

# Algo-quiz 4

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## 1 Question 1

**Ans**

- (a) C
- (b) C
- (c) B
- (d) A
- (e) B
- (f) D
- (g) C
- (h) C
- (i) D
- (j) D

## 2 Question 2

**Ans**

(a) if problem X can be reduced to problem Y, we can write as  $X \leq_p Y$ , which mean X is polynomial-time reducible to Y and Y is as hard as X, as you can solve Y so you can also solve X.

In the other word, it mean that if you have an algorithm from Y then you can use it to find an algorithm for X as well. This is called a Karp reduction.

- (b)
- (i)
- (ii)

## 3 Question 3

**Ans**

- (a)

- (i) There is a polynomial time algorithm for  $X$  and any problem in  $P$  can be solved in polynomial time.
  - (ii)  $Y$  is solvable in poly-time iff  $P = NP$ . We can say that  $Y$  is NP-complete for all  $X \in NP$ ,  $X \leq pY$ .  $Y$  is at least as hard as every problem in  $NP$ .
  - (iii)  $X$  also can be solved in polynomial time.
  - (iv) There is no information about it. But as given in the problem  $X$  can be solved.
  - (v)  $Y$  cannot be solved in polynomial time.
- (b)**
- (a) Is NP because let say we are given a list of numbers, and we can check if the number in the list is sorted or not in polynomial time.
  - (b) Is P
  - (c) Is P
- (c)** We know that  $X$  is also NP-Complete as it can be reduced to NP-Hard

## 4 Question 4

**Ans**

- (a)**
- (i) True
  - (ii) True
  - (iii) False
- (b)** NP-Complete problems are the ones that are both in NP and NP-Hard. To prove it's NP-Complete we first need to show that it belongs to NP and is NP-Hard.  
 To show that the problem is NP we can use the certificate verification strategy which will tell us the ans of polynomial algorithm that is "Yes" or "No".  
 To show that the problem is NP-Hard, we just simply choose another NP-Hard problem and reduce it to ours. As NP-Hard problem is also NP-Complete so we could use any of them. We prove that  $X \leq pY$   
 If  $X \in NP - Complete$ ,  $Y \in NP$ , and  $X \leq Y$ , then  $Y \in NP - Complete$ .
- (c)** NP-Complete are the kind of problem where you have similar category which if you are able to solve one of them then you could solve all of them. If someone managed to solve one of the problem then he/she could solve all the existing NP-Complete problem and might get famous for it.