mathbb{R}^n$$

if $v \in \mathbb{R}$ (real vector of dimension n)

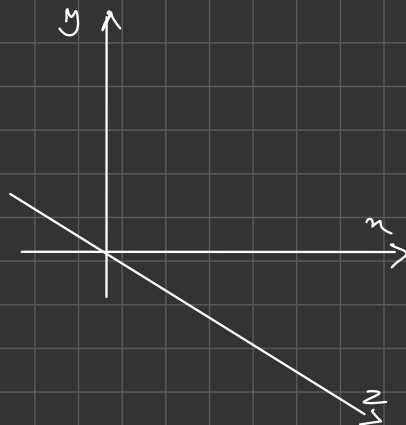
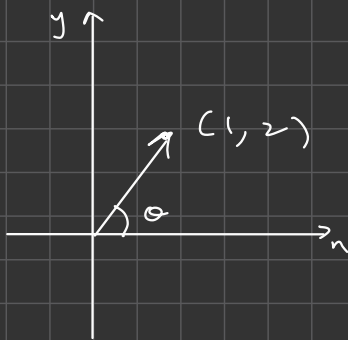
$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^n$$

Take $n=2$: \mathbb{R}^2 ($\mathbb{R} \times \mathbb{R}$)

when $n=3$ (3-D)

$$v = (1, 2, 3)$$

$$v = (1, 2)$$



VECTOR ALGEBRA

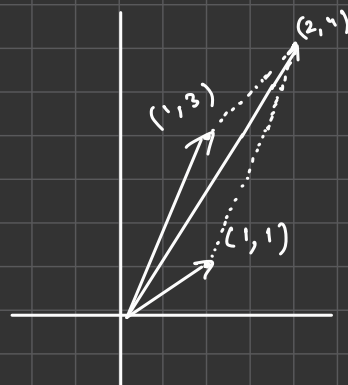
Addition / Subtraction

$$v_1 = (1, 3) \text{ and } v_2 = (1, 1)$$

$$v_1 + v_2 = (2, 4) \quad \begin{array}{l} x \text{ component of } v_1 + x \text{ component of } v_2 \\ y \text{ component of } v_1 + y \text{ component of } v_2 \end{array}$$

Similarly

$$v_1 - v_2 = (0, 2)$$



$$\mathbb{R}^n \Rightarrow \begin{array}{l} v_1 = (x_1, x_2, \dots, x_n) \\ v_2 = (y_1, y_2, \dots, y_n) \end{array}$$

$$v_1 + v_2 = (x_1 \pm y_1, x_2 \pm y_2, \dots, x_n \pm y_n)$$

Dot Product

$$v_1 = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$$

$$v_2 = (y_1, y_2, \dots, y_n) \in \mathbb{R}^n$$

$$\begin{aligned} v_1 \cdot v_2 &= x_1 y_1 + x_2 y_2 + \dots + x_n y_n \\ &= \sum_{i=1}^n x_i y_i \end{aligned}$$

$$\text{Ex: In } \mathbb{R}^3, \quad \begin{array}{l} v_1 = (1, 1, -1) \\ v_2 = (2, 3, 1) \end{array}$$

$$v_1 \cdot v_2 = (2 + 3 - 1) = 4$$

Length / Magnitude

$$\vec{v} = (x_1, x_2, \dots, x_n)$$

$$|\vec{v}| = v = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$\text{Ex} \rightarrow \vec{v} = (1, -1, 2)$$

$$\begin{aligned} |\vec{v}| &= \sqrt{1^2 + (-1)^2 + (2)^2} \\ &= \sqrt{1 + 1 + 4} = \sqrt{6} \end{aligned}$$

Angle between two vectors

$$\text{Angle: } v_1, v_2 \in \mathbb{R}^n$$

$$\alpha = \cos^{-1} \left(\frac{v_1 \cdot v_2}{|v_1| |v_2|} \right)$$

Linear Combination of vectors

Consider a set $S = \{$