Formulas used:

Relative coordinates x_{rel} , y_{rel} and z_{rel} :

$$x_{rel}(t) = x(t) - x(ref)$$

$$y_{rel}(t) = y(t) - y(ref)$$

$$z_{rel}(t) = z(t) - z(ref)$$

x(t), y(t) and z(t), are the bead's coordinates at frame interval t, x(ref), y(ref) and z(ref) is the bead position at the reference frame (currently the first frame t=0).

Normalized coordinates x_{norm} , y_{norm} and z_{norm} :

 $x_{norm}(t) = \frac{x_{rel}(t) - mean_x}{\sigma_x}$ with $mean_x$ and σ_x being the mean and standard deviation of the x relative coordinates across the timelapse.

 $y_{norm}(t) = \frac{y_{rel}(t) - mean_y}{\sigma_y}$ with $mean_y$ and σ_y being the mean and standard deviation of the y relative coordinates across the timelapse.

 $z_{norm}(t) = \frac{z_{rel}(t) - mean_z}{\sigma_z}$ with $mean_z$ and σ_z being the mean and standard deviation of the z relative coordinates across the timelapse.

Mean Squared Displacement:

$$MSD_X (frame) = x_{rel}^2(t)$$

 $MSD_Y (frame) = y_{rel}^2(t)$
 $MSD_Z (frame) = z_{rel}^2(t)$
 $MSD_{3D} (frame) = x_{rel}^2(t) + y_{rel}^2(t) + z_{rel}^2(t)$

Elapsed time:

$$elapsed time (frame) = frame * frame interval$$

1D displacement threshold values:

X and Y displacements between two consecutive frames are compared to the lateral resolution value $res^o_{x,y}$

$$res_{x,y}^o = \frac{0.51 * \lambda_{ex}}{NA}$$

Z displacement between two consecutive frames is compared to the axial resolution value res_z^o

$$res_z^o = \frac{0.88 * \lambda_{ex}}{n - \sqrt{n^2 - NA^2}}$$

3D displacement between two consecutive frames t-1 and t is compared to the reference resolution distance $res^o_{\theta, \varphi}$

$$res_{\theta,\varphi}^o = \frac{res_x^o * res_y^o * res_z^o}{\sqrt{(res_y^o * res_z^o * cos\Phi * sin\Theta)^2 + (res_x^o * res_z^o * sin\Phi * sin\Theta)^2 + (res_x^o * res_y^o * cos\Theta)^2}}$$

With

$$\Phi = \arccos \frac{x_t - x_{t-1}}{\sqrt{(x_t - x_{t-1})^2 + (y_t - y_{t-1})^2}}$$

$$\theta = \arccos \frac{z_t - z_{t-1}}{\sqrt{(x_t - x_{t-1})^2 + (y_t - y_{t-1})^2 + (z_t - z_{t-1})^2}}$$