ICCS312 Algorithms and Tractability
Term 1 2021–22
Quiz 3

Name (Print): _____

Student ID: _____

Directions:

- This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

- This exam is open-book, open-note and open-internet. You must cite your sources.
- No collaboration of any kind is allowed.
- To earn full credit on open-ended questions, you must show sufficient work to justify your answers.

- Your solution must be typeset and clearly mark the problem ID. You don't need to repeat the questions.

- You have total of 240 minutes to complete the exam. You may submit your exam anytime within the time limit. Good luck!

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	20	
Total:	60	

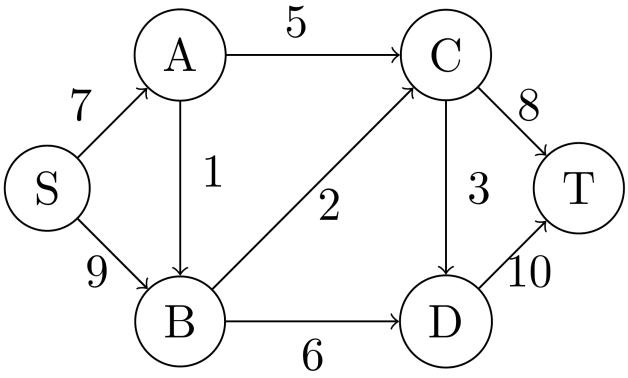
1. (10 points) Network Flows
   (a) (4 points) True or False.
      (i) If f is a maximum flow of G, then $val(f) \leq cap(A, B)$ for for any cut (A, B).

      Ans: _____
      (ii) Let (A, B) be a minimum cut of G, there exists a flow f such that $val(f) > cap(A, B)$.

      Ans: _____
      (iii) Let (A, B) be a minimum cut and f be a maximum flow, then $val(f) = cap(A, B)$.

      Ans: _____
      (iv) The running time of Ford-Fulkerson is $O(m^2n)$.

      Ans: _____
   (b) (6 points) Consider the following network (the numbers are edge capacities).



      (i) Perform Ford-Fulkerson and show the residual graph G_f for each path augmentation.

(ii) What is the value of a maximum flow f?

(iii) What is a minimum cut (A, B)? Write the partition of nodes.

2. (10 points) Customers

A server has n customers waiting to be served. The service time required by each customer is known in advance: it is t_i minutes for customer i. So if, for example, the customers are served in order of increasing i, then the i-th customer has to wait $\sum_{j=1}^{i} t_j$ minutes.

We wish to minimize the total waiting time

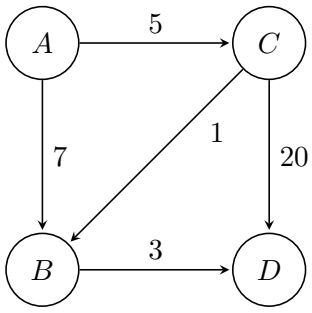
$$T = \sum_{i=1}^{n} (\text{time spent waiting by customer } i)$$

(a) (3 points) Give an efficient algorithm for computing the optimal order in which to process the customers (Same as A3)

(b) (7 points) Prove that your algorithm is optimal.

3. (10 points) Single-Source Shortest Paths

Consider the following weighted, directed graph $G = (V, E)$. Answer the questions below.



(a) (2 points) Suppose we run Dijkstra's algorithm on G, starting at node A. Fill in the following table at each iteration of the algorithm.

Iteration	Active Node	d['A']	d['B']	d['C']	d['D']
0	-	0	∞	∞	∞
1					
2					
3					
4					

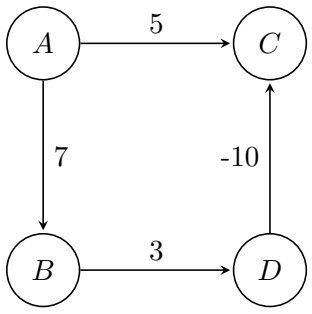
(b) (1 point) What is the running time of Dijkstra's algorithm implemented with a binary heap on a graph with n nodes and m edges?

(c) (2 points) Suppose now we run Bellman-Ford algorithm on G, starting at node A. Fill in the following table at each iteration of the algorithm.

Iteration	d['A']	d['B']	d['C']	d['D']
0	0	∞	∞	∞
1				
2				
3				
4				

(d) (1 point) What is the running time of Bellman-Ford algorithm on a graph with n nodes and m edges?

(e) (1 point) Consider another graph G'



What is a shortest path from A to C on G'?

$A \rightarrow$

(f) (1 point) Which algorithm would you use to find a shortest path in G'?

(g) (2 points) What is a negative cycle in a directed graph? Explain how to detect whether a directed graph G contains a negative cycle.

4. (10 points) Dynamic Programming

(a) (1 point) Explain what is dynamic programming in your own words.

(b) (2 points) *ROD-CUTTING*. Given a rod of length n and prices p_i for $1 \leq i \leq n$, the goal is to determine the maximum revenue r_n obtainable by cutting up the rod of the length n and selling the pieces. Define r_n in terms of its subproblems.

$$r_n = \begin{cases} \_\_\_\_\_ & \text{if } n > 0 \\ \_\_\_\_\_ & \text{if } n = 0 \end{cases}$$

(c) (7 points) *LCS-LENGTH.* Given two strings $X[1..m]$ and $Y[1..n]$, the goal is to find the length of a longest common subsequence (LCS) of X and Y. For example, the LCS of "HILOWHI" and "HOLLOW" is "HLOW" whose length is 4. To find the length of an LCS with dynamic programming, we set up a table $c[0..m, 0..n]$ to store solutions to subproblems. Complete the solution outline below.

- Let $c[i, j]$ be the length of an LCS of _____
- We can write the recurrence relation as:

$$c[i,j] = \begin{cases} \_\_\_\_\_ & \text{if } i = 0 \text{ or } j = 0 \\ \_\_\_\_\_ & \text{if } i, j > 0 \text{ and } X_i = Y_j \\ \_\_\_\_\_ & \text{if } i, j > 0 \text{ and } X_i \neq Y_j \end{cases}$$

Now fill in the table c when X is "MUIC" and Y is "MANIAC".

	∅	M	A	N	I	A	C
∅							
M							
U							
I							
C							

5. (20 points) Programming Problems

Complete the following tasks on `https://mastery.cs.muzoo.io/`

(a) (10 points) Sum of Primes

(b) (10 points) Attractions